Ontological Semantics

An Attempt at Foundations of Ontology Representation

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Preface

Abstract

The original and still a major purpose of ontologies in computer and information sciences is to serve for the semantic integration of represented content, facilitating information system interoperability. Content can be data, information, and knowledge, and it can be distributed within or across these categories. A myriad of languages is available for representation. Ontologies themselves are artifacts which are expressed in various languages. Different such languages are utilized today, including, as well-known representatives, predicate logic, subsuming first-order (predicate) logic (FOL), in particular, and higher-order (predicate) logic (HOL); the Web Ontology Language (OWL) on the basis of description logics (DL); and the Unified Modeling Language (UML). We focus primarily on languages with formally defined syntax and semantics.

This overall picture immediately suggests questions of the following kinds: What is the relationship between an ontology and the language in which it is formalized? Especially, what is the impact of the formal semantics of the language on the formalized ontology? How well understood is the role of ontologies in semantic integration? Can the same ontology be represented in multiple languages and / or in distinct ways within one language? Is there an adequate understanding of whether two expressions are intensionally / conceptually equivalent and whether two ontologies furnish the same ontological commitments?

One may assume that these questions are resolved. Indeed, the development and adoption of ontologies is widespread today. Ontologies are authored in a broad range of different languages, including offering equally named ontologies in distinct languages. Much research is devoted to techniques and technologies that orbit ontologies, for example, ontology matching, modularization, learning, and evolution, to name a few. Ontologies have found numerous beneficial applications, and hundreds of ontologies have been created, considering solely the context of biomedical research. For us, these observations increase the relevance of the stated questions and close relatives thereof, and raise the desire for solid theoretical underpinnings. In the literature of computer and information sciences, we have found only few approaches that tackle the foundations of ontologies and their representation to allow for answering such questions or that actually answer them.

We elaborate an analysis of the subject as the first item of central contributions within this thesis. It mainly results in the identification of a vicious circularity in (i) the intended use of ontologies to mediate between formal representations and (ii) solely exploiting formal semantic notions in representing ontologies and defining 'ontology-based equivalence' as a form of intensional / conceptual equivalence. On this basis and in order to overcome its identified limitations, we contribute a general model-theoretic semantic account, named *ontological semantics*. This kind of semantics takes the approach of assigning arbitrary entities as referents of atomic symbols and to link syntactic constructions with corresponding ontological claims and commitments. In particular, ontological semantics targets the avoidance of encoding effects in its definition. Therefore we argue that this semantic account is well suited for interpreting formalized ontologies and for defining languages for the representation of ontologies. It is further proposed as a fundament for envisioned novel definitions of the intensional equivalence of expressions, in potential deviation from only being formally equivalent under set-theoretic semantics. The thesis is defended that a particular usage of a formalism and its respective vocabulary should be accompanied by establishing an ontological semantics that is tailored to that *use of the formalism*, in parallel to the formal semantics of the language, in order to capture the ontological content of the formal representation for adequate reuse in other formalisms. Accordingly, we advocate ontological semantics as a useful framework for justifying translations on an intensional basis. Despite all deviations of ontological semantics from its set-theoretic blueprint, close relationships between the two can be shown, which allow for using established FOL and DL reasoners while assuming ontological semantics.

Just having outlined the most important aspects of this work, let us elucidate its contributions in greater detail, thereby following and sketching its structure. Our interest and motivation for dealing with the questions above arises from ontological analysis and ontology development, first and foremost our contributions to the General Formal Ontology (GFO). This is a top-level ontology, i.e., it covers notions of high(est) degree of abstraction and wide applicability and relevance for many domains. Further motivation accordingly results from applications of GFO in several settings. Against this background, the above and similar

questions are identified initially, in relation to the use of logical languages for representing ontologies and employing ontologies in accounting for the problem of intensional / conceptual equivalence. In both connections we have a particular interest in predicate logic, as concerns languages, and in top-level ontologies, as concerns kinds of ontologies. This set of open issues inspired our objectives for this work, based on FOL as a case of reference: (1) to expound an in-depth analysis of existing solutions and remaining problems; (2) to contribute improvements or alternatives to existing approaches; (3) to provide a semantic account that is purely based on ontological notions, namely ontological semantics; (4) to relate that semantic approach to its classical counterpart; (5) to develop an ontology of categories and relations, as a component option for ontological semantics and a proposed extension to GFO; (6) to develop further contributions to GFO, with the major case of an ontology of time; and eventually, (7) to make contributions to methodological and engineering aspects of ontology development.

It is the analysis of existing notions of formal semantics and semantic translations that leads us, following some authors, to distinguish between two types of semantics. One type is referred to as *intensional*, *conceptual*, or *real-world semantics* and differs from *formal semantics*. We realize and subscribe to the independence between these two types of semantics, at least to a substantial degree. This view, however, renders the rôle of ontologies as expounded in the literature for ontology-based semantic integration problematic, because there it rests only on notions of formal, logical equivalence modulo theories (and translations that must preserve formal semantics). Moreover, we argue not to straightforwardly accept ascribing ontological neutrality to logical languages, not even to FOL. Largely for these reasons, we engage in developing ontological semantics as a novel and expressive semantic account that aims at avoiding ontological predetermination (at the level of the general account) and thus repels the need for encoding conceptual content (at the side of semantics). Beforehand, a brief systematization of how we comprehend the notions of *ontological analysis*, *foundation*, *translation*, and *reduction* is followed by a meta-level perspective on ontologies. The latter approach centers on the notion of *abstract core ontology*, which can be functionally characterized as providing categories for classifying ontology constituents. Building further groundwork for later sections, we conceptually introduce an ontology $\mathcal{CR}$ of categories and relations as an abstract core ontology.

In preparation of the actual development of ontological semantics, two non-standard views on the classical, set-theoretic semantics of FOL are examined in detail. On the one hand, we consider the set-theoretic assumptions of predicate logic semantics in terms of a specific superstructure of sets that "emerges" from a set that acts as the universe of a mathematical structure. A set hierarchy is defined in levels that correspond to predicate logics of different orders, which further links to type theory. On the other hand, we study the definition of classical FOL semantics on the basis of axiomatic set theory, which yields an understanding of semantics as first-order theory interpretation. Both considerations allow us (i) to state concisely our understanding of the ontological neutrality of the semantics of a language, and (ii) to take the case of FOL as a blueprint for developing ontological semantics.

The development of ontological semantics itself is pursued in combination with exemplifying the approach based on FOL syntax and, within that context, by utilizing the ontology $\mathcal{CR}$ of categories and relations. First, we derive a notion of an *ontological structure* by omitting any presupposed mathematical entities in its definition, resulting in the idea that any single entity or plurality of entities yields an ontological structure, if it is or they are considered for the rôle of referents of signature elements. A brief positioning regarding philosophical aspects precedes initial steps to the notions of *ontological interpretation* on to *ontological model* and the satisfaction of a formula by such an interpretation, respectively. The definition of satisfaction follows the classical definitions of FOL semantics in the cases of logical connectives and quantifiers. The central deviation concerns the satisfaction of atomic formulas. The observation that the syntactic means of predication is conceptually overloaded motivates the introduction of *predication systems* as a means to assign predication semantics individually to predicate symbols. In its definition, each predication system relies on an ontology in order to bootstrap predication semantics, which is illustrated by means of $\mathcal{CR}$. Overall and basically by design, in its FOL variant, ontological semantics constitutes an approach that is ontologically neutral (in a sense developed earlier in the thesis).

Eventually, based on preconditions that should be widely acceptable for most ontology developers, we suggest methodological principles and thereby a recourse to employing FOL with its classical formal

semantics for ontology representation in a way that mimics specifying ontologies as FOL theories under ontological semantics, i.e., corresponding predication systems can be easily obtained from such FOL theories and classical entailment transfers to ontological semantics. This recourse is highly desirable, not at least due to the multitude of available theoretical results, proof calculi, and their implementation in theorem provers for classical FOL.

For a simpler utilization of the ideas of ontological semantics and in line with the thesis of assigning conceptual semantics for particular uses of languages, *ontological usage schemes* are defined as a translation-based variant of assigning an ontological semantics to any syntax. Basically, an ontological usage scheme specifies a translation from any particular syntax into (the language of) a particular FOL theory, which is itself interpreted via ontological semantics or at least constructed according to the principles derived from ontological semantics.

The final part of this thesis comprises three major facets. Firstly, some application cases of ontological usage schemes in combination with the theory of categories and relations $\mathcal{CR}$ are discussed, largely in connection with biomedical ontologies. Corresponding contributions include the theoretical justification of the different formalization components of the biological core ontology GFO-Bio and of different representations of phenotypic information. The second facet relates to further contributions in the form of ontological analysis and of the axiomatization of component theories for GFO, in relation with the framework of ontological semantics. In particular detail, this includes two axiomatizations on notions of time and axiomatizations of $\mathcal{CR}$. The latter complete the introduction of the specific instance of ontological semantics based on $\mathcal{CR}$. Thirdly, we briefly discuss related work. This occurs already with respect to specific points throughout the thesis. But for the core part on ontological semantics we present a condensed discussion of very closely related approaches in knowledge representation and logic, besides providing pointers to a number of further areas with shared issues.

In a nutshell, we advocate the view that intensional/conceptual/real-world semantics (to be distinguished from formal semantics) can be captured by means of ontologies, but it appears insufficient to rely on (arbitrary ways of using) languages with formal semantics for formalizing ontologies. Ontological semantics is a formal, ontologically neutral account to overcome some of the corresponding weaknesses, such that one can say it allows theories to establish "their own" semantics. Moreover, it is intended to pave the way for an improved theoretical underpinning of conceptual equivalence and, hence, of semantic integration and interoperability. The applicability of the approach is demonstrated for the case of FOL. It is established by a formalization method that uses FOL for ontology representation, is provably in harmony with ontological semantics, but is based on classical FOL semantics and thus enables exploiting existing reasoners.

Contents

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Last but not least and despite omitting details deliberately, I wish to express my utmost and heartfelt gratitude to all of my family, including my own and my wife's parents. Over the long period of writing this dissertation, all of the family provided me with their help and support, above and beyond everything I could have imagined beforehand. I wish two more of them had had the chance to witness its conclusion.

Foreword

In its extremes, this thesis touches issues in several, partially distant areas. Many of those issues actually deserve a much more detailed treatment and are equipped with an extensive, more or less historical body of work. On the flip side, the temporal and spatial constraints of the thesis as a whole disallow such treatment. In most cases I try to resolve this conflict by providing a level of detail and reference that appears appropriate to me. Nevertheless, the knowledgeable reader is asked for forbearance where necessary.

The thesis is written for a wider audience, such that some explanatory remarks may seem superfluous for specialists in the respective field. However, more than once I have made the inconvenient experience as a reader that what is well-known, for example, within a community and / or at a certain period of time, excludes readers who lack that background. Contrary to those experiences, I have found some side remarks in other works very informative and helpful. Therefore, I hope that such remarks become useful for one or another reader and that they can easily be skipped by others. A related practice concerns the provision of references. I attempt to justify also minor claims based on the literature, if they are not substantiated from the text by other forms of argument. If research yields various sources, I tend to refer to all or at least several of the most adequate of them, because they may be accessible in varying degrees to different readers and I usually found it beneficial to look at matters from the perspectives of several publications.

In accordance with the approach just declared, you will immediately notice that I am a great friend of using footnotes for additional explanations, cross-referring to other sections, mentioning related works or areas, trying to further justify claims in the body of the text, etc. First of all, being aware of different tastes concerning footnote usage (and likely being part of a minority), I apologize by those who prefer no or much less footnotes. It is clear to me that their number is (very) high, especially in the background section that touches on numerous points. My recommendation for reading larger parts of the text is to ignore all footnotes, except for a single kind of case – if you disagree with the main text and see a footnote mark, there may be a qualification or further explanation in the footnote. The second use case of those remarks is to support readers who may have found a specific position in the text directly, for example, via search.

WRITING TECHNICALITIES

This and the next paragraphs are devoted to specific writing styles and elements. One of these, which I refer to as paragraph headings, is exemplified above the previous sentence. Although originally introduced and aimed at helping only during writing, for better structuring and overview, I meanwhile believe that readers may likewise benefit from those effects of paragraph headings. For similar reasons, a detailed table of contents can be found at the beginning of each chapter.

The symbol / connects words and sometimes phrases if they apply equally/alternatively in the particular context of occurrence. I think that providing multiple terms in certain cases contributes to a more precise transfer of meaning. Double quote signs " " are used for quotations as well as for highlighting term usage in a figurative and / or quizzical sense. Accentuating terms as terms is implemented using single quotes ' '. Definitions of terms within text are emphasized by italics, e.g. *ontology*. I use the term 'section' for everything below the chapter level, which gives rise to some ambiguity if no section number is mentioned. Moreover, I refer to the overall document as 'this work' or 'this thesis'. As can be noticed already, active and passive voice occurs for more variability in writing. In the abstract above and the main text below, 'we' is used instead of 'I'. Acronyms are spelled out upon first occurrence within a certain range of text only if they relate closely enough to the contents of that section. Otherwise, e.g. if just mentioned as examples, the reader is referred to the acronym listing in App. D. That appendix chapter contains also a list of abbreviations.

"Mathematical paragraphs" primarily comprise definitions, propositions, observations, and proofs. All propositions are accompanied by proofs, whereas observations are statements that are usually more straightforward to comprehend and / or are considered less important than "regular" propositions. Accordingly, some observations are followed by a proof, whereas others are justified only by some arguments in the surrounding text before or after the observation.

Footnote positioning indicates the context of reference: a footnote after a punctuation mark means that the footnote contents refers to the overall sentence or half sentence, for example. In contrast, if the footnote contents focuses on a specific term or phrase within a sentence, the footnote mark is set directly after the term or last term of the phrase in question, possibly directly before a punctuation mark.

References are provided as usual, as Appendix E.1 herein. Markers such as the one next to this line are used to indicated citations in the text that involve (co-)authorship of ourselves (unless the same source is already cited and marked in the same context). Actual indication can be by a mere list of exactly the self-citations, then in square brackets like citations in the text, whereas a number without the latter points to a footnote that comprises additional explanatory remarks. On rare occasions, the marker is empty, then indicating a paragraph of main text on the status of self-citations. Web references are also listed in order to avoid restating them as footnotes in case of multiple occurrences of the same reference. @ marks citations of web resources in the text, while their listing is in Appendix E.2, separated from literature references. Note that "extended" text documents like articles on the web (available and cited via their web addresses) are listed among literature references.

Eventually and despite any potential stylistic inaptitude, I hope that the reader finds some interest in the topics addressed in this thesis and can see and perhaps benefit from some of its contributions.

Chapter 1

Introduction

1.1   Background

SECTION OVERVIEW

Ontology has become widespread in various areas in computer and information sciences at least since the 1990s. This includes not only the field(s) named formal ontology in information systems (FOIS) or / and applied ontology (AO), but in addition areas such as knowledge representation, description logics, the Semantic Web, and conceptual modeling, as well as domain-specific branches, first and foremost, biomedical ontologies research. In this context, a considerable amount of attention has been devoted to the understanding of the notion of ontology itself. Therefore, we clarify our understanding and terminology of this notion. A rough typology of ontologies leads on to foundational ontologies, and to the top-level ontology General Formal Ontology (GFO) [@46], in particular. Its development is the driving force behind our investigations (together with experiences from its application in several contexts), thus ontological engineering is introduced as another relevant area, though tightly limited to aspects referred to in other chapters below. Altogether, the section sets the overall scene within which the present work is conducted.

1.1.1   Ontology Research in Computer and Information Sciences

ONTOLOGY RESEARCH INCREASES, AND RELATES TO OTHER FIELDS

Over that last two decades, research on *ontologies* – the term to be clarified below – has gained progressively increasing interest and can today be seen as a considerable field of investigation, cf. a.o. [338]. As Stefano Borgo and Leonardo Lesmo remark already in 2008, accompanied by a number of prominent examples, "many important conferences and specialized meetings devote considerable part of their time to ontology topics and are careful to register the new trends in ontological research" [99, p. 1]. The first introductions and handbooks have been devoted to ontologies and ontology research since the early 2000s, among them [227, 289, 668, 669, 780, 781, 787] as well as, focusing also on related fields and / or application areas, [38, 132, 134, 748, 752]. There are mutual influences and interactions with diverse disciplines within and beyond computer and information sciences. These include artificial intelligence (in particular, knowledge representation and knowledge engineering), conceptual modeling, databases, and information systems, as well as philosophy, linguistics, and cognitive science. Beyond especially the first set of fields, a broad spectrum of application areas has been opened up, with major pillars in the life sciences (including biology, biomedicine, and bioinformatics) and medicine / health care,[1] business, and engineering, but in many other domains, as well.[2]

FORMAL ONTOLOGY IN INFORMATION SYSTEMS (FOIS) AND APPLIED ONTOLOGY (AO)

Historically, ontology successfully entered[3] computer science largely through artificial intelligence (AI) due to problems in knowledge representation (KR) and knowledge-based systems (KBS), cf. [158, 309, 327, 618], that appeared in the 1990s, and on the basis of the much older philosophical discipline of ontology

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[1]The notion of *biomedical ontologies* forms an umbrella for initiatives in these fields, cf. e.g. the Thematic Series on Biomedical Ontologies [421] [@14] of the Journal of Biomedical Semantics.

[2]For instance, besides manufacturing, e-commerce, and corporate knowledge [99, p. 2] names chemistry, cultural heritage, and network management. [330] also has an extensive list, including links to geography and law.

[3] The initial appearance of the term "ontology" in computer science is traced back to George H. Mealy's article [573] from 1967 in [348, p. 57] and [351, p. 1] (themselves referring to [757], which is an extended draft of [759], where the latter does not mention Mealy). [348, p. 68] further mentions Patrick J. Hayes with [382, 383] (from 1978, 1985) as one of the very early proponents of ontology in computer science. Similarly, [422, p. 67] refers to [567] for borrowing 'ontology' from philosophy, written by McCarthy in 1980.

and the problems it addresses, cf. e.g. [165, 424, 759].[4] From this research, the field of *formal ontology in information systems* (FOIS) emerged during the mid to the end of the 1990s, cf. [330, 340],[5] which aims at an interdisciplinary, principled, and formal approach to describing/modeling reality. Derived from a journal [@6] that is devoted to at least very similar ideas [338], the subject term *applied ontology* (AO) is becoming equally established for the field. *Ontological analysis* and *conceptual modeling* are core tasks in this area [338, p. 2]. Sect. 2.3.1 below presents our reflections on ontological analysis and associated concepts. For a more detailed survey of the field and an introduction to its core aspects, besides the books mentioned above we refer to the section "Ontology in Computer and Information Sciences" by Giancarlo Guizzardi [348, sect. 3.2]. Similarly, Rinke Hoekstra's [422, esp. ch. 2–4] is informative regarding the relationship to knowledge representation and to the more recent development of and in the Semantic Web.

SEMANTIC WEB

Indeed, there is a second area with a meanwhile much larger community than FOIS / AO which has strongly contributed to the popularization of ontology research, from the early 2000s on: the *Semantic Web* (SW). Originally orienting at a semantically enriched web of information [80], the World Wide Web Consortium (W3C) [@131] strongly engaged in developing and pushing this idea. To a large part, this involves standardizing various languages for describing and publishing data / information on the Web,[6] first and foremost the Resource Description Framework (RDF) and the Web Ontology Language (OWL / OWL 2), cf. [@102], [406], themselves based on further, "non-semantic" standards and concepts, most importantly the Extensible Markup Language (XML) [@136] [112], its schema language XML Schema [854], and the notions of Uniform Resource Identifier (URI) and Uniform Resource Locator (URL), cf. [406, sect. 2.1 for URIs and URLs, Appendix A for XML].[7]

The general idea of the Semantic Web was prominently introduced in [79] and in [80], where the term 'ontology' is explicitly mentioned. Since then, a large number of ontologies has been established in connection with/for the Semantic Web. Ontologies play a central role in the language-stacking architecture[8] of the Semantic Web. On the one hand, this role emanates from AI / KR issues related to central topics of FOIS. On the other hand, it is caused by the strong connection to the *description logic* family of representation formalisms [34], referred to as ontology languages. Description logics are currently among the preferred representation approaches,[9] because they are formally well established, have achieved a good compromise between expressiveness and tractability[10], and corresponding reasoning software could be proved to be advanced enough for wider adoption in practice.

MAIN APPLICATION OF ONTOLOGIES

Besides the diverse application domains indicated above, ontologies are applied today in many different ways, especially as soon as technical aspects are taken into account, like languages, architectures, etc. Nevertheless, the original and central application idea in connection with ontologies — more precisely, for a single self-contained ontology — is to provide an explicit, shared foundation, on whose basis data/information/knowledge sources can be constructed, communicated and, more importantly, the *integration* of several such sources can be facilitated or at all be enabled.[11] In the same spirit, Michael Uschold and

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[4]Although this link was considered skeptically, as Nicola Guarino and Mark Musen stated in 2005: "Ten years ago, academic workers in computer and information science spoke of ontology carefully and cautiously, almost embarrassed to utter the 'o' word." [338].

[5]Cf. the formative conference series "Formal Ontologies in Information Systems" [@42], initiated in 1998 [331] and continuously held bi-annually since 2004 to the present (2014).

[6]See the W3C Semantic Web Activity website [@130] for the specifications / W3C Recommendations [@102] resulting from standardization efforts of the respective groups.

[7]In the earlier German edition, see [405, sect. 2].

[8]i.e., in the "SW layer cake", which exists in diverse variants [@111].

[9]Clearly, they are not the only type of formalism. For instance, rule-based languages [659] deserve consideration in the context of the Semantic Web, cf. OWL 2 RL [601, sect. 4], the Semantic Web Rule Language (SWRL) [432], and the discussion on rules following below. Nevertheless, the ontology language specifications of primary relevance for the Semantic Web are still based on description logics. Combinations of description logics and rules have been studied extensively, cf. e.g. [24, 218].

[10]Although much more could be said about this compromise, we believe that this statement is generally accepted today. A major property that almost all description logics exhibit is the decidability of their reasoning problems, at least of basic ones, cf. [34, sect. 2.2.4, 6.3, and ch. 3].

[11]In this regard, ontology research in computer science closely relates to (aims of) terminology science, cf. e.g. [253, 884, 885]. There are other aspects in terminology that are of less relevance, for example, concerning lexical, linguistic, or socio-terminological

Michael Grüninger argue for the key rôle of ontologies in achieving seamless, semantics-based connections among people, software agents, and various kinds of IT systems [829]. They further identify four general types of ontology application scenarios [ibid., sect. 3], namely neutral authoring, ontology as spefication, common access to information, and ontology-based search. All of these support the "promise of ontologies" [829, p. 61], there cited from [237, p. 1], to provide "a shared and common understanding of a domain that can be communicated between people and application systems".

1.1.2 Ontology Definition and Terminology

1.1.2.1 Definitions

NUMEROUS DEFINITIONS OF ONTOLOGY, LINKING TO SURVEYS

Definitions of ontology exist abundantly in the literature, even without recourse to ancient and medieval times, e.g., to Aristotle's phrasing of ontology as studying "all the species of being *qua* being and the attributes which belong to it *qua* being" (*Metaphysics* VI.1, according to [337, p. 26]). The definitional plurality is even greater if more than one of the mentioned contexts — philosophy, FOIS / AO, KR, SW, terminology sciences — are taken into account. For conceptions of ontology in the philosophical domain, let us merely refer to [165], [424, sect. 3], and [348, sect. 3.1]. In computer and information sciences, [289, sect. 1.1–1.2], [337, 349], [422, ch. 4], and [499, sect. 1], comprise elaborate collections of definitions and / or insightful discussions on understanding(s) of ontology.[12] At least, a distinction between discipline and artifact can be drawn in all contexts. A discipline 'ontology' (with the word rather seen as an uncountable noun) refers to a specific field of research, whereas an 'ontology' (as a countable noun) is a system / artifact of a certain kind.[13] The nature of those fields and systems varies greatly among the distinct contexts, however.

DEFINITIONS OF RELEVANCE FOR TERMINOLOGY HEREIN

In order to delimit the reading(s) of 'ontology' applicable herein, we collect definitions which are influential for this thesis. Starting with the discipline view on philosophy (actually, its subdiscipline formal ontology) and on FOIS / AO, as well as going beyond the understanding of ontology as the *study of what there is*, the philosopher Nino B. Cocchiarella [159] has coined a definition of *formal ontology* which can be adopted for both fields:

> [. . .], formal ontology, [. . .], is the systematic, formal, axiomatic development of the logic of all forms and modes of being. [159, p. 640]

In computer science, the certainly most cited definition of ontology is one of ontology as a system, given originally in the KR context by Thomas Gruber in 1993, which he refined recently, in 2009[14]:

> An *ontology* is an explicit specification of a conceptualization. [311, p. 199]
> In the context of computer and information sciences, an ontology defines a set of representational primitives with which to model a domain of knowledge or discourse. [314, p. 1963]

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facets. Note that some authors have critized work in terminology from an ontological point of view, cf. e.g. [764], which relates to controversies in ontology itself, however, cf. the debate in [579, 580, 762].

[12]The discrimination from other notions is closely related to the comprehension of ontology in general. Obvious candidates in this respect are the notions of *model* and *knowledge base* (facing a similar problem of meaning overload, cf. [492, 724] for 'model'), besides others. [829, sect. 2.2.] briefly contrasts ontologies with (knowledge representation and) knowledge bases, highlighting the focus of ontologies on knowledge sharing. In the same work, ontologies and database schemas are contrasted, declaring little essential difference among the languages used for expressing database schemas and ontologies, but finding other distinctions, e.g. in the integrity constraints of databases vs ontology axioms [829, sect. 2.3]. [349] discusses relations between 'ontology' and 'metamodel', among others.

[13]Nicola Guarino and Pierdaniele Giaretta have highlighted this distinction early, using 'Ontology', i.e., capitalized writing without an article, for the discipline and 'an ontology', with an indeterminate article and in lower cases, for the artifact reading [337, sect. 2, p. 26].

[14]Note that [314] refers to an encyclopedia article in the database context. That article comprises a longer paragraph as the overall definition of 'ontology', of which we quote only the first sentence. This sentence is the one that corresponds best to the frequently given quote from [311], which is itself actually embedded in further explanatory text.

The first of Gruber's definitions was widely debated at and extended by different authors [289, p. 6] and actually, both of these quotations must be considered in their respective contexts. Nevertheless, what is useful in those definitions is that they abstract from particular languages used for representing ontologies. Nicola Guarino refined Gruber's initial definition in order to clarify the distinction between ontology and *conceptualization*, providing this definition of ontology:

> An ontology is a logical theory accounting for the intended meaning of a formal vocabulary, i.e., its ontological commitment to a particular conceptualization of the world.   [330, p. 7]

Eventually, Guarino captures the notion of conceptualization formally in terms of an "intensional relational structure" [330, sect. 2.1, p. 6], cf. also [339, Def. 2.4] and [348, sect. 3.3.1][15], which involves possible worlds [576] (in the sense of modal logic) and is presented in greater detail in sect. 2.2.2 below. Without that level of detail and formality, we see the position in line with viewing a conceptualization as a language-independent ontological theory[16], whereas an ontology is already constrained to being a logical theory. In particular, language-independence refers to the difference that a conceptualization is not already tied to any specific formal language,[17] whereas it can be expected for a logical theory that it utilizes a logical formalism. In (terminological) contrast, Roberto Poli refers to a conceptualization as just attributed to Guarino as a descriptive or formal ontology (akin to Cocchiarella's sense), and calls a codification in a formal language a *formalized ontology* [666].

In the context of the Semantic Web (and description logics), the notion of ontology is also constrained to formalizations. The *SW notion of ontology* is widely understood to be equivalent to 'knowledge base' and 'theory' (in a formal, logical sense) if those are expressed in accepted *ontology languages* for the Semantic Web, cf. [405, p. 12].[18]

ONTOLOGY AS TECHNOLOGY

Although Hitzler et al. still mention that knowledge of a certain domain is modeled [405, p. 12] by an ontology, equating the notion of ontology with that of logical theories or knowledge bases creates a new view which is clearly detached from the philosophical sense(s) and loses connections even with the KR view. Accordingly, a new reading of ontology has emerged in recent years which we call a *technological reading of ontology*. There, 'an ontology' refers to a formal theory expressed in ontology languages and dealt with the technology established around those languages, but it is no longer necessarily connected with the purpose of representing a conceptualization/ontological claims. The matching discipline for this notion of ontology is termed *semantic technologies* in [405].

1.1.2.2 Terminology for Ontology and Ontology Constituents

ON THE TERM 'ONTOLOGY'

Against the background of the previous section, we adopt the following terminology on 'ontology' for this thesis. From the system perspective, the term *ontology* covers any theory (in the scientific as well as the formal senses) intended to make *ontological claims*, i.e., to represent/postulate what entities exist (in the broadest possible sense of existence,[19] and including interrelationships of all kinds among those entities).

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[15]In that section, Guizzardi largely adopts the account of Guarino in [330], providing further linkage to conceptual modeling (and metamodeling) as well as to theories of the philosopher of science Mario Bunge [126], in particular, referring to the notions of 'state space' and of 'nomological state space' [348, p. 85–86].

[16]"ontological" here rather in the philosophical sense, "theory" in the sense of a scientific theory

[17]E.g., supported by phrases such as "while conceptualizations are typically in the mind of people, i.e., *implicit*." [339, p. 8, emphasis as in source] or "In general, however, a more effective way to specify a conceptualization is to fix a language we want to use to talk of it, [...]" [ibid.]

[18]The qualification "widely" in this sentence refers to the description logic perspective, which is a major pillar of Semantic Web languages and technology. In description logic, there is a well-known distinction of TBox vs ABox, see e.g. [34, sect. 1.3]. A TBox captures general, 'terminological knowledge' by being "[...] built through declarations that describe general properties of concepts." [34, p. 17], whereas an ABox covers contingent, 'assertional knowledge' "[...] that is specific to the individuals of the domain of discourse." [ibid.]. Some authors adopt this distinction to refer to a TBox as 'an ontology', and to a pair/union of a TBox and an ABox as 'a knowledge base', cf. e.g. [480, p. 76]. Similarly, [83, sect. 2] calls a (finite) set of (presumably non-factual) FOL sentences 'an ontology', which paired with a (finite) set of ground facts yields a 'knowledge base'.

[19]See sect. 1.1.5.1 for recourse to Roman Ingarden's modes of existence [443]. Further comments on existence occur wrt the reference of constants in languages in sect. 2.1.2 and further below in sect. 4.2.

Accordingly, such theories may be presented in diverse forms, e.g. ranging from natural language texts to formal logical theories.[20] If we refer explicitly to the language-independent content of an ontology, *conceptualization* will be used. The natural language presentation[21] of an ontological theory is a *verbalized ontology*. Complementary to that, *formalized ontology* refers to a formal or semi-formal representation of such a theory,[22] i.e., there must be a conceptualization that is expressed / represented / formalized. If this assumption is false or irrelevant, terms such as *theory* and *formal representation* are in order. An exception to this is the term *OWL ontology* in SW related contexts, where it may be used without expecting that a conceptualization is represented. *Formal ontology* in our case refers exclusively to the discipline view, basically assuming the definition by Cocchiarella [159, p. 640] quoted in the previous section. *Formal ontology in information systems* or *applied ontology* derives from formal ontology but extends the field to include engineering aspects and applications (in arbitrary domains).

ON ONTOLOGY CONSTITUENTS: CATEGORIES AND RELATIONS (A.O.)

The second terminological note concerns ontology constituents, i.e., (language or conceptual) elements that ontologies are composed of. First, we observe that there is a frequent dichotomy between (1) notions that are meant to categorize / classify entities on the one hand, and (2) notions that capture how entities are interrelated / associated with each other.[23] Especially in the first case, the literature offers a rich variety of terms, ranging from 'category', 'class', and 'concept' over 'frame', 'kind', 'node', '(unary) predicate', '(monadic) property', and 'set' to 'term'[24], 'type', 'unit of thought', and 'universal'.[25] There is less diversity in the second case, with terms such as 'association', 'property', 'role', and 'relation', but in any case, there is no standard choice either. In both cases, most or all of the terms mentioned have various understandings and come with connotations. Aiming at terms with few(er) connotations and ambiguity, primarily within computer and information sciences, we prefer 'category' and 'relation' as our default choice of technical terms for the distinction at hand.[26] A *category* can be predicated of entities, which are called the *instances* of the category. A *relation* connects entities. More precisely, several entities together may bear a certain relation to each other, in which case they are called the *relata* or *arguments*. This may further be the case for multiple such groups of entities and the same relation. Providing an example, an ontology capable of analyzing the sentence "Lion Leo stands on the trunk of a fallen tree." may comprise a category Lion and a relation stand-on. Categories and relations play an important role in the overall thesis, already indicated by the fact that sect. 1.1.5, 2.4, and 6.1 discuss this topic with increasing degree of detail and formality. Again on the choice of 'category' as a term, our use of 'category' deviates from the usual understanding in philosophy, where 'category' has a much more restricted reading as "highest kinds or genera" [821]. The Lion example demonstrates this aspect, as well. Overall, the distinction between categories and relations is just one basic choice. There may be further kinds of ontology elements, e.g. *individuals*, for which Leo furnishes a case; for the next level of detail in considering individuals see sect. 1.1.5, but note also individuals in the case of OWL ontologies, cf. sect. 1.1.3.[27]

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[20]The Ontology Definition Metamodel (ODM) [@75] adopts a range that is almost as broad, covering a spectrum from taxonomies to logical theories [646, sect. 9, p. 31]. Its "actual" introduction herein is in sect. 1.1.3.4. Classifying ontologies based on languages / expressiveness aspects is discussed in sect. 1.1.4.1.

[21]primarily, in the form of texts

[22]Note a richer distinction of ontologies into being 'highly informal', 'semi-informal' (expressed in restricted forms of natural language), 'semi-formal', or 'rigorously formal' in [289, sect. 1.3, p. 9], with 'highly informal' and 'semi-informal' corresponding to our notion of 'verbalized', whereas our 'formalized' covers jointly 'semi-formal' and 'formal'.

[23]NB: This is a starting point for sect. 2.4 and is treated there in greater detail.

[24]'Term' is to be read in a linguistic rather than a formal logical sense here, cf. the end of the next sect. 1.1.3.

[25]This list is likely incomplete and most, if not all, terms can be read in several ways, arising from different contexts. Sect. 1.1.3 provides languages (and references) for several of the terms named here. Moreover, they vary in the degree by which they are considered as "pure syntax" or as having a reading that is less closely associated with the syntax of a particular language / language family.

[26]Others of the mentioned terms are also used, e.g., if it appears more appropriate in a given context, and occasionally also for more varied language in the text.

[27]As an anticipatory warning, individuals in OWL are a more formal / technical notion than the one mentioned here before, although we conjecture that the latter inspired the naming of individuals in OWL.

TAXONOMIC RELATION(S)

Two important relations shall be mentioned at this point already. The first is called *is-a*, *generalization*[28], *specialization*, *subsumption* and / or *taxonomic relation*[29] and connects categories with each other, such that a *subcategory* specializes a *supercategory*, such as Lion specializes Animal. We assume familiarity of the reader with "the" is-a relation[30], in particular with its extensional reading as characterized by "all instances of the subcategory are also instances of the supercategory", cf. e.g. logic-based formulations in [109, sect. 3.5, p. 36–37] and [345, sect. 1, p. 210], and we leave further considerations to later sections.[31] In the extensional reading, we also use the term *subclass relation*, in line with OWL. Finally, by the *taxonomy* or *taxonomic backbone of an ontology* we mean the structure that consists of all categories in the ontology together with the is-a relation/relational links among those categories, as given in the ontology[32]. Moreover, an ontology is called, more specifically, a *taxonomy* if it is represented solely as a hierarchy of categories, cf. [637, p. 30–31], by interpreting / viewing that hierarchical relation as the is-a relation.[33] [34]

MEREOLOGICAL RELATION(S)

The second important relation connects parts with wholes and is therefore called *part-whole*, *part-of*, *part-hood*, and / or *mereological* relation. Leo's paw is a part of Leo, for example. "This" relation is likewise very common in ontologies (as well as in modeling[35]), where the Foundational Model of Anatomy (FMA) [@38] [714, 715] may serve as a widely known showcase. Numerous works in mereology[36] suggest that several relations may be meant and diverse views can be held, cf. e.g. [143, 394, 707]. In the general context of this thesis we employ these terms with a very vague / weakly restricted interpretation only and without a specific mereological theory in mind.[37]

NOTIONS OF DOMAIN

Final remarks concern the term 'domain', which is used throughout this thesis and with contextually differing meanings. The first of these is a *domain of knowledge / reality*, with examples such as biology, mathematics, medicine, surgery, time, functions, Arabidopsis thaliana[38] [@7], etc. 'Domain' in this sense may be rephrased as 'area of knowledge / interest' or, from the point of view of a theory on a domain, as 'subject matter'. Initial theoretic accounts on this notion of domain can be found in [392, sect. 14.7.1], [393, sect. 16.3], and [667]. Associated with a domain of knowledge / reality, there are entities. These are usually understood as individuals / particulars. Accordingly, it is meaningful to speak of a *domain of entities / individuals*, referring to a set / collection of entities associated with a domain of reality.

Remembering our general understanding of ontologies as theories, the previous notions of domain can be applied to the *domain of an ontology*. This term can thus refer to the domain of reality that is the subject matter of the ontology. Alternatively, the domain of an ontology can mean the domain of individuals that is

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[28]Listing these terms on a par may seem problematic, e.g. if 'specialization' and 'generalization' are perceived as a pair of inverse relations, in a formal sense. However, we can neglect the "direction" of the relation(s) here, and shall conceive of relations in the later sections mentioned above in a way that supports viewing 'specialization' and 'generalization' as terms that refer to the very same relation (though "read from different ends").

[29]We use all those terms synonymously and, regarding direction, as is usual for natural language. Leo Obrst draws more fine-grained distinctions in [637, esp. sect. 2.2.3, p. 30–32].

[30]See especially [107] for a discussion of various semantics that had emerged already until 1983. Some pointers to more recent sources can be found in [345, sect. 1]. Cf. also the characterization in the context of terminological systems in [189, sect. 2.2].

[31]For example, taking into account criticism of the extensional reading from [758, esp. sect. 7].

[32]The phrase "as given in the ontology" targets its explicit content by default, assuming a formalized or verbalized ontology. Where implicit is-a links are intended, this shall be mentioned.

[33]Consequently, a taxonomy is identical with its taxonomic backbone.

[34]Our use of 'taxonomy', although related, must be distinguished from more specific notions of taxonomy in the context of terminological systems [189] or as discussed in connection with classification in [556]. If the is-a relation is distinguished into different kinds, e.g. in [637, sect. 2.2.3, p. 31–32], this yields subtypes of the notion of taxonomy as introduced here.

[35]E.g., thinking of the relations / associations of *aggregation* [720, p. 164–168] and its subtype *composition* [720, p. 264–270], which are explicitly contained in the Unified Modeling Language (UML) [@126] and are equipped with dedicated syntax elements.

[36]in philosophy understood as "the theory of parthood relations: of the relations of part to whole and the relations of part to part within a whole." [843].

[37]In contrast, specific sections below involve theories that contain parthood relations in an axiomatic context, e.g. the theories of time in sect. 6.3.

[38]A plant species that serves as one of the model organisms in genetics [@7] and that is therefore captured in biological ontologies, e.g. [@9] in the NCBI Taxonomy Database [@69], [@8] in GFO-Bio [@47] [418].

conceptualized through the ontology. The latter view suffices for most of our purposes. A more elaborate notion of 'domain' starting from a domain of entities and augmenting it by sets of views and of classification principles is described in [392, sect. 14.7.1].

The third reading of 'domain' found in this thesis is the *domain of discourse* of formal logical theories, which we use synonymously with *universe (of reference)* wrt model-theoretic semantics, see sect. 1.5.2. In this connection, let us stress a necessary distinction between an ontological notion of *individual / particular* and that of a *logical individual*. The latter is merely equivalent to being a member in the *domain of discourse* of the model of a logical theory.

1.1.3 Languages and Representation of Ontologies

BROAD SPECTRUM WITH REPRESENTATIVES RELEVANT FOR THIS THESIS

Various languages have been used for authoring formalized ontologies, covering a broad spectrum wrt the degree of formality of the languages and underlying logical capabilities.[39] That spectrum ranges from plain lists / catalogs of terms over taxonomies and graph-based representations to full-fledged logical languages. In line with the previous sect. 1.1.2.2, ontologies / ontological theories that are stated in the form of natural language texts could be attached to the "less formal end" of that spectrum, yet without extending the notion of formalized ontology to include them. Starting in the SW context, we first introduce a number of logical languages (and some of their interrelations) that are relevant for this thesis, followed by covering pertinent general modeling languages and specific formats.

1.1.3.1 Semantic Web Languages, FOL, and Common Logic

OWL, ITS RELATION TO DESCRIPTION LOGIC AND RDF, AND OWL TERMINOLOGY

Work on the Semantic Web has led to a set of dedicated ontology languages, which most clearly comprises OWL / OWL 2 [@91], [572, 856], and in a wider sense RDF Schema (RDFS) [119],[40] RDF [@105], [735] and the Semantic Web Rule Language (SWRL) [432, 434], to name representatives that are well known and established today.[41] Note that from here on, the term 'OWL' / 'Web Ontology Language' may refer to either standardized version of OWL. Where an explicit distinction is required, we use OWL 1 for the corresponding set of 2004 standards and OWL 2 for those of 2012.

On the one hand, OWL is primarily grounded in (particular) description logics [34]. On the other hand, it is tied to / based on RDFS and thus RDF. This is reflected in the fact that for OWL 1 as well as for OWL 2 two distinct semantics are defined. The "direct semantics" [OWL 1: 661, sect. 3], [OWL 2: 603] is closely related to description logics, and the "RDF-compatible / RDF-based semantics" [OWL 1: 661, sect. 5], [OWL 2: 732] extends the genuine semantics of RDF [380, 384]. These two variants of semantics give rise to the distinction between *OWL Full* and *OWL DL*, referring to "RDF graphs considered as [OWL] ontologies and interpreted using the RDF-Based Semantics" in the case of OWL Full, and for OWL DL to "[OWL] ontologies interpreted using the Direct Semantics" [856, sect. 2.3], cf. also [732, sect. 9]. The understanding of OWL DL as a description logic[42] requires some syntactic conditions to be satisfied [856, sect. 2.3], therefore, syntactically, it forms a sub language of OWL Full. Regarding semantics and roughly speaking, both semantics agree with each other regarding entailments from OWL DL ontologies, captured

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[39] Similarly to the recurrence of the SW layer cake [@111], cf. FN 8 on p. 3, a common illustration of this "ontology language spectrum" (in several variants) pervades the literature. Presumably it was first published as [506, Fig. 1], where a note traces it back to a conversation in preparation for an ontology panel at the AAAI 1999 conference, cf. [@2] (slide 3; p. 12 of the guide for the panel). Further versions of mutually distinct authors are, ordered by publication year, [767, p. v], [829, Fig. 2], [289, Fig. 1.10], [348, Fig. 3.14], [339, Fig. 4], and [637, Fig. 2.2].

[40]Notably, the RDFS specification [119] provides a vocabuly extension for RDF, whereas all other pairs of languages differ substantially, e.g., in their abstract syntax (cf. sect. 2.1.1 for that notion).

[41]Besides other languages of current relevance and use, the development in the Semantic Web has witnessed a variety of predecessors and alternatives, cf. [289, ch. 4], [435, sect. 4].

[42]OWL 2 DL is strongly related to / compatible with and in many works identified with the description logic $\mathcal{SROIQ}$ [429], cf. the remarks in [856, sect. 2.3] and [603, sect. 1] on their relation. Similarly, OWL 1 corresponds most closely to the description logic $\mathcal{SHOIN}$(D) [431, sect. 3], cf. [431, esp. sect. 4] and [435, sect. 7.1] for their relation, and see [170, p. 27–32] for a clear presentation of minor differences between the two.

with more precision in correspondence theorems for the two types of semantics [OWL 1: 661, sect. A.1.5], [OWL 2: 732, sect. 7.2]. In the sequel, 'OWL' (without any accompanying adjunct) refers implicitly to OWL DL.

MAJORITY OF SW ONTOLOGIES IS REPRESENTED IN LOGICAL LANGUAGES

Not only OWL but all of the SW languages named above are equipped with a formal, model-theoretic semantics (cf. sect. 2.1.3) on the basis of which entailment relations are defined. Therefore, these languages exhibit a formal logical character, which enables reasoning as well as meta-logical analysis, a.o. Accordingly and at least in the context of the SW, nowadays the majority of ontologies is represented in terms of logical formalisms. While their utilization for ontology representation has become widespread over recent years, this may meanwhile be hidden in the technology provided in the SW.

FOL AND FURTHER LOGICAL LANGUAGES ARE USED FOR ONTOLOGY REPRESENTATION

The development of description logics (DLs) themselves [34] was originally closely tied to the goal of representing "terminological knowledge" [109, esp. sect. 3.5 and sect. 9.7].[43] Many other logical formalisms have been employed for the representation of ontologies, usually outside of and earlier than in the SW context. First and foremost, classical predicate logic, in particular first-order logic (FOL) is to be named in this respect, in different varieties of presentation and minor detail. To a much smaller extent, higher-order logic, cf. [223], and modal logics, cf. [267], have been employed, e.g., see the use of the latter in [558].

FOL is at the heart of this thesis and will be discussed from various angles in detail subsequently. We assume a certain conceptual familiarity with FOL at the side of the reader. Therefore, for introductory purposes we only mention some standard texts from which we profited particularly, namely those[44] of Heinz-Dieter Ebbinghaus, Jörg Flum, and Wolfgang Thomas [214], Herbert B. Enderton [221], Wilfried Hodges [411], Wolfgang Rautenberg [691], Philipp Rothmaler [717], and Wolfgang Tuschik and Helmut Wolter [826]. Of course, a technical introduction of notation follows below, in sect. 1.5.2.

COMMON LOGIC AND SEMANTICS OF SW LANGUAGES

In computer science and especially in KR, there are FOL derivatives such as the Knowledge Interchange Format (KIF) [272–274][45] and its more recent "successor"[46] Common Logic (CL) [452], [@18], standardized by the International Organization for Standardization (ISO) [@61] in 2007, that researchers have resorted to for ontology representation.

The CL standard "specifies a family of logic languages designed for use in the representation and interchange of information and data among disparate computer systems. [...] It defines an abstract syntax and an associated model-theoretic semantics for a specific extension of first-order logic." [452, p. 1]. We call CL a 'FOL derivative' because it features more syntactic freedom, e.g. allowing all non-logical symbols in predicate as well as in argument positions[47], and a distinct semantic approach. Chris Menzel lists "type-freedom, variable polyadicity, and "higher-order"quantification" [577, p. 2] as the major semantic differences and elaborates on them concisely in [577, sect. 2]. Herein, more details on CL itself are covered in sect. 7.2.2.2.

In connection with the SW, there are some corresponding elements / aspects between CL, RDF, and Lbase [347], a language intended to serve as "a framework for specifying the semantics for the languages of the Semantic Web." [ibid.] by a translational approach (which has not materialized, however, since its introduction in 2003).[48] These correspondences mainly concern their underlying "philosophies" / basic approaches to defining (the semantics of) all three languages, for all of which Patrick J. Hayes acted as a contributor and proponent. The deviations from the classical blueprint in the form of (standard) FOL led to

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[43] At that time, several years around 1990, DLs were called 'term subsumption languages', 'terminological logics' [617], 'terminological representation languages' [731], 'concept languages' [30], or 'KL-ONE based KR languages' [29]. For the latter, cf. the remarks on semantic networks and KL-ONE [110] below.

[44] in alphabetical order of first author surnames, without expressing any preference here

[45] For example, KIF has been used in the Ontolingua system [235], [311, sect. 3] (the paper of Thomas Gruber's ontology definition, cf. sect. 1.1.2.1), [@83], cf. also [211, sect. 4.1], developed at the Knowledge Systems Laboratory of Stanford University in the U.S. in the early 1990s. Both projects appear tightly coupled, cf. e.g. [272, 309, 310].

Note further that the KIF effort had already been criticized during early phases of its development, e.g. in [281].

[46] Cf. [577, p. 2] for a similar remark on the relationship between KIF and Common Logic.

[47] using classical FOL terminology

[48] For an analysis and comparison of the model-theoretic semantics of CL and RDF (and Extended RDF [14]), see [439].

feisty / lively discussions in early years of the SW, cf. e.g. [433], and to the semantic definitions for OWL in its two flavors, direct and RDF-based.

1.1.3.2 Description Logics: A Brief Introduction

BASIC CHARACTERIZATION AND EXAMPLE

Bridging from the direct semantics of OWL back to description logics (DLs), we return to logical languages with their classical semantics, in general. Essentially, the field of description logics is concerned with a meanwhile large family of variable-free logical languages, i.e., their development and analysis. The formal expression (1.1) is a typical example of a DL statement in the area of family interrelationships, namely that the notion of 'all women that have a child and that have only female children' is a specialization of / subsumed by the notion of 'mother'.[49]

(1.1)   Woman $\sqcap$ $\exists$hasChild.$\top$ $\sqcap$ $\forall$hasChild.Female $\sqsubseteq$ Mother.

MAJOR NOTIONS (CONCEPTS, ROLES, INDIVIDUALS), AXIOMS, AND NAMING

The basic ingredients in DLs are *concepts* (e.g., Woman and Mother), *roles* (e.g., hasChild),[50] and *individuals* (not contained in (1.1)), as well as *constructors* of logical or similar kind (e.g., $\sqcap$ as conjunctive concept constructor, and the quantificational concept constructors $\exists$ and $\forall$ that take a role and a concept as arguments). Iteratively, these ingredients can be combined into *concept descriptions* / new *concepts* (e.g., $\exists$hasChild.$\top$ in (1.1)) and *complex roles*. The concepts and roles that these iterations start from are called *atomic* concepts or roles, or, from a merely syntactic perspective, concept or role *names*. Regarding the meaning of these ingredients, one should start from the background assumption of a domain D of discourse entities / a set D of objects under consideration.[51] The modeling intuition for concepts is to capture categories / classes / types of the entities in D. Roles are intended to express relations among objects in D, commonly binary relations only. Individuals refer to single elements of D. Accordingly, individuals can be typed by concepts / can be *instances* of concepts.

Actual statements in description logic are made by specifying *DL axioms*, which are distinguished into *terminological* axioms/*TBox* axioms and *assertional* axioms/assertions/*ABox* axioms.[52] TBox axioms such as (1.1) declare subsumption or equivalence relationships between concept descriptions. ABox axioms state which concept descriptions or which roles apply to an individual or to two individuals, resp. In the sequel, all of the terms *DL theory*, *DL knowledge base*, and *DL ontology* refer to sets of arbitrary DL axioms. By the above distinction, every DL theory T can be viewed as being composed of a TBox, the set of all terminological axioms in T, and an ABox, comprising all assertions in T.

DL LANGUAGE FAMILY

The DL "family of [. . .] languages" constitutes itself on the grounds that allowing for different sets of logical constructors in concept descriptions and complex roles yields different languages with varying metalogical properties, e.g., regarding their decidability and the complexity of reasoning problems. The DL $\mathcal{ALC}$ and the above-mentioned $\mathcal{SROIQ}$ (OWL 2) and $\mathcal{SHOIN}(D)$ (OWL) are examples in this respect.[53] More

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[49] More naturally speaking, (1.1) reads as: 'All women that have only daughters as their children are mothers.'

[50] Below we shall use 'DL role' where confusion with other readings of 'role' may arise, e.g. with role in an ontological sense as in [530, 561]. In any non-technical use 'rôle' is employed.

[51] The concept $\top$ in (1.1) is interpreted by this set D.

[52] Accordingly, TBox stands for 'terminology box', ABox for 'assertional box'. Cf. [33, p. 12–13], which refers to "taxonomy" as an alternative wrt TBox.

[53] The acronym $\mathcal{ALC}$ can be derived from "Attributive concept Language with Complements" [35, p. 139], whereas variants can be found elsewhere, e.g., "Attributive Language with Complement" [781, p. 261], [@26] or similarly "Attributive Language with full Complement" [781, p. 510]. The language itself was originally introduced in [731] in 1991, based on earlier technical reports dating back to 1988 with [730]. Wrt [731], $\mathcal{AL}$ can be linked with "Attributive concept description Languages" [731, p. 7], then "$\mathcal{ALC}$ is obtained from AL by adding general complements" [731, p. 6]. Since then, the names of many other DLs, i.e., those that extend $\mathcal{AL}$ by additional constructors (including $\mathcal{SROIQ}$ and $\mathcal{SHOIN}(D)$), follow a scheme of letters associated with constructors available in a language, cf. [33, Appendix, esp. Table A.1 and sect. A.4], [35, p. 143], [@26], or sect. 2.2 of [@25] for overviews.

$\mathcal{ALC}$ is of fundamental character in that (a) it is propositionally closed, i.e., it covers all Boolean operators ($\sqcap, \sqcup, \neg$ in DL), and (b) it comprises both, universal and existential quantification in role restrictions. Moreover, it is widely presented in introductory teaching on DL.

elaborate alternatives to the present passage can be found, a.o., in early chapters of the thorough *Description Logic Handbook* [33, 34] and in [35, sect. 3.1], which gives a gentle, 5-page introduction, including remarks on the historic development of DL as a research area.

DLs as FOL fragment with specific features

Regarding the relationship between description logic and first-order logic, decidability is a crucial aspect. For unrestricted FOL, important reasoning problems are undecidable, first and foremost[54] whether a given theory entails a given formula and whether a given theory is consistent, cf. [693, sect. 3.5 and ch. 6]. In contrast, decidability is a major criterion in the DL community, aiming at the practical feasibility of the formalisms developed.[55] 'Decidability' refers to typical reasoning problems wrt DLs, such as checking for the consistency of a DL theory or for the satisfiability of a concept defined within a DL theory. In a coarse-grained view, most[56] DLs developed to date can be perceived as determining / corresponding to decidable fragments of FOL. [33, sect. 4.2] expounds the relationship between DL and FOL in some detail, partially summarizing from and further referring to [96]. Most DLs turn out to be sublanguages of the 2-variable fragment of FOL that employ only unary or binary relations.[57] This can be seen by a natural[58] translation function of DL expressions into FOL expressions, e.g., see [33, sect. 4.2, p. 150] for $\mathcal{ALC}$ or [405, sect. 5.2.2] for $\mathcal{SROIQ}$.

1.1.3.3 Horn Logic, Logic Programming and Rule Languages

The Horn fragment of FOL and efficient query computation

Horn logic is another fragment of FOL that allows for efficient computations of certain specific kinds, named after Alfred Horn [@5] due to considering the corresponding type of formulas in his [427]. By common FOL terminology, a formula that is (at least equivalent with [693, p. 140]) a disjunction of literals of which at most one is a non-negated atomic formula and that can be preceded by variable quantifications is called a *Horn formula*. A universal Horn sentence is a Horn formula where all variables of the disjunction are universally quantified. Querying a set / theory T of universal Horn sentences by another formula, the query Q, means to determine whether T entails Q, where Q itself is a Horn formula with negated atomic formulas only and only existentially quantified variables. The usual response to such an entailment is a plain yes or no in the case of general FOL theories T and formulas Q. In addition, based on the notion of *Herbrand models* (or, more generally, term models), Herbrand's theorem, see e.g. [693, Theorem 4.1.2, p. 139], and the notion of variable substitutions, querying in the Horn fragment in the above way enjoys the existence of answer substitutions if and only if the corresponding entailment holds, where an answer

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[54]In our opinion and from the perspective of utilizing FOL for ontology representation.

[55]Notably, the notions of 'decidability' and 'tractability' should not be equated. Polynomial algorithms are commonly still seen as tractable, but whether and what is accepted beyond that is subject to diverse views. For instance, [135, p. 59, 61, and esp. 65–66] motivate the introduction of a specific description logic DL-Lite with the assessment that a worst-case exponential behavior of reasoning (and thus, implicitly, behavior with even higher computational complexity) of expressive DLs is infeasible in practice, which is particularly considered wrt data complexity and query answering.
Regarding DL-Lite, [26] of 2009 contains a survey and systematic study of extensions of DL-Lite, referred to as the DL-Lite family. [137] is the earliest work on DL-Lite mentioned in [26, esp. p. 18], whereas [135, p. 65] connects DL-Lite already, a.o., with [136]. The latter work is based on a language L_K^O in FOL-style notation, characterized as "a subset of FOL that captures the fundamental features of frame-based knowledge representation formalisms and of ontology languages for the Semantic Web." [136, p. 471], only mentioned to correspond to a DL in [136, p. 472, FN 4]. That association with frame-based KR formalisms, as well as the goal, formulated for DL-Lite, to be "capable of representing some basic features of conceptual modeling formalisms" [26, sect. 4, p. 18] is interesting, because these kinds of formalisms will be introduced and related to description logics nearby below.

[56]One constructor that exceeds the capacity of FOL is the role constructor $\cdot^+$ for the transitive closure of roles, cf. [33, Def. 2.28, p. 91], introduced in [30]. Its semantics requires that a role expression R^+ is interpreted by the transitive closure of the interpretation of R. This constructor should not be confused with the (FOL-compatible) role constraint of "transitive roles" [33, Appendix, p. 487], where the latter, applied to a role S, only requires that S is interpreted by a transitive relation. Although $\cdot^+$ transcends FOL expressiveness [33, p. 149], it can be combined with other constructors and still yield a decidable DL [30], cf. also [@26].

[57]Although there are exceptions in both regards, being a FOL fragment and allowing for only binary relations. Transcending FOL expressiveness is exemplified by $\cdot^+$ in the previous FN 56. $\mathcal{DLR}$ is an example presented in [33, sect. 5.7] as one alternative of defining a DL with n-ary relations. Further, there are "too expressive" description logics to the extent that standard reasoning problems are undecidable. [@26] identifies $\mathcal{ALC}$ extended with intersection and complement on roles together with role chaining as a language where concept satisfiability and ABox consistency are undecidable, referring to [727].

[58]"natural" by taking the formal semantics of the respective DL and FOL languages into account.

substitution maps the existentially quantified variables of Q to ground terms (terms without variables)[59] in the FO language. Moreover, answer substitutions can be computed efficiently, typically on the basis of resolution (and involving unification). For more detailed expositions we refer to [288, ch. 4–7], [693, ch. 4], [70, sect. 3.5 and ch. 9], and [109, ch. 5][60].

RULE LANGUAGES AS ANOTHER RELEVANT CLASS OF REPRESENTATION LANGUAGES

One may say that the Horn fragment of FOL forms the purely logical representative of a much wider class of languages that we refer to as *rule languages / rule-based formalisms*. Horn logic yields a simple form of *logic programming*[61], the semantics of which is still defined model-theoretically, while a deviation from pure FOL semantics can already be observed, e.g., by reference to Herbrand models. Extensions of those simple (or "classical") logic programs have required new semantic definitions, thereby increasingly leaving the framework of classical logic. In these regards, we can recommend [70, ch. 9] to readers capable of German, which proceeds from Horn logic to logic programming with default negation and further to answer set programming, well clarifying the relationships between the corresponding semantics based on Herbrand models, stable models[62], and answer sets.

Syntactically, logic programming is based on *rules*, constructs that can be read as *if-then*-sentences, e.g., "If x is a bird, then x flies.",[63] where the *if*-part is called the *body* of the rule, the *then*-part its *head*. More formally captured and limited to Horn logic, rules collapse with FOL implications with at most one atomic formula in their head and none or possibly multiple ones in their body.

In general, rules appear as a fairly intuitive way of representing interrelationships / knowledge. Beyond logic programming or, more generically, inferencing (and typically departing even further from classical logic), there are diverse further rule languages that share the general approach of rule-based representation from a syntactic perspective, but rely on a different semantics / interpretation and / or different modes of how computing with rules works. Prominent examples in these regards are *production rules*, cf. [109, ch. 7], as well as *reactive* (or *event-condition-action*) rules, cf. the mentioning in [475, sect. 2].[64] Altogether, a plurality of types of rule languages and systems with associated inference / computational mechanisms / engines exist. They are relevant in connection with ontologies, directly as languages for ontology representation as well as in situations where users of rule languages may wish to utilize ontologies represented in other formalisms.

RULE LANGUAGES IN THE SEMANTIC WEB AND THEIR LINKAGE TO DLs

The importance of rule languages has a strong reflection in the family of languages associated with the Semantic Web. Due to the differences between the semantics of DLs (and FOL) and the logic programming semantics of rules[65], it was neither clear nor uncontroversial in the early 2000s how a "rules layer" should be integrated into the Semantic Web language stack [@111]. [430] discusses effects of adopting the different semantics even for Description Logic Programs (DLP) [308] only. Roughly speaking, DLP refers to the overlap / intersection of DLs and Horn logic, both perceived as fragments of FOL (and thus under FOL(-

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[59]This applies immediately only to theories without equality, whereas matters are more sophisticated if equality is involved, cf. [693, Remark 4.1, p. 142, and Theorem 4.2.2, p. 143], in greater detail [691, p. 122–123] and [288, sect. 4.3 and ch. 5].

[60][109, sect. 5.4] comprises some historical notes about the route from Horn formulas to logic programming (cf. FN 61) and the language *Prolog* [@96], in particular.

[61]Logic programming is viewed here as a term for a family of approaches / languages that is, roughly speaking, more general than Horn logic, but more specific than the family of rule-based languages. Michael Gelfond and Nicola Leone provide a valuable, compact survey section (with many further references) on the history of logic programming (and connections to nonmonotonic reasoning) in [269, sect. 1]. Moreover, they introduce logic programming under answer set semantics (in 2014 terminology) in [269] as a basis for 'logic programming and knowledge representation', in and for an equally named and oriented special issue of Artificial Intelligence [@4] of 2002. One of the very early works on logic programming cited in their article, which is also referred to in [839, ch. 1, sect. 1.4.4, p. 65], is Robert Anthony Kowalski's comprehensive exposition [484] of the paradigm from 1979.

[62]introduced by Michael Gelfond and Vladimir Lifschitz in 1988 in [270].

[63]Typical formal notations for this rule example are $\mathsf{Flies}(x) \leftarrow \mathsf{Bird}(x)$, cf. e.g. [70, Def. 9.1, p. 270] and [218, sect. 2.1], or in a common Prolog [@96] notation `Flies(X) :- Bird(X).`

[64][659, p. 10–11] distinguishes six kinds of rules, five of which we consider to rest on distinct basic principles, named derivation, transformation, integrity, deontic, and reaction rules. (*Facts* are named as a sixth class, yet we find that facts belong to a different dimension of classification.)

[65]Already ignoring other types of rules.

compatible) semantics).[66] It served as one source of inspiration for the OWL 2 RL profile, which defines "a syntactic subset of OWL 2 which is amenable to implementation using rule-based technologies" [601, sect. 4].

Overall, however, there is no unique solution to integrating rules and DLs for the Semantic Web, nor do we see that one of the options identified in the literature[67] were adopted as "the" de facto standard.[68] In [659] of 2009, Adrian Paschke and Harold Boley survey the landscape of rule languages in the Semantic Web, distinguishing rule markup languages (basically, languages with an XML dialect as (one) concrete syntax) from "Semantic Web rule languages" that use an ASCII-based syntax[69]. Let us instead distinguish between languages for rule interchange, aiming at a broad coverage of rules of distinct kinds (as mentioned above), and specific rule languages (within particular kinds). The Rule Interchange Format (RIF) [475] [@106] and the Rule Markup Language (RuleML) [92] [@109] are noteworthy representatives of interchange languages. Specific languages that focus on inference and are close to FOL and / or logic programming are most closely related to this thesis. An early proposal in this direction is SWRL, the Semantic Web Rule Language [432, 434] already mentioned above, that "extends the set of OWL axioms to include Horn-like rules" [432, sect. 1] and is dubbed "as roughly [the] 'union' " of description and Horn logic in [659, p. 3]. SWRL presents a solution that is semantically oriented at DLs and expressive, and that has been implemented in software tools. For example, the latter is supported by (the bundled plugin SWRLTab [@99] [638] for) the well-known and broadly used[70] ontology editor Protégé [@97]. From a reasoning complexity point of view, SWRL's expressiveness comes at the price of being undecidable [434, esp. sect. 6, cf. also sect. 9]. Since its proposal, decidable/tractable rule extensions have therefore been subject to much research, cf. e.g. [40, 490].

1.1.3.4 Further Languages in Knowledge Representation and Conceptual Modeling

GENERAL LANGUAGES IN KR AND CONCEPTUAL MODELING

What may be called "general KR and modeling languages" forms a large class of languages that are relevant wrt ontologies and ontology representation, and to some extent for this thesis. All or many of the languages of the previous sections can be seen under this umbrella. In this section, we focus on "remaining" approaches, linking some of them to those introduced before.

SEMANTIC NETWORKS

In connection with description logics, observe that they grew out of reasoning systems such as KL-ONE [110, 883].[71] Those reasoning systems themselves fall into or at least emerged from the area of *semantic networks* / *semantic nets*, cf. [516]. This area, in turn, refers to a large family of graph-based representation approaches, where the graphs are meant to consist of conceptual units and relations among them, and can be utilized to automatically derive implicit knowledge, usually through specific graph-based methods. The field was studied and developed primarily during the 1960s-1980s.[72] Interestingly, originally it was introduced in AI rather as a renunciation of KR based on classical logic, before preparing the ground for description logics. Related to ontology and ontological issues, the field of semantic networks has led to interesting and inspiring works on their semantics, e.g. by William A. Woods [882] and Ronald Brachman (a core figure in developing KL-ONE) and his colleagues, cf. [105, 107, 108]. Notably, and "although" equipped with a formal semantics, we believe that RDF(S) may be seen as a modern semantic network

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[66][308] introduces DLP initially and in detail, actually and more precisely distinguishing between DLP and *Description Horn Logic*, see [308, esp. sect. 2 and 5]. [389, sect. 21.4] comprises a condensed exposure, [24, sect. 4] is an in-depth recapitulation.

[67]Cf. e.g. [24, 430], [218, esp. sect. 4], [219, esp. sect. 4–5], and [659, p. 13–16].

[68]Nevertheless and naturally, some approaches have become more widespread than others. This certainly includes the RL profile in OWL 2 [601, sect. 4] as well as SWRL [432, 434] (see below in the main text).

[69]ASCII stands for American Standard Code for Information Interchange [22], cf. [@10]

[70]Quoting [@97] on Aug 12, 2014: "Trusted by over 244,817 users".

[71]This explains their naming as "KL-ONE based representation language", cf. FN 43.

[72]John Mylopoulos [614, p. 130] attributes the proposal of semantic networks to M. Ross Quillian's PhD thesis [678] (1966), which uses the term "Semantic Memory", while the 1968 publication [679] is named "Semantic Networks". Fritz Lehmann [516, sect. 2.2] traces back the roots of semantic networks further to Richard H. Richens' system Nude, cf. [776], in the context of machine translation (similarly, Quillian's work relates to text understanding, a.o.), where Richens mentions an "interlingua" that is left with "a 'semantic net' of 'naked ideas'" [706, p. 23] (1956).

approach.[73] Fritz Lehmann's survey [516, sect. 1.1, p. 3] points out 8 sub families of semantic network systems and also mentions directions that became more popular later, such as rule-based (expert) systems and object-oriented systems.

FRAMES / FRAME-BASED LANGUAGES

Yet in 1992, he states "The standard representation of semantic networks in conventional computers uses frames as data structures." [516, sect. 4, p. 8]. This leads us to *frame*-based languages in knowledge representation, cf. [109, ch. 8] and [773, sect. 3.2]. They are named after the notion of frames introduced by Marvin Minsky in [587, 588] as a "data-structure for representing a stereotyped situation" [587, p. 1], actually in the context of computer vision. Though some core ideas may still be shared, the more modern understanding of frames has developed significantly away from the original work and its context. "The fundamental idea of a frame system is rather simple: A frame represents an object or a concept. Attached to the frame is a collection of *attributes* (*slots*), potentially having types (or *value restrictions*) and potentially filled initially with values. When a frame is being used the values of slots can be altered to make the frame correspond to the particular situation at hand." [506][74]. Originally the Protégé ontology editor followed a solely frame-based approach [224],[75] which is meanwhile accompanied side-by-side with a genuine DL/OWL-oriented reimplementation, cf. [864] and [@98]. Indeed, according to our perception, the immediate relevance of frame-based languages/systems for issuing ontologies has diminished substantially since the early 2000s.[76]

F-LOGIC AND HILOG

Yet, there is another reason of why frame-based KR is worth mentioning. The short characterization of frames above already indicates a great similarity to concepts in object-orientation (OO),[77] At the crossroads of frame-based, object-oriented, and logical (KR) languages/systems, we find F-Logic (derived from 'Frame Logic') [@36] [474, 476] is a well-known (cf. [@36]) formalism to account "[...] in a clean and declarative fashion for most of the structural aspects of object-oriented and frame-based languages." [476, p. 741], based on overlapping features in these paradigms.[78] F-Logic plays a rôle in ontology representation [20], though a minor one, esp. since the standardization of OWL. The related and more recent FLORA-2 project [@37] develops a knowledge base language and a corresponding application development environment that rest on combining aspects of the rule-based and the object-oriented paradigms. It is proposed as a KR and inference framework for the Semantic Web in [888], as one of its major intended applications. The FLORA-2 language is based on F-Logic, HiLog [151], and Transactional Logic [93] (a logic to capture state changes in logic programs and databases).

HiLog [151] is a logical framework with metaprogramming capabilities that was suggested to substitute predicate logic as the basis of logic programming (insofar rather subject to sect. 1.1.3.3). It is motivated by the tradeoff between, on the one hand, the manipulability of predicates, functions, and atomic formulas within the language and, on the other hand, the goal of a sound and complete proof procedure. [713] (before presenting extensions of HiLog to allow for negation in its rule bodies as the actual contribution) nicely

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[73]Going another step to viewing even the Linked Open Data (LOD) cloud [84] as a semantic network might be tolerated from a very abstract point of view and depending on a positive evaluation of the question of how much semantics is attributed to the data items published there. On the other hand, it seems to overexpand the concept of semantic networks, e.g. by ignoring the extensive technological background in Linked Data.

[74] For actual frame-based systems / KR approaches, cf. three references in [506] to works from the 1980s and 1990s. These include [148] on Open Knowledge Base Connectivity (OKBC) [149], "an application programming interface for accessing knowledge bases stored in knowledge representation systems" [@77] and "a successor of Generic Frame Protocol [@50]" [@77]. Notably, we find the stance instructive that relates frames to classes, individuals, slots, and facets in OKBC, as described in [148, sect. "The OKBC Knowledge Model"].

[75]Protégé has a long development history from its first version mainly due to Mark Alan Musen in the late 1980s [611, 612] via Protégé 2000 [224], cf. also [211, esp. sect. 4.3] to Protégé 5.0 under development during 2014, cf. [@98]. Wrt FN 74, Protégé 2000 was designed to be OKBC [@77] conformant, see [633, sect. 1].

[76]According to our knowledge, the Foundational Model of Anatomy (FMA) [@38] [714] is the latest large ontology that is based on frames (and was developed by means of Protégé). Since its publication it has been converted into OWL at least three times, with differing goals and approaches [286, 287, 634, 635], [@40]. The OWL version [@39] that is offered (and used by several other projects) through BioPortal [@15] is described to rely on [634].

[77]Cf. also the remarks in [506, p. 96], providing further references.

[78]See [476, sect. 1] for few corresponding remarks.

summarizes a feature of HiLog that it shares with Common Logic (CL) (sect. 1.1.3.1), namely allowing for predicate names in argument positions,[79] while maintaining a first-order semantics by a distinction between predicate names and predicate extensions. We return to this aspect in sect. 7.2.2.2.

CONCEPTUAL MODELING LANGUAGES, FIRST INCL. OO AND UML

Turning to object-orientation itself, as it emerged in software engineering in connection with object-oriented programming (OOP). We do not concern ourselves with programming languages as such, but with related modeling languages. More broadly, conceptual modeling (CM) languages in fields such as information systems, databases, software engineering, and business process management yield the second branch of "general KR and modeling languages", because many have been studied and utilized in connection with ontologies and their (semi-)formal representation. A prime example in these regards is the Unified Modeling Language (UML) [@126] [94, 95, 640, 643, 644, 648, 719, 720],[80] as standardized by the Object Management Group (OMG) [@78]. Similarly to the case of the different OWL versions, we refer to UML as an umbrella term for its various versions, only naming a specific version where necessary. Examples of works utilizing or suggesting UML as ontology specification language include the Ontology Definition Metamodel (ODM) [@75] [646] by OMG itself[81], and various individual publications, such as [168, 230, 479, 655].

Regarding the semantics of UML, note that some caution is in order wrt whether or not to attribute UML a formal semantics. The language and its modeling elements are determined in its specifications,[82] the normative parts of which specify the semantics of UML in terms of paragraphs in natural language text. Regarding formal semantics for UML, there are and were group initiatives with this objective, like the UML 2 Semantics Project [122] or the precise UML group (pUML) [229], besides diverse individual, often aspectual proposals, e.g. [495, 783]. [371] aims at guiding basic understandings in this context. Meanwhile, there is a standardized semantics for executable UML models that employ a specific subset of model elements [650]. Wrt the present work, however, we are not aware of any declarative, e.g., referential semantics (see sect. 2.1 below).

ANOTHER CONNECTION: ONTOLOGICAL FOUNDATION OF CONCEPTUAL MODELING (LANGUAGES)

Moreover, research on the ontological foundations of conceptual modeling as well as ontology-driven information systems engineering appears to be increasing over recent years. Its roots date back (at least) to a similar time at which ontologies found their way into KR and KBS, cf. especially the work of Yair Wand and others, e.g. [800, 859–861] and further references therein, where already the 1989 work [859] utilizes the ontology of Mario Bunge [126, 127] for modeling objects as representation constructs[83] For a shorter time, but in a similar spirit, Giancarlo Guizzardi with various colleagues, cf. e.g. [11, 186, 348–350, 352, 353, 357, 359], and other groups, see e.g. [231–234, 299–301, 651, 861, 863], have been pursuing similar efforts, making explicit use of ontological analysis and ontological theories / proposed ontological systems[84]. Two major objectives are applications of ontological work "[. . .] to the evaluation of conceptual modeling languages and frameworks [. . .] and to the development of engineering tools (e.g., methodological guidelines, modeling profiles, design patterns) that contribute to the theory and practice of this discipline [of conceptual modeling]." [351, p. 2]. In several cases this applies directly to UML, e.g. in [231, 352, 353, 359]. Notably, this has led to the first ontologically founded UML editor OntoUML [@87] [75, 76], originating from [348].

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[79]Actually, neither HiLog nor CL draw any syntactic distinction between predicate, function, and constant symbols [151, sect. 2.2], [713, p. 2], [452, sect. 6.1, esp. clauses 6.1.1.9–6.1.1.12].

[80]These citations refer to the main UML sources we have partially dealt with / consulted while working on topics of this thesis, with a major focus on classes / classifiers (in connection with class diagrams), and some, but less consideration of diagrams for use cases, activities, and state machines.

[81]ODM was standardized as [646] in 2009, after an extended period of previous versions, incl. [209] (2004) and [441] (2006), and foregoing works such as the UML to OWL conversion in [268].

[82][647] (2011) is the most recent version formally adopted. Moreover, the "Beta 2" version [648] of its (intended) formal successor is available since 2013. According to its p. i, it is a minor update that mainly results from simplifications to [647].

[83]With some deviations from OOP characteristics of objects, e.g. considering message passing not as a fundamental modeling construct, but rather as implementation metaphor [860, p. 289].

[84]Some of those being covered in sect. 1.1.4.2 below.

THUS MANY MODELING LANGUAGES ARE RELEVANT, IN PRINCIPLE

According to the observations on conceptual modeling languages in general, various other languages / modeling approaches enter the broader context of this work, as languages for expressing ontological content or as subject to ontological evaluation, foundation, and enrichment. These include Entity-Relationship (ER) modeling[85] and its descendants, further Object Role Modeling (ORM) [@89] by Terry Halpin et al. [367, 368], the Object Process Methodology (OPM) by Dov Dori [206], Conceptual Graphs (CGs) as fostered by John Sowa [772],[86] the approach of Formal Concept Analysis (FCA) [261], and process-oriented modeling formalisms such as the Business Process Modeling Notation (BPMN) [645]. Without going into any details, domain-specific modeling (languages) may also be considered, cf. [466].

While we became aware of the named formalisms in varying degrees of detail during the period of working on this thesis, most of them will not be treated in detail subsequently. Nevertheless, the purpose of naming them here is to explicate and clarify the range of entities that is meant by a general reading of the terms 'language' and 'modeling language' herein. Moreover, some of the established interrelations among them are indicated on purpose, e.g. between FOL and DLs or Horn formulas, between the latter and logic programming, etc.

1.1.3.5 A Domain-Specific Format for Biomedical Ontologies

OBO (AND OBO FORMAT) IN THE CONTEXT OF BIOMEDIAL ONTOLOGIES

We conclude the brief survey of "ontology-related languages" (in a loose sense) with a specific format / language that was developed for the semi-formal representation of *biomedical ontologies*[87] and thus relates to sect. 5.4 below. The OBO Flat File Format [182], cf. also [285, 428], was developed in the mid 2000's in connection with the OBO-Edit "ontology editor for biologists" [183], which itself spawned[88] from the Gene Ontology (GO) project [@51] [27, 820]. The label "Open Biomedical Ontologies" (OBO), nowadays extended to "Open Biological and Biomedical Ontologies" [@73], gave rise to the name of the format, denoting an effort initiated by leading GO developers to support the bio-ontology community that was rapidly emerging during the 2000s.[89] The format became the de facto standard when various projects started the development of further bio-ontologies, inspired by the GO success. By definition, it is a computer processable, plain text format, also intended to be human-readable. However, conceptually, ontologies in the OBO format can be considered as graphs, possibly directed acyclic graphs (DAGs),[90] the nodes of

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[85] as introduced by Peter P.-S. Chen in [150].

[86] A compact overview can be found in [773, Appendix A.2]. In [400, part III], Alexander Heußner summarizes their proximity to Existential Graphs (EGs) of Charles Sanders Peirce and links to other sources, introduces a CG syntax and discusses several semantic approaches that have been proposed for CGs. Notably, [179] aims at a proper introduction of EGs as a (diagrammatic) mathematical logic.

[87] The term refers to ontologies which cover (primarily) entities in the biological, biomedical, and health care domains, such as cells and their components, anatomic structures, phenotypic features, diseases, etc. It may be understood to include further bio ontologies based on the fact that such former ontologies rely on them. For example, as of November 2014, the top-level ontology Basic Formal Ontology (BFO, see sect. 1.1.4.2) is listed among the OBO Foundry candidate ontologies [@73].) We see no clear terminology in literature. In this work, we use 'biomedical ontology' and 'life science ontology' as synonyms. Their constituents may be from any of the mentioned domains. In contrast, 'bio-ontology', 'medical ontology', and 'health care ontology' are narrower notions for us, having differing foci.

[88] e.g., as noted in [285, p. 1].

[89] In particular, a corresponding website was launched in 2001 (later accessible via [@72]) in order to register ontologies available or under development in this context, e.g. for easier dissemination and discovery, or, as stated in [760, sect. 2, p. 22], "[...] as a means of providing convenient access to GO and its sister ontologies [...]". Four years after its inception (ibid.), the OBO Foundry [@73] [760, 761] was established in order to coordinate and regulate the lively proliferation of bio-ontologies in OBO by a set of principles, cf. also [467, sect. 15.2.1], that significantly extended the initial OBO guidelines, not uncontroversially, cf. [392, esp. p. 299]. Meanwhile the OBO Foundry assigns participating ontologies the status of either member ontology or candidate ontology. Notably, the set of OBO Foundry ontologies is also accessible through Ontobee [@79] [886], a linked data portal for ontologies that has emerged in the context of the OBO Foundry, and through the NCBO BioPortal [@15] [636], which is similarly focused on ontologies in life sciences and provides a larger and less controlled collection of ontologies than in the OBO Foundry (among other features).

[90] One of the early GO publications from 2000 describes the GO ontologies as DAGs [27, p. 28], because at that time the only relations that occurred in those graphs were is-a and part-of. This has also been adopted for other ontologies developed in the OBO format in for a number of years, maintaining the view of OBO ontologies as DAGs. However, with many more relations available today, i.e., 382 as of November 9, 2014 (determined via [@108], [@79]), we do not see that acyclicity were necessarily maintained in such graphs (assuming no distinctions among the relations in establishing edges in the graph).

which represent (primarily biological) categories, among which relations can be expressed. While the OBO format is not equipped with any formal semantics, note that a kind of semantics, maybe more a kind of pragmatics (cf. the beginning of ch. 2), was originally established for using GO for annotations. This is known as the True Path Rule [819, p. 1429], stating that annotating a data entry with any node n in GO justifies the (implicit) annotation with all parent nodes of n in the DAG.[91] Later, [285, 428] proposed a formal semantics for the OBO format in terms of a generic mapping into OWL. However, although this captures many biological relations adequately, it fails for some relations. One case that requires a different solution is the family of "lacks relations", e.g. lacks-part,[92] reported by Robert Hoehndorf and colleagues, including ourselves, in [416]. More recently, the fraction of bio-ontologies that is developed directly in OWL is clearly growing [467, sect. 15.2.3, p. 354]. Nevertheless, the OBO format is still in use today and many ontologies are offered in both, the OBO format and OWL.

[416]

LEADING TO OUR MOTIVATIONS

Finally, nowadays and without limitation to the bio-ontology community, it is a frequent case that a group or community is developing "an" ontology and offers "this" ontology in several languages. A very important question in this regard is whether the different variants do actually render that "one" ontology – or whether several ontologies arise that differ in content. Indeed, it is one of the central motivating questions that this thesis addresses, catered for in sect. 1.2, in particular 1.2.1.

1.1.4 Types of Ontologies

VARIOUS TERMS SUGGEST DISTINCTIONS AMONG ONTOLOGIES

The more widespread occurrence and use of ontologies leads to the need for classification, i.e., distinguishing different kinds of ontologies, and the need for characterizing certain aspects of them. Indeed, meanwhile many such terms can be found in the literature, including 'ontology' being prefixed with terms such as 'domain', 'foundational', 'reference', 'lightweight', 'linguistic' and 'scientific'. It is not the purpose of this section to provide a fully systematic and (close-to-)exhaustive survey of these terms and their meanings. Instead, we intend to cover only those notions that occur in the remainder of the work, thereby clarifying our terminology of ontology types. Subsequently, we briefly focus on foundational ontologies, providing some examples.

1.1.4.1 Dimensions, and Types by Language / Expressiveness

ACCOUNTS IN THE LITERATURE AND THREE DIMENSIONS

[289, sect. 1.4.1] comprises an overview on ontology types developed on the basis of, a.o., [330, 507, 841]. Asunción Gómez-Pérez et al. [289, sect. 1.4.1] focus on two dimensions of classification, one of which is (1) "based on the richness of the internal structure of an ontology" [ibid., p. 28], basically following the ontology language spectrum in [507][93]. Since the latter is organized by (more of an intuitive notion of) expressiveness, we also call it the *language / expressiveness dimension*. The other one is (2) "based on the subject of the conceptualization" [289, p. 29]. Below we focus on a narrower idea, labeled the *dimension of abstraction*. Notably, we prefer *subject dimension* for dealing with classifying ontologies by (reasonably broad) subject matters. Some classes in that dimension are 'biomedial ontologies', e.g. GO [@51] and FMA [@38], 'legal ontologies', see [726], and 'ontologies of physics', e.g. [382, 383], 'ontologies of ecology', e.g. [547], etc, cf. another sample in [348, sect. 3.2.3, p. 68–70]. Returning to (1) and (2), these are also the guiding dimensions in [718]. They can also be seen to apply implicitly to a foundational chapter on ontologies / "ontological architectures" [637] by Leo Obrst.

Let us further mention a recent, more feature-based account to characterize ontologies, in which Pawel Garbacz and Robert Trypuz pursue the goal "to provide a meta-ontological schema in which to detail the philosophically outstanding aspects of engineering ontologies." [262, sect. 3, p. 5], starting from a schema

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[91]"The pathway from a child term to its top-level parent(s) must always be true." [819, p. 1429], cf. also recapitulating variants in [832, sect. 2.2, Fig. 1] and [413, esp. p. 46, 82].

[92]For instance, mice without a tail would be represented by relating a corresponding category via lacks-part to the category Tail.

[93]Remember the beginning of sect. 1.1.3 and FN 39 there.

developed for the analysis of philosophical ontologies. The approach deviates from our purposes in that it does not straightforwardly lead to types of ontologies that we aim at in this section. Notably, there is some overlap with the dimensions introduced so far in that an ontology description includes the language and the domain of an ontology as components. However, also in those respects the authors discuss some specifics that may run contrary to established conventions, e.g. conceiving of distinct syntactic formats of an underlying language[94] as distinct languages [262, sect. 3.1, p. 6], such that an OWL ontology rendered in two of its concrete syntaxes [856, see Fig. 1 and sect. 4] yield two distinct ontologies.

LANGUAGE / EXPRESSIVENESS DIMENSION

Four of the 9 types in [289, sect. 1.4.1] along the first dimension are 'controlled vocabulary' (a finite list of terms), 'thesaurus'[95], 'formal is-a hierarchy' (corresponding to the notion of taxonomy herein), and 'logical theory'[96], with an increase of expressiveness[97] from the former to the latter. Obrst's categorization can largely be viewed as a more coarse-grained variant. Starting from the four types mentioned wrt [289, sect. 1.4.1], it deviates by not covering 'controlled vocabulary', positioning 'taxonomy' slightly differently,[98] and adding 'conceptual model' as immediate predecessor of 'logical theory' (wrt that expressiveness ordering). For our purposes, we have already introduced two broad kinds of ontologies along this dimension, namely 'verbalized' and 'formalized ontology' in sect. 1.1.2.2, and we have the notion of 'taxonomy' from the same section. In order to make more specific statements than just using the term 'formalized ontology', we treat the language in which any particular formalized ontology is expressed as an attribute / feature of that ontology, possibly commenting on expressiveness.[99]

LIGHTWEIGHT VS HEAVYWEIGHT ONTOLOGIES

As final notes regarding the language / expressiveness dimension, two frequently contrasted classes of ontologies are *lightweight* vs *heavyweight* ontologies, cf. e.g. [668, sect. 9.2], [289, sect. 1.2, p. 8], or [589, sect. 4.3]. The latter almost coincide with logically formalized ontologies, but further include conceptual models that are equipped with explicit formal constraints, e.g. UML with constraints expressed in the Object Constraint Language (OCL) [@74] [455].[100] Lightweight ontologies cover the less expressive classes. Based on the position that even a controlled vocabulary without any structure can be seen as establishing ontological claims, e.g., of the existence of categories that are intended to be named by terms in the vocabulary, our use of 'ontology' in general accepts even such systems. We admit, however, that one must be very attentive in relating such systems / ontologies to the actual conceptualization that is expressed by them / that can be extracted from them. Moreover, "very lightweight" ontologies are rejected as being considered as ontologies proper in parts of the formal ontology literature, e.g. in [589, sect. 5.2]. On the other hand, none of the languages considered in sect. 1.1.3 is bound to representing conceptualizations. Insofar the question arises even for logical languages, how their expressions actually relate to the conceptualizations that they are meant to capture – foreshadowing another important motivating concern in sect. 1.2.

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[94]defined by its abstract syntax; see sect. 2.1.1 for this notion

[95]The notion of thesaurus has a more linguistic or terminological flavor than the other kinds. In agreement with [637, p. 31], we see a thesaurus as a collection of terms with synonyms, where the terms (representing categories / concepts) may be hierarchically arranged and may be associated with other terms. Importantly, the hierarchical arrangement is not necessarily interpretable as an is-a relation. Usually it is more loosely defined and is frequently referred to as 'broader-than' / 'narrower-than'. A well-know thesaurus in the field of biomedical ontology is Medical Subject Headings [@64], maintained by the U.S. National Library of Medicine (NLM) [@71] for indexing the PubMed database [@103].

Notably, at the "crossroads" of thesauri (and more generally, knowledge organization systems) and the Semantic Web there is the Simple Knowledge Organization System (SKOS) [@114], a W3C Recommendation for "concept schemes" [446], including thesauri. Moreover, ISO 25964 is devoted to "thesauri and interoperability with other vocabularies" [451, part of title].

[96]referred to as "ontologies that express general logical constraints" in [289, sect. 1.4, p. 29]

[97]Expressiveness refers to the formal expressiveness of the languages that are used for stating those category systems / ontologies.

[98]Taxonomies are rated less expressive than thesauri in [637, Fig. 2.2, p. 31], contrary to the ordering of thesauri and formal is-a hierarchy in [289, sect. 1.4]. But in the text Obrst differentiates 'weak' and 'strong taxonomies', where the latter are based on the subclass relation, thus corresponding to our taxonomy.

[99]E.g., a mere taxonomy should be identified as such, even when written / implemented in any FOL or DL variant.

[100][455] provides a formal semantics for OCL in its (informative) Annex A. For reasoning with OCL constraints, there are various approaches / proposals, cf. e.g. [677, sect. 1–2].

1.1.4.2 Types by Abstraction and the Notion of Foundational Ontology

DIMENSION OF ABSTRACTION

The dimension of abstraction can be seen as a restriction of (2) above. For (2), [289, sect. 1.4.1] briefly presents notions of ontologies based on these prefixes in their labels: KR, general / common,[101] top-level / upper-level, domain, task, domain-task, method, and application. We find that several aspects are involved in this collection that should be further separated. Therefore, let us focus only on the criterion of which level of abstraction the categories exhibit. In line with three levels of abstraction in [637, sect. 2.3], we distinguish top-level / upper-level, mid-level / core, and domain / domain-specific ontologies.[102] Most of the terms used for these classes are certainly inspired by the conventional way of visualizing taxonomies such that the most general categories are positioned at the top, the most specific (within the taxonomy) at the bottom.[103] The subsequent characterizations of the three former notions emanate primarily from [392, sect. 14.1], [397, sect. 1], [637, sect. 2.3], [289, sect. 1.4.1 and 2.2].

[397]

TOP-LEVEL AND CORE (= FOUNDATIONAL), AND DOMAIN-SPECIFIC ONTOLOGIES

Top-level ontologies (TLOs) cover entities of high / highest degree of abstraction and / or areas of abstract[104] entities of very wide applicability to and relevance for many fields[105] (and thus to numerous domain-specific ontologies). Sample categories include 'object' and 'process' at a high level of abstraction, and 'time point' and 'spatial region' with a very high degree of relevance for various domains. One major purpose of top-level ontologies is to provide a framework for categorizing entities in domain-specific ontologies, e.g. for their alignment or for their development at the outset. *Domain-specific ontologies* complement TLOs by being oriented at / primarily comprising more specialized notions, accordingly usually having much more limited domains of entities (in the sense of sect. 1.1.2.2). The degree of specialization across domain ontologies varies significantly. With their proliferation and given ontologies that address very specific domains of reality, *core ontologies* were introduced and provide a bridging level, often at the level of disciplines such as biology, medicine, or law.[106] For convenience, we introduce the term *foundational ontology* as a generalization of top-level and core ontology.

FOUNDATIONAL ONTOLOGIES IN RELATION TO THIS THESIS

Later sections and chapters reveal that this thesis is closely connected with the development of the top-level ontology General Formal Ontology (GFO) [@46], see esp. sect. 1.1.5. We adopt and defend the position that foundational ontologies of both kinds, top-level as well as core ontologies, can be valuable resources wrt several purposes. One important case, as mentioned above, is to support the integration of domain-specific ontologies. We omit any recapitulation of detailed arguments from the literature. Several works describing the foundational ontology examples that are introduced next address this issue. Moreover, there are a number of dedicated publications or sections, a.o. [99][107], [637, sect. 2.3.2], [857, sect. 4.1, p. 112].

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[101][289, p. 29] refers to "general ontologies" with recourse to [841], which presents the notion of "generic ontologies" [841, sect. 3.1, p. 193] in connection with application, domain, and representation ontologies. Notably and on the one hand, the characterization of generic ontologies there fits our understanding of top-level ontologies equally well or better than justifying a separate category. The same applies to most parts of the description provided for general / common ontologies in [289, sect. 1.4.1], which may thus be unified with top-level / upper-level ontology. On the other hand, the examples of space, time, and units in [ibid., Fig. 1.14] suggest strong similarity with a two-case distinction in our characterization of top-level ontology below, just that we include both cases a priori. [637] assigns these examples to mid-level ontologies.

[102]I.e., we maintain two labels per kind of ontologies here, because these are frequent in the literature. Perhaps core ontology is less known today – see, e.g., [831]. Further terms exist, e.g. 'foundational' is used synonymously with top-level ontology in [98, cf. sect. 2.4], 'top domain' and 'upper domain' ontology for the core ontology BioTop [@16] [738].

[103]In the 1990s, the notion of domain ontology was complemented by that of task ontology, cf. e.g. [329, esp. sect. 2.3 and 3], which appears much less common today, however. Ignoring that "legacy", 'domain ontology' appears disadvantageous insofar that each ontology is devoted to a certain domain. For this reason we shall prefer the longer 'domain-specific ontology'.

[104]in a loose sense, e.g., immaterial

[105]up to *all* fields

[106]Grasping the notion of core ontology primarily wrt the dimension of abstraction is certainly a simplification compared to "principled core ontologies" as presented in [831], cf. also [422, esp. sect. 5.4.1 and ch. 6].

[107][98, sect. 2.4] clarifies that [99] uses 'foundational ontology' rather in the sense of top-level ontology herein, characterizing them briefly as "[. . .] well designed and general heavyweight ontologies whose aim is to capture a clear perspective on reality by modeling philosophical positions" [99, p. 2].

PHASES IN THE HISTORY OF TOP-LEVEL ONTOLOGIES, PHASE I
In the development of GFO, related efforts need to be taken into account, of course. In our perception of the history of top-level ontologies when restricted to information and computer sciences, we next distinguish three unequal phases, including some examples. In the first phase, ca. in the decade of 1990–2000, the first TLOs (or component theories of TLO character) accrued at the time when ontologies started to gain interest in computer and information sciences. Frequently and naturally, those efforts were part of other fields, such as knowledge-based systems, cf. sect. 1.1.1. Representatives of this first phase are the upper level [668, Fig. 12.1, p. 261] of Cyc [@23] [519, 520], cf. also [668, ch. 12] and [289, sect. 2.2.3], a huge knowledge base of "all common sense knowledge" (among other formulations) [668, sect. 12.1, p. 259][108], and the Component Library (CLIB) [@19] [50], "a hierarchy of reusable, composable, domain-independent knowledge units" [50, p. 14] developed in the Rapid Knowledge Formation (RKF) project [@107] and reused in Project Halo [@95] [360, see p. 37 for the fact]. One might also mention the Frame Ontology [311, esp. sect. 3.6] as a member in the first phase, developed by Thomas Gruber in connection with the Ontolingua system[109] [235], although this can also be seen from another perspective that is mentioned the next section (1.1.4.3).

INTERMEZZO: LINGUISTIC ONTOLOGIES
Based on the fact, cf. [50, esp. p. 15-17], that CLIB is significantly influenced by, a.o., the lexical database WordNet [@133] [236], [668, ch. 10], a short intermezzo lends itself to introducing the term *linguistic ontologies*, which we adopt although it does *not* conform to our terminology along the subject dimension as introduced above. That means, linguistic ontologies are not ontologies with the field of linguistics as their domain of knowledge / reality, as was the case for medical and legal ontologies, for example. Instead, 'linguistic ontology' is typically used for referring to certain resources from (computational) linguistics and natural language processing that are similar to / seen as some forms of ontologies, e.g., due to hierarchical arrangments of constituents that serve as units of meaning, such as WordNet's synsets (sets of words that are synonyms in certain contexts). Notably, we proceed with a broader notion of linguistic ontology than the one with the same term in [668, ch. 17, esp. sect. 17.3]. There, a linguistic ontology is one in which categories are introduced by linguistic evidence, primarily based on grammatical information. In contrast, WordNet belongs to the class of lexical / lexical-semantic ontologies in the terminology of [668, sect. 17.2]. The notion of linguistic ontology herein, in line with [289, sect. 2.3], subsumes the former two into a single category. Both, [668, ch. 17] and [289, sect. 2.3] describe / survey a number of linguistic ontologies, e.g., besides WordNet, the Generalized Upper Model (GUM), Mikrokosmos, and SENSUS.

PHASE II ONTOLOGIES STARTING WITH THE SUO EFFORT
Versions of linguistic ontologies that were available at the last turn of the millenium, like those mentioned, received some attention in the development of top-level ontologies in the second phase of TLO "production" that we see. The same applies to a number of verbalized ontologies, i.e., ontological theories in / as scientific publications, typically including formal fragments. Examples in this respect and in the context of AI are the top-level ontologies in John Sowa's book on knowledge representation [773, esp. ch. 2 and App. B] and in the AI book by Stuart J. Russell and Peter Norvig [721, sect. 8.4, cf. Fig. 8.2, p. 229], as well as Allen's theory of time intervals[110] [6]. In the at those days emerging field of Formal Ontology, examples include Nicola Guarino's work on identity conditions [332] and work on holes [142] by Roberto Casati and Achille C. Varzi. Indeed, all works mentioned, including the three linguistic ontologies above, were part of a collection meant to prepare a top-level ontology standard. The Standard Upper Ontology (SUO) [@121] effort started with a mailing list in May 2000 and was shaped as the Working Group P1600.1 of IEEE [@59] in December 2000 [626, p. 3]. The working group has not succeeded in standardizing a TLO eventually. However, the Suggested Upper Merged Ontology (SUMO) [@119] [662] arose from those efforts [626, 663], and in parallel the time between ca. 2000 and 2005 spawned TLO proposals from a number of groups. Therefore, we view the inception of the SUO effort as the starting point of the second phase in TLO history. Finally, let us mention another ontology generated in phase I, namely the (verbalized) ontology of Bunge,

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[108]While Cyc is a proprietary, commercial product since 1994, restricted versions are available at no cost for research purposes (ResearchCyc, since 2006) and in general (OpenCyc, since 2002), see [@24]. [@24] comprises various links to Cyc-related publications, including [688] on extracting / producing a corresponding FOL version from Cyc.

[109]Cf. [211] for a comparison of Ontolingua with other systems available until 1999.

[110]which is further of relevance in connection with sect. 6.3 below

Acronym	Name	Origin	References
BFO	Basic Formal Ontology	2002 [@12]	[@12], [25, 302–306, 777, 823], [558, ch. 7–8]
DOLCE	Descriptive Ontology for Linguistic and Cognitive Engineering	2002 [259]	[@31] [100, 101, 259, 558–560]
GFO	General Formal Ontology	1999 [392]	[@46], [194–196, 385–387], [392, 396–398]
SUMO	Standard Upper Merged Ontology	2001 [626]	[@119], [626, 662, 663]
UFO	Unified Foundational Ontology	2004 [355]	[@125] [348, 354, 355, 358]
YAMATO	Yet Another More Advanced Top-level Ontology	1999 [@137]	[591]

Table 1.1: Examples of top-level ontologies (phase II) and those that are most closely related to this work.

Wand, and Weber (BWW) which originally refers to an application of Mario Bunge's work [126, 127] in ontology (in philosophy) in a series of publications by Yair Wand and Ron Weber wrt information systems and conceptual modeling, see [861] and its sect. 2, p. 497, for further references, dated between 1989 and 1997. Work on and with BWW has continued during the time of phase II and beyond, e.g. in [230–232] by Jörg Evermann et al., and some of the ontologies that we turn to now have paid regard to it.

EXAMPLES OF PHASE II ONTOLOGIES

The General Formal Ontology, which is to be introduced in the next section (1.1.5), is one of the ontologies originating early in that second phase. Attending the GFO development since 2001, we have been interested and studying other topical proposals since that time. Table 1.1[111] lists those ontologies together with sources that present the respective overall ontology or substantial parts / aspects of it. Their examination certainly had an impact on this thesis, directly or indirectly. There are further ontologies that can be numbered among phase II ontologies, where we only mention the ontology in [449] (2003) associated with Matthew West et al. and the ontology of the Process Specification Language (PSL) [@101] [728] (2000), standardized in [450] (2004). The PSL ontology is described by Michael Grüninger in [317], for instance. Moreover, we are aware of some works that cover surveys or comparisons of TLOs, namely [473, 557], [289, sect. 2.2], [398, ch. 19], or that present several of them [668, one each in ch. 8, 13, 14], [558, in ch. 2–3, 5, 7–8]. 📖 [398]

PHASE III: CONSOLIDATION IN AO AND NOVEL CORE ONTOLOGIES

Phase III in our consideration, starting around 2005 and continuing until today, has *not* resulted in widely visible novel TLOs within Formal / Applied Ontology, according to the best of our knowledge. Instead, existing systems, e.g. those in Table 1.1, have been gradually further developed and been applied, both to differing extent across the different systems. Regarding mutual interrelationships, a recent workshop [@134] on the Four-Category Ontology [539] by Edward Jonathan Lowe indicates that commonalities among the proposed systems might lead to shared theory components in the future. Moreover, broadening the scope to core ontologies, several TLOs have been used as a basic layer of newly proposed core ontologies. For instance, SUMO is accompanied by mid-level ontologies for a whole range of fields listed at [@119]. Moreover, a number of core ontologies have been proposed in the biomedical area [88], including BioTop [@16] [71, 736–738], the Simple Bio Upper Ontology [697], both of which are inspired by BFO and DOLCE, and GFO-Bio [@47] [415, 417, 418], [413, sect. 5.1] on the basis of GFO. GFO-Bio plays a rôle in sect. 5.4 of 📖 [417, 418] this thesis.

PHASE III: NEW TLOs IN THE SEMANTIC WEB

With 2004 marking the publication year of the OWL 1 standard and with the growth of the Semantic Web (SW) during the 2000s, the situation there is quite different regarding TLOs. Firstly, we observe that

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[111]Regarding the year of origin and sources of UFO, [@125] traces back the history of UFO further to 2001 and earlier publications such as [352, 353] that led to the development of UFO, yet without establishing a distinct name and acronym. In the case of GFO, work on the General Ontological Languange (GOL) preceding the introduction of 'GFO' is included, because the change was a mere renaming of the ontology part of the GOL project, cf. [392, p. 298] and sect. 1.1.5 below.

parts of the Semantic Web community appear rather skeptical about top-level ontologies. Nevertheless, the Semantic Web has not only generated numerous ontologies and (even more) OWL documents, but some of them are either intended as top-level ontologies or comprise corresponding categories, respectively. One example in this connection, labeled 'basic upper ontology' by its creators is the PROTo ONtology (PROTON) [817] developed in the context of the EU project Semantic Knowledge Technologies (SEKT) [@110], which comprises some very general categories, e.g., 'Entity', 'Abstract', 'Happening', and various categories that may also be suitable for a mid-level business ontology. Overall and as far as we are aware of SW TLOs, they are typically lightweight ontologies.

1.1.4.3 Reference Ontologies and Remarks on Further Kinds

NOTIONS OF REFERENCE ONTOLOGIES BY GUARINO, GUIZZARDI, AND IN FMA

In order to assign foundational ontologies a more purpose-driven label, let us first observe again that our notion of domain ontology merely refers to a medium or lower level of abstraction in the ontology. This characterization differs from what is called a domain ontology in [348]. Giancarlo Guizzardi uses 'domain ontology' to abbreviate the term 'domain reference ontology' [348, see p. 37], later characterizing a 'reference ontology' as "the best possible representation of a conceptualization, with the sole purpose of being a truthful representation of the domain in reality." [348, p. 46]. However, this is not meant to entail the existence of a *single* best ontology for a domain: "The idea is that a reference ontology should be constructed with the sole objective of making the best possible description of a domain in reality [wrt] a certain level of granularity and viewpoint." [348, see p. 37]. Moreover, a reference ontology should be constructed according to formal ontological principles, and "it should be application independent and not biased towards a specific mathematical model or formal theory." [ibid.]. Insofar there is an overlap with the "design criteria for ontologies" [313, sect. 3, p. 909–910] proposed by Thomas Gruber (which are clarity, coherence, extendibility, minimal encoding bias, and minimal ontological commitment), although Gruber advocates an opposing stance for ontologies in general, which are governed by purpose according to him.[112]

Guizzardi's notion is associated with Nicola Guarino's 'reference ontology' in [330, sect. 2.3]. Guarino similarly suggests a rich axiomatization based on a domain of entities and a conceptual apparatus that are adequate and sufficiently rich for the targeted domain of reality. The fact that this may lead to computational problems is less relevant due to the main purpose of such a reference ontology to foster *human* understanding and the establishment and explication of a conceptualization that is understood and shared by different partys involved. Notably, Guarino contrasts reference ontologies with another form of ontologies that may be geared towards computational efficiency for being usable in software systems, referred to as 'online' or 'sharable ontologies' in [330, sect. 2.3]. However, these actually suppose a shared conceptualization in order to work successfully. The Friend of a Friend (FOAF) vocabulary [@41] [120] may serve as an example from the Semantic Web.

A quite similar case for 'reference ontology', also referred to in [760, sect. 2.5, p. 25], is made by the developers of the Foundational Model of Anatomy (FMA) [@38],[113] which is proposed as a reference ontology in biomedical informatics [714, 715]. More precisely, "[. . .] the FMA is not intended to meet the needs of any particular user group or support any particular task, [. . .]. Rather, the FMA ontology is being developed as a reference ontology, intended to be reused in application ontologies designed to support any computational tool – with or without advanced inference capabilities – which calls for anatomical information." [715, p. 60].

FOUNDATIONAL ONTOLOGIES SHOULD BE REFERENCE ONTOLOGIES

The characterizations of reference ontology are certainly consistent in combination, and we adopt them jointly as the notion of reference ontology for this thesis. Moreover, we argue that foundational ontologies

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[112]"Formal ontologies are *designed*. [. . .] To guide and evaluate our designs, we need objective criteria that are founded on the purpose of the resulting artifact, rather than based on *a priori* notions of naturalness or Truth." [313, emphasis as in the source].

[113]Comparing 'Foundational' in the name of FMA with the notion of foundational ontology herein, note that FMA is a domain-specific ontology. The term 'foundational' is meant and justified differently for FMA, namely based on (1) the fundamental character of anatomy for "all biomedical domains" and the fact that (2) "the anatomical concepts and relationships encompassed by the FMA generalize to all these domains." [714, p. 480].

should be reference ontologies. This corresponds very well to the main purpose that we attribute to foundational ontologies: to serve as frames of reference, e.g. when establishing or integrating domain-specific ontologies, i.e., to account for a conceptualization of very general and broadly applicable notions as well as the possibility for humans to understand and thus share that conceptualization. Nevertheless, it remains desirable for us to implement/represent foundational ontologies in machine processable languages, possibly in different versions in different languages.

FURTHER NOTIONS IN THE LITERATURE

Concluding the overall section on types of ontologies, we confine ourselves to mentioning that various further distinctions can be drawn and reoccur in the literature, e.g., the one between descriptive and revisionary metaphysics / ontologies, making allowance for commonsensical conceptualizations vs aiming at strict(er) adherence to scientific and philosophical theory [348, sect. 3.3.3, p.91–92], [637, sect. 2.3.3.1]. To the best of our knowledge, we cannot point to any (close-to-)exhaustive systematic account of all those distinctions.

Let us very briefly characterize the term 'application ontology' that occurred in the FMA-related quotation two paragraphs above. In the present thesis that term is left with a rough, informal understanding as an ontology that is intended to capture ontological content for a particular (software) application. We see special cases based on the degree of the involvement of the application ontology in the actual application. By default, one may expect today that an application ontology is formalized in a machine-processable language and that it is a component of the software application and thus participates in driving the system; cf. also 'online ontology', 'ontology-driven', and 'ontology-aware information system' in [330, sect. 2.3 and 3.2.1]. While the notion of application ontology herein is still weakly based on a reading of 'application ontology' expounded in greater detail in [329, esp. sect. 3.2], where an application ontology is generated from (1) a domain-specific ontology and (2) a method ontology [ibid.] / task ontology [330, esp. sect. 2.4], [289, sect. 1.4.1], it seems to us that task ontologies (meant to capture notions for problem-solving / solving tasks) are rarely in use today, in contrast to former times, where "task analysis [was] strongly privileged" [329, sect. 1, p. 294] in the context of KBS.

Our final remark here concerns the term 'knowledge representation ontology' that is meant to "capture the representation primitives used to formalize knowledge under a given KR paradigm." [289, sect. 1.4.1], adopted from [841][114]. [841, p. 193] refers to [181] in the context of introducing that term, presumably because [181, esp. p. 19–21] discusses a general, unavoidable connection of knowledge representation (formalisms) and ontological commitments under the heading "Role 2: A Knowledge Representation Is a Set of Ontological Commitments". Moreover, the term and its notion seems to correspond to Guizzardi's meta-conceptualization [348, esp. sect. 2.3, p. 36–37; sect. 2.3.2 and 3.3.3]. The latter notes that this involves an ontological view on language constructs. A similar remark linking 'representation ontology' to 'meta-ontology' is made by Guarino in [329, p. 300]. We agree with these views and see ourselves a greater difference between a KR ontology and a top-level ontology than between a TLO and a domain-specific ontology, in line with our remark at the beginning of this section on distinct aspects in the classification by "subject of the conceptualization", as well as on the 2nd perspective that can be taken on the Frame Ontology [311, esp. sect. 3.6]. A corresponding analysis wrt ontology representation has appeared in [399] and is included in part and enhanced in sect. 2.3 below. [399]

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[114]Cf. also the commentary paper on [841] by Nicola Guarino [329].

1.1.5 General Formal Ontology (GFO)

1.1.5.1 Nature, Origin, and Sources on GFO

CHARACTERIZATION AND ORIGIN

The General Formal Ontology (GFO) [@46] is a top-level ontology intended to act as a reference ontology for basically arbitrary areas, certainly including applied ontology and conceptual modeling. Therefore, it exhibits a descriptive rather than revisionary attitude. GFO adopts a realist position called *integrative realism* [392, sect. 14.2.5], [395, sect. 6], which is itself being refined in the course of the development of GFO and in the light of topical related work, such as the debate on realist accounts in [579, 580, 762]. Moreover, GFO is embedded into a broader context targeting at an Integrated Framework for Development and Application of Ontologies [398, sect. 1]. This work was initiated in 1999 by Barbara Heller[†] and Heinrich Herre at the University of Leipzig, Germany, geared to develop a / the General Ontological Language (GOL), cf. e.g. [194]. Nowadays, led by Heinrich Herre, the integrated framework and especially GFO as its part are under constant development by members of the research group Ontologies in Medicine (Onto-Med) [@86] at the University of Leipzig, Germany, as well as associated contributors.

CONCEPTUAL ACCESS TO GFO

The most complete picture of GFO is available in verbalized form throughout various publications. The publications that encompass an overall outline and large fragments of GFO are those specified in Table 1.1.
[194–196, 385–387, 396] precede the latest comprehensive technical reports on GFO, [397] in published

[397, 398] 📖 form and [398] as the next updated draft version. The latest survey with amendments is [392]. Accordingly, our presentation of GFO content in this section is mainly based on these three sources, where we omit detailed references if statements rely on them. Beyond those three, there are many publications dedicated to particular top-level notions / issues, such as causality [582], functions [128, 131], mereology [394], roles[115]

116 📖 [526, 527, 530], space [59, 60], and time [61, 62].

AVAILABILITY IN DIFFERENT FORMATS

In the field of applied ontology, GFO must be made available in formalized versions. In this connection, a pluralistic approach is taken [392, p. 299], in that several axiomatizations may be developed and compared, even for the same set of basic categories and relations. Of course, formalizations should not remain human-readable only, in the way that they are distributed across those publications as just mentioned. Two OWL versions [@48] are available from the GFO website [@46], mainly comprising the taxonomy of GFO with few axioms (`gfo.owl`, corresponding to [397] from 2006), and similarly, but with a selection of central GFO categories only, in the version `gfo-basic.owl`, created in 2008. While most formalizations in GFO publications are presented in first-order logic (FOL), only the time theory $\mathcal{BT}^\mathcal{C}$ [61] is currently publically available in machine-readable format [@49]. Notably, much of this thesis is inspired / triggered by the problem of representing GFO in formal languages, see our motivations in sect. 1.2.

FUNDAMENTAL ASSUMPTIONS, 'ENTITY', AND META-ONTOLOGICAL ARCHITECTURE

Compared to other ontologies from Table 1.1 above, two outstanding features of GFO that are of major relevance below are (1) the inclusion of *categories* in its domain of entities, in contrast to focusing on individuals / particulars only, and (2) taking into account the theory of *levels of reality / ontological levels*, as discussed by Roberto Poli [664], [665, esp. sect. 4.1.2], related to earlier work by Nicolai Hartmann [374]. Poli distinguishes two kinds of levels (equipped with two kinds of relations that hold among levels of each kind), one of which is called *stratum*, the other one *layer*. To raise some intuition, according to [665, Fig. 1, p. 644], the 'material stratum' (put simply and reduced to entities, the domain of material entities) encompasses three layers, namely biology, chemistry and physics. Biology itself comprises sublayers corresponding to subdisciplines (in common sense parlance), e.g. genetics, cytology, and ecology. The notions of level, stratum, and layer are adopted for GFO, likewise assuming "that levels are characterized by integrated systems of categories" [392, p. 305]. At the present stage, four strata are considered. Besides

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[115]Since 'role' is frequently used as a technical term herein, we write 'rôle' if it is used in common natural language use, cf. e.g. FN 116.

[116]📖 We have contributed directly to [61, 62, 131, 527, 530] during the time of preparing this thesis. Roles play an important rôle herein and pervade the chapters. [62] forms sect. 6.3 below.

1. *categories*
2. *space regions* and *time regions*, incl. their mereology and respective boundaries
3. *material structures*, incl. their mereology and boundaries
4. *processes* and similar notions
5. *attributives*, incl. properties and property values as well as relations and relational roles
6. *propositions* and *situations*, incl. facts, configurations, and situoids

Table 1.2: Coarse-grained topics in GFO.

the material stratum there are the mental-psychological, the social, and the ideal stratum[117]. The latter is a more recent addition to account for certain abstract individuals, including mathematical entities such as numbers[118]. Moreover and (3), GFO accepts the idea of entities in different modes of existence, following Roman Ingarden [443]. Indeed, the term *entity* can be predicated of anything that exists, taking existence wrt arbitrary modes and thus in the broadest conceivable sense. (1-3) complement each other to some degree, e.g. notions that are associated with distinct levels of reality, as well as different kinds of categories may be related to different modes of existence. Overall, we argue that these fundamental assumptions are useful, e.g. in analyzing higher-order categories and the organization of categories within fields / domains, cf. also [418, sect. 2.2–2.3]. Another basic aspect of ontologies in general is their meta-ontological architecture, akin to meta-modeling in conceptual modeling. [399] is an earlier proposal in this regard to which we have contributed and which has been adopted for GFO. Its introduction herein is postponed until sect. 2.3, where it is slightly updated and embedded into related considerations. [418] [399]

1.1.5.2 Areas and Selected Categories and Relations

OVERVIEW OF GFO TOPICS

In this section we provide a brief summary of those categories and relations that are most important in the sequel: categories, relations and relational roles, as well as situations and related notions. For a more comprehensive understanding of GFO, the reader is asked to consult one of the three sources on which this summary is based, [392, 397, 398]. Nevertheless, we list the subareas / topics of GFO in Table 1.2, which allow us to at least roughly locate GFO notions thematically, associated with a few key notions. More precisely, we denote those topics by one or more of their central categories (in *italics*), and mention further aspects in that topic. In accordance with [392, 397], especially topic nr. 3 is labeled with a focus on the material stratum. Notably, time entities of topic nr. 3 are treated in much detail in sect. 6.3.

CATEGORIES, INDIVIDUALS, AND INSTANTIATION

Categories in GFO are "entities that are expressed by predicative terms of a formal or natural language that can be predicated of other entities." [392, p. 301]. The assumption is held that predicative terms are associated with certain conditions, such that the relation of *instantiation* is characterized by this requirement: an entity e instantiates a category c iff[119] e satisfies the conditions of the predicative term that is used for expressing c. However, in formalizations those predicative terms, conditions, and the notion / relation of satisfying those conditions are usually not made explicit. Accordingly, instantiation is treated as a basic relation in formalizations. Texts in the context of GFO typically use the symbol :: for instantiation, e.g. in FOL as a binary predicate with an instance-to-instantiated reading, i.e., $x :: y$ corresponds to "x instantiates y". We adopt the same convention for this thesis. Moreover, the *extensional is-a* relation among categories already mentioned in sect. 1.1.2.2 is denoted by the symbol $\twoheadrightarrow$ and defined such that $x \twoheadrightarrow y$:iff all instances of category x are instances of category y.

Categories are instantiable by nature, but an *empty category* is any category that cannot have any instances. 'Round square' yields a common example (if empty categories are accepted). Another kind of

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[117]The ideal stratum is not considered in [394, 397, 398], but it is mentioned in the more recent [60, sect. 2, p. 4].
[118]viewed as genuine entities, not equating them with any set-theoretic reconstruction
[119]From here on and following common conventions, *iff* abbreviates the phrase "if and only if" because this phrase occurs frequently.

entities that do not have instances are *individuals*. The notion of individual is (at least partially) character-ized by non-instantiability. Accordingly, category and individual are mutually exclusive. With the notion of individuals available, *simple* or *primitive* categories are exactly those that have only individuals as in-stances, whereas *higher-order* categories have categories among their instances. Finally, the existence of several kinds of categories, e.g. in the style of Jorge J. E. Gracia's work [295], is accepted for GFO. Cur-rently, 'category' is divided into 'concept', 'universal', and 'symbol', but we do not need to go into details of that.

RELATIONS, RELATORS, AND THEIR ROLES

Relations are conceptualized rather differently to what one may be used to from mathematics, possibly closer to an intuitive understanding of relations / associations in conceptual modeling. For that reason, let us introduce the term 'mathematical relation' to refer to any set of tuples (of fixed arity, usually) formed over any given set, before turning to the ontological notion of relation in GFO. First of all, relations are attributed a categorial character in that the same relation may apply to different groups of entities (then called *arguments* or *relata* of the relation, following common terminology). For examples, it may be the case that John knows Mary and that John knows Sue. For each such group (John and Mary, John and Sue), an individual entity is postulated, called a *relator* or *relation individual*, with the capacity to connect the entities in the group, thereby mediating a relation due to instantiating it. Hence *relations* are categories, namely categories of relators. Relators are specifically existentially dependent entities, i.e., they cannot exist without the entities that they connect. In the example setting, there is a relation 'knows' that is instantiated by two distinct relators, one of them mediating 'knows' between John and Mary, the other between John and Sue. Relators themselves are equipped with an internal structure by being composed of *roles*, more precisely *role individuals*, which may be seen as the "connecting ends" to the arguments and to correspond to the way in which the respective argument participates in the relation. The 'knows' relator between John and Mary thus consists of two individual roles, one "for" John, the other "for" Mary. The facts that John is the one who knows (sb.) and Mary is the one who is known (to sb.) is reflected by John's role individual being an instance of a *role category* that may be labelled 'the knower', whereas Mary's role individual instantiates 'the known', a complementary role category. All complementary role categories of a relation are required for the overall understanding of the relation. Technically speaking, 'the knower' and 'the known' must form a *role base* for 'knows', cf. [526, sect. 3.3.3, esp. Def. 3.2, p. 52]. Two basic relations are used to link a relation argument to an individual role that it *plays* and to link roles to the relator that they are a *role-of*. We stick to this terminology, while noting that the plays relation, more precisely *played-by* (plays in reverse reading direction), is subsumed by inherence, and role-of may be understood as a part-of relation, although some arguments cause hesitation, cf. [530, sect. 2.2.1]. Overall, the account of relations involving the notion of roles is connected with a more general theoretical account centering on the notion of 'role' (in a broader understanding) that we had developed until 2003, see [526], and that has been expanded [527, 530] while working on this thesis, see sect. 6.2.1. Accordingly, we note that 'role' as used herein denotes by default *relational roles* in the context of, e.g., [530], where further types are discussed, including *processual* and *social* roles.

[527, 530] 📖

FACTS, CONFIGURATIONS, AND SITUATIONS

A sentence such as "John knows Mary." from the examples above is easily taken to report a fact, cf. also [180], if it is assumed to be true. There is a close link between relators and facts, in that a *fact* in GFO is composed of a relator and its arguments by a relation called *constituent part*. Facts are understood as entities sui generis that are parts of the world. They need not only have individuals as constituent parts, e.g., "John instantiates the category human." may express another fact one of whose constituents is the category human. Depending on the involvement of individuals, *individual facts* must have at least one individual among the relata, otherwise a fact is called *abstract*.

Via relations, entities aggregate into larger parts/"chunks" of the world, where facts are just the simplest form. Several facts aggregated into a whole by the constituent part relation form a *configuration* in GFO. *Situation* is a category that specializes configuration in that situations must be compehensible as a whole and must therefore satisfy additional conditions, e.g. of unity. In order to account for this, situations are as-sociated with categories (including relations), that determine certain types of constituents within a situation.

For example, to catch a situation within an office room, categories like desk, chair, computer, indoor plant, and a relation like stands-on may be associated with the situation, thereby providing information about its perspective and granularity. As exposed so far, no distinction between situations at a fixed time point and temporally extended situations has been made. While this is available in [392, 397, 398], where temporally extended situations are named 'situoid', we shall continue without the distinction and thus use 'situation' in a generalized reading.

1.1.5.3 Ontological Engineering of GFO and the Axiomatic Method

PLURALISTIC BACKGROUND AND TARGETTING COHERENCE AND COVERAGE

From the beginning, components of GFO and GFO as a whole are built with a pluralistic attitude insofar that there may be several alternative conceptualizations and alternative axiomatizations for the same domain of reality, the latter even if the same set of categories and relations is considered. Therefore GFO is considered within an evolving Integrated System of Foundational Ontologies (ISFO) [392, esp. sect. 14.1], similar in spirit to the idea of a library of TLOs in [558, sect. 1.2]. Remembering that GFO is intended to serve as a reference ontology, the main focus in its development is to ensure a coherent / consistent view within its theory. This refers to the informal, verbalized version as well as to formalizations of the former. The second major aspect is coverage, in the sense that arbitrary phenomena should be analyzable by means of the conceptual inventory that GFO offers.

ONTOLOGICAL ENGINEERING METHODOLOGIES AND THEIR ADOPTION

In connection with the rising interest in ontologies, several development methods and engineering methodologies have been reported and proposed, see [792] (2006), [289, ch. 3] (2004), and [590] (2004) for surveys. Some approaches or elements of them have received wider attention, e.g., competency questions as proposed by Michael Grüninger and Mark S. Fox in [318], e.g. mentioned in [621, sect. 4, p. 184]. Moreover, several of the methodologies, e.g. METHONTOLOGY [289, sect. 3.3.5], are based on analogies to software engineering [289, p. 125], [199].[120] Nevertheless, broad adoption remains limited and we agree with the authors of [5] (2012): "Despite various attempts to create methodologies for developing ontologies, practice shows that most research groups create their own method of development, according to their application characteristics." Indeed, this appears to be in good harmony with the authors and supporters of the Communiqué of the Ontology Summit [@85] 2013: "Currently, there is no agreed on methodology for development of ontologies, [...]" [621, p. 179], [@84].

METHODOLOGY INTRODUCTION FOR GFO

At a coarse-grained level the development of GFO follows five core development steps expounded in [194],[121] namely identification, conceptualization, formalization, implementation, and test. After a problem, notion, or wider (sub) domain of interest is identified for inclusion[122] into GFO, the starting point of covering a specified (sub) area of interest to GFO is to conduct an *ontological analysis*, initially informally. 'Ontological analysis' refers to the process of developing a conceptualization, broadly speaking. Besides the actual problem area, it is further based on results established in other fields, and / or it draws inspiration from other fields, including philosophy, cognitive science, and conceptual modeling. We elaborate on ontological analysis and related notions further in sect. 2.3.1. The resulting initial theory of the area is usually expressed via natural language text first, e.g. in reports and publications on GFO, cf. sect. 1.1.5.1.

AXIOMATIC METHOD FOR FORMALIZATION

In order to furnish a formalized ontology on the basis of those verbalized versions in texts, the axiomatic method is adopted, cf. [392, sect. 14.2.3], [397, sect. 2.2], originating from mathematical logic. David Hilbert formulates basic ideas in [401] (1918), and Alfred Tarski discusses it in [812, ch. VI], originally in the corresponding first edition in German in 1937. Georg Kreisel and Jean-Louis Krivine discuss applications in mathematics [486, App. I]. In [790] Patrick Suppes summarizes applications of the axiomatic method and model theory in the empirical sciences.

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[120]See [791] for a methodological approach in the context of knowledge management.

[121][289, sect. 3.1, incl. Fig. 3.2] is very similar with specification, conceptualization, formalization, and implementation as subtasks of ontology development, and evaluation as supporting activity

[122]referring to the present stage at which a version of GFO is already present

In order to develop a theory of a domain of knowledge / reality according to this method, a set of categories (and relations) is stipulated as *primitive* or *basic*. All other notions of the domain are to be introduced by *definitions* that use only previously established notions, i.e., without circularities, thus eventually being based on the set of primitive notions. The latter are not defined by explicit definitions, avoiding circularities / an infinite regress of defining. Instead, a set of sentences is stipulated for them "whose truth appears to us evident" [812, p. 110] in the domain under consideration. Such sentences are called *axioms*. The meaning of all primitive notions is constituted at once / in parallel by the set of axioms adopted for a particular theory.

Four major problems must be considered in this regard, if the conceptualization itself is included (quoted from [60, sect. 2], [62, sect. 3]):

[62]

1. What are adequate / appropriate concepts and relations of a domain?   (*conceptualization problem*)
2. How may we find axioms?   (*axiomatization problem*)
3. How can we know that our axioms are true in the considered domain?   (*truth problem*)
4. How can we prove that our theory is consistent?   (*consistency problem*)

The justification of axioms, which is desirable despite postulating their evidence (according to the method), seems to be the most difficult methodological problem. One approach for empirical theories is to find supporting experimental data. The consistency problem is a focal point in the meta-theoretic analysis of the resulting formalized ontology, besides other properties. Two further interesting cases, also discussed in [812, sect. 39–40], are the completeness of the theory and the minimality of the axiom set, which is present if no axiom follows from any other axioms in the overall axiomatic system.

1.1.6 Background Essentials

SUMMARY

The field Applied Ontolog (AO) / Formal Ontology in Information Systems surrounds the notion of ontology. In this section we outlined its development and established our terminology for 'ontology' itself and for constituents of ontologies, such as categories and relations. A survey of languages that are relevant in connection with ontologies then covered Semantic Web and logical languages as well as general languages in knowledge representation and conceptual modeling. A few ways of classifying ontologies and resulting types of ontologies were considered, leading a.o. to the notions of top-level ontology and reference ontology. An instance of both kinds whose development and application forms the overarching background of our work is the General Formal Ontology (GFO) [@46]. Its brief introduction focused on those areas, categories, and relations that are of relevance below, and presented the axiomatic method, which is applied in its development.

TWO MAIN LINES OF OBSERVATION

During the last 25 years ontologies have been gaining increasing interest since entering computer and information sciences at a similar time[123] in several subdisciplines, for example, artificial intelligence/knowledge-based systems as well as conceptual modeling. They are successfully applied in (at least) two major ways. Firstly, ontologies are employed in describing the conceptual foundations of a variety of systems, where it is good practice to use a semi-formal or formal representation language in order to express those ontologies. The immediate purpose of those ontologies is mainly the explication of knowledge and the support of human communication. On that basis, other goals/tasks can be pursued. Data and systems integration is at the forefront of examples in this regard. Secondly, ontologies represented in machine-readable languages are utilized as components of software systems, for example, based on semantic technologies. Notably, many of those languages used in such a way are equipped with formal semantics. Ontologies as technological components of systems are beneficial wrt diverse purposes. The latter include the facilitation of search and navigation, querying, reasoning / computing implicit information, all of which are applicable in various domains. Moreover, ontologies were proved useful in domain-specific applications, with the shining example of genome annotation with ontology terms / categories in the biomedical domain.

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[123]ignoring sporadic earlier occurrences here, but see sect. 1.1.1

Finally, let us highlight the general connections between ontologies and languages. There is a myriad of representation languages, among them many cases of formal languages which exhibit interrelations concerning expressiveness and problem reductions according to their formal semantics. Languages relate to ontologies in at least two important ways. Firstly, a large fraction of representation languages has been considered and used for ontology representation. Secondly, any kind of representation is subject to reformulation in another language, and we argue that any such reformulation that aims at maintaining the conceptual content of the source representation involves, at least implicitly, some kind of ontological analysis. It is therefore amenable / potentially subject to ontology-based translation or integration, i.e. / thereby making use of explicit ontologies. While this line of thoughts transitions quickly to major theses that we wish to defend in this work, stated in sect. 1.3, further motivating aspects shall be presented prior to this.

1.2 Motivations

FROM THE USE CASE OF GFO TO FOUNDATIONAL ISSUES IN ONTOLOGY REPRESENTATION

Practical goals behind our work are intimately tied to the development of the General Formal Ontology (GFO) [@46] in general, and they include the formalization of its components and of extensions, for diverse communities and in distinct artificial, typically formal languages, in particular. Accordingly, the provision of ontological theories and their application motivates parts of this thesis. However, pursuing those goals led to recurring underlying issues that we expound in this section. These issues generalize to foundational, if not arbitrary ontologies and they form the basis for a more generic target for a major part of this thesis itself, beyond the use case of GFO. We wish to provide or at least contribute to a solid foundation for the representation of ontologies, which we do not yet see to have been established conclusively. Rephrased from the point of view of languages, we study the use of languages in capturing / representing conceptualizations and how that relates to core purposes of ontologies.

INTEREST IN LOGICAL FORMS OF REPRESENTATION

The primary scope wrt languages are the logical languages outlined in sect. 1.1.3 – FOL/HOL and description logics (DLs), moving on to rule-based languages, but likewise having an eye on conceptual modeling languages like UML [@126]. At least for foundational ontologies intended to act as reference ontologies, a declarative, logical representation is quasi obligatory in line with the current state of the art. Moreover, logic is desirable for well-known reasons. Reasoning over a logically formalized ontology is then well-defined and can unveil implicit consequences of importance, either by detecting unintended consequences and thus the need for theory revision, or by ensuring desirable conclusions or gaining new insights. Both of the latter can be recorded with proofs for justification. Furthermore, meta-logical, more precisely, meta-theoretical properties play an important role in the adoption of ontologies, maybe even a crucial role for foundational ontologies. This includes first and foremost proving the consistency of an ontological theory. Meta-theoretic analysis may also lead to more in-depth and new knowledge wrt a theory. Notably, we adhere to the axiomatic method [812, ch. VI], introduced at the end of sect. 1.1.5.3, for any language where this is feasible (and up to verbalized ontologies, to some extent). Eventually, using logical languages allows for relying and building on established results in logic and knowledge representation.

1.2.1 Language Variety and Resulting Problems

ASPECTS FOR CHOOSING A LANGUAGE AND LINK TO AUTOMATED REASONING

Proceeding straightforwardly with the goal of ontology representation, one faces the problem of choosing one or several languages. Besides the logical ends just stated, let us take a general-purpose perspective for providing a (foundational) ontology. Such language choice would need to consider the trade-offs between several aspects of the languages themselves:

- their degree of expressiveness
- decidability and complexity of the required reasoning tasks
- existence of theoretical analysis and results
- availability of implemented reasoning support

Work in automated reasoning can be utilized wrt meta-logical analyses of logically formalized ontologies. Consistency checking of DL reasoners is a prime example in this regard. The term *automated reasoner* is meant to refer to arbitrary software that implements deduction algorithms based on any logical calculus. In particular, besides DL reasoners this includes theorem provers. The latter, which are also referred to as automated theorem proving (ATP) systems, usually target classical predicate languages of first- or higher-order expressiveness. It is highly desirable to employ existing reasoners / theorem provers because their establishment requires very substantial theoretical and implementational work, not at least state-of-the-art optimizations.

MULTIPLICATION BY INPUT FORMATS OF A VARIETY OF REASONERS

Similarly to the landscape of representation languages, there is a large variety of automated reasoners. Several of these can be considered (more or less) ready-to-use, which includes DL reasoners, e.g. Fact++ [@35] [824], HermiT [@53] [604], Pellet [@94] [756], Racer [@104] [363][124], and theorem provers[125] like OTTER [@90] [571][126] / Prover9 [@100][127], SPASS [@116] [866], and Vampire [@128] [483], as well as many others, cf. e.g. [477, Appendix C] (related to the development of GFO) or [@11]. Hence, the problem of multiple logical languages repeats itself / multiplies to some extent with the combined or parallel use of multiple reasoners[128], primarily in terms of diverse kinds of input syntax. For instance, [477, sect. 2.4] comprises a comparative overview of six FOL syntax formats considered in connection with GFO development, including those of KIF (cf. sect. 1.1.3.1), OTTER, SPASS, and a format designed for theorem proving competitions and its problem library "Thousands of Problems for Theorem Provers" (TPTP) [@124] [793]. This multiplication likewise affects the aim of providing a theory in a form suitable for use with several different reasoners.

APPROACHES TO COPE WITH LANGUAGE VARIETY: CONVERTERS AND APIS

Of course, the problem of language heterogeneity is well-known and tackled in several ways (indeed, with progress in available proposals, also regarding their ease of use). Starting at the practical level, there are several approaches, e.g., providing conversion tools, standardizing languages, and standardizing application programming interfaces (APIs). *Converters* probably offer the quickest solution to a particular translation problem, and a number of conversion tools is available from reasoner websites or is bundled with a reasoner, e.g. in the distribution of the SPASS theorem prover. Nevertheless, converters usually work uni-directionally between two syntaxes and do not allow for "ideal roundtrip-conversion" that is supposed to yield the exact same result after converting syntax s into syntax s' and the result of that back into syntax s. Converters may also be designed to translate only a specific class of theories rather than being a fully capable translator among two languages, especially if there are significant differences among the languages, e.g. thinking of FOL and DL. APIs like the former DIG[129] [@28] [825] and its successor OWLlink [@92] [524], merely support the programmatic access to reasoners (or servers) that support that API. However, they are not under consideration in connection with determining format(s) in which to publish ontologies.

STANDARDS HELP IN A LIMITED FORM, NEED TO HANDLE MULTIPLE LANGUAGES REMAINS

Concerning the latter task, *standardized languages* like RDF, OWL, or Common Logic (CL), remember sect. 1.1.3.1, are clearly much more suited, and they reduce the number of inter-system translations. RDF and OWL are certainly two successful standards, considering their adoption in practice and the attention that they have received in research. The history of CL and the preceding and related efforts on KIF in the mid 1990s is different, and only now CL adoption appears to be gradually increasing. In general, standardized languages require quite some time to develop and run the risk (1) of not being supported due to various

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[124]'Racer' is declared at [@104] as the name of the successor of RacerPro that is described in [363], which itself followed a system called 'Racer' in the early 2000s, cf. e.g. [364].

[125]Cf. e.g. [242] for an introduction to logic and theorem proving.

[126]OTTER is an acronym for "Organized Techniques for Theorem-proving and Effective Research" [571, p. 1].

[127]Prover9 is the successor of OTTER. We are not aware of a descriptive publication, while a link to a manual is included under [@100].

[128]The use of several reasoners is desirable for ontology developers as users of such provers due to the distinct strengths and weaknesses of reasoners. In the case of FOL, a logic with, e.g., undecidable entailment problem, ATP systems can vary significantly in performance wrt specific problems.

[129]DIG was also referred to as DIG interface. DIG stands for DL Implementation Group, which provided a standardized XML interface to DLs.

reasons or (2) of being adapted to a special purpose. The TPTP format might be taken as such a case, with its readability for humans in mind.[130] Furthermore, language standardization must remain limited, since it appears impossible that a global representation standard will emerge that everyone can and would adopt, e.g. for ontology representation. There is a point in all the various notations and formats that exist in the case of RDF and OWL alone, cf. e.g. [856, Fig. 1], based on their fitness for distinct purposes. Hence, the general problem of translating between languages remains, despite language standardization in certain communities.

FROM HETS TO ONTOHUB: TRANSLATIONS BETWEEN LOGICAL LANGUAGES

Acknowledging exactly this problem, cf. [499], a very remarkable related effort has been pursued and led by Till Mossakowski for many years: the establishment of the theory of heterogeneous specifications and of the Heterogeneous Toolset Hets [@54] [594, 595]. Originating from the context of formal software specification and verification, its core idea is that specifications need not be written within a single (logic-based) language, but they may consist of several, integrated components possibly written in different languages / logics. In order to support this goal, Hets can parse corresponding heterogeneous specifications and allows for reasoning over them by means of connections to various automated reasoners, including Fact++, Pellet, SPASS, and Vampire, cf. above. The corresponding logics are interrelated by precise mappings based on the formal semantics of the included logics[131], cf. [597, incl. Fig. 1, p. 105], [598, incl. Fig. 5, p. 9]. Taking the step from "specifications" to "ontologies", the theory of heterogeneous specifications and its tools have transitioned into AO through a number of recent and very recent interlinked works / proposals, initiatives, and software systems. These include the idea of *heterogeneous ontologies*, i.e., very loosely speaking, ontologies consisting of components specified in (possibly) distinct logical formalisms. They were first proposed in 2008,[132] a.o. in [496], and have been promoted / expedited in a series of publications, including [499, 597, 599]. On this basis the proposal of the Distributed Ontology Language (DOL) [@30] [599] is made and prepared as a response to a request for proposals for standardizing "Ontology, Model and Specification Integration and Interoperability" (OntoIOp) [@82] [649]. Eventually, much of this converges into a "semantic repository for heterogeneous ontologies" named Ontohub [@81] [598], which was established in 2013. The availability of this resource clearly means tremendously more support in dealing with ontologies in different languages than just a decade ago.

SEMANTIC EQUIVALENCE IN TRANSLATIONS

A central aspect in connection with such translations is to arrange for maintaining the semantics from the source to the target language. That means the result of a translation should be *semantically equivalent* to the original source, i.e., "the same content" is expressed in the target language after a translation. At least, this is clearly an immediate desideratum for the practical task of providing foundational ontologies in different languages. Moreover, it links to the theoretical level of dealing with language heterogeneity. In particular, translations like in Hets / Ontohub take the formal semantics into account and pay attention to preserve that. "Of course", one can say, with the justification that the formal semantics of a logical language is *the* semantics of that language. We would even accept that this is incontrovertible wrt the transfer of reasoning results and logical problems between those languages.

However, we see room for doubts regarding the "conceptual content" that is *formalized* by means of a logical language. If one accepts that there is conceptual content prior to the process of formalization, this leads to questions on the relationship between that content, the syntax of a formal language, and its formal semantics. Another question is, in which way(s) conceptual content is captured by means of a logic. Corresponding answers would surely affect the view on "preserving the semantics" during translations.

We find further support for the stated hesitation in the motivation for introducing ontologies in computer and information sciences. In AI, the latter step was related to the problem of sharing and reuse of knowledge between knowledge bases, a.o. taking into account "model mismatches at the Knowledge Level" [618,

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[130]Admittedly, this is largely a matter of subjective impressions / feelings.

[131]Meanwhile, among the languages included in "The Onto-Logical Translation Graph", as [597] is entitled, there are many of the languages surveyed as ontology-related languages in sect. 1.1.3, notably FOL, HOL, and CL, as well as RDF, RDFS, the main profiles EL, QL, and RL of OWL 2, and the OBO format; more recently (parts of) UML class diagrams and F-Logic are in preparation [598, esp. sect. 1].

[132]Based on prior work associated with these issues, including [541, 595].

p. 38], cf. also [312, 330], to be cured by making ontological commitments "behind" language elements explicit. Put differently, ontologies are meant to enhance interoperability, e.g., of knowledge and software systems, by acting as frames of reference, according to which system / language constituents can be understood. This may suggest another notion of "semantic equivalence", roughly, by being associated with equal ontology constituents.

MULTIPLE ONTOLOGIES AND ONTOLOGY MATCHING

At this point, it is interesting to note where the success of ontologies as a field seems to hamper the value of individual ontologies. If multiple ontologies are available for a domain of knowledge / reality, the problems formerly at the "language level" seem to reappear for ontologies. Michael Johnson and Robert Rosebrugh thus notice: "The irony of ontology engineering needing to focus on constructing individual ontologies [...] is of course that ontologies themselves were introduced to aid system interoperability." [668, sect. 24.1, p. 565]. Indeed, the increasing adoption of building / using ontologies (in a methodic sense) has produced ontologies which either target the same domain of knowledge or which have at least some overlap in the same regard. As a consequence, *ontology matching*[133] was identified as a relevant task and initiated numerous research activities. With the appearance of dedicated workshops and survey articles [463, 632, 687, 751][134] in the early 2000s and the first book [227] in 2007, now it constitutes a field of research of its own. In line with the quote above, the very first paragraph of the introduction of [227] observes: "Thus, merely using ontologies, [...], does not reduce heterogeneity: it raises heterogeneity problems to a higher level." [227, p. 1]. Focusing on logic-based approaches for ontology matching, several sophisticated proposals are available in the literature, incl. [498], relating to Hets / Ontohub, and work on an information- or channeltheoretic basis, e.g. [472, 734]. But similarly to the case of translations above, much attention seems to paid to the formal semantics of the (formalized) ontologies, linking them as logical theories. While it appears very useful to us if this can be done, nevertheless and in contrast, we see the relationship to conceptual content very rarely addressed.

From the discussion in this section we raise the subsequent questions $\mathcal{Q}_1$ and $\mathcal{Q}_2$ for further treatment in the sequel, e.g., in deriving objectives for this thesis in sect. 1.3.2.

$\mathcal{Q}_1$ What are the exact relationships between ontology, the semantics of a language and semantic translations?

$\mathcal{Q}_2$ Is the current understanding of ontologies and semantic translations sufficient to justify claims that different expressions state "the same content" or which enhancements would be useful?

1.2.2 Reference Case: FOL-based Representation of Ontologies

CHOOSING FOL FOR REFERENCE & RATIONALE I: REASONING SUPPORT

Even in the presence of language variety and the foundational issues that may require further investigation, it appears useful to choose a language for reference. 'Reference' is here to be understood in terms of a sample case for analysis and application, as well as a point of reference wrt relations to other languages. Our choice for such reference language is first-order logic (FOL).

There are various reasons for our primary interest in FOL. First of all, although FOL is undecidable for several relevant problems like checking consistency or entailment among theories, at least it is recursively enumerable and amenable to reasoning algorithms. Because establishing and analyzing large theories will profit from computer-based support, it is also important that according theorem provers are available. For the latter and as seen in the previous sect. 1.2.1, there is a fair number of options for FOL, in contrast to many other, more specialized logical formalisms. This counterargument does no longer apply to work on description logics, especially since its boost in connection with the Semantic Web (SW), whose selection of reasoners is competitive. If decidable description logics are considered, corresponding reasoners even exceed FOL theorem provers wrt principle capabilities such as completeness of reasoning (in contrast to

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[133]On variations and more fine-grained distinctions like *ontology translation*, *ontology transformation*, see [227, sect. 2.4].

[134]We include [687] among the relevant works due to the close relationship between ontology and schema matching, which is also stated in that article. Notably, work on schema matching precedes ontology matching to some extent, starting in the early 1980s [687, sect. 2.1, p. 335].

refutation completeness of typical FOL provers) and consistency checking. However, the close interrelations between most DLs and FOL may be used to compile DL theories from FOL theories, cf. e.g. [540, sect. 3.3]. Another aspect is of more technological flavor, namely the speed of reasoning. Usually, DL reasoners are expected to be faster on DL theories than a FOL theorem prover would be on (naïve) conversions. However, some authors claim that FOL theorem provers are / can be competitive with DL reasoners, cf. [436]. On the other hand, it remains questionable whether present-day reasoning performance is not overall still in a problematic state, because the size of some theories / formalized ontologies is not or only hardly tractable for basically all existing DL reasoners. As a real-world benchmark, DL reasoner developers frequently adopt large medical terminologies (in DL versions) like SNOMED [@115] [167, 204, 442, 675] and GALEN [@88] [696, 699, 711] with tens or hundreds of thousands of concept descriptions, e.g. see [36] for two reasoners. Matters become even more difficult if ABoxes are involved.[135] This causes reasoning with expressive OWL ontologies to be used rather cautiously in SW applications, e.g. in semantic wikis[136] [124, 414, 420, 491]. Focusing on the development and application of top-level ontologies, at least their [414] development involves a large fraction of manual effort such that reasoning performance is no major concern and longer times of reasoning can be tolerated.

RATIONALE II: THEORETICAL ROLE OF FOL & RATIONALE III: EXPRESSIVENESS

There are a number of further aspects which attribute FOL a special status in the logical realm (of course, besides or after propositional logic, which we elide here due to its different "representation granularity").[137] FOL is – if not *the*, then among – the best-researched logics, such that many theoretical results are available. Many other declarative formalisms establish relations to FOL, which is valuable if translations to such languages are required (possibly modulo the issues of semantic translation mentioned above). In addition, FOL is widely perceived as a logic of central relevance and acceptance, e.g., in scientific terms / philosophy of science. With respect to more formal relatives, it is more widely accepted than specialized formalisms proposed in knowledge representation, frequently addressing some insuffiency of FOL. This does not derogate the value of specialized formalisms, but we would argue that FOL is a prime candidate to be chosen if results gained by various methods are integrated in order to develop a theory. Regarding even more formal properties, FOL also takes a special position among model-theoretic logics (viewed as abstract logics, the subject of abstract model theory [55]), e.g. as indicated by Lindström's theorem, characterizing FOL as a maximal logic satisfying the Compactness Theorem, e.g. [147, Theorem 1.3.22, p. 33], and the Downward Löwenheim-Skolem Theorem, e.g. [147, Corollary 2.1.4, p. 66].

A usually obvious justification for FOL in comparison with DLs which has not yet been mentioned is the antagonist of reasoning performance, namely expressiveness. Indeed, expressiveness is of vital importance for representing foundational ontologies, hence this is a tempting argument. However, it remains superficial unless the notion of expressiveness is clarified. While this can be considered from a purely formal perspective, we argue that the notion of expressiveness is intimately tied to the previous question of the relationship between ontology and language semantics. Some issues are listed in the next section.

1.2.3 Expressiveness and (Meta-)Logical Issues

1.2.3.1 Universe of Discourse, Metamodeling, and Intensions

UNIVERSAL DOMAIN PROBLEM VS. LIMITED DOMAIN ASSUMPTION

Especially top-level ontologies face a problem that could be called "the problem of the universal domain": intended to provide general categories which are applicable in / foundational for a very broad range of domains, the domain of entities of a top-level ontology is potentially unlimited, in which case it should cover everything that exists, in the broadest sense of that phrase, remembering the GFO position on existence mentioned at the end of sect. 1.1.5.1. First of all, this is contrary to the usual assumption of employing

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[135]The size of DL TBoxes is usually rather small compared to the sizes of ABoxes that occur / should be supported in the Semantic Web, cf. also the connection to performance benchmarks of SW knowledge base systems, e.g. [361] (an early proponent).

[136]See [708, 851] for initial work on semantic wikis, and [521] for wiki technology in general.

[137]Cf. also the general acknowledgement of the dominance of FOL in symbolic logic given in [755, p. 254], including "relative simplicity", "nice logical behavior", but also "energetic lobbying of great logicians for [FOL]".

FOL, for example, where the (intended) universe of reference is assumed to be given by the objects of one or several types / kinds. The theory of Peano arithmetic[138] is stated presuming that the (intended) universe is the set of natural numbers, for instance. Extending the (intended) domain of discourse of a theory to the universal domain[139] is problematic insofar as it should contain literally everything. Accordingly, it should be possible to refer to individuals like Leo, to categories like lion, as well as to higher-order categories[140] of which species is still among the prime examples. Likewise, relations may be subject to discourse, possibly with features (seemingly) contrary to FOL like variable arities, *anadic relations*.[141]

LINK WITH METAMODELING

Obviously, this opens the door to the subject of *metamodeling*, mainly discussed in this terminology in conceptual modeling and (model-driven) software engineering. We refrain from providing an in-depth characterization of metamodeling and only refer generally to [28, 290, 492]. For our purposes, it must suffice to distinguish two types of metamodeling. The first applies to the examples above and considers higher-order categories[142] (wrt certain entities) as well as the *reification* (of categories as objects) in knowledge representation [109, sect. 3.7]. This first view is summarized in [600] by "[m]odeling with metaconcepts is called metamodeling, [. . .]" [600, p. 618]. Boris Motik also mentions the second kind of metamodeling that we have in mind,[143] which is concerned with the relationship between (object) language(s) and meta-language(s), cf. also [348, sect. 2.3.2]. A metamodel (a language definition / an expression of a metametalanguage / an "instance" of a metametamodel) "defines the language for expressing other models" [720, p. 459].[144] That second reading of metamodeling appears to be more frequently adopted, especially in model-driven software development / engineering. Both types deliver issues relevant in the present context.

SETS IN THE DOMAIN OF DISCOURSE AND THE RELATION OF ABOUTNESS

The standard use of FOL assumes a direct tie of predicates (as syntactic constructs given by the formal language definition) with sets / (mathematical) classes as their semantic counterpart. Moreover and without adopting this position, the link between predicates and sets is often equated with a tie between predicates and categories, identifying instantiation with set / class membership. In particular, Christopher A. Welty and David A. Ferucci argue [869, esp. sect. 2.1], by referring to Rudolf Carnap [140] and Willard van Orman Quine [683], that these ties should not be broken while using FOL and should not be "circumvented" by reification with predicates à la $instance\text{-}of(x, y)$. Independently of accepting or rejecting the identification of the two relations, the problem that arises in formalizing a top-level ontology (when aiming for the universal domain) is to cover categories as well as sets in the universe of discourse. The position of Welty and Ferucci leads immediately to a need for higher-order logic. Even this may not suffice, if self-instantiation needs to be considered / stated. On the other hand, the most commonly accepted axiomatizations of set theory, in particular Zermelo-Fraenkel's (ZF/ZFC)[145], are stated in FOL (distinguishing between fore- and background set theory[146] wrt model-theoretic interpretations of those axiomatizations), cf. e.g. [212]. The adequacy of either position motivates a closer inspection of predicate logic and its set-theoretic semantics in ch. 3 herein.

A fairly related issue under the umbrella of "expressiveness" (in an informal reading[147]) arises in providing top-level foundations for information / library sciences and knowledge management. In particular,

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[138]See sect. A.2.2 in the appendix for a recapitulation and references.

[139]Note that referring to "the universal domain" implies the ontological assumption that there is an entity that "contains" everything. We discuss this further in the section on our proposal of an ontology of categories and relations, sect. 2.4.

[140]introduced in sect. 1.1.5.2

[141]For the latter, valuable historical and terminological remarks can be found in [889, endnote 17, p. 110]. NB: In [297] and [298, ch. XIV], an 'anadic logic' is proposed in order to improve the formalizability of the English language.

[142]Higher-order categories may also be termed *metacategories* and they relate immediately to the UML notion of metaclasses [720, p. 458].

[143]Cf. also Kühne's distinction of ontological and linguistic classification [492, esp. sect. 3].

[144]Note the association, but non-synonymy with object language and metalanguage in the logical context (in the sense of Tarski), see below.

[145]ZFC for 'Zermelo-Fraenkel with Axiom of Choice'. These are recapitulated in sect. A.2.1 in the appendix, see there for references.

[146]See below, in the context of Tarski's distinction of object language and metalanguage.

[147]There are established precise accounts of expressiveness, e.g. as defined in abstract model theory on the basis of model structures [214, Definition 1.2, p. 278] or the notion of information capacity of relational database schemata in [438]. However, they presuppose their formal foundation, which would interact with the issues just discussed.

we think of relations which link content-bearing entities with their topics, required for statements like "this book is about biology".[148] An ontological analysis of notions such as 'topic' and 'domain' may easily lead to categories, e.g., cf. the utilization of the theory of levels of reality (sect. 1.1.5.1) in the biological core ontology GFO-Bio [418], where levels (as broad kinds of domains) are formalized as subtypes of 'category'. Thus a corresponding foundation for content-bearing entities would have to allow for categories in its domain of entities.

📖 [418]

INTENSION VS. EXTENSION

Above we mention the identification of instantiation and set membership, which goes together with equating categories and sets / (mathematical) classes. Such identification or, contrariwise, their distinction as well as respective effects are important in a number of fields, including theories of concepts in philosophy of mind [508], accounts of intensional logics in logic [243, 834], as well as semantics and semiotics in general. Labeled differently, the matter is a frequently met separation between *intension* and *extension*.[149] In views which draw the distinction, categories are usually equipped with intension and they are seen as intensional entities.[150] In contrast, sets are purely extensional entities. They obey the Principle of Extensionality, i.e., two sets are strictly identical iff they have the same members. Moreover, especially axiomatic set theories postulate the existence of sets which provably lack any description / intension of a user of a language.[151] Widely discussed examples supporting the need for intensional entities include referring to the planet Venus as 'Venus', 'morning star', and 'evening star'; and the coextensionality of the categories of equilateral triangles and equiangular triangles, see [243, sect. 1], for example.

In general/commonly, FOL is understood to be extensional: "In classical first-order logic intension plays no role. It is extensional by design since primarily it evolved to model reasoning needed in mathematics." [243, sect. 1]. This is a usual justification for the need for intensional logic[152]. However, we see limitations of particular systems of intensional logics compared to FOL, less concerning their theoretical analysis which is well-developed for some logics, but certainly in terms of implemented reasoning systems. Given the – albeit practical – advantage of available theorem provers for FOL, it appears worthwhile to reconsider the appropriateness of the judgement about FOL. As a result, we should better be able to weight the theoretical capabilities / expressiveness of intensional logics in comparison to FOL against the need and availability of theorem proving software.

Another aspect relating to the distinction of intension and extension and of particular relevance for foundational ontologies is the relationship between general and specific categories. If we assume that categories are formalized as predicates, categories inherit the "undirected mutual influence" that predicates within the same theory exert on each other. To indicate our concerns by a simple example, the use of a top-level ontology for the foundation of a domain-specific ontology should not alter the meaning of (constituents of) the TLO, but contribute to the meaning of the domain-specific notions. In particular when considering the dynamics of theory development or the modular structuring of theories, it must be clarified which and how categories / predicates influence possibly associated intensions. These issues strongly relate to the subarea of modular ontologies, cf. [320, 532, 787].

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[148] A recent representative in this regard is the Information Artifact Ontology (IAO) [@57] originating from the context of the Open Biomedical Ontologies [@72] which comprises a relation "is about" / aboutness. (The IAO is available with some documentation from its website [@57]. As of November 2014 and to the best of our knowledge, any dedicated publication presenting the IAO itself does not seem to exist. However, [144, 765] are closely related and refer to the IAO.)
Further examples of overall domains centering around formalisms that require a corresponding foundation are Topic Maps [@123] [448, 654] and the Simple Knowledge Organization System (SKOS) [@114] [447].

[149] For the general argument we ignore the varied meanings of these terms, especially of 'intension'.

[150] There are even strict positions like Pavel Materna's [563], who claims that any set-theoretical conception of concepts faces the problem of extensionalism (and the lack of 'individual concepts', see [141, p. 39] for this notion)).

[151] E.g. for ZFC, this is plausible by considering the Axiom of Choice and the cardinalities of powersets of large cardinality.

[152] See [243] for an overview. [65, 462] report particular accounts of intensional logic.

[153] 📖 [529, 532] are contributions to this area by ourselves, which are briefly summarized in sect. 5.3 below.

1.2.3.2 Meta-Logical Issues

STANDARD AND NON-STANDARD MODELS, DISCRIMINATORY POWER (EL. EQU., ISOMORPHISM)

Beyond the issues covered so far, there are related logical and meta-logical aspects that render the overall use of classical logic (or of its model-theoretic semantics) questionable. Thinking of a logical theory and its class of models, this mainly connects with the notions of intended / standard models and non-standard models, i.e., with the existence of non-standard models for given theories. For instance, non-standard models of Peano arithmetic which cannot be excluded by any FOL theory [691, p. 83 ff.], cf. also [478], are certainly among the famous examples in comparing the expressiveness of FOL and HOL. For FOL, the notion of elementary equivalence[154] [214, p. 101] defines the highest degree of discriminatory power wrt models. Since all pairs of isomorphic structures are elementary equivalent, but not vice versa, [214, p. 101], HOL appears preferable in this regard, capable of characterizations up to isomorphism. However, this is the bottom line for any classical logic.[155] From the point of view of specifying individual / concrete structures for ontological purposes, even the distinction between specific entities (possibly structurally isomorphic, as far as known / considered) appears desirable.

OBJECT VS. METALANGUAGE

Further puzzling issues arise due to the distinction of language levels, connecting to Tarski's theory of truth in his 1933 "The concept of truth in formalized languages" (orig. in Polish), reprinted as [805], see also [803]. On the basis of (mainly) the liar antinomy [805, sect. 1, p. 157–158; cf. further p. 157–162], [803, p. 347] Tarski concludes not to use "semantically closed languages" [803, p. 348–349] (cf. also [805, p. 164–165]), i.e., to "[...] use two different languages in discussing the problem of the definition of truth and, more generally, any problems in the field of semantics." [803, p. 349]. In modern parlance, these two languages are named *object language* and *metalanguage*, which are only relative terms wrt to each other, i.e., a metalanguage L_1 wrt a given object language L_0 may be the object language wrt another language L_2 which seeks to establish semantics for L_1. Accordingly, arbitrarily many (meta-)levels of languages arise in this account. In connection with a top-level ontology, the question arises whether every new level requires additional entities.

SKOLEM'S PARADOX

Discussing formal accounts of set theory yields the distinction between foreground and background set theory, where the former refers to sets and set-theoretic notions in the object language, the latter in the metalanguage, which usually refers to sets, as well.[156] Foreground and background set theory serve in explaining Skolem's Paradox [214, p. 121 ff.], which contrasts the existence of uncountable sets in every model of, e.g., ZFC (i.e., this is provable from the axioms that constitute ZFC) with the existence of a countable model of ZFC set theory (which is specified as a countable set of FOL sentences) according to the Löwenheim-Skolem Theorem (downwards) [214, p. 94]. Although this is no longer considered paradoxical from a logical point of view, it affects the relationship of predicates and their model-theoretic interpretations. The notion of (un)countability, for instance, is relative to either a specific model under consideration or to the domain of entities referred to from within the theory.

1.2.3.3 Summarizing Overview and Resulting Questions

PROBLEM OVERVIEW AND RESULTING QUESTIONS

Collecting the issues addressed in the two preceding sect. 1.2.3.1 and 1.2.3.2 yields the following list.

- problem of the universal domain of entities
- metamodeling and reification of categories
  (incl. higher-order categories, self-instantiation, and anadic relations)

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[154]Two FOL structures are elementary equivalent iff they satisfy exactly the same FOL sentences, see sect. 1.5.1.

[155]And usually this is where the interest of mathematically oriented users of logic ends. More precisely, the abstract notion of logic as defined in abstract model theory even includes an "isomorphism condition", which requires that isomorphic structures must satisfy the very same sentences of a logical system [214, Definition 1.1, p. 277–278]. Cf. also the remarks in [749, p. 346] to the extent that it is this property that marks the formality of a logic, supported by (a.o.) quoting Alfred Tarski [806, p. 385].

[156]Although alternatives exist, e.g., in terms of (mathematical) category theory, see [3, 544].

- categories, sets, and topics in the domain of discourse
- intensionality of categories vs. extensionality of sets
- logical interactions within large / structured theories and effects on intension
- existence of standard and non-standard models
- distinguishing elementary equivalent, isomorphic, and specific structures
- language levels and effects, e.g. Skolem's Paradox

All those topics lead us to re-examine the standard semantic account of logical languages more closely, first and foremost of FOL. Its adequacy for representing ontological claims shall be considered, answering the question under which assumptions it can be employed for representing ontologies, and foundational ontologies, in particular. In this light, we believe that even question $\mathcal{Q}_4$ below is not as trivial as it may seem at first glance, in view of (1) various ontological theories that have been presented in FOL and (2) present-day ontology languages like OWL and SWRL and their relation to FOL, recalling sect. 1.1.3.1.

> $\mathcal{Q}_3$ How are ontology, language semantics, and expressiveness related?
> $\mathcal{Q}_4$ How can logics, in particular FOL, be utilized for representing ontologies?

INTENDED UTILITY: WELL-FOUNDED FORMALIZATION & JUSTIFIED TRANSLATIONS

Potential utility in answering these questions lies not only in a well-founded approach to formalizing the General Formal Ontology, but it can be(come) a contribution to the way in which formal languages are used for representation purposes, in relationship with ontologies and their role in explicating conceptualizations. A closely associated matter of equal importance is the justification of language translations that preserve conceptual content, where we will argue esp. in sect. 2.2 that the sole reliance on formal semantics is insufficient. In the case that this applies to expressions independently of whether they are intended to represent ontological statements or other forms of knowledge/information, such justified translations should also have a bearing on ontology matching, cf. [227, 463, 632]. The need for that can be expected to increase with the number of ontologies available. That number is growing, indeed, since currently many groups in science and industry are highly active in developing ontologies for various purposes.

1.2.4 Contributions to Engineering and Applying Ontologies

GFO BACKGROUND AND ACCORDING INTERESTS

The primarily foundational aspects raised in the previous sect. 1.2.1–1.2.3 are inspired by efforts to formalize the General Formal Ontology (introduced in sect. 1.1.5). In accordance with that and besides tackling those problems, we have an equally strong interest wrt GFO and other ontologies in terms of (1) contributing to their establishment by devising ontological / conceptual content, (2) formalizing ontologies according to current practice, e.g. in FOL and OWL, thereby (3) scrutinizing and advancing methodological aspects, and eventually (4) applying ontologies, which may range from addressing representation problems by recourse to existing ontologies to employing formalized ontologies as application ontologies. While not all corresponding efforts play a major role in this thesis itself, they are valuable in providing experience and input for the foundational issues, as well.

GFO AND APPLICATION AREAS

A major part of the theory of GFO is a verbalized ontology, i.e., comprised in natural language documents. Accordingly, continuing its formalization is a natural step, which is reasonably started with topics / areas that have few dependencies wrt other areas. Moreover, a major field of application in the context of GFO relates to the biomedical and life sciences. Therefore, biomedical ontologies as well as modeling problems in this field suggest themselves as applications in our context.

MODULARITY AS AN IMPORTANT ENGINEERING ASPECT

Turning to engineering and technological challenges and regarding the development of a comprehensive axiomatization of top-level ontologies, e.g. according to the methodology outlined for GFO in sect. 1.1.5.3, one can safely expect TLO axiomatizations to comprise at least hundreds of non-uniform axioms, i.e., axioms which do not follow one or a few common schemes and that go beyond subsumption hierarchies of categories as well as they cannot be captured in description logic, at least in no obvious way. Moreover,

taking the biomedical domain as a primary application domain into account, there are terminological systems, cf. [188, 189, 393], and more recently ontologies of tremendous size in the number of categories, such as SNOMED [@115] [442], cf. also [89, 166], and the Unified Medical Language System (UMLS) [@127] [440]. Their axioms[157] are usually in restricted form, but the sheer number of categories and relational links among them presents a challenge.

MODULARITY BENEFITS

According to these observations and partially independently of the language aspect, it is important to find solutions for modularly structuring ontologies that are adequate for their characteristics. In a little more detail, Alan L. Rector identifies reuse, maintainability, and evolution as goals for formalized ontologies, and lists the following items as indispensable requirements in order to achieve these goals [695, p. 121], [698, p. 2]:[158]

- reusable modules are identifiable and separable from the whole
- maintenance can be done in parallel by different authors
- independent evolution of modules, additions with minimal side effects
- human-readable *and* formal explication of the differences of distinct categories, leaving as few content as possible implicit, e.g. to labels

We adopt these views, counting at least reuse and maintainability as benefits of a modular approach. Moreover, we believe that the comprehensibility of an ontology can benefit greatly from (exploiting) a modular structure, whereas the straightforward visualization of categorial systems by subsumption hierarchies becomes insufficient in the case of large systems. Without making a case specifically against SUMO [@119] [662] in this regard,[159] one can find an illustration by looking at the SUMO hierarchy [@119] as one case (among others) where size hampers comprehensibility and subclass browsing appears insufficient to us. Modularity may further allow for new methods in the construction of ontologies, cf. e.g. recent work of Michael Grüninger et al. [320] on ontology repositories and modularization, as well as it may offer new ways for understanding reasoning / proofs, e.g. in "debugging" ontologies.

META-ONTOLOGICAL ARCHITECTURE

[399] Another topic, which is related to the aspect of structuring ontologies, is the question of a meta-ontological architecture for TLOs. Furthermore, this is linked with some of the foundational aspects raised above. Sect. 1.1.5.1 already points to [399] as the source of the current meta-ontological architecture of GFO. This work was written early in the course of preparing this thesis. In combination with the foundational issues, the meta-analysis of TLO constituents and the relationship to set theory remain worth investigating.

1.3 Theses, Objectives and Scope

THESES LAYING THE GROUND FOR CONCRETE OBJECTIVES

Our goals for this work are formulated in two ways. On the one hand, we establish as a set of general theses, mainly on the relationship between ontology and formal semantics. They capture the major driving forces behind the first part of the thesis and are to be defended therein, primarily in ch. 2–4. After and in accordance with the theses in the present section, a number of more concrete objectives is compiled.

1.3.1 Theses To Be Defended

THESES ORIGIN AND READING GUIDE

Rather early while working on this thesis, the foundational issues discussed in the previous sect. 1.2 on motivations have been studied, more precisely and primarily their treatment/occurrence in the context of rather

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[157]if at all logically captured, which is not the case for UMLS, which is a terminological resource

[158]This is a reformulation of Rector's criteria in both places. Note further that he links this with the notion of implementation normalization for ontologies, in analogy to normalization in database theory as initiated by Edgar Frank Codd [162, 163], see e.g. [263, ch. 3] for an introduction.

[159]Rather we acknowledge the provision of even several ways of browsing SUMO easily via [@119]. Moreover, note that SUMO has a modular structure, e.g., according to lines 55–59 in [@120].

different modeling and representation languages, recalling sect. 1.1.3. Furthermore, we have made attempts to enhance our understanding by taking into account the literature, though certainly selectively / punctually, from other fields, such as logic, philosophy, and cognitive sciences. On this basis, a number of attitudes assumed shape in the form of the first subsequent statements I–III. Further influence originates from contributions to the axiomatization of the General Formal Ontology and other projects involving formalization.

A few notes may guide reading the statements. *Postulates* express convictions that will not be analyzed in greater detail, but they are assumed / taken as evident in the context of our work. In contrast, the *theses* capture the main impetus for the overall work and they shall be elaborated and defended herein. A word of caution is in order: for conciseness / sententiousness, the theses are kept compact and they are deliberately pointedly worded, possibly slightly overstated. However, we shall provide no further qualifications in this section. The reader is kindly asked to assess any actual controversy on the basis of the remainder of this work.

FOUNDATIONAL STATEMENTS ON ONTOLOGY AND FORMAL SEMANTICS

We start with the foundational part.

Postulate A (Ontology is necessary for semantic interoperability.)
Referring to a common ontology / conceptualization is indispensable for the semantic foundation and integration of different languages. ❑

Thesis I (Formal semantics is partially irrelevant for semantic interoperability.)
In the early days of the Semantic Web and perhaps to some extent nowadays, there seems to be a widely held view that using a so-called ontology language like OWL suffices to develop ontologies, such that those are suited to solve semantic interoperability problems. However, the formal semantics of languages is at least partially irrelevant from an ontological point of view. Instead, the *use* of languages for formalizing ontologies needs to be grounded in ontology itself. ❑

Thesis II (A purely ontological approach to semantics does not exist.)
Currently, no semantic approach (of formal representation languages) is available that starts by defining semantics from ontology instead of mathematical objects, i.e., instead of relying on set-theory or comparable formalisms. ❑

Thesis III (The notion of 'ontology language' is misleading.)
It is impossible and unnecessary to construct a language which could express every possible ontological distinction directly in terms of its own structure / grammar. ❑

STATEMENTS ON METHODOLOGY AND TOOLS

At the side of more practical and methodological considerations, keeping in mind the formalization of GFO as a main application case, the following statements are added.

Postulate B (Ontological engineering is akin to software engineering.)
Creating large formalized ontologies and thus axiomatizations is comparable to engineering software and should be supported by similar tools. ❑

Thesis IV (Relying on established reasoners and tools is beneficial and achievable.)
Despite the claimed irrelevance of formal semantics in Thesis I, it is possible and highly beneficial to employ present-day (and future) software that resorts to the formal semantics of languages, in particular, automated reasoning systems. ❑

Thesis V (Methodology of engineering and applying ontologies is still in its infancy.)
Although ontological engineering has similarities with software engineering and can benefit from formal approaches and corresponding tools, it does not suffice to copy methods from those fields. Instead, there is a need for genuine methods in ontological engineering and the application of ontologies. This has started to be addressed in applied ontology, but it remains insufficiently developed at present. ❑

1.3.2 Objectives Regarding Foundational Issues

RECAP OF FOUNDATIONAL QUESTIONS

Besides the first three theses in sect. 1.3.1, our aims for this work that concern foundational issues are intimately tied to the questions raised in the motivation section 1.2, $\mathcal{Q}_1$–$\mathcal{Q}_2$ on p. 32 and $\mathcal{Q}_3$–$\mathcal{Q}_4$ on p. 37, resp., repeated here as a reminder.

$\mathcal{Q}_1$ What are the exact relationships between ontology, the semantics of a language and semantic translations?

$\mathcal{Q}_2$ Is the current understanding of ontologies and semantic translations sufficient to justify claims that different expressions state "the same content" or which enhancements would be useful?

$\mathcal{Q}_3$ How are ontology, language semantics, and expressiveness related?

$\mathcal{Q}_4$ How can logics, in particular FOL, be utilized for representing ontologies?

ANALYSIS FIRST

Objective 1 is to expound a comprehensive analysis of these matters, primarily $\mathcal{Q}_1$–$\mathcal{Q}_3$, in view of the practical goal of presenting ontologies in different languages. Thus the analysis involves / overlaps with the subject of semantic translations applied to ontologies. The outcome of this analysis is already hidden / anticipated in the claimed irrelevance of formal semantics for such translations in Thesis I above. Accordingly, present accounts of semantic translations appear insufficient, which must be argued for as part of the first goal, and which led us to several further objectives.

INTENSIONAL EQUIVALENCE AND ONTOLOGICAL SEMANTICS

Objective 2 is to contribute improvements or alternatives to problematic aspects of those accounts available. Notably, 'semantic translation' should be read as referring to the notion of intensional / conceptual equivalence. Indeed, behind this work there is an overarching interest in notion(s) of intension and the search for intuitively adequate and well-founded notions of semantic / conceptual / intensional translation, or rephrased, of the intensional equivalence of expressions. Such expressions may originate from one and the same language, or from different ones. FOL can serve as a reference in any case, but in the long run various representation languages like the Semantic Web languages and UML should be supported by a generic proposal for such translations. Overall and of course, the problem of intension connects to fundamental questions and can be seen as an area itself, overlapping several fields, e.g., semantics, logic, and cognitive science. For a realistic Obj. 2, we aim at small / tiny steps oriented at this field of problems. Certainly, these are constrained and biased wrt our purposes and the overall context of applied ontology.

There are good arguments for using FOL in terms of its classical semantics, in order to rely on its solid and large theoretical basis and to exploit according reasoning software. However, early in pursuing Obj. 1 and by first thoughts wrt Obj. 2, we came to believe that contributions to intensional equivalence are much better justifiable and perhaps comprehensible if they are based on a dedicated, novel kind of semantics that should be ontologically neutral (to the greatest extent possible).[160] Hence, *Objective 3* is a semantic account that is purely based on ontological notions and its analysis in terms of the requirements (e.g. wrt expressiveness) set out in the motivations section. Such an account is established in ch. 4 under the name *ontological semantics*.[161] Since the argument of promising benefits by reusing FOL theoretical results and software remains valid and wrt Question $\mathcal{Q}_4$, *Objective 4* is to relate ontological semantics to its classical counterpart and determine the extent to which approximations or encodings may be found, e.g. of consistency and entailment.

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[160] Ontological neutrality is widely accepted for / attributed to FOL under classical semantics (as well as other logical languages). But this view is challenged in sect. 2.2.2.2, accompanied by a novel proposal for that notion in sect. 3.4.

[161] Almost unrelated works that bear the same name 'ontological semantics', e.g., one by Sergei Nirenburg and Victor Raskin [628], are briefly discussed at the beginning of sect. 7.2.3.

1.3.3 Objectives Regarding Ontology Development, Applications, and Methodology

ONTOLOGIES OF CATEGORIES AND RELATIONS; FURTHER ONTOLOGICAL CONTRIBUTIONS

There are further objectives that embody actual ontological analyses. In particular, *Objective 5* is to present an ontology of categories and relations. On the one hand, this is intended as a direct contribution to GFO. As such, it is meant to deepen / enhance this part of GFO conceptually as well as to capture this theory formally. On the other hand and with hindsight, an ontology of this kind turns out to be a valuable component in two related respects: it can serve in the rôle of an abstract core ontology wrt GFO's meta-ontological architecture, as well as it delivers a prerequisite for the application of the new semantic approach resulting from work toward Obj. 3. Through the notion of relations, this ontology is interlinked with an ontology of roles, a first version of which we had proposed in [526], prior to this thesis. Its continued development as well as contributions to other parts of GFO and further ontologies constitute the umbrella of *Objective 6*. This includes conceptual work as well as formalizations. Especially regarding the latter, a main focus is on the area of time. This area is relevant for many others as well as in applications, but it does not rely on other notions in GFO. Moreover, an initial conceptualization for time was available when starting our work on this thesis. Therefore, the GFO theory of time suggests itself to be axiomatized in the context of Obj. 6.

APPLICATIONS AND METHODOLOGY

Another initially broadly defined goal of the present work is *Objective 7*, which transcends actual ontological content and formalized theories toward methodological aspects in the engineering of the former and toward the application of developed or existing (fragments of) ontologies. Meta-ontological analysis is a subgoal of Obj. 7, actually / chronologically speaking established prior to Obj. 2–3. Moreover, in the course of preparing the thesis, the novel ontological semantics as it results from pursuing Obj. 3 and 4 provides the background for seeking methodological recommendations wrt language use and the ontology-based interpretation of languages. The resulting main concept is called *ontological usage scheme* and consists in a translation-based approach to assigning ontological semantics to languages per individual use case. As a major field of applications, we target at the formalization of biomedical ontologies and their subsequent application to representation problems in the life sciences.

ALL OBJECTIVES ARRANGED BY DEPENDENCIES

In summary and using a dependency-based order, in this work we are interested in and aim at

- an in-depth analysis (Obj. 1) of accounts and open problems of
    - ontology representation, primarily by FOL theories
    - semantic translations on the basis of ontologies
- a theoretical framework that
    - addresses the problem of formally representing ontologies themselves (Obj. 3)
    - serves as a fundament for the question of how ontologies can be used for semantics / intension-preserving translations (Obj. 2)
    - provides a solution that can reuse current reasoners to a large extent (Obj. 4)
- contributions to ontologies (Obj. 6), incl. an ontology that makes the theoretical framework applicable, e.g. to GFO (Obj. 5)
- contributions to methodological / engineering aspects and applications of ontologies (Obj. 7)

1.3.4 Scope and Related Fields

SCOPING AREA OF APPLIED ONTOLOGY, AND RELATED FIELDS

The intended subject area of the present work is applied ontology (AO) / formal ontology in information systems. While this seems to suggest a natural scope, two facets of AO are presently problematic wrt setting the scope: (1) it is an emerging, at best young area, and (2) it is itself highly interdisciplinary in nature, linking to various fields, as observed in sect. 1.1.1. Due to both facets, there is no clearly delineated body of literature. Moreover, due to the foundational topics in the first part of this thesis, various aspects

Main field(s)

- **applied ontology**, esp.
    - foundational ontology
    - biomedical ontology
    - ontological engineering

Strongly related

- knowledge representation
    - logic-based KR
    - knowledge engineering
- mathematical logic
    - first-order logic
    - model theory
- Semantic Web

Weakly related

- mathematical foundations, esp. set theory
- conceptual modeling
- information systems
- philosophy
    - logic, esp. intensional logics
    - philosophy of language
    - ontology and metaphysics
- cognitive science and psychology, esp. theory of concepts
- linguistics, semiotics, and semantics, esp. situation theory / semantics
- terminology / terminology management, esp. medical terminologies

Table 1.3: Related fields, in the sense that topics in this thesis partially draw on sources from those fields. Contributions in the present work apply solely / primarily in applied ontology.

touch other broad fields in terms of issues that have already been studied in those contexts, likely from different perspectives. Table 1.3 provides a structured overview of fields that satisfy this description as 'weakly related' and 'strongly related'.[162] Moreover, it includes AO itself, highlighting branches of major relevance. By no means could we or do we intend to claim (1) to have taken into account all relevant work in those related areas systematically, nor (2) to make contributions to these areas intentionally. We have only seen the chance to access the literature selectively, esp. in weakly related areas, while developing our theory of ontological semantics in later chapters, referring to ch. 3 and 4, in particular. Indeed, in many cases we had to cut further analysis / studies of weakly related work in order to proceed toward our main objectives herein, leaving those aspects for interesting future work. Regarding contributions, we hope to provide some to the field of AO, but shall leave the assessment of our proposals wrt related fields to specialists in them.

SCOPE WRT LANGUAGES

Let us leave broader subject matters aside and adopt a more concrete, content-based perspective on scope. First of all and regarding languages, throughout the thesis there are two levels at which considerations on/for languages are conducted. On the one hand, we aim at treating 'language' at a very abstract level, allowing for exemplifications of at least the kinds described in sect. 1.1.3, from FOL via DLs and SW languages to UML and beyond. On the other hand, there is a strong focus on FOL as a case of reference and a sample case, yet also with an eye on various KR dialects / languages "for FOL", i.e., that are more closely related to FOL than to other well-established logics. These levels of consideration are also reflected in providing a very abstract (natural language) definition of ontological semantics (Def. 4.3 in sect. 4.1) as well as a sample "instance" of it in the formal definition of *ontological model* wrt FOL syntax (Def. 4.5 in sect. 4.3.1). Similarly, ontological usage schemes are defined generically for arbitrary languages and equipped with a detailed example for DLs.

SCOPE WRT ENGINEERING ISSUES, ONTOLOGIES, AND APPLICATIONS

Our scope concerning engineering aspects comprises general methodological considerations, the application of ontological usage schemes, and modularization issues in ontologies. Regarding ontologies and their applications, considerations are focused on foundational and biomedical ontologies, also indicated in Table 1.3. More precisely, studies with top-level character are all aligned/affiliated with GFO, instead of, e.g., any of the other TLOs listed in Table 1.1. Of course, appropriate selections of the latter are taken into account as related work, e.g. in presenting two time modules for GFO in sect. 6.3. Work on core ontologies is also

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[162]Table 1.3 is an ad hoc "classification" according to our perception. Various subsets of those areas overlap in one way or another. Where subfields are specified, this is not meant to be exhaustive, but highlights only main branches related to this work.

associated with GFO and concentrated on the biomedical domain, as is the case with further applications.

1.4 Outline and Contributions

The structure of this thesis is closely aligned with its objectives, as presented in dependency-based order at the end of sect. 1.3.3. In the subsequent description we combine the outlook along the sequence of chapters with a sketch of the main contributions in each chapter, relating them to the stated objectives. Note the abstract as well as the summary in sect. 7.1 in the concluding chapter as two further places in the thesis that summarize the overall work.

PREPARATORY CHAPTERS **1–3**

The present ch. 1 establishes the background in terms of general knowledge about AO and respective terminology, as well as our particular motivations, theses, and preliminaries for this work. The latter conclude this chapter in sect. 1.5.

The second chapter is devoted to foundational aspects for the remainder of the thesis, more specifically / focused and in-depth than the introduction. It first discusses languages with syntactic and semantic notions at a general level. As the first notable contribution herein we see sect. 2.2, which addresses Obj. 1 by analyzing the role of ontologies as a means of semantic integration. The section discusses the encoding aspect of formal languages in relationship to existing proposals of ontology-based consequence and equivalence, which are based on formal representation themselves. Accordingly, sect. 2.2 identifies problems in these regards, a.o. of adopting formal equivalence within a theory as an approach to capturing intensional / conceptual equivalence, thereby refining Obj. 3 of how to formally represent ontologies and the long-term Obj. 2 of accounting for intensional equivalence. After establishing views on meta-ontological analysis, which lead to the notion of an *abstract core ontology*, a.o., a major part of the second main contribution of this thesis is the conceptualization of an ontology of categories and relations, abbreviated as $\mathcal{CR}$, which is proposed as an abstract core ontology. Altogether, as a result of this chapter we are interested in providing a modified semantic account for ontology representation.

Ch. 3 prepares such an account by studying in detail the standard semantics of first-order logic (FOL) and its "background assumptions". In particular, what we call the 'theory view of semantics' in sect. 3.3 is the understanding of the definition of FOL semantics as a theory interpretation into another theory, see Prop. 3.18. In the case of FOL that theory is an extension of set theory. These considerations inspire another contribution, namely a more precise capturing of, until then intuitively understood, notion(s) of *ontological neutrality* of semantic approaches. These are presented in sect. 3.4 at the end of the chapter.

CH. **4** ON ONTOLOGICAL SEMANTICS

The core contribution of the thesis is presented in its fourth chapter on *ontological semantics*, addressing Obj. 2–4 of providing a foundation for ontology representation, which is still justified to exploit existing reasoning machinery and is intended to pave the way for future accounts of intensional / conceptual equivalence. The exposition follows tightly the usual, modern definition of FOL semantics. The overall development is accompanied by a thorough analysis of our motivations, weighing different options and, to some extent, their effects. The attempt is made to avoid as many ontological assumptions as possible in characterizing a notion of *ontological structure* (in parallel to mathematical structures). Close proximity is sought to the classical case wrt the semantic definitions of logical connectives and quantification.

A major insight concerns the semantics of predication presented in sect. 4.4, which is intended to allow for variation based on reasons expounded in sect. 4.4.1–4.4.2. In its general description, predication cannot build on any presuppositions at the side of ontological structures, and therefore it cannot be described in a uniform and similar way compared to the set-theoretic blueprint. Instead, a parametric, signature-based definition of a *predication system* is devised in sect. 4.4.3 in order to describe the assignment of predication semantics of particular instances of ontological semantics generically. A significant variation in this regard is a relaxation of the connection between the referent interpretation of predicate symbols and the semantic evaluation of atomic formulas, argued for in sect. 4.4.4.2.

The main result regarding Obj. 4 of finding a semantic approach that allows for the reuse of current reasoners is derived in sect. 4.6. An adequate transfer of inconsistency checking and of reasoning from

classical semantics to ontological semantics can be shown, constructively to a (weakly restricted) class of ontological structures by Th. 4.44 in sect. 4.6.4.[163] Moreover, in connection with ontological neutrality, sect. 4.7 essentially shows that $\mathcal{CR}$-based instances of ontological semantics turn out to be ontologically neutral, in the sense defined in sect. 3.4.

ENGINEERING, APPLICATIONS OF, AND CONTRIBUTIONS TO ONTOLOGIES

Obj. 7 of engineering and application contributions is tackled in ch. 5 already, further preparing ground for actual ontological contributions according to Obj. 5–6 in ch. 6. In particular, sect. 5.1 proposes a formalization method, labeled $\mathcal{OS}$, which is based on ontological semantics, but can likewise be used under the assumption of classical semantics. Moreover, another important contribution is made in sect. 5.2 in the form of *ontological usage schemes*, which adapt earlier ideas of ontology-based semantics / of ontological reduction to the theory of ontological semantics via $\mathcal{OS}$ theories. Insofar ontological usage schemes are a translational approach of assigning ontological semantics to any language *in a particular usage context*, which is advocated *in parallel* to a possibly available formal semantics defined for the language generically. Completing the picture on engineering aspects herein, initial work on modularity issues wrt ontologies is sketched in condensed form in sect. 5.3.

The concepts of ontological usage schemes (and thus of ontological semantics) are applied in sect. 5.4, where we detail two settings wrt biomedical ontologies and merely sketch further applications in terms of available publications in sect. 5.4.3. As the first more detailed case, in sect. 5.4.1 the theoretical underpinning of the biological core ontology GFO-Bio [@47] [418] is enhanced further by relating it explicitly to its underlying ontological usage scheme. Secondly, the $\mathcal{CR}$ ontology lends itself to contributions to phenotype representation in sect. 5.4.2.

The final set of achievements in this thesis addresses Obj. 5–6, i.e., roughly, contributions to ontologies. First among these and described in sect. 6.1, formalizations of the $\mathcal{CR}$ conceptualization proposed in sect. 2.4 are developed. The major taxonomic fragment is implemented in OWL and attached to the thesis as App. B. The FOL fragments of $\mathcal{CR}$, which extend the OWL version by application of the "standard translation" of DL into FOL, round off the sample predication systems of ontological semantics. The second and even more elaborate case of ontology development concerns two consecutive ontologies / modules of time, jointly developed with Ringo Baumann and Heinrich Herre. Sect. 6.3 develops a full account of (1) motivating the ontologies, (2) developing their conceptualization, with inspiration of work by Franz Brentano [114], (3) axiomatizing both ontologies in FOL, which implicitly assumes a certain ontological usage scheme for FOL, and (4) investigating both theories meta-logically, e.g., proving their consistency and, for one of them, completeness.

CONCLUSION OF THE OUTLINE AND THIS CH. 1

The thesis concludes in a seventh chapter with a résumé (with hindsight) in sect. 7.1, followed by comments on related work and a rediscussion of the objectives of this thesis and ontological semantics, prior to indications of future work in sect. 7.4. The latter conclude the main body of text of the thesis, as this paragraph does for sect. 1.4, as well as the next sect. 1.5 for this introductory chapter.

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[163] A non-constructive, more general version is in prospect by Obs. 4.46.

1.5 Formal Preliminaries

In this section we introduce and refer to basic set-theoretic, logical, and mathematical notation and definitions that we presuppose in the remaining chapters.

1.5.1 Basics, and Abstract Description and Properties of Logics

BASIC LITERATURE

The following text books or handbooks on logic and set theory were used while preparing this thesis:[164]

- **logic**
  primarily those of Ebbinghaus et al. [214], Enderton [220, 221, 222], Goltz and Herre [288], Rautenberg [691, 692, 693], Rothmaler [717], Tuschik and Wolter [826],
  secondarily Barwise [53], Barwise and Feferman [55], Chang and Keisler [147], Ebbinghaus et al. [213], Fitting [242], Hodges [410, 411], Kreisel and Krivine [486], Tarski [808, 809]
- **set theory**
  those of Devlin [200], Ebbinghaus [212]

FONT AND SYMBOL TYPES; BASIC NOTATION FROM [693]

First of all, letters in most mathematical fonts, such as x, y, z and A, B, C, $\mathcal{L}$, $\mathcal{M}$, $\mathcal{O}$ as well as Greek letters like α, β, Γ, Δ and Fraktur letters such as $\mathfrak{P}$, $\mathfrak{S}$ denote variables or variable parameters by default, i.e., unless otherwise stated. The serif mathematical font is usually linked with constants, i.e., having a fixed (intended) meaning, such as the former use of Leo. However, symbols in that font may also be declared for variable use. A dot vertically in the middle of a line, '$\cdot$', is used for unnamed / underspecified arguments or "empty positions". $=$ stands for equality, $\neq$ for inequality, including in FOL (object-level) formulas. For numbers x, y, $x < y$, $x > y$ stand for x is strictly less or greater than y, resp., in the reflexive case using $\leq, \geq$.

For further basic set-theoretic [200, 212] and mathematical notation and terminology, we adopt exactly that of [693, p. xix–xxi] in all those cases of symbols used herein that are listed next, which makes their meaning accessible. In the order as appearing in [693, p. xix–xxi] these are: $\in, \subseteq, \subset, \emptyset, \cap, \cup, \setminus, \mathbb{N}, \mathbb{N}_+, \mathbb{Z}, \mathbb{Z}_+, \mathbb{R}, \mathbb{R}_+, \bigcup, \bigcap, \bigcup_{i \in I}, \bigcap_{i \in I}, \times, (\cdot, \ldots, \cdot), f : M \to N,$[165] $f : x \mapsto t(x), M^I$ for all functions from I to M, $(a_i)_{i \in I}$, M^n as the set of n-tuples or sequences of length n (for $n \in \mathbb{N}_+$). We add "inverse notation" for $\subseteq, \subset$ by means of $\supseteq$ and $\supset$, resp., and agree to let $x_1, \ldots, x_n \in M$ abbreviate $x_1 \in M, \ldots, x_n \in M$ (alternatively, $\{x_1, \ldots, x_n\} \subseteq M$). $\cong$ stands for the relation 'is isomorphic to'. We use ω for the smallest transfinite ordinal number[166] such that $(\omega, \in) \cong (\mathbb{N}, <)$. Therefore, we use the notation $i < \omega$ as an alternative to $i \in \mathbb{N}$.[167] Deviations from [693, p. xix–xxi] comprise the use of $\mathcal{P}(\cdot)$ for the power set / set of all subsets; specifying domain and range of a function f via $dom(f)$ and $rg(f)$, resp.; the identity / identical function on a set M by id_M. Moreover, vector / sequence notation uses $\bar{x}$ for the tuple $(x_1, \ldots, x_n)$ or the sequences $x_1, \ldots, x_n$ or $x_1 \ldots x_n$ if $n \in \mathbb{N}_+$, and the empty sequence otherwise.[168] The length of a tuple or a sequence t is denoted by $|t|$. An operation or notation is canonically extended to tuples and sequences by componentwise application. We refer to subsets of M^n exclusively as (mathematical) *relations* over M. The *restriction* of a relation $R \subseteq M^n$ to the subset $N \subseteq M$ is defined as $R \cap N^n$ and denoted by $R|_N$. The term *predicate* is reserved as a synonym for *predicate symbol* wrt logical syntax; constants are intuitively understood as symbols with fixed meaning instead of a 0-ary operation / function, and in the case of (logical) individual constants[169] directly as a member of the universe of a set-theoretic structure. 'iff' stands for 'if

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[164]This list partially includes distinct editions of the "same" book. This is due to working with those versions at different times, partially referring to page numbers that apply to a specific edition.

[165]By default, functions are total functions herein.

[166]Cf. Def. 3.1 for the notion of ordinal number in sect. 3.1.2.

[167]Notably, we do not identify natural numbers with sets. Insofar, $i \in \mathbb{N}$ is the more appropriate way of writing, because i is assumed to be a natural number in both statements. $i < \omega$ is merely a convenient notation, e.g. if combined with a lower bound for i.

[168]E.g., if $M = \{x_1, \ldots, x_n\}$ is specified in a context that allows for $n = 0$, the case of $n = 0$ yields $M = \emptyset$. We shall consider further particular effects directly in places where sequence or tuple "nullification" can occur.

[169]Another remark pertains to the term 'individual constant', which we find ontologically misleading, because we do not see applications of logic to be limited to individuals in an ontological sense, cf. sect. 1.1.5.2 for the notion as discussed so far. 'Entity constant'

and only if', 'wlog' for 'without loss of generality'. In proofs, e.g. involving statements S_1, S_2, S_3, 'iff-chains' of the form S_1 iff S_2 iff S_3 abbreviate the conjunction of pairwise equivalences of neighbors in the chain,[170] in the example i.e. S_1 iff S_2 and S_2 iff S_3. Equality chains are understood analogously. We use := in defining operations / functional terms and :iff in defining relations or ways of expression. They can also be applied in 'iff' / equality chains. In cases of deriving symbols for entities / notions that are similar or related to entities / notions denoted by a symbol already, we employ $\hat{}$, $\check{}$, and $'$, as well as upper and lower indices such as $^+$, and $^{(n)}$ with $n \in \mathbb{N}$ as augmenting features, e.g. $\dot{\in}$ for a predicate symbol related to the membership relation $\in$ in sect. 3.3. In contexts that determine a sequence of entities via a common symbol equipped with an index with a certain range, we agree that the same symbol with a new index refers to individual entities of that sequence. For instance, given $P_1, \ldots, P_n$, we may say "every P_i is a predicate", implicitly assuming the condition $1 \leq i \leq n$, cf. e.g. the introduction of FOL semantics in sect. 1.5.2 below.

NOTION OF A LOGIC, ABSTRACTLY DEFINED

An abstract notion of a logic is determined by its syntax and a model-theoretic semantics and / or a proof theory. We restrict our considerations to logics with a model-theoretic semantics, possibly with or without a proof theory. Such a logic $\mathfrak{L}$ can be captured in set-theoretic terms by seven constituents / constituent sets Λ, Σ, *Var*, L, $\mathfrak{S}$, $\models$, and $\vdash$, of which $\vdash$ is optional and may not be specified, as well as at most one of Σ or *Var*, in which case the latter are technically viewed as / equated with the empty set.[171] All constituent sets of a logic are mutually pairwise disjoint. Some constituents may as well be treated as parameters, i.e., underspecified. Before clarifying their nature, note that the three terms 'alphabet', 'signature', and 'vocabulary' are used synonymously for language variation in the text (primarily, 'signature' is chosen among them). Signatures are sets of symbols / signs. A symbol s can have an associated arity, denoted by $ar(s)$. Λ is a logical signature, i.e., it comprises the logical signs (if addressed, we view technical signs like '(' and ')' as being included in Λ). Σ is another vocabulary, called the non-logical or extra-logical signature of $\mathfrak{L}$. *Var* is a set of symbols that act as variables in the language of $\mathfrak{L}$. The latter is reflected by L, the set of well-formed expressions / formulas, which are constructed from / over $\Lambda \cup \Sigma \cup Var$. $\mathfrak{S}$ is a set[172] of entities / semantic entities / structures[173] according to which formulas are either satisfied or not satisfied. This is determined by the relation $\models \subseteq \mathfrak{S} \times L$, usually written $\mathcal{A} \models \phi$ for a structure $\mathcal{A} \in \mathfrak{S}$ and a formula $\phi \in L$. The derivability relation $\vdash \subseteq \mathcal{P}(L) \to L$ reflects the / a potential proof theory of $\mathfrak{L}$, where $T \vdash \phi$ declares that ϕ is derivable / provable from $T \subseteq L$.

1.1 Condition (logic $\mathfrak{L}$ based on Λ, Σ, *Var*, L, $\mathfrak{S}$, $\models$, and $\vdash$)
Let $\mathfrak{L}$ be a logic with constituents Λ, Σ, *Var*, L, $\mathfrak{S}$, $\models$, and $\vdash$ as described in the preceding paragraph, i.e., abstractly

- Λ, Σ, and *Var* are three, mutually disjoint sets of symbols,
- L is a formal language over $\Lambda \cup \Sigma \cup Var$,
- $\mathfrak{S}$ is a set of semantic structures,
- $\models \subseteq \mathfrak{S} \times L$ and $\vdash \subseteq \mathcal{P}(L) \to L$ are relations. ❑

THEORIES AND MODELS, AND RELATED NOTATION

Assume Cond. 1.1 for the next three paragraphs.[174] By $Sig(X)$ we refer to the set of extra-logical symbols in Σ that occur in X, which may be a formula $X \in L$, a set of formulas $X \subseteq L$, or L or $\mathfrak{L}$ themselves,

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appears more appropriate to us; nevertheless, we stick to established logical terminology and sometimes escape the issue by just using 'constant' or by adding '(logical)' to indicate the issue, as in the present case.

[170]Reading 'iff-chains' literally in either way of parenthesizing / grouping, $(S_1$ iff $S_2)$ iff S_3 *vs* S_1 iff $(S_2$ iff $S_3)$, is false / inappropriate because these readings yield statements that are not intended, in the sense that they are not equivalent to those intended conjunctions of equivalence statements.

[171]Due to this optionality we do not introduce any tuple-based notation for $\mathfrak{L}$ à la $\mathfrak{L} = (\Lambda, \Sigma, Var, L, \mathfrak{S}, \models, \vdash)$.

[172]Notably, for $\mathfrak{S}$ we need a more generic notion than 'set' for specifying ontological semantics in ch. 4.

[173]At the abstract level, 'structures' is used generically as a mere synonym of 'entity'. In the case of classical FOL semantics below, those entities / structures are mathematical structures / set-theoretic structures, i.e., a set as universe of reference together with (mathematical) relations, (mathematical) functions, and members of the universe.

[174]We use conditions such as Cond. 1.1 and refer to them, primarily in definitions and other technical statements that follow them in order to make assumptions and presupposed parameters explicit, and to be able to reuse them in distinct locations in the text. The present case may seem excessive in this regard, but is meant to introduce the idea.

where in the last two cases $Sig(\mathfrak{L}) := Sig(L) := \Sigma$. Similarly, $Lg(\Sigma)$ denotes the language / all formulas over Σ (and Var, implicitly), and $Lg(X) := Lg(Sig(X))$ refers to the language of formulas $X \in L$, sets of formulas $X \subset L$, and $X = \mathfrak{L}$. If the syntax of the language distinguishes terms[175] from formulas, $Tm(X)$ denotes the set of terms (1) that may be constructed over a signature X (implicitly assuming Var, unless anything else is stated), or (2) that may occur in (expressions of) X, if X is a (subset of a) language, $Tm(X) := Tm(Sig(X))$. If there is a notion of free variables wrt $\mathfrak{L}$, $fVar(X)$ denotes "the free variables of X" and is overloaded in its argument as well as its values: (1) X may be a formula or a set of formulas, and (2) depending on context, $fVar(X)$ can denote the set of free variables of its argument or, if $X \in L$, an ordered sequence of that set. Free variables come with an accompanying distinction between open and closed formulas, as those with at least one vs without any free variable. The sentences $Sen(L)$ of L are exactly all closed formulas, otherwise (if there are no free variables wrt $\mathfrak{L}$) the notion of sentence and formula are equated. Indeed, until the end of this sect. 1.5.1 we are only concerned with the sentences of L. Therefore and for better readability, we ignore open formulas in L and use L synonymously with $Sen(L)$. Hence, $\phi \in L$ refers to a *sentence* $\phi \in Sen(L)$. An $\mathfrak{L}$-theory or, where clear from context, a *theory*[176] is an arbitrary set $T \subseteq Sen(L)$ of sentences, which can be introduced as / referred to via $T \subseteq L$ due to the convention just stated.

A structure $\mathcal{A} \in \mathfrak{S}$ is called a *model* of $\phi \in L$:iff $\mathcal{A} \models \phi$. $\mathcal{A}$ is a model of a theory $T \subseteq L$:iff it satisfies / is a model of every sentence $\phi \in T$. The (mathematical) class of all models of X, with $X \in L$ or $X \subseteq L$, is denoted by $Mod(X)$. Conversely, for a class $\mathfrak{K} \subseteq \mathfrak{S}$ of structures, the theory of $\mathfrak{K}$ is defined as $Th(\mathfrak{K}) := \{\phi \in L \mid \mathcal{A} \models \phi \text{ for all } \mathcal{A} \in \mathfrak{K}\}$. Logical equivalence is defined as $X \equiv Y$:iff $Mod(X) = Mod(Y)$ for sentences or theories X, Y. $\models\; :\; \mathcal{P}(L) \to L$, the logical relation of entailment / (semantic) consequence, is defined by $T \models \phi$:iff $Mod(T) \subseteq Mod(\phi)$ for any $T \subseteq L$ and $\phi \in L$. If equivalence and entailment are indexed with L, e.g., $\models_L$, this assumes implicitly a uniquely determined logic in connection with which L is considered. Similarly, the name of a theory $T \subseteq L$ as an index indicates either notion modulo that theory T (implicitly assuming a uniquely determined logic for L), i.e., $X \equiv_T Y$:iff $Mod(X) \cap Mod(T) = Mod(Y) \cap Mod(T)$, and $X \models_T Y$:iff $Mod(X) \cap Mod(T) \subseteq Mod(Y)$. Based on entailment, we extend the use of Th to the deductive closure of X, $X \in L$ or $X \subseteq L$, by $Th(X) := \{\phi \in L \mid X \models \phi\}$. Note at this point that, herein, theories are *arbitrary* sets of sentences, whereas a *deductively closed* theory (satisfying $Th(T) = T$) must be called this way or, more briefly, 'closed theory'. A particular deductively closed theory is the set of all tautologies of the logic $\mathfrak{L}$, denoted and defined by $Taut(\mathfrak{L}) := Taut(L) := \{\phi \in L \mid \mathcal{A} \models \phi \text{ for every } \mathcal{A} \in \mathfrak{S}\}$. Furthermore, $\mathfrak{L}$ satisfies an adequacy theorem (wrt $\models$ and $\vdash$) :iff $T \models \phi$ iff $T \vdash \phi$ for any $T \subseteq L$ and $\phi \in L$.[177] If the logical connectives Λ of $\mathfrak{L}$ comprise a symbol for material implication, say '$\to$', and the consequence relation $\models$ satisfies the property that

$$(1.2) \qquad T \cup \{\phi\} \models \psi \text{ entails } T \models \phi \to \psi, \text{ for any theory } T \subseteq L \text{ and sentences } \phi, \psi \in L$$

it is said that $\mathfrak{L}$ satisfies the (semantic) deduction theorem or the deduction theorem (for entailment).[178]

METATHEORETIC NOTIONS AND ELEMENTARY EQUIVALENCE

For some further definitions, the presence of (a suitable form of) negation in $\mathfrak{L}$ is required and presupposed. Accordingly, for every $\phi \in L$ let $\neg\phi$ also be a sentence in L. A theory $T \subseteq L$ is said to be (1) satisfiable :iff $Mod(T) \neq \emptyset$; (2) consistent :iff for no $\phi \in Lg(T)$ both of $T \vdash \phi$ and $T \vdash \neg\phi$ hold; (3) complete :iff for every sentence $\phi \in Lg(T)$, $T \models \phi$ or $T \models \neg\phi$. A complete and consistent extension $S \supseteq T$ is called an elementary type of T (or type of T, where clear from context). Elementary types further correspond to equivalence classes of elementary equivalent models, resulting from defining two structures $\mathcal{A}$ and $\mathcal{B}$ to

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[175]in a logical, not a linguistic reading of 'term'

[176]More precisely, an $\mathfrak{L}$-theory, but usually it is clear from context whether a specific logic is referred to, or the generic notion is assumed.

[177]The corresponding correctness theorem states that $T \vdash \phi$ implies $T \models \phi$, the converse implication is the corresponding completeness theorem of $\mathfrak{L}$. Since correctness is commonly required for calculi in mathematical logic and computer science, there one also finds the term 'completeness theorem' for the statement $T \models \phi$ iff $T \vdash \phi$, e.g. in [693, Theorem 2.7, p. 102], [826, Satz 1.24, p. 29]. For the use of 'adequacy theorem', see [214, Satz 4.2, p. 92].

[178]Cf. e.g. [693, p. 21] for a statement in the context of propositional logic, [288, Satz 3.6, p. 62] and [826, Satz 2.3, p. 57] for FOL.

be elementary equivalent, denoted by $\mathcal{A} \equiv \mathcal{B}$, :iff $Th(\mathcal{A}) = Th(\mathcal{B})$. Given a theory $T \subseteq L$, consider an algorithm $\mathtt{A}(\cdot)$ that accepts sentences of $Lg(T)$ as input and outputs either 0 or 1 at termination. We say that $\mathtt{A}$ computes T :iff $\mathtt{A}(\phi) = 1$ iff $\phi \in T$. A theory T is called (4) a recursively enumerable set of sentences :iff there is an algorithm that computes T and stops if $\phi \in T$; (5) a decidable set of sentences :iff there is an algorithm that computes T and terminates on each $\phi \in Lg(T)$; (6) (finitely) *axiomatizable* :iff there is a (finite) decidable set of formulas $A \subseteq L$ and $Th(A) = Th(T)$; (7) recursively enumerable :iff $Th(T)$ is a recursively enumerable set of formulas.

1.5.2 Classical Logic and FOL Semantics

FOL SYNTAX

Let us instantiate the notion of a logic according to Cond. 1.1 with $\mathfrak{FOL}$, the FOL family of languages under classical set-theoretic semantics, ignoring the derivability relation $\vdash$.[179] Beginning with the syntax, Λ comprises the following logical constants: (1) the connectives $\neg$ ('not'), $\wedge$ ('and'), $\vee$ ('or'), $\dot\vee$ ('either-or'), $\to$ ('if-then', 'implies', material implication / conditional), and $\leftrightarrow$ ('iff', 'equivalent-with', biconditional), (2) the quantifiers $\forall$ ('for all', universal quantifier) and $\exists$ ('there is', existential quantifier), (3) the binary predicate symbol $=$ for equality. The extra-logical signature Σ and the set of variables *Var* remain parameters in $\mathfrak{FOL}$, limited to being countably infinite sets of symbols. We omit recapitulating the syntax of FOL and associated terminology in detail (e.g., atomic formulas / atoms, denoted by $At(X)$ for any signature, theory, or language X; ground formulas / ground instances of formulas), referring to, e.g., [693, sect. 2.2], assuming the notions of first-order terms and formulas (and thus sentences). Accordingly, for fixed Σ and *Var*, we assume that $L := Lg(\Sigma)$ is basically clear. Some notational conventions are to be mentioned, though. By $Var(X)$ and $Tm(X)$ we denote the variables and terms, resp., that occur in formulas, theories, or languages X. In order to avoid parentheses, first we adopt decreasing binding strengths for connectives in the following strict total order: $\neg, \wedge, \vee, \dot\vee, \to$, and $\leftrightarrow$.[180] Regarding quantifiers, let $\{x_1, \ldots, x_n\} \subseteq Var$ for $n \in \mathbb{N}_+$ and Q a quantifier. Then $Qx_1 \ldots x_n(\phi)$ abbreviates $Qx_1(Qx_2(\ldots Qx_n(\phi) \ldots))$. Combined with vector notation $\bar{x}$, this can be shortened to $Q\bar{x}(\phi)$. Moreover, parentheses around the matrix of a quantifier can be replaced by a single dot '.' at the beginning of the matrix, if it extends to the very end of the formula. For instance, $\forall xy.R(x,y) \to \exists z.R(x,z) \wedge R(z,y)$ is a rewriting of $\forall xy(R(x,y) \to \exists z(R(x,z) \wedge R(z,y)))$.

SET-THEORETIC SEMANTIC STRUCTURES FOR FOL

In contrast to "skipping" FOL syntax, we expound its semantic definition explicitly in order to allow for clear comparison with derived, similar definitions in chapters below, esp. 4. Furthermore, this yields another illustration of our use of conditions in, e.g., definitions. Let $\Sigma = \{P_1, \ldots, P_l, f_1, \ldots, f_m, c_1, \ldots, c_n\}$ with $l, m, n \in \mathbb{N}$.[181] The P_i are predicate symbols, the f_j function symbols, and c_k individual constants. $Pred(L) := Pred(\Sigma) := \{P_1, \ldots, P_l\}$, $Pred^=(X) := Pred(X) \cup \{=\}$, $Const(L) := Const(\Sigma) := \{c_1, \ldots, c_n\}$.[182] The semantic structures $\mathfrak{S}$ of FOL are set-theoretic/mathematical structures, which depend on Σ and for which two types of notation are used herein. Assuming Σ as just stated, a (set-theoretic) Σ structure $\mathcal{A} = (A, I)$ consists of a *non-empty* set A as its universe and an interpretation function I, i.e., a total function with $dom(I) = \Sigma \cup \{=\}$ and such that $I(=) = id_A$, $I(P_i) \subseteq A^{ar(P_i)}$ for all P_i, $I(f_j) \in A^{A^{ar(f_j)}}$ for all f_j, and $I(c_k) \in A$ for all c_k. By setting $X^{\mathcal{A}} := I(X)$ for $X \in dom(I)$ and $\mathcal{A} = (A, I)$, that $\mathcal{A}$ can alternatively be specified as $\mathcal{A} = (A, P_1^{\mathcal{A}}, \ldots, P_l^{\mathcal{A}}, f_1^{\mathcal{A}}, \ldots, f_m^{\mathcal{A}}, c_1^{\mathcal{A}}, \ldots, c_n^{\mathcal{A}})$. The (mathematical) class of all Σ structures is denoted by $\mathfrak{S}^\Sigma$. In almost[183] all subsequent technical considerations we limit

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[179]To clarify, we continue using FOL, abbreviating 'first-order logic', with an intuitive interpretation, whereas $\mathfrak{FOL}$ denotes the subscheme of the formal scheme of describing logics introduced in the previous sect. 1.5.1 that corresponds to FOL as used herein. The constituents of $\mathfrak{FOL}$ are laid out in this section.

[180]However, we usually specify parentheses to clarify subformula combinations involving $\wedge$, $\vee$, and $\dot\vee$ such that it suffices to remember a corresponding preorder of higher or equal binding strengths, with $\wedge$, $\vee$, $\dot\vee$ at one level of strength between $\neg$ and $\to$.

[181]If $\Sigma = \emptyset$, due to $=$ in Λ at least one predicate remains present.

[182]There is no notation for functions because they will be ignored shortly below.

[183]The exception is the formulation of actual FOL theories, where we occasionally use functions, cf. e.g. sect. 6.3. However, in those theories the employed functions must be understood as abbreviations wrt corresponding relations by which they are defined, instead of as signature elements that are interpreted themselves. In particular, they apply only to some elements in the domain and cannot be interpreted as total functions. For a more detailed view on this option and two others, namely extending partial functions to total

ourselves to FOL with predicates and individual constants only, without functions. Accordingly, $Tm(L) = Var \cup Const(L)$, and $\mathfrak{S}^\Sigma$ is restricted correspondingly. In addition to set-theoretic structures, variable assignments A^{Var} are required, i.e., functions of the form $\beta : Var \to A$. For any variable assignment β, variable $x \in Var$, and $a \in A$, β_a^x denotes that variable assignment that agrees with β except for fixing $\beta_a^x(x) := a$, independently of $\beta(x)$. A given structure $\mathcal{A}$ and a variable assignment β forms an interpretation $\mathcal{I}$, which is denoted by $\mathcal{I} = \langle \mathcal{A}, \beta \rangle$. In analogy to and partially based on $\cdot^\mathcal{A}$ above, we set $X^\mathcal{I} := X^\mathcal{A}$ for $X \in \Sigma \cup \{=\}$, and $X^\mathcal{I} := \beta(X)$ for $X \in Var$.

In order to define the satisfaction relation $\models$ of[184] $\mathfrak{FOL}$ for all *formulas*, $\mathfrak{S}$ must be understood as the set of all interpretations as just introduced, which is the way in which we proceed in the remainder of this section. However, note that considerations can be limited to $\mathfrak{S}^\Sigma$ in connection with *sentences*, theories, and entailment, in the sense that variable assignments and the interpretation of free formulas can be left implicit.

DEFINITION OF THE SATISFACTION RELATION $\models$ AND MODELS

Now the formal definition of the relation of satisfaction $\models$ in FOL can be provided. We dissect that into four distinct definitions (and two preparatory conditions), because these parts correspond to separate definitions in ch. 4.

1.2 Condition (Λ, Σ, *Var*, *L*, $\mathfrak{S}$ and an interpretation $\langle \mathcal{A}, \beta \rangle$)

Let the first constituents of $\mathfrak{FOL}$ be given as in the previous paragraphs, i.e., as follows.

- $\Lambda = \{\neg, \wedge, \vee, \dot\vee, \to, \leftrightarrow, \forall, \exists, =\} \cup \{(,), .\}$
- Σ and *Var* are two disjoint, non-empty sets of symbols, where Σ comprises only predicates and individual constants.
- $L := Lg(\Sigma)$
- $\mathfrak{S}$ is the class of all interpretations over Σ and *Var*

On this basis, assume an arbitrary interpretation $\langle \mathcal{A}, \beta \rangle \in \mathfrak{S}$, where $\mathcal{A} = (A, I)$ is a Σ structure with universe A, and $\beta : Var \to A$ is a variable assignment. ❏

1.3 Condition ($P(\bar{t})$, *P*, and $\bar{t}$)

In addition to and based on Cond. 1.2, we determine:

- $P(\bar{t}) \in At(L)$ is an arbitrary atomic formula, hence
- $P \in Pred^=(\Sigma)$ is an arbitrary predicate symbol (incl. equality), and
- $\bar{t} \in Tm(L)^{ar(P)}$ is an arbitrary tuple of terms over Σ and *Var* of an arity equal to that of P. ❏

1.4 Definition (satisfaction of atomic formulas)

Assume Cond. 1.2 and 1.3, where $\langle \mathcal{A}, \beta \rangle \in \mathfrak{S}$ is an interpretation involving a set-theoretic structure $\mathcal{A} = (A, I)$, and $P(\bar{t}) \in At(L)$ is an arbitrary, well-formed atomic formula in L. Then $\langle \mathcal{A}, \beta \rangle$ *satisfies* $P(\bar{t})$ is defined by
$$\langle \mathcal{A}, \beta \rangle \models P(\bar{t}) \quad :\text{iff} \quad \bar{t}^{\langle \mathcal{A}, \beta \rangle} \in I(P) \ . \quad ❏$$

1.5 Definition (satisfaction of combinations by connectives)

In addition to presupposing Cond. 1.2, where Σ structure $\langle \mathcal{A}, \beta \rangle$ is arbitrary, let $\phi, \psi \in L$. Then $\langle \mathcal{A}, \beta \rangle$ satisfies the negation of ϕ and combinations of ϕ and ψ by each of the remaining connectives as follows.

(1.3)	$\langle \mathcal{A}, \beta \rangle \models \neg\phi$	:iff	$\langle \mathcal{A}, \beta \rangle \not\models \phi$
(1.4)	$\langle \mathcal{A}, \beta \rangle \models \phi \wedge \psi$	:iff	$\langle \mathcal{A}, \beta \rangle \models \phi$ and $\langle \mathcal{A}, \beta \rangle \models \psi$
(1.5)	$\langle \mathcal{A}, \beta \rangle \models \phi \vee \psi$	:iff	$\langle \mathcal{A}, \beta \rangle \models \phi$ or $\langle \mathcal{A}, \beta \rangle \models \psi$
(1.6)	$\langle \mathcal{A}, \beta \rangle \models \phi \dot\vee \psi$	:iff	either $\langle \mathcal{A}, \beta \rangle \models \phi$ or $\langle \mathcal{A}, \beta \rangle \models \psi$
(1.7)	$\langle \mathcal{A}, \beta \rangle \models \phi \to \psi$	:iff	$\langle \mathcal{A}, \beta \rangle \not\models \phi$ or $\langle \mathcal{A}, \beta \rangle \models \psi$
(1.8)	$\langle \mathcal{A}, \beta \rangle \models \phi \leftrightarrow \psi$	:iff	$\langle \mathcal{A}, \beta \rangle \models \phi$ if and only if $\langle \mathcal{A}, \beta \rangle \models \psi$

❏

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functions or the modification of FOL semantics to accomodate partial functions see [213, p. 48], [214, p. 52].

[184]We use the same symbol $\models$ at the level of abstract logics as well as for FOL / $\mathfrak{FOL}$.

1.6 Definition (satisfaction of quantified formulas)

Given Cond. 1.2 with its arbitrary interpretation $\langle \mathcal{A}, \beta \rangle$ with Σ structure $\mathcal{A} = (A, I)$, let further $\phi \in L$ a FOL formula. The universal and existential quantifications of ϕ are satisfied according to these definitions:

$$(1.9) \quad \langle \mathcal{A}, \beta \rangle \models \forall x \,.\, \phi \quad \text{:iff} \quad \text{for every } a \in A : \langle \mathcal{A}, \beta_a^x \rangle \models \phi$$

$$(1.10) \quad \langle \mathcal{A}, \beta \rangle \models \exists x \,.\, \phi \quad \text{:iff} \quad \text{there is an } a \in A : \langle \mathcal{A}, \beta_a^x \rangle \models \phi \qquad \square$$

Clearly, wrt the formulas in L, Def. 1.4–1.6 cover L exhaustively (and disjointly). This justifies an "umbrella" definition for the notion of a FOL model of arbitrary formulas, i.e., the definition of $\models \subseteq \mathfrak{S} \times L$ from the abstract logical point of view.

1.7 Definition (set-theoretic model of a formula)

Against the background of Cond. 1.2, let L a FO language over any signature $\Sigma = Sig(L)$, $\phi \in L$ a formula, $\mathcal{A} = (A, I)$ a set-theoretic Σ structure, and $\beta : Var(L) \to A$ a variable assignment.
$\langle \mathcal{A}, \beta \rangle \models \phi$, read $\langle \mathcal{A}, \beta \rangle$ satisfies ϕ or $\langle \mathcal{A}, \beta \rangle$ is a *(set-theoretic) model* of ϕ, :iff

it satisfies ϕ according to one of the Defs. 1.4, 1.5, or 1.6.

If ϕ is a sentence, $\phi \in Sen(L)$, $\mathcal{A} \models \phi$ abbreviates: for every $\beta \in A^{Var(L)}$: $\langle \mathcal{A}, \beta \rangle \models \phi$. $\qquad \square$

This definition completes the view of $\mathfrak{FOL}$ as a family of logics in the sense of sect. 1.5.1. Accordingly, all definitions and notions from that section apply to FOL languages. This further concludes not only the present sect. 1.5, but the overall introductory chapter. We continue on foundational notions and perspectives in a focused, less condensed and more paced manner.

Chapter 2

Foundations on Languages, Semantics, and Ontology

SYNTAX, SEMANTICS, PRAGMATICS

Languages are required in order to represent, communicate, and process data, information, or knowledge [82, p. 13]. Following common views in linguistics, more precisely in semiotics, syntax, semantics, and pragmatics are three central aspects in the study of languages, according to [215, p. 204] and [348, p. 18] introduced in [592], cf. also [773, sect. 6.6] in KR, [852, sect. 3.6.2–3.6.6] related to philosophy of science, [85, part IV] in theoretical computer science and software specification, and [252, sect. 3.2.1–3.2.2] with a background in terminology standardization. We adopt this view for artificial languages, as well. Roughly speaking, *syntax* centers on the notion of symbols and their compositions to new symbols / complex representational entities. In artificial intelligence, Allen Newell and Herbert A. Simon have introduced the notion of *symbol systems* together with criteria for entities to serve as symbols in such systems [625], e.g. the possibility to rely on an "arbitrary" number of copies. The question of what is represented by a set of expressions of a language L refers to the *semantics* of L, to the study of meaning. *Pragmatics* relates to the question of *what the purpose of a representation is*, i.e., what a subject – as a user of L – intends to use a set of expressions for. Alternatively, pragmatics may be called a teleological / functional dimension of languages.

SEMANTICS: THEORY, AND REFERENTIAL SEMANTICS AND DENOTATION

The difference / relationship between symbols and meaning, between syntax and semantics puzzles researchers wrt natural language semantics at least since Platon, and it applies equally to artificial languages. This difference appears similarly basic as the subject of ontology in general, and ontology should play an important role in defining semantics. Besides works that are explicitly dedicated to semantics like [827] by Stephen Ullmann, we refer to [264, ch. 5] by Peter Gärdenfors as a valuable concise introduction.[185] Gärdenfors characterizes a theory of semantics as centering around the following four questions, quoted from [264, p. 151]:

1. What are meanings?   (the ontological question)
2. What is the relation between linguistic expressions and their meanings?   (the semantic question)
3. How can the coupling between linguistic expressions and their meanings be learned?   (the learnability question)
4. How do we communicate meanings?   (the communicative question)

Herein, only the first two questions will be relevant (to a certain extent). In the highly general context of *referential semantics*[186], *denotation* may be mentioned as a broadly discussed relation referring to the idea that a symbol denotes / means something, it stands for something else of which some subject was aware of before using the symbol. Denotation assigns meanings / meaning entities to symbols, and it serves as the foundation of referential / denotational semantics (cf. also sect. 2.1.3). The long history of semantics is further visible in the impressive number of refinements of the idea of denotation, in particular in terms of "the" *semiotic triangle* as popularized due to [191, 639, 827], i.e., in the various sets of differing terms, theories and interpretations of its three components [@112], or even wider-ranging variations of it; see [773, p. 191–193], [852, sect. 3.6.4], [252, sect. 3.2.2] for treatments in a context similar to ours.

RESTRICTION TO ARTIFICIAL LANGUAGES AND SECTION OUTLINE

In this work, we will not consider natural-language issues, but restrict ourselves to artificial languages with at least precisely defined syntax. This includes basically the range of languages that we see related to ontologies and ontology representation, as briefly sketched in sect. 1.1.3. In the sections of this chapter we discuss foundational aspects and distinctions wrt syntax and semantics, before the role of ontologies in connection with semantic interoperability / semantic integration is reviewed and analyzed. This leads to

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[185]Notably, Giancarlo Guizzardi provides useful additions for the context of computer science and foremost artificial intelligence [348, sect. 2.1.2, p. 20–21]. In [371] David Harel and Bernhard Rumpe present basic material in the same regard and with a similar background in (conceptual) modeling languages and in their case specifically inspired by work and observations on the Unified Modeling Language (UML) [@126].

[186]The distinction between referential semantics and functional semantics is a division based on the first question. *Referential semantics* assumes that there are some kinds of objects that are the meanings of linguistic expressions. In contrast, *functional semantics* views the meaning of a linguistic expression as its communicative function [264, p. 151] (based on [370, p. 101 ff.]).

additions to the four motivating questions $\mathcal{Q}_1$–$\mathcal{Q}_4$ (the latter are carved out in sect. 1.2 and listed jointly in sect. 1.3.2). Moreover, we develop our views on *ontological analysis* and related notions, which interlinks with the proposal of a meta-ontological architecture, developed and applied in the context of the General Formal Ontology (GFO) (see sect. 1.1.5). The central novel notion is that of an abstract core ontology (ACO), presented in sect. 2.3.3. The chapter concludes with a candidate proposal for such an ACO that focuses on categories and relations.

2.1 Formal Syntax and Formal Semantics

2.1.1 Grammar, Concrete and Abstract Syntaxes

TERMINOLOGY OF LINEAR LANGUAGES AND CHOMSKY GRAMMAR

Syntactically, we do not intend to restrict the discussion to particular forms of language, because logical languages like FOL as well as diagrammatic languages like UML are in our scope. Nevertheless, we will use concepts and terminology primarily established in the context of linear / word languages.[187]

Grammars are an established means to describe the syntax of languages, and it appears reasonable to restrict the formal languages under consideration to those for which any grammar-based syntax specification exists. The most general form of Chomsky type 0 grammars allows for arbitrary rules for language production. Rule elements involve the distinction between *terminal* and *non-terminal* symbols, where terminal symbols are also referred to as the alphabet of the language. The grammar "produces" the set of *expressions*, i.e., sequences (or more abstract structures) of terminal symbols, in the language in a precise manner.

ABSTRACT SYNTAXES

Grammars can be distinguished as to whether they define a *concrete syntax* or an *abstract syntax*. The former typically includes additional details over the latter, including auxiliary notation to linearize the language (e.g. parentheses) or to format expressions (e.g. whitespaces). Abstract syntax focuses on those syntactic categories which are relevant for defining the semantics for a language, cf. [85, sect. 8.7], [815, sect. 2.1–2.2].[188] According to [878, p. 484], an abstract syntax is defined by a grammar G_a which is a simplified version of a grammar G_c of a concrete syntax. Productions of G_a have the simple form

$$(2.1) \quad N \;\rightarrow\; T\, N_1 \ldots N_n$$

where T is a terminal which is uniquely assigned to the production, while the N_i are non-terminals. Abstract syntax in this form is well suited to defining a denotational semantics, because each (fragment of an) expression belongs to a *syntactic domain* of a non-terminal N, i.e., the language defined by N. A semantic domain is fixed for every non-terminal N, and a semantic function maps from the syntactic domain of N to its semantic domain (usually defined by structural induction on the abstract syntax) [878, p. 485].

Somewhat relaxed forms of abstract syntax are commonly used for the language definitions / standardizations which involve several concrete notations, for instance in languages related to the Semantic Web, e.g., RDF [@105] [381, 735], OWL [@91] [856], SWRL [432], and the Web Services Modeling Language (WSML) [@135] [184]. In the case of RDF, *RDF graphs* form "expressions" of the abstract syntax, justified by characterizing abstract syntax "as a way to describe the mathematical structure of a parsed expression, rather than the particular details of a surface notation." [381, sect. 1].[189] For our purposes, viewing languages from the point of view of abstract syntax rather than concrete ones is usually sufficient in the sequel.

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[187][348, p. 19] briefly introduces corresponding notions for diagrammatic / graphical languages.

[188]A related notion in the context of parsing is that of an abstract syntax tree as discussed in [4, 315, 316], which can roughly be seen as an abstraction of parse tree together with some annotations, cf. [315, sect. 1; e.g. p. 10, 62], [4, esp. p. 48].

[189]In [381, sect. 1] Patrick J. Hayes attributes the first abstract syntax to John McCarthy by linking to [@3], which is itself seemingly closely related to [569]. The attribution of firstness is confirmed in [85, sect. 8.3, p. 181] by recourse to [566], which shares its title with later publications of McCarthy [568, 569].

The parsing of complex concrete languages like programming languages or machine-readable syntaxes of logical languages typically proceeds in several steps, cf. [315, ch. 1], [4, 316]. Among these, *lexical analysis* produces a sequence of *tokens* that forms the input for *syntactic analysis*, the "actual" parsing. Tokens appearing in lexical analysis are symbols composed of a more limited set of terminals (often called *characters*) according to a lexical grammar. The resulting token language serves as the terminal set / input alphabet for syntactic analysis.

Several kinds of tokens can be distinguished in terms of their function, a.o., identifiers, keywords / reserved words, and separators. We will adopt a semantically inspired distinction between keywords and identifiers. By *keywords* we refer to those tokens which have an active role in the definition of the semantics of the language. In FOL, for example, logical connectives like "∧" or "→" are keywords in this sense because they are involved in semantic definitions such as Def. 1.5 (in sect. 1.5.2, p. 49), or they correspond directly to a non-terminal syntactic category used in a semantic definition. The remaining tokens that (may) appear in an abstract syntax tree are called *identifiers*. Predicate and function symbols, in totality the extra-logical signature / vocabulary can be seen as identifiers in FOL. In their case, the semantics may depend on a non-terminal such as predicate, but particular predicate identifiers are irrelevant for the semantic definition, where the syntactic category of, e.g., predicate identifiers appears as a variable. In general, parsing up to the level of abstract syntax trees "bottoms out" with keywords and identifiers.

RELATION TO THE METAPHYSICAL TYPE-TOKEN DISTINCTION

As a final remark, obviously, the notion of token established here is placed in a context based on a technological / computer science perspective on artificial languages. There is a metaphysical distinction between *type* and *token* in philosophy [873],[190] e.g., departing from examples such as counting two (letter) tokens of (letter) type 'p' in the (word) token that the reader sees in apostrophes to the left of the period of the present sentence, namely 'apple'. For disambiguation and brevity, let 'token$^\mathcal{C}$' and 'token$^\mathcal{P}$' denote the former and the latter notion of token, resp. Indeed, token$^\mathcal{P}$ is certainly closely related to token$^\mathcal{C}$, not at least "because *words* are our paradigm of types" [873, sect. 4]. On the one hand, if parsing processes are viewed as / abstracted to the recognition of entities as being instances / tokens$^\mathcal{P}$ of certain types (in philosophy) and / equated with certain syntactic domains of non-terminals, token$^\mathcal{C}$ may be seen as a specialization of token$^\mathcal{P}$ (e.g., limiting token$^\mathcal{C}$ to entities that are processable within / by computers prevents equating the two). On the other hand, in any more technological view of parsing, 'token$^\mathcal{C}$' is used for referring to (1) tokens$^\mathcal{P}$ that serve as input to the parsing process, but likewise to (2) their being terminals[191] of the grammar reflected as "parser-internal" entities. Moreover, especially types (in philosophy) involve several distinct analyses and further considerations wrt notions such as universals, kinds, and sets, a.o. and following [873]. Tokens$^\mathcal{P}$ are generally understood as particulars / (ontological) individuals, which appears to us to be rarely considered for symbols (in an intuitive reading) in computer science. Altogether, the identification or specialization of token$^\mathcal{P}$ with or by token$^\mathcal{C}$ is not straightforward and should be further investigated in another context.

2.1.2 Constants and Variables

CONSTANTS AND VARIABLES: TERMINOLOGY AND EXAMPLES WRT FOL, DLS, UML, XML

Another fundamental distinction at the border between syntax and semantics is to classify the terminals of a language into *constants* and *variables*.[192] These two notions should be understood as abstract as possible. The criterion for their distinction is whether or not their semantic interpretation is fixed for the overall language. Every language makes use of constants, and many languages allow for variables. Where the latter is the case, they can also be distinguished syntactically. Variables may occur in a *scope*, i.e., an expression or part of an expression in the language, which can be set for a variable by a declaration of it or

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[190]The category 'symbol' in GFO, mentioned in sect. 1.1.5.2, is inspired by this distinction, with a strong similarity to 'type'.

[191]More precisely, being members / instances of terminals, which we see as types.

[192]We use those terms for brevity, although *semantic constant* and *semantic variable* would be more precise, e.g., in distinguishing semantic variables from non-terminal symbols in grammars.

by other language elements. We assume that the same interpretation / value applies to all occurrences of a variable within the same scope.

Let us provide some examples of constants and variables in varied settings. Constants include (1) in a first-order setting constants of any kind (logical, individual, functional, and predicate constants[193]), (2) for DLs concept and role names, (3) in UML the names of classifiers (and classifiers themselves, possibly arguably), association names, etc, and (4) in XML element and attribute names. Variables, on the other hand, (A) can be directly found in classical predicate logic (further distinguishing individual and predicate variables), (B) cannot be found in standard DLs, and (C) are available in the form of parametrization in UML, e.g. see the entry on "template" in [720, p. 638]. Those claims apply to the common types of usage of these languages. Note that the notion of *parameter* is subsumed by the concept of variable meant above. Usually, parameters constitute a "second / further" level of variables, which appears as constant / pre-determined relatively to a first level of variables.[194] For example, [212, p. 23] presents some examples on parameters and their combined use with (another type of) variables, e.g. an expression (at the level of their text) $\phi(x, w)$ for which they declare x as an ordinary variable and w as a parameter, such that $\phi(x, w)$ should be understood to define a unary predicate. Overall, in languages like standard mathematical uses of set theory where variables as well as parameters are available, we argue that these two distinguish only in their scope (at the "highest level of interpretation").

DIFFERENCE IN SEMANTIC INTERPRETATION: FIXED VS VARIATION

Adopting a position of referential / denotational semantics, the nature of constants is simple – constants are symbols that represent something directly and with fixed interpretation for the overall language. Variables are symbols without any such fixed interpretation. Instead, their denotation can refer to several entities,[195] therefore by means of variables semantic facts can be formulated in a language in a much more compact style. Put differently, given an expression where a variable appears, the overall expression possibly denotes a plurality of circumstances. Depending on the availability of constants and by virtue of substituting constants for variables (which may be subject to restrictions), such circumstances can be represented (or derived) individually. This view on constants and variables complies directly with their general introduction by Tarski [808, p. 3–4]: "[A constant] has a well-determined meaning which remains unchanged throughout the course of the considerations. [. . .] As opposed to constants, variables do not possess any meaning by themselves.", cf. also [809, p. 17], cited in [@63].

FREE VS INTENDED REFERENCE OF CONSTANTS, ATTITUDES, AND REFERENTIAL EQUIVALENCE

Accordingly, constants can be understood as the ultimate syntactic means of reference (to anything).[196] Immediately related to reference and to the idea of "well-determined meaning", in combination with anticipating logical representations, is the question of whether that fixed reference is (expected to be) the / an intended one by someone using the language. Alternatively, that reference can be free / arbitrary in comparison with such intended entities, thereby still "fixed" and "well-determined", as is the case in individual FOL models. We shall return to these issues in ch. 4.

Another aspect concerns attitudes wrt the referents of constants (and overall theories), e.g., propositional attitudes (cf. e.g. [58, 794]), fictional entities / theories [489], etc. [739, esp. sect. 2–3], incl. a "typology of Terms of Dubious Reference" demonstrates the relevance of this question for biomedical ontologies. Put simply, the underlying question is with which attitude a constant is used (or statements are made). E.g., one approach is to assume a real, subject-independent existence of an entity that is (intendedly) referred to by a symbol in question. But clearly, languages can be used to formulate truths, beliefs, or (partially)[197]

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[193]The statement refers to FO languages that result from determining a specific signature. In contrast, software for FOL, such as corresponding parsers and automated theorem provers, is capable of processing the "overall" (modulo limitations in the naming of signature elements) FOL, as a family of languages parametrized by an extra-logical signature and a set of variables. Hence, e.g. predicates as well as variables yield identifiers, as discussed in sect. 2.1.1.

[194]One may consider iterations. We see the use of 'level' in relation to the distinction of object and meta level, cf. sect. 1.2.3.2.

[195]Which is *not* to say that there were single semantic referents of variables such that those referents were exhibiting a varying / variable character, as suggested by the notion of "variable number" which is mentioned and rejected by Tarski [808, p. 4].

[196]Regarding natural language and social interaction, a kind of exception is available to humans by means of pointing towards something.

[197]Intuitively, it appears hard to imagine completely ficticious descriptions / representations (to the extent that the *overall* conceptualization therein / underlying is unrelated to the real world), because then nothing at all should be understandable to a human reader.

fictitious situations. We do not make any particular assumptions or exclusions here, first of all. Nor will we try to accommodate an explicit account of, say, propositional attitudes, in the sense of the theories provided in [58, 794]. That means, theories in the scope of this work capture only propositional contents, without explicating the attitudinal relationship to the language user by dedicated language constructs. Yet we shall require that all referents of constants (and all statements within a theory) are based on an "intentionally coherent picture" at the side of the language user, excluding contradictory beliefs, for example.

Even if constants stand for their denoted entities, they maintain a symbolic aspect insofar that the same entity can be denoted by several constants. That yields equivalence classes of constants induced by this *co-reference*, i.e., referring to the same semantic counterpart. Similarly to constants, syntactically distinct expressions of a language can represent the same semantic situation. However, actual semantic considerations are subject to ch. 3 and 4. Next we only touch the notion of formal semantics in general, for classification and scoping purposes.

2.1.3 Formal Semantics

FORMAL SEMANTICS IN GENERAL

Formal semantics, understood as a subfield of computer science, covers approaches to precisely capture the semantics of (artificial) languages, given their clear syntactic specification, e.g. in terms of a grammar or other description of (esp. abstract) syntax. Generally speaking, formal semantics deals with establishing interpretations for syntactic expressions, where interpretations are typically mathematical entities / are expressed in a mathematical formalism, in order to achieve the desired degree of precision. In different fields, various types of interpretation objects have been introduced. Searching for types of formal semantics wrt logic, one quickly finds proof-, model-, and gametheoretic semantics as well as truth-value[198] and probabilistic semantics [@43]. [816, sect. 6, p. 295], part of a handbook in linguistics, provides a very compact survey on different kinds of formal systems in logic, thereby offering several pointers to influential works, broadly classified into proof-theoretic and model-theoretic methods. At the side of programming languages, broad established classes of semantics comprise denotational, operational, or axiomatic approaches, cf. [515, p. 337ff.], [85, ch. 3]. While the assignment / construction of mathematical objects remains the core idea of denotational semantics also in the context of programming languages, the actual mathematical objects that are utilized in this context, such as (mathematical) functions, diverge from model-theoretic structures and semantics. Therefore note that the terms 'denotational' and 'referential semantics' are read on a high level of abstraction herein, but with Tarski-style model-theoretic semantics as the default specialization.

LANGUAGE FOCUS, AND STATIC AND DYNAMIC LANGUAGE ASPECTS

Given the interest in representing ontologies, our scope is set to languages developed for representation and / or reasoning, and mainly refers to the fields of (symbolic) knowledge representation and of modeling languages. In many of the respective languages (see sect. 1.1.3), a referential, model-theoretic semantics is available, either directly like for FOL and DLs, or indirectly by translations / reductions into another language that has such a semantics already, e.g. remembering proposals for formal semantics of the OBO format [182] in sect. 1.1.3.5.

Yet another distinction regarding the semantics of languages must be mentioned. In conceptual modeling, there is a common distinction that system models cover *static* and *dynamic* aspects[199]. Inspired by this distinction we see an analogy for most languages under consideration.[200] Although not strictly / literally applicable to all cases, mathematical structures which directly serve as interpretations of syntax form a static semantic component. Moreover, in most cases there is a form of dynamics, exemplified by reasoning in logic and by computation for programming languages. For instance, while a single FOL model of some first-order theory would fall into the static realm, the consequence relation between theories and sentences

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Accordingly, at some level there seems to be the need to rely on notions which can be associated with existing knowledge.

[198]Truth-value semantics adopts a substitutional approach to quantification, see e.g. [514]. [619] falls into this category and is discussed in sect. 7.2.2.3 (among related work).

[199]In UML terminology this is refers to structural classifications and dynamic behavior, respectively [720, e.g. already on p. 3]

[200]One might also draw an analogy to the distinction of semantics and pragmatics.

bridges to dynamics (and even more does its algorithmization, as far as that is possible). Such dynamics is typically defined with recourse to the static parts of a semantics.

We conclude the very general considerations on formal semantics at this point, whereas further detailed issues on model-theoretic semantics pervade the subsequent sections and chapters, intensely until the provision of ontological semantics in ch. 4. Its justification arises mainly from the next section.

2.2 The Role of Ontologies in Semantic Integration

GOALS AND SECTION OUTLINE

Starting from Postulate A, which preceded our theses in sect. 1.3.1, namely the demand for ontology in order to enable semantic interoperability, in this section we study corresponding current formal proposals of how ontologies are employed in establishing / enabling semantic interoperability, semantic integration, and semantic translations. If not already achieved by existing approaches, a desirable long-term goal would be an approach that allows for provably sound semantic translations. These do exist (one may say), e.g., recalling just the valuable work around the Hets toolset [@54] [594] and the related Ontohub repository [@81] [598], introduced in sect. 1.2.1. Yet in the same section we indicate some hesitation that is now substantiated, starting from general observations on semantic translations and the reading(s) of 'semantics' that they (should) refer to. That theme continues to analyzing how ontologies account for intensional semantics according to the literature. Anticipating the effects of our resulting views, those views lead us to new questions and to the objective of working toward a novel kind of model-theoretic semantics, eventually.

2.2.1 Semantic Translations and Formal Semantics

TRANSLATIONS AND PROBLEMS: FORMAL SEMANTICS AND ENCODINGS

The fields of knowledge representation and modeling languages face the same situation as artificial languages in general, namely offering a tremendous number of more or less specialized formalisms, as (partially) illustrated by sect. 1.1.3. These formalisms are designed to represent knowledge and / or information in symbolic form for different, yet overlapping purposes and tasks. Due to that language diversity, there is a common need to translate between different representations. Of course, such translations should maintain the contents of what is represented using the source formalism, i.e., the target representation should be *semantically equivalent*. The usual, perhaps natural / "tautological" understanding of 'semantics' in connection with formal languages is *formal* semantics, which is to be preserved by translations.

However, at this point we see different problems arise wrt the formal semantics of the formalisms under consideration. If these semantics are similar, for instance like the standard Tarskian model-theoretic semantics of FOL and DL,[201] it is rather straight-forward to establish translations on the basis of the formal semantics that are adequate for a number of purposes / tasks. The problem becomes more involved when dealing with formalisms whose formal semantics relies on different mathematical constructions – statically, these are simply different. Commonly, it is then examined whether the dynamics of one formalism can be simulated by another. That means to find a translation such that the dynamic behavior of the target language (or corresponding relations among its expressions, e.g. consequence relations) reflects or comprises the behavior associated with the source language. For static representation languages, this refers mostly to reasoning, understood abstractly as a consequence relation between explicitly given expressions and implicit, derivable expressions (i.e., in the sense of an abstract logic as defined in sect. 1.5.1).

Especially with respect to simulating the dynamic behavior of one language within another, many constructions rely on *encodings* of language constructs of the source language by those of the target language. This technique may even be used if there is a common approach to formal semantics, e.g., in order to reconstruct (kinds / means of) expressions that are not directly / "naturally" available in the target language. For instance, polyadic / n-ary relations as available in FOL are encoded in various ways into such[202] description

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[201]cf. sect. 1.1.3.2 for their relation
[202]"such" and currently *most* DLs, remembering sect. 1.1.3.2, esp. around FN 57 on p. 11.

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logics that offer only binary relations, cf. the suggestions in [631] (2006) by Natasha[203] Noy and Alan Rector, and subsequent works [260, 307, 535, 536], incl. ontology design patterns (ODPs)[205] for n-ary relations [@66]. Notably and interestingly, those n-ary relation encodings focus on reflecting / capturing atomic facts involving an n-ary relation, instead of ensuring (more of) their overall original formal semantics.[206] But the discussion can be extended to formal approaches to encodings between languages where formal semantics is preserved, not only in the light of Hets/Ontohub mentioned above. These include, from a classical logical perspective, theory interpretations [221, sect. 2.7, p. 154–163] (see sect. 2.2.2.1 for intuitions, sect. A.1.2 in the appendix for the formal definition) and Szczerba's notion of interpretability [798, 799] between two theories within the same logic. Beyond that, there are more general frameworks abstracting from particular logics (often on a category-theoretic basis), a.o. [228, 283, 472, 734] [207]. Other types of encodings arise from relating different formalisms purely in terms of their "representation constituents" (called "representation primitives" in [289, p. 29] in connection with knowledge representation ontologies as introduced in [841], cf. sect. 1.1.4.3. One may take simple, approximate correspondences between formalisms (which can be found commonly) as a starting point for such encodings.[208] Of course, there are also elaborated accounts, e.g. wrt UML, RDF(S), OWL, CL and Topic Maps [@123] [448, 654] in the Ontology Definition Metamodel (ODM) [@75] [646] published by the OMG [@78].

DISTINGUISH INTENSIONAL AND FORMAL SEMANTICS / MEANING

Frequently, another notion of semantics is involved at least implicitly, different from that of formal semantics introduced / discussed above. It is this other kind of semantics which is also employed to explain why something is an encoding rather than a native use of a language. We refer to this type as *intensional* meaning / semantics of some representation, and for language variability likewise as *conceptual* and as *intuitive* meaning. The three terms are inspired by different connotations that we consider to be tightly interconnected. "Intensional" derives from intension in the context of semiotics as well as from shared objectives with intensional logics. "Conceptual" is borrowed from our use of conceptualization, that is characterized in brief by the phrase "the language-independent content of an ontology" in the context of the terminological clarifications in sect. 1.1.2.2). "Intuitive" is to convey the idea of being prior to formalization, in the above sense of formal semantics. This priority leads us to the encoding aspect from a different angle than translations among two formalisms. Indeed, *formalization*, by its very nature – if that is understood to be the expression of any content in terms of formal / mathematical means and entities – involves the encoding of content in *other* terms. Consequently, by viewing content itself as being conceptual in nature, the formalization of content involves encodings, unless the conceptualization is solely concerned with formal / mathematical entities itself.

Accordingly, the notion of conceptual meaning is applicable to natural and artificial languages, in particular it can also be applied to artificial languages without formal semantics. UML and XML / XML

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[203]Natasha follows the author specification in [631], while Natalya is the official / more formal first name [@67].

[204]📖 For a discussion of contributions from our side in [535, 536] see sect. 5.4.2.

[205]See, e.g., [257, 674] for the notion of ODPs.

[206]In description logics, [139, sect. 6.5] discusses theoretical issues of reifying relations by a new DL concept and as many DL roles as the arity of the relation to be captured. In particular, models may comprise multiple objects / relation instances that connect the same argument objects, but cannot be conflated without violating axioms, e.g., cardinality constraints, although there should only be one relation instance / reified tuple for each sequence of arguments. Yet the authors refer to the generalizability of a result for binary relations [139, sect. 4.4.2, Lemma 4.8, p. 1607] which justifies reasoning with this encoding. Moreover, "[b]y exploiting this idea, Calvanese, De Giacomo and Lenzerini (1997) [138] introduce the DL $\mathcal{DLR}$ [mentioned in FN 57], which extends $\mathcal{ALCQI}$ by boolean combinations of n-ary relations, and show that reasoning over $\mathcal{DLR}$ knowledge bases is again EXPTIME-decidable." [139, p. 1624]. [39] is another early publication on reification in DLs, using a role for membership with an adapted language semantics and a sound and complete reasoning algorithm.

[207]Cf. also the last but one paragraph of sect. 1.2.1 on ontology matching.

[208]For example, [505, Table 2.1, p. 23] presents the following "terminological correspondences":

OOP Systems	Frame Systems	DLs
instance	frame, instance, individual	instance, individual
instance variable, slot, attribute	slot	role, attribute
value	filler, value	filler
class, type	frame, schema	class, concept

Schema are prime candidates in this respect.[209] Similarly, translations between languages used for expressing ontologies were considered early, e.g. the conversion of UML into OWL ontologies discussed in [268] appeared in the year of the standardization of OWL 1 [572].

Clearly, the idea of intensional meaning can be found in numerous other works. First and foremost, this is a central topic of (mainly linguistic) semantic theories, as, e.g., the three volumes [550–552] on "Meaning" suggest, or cf. also [264, esp. sect. 5.1.2, p. 152 ff.]. Obviously, the distinction between intension and extension is also at the heart of the semiotic triangle[210]. Intensional logics form a separate area motivated by this distinction [243, 834]. However, there is further support from other fields not immediately concerned with the issue, at least from an application perspective. In this connection, Weidong Chen, Michael Kifer and David S. Warren, the authors of HiLog [151], cf. sect. 1.1.3.4, argue for a separation of the intension and extension of predicates, for example.[211] Further rationale in [151] traces back to [548, 549], in which Anthony S. Maida and Stuart C. Shapiro argue that extensions / the world is not directly accessible to cognitive agents, hence KR should not act on extensions directly. Similar reference to "the (real) world" or, more precisely a distinction between real-world semantics and formal semantics, is gathered by Giancarlo Guizzardi in the realm of AI, citing several sources in [348, p. 20–21]. He points out "Architectural semantics, under the term *real-world semantics* (Partridge, 2002 [658]; [further references omitted]) have been perceived as fundamental for semantic interoperability and semantic integration of information sources." [348, p. 20]. Referring to another view of further authors he concludes for modeling: "This view, supported here, emphasizes the precedence of real-world concepts over mathematical concepts in the design and evaluation of modeling languages or, in other words, the precedence of real-world semantics over purely mathematical semantics." [ibid., p. 21]. We adopt the same position here, postulating in addition that commonly the users[212] of a translation service between two representation languages are interested in the equivalence of (sets of) expressions on the basis of their intuitive semantics rather than referring to the formal semantics. Ideally, though, both cases apply in parallel (in our opinion, rarely enough, especially in cases of moderate or higher degrees of complexity[213], unfortunately).

EXAMPLE OF INTUITIVE SEMANTIC EQUIVALENCE: UML, OWL AND FOL

To illustrate the overall point more directly and set up a case for reconsideration below, consider the UML class diagram in Fig. 2.1 (ignoring its caption), the DL axiom (2.2), and the FOL sentence (2.3).

Figure 2.1:
UML representation of "Lion specializes Animal".

(2.2) $\qquad$ $\mathsf{Lion} \sqsubseteq \mathsf{Animal}$

(2.3) $\quad \forall x \,.\, \mathsf{Lion}(x) \rightarrow \mathsf{Animal}(x)$

Assuming some familiarity with the concepts of UML class diagrams, DL, and FOL, and in particular, the intended reading of the specific arrow[214] in Fig. 2.1 and the DL symbol $\sqsubseteq$ as the extensional is-a relation (cf. sect. 1.1.2.2), the reader may agree that Fig. 1 and these two sentences (are meant to) represent the same fact, namely that the category Lion specializes the category Animal. We argue that this judgement is (largely) independent of the formal semantics of each formalism, but is rather based on the fact itself that is to be expressed, in combination with the linkage / usage of the respective notational elements (like DL concepts

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[209]Qualifications regarding UML semantics are given in sect. 1.1.3.4.

[210]"The" semiotic triangle is mentioned above in the second paragraph of sect. 2.

[211]Although their reading of 'intension' seems to collapse either with names themselves or (what we would call) their referents: "For instance, in λ-Prolog, one aspect is the *meaning* of the expression as a λ-term (or, more precisely, as an equivalence class of λ-terms), which we call *intension*; the other aspect is the relation or a function associated with the expression, which we call *extension*. [...] For instance, in Henkin's semantics of Church's type theory [23] [[391] herein], it makes no difference whether predicate variables are considered to range over the domain of interpretation for predicate names (predicate intensions) or over relations (predicate extensions), since each extension now becomes associated with exactly one intension, and vice versa." [151, p. 189]. For λ-Prolog see originally [615], and [616] in the context of a general handbook on logic programming.

[212]'Users' here standing for, say, "end-users of information technology". Of course, translations meant to transfer the dynamics of formalisms, e.g., reasoning services, are intended to maintain the formal semantics.

[213]in an intuitive sense, not referring to computational complexity here

[214]In order to keep the argument simple, ignore the feature inheritance and ownership aspects of generalization [648, p. 141], in UML.

and UML classes) with certain kinds of entities from an ontological point of view (like categories). Accordingly, the interrelations between the formal semantics are of minor relevance. Now, it is the case that (2.2) and (2.3) in the example fit the common formal semantic equivalence[215] between FOL and DLs. But, in contrast to Welty and Ferucci [869, esp. sect. 2.1] (cf. sect. 1.2.3.1), we defend the position that the FOL ground sentence (2.4) is a viable alternative to represent exactly that fact, utilizing a binary predicate specializes.

(2.4)   specializes(lion, animal)

Note that, in the context of the present examples, (2.3) and (2.4) constitute two distinct FOL theories, each comprising exactly one sentence. Lion and Animal in (2.3) are predicates, but in (2.4) lion and animal are (logical) individual constants. Thus no predication over predicates is involved in (2.4). Below more must be said on the last option (2.4), see sect. 2.3.3. The point here is that the formal semantics of FOL and (standard) DLs cannot serve in justifying a translation from (2.2) to (2.4). Instead, the standard translation respecting the formal semantics would just map (2.4) (seen as an ABox statement in DL) onto itself, this time viewed as a FOL atom.

ANOTHER EXAMPLE: XML SCHEMA AND OWL TRANSLATIONS

There are further well-known cases that can be adduced as instances of desired conceptual translations. One of them is the migration of the Foundational Model of Anatomy (FMA) [@38] [714, 715] (see sect. 1.1.4.3) from a frame-based representation into OWL, which had competing proposals, cf. FN 76, p. 14 in sect. 1.1.3.4). As another detailed example, we consider relationships between (1) the Extensible Markup Language (XML) [112] and its schema language XML Schema [854] and (2) Semantic Web languages, primarily OWL. Apart from the syntactic layering of the most widely used concrete syntaxes of the latter on the former, these languages remain in two fairly separated worlds[216] – and there are sufficient differences in purpose, context, and background for the division, indeed. Nevertheless, data / information integration is an application that concerns both worlds. In this connection, several approaches to converting XML Schema files (and XML files based on them) into OWL ontologies were developed [21, 91, 555].[217] Obviously, if one converts between two such files, the formal semantics of these languages are hardly relevant. XML Schema can basically be seen as a grammar language for XML subsets and the semantics of an XML document / expression mainly yields a tree of symbols, cf. [405, p. 21].[218] OWL with its foundation in DL in general and its model-theoretic semantics is very different. Analogously to the specific expressions in the previous set of examples, the question that arises for conversions between XML and OWL is: In which case (and why) should a conversion be considered "semantically correct"? We believe that one approach of accounting for an answer is to rely on the intuitive semantics associated with the constituents of XML documents, e.g. elements and attributes.[219] Note that a dynamic / functional perspective may provide different answers. E.g., imagine an OWL visualization tool that shall be used to display an XML document. This might justify a special-purpose translation from XML to OWL, such that it yields the desired visualization. However, such cases lead to translation results that are usable in a specific, intended procedure, instead of the more general character of stating (data,) information, or knowledge that we see behind [21, 91, 555].

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[215]wrt which [33, sect. 4.2, p. 150] for $\mathcal{ALC}$ or [405, sect. 5.2.2] for $\mathcal{SROIQ}$ are cited in sect. 1.1.3.2.

[216]At least, proposals with a unifying semantics like [660] have not been widely adopted. [660] presents "a unified model for XML and RDF" [p. 443] in terms of a model-theoretic semantics for XML XQuery and XPath [240]. ([240] is a version of the specification published after [660].)

[217][555] presents work on OntoCAPE [@80], a large-scale ontology for chemical process engineering. In this context, an XML to OWL converter has been developed, see [@80].

Another XML-related branch of work of relevance to the present argument but not further discussed here concerns the distinction between conceptual XML schemata and logical XML schemata [244].

[218]A rigorous formal treatment of another XML "schema language", namely of Document Type Definitions (DTDs) [112, sect. 2.8], is presented in [81].

[219]A compilation of several aspects involved in considering the semantics of XML documents against a background of applied linguistics is [771].

ARGUMENT SUMMARY
Let us put the important points made thus far on record, together with our opinion about the relevance of semantic approaches for translations between languages.

- For languages, static and dynamic aspects are to be distinguished. Static aspects refer to "what is stated" by expressions of a language, whereas dynamic aspects capture the "interaction" of expressions (e.g. deduction / inference, computation, etc).
- Two types of semantics for languages must be distinguished, namely formal accounts of semantics and intensional / conceptual accounts. Accordingly, "semantic translations" between languages may refer to their formal semantics (if available) or to their intensional semantics.
- There are many valuable theoretical results on the formal semantics of languages and their interrelations, and, consequentially, on formal semantic encodings. Such results capture formal properties and serve well for translations wrt dynamic aspects (including simulations).
- In practice, translations according to intensional semantics and with predominance of static aspects play a role of at least equal importance as formal, dynamics-oriented translations.[220]

2.2.2 Ontologies as Intensional Semantics

This section discusses the role of ontologies in providing an account for translations based on "what is stated" by language expressions wrt the conceptual / intensional semantics discussed just above. Moreover, this type of translation suggests itself as the one of primary relevance for our intended major application of providing GFO in distinct formats.

2.2.2.1 Standard Account of Ontology-Based Semantic Integration

ONTOLOGIES FOR SEMANTIC INTEGRATION: THE CENTRAL IDEA

At the beginning of ontology research in computer science, more precisely, in the AI fields of KR and KBS, ontologies were invented to tackle the problems of *semantic integration / interoperability* and of *semantic translations* [158, 207, 311, 327, 618].[221] In the light of the previous section, 'semantic' refers to intensional semantics here. The aim was to reuse knowledge represented in one KB (with its specific kind of KR scheme) in other KBs (relying on different representation formalisms). We claim that this corresponds to an intensional and primarily static view, whereas a translation in terms of the different representation schemes / formal semantics would address the simulation of the dynamics / functional behavior of one KB in another.[222] The central idea of this ontological approach was to refer to a common ontology Ω in order to share common meaning [312, 330]. More precisely, in the terminology adopted for this thesis in sect. 1.1.2.2, one refers to a conceptualization Ω, which is specified / formalized explicitly through a formalized ontology $R(\Omega)$. It is Ω and $R(\Omega)$, respectively, that an X (a human or software agent, a program, or a language) may *commit* to, i.e., X presupposes / accepts / understands the entities referred to in Ω and their constraints and interrelations as given by Ω [311, p. 201], [313, p. 908].[223] The general idea of referring to

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[220]In a wider discussion, this connects to the philosophical debate about 'logic as language' and 'logic as calculus', see [160, esp. ch. 1] and sect. 2.2.2.2 herein.

[221]Cf. remarks on the first usage of the term in computer science and AI in sect. 1.1.1, esp. FN 3 on p. 2.

[222]Yet again, in the ideal case both static and dynamic aspects could be covered completely.

[223]In many cases, it is not strictly separated whether ontological commitment commits one to the conceptualization Ω or to the formalized ontology $R(\Omega)$. On the one hand, the central idea suggests priority for committing to the conceptualization, because this *is* the content / ontological theory that one commits to. Thomas Gruber states "Ideally, we would like to be able to specify ontological commitments at the knowledge level [624]." [311, p. 201, citation adapted]. On the other hand, formalized ontologies (or informal ontology representations) are a necessity to communicate about conceptualizations. In the context of his formal account, Nicola Guarino adopts committing to conceptualizations [330], see below in the main text.

Some remarks on the term 'knowledge level' and related notions are due. 'Knowledge level' in Gruber's article refers to an analysis of Allen Newell, originally presented in [623]. There is related earlier work by Ronald J. Brachman [105], [106] (reprinted as [108]). In [106], Brachman discusses five levels for the understanding of semantic networks (see sect. 1.1.3.4), called "the implementational, logical, epistemological, conceptual, and linguistic levels" [106, p. 4]. Brachman's work served as one of the starting points for Guarino's discussion of an / the 'ontological level' [326, 328], [327, sect. 4], recently reconsidered [333]. Certainly, our twofold distinction between the intensional and formal semantics of formalisms is much more coarse-grained, leaving the adoption and possibly adaptation of more fine-grained distinctions as a worthwhile future task.

a common ontology has become increasingly widespread since then, it is echoed in numerous publications, and has led to a large number of proposals that apply ontologies in one way or another, cf. sect. 1.1.1. A much more limited number of accounts aims at capturing that general idea more formally.

GUARINO ON CONCEPTUALIZATION, ONTOLOGY, COMMITMENT

For the case of a language L (committing to Ω), Nicola Guarino made important contributions to the understanding of and interrelationships among the notions of conceptualization, ontology, ontological commitment and others [330], cf. also the presentations in [339], [348, sect. 3.3.1], and [733]. He captures these notions formally with respect to a possible worlds approach [267, 887].

For the intuitions behind it[224], let us start from all maximal sets of states of affairs[225] [330, sect. 2.1, p. 5] over a given domain of entities, such that it is possible for those states of affairs to obtain ontologically, i.e., they are ontologically admissible, they are possible wrt one's conceptualization (in our sense of conceptualization, see sect. 1.1.2.2). Guarino refers to these maximal sets of states of affairs as possible worlds. Consider in addition a set of relations on an intuitive basis, referred to as intensional / conceptual relations (in Guarino's terminology (prior to formalization [330, p. 5]), compatible with ours), which occur in each possible world. Formally, one may consider a single possible world as a set-theoretic structure that comprises exactly one extension (set of argument tuples; called "admittable extension" [ibid.]) for each of those relations in that world. A conceptual relation as a whole, i.e., in a crosscutting view over all possible worlds, can then be captured formally by a function that maps each world to the respective extension of the relation in that world.[226] This overall setup allows Guarino to formalize ontological commitment as a mapping from a vocabulary to the intensional relations and, eventually, defining set-theoretic structures for the same vocabulary as *intended structures* :iff they correspond to a possible world (or *are* a world in the formal understanding of the term) under the given ontological commitment.

On this basis, let us briefly[227] recapitulate the account of Guarino more formally, where we partially modify and adapt the notation to fit the present work.

2.1 Definition (conceptualization according to Guarino)
A *conceptualization* is formalized as a possible world structure $C := (D, W, R)$ with these constituents:

- D is a set (or domain) of individuals,
- W is a set of worlds ("sets of maximal states of affairs" [330, p. 5]), and
- R is a set of *conceptual relations*, i.e., of functions of the form $\rho : W \to \mathcal{P}(D^n)$ where $n \in \mathbb{N}$ is an arbitrary, but unique arity associated with ρ. ❑

Based on this formalization of conceptualization, *ontological commitment* of a language L (of predicate logic, omitting functions) to a conceptualization $C = (D, W, R)$ is captured as a *particular*[228] interpretation $\mathcal{I}_{\text{int}}$ of the signature of L into C, $\mathcal{I}_{\text{int}} : Sig(L) \to D \cup R$. More precisely, each individual constant c is interpreted by a member of D (independent of worlds, which links closely to the constant domain assumption[229] underlying C), $\mathcal{I}_{\text{int}}(c) \in D$, whereas a conceptual relation in R is assigned to each predicate P, presuming compatible arities, $\mathcal{I}_{\text{int}}(P) \in R$. Eventually, the notion of an intended structure[230] can be defined as follows, based on [339, Def. 3.3, p. 10–11].

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[224]We leave the treatment of entities referred to by (logical) invidual constants to the technical exposition below.

[225]Cf. [818] for the notion of states of affairs (and related concepts and issues in philosophy).

[226]Guarino notes already that this account of intensional relations has some limitations, e.g. for necessarily co-extensional notions [330, p. 5] (the example of equilateral and equiangular triangle is also mentioned in sect. 1.2.3.1).

[227]Some "intermediate" definitions and terms from [330, 339] are omitted.

[228]In this referring to a single, individual interpretation we see a difference between Guarino's understanding of conceptualization and ontological commitment, and the notion of ontology-based semantic integration, which is discussed in detail below. In contrast, these notions are considered in close analogy in [733]. However, [330] does not suggest that concepualizations could be specified directly, in terms of languages of modal logic.

[229]That assumption is also referred to as fixed-domain or possibilist approach. See [267, sect. 13] for the definition that it "assumes a single domain of quantification that contains all the possible objects" and for further introduction.

[230]We use intended *structure* instead of Guarino's *intended model*, because we reserve the term 'model' for structures that satisfy a sentence or theory.

2.2 Definition (intended structure ("intended model" in [330, 339]))
Let $C = (D, W, R)$ a conceptualization, L a language, and $\mathcal{I}_{\text{int}} : Sig(L) \to D \cup R$ an ontological commitment of L to C. A set-theoretic $Sig(L)$ structure $\mathcal{A} = (D, \mathcal{I}_{\text{ext}})$ is an *intended structure* wrt C and $\mathcal{I}_{\text{int}}$:iff $\mathcal{I}_{\text{ext}}$ and $\mathcal{I}_{\text{int}}$ agree with each other on all constants and wrt (at least) one world in W by satisfying

- $\mathcal{I}_{\text{int}}(c) = \mathcal{I}_{\text{ext}}(c)$ for all individual constants $c \in Const(Sig(L))$ and
- there is a world $w \in W$ such that for all predicates $P \in Pred(Sig(L))$ there is a $\rho \in R$ such that
$$\mathcal{I}_{\text{int}}(P) = \rho \quad \text{and} \quad \rho(w) = \mathcal{I}_{\text{ext}}(P). \qquad \qquad \square$$

An extensive illustration in these respects is elaborated in [339, esp. p. 3–12], [348, p. 82] presents a more compact example. Eventually, the role of ontologies is defined as follows by Guarino.

> "An ontology is a logical theory accounting for the intended meaning of a formal vocabulary[footnote omitted], i.e. its ontological commitment to a particular conceptualization of the world. The intended models of a logical language using such a vocabulary are constrained by its ontological commitment. An ontology indirectly reflects this commitment (and the underlying conceptualization) by approximating these intended models." [330, p. 7]

ONTOLOGY DEFINITION AS WORKING HYPOTHESIS

We adopt this view as a working hypothesis, although some issues remain to be further analyzed and / or are reconsidered below. For instance, an important question is whether the formal notion of conceptualization above suggests ways to represent language-independent conceptualizations, our main reading of 'conceptualization' as introduced in sect. 1.1.2.2. Similarly the notions of possible objects and worlds would require ontological clarification. Leaving those questions aside here, we first pursue completing the brief survey of precisely and formally rendered accounts of ontology and their application for (conceptual) semantic integration / translations. The latter topic is only alluded to in [330].

STANDARD ACCOUNT OF SEMANTIC TRANSLATIONS IN FORMAL DETAIL: REFERENCES

The first formally established account of ontology-based semantic translations, wrt ontologies represented in FOL, is presented by Mihai Ciocoiu and Dana S. Nau in [156]. More recently, in [733] Marco Schorlemmer and Yannis Kalfoglou profoundly present the "standard" account of semantic integration on the basis of an ontology by defining the notions of *semantic integration* and *ontology-based consequence*. They first recapitulate a classical first-order form of (formal) semantic integration [733, p. 134–136] of two theories $T_1 \subseteq L_1$ and $T_2 \subseteq L_2$ wrt a reference theory $T \subseteq L$. This is based on the first-order languages L_i and L, on their formal first-order model-theoretic semantics, and on the notion of *theory interpretations* from the T_i to T, cf. for the latter sect. A.1.2 in the appendix or [221, sect. 2.7, p. 154–163], [156, p. 541], [733, p. 134]. The classical form is advanced to analogous definitions on the basis of (category-theoretic) institutions, cf. [733, sect. 4, p. 138–140] for a short, [283] for the original introduction. In both cases, the notion of (formal) semantic integration between T_1 and T_2 (wrt T) is then used to define a relation of *ontology-based consequence*, which allows for calling a formula $\phi \in Lg(T_1)$ an ontology-based consequence of the theory T_2.[231] This could be extended easily to two theories as arguments, e.g. that T_1 is an ontology-based consequence of T_2. The latter form is also employed in [156], which refer to the case of T_1 being an ontology-based consequence of T_2 (wrt T) as T_1 being an *ontology-based partial translation* of T_2 (wrt T). Ciocoiu and Nau additionally introduce *ontology-based translation* for T_1 and T_2 being mutually ontology-based partial translations.

INDICATION OF PROBLEMS

There are some idiosyncrasies of the standard approach that relate mainly to the statement that "interoperability should be formalised in terms of logical consequence" [733, p. 136]. Though we agree with this view in general, some care is required wrt logical interactions between the definition of ontology-based consequences and utilizing a theory as a reference, like in [733, Def. 1, p. 134 and Def. 7, p. 142]. We find that in some cases these definitions are insufficient for a notion of precise, intensional *semantic translation*. Such notion should enable the user of a language L_1 to translate an expression $e_1 \in L_1$ into $e_2 \in L_2$, thereby being justified in claiming that e_2 is a "proper" translation of e_1 (possibly assuming a background

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[231] Definition 2.3 provides the account in detail in modified form. Some further notes beforehand shall prelude resulting problems.

ontology Ω). Put differently, e_2 should be characterizable as being intensionally equivalent with e_1. We claim that it remains an open question in particular for ontologies themselves, how ontology-based consequence and ontology-based translations might serve this purpose. The main reason is that the use of logical consequence appears rather coarse-grained, assuming that an ontology / conceptualization is represented by a formalized ontology / logical theory. Given the idea of *first-order ontology-based consequences* [733, p. 136], it is indeed tempting to view e_1 and e_2 as sharing a common intension if they are consequences of each other. However, this has counterintuitive effects which we elaborate in the next sect. 2.2.2.2.

DEFINITIONAL VARIANT OF ONTOLOGY-BASED CONSEQUENCE AND EQUIVALENCE AS IN THE LITERATURE

For the purpose of our analysis, we provide a modified version of defining the relevant notions of ontology-based consequence and equivalence compared to those presented in [156, sect. 4] and [733, Def. 8, p. 142]. Fig. 2.2 visualizes the major setup among the languages and the ontology involved in Def. 2.3, comprising two languages L_1 and L_2 (of arbitrary kind) that are meant to be semantically integrated by means of a formalized ontology T_Ω and translations from them to the language L_Ω of T_Ω. For simplicity, we assume that L_Ω is capable of accommodating translations from L_1 and L_2 (or otherwise is extended beforehand). Note that Def. 2.3 can be read at

Figure 2.2: Assumed background in Definition 2.3 of ontology-based consequence and equivalence.

the abstract level of logical notions as introduced in sect. 1.5.1. In this connection remember that $=_T$, $\models_T$, and $\equiv_T$ refers to equality, entailment, and equivalence, resp., modulo a theory $T \subseteq L$ wrt the logic that L is formulated in.

2.3 Definition (ontology-based equality, consequence and equivalence)
Let $T_\Omega \subseteq L_\Omega$ be a formalized ontology that represents a conceptualization Ω in a logical formalism L_Ω, assuming consistency of T_Ω, i.e., $Mod(T_\Omega) \neq \emptyset$. Let L_1 and L_2 two arbitrary languages, $\tau_i : L_i \to L_\Omega$ for $i \in \{1, 2\}$ translation functions from L_i to L_Ω wrt T_Ω,[232] and extend the τ_i canonically to sets of expressions $S \subseteq L_i$ by $\tau_i(S) := \{\tau_i(e) \mid e \in S\}$.
On this basis, for any two expressions $e_1, e_2 \in L_1$ and an arbitrary theory $T_1 \subseteq L_1$ define the three subsequent notions, all of which are parametrized wrt T_Ω and the τ_i.

(2.5) e_1 and e_2 are *ontology-based equal*   :iff $\tau_i(e_i) \in Tm(L_\Omega)$ and $\tau_1(e_1) =_{T_\Omega} \tau_2(e_2)$.

(2.6) e_2 is an *ontology-based consequence* of T_1 :iff $\tau_1(T_1) \subseteq L_\Omega, \tau_2(e_2) \in L_\Omega$, and $\tau_1(T_1) \models_{T_\Omega} \tau_2(e_2)$.

(2.7) e_1 and e_2 are *ontology-based equivalent* :iff $\tau_i(e_i) \in L_\Omega$ and $\tau_1(e_1) \equiv_{T_\Omega} \tau_2(e_2)$. $\square$

A number of comments on this definition are in order, where we start with the relationship to [156, sect. 4] and [733, Def. 8, p. 142]. First of all, the notion of ontology-based equality is an addition wrt terms following the spirit of ontology-based consequence and equivalence. Apart from that, Def. 2.3 is simplified to the extent that it omits the aspect of formal semantic integration, i.e., there is no requirement that τ_1 must be a theory interpretation of T_1 (or $\{e_1\}$) into T_Ω. We expect this aspect to be considered by the authors of [156, 733] as one that is central for "semantic" integration, paying attention to the dynamics / formal semantics of the underlying logic(s). However, in accordance with the discussion in the previous sect. 2.2.1, the accomplishment of *formal* semantic integration appears as a second (and partially secondary) issue to us, besides an "ontological" transitioning from explicit L_i expressions to L_Ω (addressed in the next paragraph). This simplification yields an abstraction / generalization, which is intended to elucidate the main issue with existing accounts, while it applies likewise if (formal) semantic integration is assumed. In addition, this generalization makes / keeps the definition applicable to languages L_i without any formal semantics.[233]

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[232]See the discussion that follows this definition for comments on the phrase "translation functions ... wrt T_Ω" and for further requirements wrt the τ_i.

[233] Although Ciocoiu and Nau also take arbitrary "declarative languages" into account, "which for [their] purpose are languages L such that a function σ can be specified that converts sentences of L to sentences of a first order language $\mathcal{L}$." [156, sect. 3.1, p. 541]. σ is called "logical rendering function" [ibid.], its images of arguments the "logical render" [ibid.]. The step of theory interpretation incorporated into their approach is actually applied to such logical renders.

Another important note concerns the phrase that the τ_i are to be "translation functions [...] wrt T_Ω". This is intended to account for an ontological commitment (in an informal sense)[234] / ontological interpretation of the languages L_i wrt T_Ω (namely in terms of the functions τ_i). Translation functions "wrt T_Ω" anticipate the notions of *ontological translation* and *foundation* which are elaborated in sect. 2.3.1. Yet at this point, it is not crucial to refine the definition in these regards, because our criticism below is mainly based on reducing ontology-based consequence and equivalence to classical logical notions modulo a given theory.

From a logical point of view, at least the condition of ontology-based equivalence can be equivalently stated as $T_\Omega \models_{L_\Omega} \tau_1(e_1) \leftrightarrow \tau_2(e_2)$, if the deduction theorem (1.2) is satisfied by the logic underlying L_Ω. In the case of FOL, this form applies if the $\tau_i(e_i)$ are sentences / closed formulas, see e.g. [288, prop. 3.6, p. 62]. Finally, the translation functions could be generalized to map into sets of L_Ω expressions, i.e., $\tau_i : L_i \to \mathcal{P}(L_\Omega)$. This would cause some technical modifications, but it does not affect the subsequent overall argument. Therefore, the simpler option is presented here, also with less deviation from [733] than generalizing further.

2.2.2.2 Analysis of the Standard Account

ELABORATION OF THE EQUIVALENCE-MODULO-T PROBLEM
Without further restrictions/explication, ontology-based equivalence in the standard sense of Def. 2.3 cannot be used for justifying that two expressions e_1 and e_2 are mutual translations of the same conceptual content. The problem is that the question of e_2 being an ontology-based consequence of $\{e_1\}$ does not pay any attention to distinguishing T_Ω and $\{e_1\}$. In particular, any consequences of T_Ω that are τ_2 images of L_2 expressions are ontology-based consequences of $\{e_1\}$ – independently of what e_1 actually states, because $\models_{T_\Omega}$ is utilized. Being mutual ontology-based consequences thus yields the notion of (formal) equivalence modulo a theory as stated in Def. 2.3. For sets of expressions $E_1 \subseteq L_1$ and $E_2 \subseteq L_2$, this is not a problem if one aims at determining the mere *logical* relationship of E_1 and E_2, modulo T_Ω. The same applies to e_1 and e_2. But one must be aware that the answers to this are statements of logical equivalence modulo a theory, none that should serve as intensional equivalence of e_1 and e_2. Therefore purely logical laws invalidate the use of Def. 2.3 as an account of intensional equivalence, in particular for e_i images that follow from T_Ω *itself*.

LOGICAL BACKGROUND OF THE PROBLEM
Let us recall and delve into the relevant logical background. Stated in classical propositional logic, the problem relates to the fact that for every theory T and propositional variables p and q, $T \models p \leftrightarrow q$ is entailed by $T \models p \wedge q$, which includes the case of $p, q \in T$. More generally, the following observations apply to equivalent formulas and their determination or independence wrt a theory. We silently assume an abstractly given logic $\mathfrak{L}$ with language L that includes negation, and we consider only sentences, thus identifying L with $Sen(L)$, cf. sect. 1.5.1. In particular, all definitions and observations apply to FOL.

2.4 Definition (independence, determination, anti-sentences)
Let $T \subseteq L$ a theory, $\phi \in L$ a sentence. We agree on the following terminology and symbolic notation.

- ϕ is *determined by* T :iff T entails ϕ or its complement. $T \downarrow \phi$:iff $T \models \phi$ or $T \models \neg\phi$.
- ϕ is *independent* of T :iff ϕ is not determined by T. $T \uparrow \phi$:iff $T \not\downarrow \phi$.
- ϕ is a *sentence of* T :iff $T \models \phi$.
- ϕ is an *anti-sentence of* T :iff $T \models \neg\phi$.   ❑

2.5 Observation (on equivalences and determination)
Let $T \subseteq L$ a theory, $\phi, \psi \in L$ sentences.

1. Determination and independence are neutral wrt negation, i.e., $T \cdot \phi$ iff $T \cdot \neg\phi$ for $\cdot \in \{\downarrow, \uparrow\}$.
2. If $\phi \equiv_T \psi$, then $T \cdot \phi$ iff $T \cdot \psi$ for $\cdot \in \{\models, \downarrow, \uparrow\}$.
3. If $T \downarrow \phi$ and $T \downarrow \psi$, then $\phi \equiv_T \psi$ or $\phi \equiv_T \neg\psi$. In FOL, the consequent is equivalent to $T \downarrow \phi \leftrightarrow \psi$.
4. If $T \models \phi$ and $T \models \psi$, then $\phi \equiv_T \psi$.   ❑

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[234]Nevertheless, cf. also [733, p. 134 and sect. 3.2, p. 138] for the technical view on ontological commitment in [733].

Proof. In property 1., the case of $\downarrow$ (denoted by 1.($\downarrow$)) is immediate from Def. 2.4 (due to "symmetry" wrt ϕ and $\neg\phi$). For 1.($\uparrow$) one has $T\uparrow\phi$ iff not $T\downarrow\phi$ iff (by 1.($\downarrow$)) not $T\downarrow\neg\phi$ iff $T\uparrow\neg\phi$.

Wrt property 2., let (*) $\phi \equiv_T \psi$, which is equivalent to (**) $\neg\phi \equiv_T \neg\psi$. 2.($\models$) follows from (*) by the definition of $\equiv_T$ (see sect. 1.5.1) applied to, first, $Mod(T) \subseteq Mod(\phi)$, by which $Mod(T) \cap Mod(\phi) = Mod(T) = Mod(T) \cap Mod(\psi)$ and thus $T \models \psi$ (analogously for the converse direction). For 2.($\downarrow$), let $T\downarrow\phi$, i.e., (i) $T \models \phi$ or (ii) $T \models \neg\phi$. From (i) follows (2.($\models$)) $T \models \psi$ and thus $T\downarrow\psi$. For (ii) use (**) to find $T \models \neg\psi$ and thus $T\downarrow\neg\psi$, which yields $T\downarrow\psi$ with 1.($\downarrow$). By analogy, the converse direction is shown. The third case, 2.($\uparrow$), starts from $T\uparrow\phi$, i.e., $T \not\models \phi$ and $T \not\models \neg\phi$. Then $T \not\models \psi$ follows from 2.($\models$), which in combination with (**) also entails $T \not\models \neg\psi$, hence $T\uparrow\psi$. Again, the other direction is symmetric.

Turning to property 3., let (a) $T \models \phi$ and consider $T\downarrow\psi$, i.e., (i) $T \models \psi$ or (ii) $T \models \neg\psi$. In case (i) $\phi \equiv_T \psi$ follows, because ϕ and ψ are true in every model of T. Case (ii) entails $\phi \equiv_T \neg\psi$ by the analogous argument for ϕ and $\neg\psi$. Moreover, the case of (b) $T \models \neg\phi$ is similar and yields $\phi \equiv_T \psi$ from the same case (ii) of $T\downarrow\psi$, and $\phi \equiv_T \neg\psi$ from case (i) of $T\downarrow\psi$. In FOL, the equivalence of $\phi \equiv_T \psi$ or $\phi \equiv_T \neg\psi$ with $T\downarrow\phi \leftrightarrow \psi$ is a consequence of the available connectives (and their semantics) and the deduction theorem (1.2), which applies in FOL. In particular, note that (***) $\alpha \leftrightarrow \neg\beta \equiv_{\mathfrak{FOL}} \neg(\alpha \leftrightarrow \beta)$ for arbitrary FOL formulas α, β by sharing the model class $(Mod(\alpha) \cap Mod(\neg\beta)) \cup (Mod(\neg\alpha) \cap Mod(\beta))$. Hence, $T\downarrow\phi \leftrightarrow \psi$ iff ($T \models \phi \leftrightarrow \psi$ or $T \models \neg(\phi \leftrightarrow \psi)$) iff (involving (***) and the deduction theorem) ($\phi \equiv_T \psi$ or $\phi \equiv_T \neg\psi$).

Property 4. specializes property 3., and is also immediate from its preconditions, which entail $Mod(T) \cap Mod(\phi) = Mod(T) = Mod(T) \cap Mod(\psi)$. ❑

EFFECTS CONSIDERED ALONG THE STRENGTH OF ONTOLOGY AXIOMATIZATIONS

Notably, by these observations the set of mutually equivalent sentences is larger the stronger a theory is. In effect, the more specific ontological content a formalized ontology T_Ω captures by stronger axiomatization, the worse for ontology-based consequences and equivalences (according to Def. 2.3) on the basis of T_Ω, in these sense that, e.g., more expressions turn out to be ontology-based equivalent, but inadequately. For instance, the τ_i images of two expressions could be two contingent sentences that do not share any (extralogical) signature, nevertheless the expressions can be ontology-based equivalent according to Def. 2.3 just because the τ_i images are both consequences of the ontology T_Ω. This example also illustrates that one cannot assume, e.g., tempted by interpolation properties as for FOL,[235] that these negative effects were limited to the deductively closed subtheory in the language of the $\tau_i(e_i)$ or constrained by its strength. This is not the case, and the problem exists also for a weakly axiomatized ontology if τ_i images coincide with consequences of the ontology. Wrt a fixed language, stronger theories are more susceptible only insofar that they have more consequences than weaker axiomatizations. Following this train of thoughts, we reach the culmination of strengthening theories wrt a fixed language in theories that are *complete* (and *consistent*, to be sensible). Utilizing the notions above, they are exactly those subsets $T^c \subseteq L$ of a language that determine every sentence $\phi \in L$, see also [826, Satz 3.16, p. 98]. Alternatively, an equivalence-based characterization of consistent and complete theories can be given: a theory T^c is complete and consistent iff every sentence ϕ is T^c-equivalent with *all* either sentences or anti-sentences of T^c, i.e., for all $\phi, \psi \in L$: either $\phi \equiv_T \psi$ or $\phi \equiv_T \neg\psi$. Hence, a complete theory as an ontology would claim intensional equivalence among all expressions that are translated to sentences of the theory, and among all those that are translated to anti-sentences, resp.

PROBLEMS IN EXPLAINING ONTOLOGY TRANSLATIONS WITH THE ONTOLOGY-BASED NOTIONS

These logical properties become most problematic for ontologies themselves if these are expressed in different formalisms. From the above observations follows that formalized ontologies as a whole can be determined to be (formally / logically) equivalent, but for their constituents, e.g., their axioms, the counterproductive result is that these are arbitrarily equivalent. To illustrate this, assume the use of a description logic theory and a relational schema, akin to an example in [733, Fig. 1, p. 135]. In our case, both are meant to express "the same" conceptualization Ω. Let T_Ω be a FOL representation of Ω to which the other

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[235]In connection with FOL, we are mainly referring to the Craig Interpolation Theorem, see e.g. [53, Theorem 5.9, p. 72], although there are other interpolation theorems. For a definition of the "semantic Craig interpolation property" in connection with institution theory see [596, Def. 4.11, p. 11].

representations commit, i.e., into which they translate, in order to decide about ontology-based equivalence. Each representation as a whole is ontology-based equivalent with the others, because both map themselves to T_Ω (in total). This is an appropriate result. However, inappropriately for a translation setting, every description logic axiom is ontology-based equivalent to every (sentence-generating) fragment of the relational schema – because both result in sentences which are both true in T_Ω, and thus equivalent.[236]

THERE ARE ALSO WORKING CASES, YET UNCLEAR TO WHAT EXTENT

It must be stressed again that the problem with Def. 2.3 arises in those cases where the result $\tau_i(e_i)$ of translating an expression e_i into the ontology formalization language L_Ω is already a sentence of the formalized ontology T_Ω, $\tau_i(e_i) \in T_\Omega$. Needless to say, this class of cases is highly relevant for the objective of translating a formalized ontology (intensionally) appropriately into other languages, or of recognizing / justifying two theories as standing in such conceptual equivalence relation. On the other hand, there are cases where Def. 2.3 works (or better, its "relatives" in the literature). The example (of an ontology-based partial translation) given in [156, sect. 5, p. 543–545] is of the following kind: the (FOL-formalized) ontology captures constraints among the predicates only (similar to a DL TBox), whereas the theories examined for being mutual translations consist of ground facts (similar to a DL ABox; using additional constant symbols for individuals, thus involving a language extension). More generally, one may ask whether for all sentences ϕ, ψ such that $T_\Omega \uparrow \phi$ and $T_\Omega \uparrow \psi$ the fact $T_\Omega \cup \{\phi\} \equiv_{L_\Omega} T_\Omega \cup \{\psi\}$ captures intensional equivalence of ϕ and ψ wrt Ω in a satisfactory way. We do not dwell upon this particular question much further. Just note that this route excludes a number of sentences, e.g., all tautologies, because they are determined by any theory (of appropriate signature) and thus by T_Ω. Tautologies may not seem very interesting wrt intensional equivalence. However, pursuing the combination of this account with formal semantic integration / maintaining results from language dynamics[237] may quickly lead to them, as well as to further entailed sentences that are no longer independent from T_Ω. After all, for us the problems outlined above justify a fair amount of skepticism against Def. 2.3 even in cases where it seems to work properly.

IS CRITICISM FAIR AND / OR REMEDIABLE

It is a natural question to ask whether the criticism is "fair" and / or whether it can be remedied easily, with both questions oriented at the problematic case of translating ontologies themselves. One may object that it is not intended to apply the approach to sentences of T_Ω [238] itself. On the other hand, this position would entail that an ontology cannot commit to itself, which appears dubious to us. In any case, for "repairing" the approach for sentences of T_Ω one may select a fragment $\Pi \subseteq T_\Omega$ (instead of T_Ω as a whole) as a basis for comparing sentences outside of Π. Then the question becomes which fragments to select, since in general multiple fragments may be chosen, some of them rendering two sentences "intensionally" / ontology-based equivalent whereas others do not. In the limit case and closely related to the just-mentioned objection, one may select a uniquely determined fragment – the empty set / theory[239]. Then intensional / ontology-based equivalence (of ontological statements) would collapse with mere logical equivalence. But then we arrive at the starting point prior to seeking refuge in ontologies, and without any resolution of problems such as extensionality/intensionality, see sect. 1.2.3.[240] We conclude that there is no obvious minor revision/variation of Def. 2.3, in order to serve its intended purpose of defining conceptual equivalence.

DISTILLED MAIN QUESTION & INCREASING CHAIN OF EQUIVALENCES

Crystallizing the core issues further, two central questions arise from Def. 2.3.[241]

$\mathcal{Q}_5$ What form of equivalence is appropriate for characterizing intensional equivalence?
$\mathcal{Q}_6$ In which way should the translations τ_i be constrained wrt the source language semantics?

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[236]Accordingly, the restriction to ontology-based consequences in [733] is quite reasonable and the wording there is more careful compared to [156]. Admittedly, for the notion of ontology-based translation Ciocoiu and Nau [156] literally refers to sets of sentences that are translated, instead of individual sentences as in Def. 2.3. But considering singletons yields the same problem.

[237]as the aspect that we have ignored so far, but that is included in [156, 733] and would be beneficial, indeed

[238]In this paragraph, by 'sentences of T_Ω' we refer abbreviatingly to sentences which are τ_i-images of expressions of the source languages L_i in Def. 2.3.

[239]Whose deductive closure includes all tautologies in the respective language, of course.

[240]In a sense, that would produce a circularity of trying to solve problems attributed to working (only) with formal semantics by relying on formal semantics alone again.

[241]Their numbering continuing the motivating questions in sect. 1.2, listed compactly in sect. 1.3.2.

Concerning the first question, the above argument aims to uncover shortcomings of classical logical equivalence, i.e., of stating that ontological translations/sentences representing ontological claims are translations of each other if they are logically equivalent (possibly modulo a theory).[242] In particular, we believe that too many expressions are ontology-based equivalent in this account. But interestingly, a range of extensions of this type of equivalence have been proposed which lead to an even greater number of equivalences. These proposals originate from the fact that the direct approach of logical equivalence requires for both theories / formalized ontologies the adoption of (i) the same logic / logical framework and (ii) the same signature. Clearly, an agreement about either aspect is far from being realistic, cf. e.g. [733, sect. 3 and 6].

EXAMPLES: THEORY INTERPRETATIONS, INTERPRETABILITY, INSTITUTIONS

Notably, these extended notions of equivalence arose independently of using logical theories for representing ontologies, namely in studying methods for formal logical issues like determining the expressiveness or the decidability of theories. Their power of comparing theories and their increasing degree of abstracting from particular logics and languages, however, is obviously tempting to adopt them for the purpose of cross-language theory comparison. To provide some examples, *theory interpretations* [221, sect. 2.7, p. 154–163], see A.1.2 herein, are a well-established way of "ignoring" / "dissipating" differences in the signatures of FOL theories. The main idea of interpreting a FOL theory $T_1 \subseteq Lg(\Sigma_1)$ into another theory $T_2 \subseteq Lg(\Sigma_2)$ is to define interpreting $Lg(\Sigma_2)$-formulas for all predicates and functions in Σ_1 as well as possibly for restricting the universal quantifier to a subset of the universe of T_2 interpretations. This allows for bridging some differences in the signature. For example, [221, ibid.] presents an interpretation from the theory $(\mathbb{N}, 0, S)$ of natural numbers $\mathbb{N}$ with 0 and the successor function S into the theory of integers $(\mathbb{Z}, +, \cdot)$ with addition and multiplication. However, theory interpretations are limited in the sense that predicates and functions can only be interpreted by formulas with at most as many free variables as the arity of the interpreted predicate is (increased by 1 for functions).

For cases of greater differences, the notions of interpretability[243] [798, 799] and of institutions [283] provide further solutions. *Interpretability* still applies to FOL languages / theories, but it transcends theory interpretations by allowing one to define / encode objects of the universe of one theory by multiple objects of the universe of the interpreting theory. For instance, a point in a theory of planar geometry may be represented by a pair of real numbers, following the standard account of the Cartesian plane. Indeed, Tarski's result of the "equivalence" among the theory of the real numbers[244] and the theory of (planar) Euclidean geometry [804, 807] can be retraced and captured formally in terms of their mutual interpretability, see [605, 630]. Intuitively, this provides an account of translating sentences between the respective languages, where sentences in such translation pairs either both, each from the corresponding theory, or both do not follow.

The theory of *institutions* [283],[245] cf. also [596] or [733, sect. 4, p. 138-140], bridges an even greater distance, in that they allow for relating different logical systems with each other. More precisely, institutions capture formally the notion of a logic / logical framework,[246] e.g., FOL or a particular modal logic, and they allow for studying interrelationships among logics (as institutions). The basic mathematical framework that the theory of institutions rests upon is (mathematical) category theory, cf. [3, 52, 518, 544] for expositions. The components of an institution $\mathcal{I} = (Sign, Sen, Mod, \models^{\mathcal{I}})$ comprise a system of signatures $Sign$, well-formed sentences Sen, and interpretation structures Mod over those signatures. The fourth component $\models^{\mathcal{I}}$ assigns to each signature a satisfaction relation between its interpretation structures and its sentences. On

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[242] Note that this is the starting point adopted in [156, p. 540].

[243] For several weaker notions by other authors the reader is referred to the discussion in [799, p. 130].

[244] More precisely, of the theory of real closed fields, for the axioms see e.g. [410, p. 37–38],[147, sect. 1.4, esp. p. 41].

[245] According to [283, sect. 1.2, p. 100], institutions were first introduced in [284] by Joseph A. Goguen and Rod Burstall in 1984. [733, sect. 4, p. 138] associates their origin with the study of model-theoretic properties of logical systems in the late 1970s and early 1980s. Accordingly, there are close ties with *abstract model theory* and corresponding accounts of logics, see e.g. [55], [838, sect. 1.3].

Furthermore, Schorlemmer and Kalfoglou attribute institution theory success in providing "semantics to powerful module systems of both imperative and declarative programming languages, multi-logic specification languages, databases, and ontologies." [733, sect. 4, p. 138]. Indeed, there is a growing body of recent work on the basis of institution theory in ontologies, including [499, 597, 686], significantly mediated by the development of Hets [@54] [594, 595] and its application for heterogeneously structured ontologies in AO, cf. also the paragraph "from Hets to Ontohub" in sect. 1.2.1.

[246] In particular, different signatures do not lead to different institutions, but a range of signatures is integrated into each institution.

this basis, as [733, sect. 5.3, esp. Def. 6, p. 141] shows, the notion of theory interpretations of FOL can be lifted to institutions, which basically yields an account of equivalence of theories in possibly different institutions.

ARGUMENTS AGAINST FORMAL SEM. EQUIVALENCE WRT QUESTION $\mathcal{Q}_5$

Despite the availability of all of this formal machinery, which definitely has its merits wrt formal logical results, the question remains whether such notions of equivalence can appropriately account for intensional equivalence. We do not see how that would be possible, first of all, because any initial formalization, in the sense of expressing a conceptualization in terms of formal / mathematical means and entities,[247] involves encoding[248] at least certain entities of/within the conceptualization. This encoding is "hidden"/not available from the initial formal representation. Hence, translations that are provably correct wrt the formal semantics of a formalism can only preserve the translation source – which is already a mix of that initial encoding and the formal semantics of the language employed. Indeed, we believe that this or similar observations, back then in the context of knowledge-based systems (a.o.), constitute the / major actual reason(s) that led to dealing with ontologies in computer and information sciences, cf. also sect. 1.1.1 and 1.2.1.

As a remark, the present level of argumentation reaches / touches very basic questions and overall foundational grounds / problem areas here, such as the problem of meaning, cf. the beginning of this ch. 2, and the Symbol Grounding Problem [372] in AI and cognitive science, which may shine through.[249] From our point of view, it were more than presumptuous to attempt tackling these issues "as a whole". This motivates these remarks in order to avoid any impression to that effect. Instead, with much more modest aims, the analysis up to this stage lead us (1) to be interested in pinpointing where encodings occur, e.g., in classical logical languages and their semantics, and to study to what extent this aspect may be repressed by limited adaptations, while clearly remaining within symbolic (knowledge) representation.[250] Moreover, our Objective 4 of reusing existing theory and systems to the greatest possible extent, as introduced in sect. 1.3.2–1.3.3 remains important.

That said, let us collect some further, more concrete hindrances to accepting formal semantic translations / notions of equivalence as the (sole) basis for conceptual equivalence. Remembering the case of the mutual interpretability between the theory of geometry and the theory of the reals, the question arises which conclusions one should / could draw from these results, thereby viewing these theories as axiomatic characterizations of geometric entities and real numbers, resp. We find it highly counterintuitive to derive any ontological implications from that. To mention another extreme case (against the background of FOL), Lesław W. Szczerba defines *universal theories* [799, p. 139] as those that can interpret *every* other theory (more precisely, there are extensions of a universal theory in the same signature with this property). He names five examples, among them the logical theory of a single binary relation (also known as the *theory of (directed) graphs*) and the theory of partial order [799, p. 140]. It is unclear to us, from an intensional point of view and if intensional equivalence were reducible to formal interpretability, what the role of such theories would be. Moreover, it appears plausible that the stronger the means for formal transformations / interpretations are, the more such formal correspondences can be established (independently of the / any conceptual content that theories may have resulted from). Our reservations are reinforced by the fact that most definitions adopt existential conditions, e.g. two theories are interpretable if *there is* an interpretation between them.

τ_i RESTRICTIONS: PSEUDO-NEED FOR (FORMAL) SEMANTIC INTEGRATION

The last points indirectly touch $\mathcal{Q}_6$, the second central question here, namely whether and how the translations τ_i in Def. 2.3 should be constrained. This means also to (re)turn to the hitherto ignored first "half" of the standard approach described in [156, 733], namely the requirement for the translations τ_i to be *formal*

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[247] Cf. also the related statement in sect. 2.2.1, where we distinguish conceptual and formal semantics in the first place.

[248] Cf. the assumption of a "logical rendering" in [156, esp. sect. 3.1, p. 541], already mentioned in FN 233 with slightly more detail. Ciocoiu and Nau [156] do not address the question of *how* a logical rendering is to be produced.

[249] As formulated by Steven Harnard, the problem centrally comprises questions such as "How can the semantic interpretation of a formal symbol system be made intrinsic to the system, rather than just parasitic on the meanings in our heads? How can the meanings of the meaningless symbol tokens, manipulated solely on the basis of their (arbitrary) shapes, be grounded in anything but other meaningless symbols?" [372, p. 335].

[250] Accordingly, the Symbol Grounding Problem is not in the scope of our work. Notably, it is raised and discussed in the context of the Semantic Web in [169].

semantic integrations between the source languages L_i and the ontology language L_Ω. Commonly, it is this "half" that is taken to justify the speaking of 'preserving semantics'. This position appears fairly agreed upon, although some debate remains on the specific nature of the integration.[251] As stated above, formal semantic integration is desirable in principle, nevertheless we believe that the standard account is too restrictive in requiring full formal semantic integration. The problems described above (of equivalence modulo a theory, and of formal notions of equivalence / interpretation / interpretability altogether) recondition the importance of distinguishing formal and conceptual semantics, first discussed in sect. 2.2.1. The resulting two readings of 'preserving semantics' are simply to be kept distinct, and we adopt the position of assigning priority to conceptual semantics. In supporting this view, much of the discussion in sect. 2.2.1 surfaces again. For instance, there is the problem of how to assign translations τ_i to expressions of languages that lack a formal semantics.[252] In addition, it is not uncommon in applications that the formal semantics of a language is "misused", i.e., used intentionally in a non-standard[253] way in order to create a certain system behavior rather than producing a "statically correct" representation of knowledge in terms of the underlying formal semantics. In such cases, it can be necessary to interpret uses of the same (possibly semi-)formal type of constructs / abstract syntax category intensionally differently, depending on the particular arguments that they are applied to. Such "impure" / "mixed" translations appear hard to justify in terms of the semantics of the source language, supposing that it prescribes a uniform treatment of the construct type / abstract syntax category. A common type of example in this regard are category hierarchies / subject classifications as well known from library science [208] and have spread into Web 2.0 information systems such as wikis [521] (and semantic wikis [708, 851] in particular).

Of course, *if* both can be achieved, i.e., expressions of the source language can be captured intensionally adequately and by a translation that is also a formal semantic integration, that situation is clearly highly preferable. For instance, if the source language offers implicit knowledge, this would be reasonably maintained / supported by a formal semantic integration. Notwithstanding this desirability, mismatches between the source and the ontology language dynamics may need to be accepted as differences between functionalities of systems with differing purposes. For illustration, think of the differences of adopting the Closed World Assumption (CWA) [703, esp. p. 60],[254] cf. also e.g. [109, sect. 11.2], [118, sect. 6.2.4],[255] [116, sect. 2.1], or [406, sect. 4.1.7, p. 131ff.; App. C.2, p. 372], like in databases in contrast to, say, OWL or FOL, which are said to adhere to the Open World Assumption (OWA) [406, 703][256]. This distinction frequently plays a major role in applying Semantic Web technology, as well as in the context of logic programming and rule languages, cf. sect. 1.1.3. Works like [404, 602] nicely demonstrates the plurality of issues arising from it. Consequently, the claim that "[b]oth relational schema and DL T-Box are notational variants of

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[251]For example, [496, p. 44] critizes [733] for to few restrictions, e.g., due to the possibility of trivial integrations which is not excluded, and suggests to constrain arbitrary theory interpretations to signature morphisms or derived signature morphisms, see [725, p. 382]. Note that [733, FN 2, p. 134] also refers to derived signature morphisms while drawing inspiration from the FO case of theory interpretations. One may thus speculate about similar intentions.

[252]Although it should be acknowledged once more that [156] arrange for this in principle through their logical renders, yet without any further details. In [733] a comparable step is hidden in their examples (but not part of the formal account), where they consider FOL representations of relational database schemata, for instance.

[253]i.e., compared to a postulated connection between conceptual notions and (abstract) syntax categories, cf. Welty and Ferruci's attitude on reification in sect. 1.2.3.1 and the next sect. 2.3

[254]Assuming FOL in this footnote, the CWA is concerned with facts / ground atoms, i.e., variable-free atomic formulas, in connection with a given theory T. Intuitively speaking, it states that every fact that is not true in T is considered to be false. ([406, p. 372] has a similar formulation.) In terms of Def. 2.4, the CWA determines all T-independent facts (such that they are false). More precisely and following [118, p. 250–251], adopting the CWA wrt T means to adopt an inference rule that infers the falsity of any ground atom if it is not entailed by T. Equivalently, this amounts to reasoning over $CWA(T) := T \cup \{\neg\alpha \mid \alpha \in Lg(T) \text{ is a ground atom and } T \not\models \alpha\}$ instead of T itself. Note that $CWA(T)$ is consistent iff T has a least Herbrand model [118, Theorem 6.10] (see e.g. [693, sect. 4.1] for the notion of Herbrand model). That allows for consistent theories T such that $CWA(T)$ is inconsistent, e.g., $T = \{\alpha \vee \beta\}$, $CWA(T) = \{\alpha \vee \beta, \neg\alpha, \neg\beta\}$ for facts $\alpha, \beta \in Lg(T)$. [109, sect. 11.2.4] presents a weakened notion called Generalized CWA that maintains the consistency of T in any case and agrees with the CWA in the absence of any T-entailed disjunctions with the property that all of the disjuncts are not T-entailed.

[255][118, p. 250] attributes the introduction of the term to [703] by Raymond Reiter.

[256]The same citation details as for CWA apply in both cases. The OWA is introduced as the opposite of the CWA in [703, p. 60], i.e., adhering to / adopting the OWA is intended to simply mean not to adopt / apply the CWA. Notably, it is not mentioned in [109, 116, 118], likely due to the misleading name containing 'assumption', because actually no additional assumption is made, e.g., compared to standard FOL semantics.

fragments of first-order logic" [733, p. 135] takes a fairly long stride[257] and simplifies matters considerably.[258] There is the question for an example like [733, Fig. 1 and 2], from the perspective of adopting the relational model [162] / the Structured Query Language (SQL) [@117] [453] itself as source language (instead of its FOL-ified "notational variant" under modified assumptions), namely whether formal semantic integration would be necessary for obtaining an adequate representation of ontological assumptions "behind" the source language expressions. In this case, we do not really see benefits of that demand, in the light of the very different character of the formal semantics. Altogether, we are not convinced that formal semantic integration is strictly necessary for a revised account of intensional equivalence.

(NO) ONTOLOGICAL NEUTRALITY OF LOGICAL (AND GENERAL KR / CM) LANGUAGES

The insinuated "irrelevance" of / less emphasis on the rôle of formal semantics in defining ontological translations (the τ_i in Def. 2.3) as well as the objections against the formal notions of equivalence prior to that connect to the ontological impact attributed to the formal semantics of (general purpose, especially logical) representation languages. Indeed, the standard approach and some of its problems are based on the fundamental assumption that logic(s) and (some may add) general-purpose KR / CM languages are *ontologically neutral*, see e.g. [773, p. 492], or to quote Nicola Guarino [327, p. 631]: "First order logic is notoriously neutral with respect to ontological choices." At first glance, there seems to be an interesting shift in Guarino's position on the matter, documented in [329, p. 300]. This publication refers to an earlier argument *against* such neutrality in [334].[259] However, considering the later proposal of an ontological level [326, 328] [327, esp. sect. 4] and its main idea to tie language primitives (i.e., (certain) abstract syntax categories, in the terminology of sect. 2.1.1) to an ontologically "predetermined" meaning, later discussed in combination with ontological neutrality at the logical level, we find a coherent picture despite the distinct statements on ontological neutrality. The original considerations for establishing / discussing the ontological level have meanwhile been taken much further, not at least in connection with work on providing foundations for conceptual modeling languages, esp. [348], and the related implementation of OntoUML [@87] [75, 76], cf. near the end of sect. 1.1.3.4. They also play an eminent rôle in developing an alternative semantic approach in ch. 4 below (esp. in sect. 4.4), motivated by the overall present analysis. Nevertheless, regarding the question of ontological neutrality, this forms a separate, though closely related aspect. The open question here concerns the ontological status of formal semantic entities of a language, if that language is used to express ontologies.

For shaping our own views on the matter (expounded in ch. 3) further, we continue our survey on findings from the literature with Giancarlo Guizzardi, who likewise subscribes to the view that logical languages are ontologically neutral [348, sect. 3.3.2, p. 86 ff.]. Interestingly, he discusses the topic in connection with real-world semantics, cf. sect. 2.2.1, provides an interesting side reference to Mario Bunge [126],[260] and basically adheres to Guarino's analysis just discussed, which in [348] results in the following characterization for general CM languages: "[. . .] the meaning of structuring primitives should be characterized in terms of meta-level conditions that make explicit the ontological commitment made by the modeler when choosing a particular structuring primitive to represent a domain element." [348, p. 89]. Switching to knowledge representation briefly, e.g. Thomas Gruber considers KR languages to be independent of the represented content [311, p. 200]. Note that this position merges into the notion of KR ontologies [841, esp. p. 193], [289, p. 29].[261] In contrast, Randall Davis, Howard Shrobe, and Peter Szolovits see an immediate connection between "a knowledge representation" (language) and ontological commitment: "[. . .] in selecting

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[257]This is rather an accepted position in certain communities than a specific claim of Schorlemmer and Kalfoglou.

[258]NB: Expressiveness and built-in assumptions / theories are further subject matters that easily require more attention in the details than is sometimes attributed to them. E.g. on page [733, p. 135] the authors themselves require FOL with built-in number comparison. Moreover, recall DLs with features that trespass the capabilities of FOL in FN 56 on p. 11, i.e., not forming a FOL fragment, at least if the usual relationship (cf. sect. 1.1.3.2) is to be maintained.

[259]Interestingly, [841, p. 193] contains the statement: "[Representation ontologies] are intended to be neutral with respect to world entities (Guarino & Boldrin, 1993) [i.e., [334]]."

[260]According to [348, p. 87], Bunge [126] states that predicates can be negated or disjunctions can be expressed, but there are neither negative nor alternative entities in reality. This touches the question of comprehension axioms for categories and is therefore among our concerns, as well.

[261]Cf. our initial remarks on KR ontologies above in sect. 1.1.4 (esp. the final paragraph of sect. 1.1.4.3) and a more detailed coverage in the next sect. 2.3, esp. in connection with abstract core ontologies in sect. 2.3.3.

any representation we are in the very same act unavoidably making a set of decisions about how and what to see in the world. That is, selecting a representation means making a set of ontological commitments.[2] [FN omitted here] The commitments are, in effect, a strong pair of glasses that determine what we can see, bringing some part of the world into sharp focus at the expense of blurring other parts." [181, p. 19]. On the other hand, the idea of "neutral representations" has been adopted in other areas, as well, e.g., see [74, esp. sect. 3].

COCCHIARELLA'S PHILOSOPHICAL PERSPECTIVES ON LOGIC AND ONTOLOGY

But can ontological neutrality really be granted, refocusing on logical languages and such with a model-theoretic semantics, in particular? Agreeing with Nino B. Cocchiarella's "Logic and Ontology" [160] (and some works in KR, like [181]), we object to this point. The subject of his work [160, esp. sect. 1] is the differentiation of two distinct views / understandings of logic in connection with ontology, named 'logic as a calculus' and 'logic as a language'. According to Cocchiarella, the perspective of 'logic as a calculus' is paramountly adopted today [160, p. 118, 119, 123]. It views logic "[...] as an abstract calculus that has no content of its own, and which depends upon set theory as a background framework by which such a calculus might be syntactically described and semantically interpreted" [160, p. 118]. Logical expressions are formally interpreted in terms of set-theoretic constructs, and in particular, predication is "transliterated" / encoded into set membership at the side of formal semantics.[262] Cocchiarella acknowledges the importance and utility of this perspective [160, p. 123], which we underline. However, applying this understanding of logic in connection with ontological analysis[263] and formalization leads to viewing logic basically as a part of set theory. That position we cannot take, adopting Cocchiarella's instead:[264] "Notwithstanding the great power and utility of set theory as a mathematical theory, and of set-theoretic model theory in particular as a method for proving a number of results in formal semantics, it is not the right sort of framework in which to represent either a general ontology or our commonsense and scientific understanding of the world." [160, p. 123].

The view of 'logic as a language' as introduced by Jóseph Maria Bocheński [87] takes a different position on employing logic for doing ontology: "[...] it is a *logistic system* in which logic is a language with content of its own." [160, p. 119–120].[265] Regarding predication and set membership, Cocchiarella argues that predication is much more fundamental, and the different accounts of universals developed in philosophy lead to different alternative theories of logical forms (given the 'logic as a language' view).[266] In favor of the non-exclusiveness of the two views, he then suggests that the position of 'logic as a language' yields different logics which can be compared by means of the methods valid for 'logic as a calculus'. However, Cocchiarella himself notes that this provides one only with "an external, mathematical model of [an] ontology" [160, p. 124].

Although our main problem of intensional semantic translations and / for ontology translations is not addressed by Cocchiarella, this note of caution may support the expectation that relying solely on standard mathematical formalization would not provide a solution – all the more in the light of the arguments above.

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[262]Ch. 3 elaborates the relations between FOL theories and their set-theoretic background in detail.

[263]See sect. 2.3.1 for an explication of 'ontological analysis'.

[264]Wrt the quote in every respect, including the appreciation of formal analyses.

[265]The remaining characterization is primarily functional, i.e., referring to how a logic as a language is used: "Moreover, as a general framework by which to represent our commonsense and scientific understanding of the world (through the introduction of descriptive constants and nonlogical axioms), the logical forms of a logistic system are syntactic structures that, as it were, carry their semantics on their sleeves. It is by assigning such logical forms to the (declarative) sentences of a natural language or a scientific theory that we are able to give logically perspicuous representations of the truth conditions of those sentences, and thereby locate them ontologically within our general conceptual framework. In this regard, a sufficiently rich formal logic is the basis of a *lingua philosophica* within which conceptual and ontological analyses can be carried out, and therefore a framework for general ontology. This approach, in contrast to the view of logic as calculus with set theory as the framework for general ontology, is what is meant by the view of logic as language." [160, p. 120]. The same section concludes with establishing relations to positions of Descartes, Leibniz, Frege, and Russell.

[266]"Membership, the basic notion upon which set theory is constructed, is at best a pale shadow of predication, which, in one form or another, is the basic notion upon which thought, natural language, and the logical forms of the view of logic as language are constructed. Indeed, so basic is predication that different theories of logical form as different versions of the view of logic as language are really based on alternative theories of predication." [160, p. 123]. Note that a main inspiration for dealing with different philosophical positions wrt categories ('universals' in [160]) in the context of the General Formal Ontology (GFO) is drawn from the work of Jorge J. E. Gracia [295], as mentioned in sect. 1.1.5.2.

Moreover and despite the phrase "no content of its own" for 'logic as a calculus', we see no ontological neutrality in either view on logic, as 'logic as a calculus' presupposes set theory.[267]

COMMENT ON INTENSIONAL LOGICS AND SITUATION THEORY

It is by these arguments of Cocchiarella as well as due to the encoding aspect that (according to our knowledge) seems to be pervasively involved in accounts of intensional logics, cf. [243], [246, esp. ch. 1–2], [657, esp. sect. 4.1–4.2] and, more specifically, e.g. [47, 65, 426, 462], and the significant deviation from classical logic[268] that prevents us from adopting one of the latter straight-away for attempts in redefining intensional equivalence. By 'encoding aspect' we mean that, to the extent of our knowledge, established accounts of intensional logics involve an interpretation of certain syntactic entities into formal / mathematical entities – which produces an encoding. The work of George Bealer on theories of "Properties, Relations, and Propositions" [64–66] is partially an exception. Notably, Bealer [65, Introduction, sect. 4] briefly charts how 12 other approaches and his own behave wrt a set of 25 desiderata that he is interested in and that his account is declared to solve. Another remarkable case is situation semantics / situation theory [56, 202], which started out with similar motivations and with "ontological freedom" in its interpretations, cf. [202, p. 601–602]. However, presumably due to its attunement to providing a formal account for natural language, quite a number of fundamental assumptions differ from classical logic. Therefore, we draw some inspiration from situation theory, a.o., yet we continue our own analysis with Def. 2.3 as its starting point, together with classical logic (and FOL in particular).

REFINING OBJ. 3 OF AN ALTERNATIVE MODEL-THEORETIC SEMANTICS

Given the state of the chapter up to this point and the questions $\mathcal{Q}_5$ and $\mathcal{Q}_6$ carved out above on p. 67, initial work on $\mathcal{Q}_5$ (still ahead in sect. 2.3.3) let it appear fruitful to us to study to what extent a different form of model-theoretic semantics can be developed. As noted above, the (re)usability of existing theoretical results and automated reasoners from sect. 1.2.2 must be kept in mind, as well as the expressiveness issues in sect. 1.2.3. The driving factors for such a model-theoretic account can now be listed:

1. Avoid ontological predetermination as much as possible.
2. Maintain the ontological character of the representation to the greatest possible extent.
3. Minimize limitations wrt expressiveness.

Our proposal in this regard is elaborated in ch. 4. In order to provide further prerequisites to its development, the two subsequent sections of this chapter present and advance meta-ontological notions in sect. 2.3 (against the background of GFO) and envisage an abstract core ontology of categories and relations, presented in sect. 2.4.

2.3 Ontological Analysis and Meta-Ontological Architecture

INTRODUCTION AND QUESTIONS THAT LEAD TO THE NOTIONS TO BE CONSIDERED

To a large extent and as explained esp. at the beginning of sect. 1.2, the background of our work is associated with developing, providing, and applying foundational ontologies. Among other reasons, this has led us to formulate the goal of a novel account of intensional equivalence, seeing the need to revise Def. 2.3 in sect. 2.2.2.1. Still assuming similar preconditions of that definition, there remains the need of "extracting ontological representations" from source language expressions (the τ_i in Def. 2.3). Therefore, this as well as the enterprises named above are methodologically tightly linked with the following basic issues.[270]

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[267]Cf. more detailed investigation of this aspect in sect. 3.2.

[268]This applies to various respects in both syntax and semantics. Of course, we seek a deviation, as well, on the one hand. On the other hand, there is the objective of (ideally) "maximal" reuse of existing reasoning machinery.

[269] This section extends and revises material that was originally published in [399] (2005) and briefly revisited / touched upon in [533, sect. 1.3] (2011).

[270]Recalling the terminology of sect. 1.1.2.2, a conceptualization refers to an ontological theory perceived as language-independent content, whereas a formalized ontology is a formal representation of such content.

$\mathcal{Q}_7$ How are conceptualizations created?

$\mathcal{Q}_8$ Assuming an adequate representation language, how are formalized ontologies developed?

$\mathcal{Q}_9$ Given any representation / expression of a language, be it informal, semi-formal, fully formalized, or a mixture thereof, how can its conceptual contents be unravelled?

We establish corresponding terminology in the first section below. Its subsequent sect. 2.3.2 distinguishes two perspectives on formalized ontologies, prior to presenting the meta-ontological architecture developed in connection with GFO.

2.3.1 Ontological Analysis, Foundation, Translation and Reduction

ONTOLOGICAL ANALYSIS & ITS LANGUAGE ASPECT

The central notion regarding the above basic issues is *ontological analysis*. Intuitively and in a broad sense, ontological analysis is a process through which an ontology is developed/compiled. Given the broad understanding of 'ontology' herein (see sect. 1.1.2), the understanding of ontological analysis can be considered from a language-independent and a language-oriented point of view. In the first case, the focus is on devising a conceptualization in the course of the analysis, i.e., it is irrelevant from this perspective in which language (in the widest sense) the resulting theory is represented. Adopting a language-oriented perspective of ontological analysis still means to develop a conceptualization, but with an additional requirement to represent / formalize it in a particular (set of) language(s), which is usually predetermined.

TARGET CASES OF ONTOLOGICAL ANALYSIS: DOMAIN OR REPRESENTATION

Another aspect of ontological analysis is its target. We distinguish primarily two lines of target cases, according to our experiences in developing and applying GFO. On the one hand, ontological analysis may head for an ontology of a *domain of knowledge/reality* (and thereby a domain of entities, see sect. 1.1.2.2 for the distinction). On the other hand, ontological analysis may "extract" an ontology from a given, frequently special purpose, representation[271]. The notion of domain of knowledge / reality refers to areas of interest. As such it covers a whole spectrum in terms of possible thematic breadth. At one end, there are fields that unfold by starting from a single notion, such as function, role, part-of, or causality[272]. At the other end, there are large domains like biology, medicine, or mathematics, which may be subject to ontological analysis / for which ontologies are to be provided.[273] In any case, the goal of an ontological analysis of a domain is an ontological theory about that domain, allowing one to categorize and relate the entities in the domain, or, more generally, describe them (including connections among them). Looking at the second target case, analyzing a given representation ontologically means to develop an ontological theory of the domain of entities addressed in / described by the representation.[274]

RELATION WITH 'ONTOLOGY DEVELOPMENT'

The above characterization of ontological analysis in one sentence, namely as "a process through which an ontology is developed / compiled", requires some further limitation in relation to the terminology of other authors. In particular, we do not equate ontological analysis with the terms of ontology development in general and of the 'ontology development process' as introduced in [239] and more recently presented in [289, sect. 3.1 and 3.3.5]. In the latter terminology, our ontological analysis primarily and roughly corresponds to a combination of the activities 'conceptualization', (in the language-oriented version) 'formalization', as well as 'documentation' and 'integration' (cf. [289, Fig. 3.2, p. 110]). The last item actually refers to reusing existing ontologies in devising new ones.

[526, 527, 530]

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[271]'Representation' is here meant in a very broad sense, including expressions of particular languages up to, e.g., the contents of information systems.

[272]These examples are biased by our experience. Corresponding ontological analyses of the mentioned concepts can be found in [128, 526, 527, 530, 582, 707], a.o. But, clearly, more specific notions may equally well be chosen to be at the center of an ontological analysis.

[273]For those accepting the possibility of a / the universal ontology, the latter would form the extreme case of an ontology whose domain is literally everything. NB: While [1] bears 'universal ontology' in its title and the ambition has been stated in [562, p. 1340] in connection with Cyc [@23] (cf. sect. 1.1.4.2, near and incl. FN 108), [499, sect. 1] argues convincingly against the possibility. [265, p. 20] quotes criticism in [750] against a "global ontology".

[274]Despite recourse to domains from representations, nonetheless we distinguish these lines of target cases due to their differences in the raw material, which is unrestricted and equally treated in the case of domains, but in the analysis of existing representations includes specifically determined material with special status (although additional material may be utilized in the course of the analysis).

INTEGRATION WITH EXISTING ONTOLOGIES: ONTOLOGICAL TRANSLATION AND FOUNDATION

'Integration'/reuse is of vital importance in our setting of developing foundational ontologies. In particular, we suggest to refer to top-level and core ontologies when conducting an ontological analysis, thereby integrating domain-specific concepts "on the fly" with foundational ontologies. Of course, adequate domain-specific ontologies can/should also be reused. Especially in the target case of providing an ontological description of an existing, typically special-purpose representation, this leads to two notions of activities which are often carried out in parallel to/as "part" of ontological analysis: ontological translation and foundation.

The ontological analysis of a representation, or for simplicity say, of a set of expressions E, can be viewed as a transformation of these expressions into a formalized ontology O, whose formalization language is $Lg(O)$. Of course, such transformation must respect/preserve/elicit the (intended) intensional contents of the original representation E.[275] The distinction into two cases arises on the basis of the coverage that O provides for the conceptualization underlying E. The ontology may be such that for every constant and every expression of E an immediate ontological counterpart is available.[276] Then we speak of an *ontological translation*.

However, frequently enough this is hardly possible, because sufficiently specific ontologies are not yet available. Accordingly, the second form of reuse utilizes those notions in an ontology which are not equal/equivalent to a concept/conceptual meaning behind a language element, but which are more general than that if it were available in the ontology. For instance, one may have a term lion and analyze this with respect to top-level ontological notions such as object, process, or quality, and then assign it to be more special than object. This kind of linkage to a (usually foundational) ontology is referred to as *ontological foundation* or *ontological embedding*. We can more precisely indicate the kind of foundation by the adjunct *by specialization*. The second kind of linkage under the general name of ontological foundation is the classification of a term('s referent) into a category of the ontology, hence called *ontological foundation by classification*. This is the apparent type for (ontological) individuals, which otherwise could not be ontologically founded because generalization is not applicable to individuals. For categories, this kind/aspect of ontological foundation may appear less obvious, but it is equally appropriate if the target ontology is sufficiently rich, e.g., in terms of higher order categories.

At the level of ontological translation vs foundation, let us stress that these are to be understood as independent notions, at the same level of generality. In particular, neither is a special case of the other. While we note that the term 'ontological translation' (in a broader sense) may also be a good candidate for naming a generalization of the two, no dedicated term for this reading is introduced herein.

ONTOLOGICAL REDUCTION AS A STRICT FORM

The above paragraphs settle terminology strongly related to and selected in connection with the notion and method of *ontological reduction*/*ontological mapping* developed by Heinrich Herre, Barbara Heller and others, associated with GFO [396, prim. sect. 5], [388, prim. p. 58–59], [397, sect. 2.4.2].[277] The method of ontological reduction is primarily formulated for the ontological analysis of existing representations (e.g., terminological systems in [396]) and involves both ontological foundation and translation. Earlier work on ontological foundations for UML, e.g. [352], while not explicitly mentioning the term 'ontological reduction', account for related examples, as well.

Considering the case of a single expression e, an ontological reduct of e is an expression $\tau(e)$ resulting from a "semantic translation" τ from $Lg(e)$,[278] the language in which e is formulated, into a language in which an ontology O^+ is formalized. The ontology O^+ is assumed to be an extension of a foundational ontology O, such that O^+ is suited to account for $\tau(e)$ to be semantically equivalent with e, also with regard to $Lg(e)$ and its semantics.[279] In the general case, it is expected that O^+ cannot solely rely on the language

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[275]Despite the vagueness of the notion of intension at this point, note again the connections with Def. 2.3 in ch. 2.

[276]We neglect the notion of variables, esp. free variables here for simplicity.

[277]Although some proximity remains, ontological reduction of Herre et al. should not be identified with the philosophical notion(s) of ontological reduction (see e.g., [295, ch. 8], [892, p. 221]), a.o. because of its particular focus.

[278]The use of τ here is adapted to and intended to link to the translation functions in Def. 2.3, sect. 2.2.2.1, instead of the object to meta level translation employed in sect. 3.3 below. Due to the adaptation, it deviates notationally from the sources cited above.

[279]In [388, 396, 397] the understanding of 'semantically equivalent' to be applied here is not expounded in great detail, e.g., con-

of O, but instead requires a language extension, such that new symbols for primitive notions can be added, which should be characterized axiomatically (like O itself) and should be ontologically founded in O. Altogether and without going into further detail, the three methodological steps prescribed by ontological reduction as a method comprise the following, according to [396, sect. 5, p. 112] with minor modifications. Note the close relationship with the axiomatic method, described in sect. 1.1.5.3.

1. the construction of a set of primitive concepts and relations for the expression e (*problem of the primitive base*)
2. the construction of an extension O^+ of the selected foundational ontology O by adding new categories, relations, and axioms (*axiomatizability problem*)
3. the construction of equivalent expressions for e (*definability problem*)

2.3.2 Perspectives of Analysis Explication vs Theory Comparison

A short interlude shall provide us with a further distinction, namely of two perspectives/views on formalized ontologies (and their models[280]). We name these the 'analysis explication' and the 'theory comparison' perspectives. They recur in ch. 4, where they play a significant rôle in some arguments.

ANALYSIS EXPLICATION

In the context of ontology development in general and of ontological analysis in particular, *analysis explication* refers to the idea that a formal theory is created as a reflection / formalization of a particular (informal) ontological theory, i.e., representing a single conceptualization. In such context one can assume that the ontology engineer has specific ideas about the intended / intensional interpretations of constant symbols in the language used. Accordingly, the freedom of interpretation available in classical semantics might require further constraints – under this perspective. Moreover, we assume that such formal theory aims at capturing the intensions of the ontology engineer maximally and precisely. Clearly, this does not imply that the theory will be complete, with answers to any conceivable query in the given language, nor that it were capable of characterizing a / the "unique" model. Instead, for theories axiomatizing real-world domains, high degrees of incompleteness must be expected, i.e., a large number of sentences that remain independent of the theory that covers the knowledge of the ontology engineer rather than any "objective states of affairs".

THEORY COMPARISON

The perspective of *theory comparison* is different and more closely related to the classical approach to logical semantics, with outmost freedom in interpretations. In that perspective, a single or several formalized ontologies are given and, as formal theories, can be examined wrt metalogical properties (e.g., consistency, completeness, decidability, etc) and wrt mutual, formal interrelations (e.g., exploring whether there are theory interpretations among them). It appears reasonable for such analyses to let the interpretations of constants vary beyond their intended meaning.

LINK WITH HODGES' DISTINCTION OF AXIOMATIZATIONS AND WITH COCCHIARELLA'S VIEWS ON LOGIC

Eventually, note that there is a strong resemblance of the distinction of analysis explication vs theory comparison with the one discussed by Wilfried Hodges on axiomatizations describing particular structures vs defining classes of structures in [411, sect. 19 and 20]. To some extent one may consider contrasting the two perspectives above as an application / adaptation of Hodges' distinction in / to our problem setting. Yet again, the semantics proposed in sect. 4, though, pays more attention to this issue in its very definition. Similarly to the case of Hodges' juxtaposition, Cocchiarella's [160, sect. 1] distinction that is introduced in sect. 2.2.2.2 can be seen in parallel with analysis explication vs theory comparison. The view of 'logic as a language', which is assumed as a framework for general ontology, suggests similarities with analysis explication. Equally, 'logic as a calculus', proposed by Cocchiarella to be utilized in comparing different 'logics as a language' formally, appears to be in good harmony with the idea of (formal) theory comparison.

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cerning our distinction of conceptual and formal equivalence. Prima facie, formal equivalence seems to be at least involved, which would face the same issues as discussed in the previous sect. 2.2.2.2.

[280]Read 'model' here in an abstract, functional sense of Tarski-style semantics, but not necessarily referring to classical set-theoretic models.

2.3.3 Abstract Core Ontologies and Meta-Ontological Architecture

ONTOLOGICAL ANALYSIS OF ONTOLOGY REPRESENTATIONS

For approaching a semantic basis of languages / representations with greater intensional flavor it is a question of interest of how to access the conceptual semantics of a representation $R(O)$ of an ontology O in a formal language L, which is supposed to have a formal semantics. In the terminology that is established in sect. 2.3.1 above this leads to conducting an ontological analysis targeting at the representation $R(O)$. One may wonder why the need for an ontological analysis of $R(O)$ would arise at all, given that $R(O)$ is a representation of an ontology by assumption. The motivation is, anew, the encoding problem based on the formal semantics of L discussed in sect. 2.2, which is even more visible in what we elaborate and refer to as the theory view of semantics in sect. 3.3 below. Of course, it may well be the case that the same conceptualization is formalized in different ways. Clearly this applies wrt several languages, but also wrt one and the same language. Assuming that L is a language that is, in a sense, capable of merely expressing O through $R(O)$ (without any encodings of entities postulated by O), one may think of defining the conceptual semantics of further formalizations by ontological translations into L or $R(O)$, resp. But for $R(O)$ itself this approach is not fruitful. There would be no need to consider a language other than L and the translation "within" L should yield the very same result, namely $R(O)$. We remark that this thought forcefully anticipates sect. 3.4 and the notion of an ontologically neutral semantic approach therein. However, for $R(O)$ itself, ontological *foundation* can additionally enhance the conceptual semantics provided by such $R(O)$.

FIRST STEP: ADD A CONCEPTUAL META LEVEL

A first step in this regard is to consider whether certain ontological kinds can be associated with basic / elementary syntactic domains of the abstract syntax (cf. sect. 2.1.1). This association reflects the observation that the grammar of many (artificial) languages, esp. in conceptual modeling and knowledge representation, is implicitly tied to intensional aspects.[281] Associating syntactic domains with ontological kinds is to be understood as a form of ontological foundation, namely ontological foundation by classification. It means that corresponding intensional interpretations of expressions in these syntactic domains are instances of those ontological kinds.[282] One can specify such assignments even for general-purpose representation languages like FOL, if a sufficiently general ontology is employed. Notably, though of course, the user of a language may deviate from such general assignments and may use expressions from the same syntactic domain with intended referents in clearly differing ontological categories.

META-ONTOLOGICAL ARCHITECTURE & ABSTRACT CORE ONTOLOGY (ACO) IN [399]

The initial proposal of the meta-ontological architecture developed in the context of GFO is presented in [399]. Fig. 2.3[283] provides an overview of the architecture. One major aim in [399] is to establish an ontological meta level, with the central notion of an *abstract core ontology* (ACO), and to discuss the relationships between that meta-level with the / "its"[284] object level and the meta-meta level. In summary, the meta-level is deemed 'ontological' because it connects to the object level by means of instantiation, corresponding to the case of an ontological analysis of ontology representations mentioned above. In contrast, the formalization of the meta level rests on FOL under classical semantics in [399], hence the meta-meta level involves a formal reduction of meta-level entities.
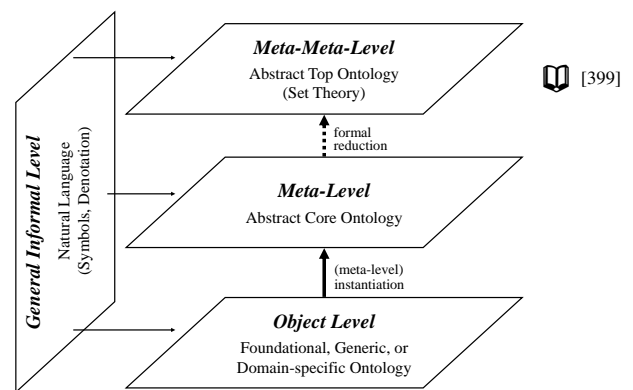
Figure 2.3: Meta-ontological architecture presented in [399], adopted and refined for GFO.

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[281] In the younger advent of domain-specific languages [578] (see FN 461 on p. 140 for slightly more background), this plays a significant role, as well.

[282] This corresponds to 'categorical abstraction' [398, sect. 3.1.2] up to a highly general level.

[283] Fig. 2.3 corresponds to [399, Fig. 1, p. 1401] (©2005 Springer, permission for reproduction granted as of Dec 6, 2014).

[284] Note the refinement of splitting the object level into *basic level* and *domain level* in [398].

Later parts of [399] discuss several particular choices of which kinds of entities ACOs may comprise, as well as an application to Formal Concept Analysis [261].

FUNCTIONAL CHARACTERIZATION OF ACOs, BASED ON ONTOLOGIES

For the purposes relevant in the sequel, we restrict ourselves to a generic, functional characterization of ACOs, similar to that in [534, p. 50–51], [399, sect. 3, p. 1403]. In our context, the need for ACOs originated from the ontological analysis of ontologies themselves, as described above. An abstract core ontology can thus be defined as an ontology of constituent entities in ontologies, i.e., an ACO provides categories for classifying ontology constituents, where 'classifying' stands for proper ontological instantiation. For instance, $\mathcal{CR}$, the ontology of categories and relations initially presented in the next sect. 2.4, may be adopted as an ACO for classifying constituents of GFO. Then constituents such as chronoid, topoid, process and presential are instances of *category*[285] in $\mathcal{CR}$, others like part − of and inheres − in would be categorized as *relations* in $\mathcal{CR}$. In order to add precision to the characterization of ACOs, the ontology must be assumed to be given as an ontology representation $R(O)$ in some language L. Regarding $R(O)$ and L, resp., we expect ACOs to capture / provide categories for those entities expressed by certain constants[286] of L. In contrast, it is not necessary that $R(O)$ as a whole can be covered by an ACO category, nor "fragments" of $R(O)$. For example, $\mathcal{CR}$ does not include any category for 'ontology' nor for 'ontology module'. Besides, no position is taken here on the object-level / meta-level distinction. That means, we do not enforce the view that the ACO level must be strictly separated from the object level that it classifies. This remark anticipates yet another issue, which is introduced in more detail in sect. 3.2 and treated on several occasions while developing approach of ontological semantics in ch. 4.

GENERALIZATION OF THE ACO CHARACTERIZATION

Given this ontology-centric ACO characterization, it turns out that this approach naturally generalizes / leads to a specific form of ontological foundation for arbitrary representations if these are equipped with, at least, a set of constants for which the language user conceives of intended referents.[287] In this regard, we extend the division of symbols / tokens (resulting from lexical analysis) of a language L (cf. sect. 2.1.1) on a semantic basis. Tokens may be categorized as *referential* and *non-referential* depending on whether or not they (are intended to) denote / refer to an entity. This is of interest for both keywords (tokens of types with functional roles in L's semantics) and identifiers (tokens of other types that possibly occur in an abstract syntax tree of L). For the latter, it is the common case, e.g., thinking of predicates / predicate identifiers in a particular use of FOL.[288] According to these observations, the generalization of the ontology-centric notion of ACOs leads to ontological foundation by classification, namely where every referential token of a language (usage) can be assigned to at least one category in the ontology such that the intended referent of the token is an instance of the category. From this perspective, an ACO as described above is an ontology that provides notions for conducting an ontological foundation by classification of ontologies, in that ACO notions serve as categories for the referential tokens / the constituent entities of an arbitrary (object level) ontology.

OUTLOOK: ONTOLOGICAL USAGE SCHEMES & OVERLAP WITH INTERPRETING SYNTACTIC CATEGORIES

Some languages will allow for a more schematic treatment in terms of their grammar. Assume that a language L is used in a way such that all identifiers of a syntactic category S of the grammar of L can be expected to be referential and to be categorized into one and the same category C in a respective ontology. Then an assignment between S and C can serve as a much more compact form of all individual assignments

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[285]More precisely, they instantiate simplecategory and non − relationalcategory, which is worthwhile stating, because category subsumes relation in $\mathcal{CR}$, see sect. 2.4.

[286]More precisely, only referential constants are under consideration here, e.g., in logical languages typically non-logical and non-technical symbols with fixed interpretation. We avoid further details on this here, but the issue re-occurs in sect. 4.5.1 below.

[287]Standard propositional logic utilizing only propositional variables and possibly logical constants $\top$, $\bot$ does not belong to this class, interestingly, at least if considered without a particular context of usage.

[288]Especially for identifiers one may stress that the grammar of a language (language family, from another perspective) involves a set of admissible identifiers (e.g., determined by a lexical grammar), unless this level is omitted as in theoretical work intended to be read by humans. Only a subset of all admissible identifiers is used in the context of a particular representation. In the case of FOL, this comment applies to a general grammar for FOL (as "one" language), while the signature is determined / implicit in expressions that can be generated from that grammar.

between identifiers "in" S and C. Moreover, one can observe a transition to ontological foundation by specialization in the case of S and C. These considerations are continued and methodologically exploited in sect. 5.2 through the notion of *ontological usage scheme*. Here only an extensional overlap shall be pointed out, namely between notions that one can expect in ACOs according to their above characterization (like category, relation, etc) and ontological interpretations of syntactic categories in general-purpose representation formalisms, including logical (FOL, DL, etc) and conceptual modeling formalisms (UML etc) as well as others (e.g., frame-based languages as originating from [588] and Formal Concept Analysis (FCA) [261]; cf. [399, sect. 4] on FCA wrt this claim). Among the latter one may count categories and relations, [399] e.g., when interpreting (unary and non-unary) predicates in FOL or alternatively concepts and roles in DLs. Other languages may require additional constructs like properties/attributes, as available in the two concrete ACO examples in [399, sect. 3.2–3.3].

ACOs vs (knowledge) representation ontologies

The transition just presented sets the notion of an ACO into tight relationship with the 'representation ontology' in [841] (based on [181]), later referred to as 'knowledge representation ontology' (KR ontology), e.g., in [289, sect. 1.4.1.2, p. 29 & sect. 2.1], as first discussed above prim. in the final paragraph of sect. 1.1.4.3. ACOs originate from and thus have a clear focus on ontologically classifying (referential) tokens. In contrast, a KR ontology "gathers the modeling primitives used to formalize knowledge in a KR paradigm" according to [289, sect. 2.1]. By further analysis of the examples given in the same section, it remains unclear to what extent a representation ontology differs from (possibly a specific subset of) the abstract syntax of a representation formalism, despite the more conceptually oriented wording in terms of, e.g., 'modeling primitives'. It is interesting to note, though, that originally, in [181] an aspect of ontological commitment was involved, which was repressed in [841] taking recourse to [334]. When coining the term 'representation ontology', [841] actually refers to [181], but at the same time states that "[representation ontologies] are intended to be *neutral* with respect to world entities [334]." [841, p. 193]. This reawakens awareness of our rejection of the claim of the ontological neutrality of even general-purpose languages in sect. 2.2.2.2. Indeed and as observed there already, [181, esp. p. 20] attributes ontological commitment to the earliest choices of representation.[289] Accordingly, the proximity between ACOs and this early (implicit) notion of representation ontology is greater than that to the later readings. Nevertheless, there remains a difference between ACOs and knowledge representation ontologies depending on the domain a language is intended for. The above restriction to general-purpose languages wrt the overlap of constituents of ACOs and KR ontologies is deliberate. We expect ACOs to comprise highly general notions, whereas a KR ontology of a language with a specific domain focus may have to commit to much more specific notions, unsuitable for classifying ontology constituents. This observation concludes our introduction of ACOs.

2.4 Conceptualization of Categories and Relations – $\mathcal{CR}$

Purpose, structure, and presentation of this early $\mathcal{CR}$ section

The notion of abstract core ontology from the previous section plays a further rôle when reconsidering the treatment of predication in the context of ontological semantics, in sect. 4.4. Preparing for a specific instance of such treatment in sect. 4.4.3.2, the present section establishes the conceptual fundament of a small, but highly general ontology abbreviated as $\mathcal{CR}$, for *ontology of categories and relations*. Furthermore, we propose this ontology for integration into the General Formal Ontology (GFO), see sect. 1.1.5. In both cases, the reason is that we consider $\mathcal{CR}$ as a candidate of an ontological theory that may be relevant, at least indirectly, in every ontological analysis.[290]

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[289][181, p. 21] briefly indicates that a difference must be made between a knowledge representation and a data structure. It comprises an illustrative, intuitively appealing example in distinguishing a semantic net, which is a representation, from a graph, claimed to be a data structure. "They are different kinds of entity, even though one is invariably used to implement the other, precisely because the net has (should have) a semantics." [ibid.]

[290]Of course, endorsing a pluralistic attitude wrt ontological analysis, other ontologies (of equal degree of abstraction) may not subscribe to these notions at all and may adopt different basic notions. In that case, we are interested in understanding how the latter account for "comparable situations", e.g., in easily comprehensible domains.

Firstly, the scope and relevance of $\mathcal{CR}$ due to the fundamental nature of categorization and relations is discussed. The subsequent sections introduce our views on / high-level analysis of categories and relations. A few further trains of thought that depart from $\mathcal{CR}$ complete the section.

The mode of presentation in this section utilizes primarily natural language and (pseudo-)UML class diagrams[291] and there are two ways of reading them. Without further knowledge of the subsequent chapters, they can largely be interpreted according to the (semiformal) UML semantics.[292] Once that ontological semantics and the notion of ontological usage schemes are established, they are primarily intended to be read with a particular ontological usage scheme for UML. Note that a logical axiomatization is given in sect. 6.1, to which an analogous duality applies, but which is a little closer associated with ontological semantics, such that interjacent parts of the work should establish that approach first.

2.4.1 The Fundamental Notions of Categorization and Relations

SCOPE OF $\mathcal{CR}$

Prior to addressing its relevance, let us specify the scope for $\mathcal{CR}$. Not at least wrt its later uses herein, we aim at a very general ontological theory that allows for *representing / expressing* two kinds of statements, namely the categorization / classification of entities, and setting them in relation to each other.[293] While the resulting ontology establishes a few claims about the nature of notions such as category, individual, relation, and role, we stress 'representing / expressing' above because we do not engage in an in-depth ontological analysis, in the sense that a strong explanatory theory of these notions were to be given, nor shall detailed systems of subtypes (and corresponding theory) be considered. In our opinion, such issues constitute very interesting topics for further work, but herein we focus on a highly general level only, which suffices in the sequel.

IMPORTANCE OF CATEGORIZATION

So, why are we interested in $\mathcal{CR}$, remembering the link with the notions of abstract core ontologies, which are intended to account for the ontological analysis of ontology constituents? Loosely speaking, though compatible with sect. 2.3.1, the aim of an ontological analysis of a subject is to determine "What entities are there?" and "How are those entities interrelated?". This characterization suggests two core ingredients: first, the notion of *category* / categorization and, secondly, the concept of *relation*. It should be safe to call the fundamentality of categories and categorization undisputed, acknowledging the broad range of philosophical literature on categories as well as the more recent development in cognitive science. In the latter field, George Lakoff has authored the following statement in [501, ch. 1], where that chapter is devoted to "The Importance of Categorization" altogether.

> "Categorization is not a matter to be taken lightly. There is nothing more basic than categorization to our thought, perception, action, and speech."　　　　　　[501, p. 5]

Categories are equally acknowledged in AI and KR, e.g. in the popular AI textbook [722] by Stuart J. Russell and Peter Norvig.

> "The organization of objects into categories is a vital part of knowledge representation."
> 　　　　　　[722, p. 322]

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[291] These elements should be available and basically unchanged in most UML versions. When compiling the diagrams, we primarily relied on the 2. ed. of the UML Reference Manual [720]. Occasionally, however, there are deviations in order to achieve more compact models, better illustration, etc., because the mere purpose of visualizing / highlighting major connections does not require a fully conformant UML specification.

[292] However, note that there are aspects of ontology representation that render pure UML class diagrams insufficient for this purpose, as described in the design principles of ODM [@75] in [646, esp. sect. 8.1].

[293] Insofar $\mathcal{CR}$ may be taken as abbreviation of (ontology for) 'categorization and relating', possibly more appropriately.

IMPORTANCE OF RELATIONS

Relations are of likewise fundamental nature. As soon as an entity is singled out from "the rest of the world", i.e., as soon as two or more entities are / can be distinguished, at some point relations must be assumed, i.e., entities with the power to mediate between those things distinguished.[294] The spectrum of the literature on relations is very rich, as well, and to some extent it overlaps with that of categories. Indeed, relations are widely construed as categories that apply to several individuals. At least, commonly in the mathematical treatment of relations (and the use of the term), one does not strictly distinguish between sets (of objects) and (mathematical) relations, i.e., sets of tuples (typically of fixed arity), usually equating / introducing $A^1 = A$ for a set A. For glancing into philosophy, note anew wrt terminology that there 'property' is the term that corresponds (best) to our 'category'. That said, Chris Swoyer and Francesco Orilia declare in [796, sect. 1.1.3] that, on the one hand, "Properties are usually distinguished from relations.", but via "Relations generate a few special problems of their own, but for the most part properties and relations raise the same philosophical issues.", they decide to "[. . .] use 'property' as a generic term to cover both monadic (one-place, nonrelational) properties and (polyadic, multi-place) relations (i.e., properties of degree higher than one)." [ibid.]

CATEGORIES AND RELATIONS IN MODELING

In computer and information sciences, both notions frequently appear together in collections of very basic concepts – with varying terminology, not surprisingly. For example, John Mylopoulos [614] reports that one novelty in M. Ross Quillian's proposal of semantic networks [678, 679] was to organize the information base in terms of 'concepts' and 'associations' [614, p. 131].[295] He further points out the distinction of 'entities' and 'relationships' in Peter P. Chen's Entity-Relationship (ER) model [150]. From both, semantic networks and ER models, it seems to be only a short step (in this regard) to the Ontology Definition Metamodel (ODM) [@75] and its meta model foundation, the Meta Object Facility (MOF) [@65], version 2.0 [641], which includes 'classes' and 'associations'. Indeed, numerous formalisms exhibit categories and relations (each under several names) as fundamental elements. This can also be read from an earlier survey in [526, p. 41], actually devoted to the occurrence of the notion of role in various formalisms, which links to the theory of relations that we adopt herein. In this connection note that the Enterprise Ontology [@33] [828, 830], [157, sect. 3.1.2], [289, sect. 2.4.4, 3.2.2] uses a meta ontology that comprises 'entity', 'relationship', and 'role', a.o. [830, sect. 3]. Similarly, Object Role Modeling (ORM) [@89] [367, 368], [815, sect. 4] is based on 'entity types', 'fact types' (with the synonyms 'relationship' or 'association' [368, e.g. p. 10]), and 'roles'. Altogether, we find strong evidence to consider categories and relations to be of fundamental character.

PRESUPPOSITIONS AND NEXT STEPS

The brief glimpse on the literature in order to substantiate the claim of the importance of the notions of category and relation may serve a second purpose in realizing that there is a plurality of accounts on each and both notions. Of course, ontological theories of them involve further intensely debated notions and well-known issues. Categories are related to entities by means of *instantiation* / exemplification / predication[296], among such instances are *individuals*[297]. Concerning relations, $\mathcal{CR}$ builds upon an earlier account of relations in the context of a theory of roles [526], which has already been incorporated into GFO, cf. e.g. [392, sect. 14.4.5.2], and has been further developed in [527, 530] in relation to this thesis. Besides / instantiating relations the approach involves *relators*, themselves consisting of *roles* ('relational roles' in the aforemen-

[527, 530]

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[294]The only (but self-defeating / -destructive) alternative that we see were a single, completely homogeneous "world entity" without any parts and features.

[295]Recalling FN 72, R. H. Richens refers to semantic nets whose "elements represent things, qualities or relations." [706, p. 23].

[296]These are seen as synonyms in our case, although especially predication may deserve a separate account. On the other hand, Nino B. Cocchiarella mentions on the connection of predication and universals: "Here, by a universal we mean that type of entity that can be predicated of things, which is essentially the characterization originally given by Aristotle.[19]" [160, p. 124], where endnote 19 points to Aristotle's *De Interpretatione* 17 a 39 for his account of universals.

[297]Especially in this sect. 2.4.1, the term 'individual' refers to the ontological / metaphysical notion of individual, see e.g. [538, ch. 3], [294], which is opposed to category in GFO, for example. The logical notion of 'individual', understood as an arbitrary *element* (in the set-theoretic sense) of the universe of a mathematical / set-theoretic structure is only addressed as 'logical individual' in this section.

tioned publications), which can have *players*, whereas the relators serve as *context* of the roles. This approach involves two corresponding relations named *plays* and *role-of*.

The reader may consider refreshing the introductory presentation in sect. 1.1.5, esp. 1.1.5.2, for better comprehensibility of the subsequent elaboration. This elaboration is challenging wrt structure, because the two domains of categories and relations are highly intertwined in CR. We attempt to dissect the matter nevertheless, focusing on categories first, then on relations, adding a section for addressing issues in combination.

As a whole, note that we see CR as one proposal for a conceptually minimal basis, to the extent that all other notions can be introduced as theory extensions and thereby can be *ontologically founded*, cf. sect. 2.3.1, in/by means of the basic notions of and around categories and relations. We argue that everything that exists (and is dealt with in theories about reality / the world, in the broadest sense, cf. sect. 1.1.5.1) can be categorized or interrelated with each other, thereby (also) making CR a good candidate for an abstract core ontology. Importantly, this is *not* to claim that every notion or entity can / could be *reduced* to those fundamental notions, in a strict sense of ontological reduction, cf. e.g. [295, ch. 8], [892, p. 221]. Ontologically, we consider everything on a par with each other.[298]

2.4.2 Categories

2.4.2.1 Background and Scope for the Notion of Category

CR **CATEGORY VS CATEGORY, PROPERTY, AND UNIVERSAL IN PHILOSOPHY**

For a gentle start and in accordance with the context of GFO, note that we apply the term 'category' to notions of arbitrary generality, hence reaching from top-level categories such as material structure and process to highly specific domain-level categories, e.g. lion, upholstery hammer, even including categories like the one described by the phrase 'lion that lives in central Africa between the years 1990 and 2000' (an *aspectual category* in terms of [392, sect. 14.7.1, p. 339]). This differs from the view of categories in philosophy where they are understood as "highest kinds or genera" [821, 1. par.], cf. also [537, p. 3], i.e., as top-level categories. The wider interpretation of category, however, follows the GFO terminology and it can be found in cognitive science literature, e.g. [501], but also in some philosophical works, e.g. [295].

Somewhat repetitive to further above, category in the CR sense may also, but partially, be understood as 'property' [796]. Then again, the notion of property in [796] includes several examples and special kinds of properties that we do *not* consider as categories, not at least the (GFO) notion of property/quality. Picking up the example of red apples from [ibid., sect. 1.1], first observe that apple is a category herein (and wrt GFO) that is instantiated by particular apples (a case which generalizes to sortal properties / categories [ibid., sect. 7.8], [348, sect. 4.1 a.o.]). The case of red is analyzed differently, however. In simplified form,[299] red can be analyzed in GFO as a category of properties. Properties are individuals that *inhere in* a[300] property bearer, say, a particular apple. Insofar, there is a different connection between apple and a particular instance a of it, compared to a property (individual) r_a that inheres in this a, while r_a instantiates the property category red, but a does not instantiate red. In [796], both of these cases are discussed as properties.

Remembering the remarks on [796, sect. 1.1.3] above, relations are perceived as polyadic properties that are *jointly instantiated* by several entities / relata, which is supposedly a widespread view. However, there is a clear differentiation between categories and relations in GFO and CR, resp., in that there is no joint instantiation by multiple entities. There are no many-place categories in that sense in CR. Instead, each CR category can have instances,[301] but each instance is on its own an instance of the category. In contrast, interconnections among entities are established by relations, to be detailed in sect. 2.4.3.

[397] in FN 299

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[298] This statement, in turn, concerns primarily existence, but is not to say that there are no / could not be any ontological dependencies among entities, or any explanatory accounts of one kind of entity in terms of others.

[299] We ignore the distinction between property and property value in GFO [392, sect. 14.4.5.1], [397, sect. 9], to be clearer on the actual argument. The argument can be applied exactly wrt the notion of color (for which red is a value, according to GFO), but then the linguistic relation changes, where 'red apple' is meaningful, but 'color apple' is not, at least not in an analogous way.

[300] More precisely, each property inheres in exactly one property bearer. To some extent being akin to tropes [565], GFO properties are *non-transferable* [565, sect. 2.4] / satisfy the *non-migration principle* [352, p. 68].

[301] This phrase is to be further analyzed below.

Consequently, the notion of category in $\mathcal{CR}$ encompasses the philosophical notion of highest kinds as a special type of $\mathcal{CR}$ categories and it overlaps the notion of properties in philosophy, yet excluding relations if those are understood as properties that are jointly instantiated by multiple entities.

Last but not least, the term 'universal' deserves consideration in the present context. We may say that the notion of universal has been dealt with in the previous paragraph already, because it appears to be taken as a synonym of property in much philosophical literature, at least as long as individualized properties such as tropes (and those in GFO) are not considered. For example, this stance is adopted in [796, sect. 1.1], apart from some remarks in [796, sect. 1.1.2] on distinguishing the possibilities of properties as universals vs properties as particulars, referring to [565] for tropes. Compared to [796], the chapters devoted to universals by Michael J. Loux [537, ch. 1–2] suggest a similar picture, being worded with the term 'universal'.[302] Insofar nothing is to be said in addition wrt relating universals to the notion of $\mathcal{CR}$ category. Notably and although we abide by the terminology of category, individual, and property as in GFO, Edward Jonathan (E. J.) Lowe provides a clearer terminology in [538, ch. 3, sect. 2, p. 77] referring to two oppositions, namely (1) universal vs particular and (2) property vs object, and a distinguished notion of individual as "something that is 'individuated'", in a metaphysical sense as presented in [538, ch. 3].

TYPES OF CATEGORIES TO ILLUSTRATE THE SCOPE OF $\mathcal{CR}$ CATEGORIES

The distinction between universals and particulars immediately prompts a central metaphysical problem, or better, a central set of related questions, starting with the one for their existence (and the mode thereof, if accepted), overall the Problem of Universals [537, ch. 1–2]. Typically *realism*, *conceptualism*, and *nominalism* are distinguished as broad streams in this regard, cf. also [796, sect. 1.1.5]. Without following any route that leads to details here, we simply take up the GFO approach in adopting a pluralistic position close to and inspired by Jorge J. E. Gracia. In [295, ch. 9, p. 177–217] he discusses seven of the many positions on the ontological status of categories that occur in the philosophical literature. These positions identify categories with transcendental entities, immanent constituents of things, similarities, collections, concepts, types, and tokens.[303] Cocchiarella adopts a comparable standpoint in his "Logic and Ontology" [160], where he discusses different types of predication that give rise to different 'logics as a language', see sect. 2.2.2.2.

Applied ontology as well as computer and information sciences have also brought about different types of categories. First of all, the ontology evaluation method OntoClean [341, 342], cf. also [289, sect. 3.8.3], is based on a set of meta properties / characteristics, incl. *rigidity*, *identity*, and *unity*, that were "imported" from / inspired by philosophical notions and distinctions, and are presented in a long series of publications by Nicola Guarino et al., incl. [258, 325, 332, 335, 336, 343–346]. Those meta properties also form the basis for a collection of meta categories (i.e., types of categories for our context), e.g., *type / kind*, *role*, *sortal*, and *mixin*, e.g., presented in [346, sect. 3–4, esp. Table 2, p. 62].[304] From another angle, the way in which categories are described / defined may bear implicit or explicit assumptions on different types of categories. Along these lines, *Aristotelian* or *genus-differentia definitions* are advocated in, e.g., [714, sect. 3.1.3, p. 482], [620], [762, p. 151], whereas prototypes are discussed in, e.g., [265, esp. sect. 6], [249], as well as, of course, in cognitive science, see e.g. [502, 712] and [610, a.o., ch. 3]. Similarly, [860, sect. 3] discusses classification theory in the context of theoretical foundations for conceptual modeling, similarly pointing to [502, 712, 769] in cognitive science.

The main purpose of this spectrum of types of / views on categories is to illustrate potential candidates that the notion of $\mathcal{CR}$ category might subsume, in terms of future extensions of the theory which should include a systematic classification of categories. But presently aiming at a theory at a highly general level suggests to equip $\mathcal{CR}$ only with few assumptions/restrictions, at least wrt category and its "closest relatives", instantiation and individuals.

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[302] Although we must note a more limited reading of 'property' described by Loux: "many realists lump all monadic universals together under the title 'property'" [537, p. 20], contrasted with relations characterized as "polyadic or many-place universals" [ibid.].

[303] One deviation from this account is to view collections, and sets in particular, as *abstract individuals* in GFO. That notion is merely mentioned in [392, sect. 14.4.2, p. 309], [397, sect. 6, p. 19] with examples of numbers such as π, but not of sets. Recall also FN 266 with a quote from [160, p. 123], where Cocchiarella denies set membership as a kind of predication.

[304] More recently, the approach was reworked into a UML profile [359], [348, ch. 8], [358, sect. 8.4] and has become integrated and further developed into the OntoUML system [@87] [75, 76].

2.4.2.2 Entity, Category and Individual

PRINCIPAL NOTIONS IN THE DOMAIN OF CATEGORY

In the domain of category we first identify four principal notions, namely *entity*, *category*, *individual*,[305] and *instantiation*.[306] The previous sect. 2.4.2.1, more precisely, several sources cited therein, such as [796] and [537, ch. 1–2], suggest as a kind of minimal consensus that instantiation relates categories with other entities, called their *instances* in this context. This is represented in Fig. 2.4. The UML association named by the double colon :: represents the instantiation relation by the same symbol which is used in sect. 6.1 as part

Figure 2.4:
Entities, Categories and Individuals in $\mathcal{CR}$.

of the FOL signature in the context of the $\mathcal{CR}$ formalization. 'Entity' refers to a notion that is applicable to what exists whatsoever, or rephrased in $\mathcal{CR}$ itself, entity is the category that is instantiated by every single object[307] that exists.[308] Since existence is understood in the broadest possible sense, a.o. with recourse to Roman Ingarden's modes of existence [443],[309] this leads to the fact that literally everything is an instance of entity. Following the GFO approach, entity is partitioned into categories and individuals, where categories are those entities that can be instantiated[310]. The latter is marked by the UML rolename[311] instantiatedRL. Its suffix "RL" abbreviates 'role', whereas the purpose of that will become clear from below. Every entity instantiates at least one category, as we just saw, Entity itself. Fig. 2.4 does not enforce instances for every category, where we leave the discussion of *empty categories* for below, as well.

EXTENSIONAL SUBSUMPTION

Already on this basis, the extensional *is-a relation* can be introduced. We partially anticipate the FOL signature of sect. 6.1, which includes a unary predicate Cat and binary predicates $\dashrightarrow$ for extensional is-a (read $x \dashrightarrow y$ as "x is a y"/"x specializes y") and, as noted, :: for instantiation. The (well-known) characterization of extensional is-a from sect. 1.1.2.2 becomes a definition for $\mathcal{CR}$ (and is applicable to the UML generalization notation[312]). Overall, while FOL formulas in this section can only be understood ontologically after ch. 4 has been read, the end of that chapter ensures by Th. 4.44 that they can be interpreted classically, i.e., according to set theoretic semantics, in order to follow any statements on entailments.

$$(2.8) \quad \forall xy \, . \; x \dashrightarrow y \; \leftrightarrow_{df} \; \mathsf{Cat}(x) \wedge \mathsf{Cat}(y) \wedge \forall z \, . \, z :: x \rightarrow z :: y$$

On its basis, *coextensionality* of categories is definable in terms of extensional is-a as in (2.9), which by the definition (2.8) immediately yields (2.10).

$$(2.9) \quad \forall xy \, . \, \mathsf{coext}(x,y) \; \leftrightarrow_{df} \; x \dashrightarrow y \wedge y \dashrightarrow x$$

$$(2.10) \quad \forall xy \, . \, \mathsf{coext}(x,y) \; \leftrightarrow \; \mathsf{Cat}(x) \wedge \mathsf{Cat}(y) \wedge \forall z \, . \, z :: x \leftrightarrow z :: y$$

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[305]In the terminology of [392, sect. 14.7.1], entity may be viewed as a *principle category* in the domain under consideration, classified into *elementary categories* category and individual.

[306]Regarding the use of font types in this section, we aim at easy visual readability by keeping changes to the font type low. Accordingly, we *emphasize* notions on their first (technical) occurrence in italics, whereas the seriffont for technical notation is limited to cases where we see potential ambiguities without it or selected cases of markings, e.g., in relating to UML diagrams.

[307]avoiding "that is instantiated by every single entity"

[308]While the theory of $\mathcal{CR}$ applies also to parts of reality, the intended domain of discourse from an analysis explication perspective (see sect. 2.3.2) is the domain of literally all entities here.

[309]Cf. esp. corresponding remarks in sect. 1.1.5.1 on GFO, the comments on reference of constants in languages in sect. 2.1.2, and for further pointers on existence herein FN 19.

[310]This formulation is refined in sect. 2.4.4, after relations are discussed in more detail.

[311]Following the model of [720, p. 576 ff.], we continue the use of 'rolename' wrt what is officially called 'association end name' in UML 2.

[312]The UML 2.0 reference manual requires generalization to be transitive and "antisymmetric" [720, p. 370]. The latter term may refer to what we call 'asymmetric', because the same page excludes any direct generalization cycles. However, we do not read those constraints from the section on generalization in the UML 2.5 specification [648, p. 141f.]. In any case, transitivity of $\dashrightarrow$ follows straightforwardly from (2.8), but also reflexivity. Moreover, below there is the need to present mutual extensional is-a connections between intensionally distinct categories, thereby breaching some constraints described in [720, p. 370].

Let us mention again (after sect. 1.1.2.2) that the characterization of is-a by means of instantiation is "the typical" extensional definition[313], whereas Ronald Brachman has gathered various approaches to understanding is-a in [107]. Other criticism has been raised, as in [758, sect. 7], leading to alternative proposals of defining is-a, e.g. "A *is_a* B if and only if: (1) A and B are universals, and (2) for all times t, if anything instantiates universal A at t then that same thing must instantiate also the universal B at t." [758, p. 79].[314] In [399, see esp. sect. 3.1 and 4], we introduce and discuss a novel, genuine relation between categories 📖 [399] for the framework of GFO, *categorial part-of*, that is meant to reflect dependencies among categories and to analyze how one category may be constructed out of others. Categorial part-of would lend itself to considering novel / modified versions of is-a, with a more intensional flavor. However, at this point we shall postpone any consideration of an intensional is-a relationship until further thoughts on the understanding of 'intension' in our context have been discussed.

2.4.2.3 Instantiation and Choices for Categories

INDIVIDUALS AND INITIAL ENDS OF INSTANTIATION CHAINS

The notion of individuals merely complements that of category (at this stage). Apart from that, individuals cannot be instantiated, as depicted in Fig. 2.4. This links to instantiation chains, more precisely, chains of individual instantiation relators which we may abbreviate by writing $x_1 :: x_2 :: \ldots :: x_n$ for the logical formula $\bigwedge_{1 \le i < n}(x_i :: x_{i+1})$ where $n \in \mathbb{N}$ (or $x_1 :: x_2 :: \ldots$ for the theory $\{x_i :: x_{i+1} \mid 1 \le i < \omega\}$). Individuals, yet together with *non-instantiated* categories, are the initial objects of instantiation chains. Asking for the structure of multiple, up to all entities and their interrelatedness in terms of instantiation leads on to "more advanced" issues that can largely be attributed to the domain of categories. Not at least due to the use of logic(s) with set-theoretic semantics in AO, KR, and SW, a.o., set theory suggests itself (to some extent) as a sample theory to be considered in connection with instantiation.

ON A UNIVERSAL CATEGORY, REMEMBERING RUSSELL'S PARADOX

In such a "comparison" with set theories, let us think about Entity first, which above was just introduced as the category that is instantiated by every existing object, i.e., entity is instantiated by everything, which allows for calling it a[315] *universal category*, i.e., a category that is instantiated by everything.

The existence of a universal category may appear dubious from the point of view of set theory, where various systems do not allow for a universal *set*. In ZFC, the Axiom Schema of Separation (see **Sep** in sect. A.2.1) yields this consequence, see e.g. [212, Satz 1.3, p. 28]. The schema itself is formulated in order to escape Russell's paradox [445]. The latter emerges from the assumption of a schema of unrestricted comprehension axioms of the form[316] (2.11), in which ϕ is an arbitrary formula in which s is not a free variable, by considering the case of $\phi = \neg(x \in x)$, which leads to the instance (2.12) of the schema.

(2.11) $\exists s \forall x . x \in s \leftrightarrow \phi$

(2.12) $\exists s \forall x . x \in s \leftrightarrow \neg(x \in x)$

(2.12) is contradictory in itself, as is seen by its entailment $\exists s . s \in s \leftrightarrow \neg(s \in s)$ that results from substituting s for x in the universally quantified part, cf. also [693, Ex. 1, p. 73]. The question of comprehension formulas is discussed below, let us first return to the issue of a universal set (in analogy to category, as in our case).

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[313] The use of 'extensional' evokes associations with set theory and, indeed, there is a very close set-theoretic formulation as "$x \twoheadrightarrow y$ iff the set of instances of x is a subset of the set of instances of y", cf. e.g. [758, sect. 7, p. 79]. However, (2.8) and (2.9) take no recourse to any sets at this stage (and in the case of extensional is-a, together with hindsight from ch. 4, this should not even be postulated from the set-theoretic semantics of FOL, because this need not be assumed in order to draw the same conclusions). Once that sets / (mathematical) classes are explicitly referred to, e.g. by introducing the *extension* of a category as the set or class of its instances, only then there is a clear set-theoretic connection (and an adapted formulation of $x \twoheadrightarrow y$ iff the extension of x is a subset of the extension of y).

[314] This definition is clearly narrower. Moreover, the use of 'universal' in the context of [758] suggests a meaning of that term that is more specific than our broad use of 'category'.

[315] The article 'a' (vs 'the') is justified by the fact that below further categories occur that are coextensional with entity, for example, Instance.

[316] Cf. [445, sect. 4] for a set-theoretic formulation that arises from the one shown here by renaming some symbols. [222, p. 284], for example, provides an analogous formulation in second-order predicate logic.

Indeed, there are set-theoretic accounts that prevent Russell's paradox in other ways, such that a universal set is admitted. One approach among these is Willard Van Orman Quine's system New Foundations (NF) [680] a modification of which by Ronald Björn Jensen to include urelements, called NFU, could even be proved to be consistent, first in [461].[317] M. Randall Holmes provides an elementary introduction to NFU [425], including further arguments for a set theory with a universal set [425, ch. 1], though mainly in the context of mathematics and set theory itself.

From the perspective of AO itself and other related fields, permitting a universal category appears less problematic. In particular and "[m]oreover, many KR systems have a top level class ENTITY which is itself considered an entity. Hence, 'ENTITY is an ENTITY', or the like, is typically taken to be a theorem of these systems, suggesting a breach in the syntactic division between individual constants and predicates as well." [577, p. 3].[318] The notion of Resource in RDF / RDFS [@105] shares the character of a universal category, e.g. (and without further elaboration), cf. the semantic condition $\mathsf{ICEXT}(I(\mathtt{rdfs:Resource})) = IR$ in [384, sect. 9]. Considering a case of a top-level ontology, SUMO [@119] [662, esp. ch. 4] comprises a category named 'Entity': "The universal class of individuals. This is the root node of the ontology." [@120], lines 724–728.[319] We follow the same route with Entity in CR, as is described above.

ISSUES IN COMBINATION WITH SET THEORY

This choice of including a universal category Entity has immediate effects if combinations of CR with set theories are considered, which needs to be kept in mind where set-theoretic notions are referred to. A highly relevant case is the notion of the *extension* of a category c, which is usually defined as the[320] *set* of all instances of c.[321] As soon as sets are included explicitly in the domain of (considered) entities, sets themselves are subject to being instances of entity. Following the definition of extension, the extension of entity had to be a set that includes itself as a member. Hence, either (1) there is no set that could be an extension of entity, e.g., if sets follow the regime of ZFC, and then not every category has an extension, or (2) the extension of entity exists and is a member of itself, which requires a different kind of set theory than ZFC, for example. Before we meet such kinds of set theory shortly below and in analogy to instantiation, potential cycles of instantiation are discussed. In doing so we step back from considering CR together with sets made explicit, in order to focus on the core notions of categories and relations first.

INSTANTIATION CYCLES AND META CATEGORIES

This sect. 2.4.2.3 starts with observing individuals and non-instantiated categories as initial ends of instantiation chains. In contrast, the category entity just discussed *seems to* represent the terminal end of maximal instantiation chains, because each item in such a sequence is itself related to entity via instantiation. However, the same is true for entity itself. The reason for that is that we consider / require entity to be in the domain of discourse. Indeed, this is the case for all notions that we discuss here, e.g., category, individual, and instantiation.[322] Hence, entity instantiates itself, requiring the possibility of instantiation *cycles* of length 1 (i.e., there is one particular instantiation relator in the cycle).

Fig. 2.5 accounts for a more precise picture and introduces further distinctions in CR that we find useful for ontological analysis. It displays in particular three subcategories that partition[323] Category into those

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[317]NFU is a weaker theory than NF, e.g., NF refutes the Axiom of Choice, whereas NFU does not [245, sect. 4]. The same source refers to further consistency proofs of NFU. For an extensive bibliography on the matter, see [@13].

[318]The part "suggesting a breach . . ." is due to the overall context of this quote, in which arguments for the utility of type-free logics / logics with a single type are gathered. The discussion in sect. 7.2.2.2 addresses that issue.

[319]Notably, the notion of THING in the Frame Ontology seems to pay tribute to (mathematical) classes, called 'unbounded objects' in its terminology, where 'class' within the Frame Ontology refers to an ontological reading of class akin to our category: "THING is the class of everything in the universe of discourse that can be in a class. [. . .] There are entities in the universe of discourse for KIF that cannot be instances of THING. These entities are unbounded objects, which by definition cannot be members of any set." [@45], page THING.

[320]If such a set exists, its uniqueness (justifying "the") follows from the principle of extensionality (of sets), which we assume to be valid in every set theory.

[321]This could be generalized to the notion of mathematical classes (instead of considering sets), but the problematic point remains the same.

[322]Concerning logical aspects, the relationship between object and meta level as well as how this assumption may be "implemented" in available logical syntax is primarily discussed in ch. 4.

[323]Not indicated in the diagram.

categories that (1) (can) have only individuals as their instances (*individual-category*)[324], (2) (can) have only categories as their instances (*category category*), and (3) the remaining categories, which intermingle individuals and categories among their instances (*mix-category*) – such as Entity. Note that we use *meta category* (not shown) as a convenient term to refer to categories that are not individual-categories, thus to category categories or mix-categories. The stereotypes in Fig. 2.5 declare the corresponding meta categories for the nodes in the diagram, e.g., MixCategory :: CategoryCategory.

In this setup and in addition to instantiation loops at entity and category (via generalization), and at category category, we observe instantiation cycles (without repeating inner "nodes") up to length 4, e.g. Entity :: MixCategory :: CategoryCategory :: Category :: Entity. Moreover, one could repeat a similar move as just applied to category iteratively, e.g., to category category in the next step, which yields longer instantiation cycles, without repeating categories in between. Taking also repetition into account, arbitrarily long instantiation chains (ascending or descending) are immediate. Regarding the question of whether any restrictions should be enforced, a "side look" toward set theory shows that, besides theories with a universal set, set theory has grown another, though related, branch of research on axiomatic systems: *non-wellfounded sets*, prominently and uniformly treated in [2], cf. also [801, esp. sect. 1] and [593]. In a narrow interpretation,[325] corresponding research starts primarily from ZF/ZFC,

Figure 2.5:
Instantiation-based metacategories in $\mathcal{CR}$.

but rejects the Axiom of Foundation (Found in sect. A.2.1) which ensures that the membership relation is well-founded, i.e., there are no infinite descending membership chains, cf. e.g. [212, sect. III.6]. However, apart from the Axiom of Foundation, for several other axioms of ZFC it is not obvious that their reformulation in terms of instantiation would be appropriate / ontologically adequate (for integration into $\mathcal{CR}$). For example, using axiom identifiers from sect. A.2.1, we doubt that for Inf, $\bigcup$-Ax, and even $\cup$-Ax. For reasons wrt the latter, see the comments on disjunctive categories below. Of course, extensionality is clearly inacceptable for instantiation and categories. Nevertheless, we see future investigations of set-theoretic systems as a relevant source of further inspiration. Besides set theory, we remark that instantiation chains, loops, and cycles are present in the same KR / SW approaches that we have considered wrt the universal set, e.g., cf. the quote from [577, p. 3] above. Eventually, at least as a first stage, we conclude not to adopt restrictions on the structure of instantiation (in isolation), but only maintain the argument limitation to categories in its InstantiatedRole. Naturally, less constraining axioms yield a weaker theory, but this may even be advantageous in the light of leaving room for distinct views on categories, cf. sect. 2.4.2.1 above.

ON COMPREHENSION AXIOMS

A similarly hesitant position is adopted for $\mathcal{CR}$ wrt the issue of category comprehension. On the one hand, there are clearly various cases of *conjunctive categories*, by which we mean categories whose extension equals the intersection of the extensions of two or more "parent categories" and which arise from those "parent categories" "by means of" logical conjunction, e.g., associated with phrases of the form 'X that is also a Y', cf. [610, esp. ch. 12, p. 464ff.].[326] Similarly, there are *disjunctive categories*, two examples of which are considered above, namely meta category corresponding to category category *or* mix-category, and foundational ontology in sect. 1.1.4.2, which can be rephrased as top-level ontology *or* mid-level ontology. On the other hand, there is much criticism on logical combinations in category "formation", within AO, as can be illustrated by criticism against logically constructed cases of class subsumption in [758, p. 79], and elsewhere, in particular wrt the abandonment of 'the classical view of concepts' in cognitive science and psychology, cf. e.g. [610, esp. ch. 2]. We shall avoid further discussion of specific kinds of categories

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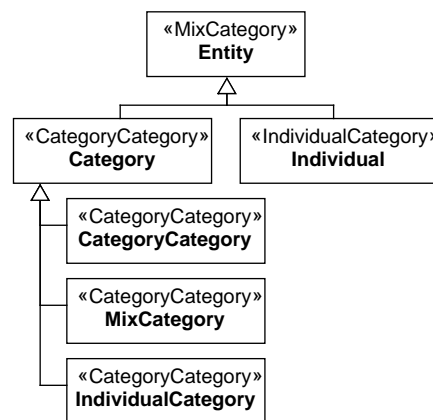
[324]In the context of established GFO terminology, *simple* or *primitive* category are synonyms to 'individual-category'.

[325]A wider view would admit any set of set-theoretic axioms that allows for some non-wellfounded sets, then including NF/NFU.

[326]Indeed, the Aristotelian definitions mentioned in sect. 2.4.2.1 are closely related to such forms.

here, as well as further details on conceptual combinations, the analysis of which has seen "considerable progress", but "is still at a relatively early stage of development" [369, p. 155].[327]

Instead, due to these open questions, we do not formulate comprehension axioms for categories / instantiation. For later definitions of language semantics this suggests a potentially interesting side effect in combination with predicate logic and the classical first- vs second-order distinction (cf. sect. 3.1), inspired by Herbert B. Enderton's summary of the (set-theoretic) *general* or *Henkin semantics*:[328] "Second-order logic with the general semantics is nothing but first-order logic (many-sorted) together with the comprehension axioms." [223, sect. 3]. In any case, on category comprehension we adopt a position that is analogous to a view reported wrt Common Logic: "Hence, there are no valid, general comprehension principles; there are no principles telling us that, for any expressible condition there is a thing whose relation extension includes exactly the (n-tuples of) things that satisfy that condition [...]" [577, p. 7]. The analogy deviates in the nature of relations, which is to be seen in next section.

BOOLEAN RELATIONS AMONG CATEGORIES

Beforehand, the final consideration in this section shall provide us with useful terminology for the text below, thereby making the use of 'conjunctive' or 'disjunctive' category above more precise, though in an extensional rather than intensional sense. An ontological account of what intensions are, or at least of what intensions of categories are, should be a prerequisite for the latter sense, even if a preliminary one. But having just identified insufficiencies in the available definition(s) of intensional / conceptual equivalence of sentences, which in itself is not equipped with corresponding characterizations of intensions for any constituents of those sentences, we do not have such an account readily available and thus cannot pursue an intensional approach immediately.

Instead, the matter here concerns relationships among categories that emanate from the Boolean operations in set theory by close analogy. We define these relational versions of them for categories / instantiation.

2.6 Definition (basic extensional categorial Boolean relations)
In the context of the FOL axiomatization of CR, we define four predicates capturing the *extensional categorial intersection*, *union*, *absolute complement*, and *relative complement* (for short, *ec-intersection*, *ec-union*, *ec-acomplement*, and *ec-rcomplement*):

$$(2.13) \quad \forall cde \,.\, \mathsf{ecintsect}(c,d,e) \leftrightarrow_{df} \forall x \,.\, x :: c \wedge x :: d \leftrightarrow x :: e$$

$$(2.14) \quad \forall cde \,.\, \mathsf{ecunion}(c,d,e) \quad \leftrightarrow_{df} \forall x \,.\, x :: c \vee y :: d \leftrightarrow x :: e$$

$$(2.15) \quad \forall de \,.\, \mathsf{ecacompl}(d,e) \quad \leftrightarrow_{df} \forall x \,.\, \neg x :: d \leftrightarrow x :: e$$

$$(2.16) \quad \forall cde \,.\, \mathsf{ecrcompl}(c,d,e) \leftrightarrow_{df} \forall x \,.\, x :: c \wedge \neg x :: d \leftrightarrow x :: e \qquad \Box$$

There are some immediate deviations from the set-theoretic blue print, of course. Firstly, without having (set) extensionality available wrt instantiation, these relations are not functional in their last argument in CR. This can be seen with any two distinct, but coextensional categories, of which examples such as Entity and Instance arise below. Accordingly, there is no operation that could yield, e.g., a unique ec-union of two given categories. Secondly, the relations are not necessarily total wrt to their first (for ecacompl) and first two (for the others) arguments, resp. Due to the lack of comprehension axioms, the existence of ec-intersections, ec-unions, and/or ec-complements is not necessarily the case for arbitrary categories. Indeed and remembering (2.12) on p. 85, if its "positive relative" is transferred to instantiation and the existence of "the" category of self-instantiating categories is assumed by $\exists s \forall x \,.\, x :: s \leftrightarrow x :: x$, this constitutes the case of a category that cannot have an extensional categorial absolute complement. Note that we adopt no position on the existence of a category of all self-instantiating categories within CR herein. Despite those "gaps" as just observed and postponing further elaboration, some of the laws of set algebra, cf. e.g. [200, sect. 1.2], transfer straightforwardly to those predicates / relations introduced. For example, the symmetry of ecintsect and ecunion in the first two arguments corresponds to the commutativity of $\cap$ and $\cup$ and is immediate from the commutativity of conjunction and disjunction, as used in the definitions.

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[327]Despite the fact that the quote is from 1997, [610] (2002) supports the same view. At least, we believe there is no conclusive "solution", but much room for further research.

[328]Henkin semantics is described briefly in sect. 3.1.4.

2.4.3 Relations

2.4.3.1 Background and Scope for the Notion of Relation

SPECIFICITY AND GENERALITY IN THE SCOPE OF 'RELATION'

First of all, in some contrast to the notion of category, where we aim at a very broad understanding in order to maintain the potential of covering various accounts of categories, a rather specific view on relations and their way of relating entities has emerged in the time of working on this thesis as well as some time before, during an initial ontological analysis of the notion of roles [526]. Nevertheless, we shall argue for the generality of this account of relations insofar that we have not yet come across cases that withstand a corresponding analysis.

SKETCH ON RELATIONS IN THE LITERATURE

Prior to detailing our approach, let us briefly point to related elements in the literature, starting in philosophy. The above discussion in sect. 2.4.1 on the fundamental nature of categories and relations reveals with recourse to [796, sect. 1.1.3] that the philosophical analysis of relations is frequently conducted in combination with properties, for which we may substitute 'categories' in this connection. This notwithstanding there is a body of work that is explicitly dedicated to relations. For example, Nino B. Cocchiarella reviews the accounts of relations in the work of Gottlob Frege and the work of Bertrand Russell, also in connection with predication [160, sect. 4], i.e., instantiation in our terms. Similarly, i.e., incl. reference to Frege and Russel (a.o.), Kevin Mulligan discusses relations and draws a distinction between what he calls *thick* and *thin* relations [607], cf. also the earlier [606]. This distinction has found its way into AO, e.g., as *material* and *formal* relations in [348, esp. sect. 6.2.7], [357] and [387, sect. 6.1.2], and it was used in an early account of relational roles, cf. [526, sect. 3.3.1]. However, developing the latter approach further, we do not need to rely on that distinction for $\mathcal{CR}$.[329]

Treatments of relations are available in cognitive science and psychology, as well, cf. e.g. the work on inferencing with relations in [291]. For the areas of computer and information sciences, the featuring of relations in a number of approaches in KR and CM can already be found in [526, esp. ch. 2].[330] Frequently, however, relations are primarily identified with the mathematical notion of relations as sets of tuples (themselves encoded into sets in specific ways). This is partially the case in works in AO, e.g. tuples are considered as instances of material relations in [348, p. 241]. Apart from that, some ontological analyses of the relationship construct in modeling languages are available, cf. [861] on the basis of ontological work by Mario Bunge [126], and [348, 357] in connection with the Unified Foundational Ontology (UFO) [@125], see sect. 1.1.4.2. Note that the latter is a very closely related account (to what follows below) which is based on relators in the case of material relations.

RELATIONS AS "ANALYSIS TERMINATION"

We afford an explanatory comment at the meta level, i.e., it is not (yet) to be included in the actual theory of $\mathcal{CR}$ and we cannot substantiate the view much further here. We observe that (ontological) relations are useful "devices" that can be employed at the point at which no further, more detailed analysis of interconnections / circumstances is required, thereby "terminating the analysis". For example, "John is a parent of of Mary" may be taken as a relational statement about John and Mary, utilizing a parent-child relationship (and possibly leading to a logically atomic sentence in predicate logic). On the other hand, one may analyze with scrutiny the circumstances / intensions that are underlying this statement, involving John and Mary. The resulting analysis may comprise many more entities and, in turn, (primarily other) relations. Let us look at another example, inverting the direction to go from fine-grained to coarse-grained analysis now. Assume that "John, Mary, and Sue play together at the playground with a ball by kicking it." Then one may understand that as describing a situation (a situoid in GFO [392, esp. sect. 14.4.6 and sect. 14.5.9], [397, esp. ch. 11 and sect. 14.9]) that is constituted by a process that (1) is spatially framed by the playground, (2) counts as playing together, (3) in which John, Mary, and Sue participate as human players as well as (4) the

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[329]Yet we conjecture that the distinction can still be introduced on the basis of $\mathcal{CR}$. Material relations would be such that they are derived from other ontological entities, i.e., their relators are founded / dependent on further entities, e.g., processes. This may not be the case for formal relations. But we focus on the main route toward relations first, leaving the issue as interesting future work.

[330]possibly somewhat hidden among the notions of roles, which [526] focuses on

ball participates in it as a toy, and (5) the process has parts that instantiate kicking – altogether, an analysis with several entities of various ontological kinds and with several relators between them ("stopping" the analysis at a certain level as available in the sentence). Alternatively, one may abstract a relation from the whole situation such that one relator of it has seven roles which are played by John, Mary, Sue, the ball, the playground, and kicking.[331] Depending on the required level of detail, instead of a relator with six roles (and six arguments), one may abstract a relation such that its relator that corresponds to / derives from the sentence has just three roles filled by John, Mary, and Sue, and a fourth role filled by the ball, accounting for "John, Mary, and Sue play together with a ball". Again, this paragraph is only meant to illustrate our current (and certainly preliminary) view on the rôle of relations in ontological analysis. Further we argue that all such relators are on a par with each other (as relators), while there may be interrelations between them, which – at least in some cases – "appear" / can be captured as logical connections in formalizations.

2.4.3.2 Relator, Role and Non-Relating

INTRODUCTORY AND TERMINOLOGICAL COMMENT

While the previous subsections of sect. 2.4 rely on the brief introduction of the GFO approach of relations presented in sect. 1.1.5.2, let us now start over and construct this account systematically and in combination with the views on categories and instantiation expounded so far. At this point, let us mention anew that the use of the term 'role' herein corresponds to 'relational role' in our work on roles (in a general reading), i.e., in [526–528, 530, 531].

RELATORS COMPOSED OF ROLES

The basic idea of *relations* in $\mathcal{CR}$ is that they are categories that are instantiated by individuals called *relators* that have the power of relating / mediating / gluing together arbitrary entities, thereby establishing those relations between the entities that the relators mediate between. For a concrete example and in order to demonstrate a case that is commonly seen as a formal relation (and is nevertheless subject to an analysis by means of relators in $\mathcal{CR}$), take the fact from the domain of natural numbers that 1 is less than 2, commonly written $1 < 2$. In analyzing this fact in $\mathcal{CR}$, we first distinguish four entities, namely 1 and 2, the arguments that are related, then the less-than relation $<$, and the particular relator that mediates $<$ between 1 and 2, for which we choose $r^<$ as a (logical) individual constant symbol.

However, just with $r^<$ available, it is not clear which rôles 1 and 2 have wrt $<$. This problem is solved by *roles* (and *base role categories*), of which there is one role per argument. Roles are individuals that can be seen as parts of relators, but within $\mathcal{CR}$ we use the term 'role of' and in its FOL signature the binary predicate symbols $\multimap$ and $\circ\!\!-$ for the infix notation $x \multimap y$ of 'x is a role of y' and $x \circ\!\!- y$ for 'x has role y'.[332] In our example, we have two roles of $r^<$, call these q_g and q_l. However, the "semantics" of the two wrt $<$ results from the role categories they instantiate. Indeed, the overall semantics of $<$ depends on determining role categories such that relators composed of roles of those categories adequately reflect in which way each single argument participates in the relator (and thereby/loosely speaking, "in the relation"). We say in such cases that a role category is a *base role category* for a relation. In the case of $<$, we denote its base role categories by ᐧless and ᐧgreater. Further assume that $q_g ::$ ᐧgreater and $q_l ::$ ᐧless.

As a terminological note wrt natural language[333], with the discussion of roles / role individuals as well as role categories, our terminology now shifts to let 'role' be indeterminate wrt 'role individual' and 'role categogy'. However, often the context will allow for disambiguation. If otherwise necessary, we shall use one of the precise terms.

ENTITIES PLAYING ROLES

Another connection is required to / from a role wrt the corresponding argument of the relator, e.g., from q_l to 1. This connection is called *plays* / *plays role* / *fills role*, with this FOL notation: $x \rightsquigarrow y$ for 'x plays (role) y'

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[331] The roles of John, Mary, and Sue may instantiate the same role category. We omit further qualifications or explanations, since the analysis in itself is of almost no relevance for the point to be made.

[332] In common mathematical terminology and viewing $\multimap$ and $\circ\!\!-$ as binary mathematical relations, these are intended to be inverses of each other. Forestalling the approach of ch. 4, in an ontological reading, both symbols refer to exactly the same entity, namely the 'role of' relation.

[333] as opposed to the languages used in formalization

(or 'x is the *player* of y'), and $x \leftarrow\hspace{-0.3em}\shortmid\; y$ for 'x is played by y'. To conclude our example coherently, we need $1 \rightsquigarrow q_l$ and $2 \rightsquigarrow q_g$. Overall, the role analysis of $1 < 2$ yields formula 2.17. This uses a similar abbreviating notation as in the case of instantiation cycles above, namely that alternating sequences of terms and binary infix predicates are to be read as the conjunction of all FOL atoms that are contained in that sequence. Moreover, with $<$ from our example, we hit a first case of a predicate constant that we (would) need to use as an argument of other predicates. In expectation of the forthcoming chapters and adhering to the usual syntactic restrictions of FOL, we shall introduce a (logical) individual constant which, considered ontologically, denotes the same entity that we refer to (ontologically) by means of the predicate symbol.[334] We adopt the convention of deriving the name of such individual constants from predicate names / constants by prefixing them with a dot in the center of the line, e.g., in the present case, $\cdot<$.

(2.17) $r^< :: \cdot< \;\wedge\; q_l :: \cdot\text{less} \;\wedge\; q_g :: \cdot\text{greater} \;\wedge\; 1 \rightsquigarrow q_l \multimap r^< \multimapinv q_g \leftarrow\hspace{-0.3em}\shortmid\; 2$

RELATION FORMATION DESCRIBED AT THE CATEGORIAL LEVEL

If one compares only the atomic formula $1 < 2$ to formula (2.17), this may suggest that the $\mathcal{CR}$ view on relations merely adds complexity in the analysis of relations (and "adds" entities, namely relator and role individuals), whereas not much or nothing novel appears to be derivable from it. However, we see the strength of the approach at the categorial level (wrt relation analysis), e.g., in specifying relations precisely. Fig. 2.6 takes us to that level by means of a UML diagram that serves as a schema for interconnections among relators, their roles, and players of these roles, such that cases like $1 < 2$ / (2.17) are instances (in the UML sense) of that

Figure 2.6: Relation formation in $\mathcal{CR}$.

schema. More precisely, the part $1 \rightsquigarrow q_l \multimap r^< \multimapinv q_g \leftarrow\hspace{-0.3em}\shortmid\; 2$ of (2.17) accounts formally for the relator $r^<$ that hasRole q_l playedBy 1, and that hasRole q_g playedBy 2. Thus, although just to renew what was assumed above, $r^<$ consists of two roles. The multiplicity of '2..*' at the role role (roleRL)[335] of Role clarifies that *every* relator consists of at least two roles. This appears sensible, because relating / mediating should involve at least two distinct aspects (i.e., roles), even if these may refer to (i.e., the roles may be played by) exactly the same entity.

A FIRST CLASSIFICATION OF CATEGORIES CONCERNING RELATIONS

Before dwelling on categorial-level distinctions it is worthwhile to systematize the main vocabulary established for relations so far, and to interlink it with the categories of Fig. 2.4. This is supported by Fig. 2.7. It visualizes that relators and roles are individuals. Both are genuine types of entities in $\mathcal{CR}$, and both are existentially dependent entities, similar to properties in GFO (which inhere in some bearer and cannot exist without that bearer) and to boundaries, e.g., of temporal entities, cf. sect. 6.3, esp. sect. 6.3.5.1. Relators depend specifically on those entities which they mediate. Consequentially of that, roles depend specifically on their players. But likewise roles depend on the *complementary roles* within their relator and on that relator.
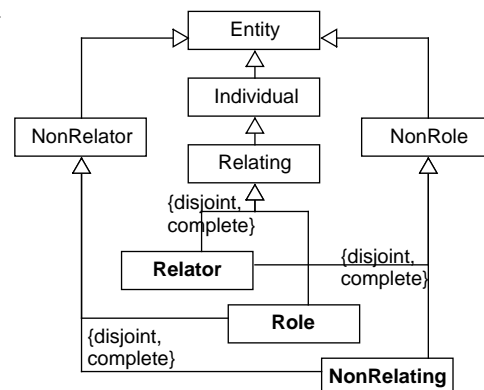
Figure 2.7:
Classification of Relator, Role, NonRelating.

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[334]Clearly, the classical semantics of FOL can assign two different entities to the two symbols (and it does so at least in all mathematical structures where no element of the universe of reference is a member of any other such element), cf. ch. 3.

[335]Obviously, we apply the same kind of relation analysis to hasRole. See sect. 2.4.4.3 for further thoughts in this regard.

We assume that the relator of a role is uniquely determined, cf. the multiplicity at contextRL in Fig. 2.6.[336] Due to these commonalities and because all relators and roles are entities that are involved in connecting / relating arbitrary entities, they are subsumed by the category *relating* (partitioned by relator and role), whereas all other entities that do not exhibit this character fall into the category *non-relating*.[337] Hence, entity is three-partitioned into relator, role, and non-relating. The latter extensionally subsumes (e-subsumes) category, because categories neither mediate entities nor are they individual roles of relators. This observation can also be derived from Fig. 2.4 and Fig. 2.7 by the facts that Relating is e-subsumed by individual, which is disjoint with category. In addition to Relating, the notions of non-role and non-relator are defined by the (extensional categorial) absolute complements of the named categories, which is equivalent to being the remaining two unions over the three-partitioning.

ENTAILED RESTRICTIONS ON THE THREE CENTRAL RELATIONS ::, ⤳, ⊸

The disjointness of the partitionings suggest a number of restrictions on the three major relations of instantiation, plays, and role-of, based on their domains and ranges / codomains. On the one hand, the domains impose no restrictions, because instantiation and plays-role relators can "start" at arbitrary entities, hence including the roles that constrain the case of role-of. On the other hand, all three relations can be distinguished due to having disjoint ranges. Instantiation has categories and thus non-relating as its range, whereas plays-role with roles and role-of with relators both have a range that is e-subsumed by individual. Comparing the latter two relations, their ranges are also disjoint categories.

While it remains correct in effect, this step of concluding the distinctness of the three relations by means of taking especially their disjoint ranges into account can be refined wrt role-based relations in *CR*. This potential unfolds especially at the level of categories of relators, roles and entities playing those roles.

2.4.3.3 Relations and Role Bases

EXTENSION(S) OF RELATIONS

The last but one paragraph of the previous sect. 2.4.3.2 discussed domains and ranges of the relations ::, ⤳, and ⊸, much in the usual terminology of mathematical relations. A mathematical relation is a set of tuples. In contrast, if relations are perceived as a genuine kind of entities and, in particular, as polyadic properties that can be exemplified / instantiated (by multiple entities in each case of exemplification), the distinction between intension and extension can be applied to relations, such that typically mathematical relations form the extension of relations (of genuine character).

In the case of *CR*, with relations that are instantiated by relators, the term 'extension' can be given two meanings. On the one hand, relations are *relator categories* (RelatorC in Fig. 2.8). The latter are those categories that (can) have only relators as instances. Accordingly, applying the usual notion of the extension of a category[338] to a relation yields a set of relator individuals as the extension of that relation. Where we need to highlight this character, we call this the *relator extension* of a category. In addition to the notion of relator extension, it appears useful to find an alternative notion that mimics / links to mathematical relations as extensions for relations. These are called *tuple extensions* and can roughly be characterized as sets of tuples that are uniformly derived from the relators instantiating a relation.

ADIRECTIONALITY OF RELATORS WITH NOVEL NOTIONS OF SYMMETRY

At this point, it is important to reflect that relators have roles, but there is no "direction of reading" among those roles (and thus of relations), there is no preeminent order among them. Moreover, as stated above in sect. 2.4.3.2, an individual role q in any relator r of relation R must be understood as an instance of a *base role category* Q of that relation R. The base role category accounts for the particular aspect of the role player of q in the overall relator r *as an instance of* R. As noted earlier in [526, sect. 3.3.4], multiple instances of the same base role category can be roles in one and the same relator, which leads to intensional notions

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[336]Considering parthood relations among relators is left for future consideration.

[337]The question may arise whether those two categories of relating and non-relating (entities) exist, due to the lack of general comprehension principles for categories. In our view, referring to them (as categories in *CR*) postulates their existence (within *CR*), which appears adequate in these cases and coherently possible. The same position is taken for non-role and non-relator.

[338]defined as the set of its instances, cf. sect. 2.4.2.3 above

of symmetry of / for relations which are novel compared to the classical formulation of an (extensionally) symmetric relation that, formalized by a binary predicate P, obeys the axiom $\forall xy \,.\, P(x,y) \to P(y,x)$.

DEFINING TUPLE EXTENSION

Sticking to the intimately related issues of role ordering and tuple extension, what is required for defining a tuple extension of a relation R is basically a surjective mapping π from tuple positions to base role categories. Then relators instantiating R can justify tuples of their specific arguments.

2.7 Definition (π-tuple extension (for fixed arity))
For $m, n \in \mathbb{N}_+$ with $n \geq 2$ let R an n-ary (ontological) relation with the base role categories $Q_1, \ldots, Q_m$. Then every surjection $\pi : \{1, \ldots, n\} \to \{Q_1, \ldots, Q_m\}$ determines the π-tuple extension $\mathsf{tpext}_\pi(R)$, which satisfies $(x_1, \ldots, x_n) \in \mathsf{tpext}_\pi(R)$:iff there is an R-relator r that bears witness to the fact that those x_i are "appropriately related by" R, i.e.,

1. r is an instance of R and
2. r has exactly n individual roles $q_1, \ldots, q_n$ s.t. q_i is played by x_i and is an instance of $\pi(i)$, for all i. ❑

MULTIPLE TUPLE EXTENSIONS AND INVERSE (MATHEMATICAL) RELATIONS IN CASE OF $<$

To illustrate this, we return to the example of the less-than relation[339] $<$, for which now a clear distinction between an ontological understanding of less-than (as a relation) and the mathematical / set-theoretic view on less-than as a set of tuples. $<$ denotes the former, $<^\in$ the latter entity. The ontological relation $<$ is equipped with two base role categories, namely less and greater, cf. formula (2.17). Then $\pi^< := \{(1, \mathsf{less}), (2, \mathsf{greater})\}$ yields $\mathsf{tpext}_{\pi^<}(<) = \{(x,y) \mid x < y\} = <^\in$, where $(1,2) \in \mathsf{tpext}_{\pi^<}(<)$ is justified by the existence of the relator $r^<$, which is (partially[340]) described in (2.17). Likewise one may consider the tuple extension that derives from $\pi^> := \{(1, \mathsf{greater}), (2, \mathsf{less})\}$, namely $\mathsf{tpext}_{\pi^>}(<) = \{(x,y) \mid y < x\} = >^\in$, well-known as the 'greater-than' relation.

This leads us to observe not only that $<$ has two distinct tuple extensions, but that the two that we have defined correspond to what in the usual mathematical setting are *two distinct* and even disjoint *mathematical relations*, namely $<^\in$ and $>^\in$. To stress that again, from the perspective of $\mathcal{CR}$ there is only *one ontological relation* "underneath", from which two tuple extensions originate by choosing different orders of the base role categories of the relation. While it is useful / efficient to employ symbols, e.g., $<$ and $>$, or natural language terms / phrases, e.g., 'less than' and 'greater than', with a clear direction of reading, we argue that both symbols and both terms refer to one and the same ontological / semantic counterpart.[341] Moreover, whereas the sets $<^\in$ and $>^\in$ constitute a pair of inverse (mathematical) relations, there cannot be any inversion of an ontological relation, because that lacks any directionality, as mentioned above.[342] Although various further thoughts emerge at / depart from this point, e.g., on *anadic relations*[343], their base role categories and options of deriving tuple extensions, we return to our main track and focus further on the "understanding" of relations in terms of base role categories and additional notions.

EARLY NOTION OF A ROLE BASE IN [526, DEF. 3.2]

Fig. 2.8 lends itself to the opportunity of introducing, in addition to base role categories, the analog of the domain(s) of a mathematical relation, i.e., the sets of the Cartesian product that a mathematical relation is a subset of. We remark that this yields a revised, now fully ontological account of a *role base*. Under that term Def. 3.2 of [526] referred to a set of role categories that satisfy the subsequent conditions, which we recapitulate in adaptation to the present context, i.e., only for the case of relational roles (wrt [526])

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[339] For simplicity, assume $\mathbb{N}$ as the domain of numbers over which less-than is considered, yet the actual domain is not relevant here.

[340] To be precise, 2.17 does not account for the aspect that q_l and q_g are *exactly* the roles of $r^<$, as required in Def. 2.7.

[341] Notwithstanding, $x < y$ (as a short-hand for 'x is less than y') is to be interpreted differently from $x > y$, of course.

[342] There is a certain similarity between (1) the concept of (mathematical) relations being inverse to (2) that of roles being complementary, which can be lifted to base role categories.

[343] Anadic relations have no fixed arity. In $\mathcal{CR}$, anadic relations have relators as instances that differ in the numbers of their arguments among which they mediate. Anadic relations are also called *variably polyadic* or *multi-grade* relations. [889, esp. endnote 17, p. 110] briefly reviews the historic roots of the three terms, referring to works of Richard E. Grandy as early sources wrt the term 'anadic predicate', e.g. in [297, esp. p. 399] (with the overall title "Anadic Logic and English").
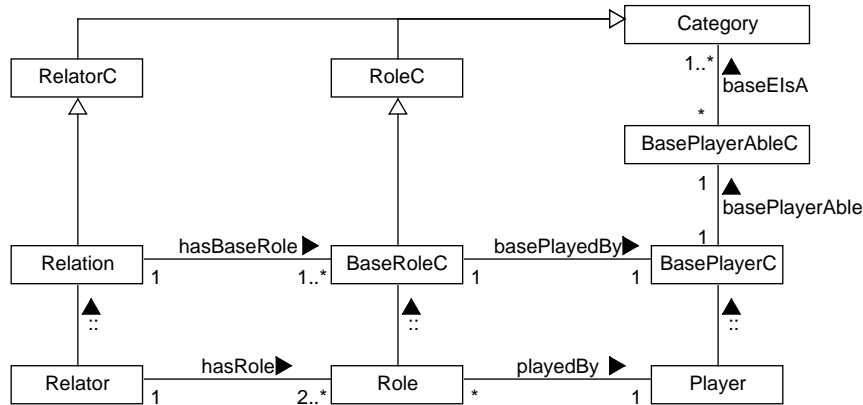
Figure 2.8: Relations with role base notions and corresponding individual categories.

and slightly modified in terminology.[344] Although referring to categories, these conditions are partially reminiscent of those required for tuple witnesses in our Def. 2.7 of tuple extensions above. Given a relation R as a context entity / category for which a role base B is defined as a set of role categories, the conditions are:[345] (1) the categories in B are mutually disjoint, (2) each individual role of an R-relator instantiates exactly one role category in B, (3a) for each role category $Q \in B$ the instances of Q can only be roles of R-instances, and (3b) for all $Q \in B$, each R-instance has at least one role individual that instantiates Q.

CHARACTERIZATION OF BOTTOM LEVELS AND BASE ROLE CATEGORIES

We start reworking the core ideas of a role base systematically along Fig. 2.8, which shows the core connections for ontological relations in CR at two levels of instantiation. The bottom level repeats with less detail what is depicted in Fig. 2.6, though with Entity there being labeled Player in Fig. 2.8. This relation-centric CR category is more plausibly used wrt the levels above, and for the present purposes we can presume that entity and player are coextensional categories, because every entity plays a role in relation to some other entities.[346] The second level from the bottom links to "the semantics" of a relation, in the sense that the mere fact that there is a relator (which is an individual) with individual roles that mediates between some relata does not account for more than allowing one to know that those relata are related – but not allowing to see by which relation and in which way. For the latter, the relator must be recognized as an instance of some specific relation, and the ways in which the relata are involved wrt that relation must be known by playing roles that instantiate corresponding base role categories (BaseRoleC in Fig. 2.8).

TRICHOTOMY IN INTERPRETING ROLE TERMS

Even with having arrived at base role categories, there is something missing in the understanding of a relation (and in the previous definition of a role base discussed above), in the light of the typical domain constraints / statements for relations. The missing element is a characterization / a characterizing category that embraces all those entities that are subject to the relation – which may mean actually and potentially. In this connection, first we observe an overloading of what may be called 'role terms' in natural language. For example, wrt the statement "The lion's paw is a part of the lion." we argue that 'part' can be analyzed with two or three senses[347] First, it refers to a role (more precisely, a role category) that the paw plays in the context of a specific relation to the lion. Secondly, 'part' can be seen as a category that classifies the paw

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[344]Literally, [526] employs 'universal' where we use 'category' now, but there is actually no shift in understanding. Moreover, its Def. 3.2 is worded wrt roles in general, incl. relational, processual, and social roles in [526], cf. more recently [530].

[345]The numbering adheres to the numbering in [526, Def. 3.2].

[346]On the one hand, one may refer to that fact that everything instantiates entity, thereby playing an instance role wrt entity. Note the connection with self-analysis of CR and its issues, briefly addressed in sect. 2.4.4.3. On the other hand, it may appear desirable to ignore the basic relations ::, ⇝, and ⊸ wrt this question, or consider the statement relative to a certain set of relations. In such cases, player is even more appropriately used, compared to entity, because then it could be more specific (extensionally).

[347]Those senses refer to the rôle that the term has wrt other components of the sentence. Dealing with distinct understandings of notions of parthood is an orthogonal matter which is not addressed here.

Relation	BaseRoleC	BasePlayerC	BasePlayerAbleC	Category
::	InstanceRL	Instance	Categorizable	Entity
::	InstantiatedRL	Instantiated	Instantiable	Category
⤳	PlayerRL	Player	PlayerAble	Entity
⤳	PlayedRL	Played	Playable	Role
⊸	RoleRL	RoleRole	RoleBeAble	Role
⊸	ContextRL	Context	RoleHaveAble	Relator

Table 2.1: Role base notions of the three basic $\mathcal{CR}$ relations, following Fig. 2.8. Regarding naming conventions, the suffix C in three column heads stands for 'category'. The RL suffix in the base role column originates from the term 'role', having the function to distinguish the base role reading from the base player reading.

(instead of its role), i.e., the argument of the relation, in general and from the point of view of the relation itself. Although this is likewise based on the same specific relation to the lion implicitly, it yields a different kind of category, typically a subcategory of NonRelating. Thirdly, one may read 'part' to already refer to the overall relation that a relator mediates between the lion and the paw under consideration. This third case might be eliminated by attributing that sense to the word combination 'part of' rather than 'part', but we see no particular effects to decide on one or the other view.[348] Overall, we observe this, say, trichotomy in the interpretation of role terms. It is reflected in Fig. 2.8 by Relation, BaseRoleC, and BasePlayerC. The last item accounts for the argument-classification reading of a role term. The first three columns of Table 2.1 comprise corresponding examples / instances wrt the three basic $\mathcal{CR}$ relations ::, ⤳, ⊸.

POTENTIAL OF BASE PLAYERSHIP AND RELATION-INDEPENDENT CATEGORIES

The base player category is understood to refer to entities that are playing a base role in a given relator. However, this only accounts for *actually* related entities. Another aspect emerges from recalling the intention to characterize the domains of a relation R, in the sense of categories of those entities that are potentially related by that relation. It is important to understand 'potentially' here merely as "to be subject to being related by R", instead of a more modal / possibility-driven reading of "possibly being related by R". The former reading is declarative in nature for a relation, and forms a precondition for the latter. The example of empty categories in sect. 2.4.4.2 elucidates that distinction, but we need to complete the actual conceptualization of the present matter beforehand. The aspect of characterizing a domain of entities that are subject to / potential arguments of a particular relation is covered by the categories BasePlayerAbleC and Category in Fig. 2.8, because categories capturing this potential character can result under one of two views: (1) either from the point of view of the relation or (2) from the point of view of categories of (typically, but not necessarily) non-relating entities. That means, BasePlayerAbleC has categories as its instances that are defined in association with the relation and are meant to capture all actual players / arguments of the relation, whereas BasePlayerAbleC is similar in that its instances are categories defined from the perspective of the relation, but deviate by capturing all potential players of roles in the relation. The relationship baseEIsA (from 'base extensional is-a') models extensional subsumption relations to categories that are given independently of the relation under consideration. Analogously, basePlayerAble expresses extensional subsumption of the category of actual players of a role and the category of its potential players.

For example, at the level of an individual relator such as $r^<$ from (2.17) above, each of its arguments can be categorized as an actual player or a potential player of a role (i.e., a role individual instantiating a base role category) of $<$. It can further be classified by a category that extensionally subsumes (or may even be coextensional with)[349] the category of all potential arguments of a relation. Utilizing descriptions due to lacking dedicated words, 1 is an instance of "entity that is less than another" (that category being an instance of BasePlayerC in Fig. 2.8), an instance of "subject to the role of the less in being related by less-than to another entity" / "entity potentially less than another" (a BasePlayerAbleC), and it is an instance

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[348]Accordingly, making that decision is not of importance here.

[349]Coextensionality is a common case with mathematical relations, "by design" / by the nature of the subject.

of number (a Category that subsumes the BasePlayerAbleC category named immediately before). Table 2.1 shows further examples in columns three to five.[350] Note that we adopt the position for *CR* that all pairs of categories in columns four and five in the same row present cases of pairs of distinct, but coextensional categories.

UPDATED VIEW ON ROLE BASE AND FURTHER USE OF 'DOMAIN' AND 'RANGE'

Against the overall background of Fig. 2.8, our revised view on a *role base* is one of a configuration (cf. sect. 1.1.5.2 for this GFO term) among categories as specified by Relation, BaseRoleC, BasePlayerC, BasePlayerableC, Category and the relations / UML associations among them. We use the concept flexibly in specifications, to the extent that it may be only partially specified and / or include additional constraints. However, every relation must be equipped with a number of / at least one base role category. Overall, awareness of the role base of a relation is an important aspect in order to comprehend the relation.

Moreover, we continue employing the usual parlance of the 'domain' and 'range' of binary relations. However, this happens with the conceptualization *CR* in mind. By default, we take it for granted to let 'domain' and 'range' refer to (instances of) Category in Fig. 2.5, as categories given / defined independently of the relation, yet together with a coextensionality assumption wrt the BasePlayerAbleC category that corresponds to the base role categories associated with the first or the second argument position of the binary relation. In analogy, this applies to talking of 'relations over' a set or a kind of entities. E.g., declaring $<$ as a relation over the natural numbers allows for the assumption that all of them and only they are subject to the relation under consideration. Similarly, statements on the domains and ranges of the *CR* basic relations made above this section 2.4.3.3 remain valid.

2.4.3.4 Relation Comprehension and Formation

NO CYCLES IN PLAYS AND ROLE-OF RELATIONSHIPS

In analogy to the consideration of instantiation chains and cycles, one may study / think about the question to what extent chains of $\rightsquigarrow$ and chains of $\multimap$ may occur. In the latter case, chaining is prevented by the domain and range of role-of, which are role and relator, cf. Table 2.1 or Fig. 2.6, two disjoint categories. Hence, there are only "singular" role-of relators, or chains of length 1, put differently.

In the case of the plays relation, its domain and range, namely entity and role, do not prevent chaining or even cycles. However, we argue that a relator that accounts for some aspect(s) of the interconnection of two entities is best understood as an entity that is completely separated from those entities that it mediates between. In accordance with that, we defend the view that there cannot be loops or cycles of plays. Chaining appears possible, however, due to repeated role analysis of relators. This is discussed further in sect. 2.4.4.3 below.

RELATION COMPREHENSION

Similarly few words on relation comprehension shall end the present section 2.4.3 on relations. First of all, we do not see any restrictions concerning the question, at an abstract level, of whether one entity may be related to another one. All entities are relatable, there are no entities that are "isolated" in principle.

On the other hand, it appears even harder than in the case of non-relator categories to adopt general comprehension principles. Relations in *CR* are categories of relators, genuine entities themselves. While tuple extensions may be derived from the relator extension of a relation, it is at best not obvious that arbitrary sets of tuples yield an ontological relation necessarily.

Insofar, presently relation comprehension in *CR* is treated equally as category comprehension in sect. 2.4.2.3, by not making assumptions on the existence of relations unless these are explicitly named – and then they should be introduced with a corresponding role base. However, at a more concrete level, certain schemes in the formation / derivation of relations can be identified, e.g. in connection with semantic networks and, more recently, in bioontologies. Since these schemes concern transitions along the instantiation relation, they are discussed in sect. 2.4.4.1. For future considerations on relation comprehension, one may extend these lines of thought to the derivation of role bases from defining formulas. With a feasible solution, comprehension axioms for relations may be reconsidered at a very abstract level.

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[350]We must admit that the terms used in that table are not natural in several cases, but we believe this is due to the fact that the minor shifts in meaning are handled by overloaded words in natural language.

2.4.4   Categories and Relations in Interaction

Leaving behind the study of instantiation, plays, and role-of (mostly) apart from each other, interaction among categories and relations can be interpreted as looking at the interplay of the basic $\mathcal{CR}$ relations. Along these lines, we have identified few generally valid restrictions so far that involve two or more of the basic $\mathcal{CR}$ relations ::, $\rightsquigarrow$, and $\multimap$. One observation from the end of sect. 2.4.3.2 is that all three relations must be pairwise disjoint already due to their domain and range categories. Besides that, some first-order axioms are formulated and discussed in the axiomatization section of $\mathcal{CR}$, see sect. 6.1.

PURPOSE AND CONTENT OF THE SECTION

Because they are not essential for the basic understanding of $\mathcal{CR}$ for the next chapters, we shift our interest to other kinds of interaction among the notions associated with categories and relations already presented. The remainder of this section is divided into short subsections dealing with $\mathcal{CR}$ notions in connection with first (theoretical) applications, also for the purpose of creating some intution on the kind of possibly analyses and their value. Those applications concern mostly extensions of its own theory, despite a case from the field of biomedical ontology that we enter immediately.

2.4.4.1   Schemes for Lifting Relations to the Categorial Level

OBSERVATION ON RELATIONS AMONG CATEGORIES AND THEIR INTERPRETATION

An observation that can easily be made in various formalisms, incl. semantic nets like that of the Unified Medical Language System (UMLS) [@127], cf. [570, a.o. Fig. 1, p. 83], modeling languages such as UML, also referring to the class diagrams in the previous section, as well as the OBO format (see sect. 1.1.3.5) that is used for ontology representation in the biomedical domain, is that relations among categories are expressed, e.g., graphically by nodes (labeled with category names) and edges / links (labeled with relation names) between them. Alternatively, this may be perceived as atomic statements in a FOL language, e.g., causes($\cdot$Bacteria, $\cdot$Infection), an example discussed in [465], or partOf($\cdot$Paw, $\cdot$Leg). Independently of the form of representation, an important question in this regard is what those representations mean, what may be derived from them.

In particular, most of these relations must be expected to actually relate individuals with each other, e.g., Leo's paw would be a part of Leo's leg. [465] discusses in detail that various interpretations at the categorial level can be given by means of FOL or description logics, e.g., to causes($\cdot$Bacteria, $\cdot$Infection). These include all four quantifier combinations equal or analogous to the option "all/some"[351] ($\forall\exists$) (2.18), i.e., "all/only" ($\forall\forall$), "some/some" ($\exists\exists$), "some/all" ($\exists\forall$), but also other schemes, where [465, p. 7] describe "all/each" (2.19) in addition.

(2.18) $\forall x \,.\, \mathsf{Bacteria}(x) \rightarrow \exists y \,.\, \mathsf{Infection}(y) \wedge \mathsf{causes}(x, y)$

(2.19) $\forall xy \,.\, \mathsf{Bacteria}(x) \wedge \mathsf{Infection}(y) \rightarrow \mathsf{causes}(x, y)$

Notably, most of these – although not all for the same original link between two categories – seem to have some justification in the domain. Indeed, it is not our goal here to advocate one or another specific option.

NAMING SCHEME AND DEFINITION SCHEMES

Instead, we are interested in seeing how $\mathcal{CR}$ may help in clearly and explicitly specifying (1) the statements at the categorial level and (2) which options/definitions are associated with them. In terms of the distinctions already available, especially of entity (E), category (C), and individual (I), a systematic naming scheme can be introduced in order to separate relation names at the different levels. This may be combined with other indicative components in relation names.

Emerging from joint work on the biomedical core ontology GFO-Bio [418], see sect. 6.2, we describe [416, 418] such patterns explicitly in [416], there only based on the distinction of categories and individuals. Table 2.2, reproduced from [416, Table I], presents the relations that are defined there. The overall work delivers proposals and a prototype implementation for the problem of integrating biomedical ontologies,

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[351]This is the main reading proposed wrt an OWL-based semantics to OBO in [428]. [465, p. 12ff.] sees room for several variants.

Relation	Domain:Range	Definition
x II-R y	Individual:Individual	The individuals x and y stand in the relationship II-R.
x IC-R y	Individual:Category	There exists an individual z, such that z IC-instance-of y and x II-R z.
x CC-R y	Category:Category	For all individuals a such that a IC-instance-of x, a IC-R y.
x CC-canonical-R y	Category:Category	For all individuals a such that a IC-instance-of x, by default, a IC-R y.
x II-lacks-R y	Individual:Individual	The individuals x and y do not stand in the relationship II-R.
x IC-lacks-R y	Individual:Category	The individual x does not stand in the relationship IC-R to y.
x CC-lacks-R y	Category:Category	For all individuals a such that a IC-instance-of x, a IC-lacks-R y.

Table 2.2: (published in [416, there Table I], here with minor changes in formatting and new caption) Some schematic definitions derived from a relation between individuals, II-R (hence, column three in the first row specifies a reading instead of a definition). The relation IC-instance-of names instantiation in the context of [416], limited to individuals in its domain. Relation prefix components I and C indicate whether the BasePlayerAbleC categories of the first and second argument, resp., are extensionally subsumed by individual or category. Note that the CC-canonical-R relationship is defined as a default relation.

aiming at the interoperability among canonical and phenotype ontologies, e.g., the Mouse Anatomy ontology [378] and the Mammalian Phenotype ontology [768]. Besides definitions as those seen above, the relation CC-canonical-R takes recourse to a default formulation in its definition. Besides other major contributions, Robert Hoehndorf developed the corresponding implementation of the approach, primarily based on OWL [@91] in combination with answer set programming (ASP) [117], [70, ch. 9, esp. sect. 9.7-9.11] for the semantics of defaults, utilizing the DLVHEX system [@29] [217].

A similar typology with class-class, class-instance, and instance-instance relations can be found in [763, p. R46.3-4], there without addressing definitions. [145, sect. 3] and [146] uses that typology with definitions in the context of negative findings in electronic health records.

2.4.4.2 Empty Categories

ROLE BASE NOTIONS OF INSTANTIATION

In sect. 2.4.3.3 above we introduce the three-fold distinction of base player category, base playerable category, and relation-independent category as role base notions. Moreover, Table 2.1 includes in the first two rows instances of those notions for instantiation itself. There are the base player categories *instance* and *instantiated*, the base playerable categories *categorizable* and *instantiable*, and entity and category itself subsuming the latter correspondingly.[352] These categories are now depicted in Fig. 2.9, together with the instantiation relation.
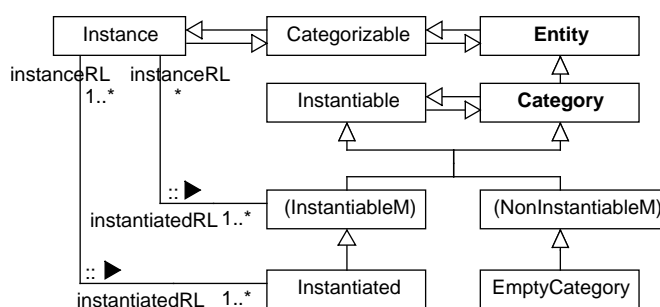
Figure 2.9: Role base notions of :: and *empty category*.

We have added two notions that we can describe in the text but that are not part of the CR formalization – the parentheses around *instantiableM* and *non-instantiableM* are there to mark that fact. Their characterization should be highly reminiscent of a discussion already given near Table 2.1, when introducing the idea

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[352]Those subsumptions are reflected by a baseEIsA relator from instantiable to category in the role base of instantiation, not shown here. Cf. Fig. 2.8 for baseEIsA.

of base playerable categories. A category c is classified under instantiableM if it is not only *subject* to the instantiation relation (which every category is by definition), but if in addition it is logically possible that c has instances. The suffix 'M' in the category names indicates this modal aspect.

ANALYSIS OF EMPTY CATEGORY ON THAT BASIS

Let us apply this framework in the characterization of the notions of *empty category* that is postponed to the present point already in sect. 2.4.2.2. Importantly, by 'empty category' we understand categories that are not only not instantiated (which would be a *non-instantiated* category), but akin to the frequent example of the round square. One interesting question wrt to this kind of categories is: Are empty categories instantiable?

This question seems to allow for a clear affirmation or rejection. However, especially with the availability of instantiable and instantiableM in Fig. 2.9, a solution that accounts for 'yes' and 'no' is lurking (and is already visible in the figure, of course). Indeed, we wish to defend the view that (1) on the one hand, empty categories are instantiable to the extent that they are subject to the instantiation relation just as all other categories. (2) On the other hand, empty categories are not instantiable in the sense that it is logically impossible for them to have instances. With both notions of instantiability available both aspects can be accounted for coherently.

Note that the notion of non-instantiated category, in a general reading, is neither subsumed by instantiableM nor by nonInstantiableM, although it is subsumed by instantiable. Due to the partitioning of instantiable into instantiableM and non-instantiableM (not stated in Fig. 2.9), a partitioning of non-instantiated category results, namely into (1) those non-instantiated categories subsumed by instantiableM and (2) non-instantiated categories subsumed by non-instantiableM. The latter is coextensional with empty category.

FORMALIZATION OUTLOOK

Again, we do not include instantiableM and non-instantiableM in the formalization of $\mathcal{CR}$ at the present stage. Insofar, instantiable has just two subcategories, namely instantiated and non-instantiated, whereas empty category is extensionally subsumed by the latter. Finally for this topic, the subsequent two formulas illustrate a distinct characterization of the notions of instantiated and instantiable in $\mathcal{CR}$.

(2.20) $\quad \forall x \,.\, \mathsf{Instantiated}(x) \,\leftrightarrow_{df}\, \exists y \,.\, y :: x$

(2.21) $\quad \forall x \,.\, \exists y(y :: x) \rightarrow \mathsf{Instantiable}(x)$

2.4.4.3 Self-Analysis and the Notion(s) of Order of $\mathcal{CR}$

REMARKS ON NOTION(S) OF ORDER

With two final remarks we hope to round off our conceptual picture of the ontology $\mathcal{CR}$ of categories and relations herein, although they rather touch future work than they establish "hard facts". The first concerns the question of whether there is a natural notion of order, e.g., in the light of the distinctions of first- and higher-order predicate logic as well as notions of individual and category categories in $\mathcal{CR}$. It is certainly tempting to think of instantiation to provide a natural order, especially if one draws an analogy between instantiation and set membership, possibly due to a similar use/view in connection with especially monadic predicates.

However, we do not see matters as straightforward, because relations – as prime candidates for the interpretation of polyadic predicates – can be recognized as genuine entities, they are significantly different from standard mathematical relations/sets of tuples, and they are not "synchronized" with instantiation. The latter means that a relator, i.e., an individual and first order entity from the point of view of instantiation, can connect two categories of high instantiation order. In addition, role nesting in terms of the plays relation can be considered as an alternative way of counting an order of entities. In this respect, first-order entities are non-role entities, which are not played by any other entity. A role is of the next order given the order of its player, and a relator shares its order with those of its roles, e.g. by taking the highest order among all roles of that relator to be the order of the latter itself. Yet this is just one variant for role-nesting. Setting up a specific priority among this one and others without any clear justification does not appear beneficial to us at the present stage. Furthermore, the question remains open of how an instantiation-based notion of order would be combined with the understanding of relations in $\mathcal{CR}$, where an is-a relation appears to be on a par with relations among individuals.

Overall, this matter remains future work and is of no crucial importance for what follows. However, the case of the is-a relation, understood as a genuine ontological relation, furnishes us with a fit occasion for mentioning the second intricate and mostly future issue – the question of self-analyzing capabilities of $\mathcal{CR}$.

SELF-ANALYZABILITY OF CATEGORIES AND RELATIONS

Indeed, we consider the three basic $\mathcal{CR}$ relations, $::$, $\rightsquigarrow$, and $\multimap$ to be sufficient for their mutual ontological analysis. By that we mean that in order to analyze, for example, instantiation ontologically, it can be studied as a relation – just like any other relation. Likewise, relations / the "construction" of relating entities in terms of relators and roles can be explained with only categories and instantiation, as well as relations and plays and role-of themselves available. For this reason, we propose $\mathcal{CR}$ for consideration as a minimal conceptually-semantic basis.

One important point in those regards is, however, that the mutual analyzability does certainly not entail that analyses of the involved notions could not be tied to other entities/notions, as well, which are not among those referred to as 'basic' above. Say, for certain categories and the relation of instantiation (possibly limited to them), a detailed analysis with respect to similarity may be reasonable. Likewise, an advanced analysis of relations might identify these as being subject-dependent, and such an account would involve additional entities.

Eventually, we remark that self-analyzability first utilizes a position where relations and properties can be subject to predication, which is close to that of the early Russell according to Nino B. Cocchiarella [160, p. 129]. Cocchiarella notes that this leads to Bradley's infinite regress argument [111], cf. [796, sect. 7.7] for a version wrt instantiation. Clearly self-reference is involved if we analyze instantiation in terms of relators that require instantiation for their comprehension. We shall not deal with this issue in depth here. It appears more plausible to us to apply the same kind of analysis to all relations, instead of assuming a second kind of relation with "special powers", e.g. a 'nexus', 'non-relational tie', or anything the like, cf. [796, sect. 7.7]. Arguments have been raised against viewing that regress as one of the vicious kind. This is the case in Donald W. Mertz's [581, cf. p. 43–50 and ch. 7–8], for instance. Moreover, a crucial distinction is pointedly presented in [796], starting from the example to be analyzed that a instantiates F (or has the property F):

> "This disagreement may perhaps be avoided if we distinguish an 'internalist' and an 'externalist' version of the regress (in the terminology of Orilia 2006a [652]). In the former, at each stage we postulate a new constituent of the state of affairs, s, that exists insofar as a has the property F, and there is viciousness because s can never be appropriately characterized. In the latter, at each stage we postulate a new, distinct, state of affairs, whose existence is required by the existence of the state of affairs of the previous stage. This amounts to admitting infinite explanatory and metaphysical dependence chains, but, since no decisive arguments against such chains exist, the externalist regress should not be viewed as vicious (Orilia 2006a, §7 [652]). An extensive defense of a similar approach can be found in Gaskin 2008."   [796, sect. 7.7]

We find the view of the second, externalist version attractive and apposite to $\mathcal{CR}$. Nevertheless we shall (primarily) consider a version of the theory $\mathcal{CR}$ without enforcing this "feature" explicitly, and pay attention as to when it is applied or possibly required.

2.5   Summary of the Analysis and Next Steps

After the rather terminological and preparatory character two sections above, and a substantial intermezzo of presenting the conceptualization of a theory $\mathcal{CR}$ of categories and relations, let us conclude this chapter by recapitulating its central observations and compiling its main arguments in a distilled form. This is complemented with next objectives and steps to drive / inspire the remainder of the work.

DEFINING SEMANTICS & TYPES OF SEMANTICS

Defining a semantics for a language L involves dealing with symbols that form the alphabet of L and it compasses a treatment of the grammatical rules of L. From an abstract point of view, grammar rules establish syntactic relations between expressions (symbols and words / composed symbols). The (specification

of the) semantics of a language defines how syntactic relations are transformed / interpreted into semantic counterparts of expressions, in the referential model that we maintain. In this connection, we draw a clear distinction between two readings of the term "semantics". The first is accessible for any natural language speaker and refers to semantics as the intuitive / *intensional* meaning of some expression. The second is the view of *formal semantics* that is the major paradigm in the context of formal languages. We note that the former appears also applicable to languages without a formal semantics.

THE "VICIOUS CIRCLE" OF ONTOLOGIES IN AO / FOIS

We argue that ontologies were introduced originally into computer and information sciences for the purpose of reuse of formalized knowledge. In this context, at least to some extent, the preservation of formal semantics did not provide a comprehensibly satisfying solution. However, ontologies must themselves be represented in order to apply them for any purpose. The standard account of defining ontology-based equivalence[353] expects the ontology to be represented by a logical language (with formal semantics). Ontology-based equivalence is then defined in terms of / reduced to formal equivalence according to the formal semantics of that language, and wrt the ontology as a formal theory, in addition. At this general level, this is strongly reminiscent of a 'vicious circularity', because problems of formal semantics are to be solved by ontologies, which / the representation of which in turn rely on formal semantics – and thereby on encodings of ontological notions in terms of formal entities. In greater detail, a number of problems with the standard approach to ontology-based equivalence are identified esp. in sect. 2.2.2.2:

- Defining ontology-based equivalence basically as logical equivalence modulo a theory leads to the equivalence of all statements of the theory itself. Therefore, ontology-based equivalence cannot be directly applied to ontologies themselves.
- There is no obvious variation of the standard account resolving the first problem. Nor do more liberal formal equivalence relations like theory interpretations or interpretability seem to provide a better solution either.
- Well-known criticism to extensional accounts of logic (see sect. 1.2.3) still applies.
- Using logic for representing ontologies suggests to adopt the viewpoint of 'logic as a language', in contrast to 'logic as a calculus' (both described in [160]). However, under this view the claim of an 'ontological neutrality' of logic is not tenable and the formal semantics of the representation language deploys ontological impact.

OBJECTIVE AND OUTLINE

Consequently, we consider it an open problem to explain how ontologies can be represented (in a way that repels the encoding problem as much as possible). According steps on representations can be taken in the spectrum between a single standard representation language and a set of diverse languages which allow for representing the very same contents. In any case, if several distinct representations are possible, the demand is immediate to explain / justify why / in which sense these are conceptually equivalent.

Forestalling later development and maintaining realistic expectations, ch. 4 comprises our attempt in contributing and working towards these aims. Our focal effort and achievement there is to develop a novel account of language semantics termed *ontological semantics*. For this account and based on experiences of early phases of defining such semantics in analogy to the classical semantics, it is advisable and instructive to study the "nature" of the standard / set-based model-theoretic account of classical predicate logics from different angles. In particular, its ontological implications under the 'logic as a language' view (cf. sect. 2.2.2.2) are in the focus of ch. 3.

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[353] as well as ontology-based consequence

Chapter 3

Views on Set-Theoretic Semantics of Classical Predicate Logics

The aims of this chapter are a closer inspection of classical Tarskian model theory for predicate logic, in particular its interconnections with set theory / set-theoretic assumptions, as well as a study of options for (ontologically) postulating entities / types of entities by using a language (say, FOL) with this classical semantics. Readers well educated in (mathematical) logic and its relationship with set theory may find much of the contents of this chapter familiar, possibly exposed in more detail than in textbooks and in uncommon notation. The latter may result from the intention of a style of writing that is accessible with basic logical training. Overall, the chapter sets the scene for deriving ontological semantics in ch. 4, e.g. by a proposal of more precisely capturing the notion of ontologically neutral semantic accounts in Def. 3.21 (sect. 3.4), in contrast to a first discussion in sect. 2.2.2.2.

3.1 Tarskian Model Theory and Set-Theoretic Superstructure

3.1.1 Basic Remarks on Set Theory and Σ Structures

INTRODUCTORY REMARKS ON TARSKIAN SEMANTICS

To see in greater detail how, under the view of 'logic as a calculus' [160] introduced in the previous sect. 2.2.2.2, set theory imposes ontological assumptions on classical predicate logic (of first and higher order), the standard Tarskian approach to defining model-theoretic semantics shall be recalled. In addition, we are interested in correspondences of the logical notion(s) of *order* in set theory.

There is a very tight link between standard FOL semantics and set theory. Typical introductions to Tarskian semantics build on previously defined notions of (mathematical) structures, cf. the one presented in sect. 1.5.2. Structures, in turn, are based on some variant of set theory.[354]

SET THEORY IN BRIEF

For this reliance, there is an (at least implicit) assumption of *sets*, either understood on a (more) intuitive basis[355] or as axiomatically characterized [459],[356] e.g. by systems like Zermelo-Fraenkel (ZF) [460] or von Neumann-Bernays-Gödel (NBG), see [200, 212, 459] for introductions. In the appendix, sect. A.2.1 lists the axioms of ZF and related systems, namely ZF extended with the Axiom of Choice (ZFC) and ZF without the Axiom of Foundation (ZF^0). Notably opinions on a / the "standard set theory" differ. Herein, following [212, p. 12] we consider ZFC and NBG as standard set theories.[357] For sets themselves, both theories are equivalent, in the sense precisely captured and proved in [212, ch. XII]. The phrase 'for sets themselves' refers to the distinction of sets and (mathematical) *classes* that is available in the theory of NBG, see e.g. [212, p. 11–12 and ch. XII], [789, esp. p. 11–12]. Classes split into sets and *proper classes*. Sets are classes that can be members / elements of classes, while proper classes are not subject to the element role in $\in$, speaking in terms of $\mathcal{CR}$ (cf. sect. 2.4). This distinction enables allowing for unrestricted comprehension in NBG. In that sense, proper classes are "set"-theoretic entities / collections that are "too large"[358] to be sets / elements of classes, e.g., the class of all sets [212, p. 11]. Another branching point for set theories is the question of whether they allow for, following Zermelo, *urelements* [212, p. 10].[359] From the point of view of set theory, urelements are all entities that are not sets, but that can be members of sets.[360] Modern accounts of set theory typically neglect urelements.[361] This is also the case in this section although, clearly, urelements will be required for ontological purposes.

Σ STRUCTURES

Based on the notion of mathematical structures, the first-order semantics for "standard" FOL syntax over a (finite) signature $\Sigma = \{\tilde{R}_1 \ldots, \tilde{R}_l, \tilde{f}_1, \ldots \tilde{f}_m, \tilde{c}_1, \ldots, \tilde{c}_n\}$, with $l, m, n \in \mathbb{N}$ and predicate symbols $\tilde{R}_i$, function symbols $\tilde{f}_j$, and logical individual constants $\tilde{c}_k$, is defined wrt Σ structures, see sect. 1.5.2. Adapted to the present notation, $\mathcal{A} = (A, R_1, \ldots, R_l, f_1, \ldots, f_m, c_1, \ldots, c_n)$ is a Σ structure :iff A is an arbitrary

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[354]For theoretical purposes, it often suffices to consider relational structures, cf. [214, sect. VIII.1] for a detailed reduction / rewriting of functions and individual constants into relations (capturing the graphs of the former). Moreover, see [55, ch. II.1] for a general notion of logic for the relational case. Other mathematical structures than relational structures alone have been studied [55, ch. I.1.1], e.g., topological logics [55, ch. 15]. We return to this issue in sect. 3.1.4.

[355]This can be associated with the term 'naive set theory', e.g., see the introductory book "Naive Set Theory" by Paul R. Halmos [365, 366].

[356]Presently, the daily use of sets appears to be predominantly on an intuitive basis in mathematics and theoretical computer science. Nevertheless, according to [459] it is considered standard in mathematics that sets are captured axiomatically. The latter view also fits the axiomatic-deductive approach adopted herein, see sect. 1.1.5.3.

[357]There are diverse further set theories. Some of these are mentioned in connection with the discussion of a universal category and instantiation cycles in sect. 2.4.2.3, e.g. the New Foundations (NF) of Willard Van Orman Quine [245, 680] and non-wellfounded sets [2, 593].

[358]Cf. [423, sect. 3.2, esp. p. 145–146] on the formulation, but also [212, p. 11].

[359]The GFO term 'item' (at the abstract top level, cf. sect. 2.3.3) is synonymous with 'urelement' [397, p. 11].

[360]In contrast to proper classes which are "too large" to be elements, Dirk W. Hoffmann characterizes urelements as "too small" to have elements themselves [423, p. 146].

[361][200, p. 145] states that it is easy to amend ZFC with urelements. However, in other set theories the addition of urelements has respectable consequences, e.g. in the case of Quine's New Foundations [245], cf. [425].

non-empty set, the *universe*[362] of $\mathcal{A}$,[363] the R_i are (mathematical) relations over A, the f_j (mathematical) functions over A, and the c_k are members of A, cf. also [214, Def. 1.1, p. 31]. Since our focus is on structures themselves, in this sect. 3.1 signature elements related to/interpreted by a structure element X are denoted by $\tilde{X}$, i.e., the same symbol but carrying a $^{\sim}$ marker. This establishes the underlying interpretation function implicitly. As usual, relations and functions in Σ structures have fixed arities.

3.1.2 Cumulative Hierarchy of Sets and Order in Logic

THE UNIVERSE WRT BACKGROUND SET THEORY

As already stated, entities occurring in Σ structures (the universe A, (mathematical) relations and functions) are considered as sets.[364] Asking the question of *which* sets (and therefore relations, functions, and Σ structures) are available for forming interpretation structures requires one to consider the set theory that is adopted for the semantics of FOL, the *background set theory*. 'Background' may simply refer to the semantic background. In the special context of formalizing (a) set theory by means of FOL syntax, the syntactically captured set theory yields the *foreground set theory* whereas interpretations of that theory are based on the background set theory, because the latter provides the foundation for the formal semantics of the logical language, cf. also [214, sect. VII.2, esp. p. 116].

Actually, all non-logical constant symbols are interpreted by entities of the background set theory. In order to support this claim and in parallel to rendering it more precisely, we consider a different universe for the interpretation structures. The "defining" requirement for that universe is to cover all entities referred to by predicate, function, and constant symbols. Moreover, a clear and close relation to the notion of order, i.e., to distinguishing logics of first, second, third, etc. order is desirable.

IMMEDIATE CANDIDATE: ALL (WELL-FOUNDED) SETS, CUMULATIVE HIERARCHY

The most immediate idea for this new universe is to rely on the class of all sets. Assuming ZFC as standard well-founded set theory and having in mind the notion of order of classical logic, an attempt to relate sets and orders could be made by referring to the class of well-founded sets, which has close ties with the *cumulative hierarchy* of sets $\mathcal{V}$.[365] This hierarchy, or actually class, of sets, is defined on the basis of *ordinal numbers* [212, sect. VI.2], [198, sect. 2.6]. The following characterization is compiled from [212, Def. 1.11 on p. 51, Def. 2.1 and Satz 2.2, p. 90] and [198, p. 225, 258], defining ordinal numbers as transitive sets that are well-ordered wrt $\in$.

3.1 Definition (ordinal number)
For an arbitrary set x the restriction of $\in$ to x is defined by $\in_x := \{(u, v) \mid u, v \in x \text{ and } u \in v\}$.
A set α is an *ordinal number* :iff
- α is a transitive set, i.e., its elements are also subsets of it: for all $x \in \alpha$: $x \subseteq \alpha$, and
- α is well-ordered by $\in$, i.e.,
    - $\in_\alpha$ is a linear ordering and
    - every non-empty subset of α has a least element wrt $\in_\alpha$:
      for every $v \subseteq \alpha$ with $v \neq \emptyset$ there is an $m \in v$ such that $m \in_\alpha u$ for all $u \in v$.   ❑

Ordinal numbers originate from modeling counting and natural numbers in set theory. The empty set is defined as the initial ordinal **0**. Then there are two ways to form new ordinals. Given an ordinal number x, the *successor ordinal* of x is defined by $x \cup \{x\}$. Obviously, this construction is repeatable infinitely many times (in principle). Infinite chains of successor ordinals are transcended by *limit ordinals*, defined as all

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[362]alternatively, 'universe of reference', 'universe of discourse', 'domain of discourse', 'domain' (if unambiguous from context wrt other notions of domain, cf. the end of sect. 1.1.2.2), or 'carrier set' / 'carrier'

[363]According to [@118], some authors consider also structures that allow for classes as their universe, e.g. Bertrand Russell and Alfred North Whitehead in the "Principia Mathematica" [875].

[364] Notably, this requires well-known set-theoretic reductions/encodings of functions and relations, and thus of the Cartesian product and n-tuples to purely set-theoretic constructions [212, sect. IV]. Familiar reductions today are based on, eventually, the encoding of an ordered pair (a, b) that is attributed to Kazimierz Kuratowski, dated in 1921 [493], where $(a, b) := \{\{a\}, \{a, b\}\}$ [197, p. 48].

[365]The cumulative hierarchy is also called the 'von Neumann hierarchy' [212, sect. VII.3] by Heinz-Dieter Ebbinghaus. Keith Devlin refers to it as 'Zermelo hierarchy' [200, p. 38], Oliver Deiser introduces it as 'Neumann-Zermelo hierarchy' [198, p. 276] [own transl.].

ordinals that are not successor ordinals.[366] Limit ordinals are exactly those ordinals such that $\bigcup x = x$ [197, p. 260], i.e., they are formed by the union / as the supremum of infinite sequences of ordinals (all starting from **0**). Due to limit ordinals, "counting" continues transfinitely. Ordinals provide a form of "numbers" that give rise to cumulative levels / layers of sets, leading to the definition of the cumulative hierarchy, which we present as a variation from [212, Def. 3.1 and 3.3, p. 107] and [198, p. 276].

3.2 Definition (von Neumann hierarchy / cumulative hierarchy of sets $\mathcal{V}$)
Define *von Neumann layers V.* by

$$
\begin{aligned}
V_x &= \emptyset && \text{if } x \text{ is not an ordinal number} \\
V_{\mathbf{0}} &= \emptyset && \\
V_{\alpha'} &= \mathcal{P}(V_\alpha) && \text{if } \alpha' \text{ is the successor ordinal of } \alpha \\
V_\delta &= \bigcup \{ V_\beta \mid \beta \in \delta \} && \text{if } \delta \text{ is a limit ordinal}
\end{aligned}
$$

The *von Neumann hierarchy* $\mathcal{V}$, alternatively referred to as *cumulative hierarchy of sets*, is the class of all sets x such that there is an ordinal number α with $x \in V_\alpha$. ❑

Adopting ZFC as background set theory, the class of *all* sets and class $\mathcal{V}$ are equal. This follows from the fact that the Axiom of Foundation (Found in sect. A.2.1) is equivalent to the universality of membership in $\mathcal{V}$, i.e., ZF^0 entails Found $\leftrightarrow \forall x . x \in \mathcal{V}$ [212, Prop. 3.7, p. 110].[367] Furthermore, closing the circle to the cumulative hierarchy, note that the term 'cumulative' is justified by the property that each level comprises among its elements the elements of *all* previous levels.

CUMULATIVE HIERARCHY NOT ADEQUATE FOR LOGICAL ORDER

Although the cumulative hierarchy plays an eminent role in the understanding of the structure of the set-theoretic universe [212, sect. VII.3 & X.3], there are several reasons not to adopt it for the purpose of an interpretation universe as that aspired to above. Firstly, the cumulativity of the levels prevents close resemblance with the notion of *order* in classical logic, at least for the first and second orders, where members of the universe should be kept separate from relations over them, and analogously for referents of second-order predicates. Cf. also the remarks in [212, sect. VII.3] on the connection of $\mathcal{V}$ with Russell's type theory [875]. In the context of logic, on the one hand, order is more closely tied to individual "steps" of the membership relation between sets and the question of whether and which sets can be quantified over, cf. [214, ch. IX] and below.[368] In addition, the syntactic limitations of predicate logic can avoid addressing *arbitrary* sets, depending on the "ranges" of higher-order predicates, variables, and corresponding quantification. Assuming a universe of urelements, first-order predicates capture only relations, purely over urelements, whereas higher-order predicates may do so purely for relations over relations of urelements.[369] On the other hand, an aspect that is neglected here are effects of mathematical relations being sets of tuples, which are themselves encoded as sets, which yields membership chains of varying lengths for the same logical order.

3.1.3 Tuple and Powerset Superstructure $\mathcal{A}^{\mathcal{P}^\omega_{\text{fix}}}$

TUPLE AND POWERSET HIERARCHY

Accordingly, a different, more straight-forward set-theoretic structure shall serve as the universe for all interpreting entities. Standard set theories allow for the construction of tuples / the Cartesian product and the powerset formation (of a given set). This allows one to consider the following infinite hierarchy of sets

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[366] According to [197, p. 260] it is common to view the empty set as a limit ordinal. [212, esp. Def. 2.8, p. 93] distinguishes them.

[367] Since $\mathcal{V}$ is a proper class wrt ZFC / NBG, the expression '$x \in \mathcal{V}$' in the formula cannot be understood as an object-language statement wrt ZFC. Instead, it should then be seen as an abbreviation of stating the membership of x in class $\mathcal{V}$ according to Def. 3.2. Cf. the remarks in [212, sect. II.2, esp. p. 23].

[368] Cocchiarella points out that historically there were different views on the notion of order, e.g. among Frege and Russell [160, sect. 4, p. 128–129].

[369] In contrast, in [223, sect. 4] Herbert B. Enderton describes higher-order predicates, thereby, in particular, allowing for "super-predicate symbols" that take as arguments individual or predicate symbols.

that starts with an arbitrary set A (with or without *urelements*) and adds levels of relations, each time being relations over the previous level.

3.3 Definition (tuple-powerset hierarchy $\cdot^{\mathcal{P}^n_{\text{fix}}}$)

Let A an arbitrary set. The *tuple-powerset hierarchy* over A is the infinite sequence of sets that has A as its initial item $A^{\mathcal{P}^0_{\text{fix}}}$, and every other item $A^{\mathcal{P}^{n+1}_{\text{fix}}}$ comprises the set of all relations of all finite and fixed arities over elements of the previous item $A^{\mathcal{P}^n_{\text{fix}}}$, concluded by the union over all prior sequence items:

$$A^{\mathcal{P}^0_{\text{fix}}} := A$$
$$A^{\mathcal{P}^{n+1}_{\text{fix}}} := \bigcup_{1 \leq k < \omega} \mathcal{P}\left(\left(A^{\mathcal{P}^n_{\text{fix}}}\right)^k\right) \qquad \text{for } n < \omega$$
$$A^{\mathcal{P}^\omega_{\text{fix}}} := \bigcup_{n < \omega} A^{\mathcal{P}^n_{\text{fix}}} \qquad\qquad\qquad\qquad \square$$

Note, from a purely set-theoretic perspective and assuming the standard encoding of Cartesian products of arbitrary power, cf. e.g. [212, Def. 1.9] and FN 364, a level of this kind comprises sets with membership paths of highly differing lengths. We only mention that the levels are not cleanly separated under this perspective, e.g. $\{\{\{a\}\}\} \in A^{\mathcal{P}^1_{\text{fix}}} \cap A^{\mathcal{P}^3_{\text{fix}}}$. Its membership in $A^{\mathcal{P}^3_{\text{fix}}}$ is clear from $a \in \{a\} \in \{\{a\}\} \in \{\{\{a\}\}\}$ by three applications of $\mathcal{P}$. The encoding $(a, a) = \{\{a\}\}$ and $\{(a, a)\} \in A^{\mathcal{P}^1_{\text{fix}}}$ yield the second fact to account for membership in the intersection. This gets blurred / obliterated further if A comprises (also) sets, e.g. for $A = \{a, b, c, \{a, b, c\}, \{a, b\}\}$. However and despite we are left with some doubts on the general innocuousness of such effects, we adopt the $\cdot^{\mathcal{P}^\omega_{\text{fix}}}$ construction for further consideration, whose argumentation appears unaffected by that – at the very least, for the case of FOL.

INTERPRET ALL SYMBOLS AS CONSTANTS, SET THEORY DEPENDENCE

With respect to the final item of the tuple-powerset hierachy over some A, every symbol in Σ (from sect. 3.1.1) can be interpreted as a constant in the following structure, i.e., as a member of its universe.

$$(3.1) \qquad \mathcal{A}^{\mathcal{P}^\omega_{\text{fix}}} = \left(A^{\mathcal{P}^\omega_{\text{fix}}}, R_1, \ldots, R_l, f_1, \ldots, f_m, c_1, \ldots, c_n\right)$$

The relationships between those relation and function constants are governed by the set theory chosen to "produce" $A^{\mathcal{P}^\omega_{\text{fix}}}$. Only the (logical) individual constants c_i are not constrained by the set theory, because they denote members of the set A initiating $A^{\mathcal{P}^\omega_{\text{fix}}}$. The c_i may therefore be related (intensionally and extensionally) in arbitrary ways. The dependence / commitment to set theory is even more explicit if set membership is made explicit itself through a relation $\bar{\in}$ (3.2).

$$(3.2) \qquad \mathcal{A}^{\mathcal{P}^\omega_{\text{fix}}}_{\bar{\in}} = \left(A^{\mathcal{P}^\omega_{\text{fix}}}, \bar{\in}, R_1, \ldots, R_l, f_1, \ldots, f_m, c_1, \ldots, c_n\right)$$

In the structure $\mathcal{A}^{\mathcal{P}^\omega_{\text{fix}}}_{\bar{\in}}$, $\bar{\in}$ denotes the set $\{(x, y) \mid x, y \in A^{\mathcal{P}^\omega_{\text{fix}}} \text{ and } x \in y\}$. It is interesting to observe the relationship between $\bar{\in}$ and the "complete" background membership relation $\in$, cf. also analogous remarks in [212, sect. IV.1]. First of all, $\bar{\in}$ is a restriction of $\in$ to all members of $A^{\mathcal{P}^\omega_{\text{fix}}}$, by definition. Secondly, however, $\bar{\in}$ cannot cover $\in$ completely in well-founded set theories. $A^{\mathcal{P}^\omega_{\text{fix}}} \in A^{\mathcal{P}^\omega_{\text{fix}}}$ is set-theoretically excluded, for instance, hence for every $x \in A^{\mathcal{P}^\omega_{\text{fix}}}$ the fact $\bar{\in}(x, A^{\mathcal{P}^\omega_{\text{fix}}})$ cannot hold. A similar argument applies to $\bar{\in}$ itself. If it were contained in $A^{\mathcal{P}^\omega_{\text{fix}}}$, for every pair $p \in \bar{\in}$ another member of $\bar{\in}$ would be required, namely $(p, \bar{\in}) \in \bar{\in}$. Obviously, this would entail a – not admissible – membership cycle starting and ending at $\bar{\in}$. Eventually, note that $\bar{\in}$ is defined "semantically" above, i.e., from a point of view outside of $A^{\mathcal{P}^\omega_{\text{fix}}}$. It is a very different question to what extent any syntactic theory over the corresponding signature Σ may characterize $\bar{\in}$ (or $\in$).[370] .

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[370]Notably, such characterizability questions are a common issue in model theory. The famous example of non-standard models of Peano arithmetic that cannot be excluded by FOL [691, p. 83 ff.] is already mentioned in sect. 1.2.3.2. Cf. also Wilfried Hodges' discussion of axiomatizations defining classes of structures vs. describing particular structures [411, sect. 19 and 20].

Before proceeding to the relationship of $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$ and logical order and further alternatives in semantics, some notes on connections to type theory are overdue. *Type theory* is a multi-faceted area with origins in connection with higher-order logic and the foundations of mathematics, cf. e.g. [586], [223, sect. 4] and Russell's type theory [875], but which has found novel applications in connection with intensional logics, see e.g. [246, 462], with linguistics, e.g. Montague Grammar [656] (itself highly related to intensional logics), and in computer science, e.g. in combination with functional programming [822].

Readers familiar with *type theory* may already have noticed a strong resemblance of the presentation above to the borderland between type theory (primarily in the context of higher-order logics) and set theory. Indeed, Johan van Benthem and Kees Doets present a type system defined inductively by letting 0 be a type of individuals, and letting finite sequences $(\tau_1, \ldots, \tau_n)$ of types be types again, namely for relations between objects of types $\tau_1, \ldots, \tau_n$ [838, sect. 3.1, p. 217]. Types are assigned a natural number called their *order*,[371] namely 1 for type 0 and $k+1$ for $(\tau_1, \ldots, \tau_n)$, if k is the maximal order among the τ_i. They continue with the introduction of a logic L_ω, where, based on those types, a.o., variables for each type are present, and universes for each type, starting from a classical first-order structure, denoted by $\mathcal{A} = \langle A, * \rangle$, i.e., with universe A and interpretation function $*$. Wrt $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$, the interesting connections reported for L_ω are as follows.

> As for the connections between L_ω and set theory, notice that the present logic is essentially that of arbitrary models A provided with a natural set-theoretic superstructure $\bigcup_n V^n(A)$; where $V^0(A) = A$, and $V^{n+1}(A) = V^n(A) \cup \mathcal{P}(V^n(A))$. [...]

> We will not go into the exact relations between the logic of L_ω and ordinary set theory, but for the following remark.

> Given a structure $\mathcal{A} = \langle A, * \rangle$, the structure $\mathcal{A}^+ = \langle \bigcup_n V^n(A), \in, * \rangle$ is a model for a set theory with atoms. There is an obvious translation from the L_ω theory of $\mathcal{A}$ into a fragment of the ordinary first-order theory of $\mathcal{A}^+$. The reader may care to speculate about a converse (cf. [Kemeny, 1950] [[469] herein[372]]). [838, p. 218–219]

These comments support our investigation above. On mutual relationships we only notice that $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$ is limited to what we might call "order-homogeneous" types, i.e., it addresses only types $(\tau_1, \ldots, \tau_n)$ s.t. all τ_i share the same order, due to not mixing / uniting the powerset operation with the previous level. On the other hand, $\bigcup_n V^n(A)$ ignores tuple formation (and encodings), such that also sets that are contained in both classes can appear with different indices.

3.1.4 Alternatives and Remarks

We believe that the position of ontological commitment in classical logic is applicable to Tarskian semantics and the structure $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$ provides some clarification in this respect. However, the adoption of the ontological commitment must remain to some extent deliberate in the context of philosophical logic and alternative semantics for FOL. A detailed analysis of alternatives that have been debated about is too far from our goals and thus shall not be pursued further here. But some pointers into the respective literature may be useful. For instance, Quine in [684, ch. VI] critizes the view of postulating the existence of sets from the mere use of predicates [684, p. 112 ff.]. Quine's own position in this respect has undergone some development between his earlier and later writings [879, ch. 8, p. 189]. His later views connect to alternative semantics based on *substitutional* interpretations of quantification, among them substitutional semantics

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[371]This corresponds to the logical understanding of order.

[372]To resolve the speculation first, [469] refers to a short abstract that is based on the PhD thesis "Type Theory vs. Set Theory" of John G. Kemeny [468]. The abstract is actually a part of [874] and reports on the relation between "[t]he simple theory of types, T, and Zermelo set-theory, Z [...]" [469], for which it states "The fundamental result is that a truth-definition can be given for T within Z. From this it follows that the consistency of T can be proved in Z. This shows that Z is much stronger than T in three senses: (1) Theorems can be proved in Z which cannot be proved in T. (2) Sets of integers can be defined in Z which cannot be defined in T. (3) One can prove by elementary methods that the consistency of Z implies the consistency of T, but not the converse." [ibid.].

and truth-value semantics [514]. In contrast, the interpretation of quantifiers usually attributed to Tarskian semantics is *objectual* [514, p. 53, FN 1]. There are exceptions like Shaughan Lavine's work that discusses Tarskian semantics but under the regime of substitutional quantification [509]. The compatibility of (logical formalizations of) set theory and the substitutional account is discussed by Dale Gottlieb and Timothy McCarthy [292]. They provide a substitutional interpretation of quantification for ZF which is consistent with ZF itself, as a reply to according objections to the substitutional account. Regarding specifics of higher-order logic, Peter Simons tackles the neutralization of the ontological commitment to abstract entities that higher-order logic appears to involve [755]. Finally, we refer the interested reader to the valuable collection of many further related references compiled by Ted Sider [754].

QUANTIFICATION OF FIRST AND HIGHER ORDER

Respecting the necessary precautions, we return to the adopted account. The structure $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$ can be used to analyze (objectual) quantification of first and higher orders in predicate logic. Regarding that distinction, a frequently observable confusion is between syntactic and semantic higher-orderness. Examples of *syntactic higher-order* features of a language are that predicates can appear in argument positions of predicates or functions, and that predicates or functions can be bound by quantifiers. These syntactic deviations from FOL need not imply that the corresponding language is *semantically higher order*. The latter notion depends on the set members of $A^{\mathcal{P}^{\omega}_{\text{fix}}}$ that can be quantified over. Quantification is *first-order (semantically)* iff it ranges only over members of $A = A^{\mathcal{P}^{0}_{\text{fix}}}$. *Higher-order quantification (semantically)* is exhibited by logics which admit quantifiers ranging over other members of $A^{\mathcal{P}^{\omega}_{\text{fix}}}$ than just A. If those other members are all subsets of A this results in monadic second order logic. For arbitrary members of $A^{\mathcal{P}^{1}_{\text{fix}}} = \bigcup_{1 \leq k < \omega} \mathcal{P}\left(A^k\right)$ this leads to (full) second-order logic (SOL). At this point, the suitability of the chosen hierarchical construction should be clear from the correspondence of the first levels in $A^{\mathcal{P}^{\omega}_{\text{fix}}}$ and the counting of orders in logic. Higher orders than the second correspond to further levels, namely $A^{\mathcal{P}^{n}_{\text{fix}}}$ to order $n + 1$, if higher-order predicate variables are limited to valuations from the level directly below them. Alternatively and more in line with type theory as mentioned in connection with [838, sect. 3.1] at the end of the last section, the definition of $A^{\mathcal{P}^{n+1}_{\text{fix}}}$ can be adapted to be fully analogous to $V^{n+1}(A)$ in the quote from [838, p. 218–219]. However, the difference between these two approaches of higher-order logic has no important consequences for what follows below, such that these observation shall complete the picture of orders in classical logic for our purposes.

The above-mentioned confusion of syntactic and semantic higher-orderness is primarily observed outside of logic, e.g. in AI and the Semantic Web.[373] Already in 1993, Weidong Chen, Michael Kifer and David S. Warren discussed the notion of higher-orderness in the context of logic programming [151, sect. 1, p. 187–190)], listing logics of all combinations of FO/HO syntax and FO/HO semantics. They refer to [220, p. 282] in pointing out that logicians are long aware of the distinction.[374]

DIFFERENT STANDARD INTERPRETATIONS AND HENKIN SEMANTICS

Eventually, let us reconsider interpretations in classical logic on the basis of $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$. *Different* interpretations for a formula or a theory basically appear only by choosing a different universe A or assigning different members from $A^{\mathcal{P}^{\omega}_{\text{fix}}}$ to symbols. For a fixed universe A, the underlying structure as outlined in the above construction is completely determined by A. An exception in this regard are Leon Henkin's *general structures* [391] of (syntactically) higher-order formulas, also called Henkin structures. Henkin semantics "relaxes" $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$ in the following sense, mainly based on [223, sect. 3].[375] While $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$ uses the power set operation of its underlying set theory, Henkin semantics does not enforce the existence of *all* members of those power sets. Only relations that are *definable* in terms of the language must be present, and more precisely,

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[373]Recently, languages with higher-order syntax like Common Logic [452] have reflated discussion of these issues (e.g., cf. [@34]), as well as meta-modeling approaches that are similar in spirit, see [600].

[374]Moreover, Chen et al. [151] claim that a third aspect of higher-orderness is the equational theory built into the semantics of the logic, which in an extreme case may make the first and higher-orderness of the semantics indistinguishable.

[375][391] is the initial journal publication of general semantics for higher-order logic, with results that are initially presented in Leon Henkin's PhD thesis [390]. [223] presents the approach for second-order logic in terms of a classically oriented account. [586] covers the topic directly for higher-order logic in general, in a more type-theoretic spirit. As further reading, earlier editions of [18] and [838] are recommended in [586].

all definable relations, which amounts to so-called *full comprehension*.[376] Consequently and interestingly, sentences in second-order syntax under this semantics are valid iff they are valid under a first-order, many-sorted interpretation that satisfies all comprehension axioms.[377] This leads immediately to very different metalogical properties, e.g., a completeness theorem for Henkins higher-order semantics has been proved [223, sect. 3]. Altogether, Henkin's approach is a good example for the leeway that is available for defining higher-order interpretations. Because the choice of relation and function universes must be made within the overall powersets considered in constructing $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$, the constraints of the freedom of interpretation are again set by the set-theoretic basis adopted for a semantics.

3.2 Formal Semantics and Choices for Entity Postulation

OBJECT AND METALANGUAGE, AND FORMAL SEMANTICS

In sect. 1.2.3.2, the distinction between object language and metalanguage as discussed by Tarski [810, p. 152–278], [803] is briefly mentioned. That distinction can be sketched in slightly more detail and in generalized form as follows. The main purpose of defining languages (relevant for this work) is to represent information about (arbitrary) domains of reality / knowledge[378], at least in the outmost number of cases.[379] Let L be a formal language for a domain of reality / knowledge D. Then L is said to be an *object language* for D, and the expressions of L form *object-level statements* wrt D. In order to define L, another language L' should be used, in the tradition of Tarski. L' is then called the *metalanguage* for L. Defining L in terms of L' comprises specifying the syntax and semantics of L. Tarski establishes precise lists of object and metalanguage within his case study of defining the (object) language of the calculus of classes by a respective metalanguage [805, p.168–169 for the object language, p. 169–174 for the metalanguage]. Nowadays, for logical languages, L' is usually a mixture of (mathematical) natural language, possibly some grammar specification formalism such as the Extended Backus-Naur Form (EBNF) [456], and set-theoretic notation. The function of set theory in this respect derives basically from the motivation of establishing *formal* semantics in general.[380] The aim is to provide a rigorous and mathematically precise description of how to interpret syntactic expressions. This is achieved by defining an interpretation function from syntactic expressions to semantic / set-theoretic constructions. The specification of an interpretation function usually exploits the grammatical categories / syntactic domains / non-terminals of the abstract syntax [878, p. 485].

DISTINGUISH SYNTACTIC AND SEMANTIC ASPECTS

Three aspects of the metalanguage as just described should be separated. The distinction orients itself at the division of syntax and semantics, which corresponds roughly to different types of entities involved, and includes their systematic relatedness / combination as the third, overarching aspect. The *syntactic "part"* of the metalanguage captures the definition of the syntax. Considering the kinds of entities required in this respect, it involves mainly *symbols / symbol structures / expressions* and corresponding relations, e.g. of composing expressions to new expressions. We shall not develop this aspect further herein. Much more relevant to our aim of establishing a semantic foundation for representing ontologies is the *semantic "part"* of the metalanguage. As just stated (and analyzed and criticized in ch. 2, esp. sect. 2.1–2.2), set-theoretic entities are usually employed,[381] possibly in combination with other entities (referred to as *urelements* from the set-theoretic perspective, cf. [212, p. 10], [200, p. 145]).

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[376]Formula (2.11) on p. 85 shows a simple form of comprehension axiom. More sophisticated / generic versions can be found in [222, p. 284], [223, sect. 3], and [445, sect. 4], for instance.

[377]If comprehension is weakened further, this connection ultimately leads to the "coincidence" of (modified) second-order semantics on the one hand and first-order, many-sorted semantics on the other hand. That case applies if no comprehension axioms are assumed at all. In that case, arbitrary sets of, e.g., relations can be chosen as the relation universe in an interpretation structure. Cf. [223, sect. 3].

[378]cf. sect. 1.1.2.2 on notions labeled 'domain'

[379]Accordingly, formal language theory, for instance, is out of scope, where languages are studied without an immediate connection of using the languages for representing something.

[380]Set theory is a typical representative of a mathematical theory applied in defining the semantics of languages formally. Other abstract entities may be employed for this end, as well, e.g., (mathematical) category theory (see [3, 52, 544]).

[381]The notion of set-theoretic entity cannot be strictly defined here. But it covers at least the membership relation ($\in$), all sets and (mathematical) classes, as well as further entities definable in (an adopted) set theory, for instance the subset relation ($\subseteq$).

Taking an ontological stance, the question arises which entities are accounted for by a formalized ontology (see sect. 1.1.2.2 for the term). First of all, we strongly defend the position that the answer must be determined at the side of the semantics of the language used for the formalization. In general, we presuppose a language L with a Tarski-style semantics, where, in particular, the syntax is mapped in terms of an interpretation function into a certain 'interpretation structure'[382]. For immediate intuitions let us restrict ourselves to an ontology formalized by a FOL theory $T \subseteq L$.

The bottom line for an answer to the question of ontological commitment just stated may comprise at least those entities whose existence can be expressly enforced within the theory itself, e.g. by the use (incl. entailments) of existential quantifiers or of logical individual constants.[383] Their selection appears reasonable, because they are guaranteed to exist in all models of the theory. However, if these were the only entities committed to, in the case of FOL (under classical semantics) that would *ex*clude all entities denoted by predicates. Another point of view is to include additionally all those entities that are denoted by any constants of the language (in the sense of all symbols with an "invariant evaluation" within an interpretation, cf. sect. 2.1.2; this would include those entities denoted by predicates), except for logical constant symbols like $\wedge$, $\vee$, etc. However, there "are" even more entities involved at the side of classical FOL semantics, namely those governed by the meta-theory T' (which is expressed in the metalanguage L'), i.e., the theory underpinning discussion and reasoning about the semantics of the theory T (as an expression / set of expressions / syntactic object). Then all those entities within the structure $\mathcal{A}^{\mathcal{P}^{\omega}_{\mathrm{fix}}}$ (cf. sect. 3.1) must be supposed to exist whose existence is enforced by T' (possibly incl. the overall membership relation in terms of which $\mathcal{A}^{\mathcal{P}^{\omega}_{\mathrm{fix}}}$ is "constructed" and by which those entities are internally related). Altogether and in terms of $\mathcal{A}^{\mathcal{P}^{\omega}_{\mathrm{fix}}}$ (arising from the initial set A, see sect. 3.1), the three options for entity postulation just discussed can be summarized as follows.[384]

1. the subset of A derived from logical individual constants of L and existential claims[385] entailed by T
2. the subset of $\mathcal{A}^{\mathcal{P}^{\omega}_{\mathrm{fix}}}$ derived from all non-logical constants of L and existential claims entailed by T
3. the subset of $\mathcal{A}^{\mathcal{P}^{\omega}_{\mathrm{fix}}}$ (possibly augmented by the membership relation) derived from all non-logical constants of L and existential claims entailed by T'

3.3 Theory View of Semantics

3.3.1 Introduction and Preview

The choice between the options listed at the end of the previous section and, maybe to an even greater extent, the third option itself are linked to another instructive consideration of classical models of a FOL theory, which we refer to as the *theory view* of (FOL) semantics. In terms of that view, the distinction between A and $\mathcal{A}^{\mathcal{P}^{\omega}_{\mathrm{fix}}}$ in sect. 3.1 becomes visible in a different manner. Moreover, the ontologically postulated entities

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[382]This term should not be linked with particular mathematical structures here, but rather accounts for anything in the rôle of being that into which the syntax is interpreted, i.e., anything that one may utilize in order to define a notion of satisfaction of a formula.

[383] This option could be divided further on the question of how to treat logical individual constants. However, we adopt a classical logic position here, in particular contrast to free logics [73, 517, 629]. The alternative of free logics is deliberately ignored thus far for mainly two reasons. The first is compatibility with classical logic, remembering the rationale of reusing existing results and technology, e.g., in sect. 1.2.2. The second reason is a distinguished position wrt the link between existence and denotation / reference compared to literature on both classical and free logic. We would suggest to aim at solving issues of so-called non-denoting terms by means of a classical logical approach (in this regard) combined with a very general notion of existence that encompasses claims of existence for fictitious entities, for instance, and distinguishes different modes of existence, cf. e.g. the respective analyses by Roman Ingarden [444], already referred to in sect. 1.1.5.1. However, it remains future work to elaborate on this issue. On suitable occasions further comments appear below. A quotation from [876] in [422] indicates an AI reading of this position: "An (AI-)ontology is a theory of what entities can exist in the mind of a knowledgeable agent." [422, p. 78].

[384]Note that this adopts the point of view of a single structure. The question for the status of the universe may be posed in this connection, but this complication is ignored here because the continuation of the overall issue in sect. 3.4 is independent of answering that question.

[385]In the case of several entities witnessing an existential claim, we include all of them, justifying the uniqueness of the subset of A. This applies to all options stated here.

according to the third option can be gained through adopting the first or second options only (depending on the treatment of the membership relation).

But what does 'theory view' actually mean here? In modern accounts of semantics (more exactly, of defining interpretations), set-theoretic assumptions and constructions are usually treated in terms of natural language statements with embedded precise formal expressions. The first step that leads to the theory view is to think of these assumptions and constructions in a fully formalized way. The satisfaction relation between an interpretation structure and a theory T (which propagates to the entailment relation, cf. sect. 1.5.1) can then be recast in terms of the following "ingredients":

1. an adopted set-theoretic formalization[386]
2. general assumptions reflecting the construction of interpretations (and structures)
3. a translation τ of the original theory T, i.e., a new theory $\tau(T)$

In one part, the term 'theory view' can be attributed to the third item. The other part originates from the fact that all three items form a joint theory that founds the (formal) semantic theory of the language of T, as well as (formal) semantic considerations regarding T itself.

Basically, we show next that classical set-theoretic semantics allows one to consider a theory T as a modified version of an "actually stated / meant" theory $\tau(T)$, in the sense that $\tau(T)$ formalizes the semantics of T in the context of an extension of set theory (with the main idea behind τ being to translate predication into set membership), altogether denoted by $\tau^+(T)$ below. We prove Prop. 3.15 and 3.17 that consistency and entailment are "equivalent" wrt T and $\tau^+(T)$, i.e., T is consistent iff $\tau^+(T)$ is consistent, and $T \models \phi$ iff $\tau^+(T) \models \tau(\phi)$. On this overall insight the above claim is held that the third option in the final part of sect. 3.2 can be identified with the second, to some extent.

As common as those connections between T and $\tau^+(T)$ may appear, following the impression caused by the quote involving "an obvious translation from the L_ω theory of $\mathcal{A}$ into a fragment of the ordinary first-order theory of $\mathcal{A}^+$" [838, p. 219], see the end of sect. 3.1.3, the remainder of this section 3.3 is concerned with the precise version of this summary ex ante. Not at least, this is due to requiring some technical "machinery".

3.3.2 Preliminaries and Basic Observations

For simplicity of exposition and readability, abbreviations for sequences are widely used, e.g. $\bar{P}$ for a sequence $P_1, \ldots, P_n$ (where the precise value of $n \in \mathbb{N}$ remains open). The length of a sequence is denoted by $|\cdot|$, e.g., $|\bar{P}| = n$ may be assumed or observed. If operations or indices are applied to a sequence abbreviation, these must be understood to be applied componentwise, e.g., $\bar{P}^{\mathcal{A}}$ stands for the sequence $P_1^{\mathcal{A}}, \ldots, P_n^{\mathcal{A}}$. Signatures are specified as sets of symbols, possibly in abbreviated form. The arity aspect of symbols is usually left implicit, but will become clear from the context or dedicated remarks. Unless otherwise noted, the term 'constant' is to be read as 'logical individual constant' until the end of ch. 3.

A special type of superscript indices $^{(\cdot)}$, called *level indices*, is employed to indicate the language level of symbols / entities. 0 refers to the object level, 1 to the meta level, 2 to the metameta level, etc. For instance, $P^{(0)}$ marks that P is a predicate symbol of the object-level language / syntax. There is *no* connection to the logical notion of order here, i.e., if HOL were regarded, $P^{(0)}$ could be a third-order predicate at the object level. Where symbols are unique / disambiguated without a level index, the use of the latter is merely informative and, in order to support readability, it is often limited to the introduction of the symbol and to highlighting its level. For symbols of this type, say, P and its indexed variant $P^{(0)}$, both variants have the very same referent (from the point of view of this text). There are some exceptions wrt notions that occur on each level, e.g. the symbol $\models$ and the term 'consistent', where the level index is used to

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[386]In many cases, such formalization is assumed at most implicitly, whereas actual work, e.g. proofs, are carried out wrt naïve set theory.

separate relationships of satisfaction or entailment[387] wrt the language levels, as well as different "versions" of consistency are tied to the different levels. Another exception refers to the membership relation $\in$ that occurs separately at levels 1–3. Level 1 further includes a predicate symbol $\dot\in$ for the formal theory of membership at level 1, where $\in^{(1)}$ and $\dot\in$ (and thus $\dot\in^{(1)}$) refer to the same metalevel relationship, despite their different rôles.

Regarding the distinction between 'consistency' and 'satisfiability', at each level we act on the assumption of a first-order setting that supports correctness and completeness theorems, cf. sect. 1.5.1 or e.g. [221, sect. 2.5]. Therefore the two notions are deemed equivalent and are used interchangeably (often giving preference to writing 'consistency', following some authors in AI / KR / DL, see e.g. [83, e.g. sect. 2]).

Another abbreviation used in this section is the predicate symbol ΣStr, intended to apply to a number of sets and possibly urelements iff they are related in such a way that they form a Σ structure, given a specific signature Σ.[388] Finally, note that everything that follows is formulated in the context of FOL restricted to predicates and constants, again for simplification.

CONDITIONS FOR SUBSEQUENT STATEMENTS

Now the coarse sketch above can be detailed. To start this, a number of symbols and preconditions must be established. The first step provides two languages based on two distinct signatures. $L_{\dot\in}$ serves as the language into which a theory in L can be translated via τ.

3.4 Condition (determining two languages L and $L_{\dot\in}$)
Let two FOL signatures be given and define the languages $L^{(0)}$ and $L^{(1)}_{\dot\in}$ on their basis as follows.

- $\Sigma^{(0)} := \left\{ \bar{P}, \bar{c} \right\}$ is a finite, non-empty FOL signature comprising a number of predicates $\bar{P}$ and a number of constants $\bar{c}$.
- $\Sigma^{(1)}_{\dot\in} := \left\{ \dot\in, \Sigma Str, {\cdot}U, {\cdot}\bar{P}, {\cdot}\bar{c} \right\}$ a FOL signature disjoint with $\Sigma^{(0)}$, comprising a binary predicate $\dot\in$, a predicate ΣStr, a constant ${\cdot}U$ and a number of constants ${\cdot}\bar{P}$ and ${\cdot}\bar{c}$, such that $|\bar{P}| = |{\cdot}\bar{P}|$ and $|\bar{c}| = |{\cdot}\bar{c}|$ and $ar(\Sigma Str) = |\bar{P}| + |\bar{c}| + 1$.
- $L^{(0)} := Lg(\Sigma)$.
- $L^{(1)}_{\dot\in} := Lg(\Sigma_{\dot\in})$ such that $Var(L) \subseteq Var(L_{\dot\in})$. ❑

The membership symbol $\dot\in^{(1)}$ is distinguished from $\in^{(1)}$, $\in^{(2)}$, and $\in^{(3)}$, where the former belongs to a particular FOL signature / syntax, while the latter three augment the mathematical textual descriptions of this text which are, a.o., about the former language and theories therein. For ease of presentation, besides leaving the arities of predicates in $\bar{P}$ implicit, similarly the encoding of tuples into sets is ignored / not decoded into pure set expressions. The arity of ΣStr is tied to $|\bar{P}|$ and $|\bar{c}|$, because that predicate needs to be applicable to all constants of $L^{(1)}_{\dot\in}$, a.o. cf. Obs. 3.8.

The next condition enforces the availability of a fully formalized set theory that consistently[389] accounts for the common notion of Σ structures and the existence of such.

3.5 Condition (set theory and definition of ΣStr)
Given Cond. 3.4, let $ST \subseteq Sen(Lg(\{\dot\in\}))$ be a FOL axiomatization of a set theory with the following properties, considered wrt $L_{\dot\in}$ that is derived from an arbitrary, but fixed finite signature Σ.

1. $ST^{(1)}$ is assumed to be consistent$^{(2)}$.

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[387] I.e., the usual symbol overloading of $\models$ applies, in addition, cf. sect. 1.5.1.

[388] Some qualifications apply to calling ΣStr a predicate. The symbol is used below in parameterized form that depends on an arbitrary finite signature Σ. In our overall account, this allows for addressing a family of predicates in the corresponding rôle as described. Nevertheless, in the theories that are discussed, ΣStr is a single predicate with unique arity itself. Any usage of the symbol below refers only to an arbitrary, but single finite signature of the object level language, which is fixed in advance, hence the symbol rests on neither variable arity nor does it require multiple interpretations.

[389] Which is to say, for the set theory itself, that one can at least plausibly assume its consistency, e.g., as in the case of ZFC. As discussed, e.g., in [212, sect. XI.1], ZFC and, more generally, all extensions of Z (see sect. A.2.1) cannot be proved to be consistent by any means up to the strength of ZFC itself [212, p. 179].

2. It allows for defining the notion of a mathematical structure wrt Σ on the basis of sets.[390] In connection with Cond. 3.4, that means there is a formula $\delta_{\Sigma Str}(x, \bar{y}, \bar{z}) \in Lg(\{\dot{\in}\})$ with $|\bar{y}| = |\bar{P}|$ and $|\bar{z}| = |\bar{c}|$ that characterizes which sets and urelements, if any, together form a mathematical structure for this fixed[391] signature Σ, such that $\Delta^{(1)}_{\Sigma Str} \subseteq L_{\dot{\in}}$ captures the notion via the predicate ΣStr:

$$\Delta_{\Sigma Str} := \{\forall x \bar{y} \bar{z} \, . \, \Sigma Str(x, \bar{y}, \bar{z}) \leftrightarrow \delta_{\Sigma Str}(x, \bar{y}, \bar{z})\} \ .$$

3. $ST \cup \Delta_{\Sigma Str} \models^{(2)} \exists x \bar{y} \bar{z} \, . \, \Sigma Str(x, \bar{y}, \bar{z}) \ .$ ❑

Clearly, the notion of Σ structure should accommodate the usual characterization by requiring the non-emptiness of the universe, predicate symbol interpretations that are (mathematical) relations over that universe, and by having elements of the universe as constant interpretations.

ZFC is an obvious example of a first-order axiomatization that satisfies Cond. 3.5. Moreover, it would be of interest to provide a stronger minimal characterization of theories that "reasonably" constitute a theory of sets. For instance, one may require the Axioms of Extensionality (Ext), of Small Union ($\cup$-Ax), and the Power Set Axiom (Pow), cf. A.2.1 in the Appendix. The background of Dana Scott's axiomatization [742] as discussed in [212, sect. X.3] yields alternative inspirations for such enterprise.

DEFINITION AND EXISTENCE OF ALIGNED OBJECT AND META LEVEL STRUCTURES

For the purposes of our main route, however, the formulation in Cond. 3.5 shall suffice and the rôle of ST becomes apparent in Cond. 3.6.[392] Moreover, a notion of alignment between structures based on $L^{(0)}$ and $L^{(1)}_{\dot{\in}}$ is useful, in preparation of Obs. 3.8.

3.6 Condition (ST as background set theory at all levels)

Based on Cond. 3.4–3.5, let ST be adopted as background set theory at all levels, i.e., at level $^{(1)}$ for $L^{(0)}$, at level $^{(2)}$ wrt $L^{(1)}_{\dot{\in}}$, as well as at level $^{(3)}$ and possibly beyond. ❑

3.7 Definition (aligned L and $L_{\dot{\in}}$ structures, and aligned variable assignments)

Assume Cond. 3.4, and let $\mathcal{A}^{(1)} = (U^{\mathcal{A}}, \bar{P}^{\mathcal{A}}, \bar{c}^{\mathcal{A}})$ a Σ structure, $\mathcal{V}^{(2)} = (V^{\mathcal{V}}, \dot{\in}^{\mathcal{V}}, \Sigma Str^{\mathcal{V}}, \cdot U^{\mathcal{V}}, \cdot \bar{P}^{\mathcal{V}}, \cdot \bar{c}^{\mathcal{V}})$ a $\Sigma_{\dot{\in}}$ structure, and $\beta : Var(L) \to U^{\mathcal{A}}$ and $\gamma : Var(L_{\dot{\in}}) \to V^{\mathcal{V}}$ corresponding variable assigments. Then

- $\mathcal{A}$ and $\mathcal{V}$ are *aligned* with each other   :iff $U^{\mathcal{A}} \subseteq V^{\mathcal{V}}$, $\cdot U^{\mathcal{V}} = U^{\mathcal{A}}$, $\cdot \bar{P}^{\mathcal{V}} = \bar{P}^{\mathcal{A}}$, and $\cdot \bar{c}^{\mathcal{V}} = \bar{c}^{\mathcal{A}}$.
- β and γ are *aligned* with each other   :iff   they agree on $Var(L)$, $\gamma|_{Var(L)} = \beta$. ❑

3.8 Observation (existence of structures at object and meta level)

Assuming Cond. 3.4–3.6, then the following statements are equivalent[3] and all true[3].

1. There is a Σ structure $\mathcal{A}^{(1)} = (U^{\mathcal{A}}, \bar{P}^{\mathcal{A}}, \bar{c}^{\mathcal{A}})$.
2. $ST \cup \Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \cdot \bar{P}, \cdot \bar{c})\}^{(1)}$ is consistent[2].
3. There is a $\Sigma_{\dot{\in}}$ structure $\mathcal{V}^{(2)}$ such that $\mathcal{V} \models^{(2)} ST \cup \Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \cdot \bar{P}, \cdot \bar{c})\}^{(1)}$. ❑

Truth and equivalence of all statements are immediate from definitions and the three assumptions, with a central rôle of Cond. 3.5, and especially requirement 3 therein. Basically, the same fact is described in the first and second statement, once in terms of the standard semi-formal style of meta-level description, and then in a fully formalized way as regards the first meta level, employing the notion of consistency at the second meta level. The latter corresponds to the third statement by definition (and the general assumption on the equivalence of consistency and satisfiability at all language levels). Concerning the seeming, but not

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[390]The requisites on Σ structures are repeated from sect. 1.5.2 below this condition, see also standard expositions such as [214, sect. III.1, esp. Def. III.1.1] and [693, sect. 2.1].

[391]The repetition of "fixed" is there to stress the fact that the Σ in the phrase 'mathematical structure wrt Σ', in it's shorter version 'Σ structure', as well as in the predicate symbol ΣStr is a proper parameter that should be viewed to have a *single*, specific value assigned to it, namely as assumed by Cond. 3.4. Accordingly, there is no issue of possibly varying arities in the definition of Σ structure in $\Delta_{\Sigma Str}$, which could only arise between *multiple distinct* signatures.

[392]The adoption of ST at all levels is discussed further with Obs. 3.16 on p. 116.

necessary linkage between the two structures $\mathcal{A}$ and $\mathcal{V}$ regarding $U^{\mathcal{A}}$, $\bar{P}^{\mathcal{A}}$, and $\bar{c}^{\mathcal{A}}$, the following observation involving actual alignment relationships (see Def. 3.7) can be added.

3.9 Observation (existence of aligned structures)
Let Cond. 3.4–3.6 and Def. 3.7 be given.

1. For every Σ structure $\mathcal{A}^{(1)}$ there is a $\Sigma_{\dot{\in}}$ structure $\mathcal{V}^{(2)}$ such that $\mathcal{A}$ and $\mathcal{V}$ are aligned with each other and $\mathcal{V} \models^{(2)} ST \cup \Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \cdot \bar{P}, \cdot \bar{c})\}^{(1)}$.
2. For every $\Sigma_{\dot{\in}}$ structure $\mathcal{V}^{(2)}$ such that $\mathcal{V} \models^{(2)} ST \cup \Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \cdot \bar{P}, \cdot \bar{c})\}^{(1)}$,
   there is a Σ structure $\mathcal{A}^{(1)}$ that is aligned with $\mathcal{V}$. ❑

The first statement in this observation is justified by the idea that every meta-level $^{(1)}$ structure $\mathcal{A}$, from the point of view of the metameta level $^{(2)}$ and a set theory formalized at level $^{(1)}$, resides within a model $\mathcal{V}$ of that set theory that comprises all entities in $\mathcal{A}$, i.e., all components of $\mathcal{A}$ and all members of $U^{\mathcal{A}}$. Accordingly, alignment can be established between these two. The second statement claims that a Σ structure can be "extracted" from any model $\mathcal{V}$ of $ST \cup \Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \cdot \bar{P}, \cdot \bar{c})\}^{(1)}$. This is a kind of transcription of that theory (at level $^{(1)}$) into a structure (at level $^{(1)}$, as well) that is suitable for interpreting object-level syntax. The defining equalities for aligned L and $L_{\dot{\in}}$ structures can be taken as the defining rules of this transcription.

The next observation is straightforward due to $Var(L) \subseteq Var(L_{\dot{\in}})$ in Cond. 3.4, $U^{\mathcal{A}} \subseteq V^{\mathcal{V}}$ if $\mathcal{A}$ and $\mathcal{V}$ are aligned, and under the presupposition that ST allows for extending and restricting relations.

3.10 Observation (existence of aligned variable assignments wrt aligned structures)
Assume Cond. 3.4 and Def. 3.7, and let Σ structure $\mathcal{A}^{(1)} = (U^{\mathcal{A}}, \bar{P}^{\mathcal{A}}, \bar{c}^{\mathcal{A}})$ and $\Sigma_{\dot{\in}}$ structure $\mathcal{V}^{(2)} = (V^{\mathcal{V}}, \dot{\in}^{\mathcal{V}}, \Sigma Str^{\mathcal{V}}, \cdot U^{\mathcal{V}}, \cdot \bar{P}^{\mathcal{V}}, \cdot \bar{c}^{\mathcal{V}})$ be aligned with each other.

- For every variable assignment $\beta : Var(L) \to U^{\mathcal{A}}$ there is a variable assignment $\gamma : Var(L_{\dot{\in}}) \to V^{\mathcal{V}}$ such that $\gamma|_{Var(L)} = \beta$, i.e., β and γ are aligned.
- There is a variable assignment $\gamma : Var(L_{\dot{\in}}) \to V^{\mathcal{V}}$ such that $\beta := \gamma|_{Var(L)}$ is a (total) function $\beta : Var(L) \to U^{\mathcal{A}}$, thus being a variable assignment for L which is aligned with γ. ❑

3.3.3 Object- to Meta-Level Translation

DEFINING A TRANSLATION FROM OBJECT- TO META-LEVEL SYNTAX
Obs. 3.8–3.10 pave the way towards the theory view, but next the actual translation from an object level theory into a formalized semantic theory must be defined. This is nothing else than a formal rewriting of standard definitions of Tarski-style satisfaction conditions for structures and formulas. For simplicity, we restrict the definition to a minimal set of logical connectives and quantifiers in terms of which other classical representatives can be defined, in particular those introduced in sect. 1.5.2. Based on sect. 1.5.1, we recall that notation and operations applied to sequences and tuples are implicitly assumed to be applied componentwise by default, unless anything else is stated. For example, for a tuple $\bar{x} = (x_1, \ldots, x_n)$, $n \in \mathbb{N}$, $\tau(\bar{x}) := (\tau(x_1), \ldots, \tau(x_n))$, and $\cdot \bar{x} := (\cdot x_1, \ldots, \cdot x_n)$. Further, $Tm(L) = Const(L) \,\dot{\cup}\, Var(L)$.

3.11 Definition (translation τ of object-level to meta-level syntax wrt set-theoretic semantics)
Based on Cond. 3.4 and setting $\mathcal{E} := \Sigma \,\dot{\cup}\, Var(L) \,\dot{\cup}\, L$, the translation function $\tau : \mathcal{E} \to \mathcal{E}$ is defined by[393]

1. $\tau(x) := x$ for every variable $x \in Var(L)$
2. $\tau(c) := \cdot c$ for every constant $c \in Const(L)$
3. $\tau(P) := \cdot P$ for every predicate $P \in Pred(L)$

for every $\phi \in L$
4. if $\phi = {}'x = y{}'$, $\tau(\phi) := {}'\tau(x) = \tau(y){}'$ for any $x, y \in Tm(L)$

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[393]Note that $\tau = \tau_{\Sigma} \,\dot{\cup}\, \tau_{Var} \,\dot{\cup}\, \tau_L$ is partitioned into $\tau_{\Sigma} : \Sigma \to \Sigma_{\dot{\in}}$, $\tau_{Var} : Var(L) \to Var(L)$, and $\tau_L : L \to L_{\dot{\in}}$. Using a joint function name eases presentation, while τ_L is the target of this definition, whereas τ_{Σ} and τ_{Var} are more of auxiliary nature.

5. if $\phi = P(\bar{x})$, $\tau(\phi) := \tau(\bar{x}) \mathbin{\dot{\in}} \tau(P)$ for any $P \in Pred(L)$ and $\bar{x} \in (Tm(L))^{ar(P)}$
6. if $\phi = \neg\psi$, $\tau(\phi) := \neg\tau(\psi)$
7. if $\phi = \psi \wedge \psi'$, $\tau(\phi) := \tau(\psi) \wedge \tau(\psi')$
8. if $\phi = \exists x \,.\, \psi$, $\tau(\phi) := \exists\tau(x) \,.\, \tau(x) \mathbin{\dot{\in}} \cdot U \wedge \tau(\psi)$

τ is naturally extended to sets of formulas: $\tau(T) := \{\tau(\phi) \mid \phi \in T\}$ for any $T \subseteq L$.   ❑

Firstly, all conditions involving variables, namely items 1, 4, 5, and 8, yield immediate consequences.

3.12 Observation (variable stability of τ)

Def. 3.11 entails $\tau|_{Var(L)} = id_{Var(L)}$ and $Var(\phi) = Var(\tau(\phi))$ for any $\phi \in L$.   ❑

CHARACTERIZING THE THEORY VIEW OF CLASSICAL SEMANTICS BASED ON SET THEORY

In order to establish those connections between an L-theory T and its translation that are at the heart of the theory view of semantics, a small extension of $\tau(T)$ based on Cond. 3.5–3.6 is required, before first connections can be stated in Prop. 3.14.

3.13 Definition (augmented translation τ^+)

Assume Cond. 3.5–3.6 and let τ be defined according to Def. 3.11.
The set-theoretic augmentation function $\alpha : \mathcal{P}(L) \to \mathcal{P}(L_{\dot{\in}})$ is defined for every $T \subseteq L$ by

$$\alpha(T) := ST \cup \Delta_{\Sigma Str} \cup \big\{\Sigma Str(\cdot U, \cdot\bar{P}, \cdot\bar{c})\big\} \cup \{x \mathbin{\dot{\in}} \cdot U \mid x \in fVar(T)\} \ .$$

The augmented translation function $\tau^+ : \mathcal{P}(L) \to \mathcal{P}(L_{\dot{\in}})$ is defined for every $T \subseteq L$ by

$$\tau^+(T) := \alpha(T) \cup \tau(T) \ . \hspace{2em} ❑$$

3.14 Proposition (theory view equivalence of formula satisfaction)

Given Cond. 3.5–3.6 and Def. 3.7, 3.11, and 3.13, let $T^{(0)} \subseteq L$ and $\mathcal{A}^{(1)} = (U^{\mathcal{A}}, \bar{P}^{\mathcal{A}}, \bar{c}^{\mathcal{A}})$ a Σ structure.
Let $\mathcal{V}^{(2)} = (V^{\mathcal{V}}, \dot{\in}^{\mathcal{V}}, \Sigma Str^{\mathcal{V}}, \cdot U^{\mathcal{V}}, \cdot\bar{P}^{\mathcal{V}}, \cdot\bar{c}^{\mathcal{V}})$ $\Sigma_{\dot{\in}}$ structure s.t. $\mathcal{V}$ is aligned with $\mathcal{A}$ and satisfies the cond.
(*): $\mathcal{V} \models^{(2)} ST \cup \Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \cdot\bar{P}, \cdot\bar{c})\}^{(1)}$.
Moreover, let $\beta : Var(L) \to U^{\mathcal{A}}$ and $\gamma : Var(L_{\dot{\in}}) \to V^{\mathcal{V}}$ aligned variable assignments, i.e., $\gamma|_{Var(L)} = \beta$.

$$\text{Then:} \quad \langle \mathcal{A}, \beta \rangle \models^{(1)} T \quad \text{iff} \quad \langle \mathcal{V}, \gamma \rangle \models^{(2)} \tau(T) \ . \hspace{2em} ❑$$

Proof (structural induction). We show $\langle \mathcal{A}, \beta \rangle \models \phi$ iff $\langle \mathcal{V}, \gamma \rangle \models \tau(\phi)$ for any formula $\phi \in L$. On this basis the equivalence for T and $\tau(T)$ is immediate from the definition of $\tau(T)$ in Def. 3.11.

For better readability due to less complex indices, we write $\mathcal{A}$ also for $\langle \mathcal{A}, \beta \rangle$ and $\mathcal{V}$ also for $\langle \mathcal{V}, \gamma \rangle$ within this proof. Observe first that (**): $\tau(x)^{\mathcal{V}} = x^{\mathcal{A}}$ for all $x \in Tm(L)$, as follows. A term $x \in Tm(L)$ can be (i) a constant or (ii) a variable. For (i), if $x \in Const(L)$, then $\tau(x)^{\mathcal{V}} = \cdot x^{\mathcal{V}} = x^{\mathcal{A}}$ by definition of τ and by $\mathcal{A}$ and $\mathcal{V}$ being aligned. In case (ii), for $x \in Var(L)$ we have $\tau(x)^{\mathcal{V}} = x^{\mathcal{V}} = \gamma(x) = \beta(x) = x^{\mathcal{A}}$, due to the definition of τ and β and γ being aligned.

The induction starts with atomic formulas.

- Let $\phi = {}'x =^{(0)} y'$. $\mathcal{A} \models^{(1)} x =^{(0)} y$ iff(1) $x^{\mathcal{A}} =^{(1)} y^{\mathcal{A}}$ iff(2) $\tau(x)^{\mathcal{V}} =^{(2)} \tau(y)^{\mathcal{V}}$ iff(3) $\mathcal{V} \models^{(2)} \tau(x) =^{(1)} \tau(y)$, where (2) is due to (**) since $x, y \in Tm(L)$ and (1, 3) follow from interpreting $=$ by identity on all levels, i.e., $(=^{(0)})^{\mathcal{A}} = id_{U^{\mathcal{A}}}$ and $(=^{(1)})^{\mathcal{V}} = id_{U^{\mathcal{V}}}$, resp.
- For $\phi = P(\bar{x})$, where P is from the $\bar{P}$ but not '$=$', we must show $\mathcal{A} \models^{(1)} P(\bar{x})$ iff $\mathcal{V} \models^{(2)} \tau(P(\bar{x}))$. Starting at the right-hand side, the following chain of equivalences applies:
  $\mathcal{V} \models^{(2)} \tau(P(\bar{x}))$ iff(1) $\mathcal{V} \models^{(2)} \tau(\bar{x}) \mathbin{\dot{\in}} \tau(P)$ iff(2) $\mathcal{V} \models^{(2)} \tau(\bar{x}) \mathbin{\dot{\in}} \cdot P$ iff(3)
  $(\tau(\bar{x})^{\mathcal{V}}, \cdot P^{\mathcal{V}}) \in^{(2)} \dot{\in}^{\mathcal{V}}$ iff(4) $\tau(\bar{x})^{\mathcal{V}} \in^{(1)} \cdot P^{\mathcal{V}}$ iff(5) $\tau(\bar{x})^{\mathcal{V}} \in^{(1)} P^{\mathcal{A}}$ iff(6) $\bar{x}^{\mathcal{A}} \in^{(1)} P^{\mathcal{A}}$
  iff(7) $\mathcal{A} \models^{(1)} P(\bar{x})$.
  Steps (1, 2) apply the Def. 3.11 of τ, (3) that of $\models$. Step (4) relies on Cond. 3.6 and on (*), (5) on the precondition that $\mathcal{A}$ and $\mathcal{V}$ are aligned. Step (6) requires showing $\tau(\bar{x})^{\mathcal{V}} = \bar{x}^{\mathcal{A}}$, which means to prove $\tau(x_i)^{\mathcal{V}} = x_i^{\mathcal{A}}$ for all x_i in $\bar{x}$. This is immediate from (**). Step (7) applies the definition of $\models$. This concludes the base case.

The following cases form the concluding induction step.

- Let $\phi = \neg\psi$. $\mathcal{A} \models^{(1)} \phi$ iff $\mathcal{A} \not\models^{(1)} \psi$ iff, by the induction hypothesis, $\mathcal{V} \not\models^{(2)} \tau(\psi)$ iff $\mathcal{V} \models^{(2)} \neg\tau(\psi)$ iff $\mathcal{V} \models^{(2)} \tau(\phi)$ by the definition of τ.
- Let $\phi = \psi_1 \wedge \psi_2$. $\mathcal{A} \models^{(1)} \phi$ iff $\mathcal{A} \models^{(1)} \psi_1$ and $\mathcal{A} \models^{(1)} \psi_2$ iff, by the induction hypothesis, $\mathcal{V} \models^{(2)} \tau(\psi_1)$ and $\mathcal{V} \models^{(2)} \tau(\psi_2)$ iff $\mathcal{V} \models^{(2)} \tau(\psi_1) \wedge \tau(\psi_2)$ iff $\mathcal{V} \models^{(2)} \tau(\phi)$.
- Let $\phi = \exists x \,.\, \psi$. $\mathcal{A} \models^{(1)} \phi$ iff (***) there is an $a \in^{(1)} U^{\mathcal{A}}$ s.t. $\langle \mathcal{A}, \beta_a^x \rangle \models \psi$. Since β_a^x and γ_a^x are aligned if β and γ are, and because $\mathcal{A}$ and $\mathcal{V}$ are aligned s.t. esp. $\cdot U^{\mathcal{V}} = U^{\mathcal{A}} \subseteq V^{\mathcal{V}}$ (and relying on precondition (*)), the induction hypothesis can be applied wrt $\langle \mathcal{A}, \beta_a^x \rangle$ and $\langle \mathcal{V}, \gamma_a^x \rangle$, i.e., (***) is equivalent with: there is an $a \in^{(2)} V^{\mathcal{V}}$ with $a \in^{(2)} \cdot U^{\mathcal{V}}$ and s.t. $\langle \mathcal{V}, \gamma_a^x \rangle \models^{(2)} \tau(\psi)$ iff $\mathcal{V} \models^{(2)} \exists x \,.\, x \,\dot{\in}\, \cdot U \wedge \tau(\psi)$ iff (by $\tau(x) = x$ for variables $x \in Var(L)$) $\mathcal{V} \models^{(2)} \tau(\phi)$. $\quad\Box$

Another connection between T and $\tau^+(T)$ results immediately from Prop. 3.14.

3.15 Proposition (theory view characterization of consistency)
Assuming Cond. 3.5–3.6 and Def. 3.7, 3.11, and 3.13, let $T \subseteq L$ and $\tau^+(T)$ its meta-level translation.

$$T \text{ is consistent}^{(1)} \quad \text{iff} \quad \tau^+(T) \text{ is consistent}^{(2)}. \qquad\qquad \Box$$

Proof. $\Rightarrow$) Let T be consistent, thus there is a model $\langle \mathcal{A}, \beta \rangle \models T$. Due to Obs. 3.9 and 3.10 there is an interpretation $\langle \mathcal{V}, \gamma \rangle$ s.t. $\mathcal{V}$ is aligned with $\mathcal{A}$, (1) $\mathcal{V} \models^{(2)} ST \cup \Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \bar{P}, \bar{c})\}^{(1)}$, and γ is aligned with β. By Prop. 3.14 we get (2) $\langle \mathcal{V}, \gamma \rangle \models^{(2)} \tau(T)$ from $\langle \mathcal{A}, \beta \rangle \models T$. $\beta(x) \in^{(1)} U^{\mathcal{A}}$ for all variables $x \in Var(L)$ and for all free variables, in particular, which together with Cond. 3.6 and $U^{\mathcal{A}} = \cdot U^{\mathcal{V}}$ due to the alignment of $\mathcal{A}$ and $\mathcal{V}$ entails (3) $\langle \mathcal{V}, \gamma \rangle \models^{(2)} \{x \,\dot{\in}\, \cdot U \mid x \in fVar(T)\}$. (1–3) in combination yield $\langle \mathcal{V}, \gamma \rangle \models \tau^+(T)$, verifying the consistency of $\tau^+(T)$.

$\Leftarrow$) Let $\tau^+(T)$ be consistent, then there is a model $\langle \mathcal{V}, \gamma \rangle \models^{(2)} \tau^+(T)$. Due to $ST \cup \Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \bar{P}, \bar{c})\}^{(1)} \subseteq \tau^+(T)$ and by Obs. 3.9, for $\mathcal{V}$ there is an aligned Σ structure $\mathcal{A}$ with universe $U^{\mathcal{A}} = \cdot U^{\mathcal{V}}$. A variable assignment β for L remains to be constructed. By Obs. 3.10, there is a variable assignment $\gamma' : L_{\dot{\in}} \to V^{\mathcal{V}}$ such that $\beta' := \gamma'|_{Var(L)}$ is a variable assignment for L. Then define

$$\gamma''(x) := \begin{cases} \gamma(x) & \text{if } x \notin Var(L) \\ \gamma(x) & \text{if } x \in Var(L) \text{ and } \gamma(x) \in U^{\mathcal{A}} \\ \gamma'(x) & \text{otherwise} \end{cases} \qquad \text{and set } \beta := \gamma''|_{Var(L)}.$$

Thereby γ is modified to achieve that β is an assignment for L wrt $U^{\mathcal{A}}$ (note $\beta = \gamma|_{Var(L)}$ if the range of that restriction is a subset of $U^{\mathcal{A}}$.) From $\langle \mathcal{V}, \gamma \rangle \models^{(2)} \tau^+(T)$ we conclude $\langle \mathcal{V}, \gamma'' \rangle \models^{(2)} \tau^+(T)$, for the following reasons. First of all, by the coincidence theorem for FOL, see e.g. [693, Theorem 3.1, p. 65], only free variables could lead to a change wrt satisfaction. Hence, for all sentences $\phi \in L$ we have $\langle \mathcal{V}, \gamma \rangle \models^{(2)} \phi$ iff $\langle \mathcal{V}, \gamma'' \rangle \models^{(2)} \phi$ due to the general argument of a sentence being either satisfied or not by $\mathcal{V}$, equally with any variable assignment. This, in particular, covers all sentences in $\tau(T)$ as well as the component(s) $ST \cup \Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \bar{P}, \bar{c})\}^{(1)}$ of $\tau^+(T)$, because they are theories / sets of sentences. It remains to consider formulas with free variables. For all free variables in $\tau(T)$ we have $\langle \mathcal{V}, \gamma \rangle \models^{(2)} \{x \,\dot{\in}\, \cdot U \mid x \in fVar(\tau(T))\}$ and thus $\gamma(x) \in U^{\mathcal{A}}$ due to Cond. 3.6 and alignment between $\mathcal{V}$ and $\mathcal{A}$, which further yields $\gamma''(x) = \gamma(x)$ for all $x \in fVar(\tau(T)) = fVar(T)$ with Obs. 3.12. From that follows at once $\langle \mathcal{V}, \gamma'' \rangle \models^{(2)} \{x \,\dot{\in}\, \cdot U \mid x \in fVar(T)\}$ and $\langle \mathcal{V}, \gamma'' \rangle \models^{(2)} \psi$ for any formulas $\psi \in \tau(T)$ with free variables, and in conclusion $\langle \mathcal{V}, \gamma'' \rangle \models^{(2)} \tau^+(T)$.

With $\langle \mathcal{V}, \gamma'' \rangle$ available, where $\mathcal{V}$ is aligned with $\mathcal{A}$ and satisfies $ST \cup \Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \bar{P}, \bar{c})\}^{(1)}$, and γ'' is aligned with β, Prop. 3.14 can be utilized to prove $\langle \mathcal{A}, \beta \rangle \models T$, witnessing the consistency of T. $\Box$

POTENTIAL CONFLICTS IN CASE OF INCOMPATIBLE SET THEORIES

With Prop. 3.15 available, we can further illuminate the relevance of Cond. 3.6 on accepting the same (or at least compatible) set theories as background set theories at different levels. The interplay of consistency of $\tau(T)$ and $\tau^+(T)$ is of interest in this regard.

3.16 Observation (interaction of ST and ST' wrt $\tau(T)$ and $\tau^+(T)$)
Assume Cond. 3.5 and Def. 3.7, 3.11, and 3.13. Instead of Cond. 3.6 require only that ST is the background theory for $L^{(0)}$, but allow for a set theory ST' as background theory for level $^{(1)}$ that is inconsistent$^{(3)}$ with

ST. Then it is possible that $\tau^+(T)$ is inconsistent$^{(2)}$ and T is inconsistent$^{(1)}$, albeit $\tau(T)$ is consistent$^{(2)}$.□

Proof. For an example, let $ST = \{\forall xy \, . \, x \neq y \rightarrow \neg\exists z(z \neq x \wedge z \neq y \wedge \forall u(u \; \dot\in \; z \leftrightarrow u \; \dot\in \; x \wedge u \; \dot\in \; y))\}$ be a set theory that prevents the existence of exact intersections for all pairs of distinct sets neither of which is a subset of the other.[394] ST is itself on level $^{(1)}$ and the background theory for $L^{(0)}$. As background set theory for level $^{(1)}$ assume ZFC, hence the union of ST and ZFC as theories in a single level $^{(n)}$ are inconsistent$^{(n+1)}$.

Moreover, let a theory $T \subseteq L$ comprise the following formulas, where P, P', P'' are predicate symbols, a, b, c constants.

(3.3) $\forall x \, . \, P(x) \; \leftrightarrow x = a$

(3.4) $\forall x \, . \, P'(x) \leftrightarrow x = a \vee x = b$

(3.5) $\forall x \, . \, P''(x) \leftrightarrow x = a \vee x = c$

(3.6) $a \neq b \wedge a \neq c \wedge b \neq c$

This entails $\forall x \, . \, P(x) \leftrightarrow P'(x) \wedge P''(x)$, which must be unsatisfiable$^{(1)}$, given that ST excludes the existence of an intersection set of two distinct sets, unless the intersection is one of them already. Accordingly, T is inconsistent$^{(1)}$.

In the formalization at the meta level, i.e., regarding $\tau(T)$ and $\tau^+(T)$ this displays as inconsistency$^{(2)}$ of $\tau^+(T)$ despite of the consistency$^{(2)}$ of $\tau(T)$ as follows, where we start by specifying $\tau(T)$.

(3.7) $\forall x \, . \, x \; \dot\in \; \cdot U \; \rightarrow \; (x \; \dot\in \; \cdot P \leftrightarrow x = \cdot a)$

(3.8) $\forall x \, . \, x \; \dot\in \; \cdot U \; \rightarrow \; (x \; \dot\in \; \cdot P' \leftrightarrow x = \cdot a \vee x = \cdot b)$

(3.9) $\forall x \, . \, x \; \dot\in \; \cdot U \; \rightarrow (x \; \dot\in \; \cdot P'' \leftrightarrow x = \cdot a \vee x = \cdot c)$

(3.10) $\cdot a \neq \cdot b \wedge \cdot a \neq \cdot c \wedge \cdot b \neq \cdot c$

$\tau(T)$ is consistent$^{(2)}$ with ZFC in the background, where one can specify a model $\mathcal{V}^{(2)}$ for it, e.g. based on the universe $V = \{\cdot a, \cdot b, \cdot c, \cdot P, \cdot P', \cdot P'', \cdot U\}$ and assigning pairs from $V \times V$ to the interpretation of $\dot\in$ s.t. every formula is satisfied. However, a model of $\tau^+(T)$ cannot be given, as long as ΣStr is defined in $\Delta_{\Sigma Str}$ with minimal requirements as usual (essentially, non-emptiness of the universe, together with interpretations of unary predicate symbols (as is sufficient for the present case) being subsets of the universe).

To see this, assume a model $\mathcal{V}'^{(2)} \models \tau^+(T)$. From (3.7)–(3.9) follows $\mathcal{V}' \models \forall x \, . \, x \; \dot\in \; \cdot U \; \rightarrow (x \; \dot\in \; \cdot P \leftrightarrow x \; \dot\in \; \cdot P' \wedge x \; \dot\in \; \cdot P'')$. Assuming $\forall x \, . \, x \; \dot\in \; \mathbf{X} \rightarrow x \; \dot\in \; \cdot U$ to follow from $\Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \cdot P, \cdot P', \cdot P'', \cdot a, \cdot b, \cdot c)\}$ for $\mathbf{X} \in \{\cdot P, \cdot P', \cdot P''\}$, mainly propositional reasoning justifies all of the following to hold in $\mathcal{V}'$.

(3.11) $\forall x \, . \, x \; \dot\in \; \cdot P \; \leftrightarrow x \; \dot\in \; \cdot U \wedge x = \cdot a$

(3.12) $\forall x \, . \, x \; \dot\in \; \cdot P' \leftrightarrow x \; \dot\in \; \cdot U \wedge (x = \cdot a \vee x = \cdot b)$

(3.13) $\forall x \, . \, x \; \dot\in \; \cdot P'' \leftrightarrow x \; \dot\in \; \cdot U \wedge (x = \cdot a \vee x = \cdot c)$

(3.14) $\forall x \, . \, x \; \dot\in \; \cdot P \; \leftrightarrow x \; \dot\in \; \cdot U \wedge x \; \dot\in \; \cdot P' \wedge x \; \dot\in \; \cdot P''$

(3.15) $\forall x \, . \, x \; \dot\in \; \cdot P \; \leftrightarrow x \; \dot\in \; \cdot P' \wedge x \; \dot\in \; \cdot P''$

Moreover, assuming identity for any pair over $\{\cdot P, \cdot P', \cdot P''\}$ leads to $\mathcal{V}' \models \neg\exists x \, . \, x \; \dot\in \; \cdot U$ due to (3.10), e.g., suppose $\cdot P = \cdot P'$, then $\mathcal{V}' \models \forall x \, . \, x \; \dot\in \; \cdot U \rightarrow (x = \cdot a \leftrightarrow (x = \cdot a \vee x = \cdot b))$. However, emptiness of $\cdot U$ contradicts the standard notion of Σ structure, i.e. here, $\Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \cdot P, \cdot P', \cdot P'', \cdot a, \cdot b, \cdot c)\}$. Hence,

(3.16) $\cdot P \neq \cdot P' \wedge \cdot P \neq \cdot P'' \wedge \cdot P' \neq \cdot P''$

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[394]Of course, this is a highly artificial example, merely for the sake of this proof. More substantial minimal requirements for set theories, as touched upon above, after Cond. 3.5, may therefore invalidate this specific example. Further, the restriction to $x \neq y$ prevents the axiom from being inconsistent itself, whereas $x \neq z \wedge y \neq z$ allows for having sets in subset relation in that set theory, which would be excluded through the cases $z = x$ and $z = y$. Sets in subset relation are necessary for the definition of ΣStr.

must be satisfied in $\mathcal{V}'$. But (3.16) together with ST and (3.11)–(3.15) yields a contradiction, as well. ❑

The example in the proof with no structure $\mathcal{A}^{(1)}$ s.t. $\mathcal{A} \models T$, but a structure $\mathcal{V}^{(2)}$ with $\mathcal{V}^{(2)} \models \tau(T)$, demonstrates the relevance of Cond. 3.6[395] in connection with aligned structures and Prop. 3.14. Note that ZFC is a mere (and known) sample instance for a set theory in conflict with ST.

3.17 Proposition (theory view characterization of entailment)
Assume Cond. 3.5–3.6 and Def. 3.7, 3.11, and 3.13. Entailment wrt theories $T \subseteq L$ and $\tau^+(T) \subseteq L_{\dot\in}$ is equivalent:
$$T \models^{(1)} \phi \quad \text{iff} \quad \tau^+(T) \models^{(2)} \tau(\phi) \quad \text{for every } \phi \in L.$$ ❑

Proof. By the definitions of satisfaction and (semantic) inconsistency, $T \models^{(1)} \phi$ iff $T \cup \{\neg\phi\}$ is inconsistent$^{(1)}$. Prop. 3.15 entails equivalence to the inconsistency$^{(2)}$ of $\tau^+(T \cup \{\neg\phi\})$. This, in turn, is equivalent to inconsistency$^{(2)}$ of $\tau^+(T) \cup \{\neg\tau(\phi)\}$ by the Def. 3.11 of τ for theories and formulas (esp. negations). Analogously to the very first step (reversed), the equivalence to $\tau^+(T) \models^{(2)} \tau(\phi)$ is established. ❑

3.3.4 Semantics as Theory Interpretation

Prop. 3.17 is illuminating if we set this in relation to the notion of theory interpretations. The latter was already met in sect. 2.2 in the context of the established definition of ontology-based semantic integration (Def. 2.3; see sect. 2.2.2 as a whole). In brief, for two FOL theories T and T', a translation function $\alpha : Lg(T) \to Lg(T')$ is a (faithful) theory interpretation :iff the theorems of T are preserved under α as theorems of T' (plus, for faithfulness, vice versa), i.e., $T \models \phi$ implies (iff, for faithfulness) $T' \models \alpha(\phi)$ for every $\phi \in Lg(T)$, cf. Def. A.4 in sect. A.1.2 of the Appendix.

3.18 Proposition (τ yields theory interpretations)
Let Cond. 3.5–3.6 apply and Def. 3.7, 3.11, and 3.13 be given. For every language L and theory $T \subseteq L$, the function $\tau : L \to L_{\dot\in}$ is a faithful theory interpretation of T into $\tau^+(T)$. ❑

Proof. We prove that τ is a faithful first-order theory interpretation in the sense of Enderton, see Def. A.9 in A.1.2 or [221, sect. 2.7, p. 154–163]. Let L a language and $T \subseteq L$ an arbitrary theory, under Cond. 3.5–3.6 and given Def. 3.7, 3.11, and 3.13. Observe first that Prop. 3.17 establishes the preservation of theoremhood between T and $\tau^+(T)$ by means of τ. It remains to be shown that τ is a formula interpretation translation, thus based on a signature interpretation translation (see Def. A.8 for both notions). The defining items for τ regarding non-atomic formulas in its Def. 3.11 agree literally with the definitions concerning non-atomic formulas in Def. A.8, i.e., $\alpha(\phi) = \tau(\phi)$ for all non-atomic formulas ϕ, provided the same equation applies to atomic formulas. Therefore, it suffices to specify a signature interpretation translation $\pi : Sig(L) \cup \{\forall\} \to L_{\dot\in}$, derived from Def. 3.11, that allows for understanding $\tau(\phi)$ for *atomic* formulas ϕ as the corresponding case of atomic formulas in a formula interpretation translation $\alpha : L \to L_{\dot\in}$.

For this purpose and for every predicate $P \in Pred(L)$ (and $=$ and $\forall$) and constant $c \in Const(L)$, define π as follows.[396]

(3.17) $\pi(\forall) := x \dot\in \cdot U$

(3.18) $\pi(=) := x = y$

(3.19) $\pi(P) := (x_1, \ldots, x_n) \dot\in \cdot P \quad \text{where } ar(P) = n$

(3.20) $\pi(c) := x = \cdot c$

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[395]at least for the object and first two meta levels
[396]One should read the use of the tuple notation in the case of $\pi(P)$ as a function application that yields a set encoding of that tuple, in order to accept this as a plain first-order formula.

This function π is a signature interpretation translation by virtue of (i) $\tau^+(T) \models \exists x \,.\, x \,\dot{\in}\, \cdot U$ and (ii) $\tau^+(T) \models \exists x \,.\, x \,\dot{\in}\, \cdot U \wedge \forall y(y = \cdot c \leftrightarrow y = x)$, where (ii) simplifies to $\tau^+(T) \models \exists x \,.\, x \,\dot{\in}\, \cdot U \wedge x = \cdot c$. Both follow from $\Delta_{\Sigma Str} \cup \{\Sigma Str(\cdot U, \bar{P}, \bar{c})\} \subseteq \tau^+(T)$.

Finally, reading the rewriting of constants between L and $L_{\dot{\in}}$ as a bijection $\rho(c) = \cdot c$ (for all constants $c \in Const(L)$), Def. A.8 applied to atomic formulas (under rewriting of constants) results in $\alpha(P(t_1, \ldots, t_n)) = \pi(P)[x_i/t_i]_{1 \le i \le n} = (t_1, \ldots, t_n) \,\dot{\in}\, \cdot P = \tau(P(t_1, \ldots t_n))$ and in $\alpha(t_1 = t_2) = \pi(=)[x_i/t_i]_{1 \le i \le 2} = {}^\backprime t_1 = t_2{}^\prime = \tau(t_1 = t_2)$, resp. Hence, $\alpha(\phi) = \tau(\phi)$ for all $\phi \in L$. This reveals τ as a first-order formula interpretation translation of L into $L_{\dot{\in}}$ and, with Prop. 3.17, as a faithful first-order theory interpretation of T into $\tau^+(T)$. ❏

In a nutshell, Prop. 3.18 declares the standard set-theoretic semantics of any FOL theory T to be nothing else than a specific theory interpretation into a set-theoretic rewrite of T (uniformly defined for arbitrary theories) augmented with set theory itself.

DIFFERENCE TO DIRECT THEORY INTERPRETATION INTO SET THEORY

Note that this kind of theory interpretation differs from another, more widely found type of interpreting, especially mathematical, theories and notions within set theory. Examples of the latter are the Kuratowski encoding of an ordered pair, [493], cf. also [198, p. 48], or the von Neumann encoding of natural numbers[397] [853], cf. also [198, p. 257, esp. p. 260]. While theory interpretations based on formula interpretation translations $\alpha : L \to L'$ are brought to bear in these cases, as well, these interpretations use solely set theory as their target / co-domain, i.e., $L' = Lg(\{\in\})$, and they vary among each other, in contrast to the universal character of τ wrt arbitrary theories. Following Rautenberg's definitional variant of theory interpretations [693, sect. 6.6], such interpretations are usually described by a set $\Delta(T)$ of definitions of the basic notions of the source theory $T \subseteq L$, in order to then prove (the axioms of) T as a set of theorems from the chosen set theory together with those definitions.

3.3.5 Additional Observations

MULTIPLE MODELS OF T VS. $\tau^+(T)$

Let us conclude the exposition of the theory view with a few more remarks. One aspect that is not addressed above is the existence of multiple (non-isomorphic) models of a (consistent) object-level theory T, as is potentially the case and is usual for formalized ontologies due to underspecified, incomplete theories. Of course, a single interpretation structure $\mathcal{V}$ that satisfies the chosen set theory ST encompasses a multiplicity of Σ structures.[398] By construction the interpretation of the constants $\cdot U, \bar{P}, \bar{c}$ in the signature of $L_{\dot{\in}}$ fixes / links to a particular Σ structure within every model $\mathcal{V}$ of $\tau^+(T)$. Merely changing the interpretations of those constants potentially yields further models of $\tau^+(T)$, without considering a different set-theoretic model (of ST). Apart from that, indeed one must expect that ST is incomplete in the general case[399] and thus ST has multiple models being neither isomorphic nor elementary equivalent. Further discussion of the existence of multiple models is contained in sect. 4.2 and 4.3, in connection with an ontological perspective.

META-LEVEL REASONING IS USUALLY SYNTACTIC / NATURAL DEDUCTION

Eventually, another aspect of the theory view concerns the degree of formality that is actually applied in reasoning at the meta level. Above everything is expounded in terms of a fully formalized meta theory.

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[397] More precisely, this refers to the definition of ordinal numbers in [853], including and focusing on transfinite ordinals.

[398] More precisely, regarding $\mathcal{V}$ itself, it comprises *all* Σ structures within that particular model of ST. However, if all models of ST are considered (at the meta-meta-level), not every Σ structure (modulo isomorphism) will be available in each model, e.g. thinking of ST models of countably vs. uncountably infinite cardinality (and potential choices for Σ structures within either). Hence the use of "multiplicity". The variability under the object-level perspective remains open for us.

[399] At least, if one assumes that ST is strong enough to encode Peano Arithmetic (PA), in connection with Gödel's First Incompleteness Theorem. The axioms of Peano Arithmetic are captured in sect. A.2.2 in the Appendix. As sources for the Incompleteness Theorems of Gödel, we refer to the presentations by Wolfgang Rautenberg [693, ch. 6] and by Dirk W. Hoffmann [423, ch. 4]. While we have just followed the familiar reference to Peano Arithmetic, it should be mentioned that the weaker system of Robinson Arithmetic suffices for Gödels proof of the First Incompleteness Theorem, cf. [693, system Q in sect. 6.3] and [423, sect. 4.4, esp. p. 229–230]. Robinson Arithmetic adopts all axioms of PA except for the induction axiom, which is replaced by requiring that every natural number except for 0 is a successor: $\forall x \,.\, x \neq 0 \to \exists y(x = S(y))$.

entity postulation wrt $\mathcal{A}^{\mathcal{P}_{fix}^{\omega}}$	entity postulation wrt theory view
the subset of A derived from logical individual constants of L and existential claims entailed by T	$\bar{c}$ and existential claims entailed by T
the subset of $\mathcal{A}^{\mathcal{P}_{fix}^{\omega}}$ derived from all constants of L and existential claims entailed by T	$\bar{c}, \bar{P}$, and existential claims entailed by T
the subset of $\mathcal{A}^{\mathcal{P}_{fix}^{\omega}}$ (possibly augmented by the membership relation) derived from all constants of L and existential claims entailed by T'	$\cdot\bar{c}, \cdot\bar{P}$, and existential claims entailed by $\tau^+(T)$ (and possibly inluding $\dot{\in}$)

Table 3.1: Options for entity postulation wrt to a structure $\mathcal{A}^{\mathcal{P}_{fix}^{\omega}}$ (see sect. 3.1) and syntactic reformulation in terms of the theory view (sect. 3.3).

However, what must typically be expected is that $\tau^+(T)$ is usually treated in an informal or mathematically semi-formal style to which natural deduction is applied. This corresponds more to the application of a calculus / syntactic consequence operator ($\vdash$) at level $^{(2)}$ instead of semantic entailment ($\models^{(2)}$).[400]

3.4 Aims for an Ontologically Neutral Semantic Account

RETURN TO ENTITY POSTULATION ISSUE AND RECAST OPTIONS

Returning to the problem of entity postulation raised in sect. 3.2, the theory view expounded in the previous section allows for recasting the choice of the three options at the end of sect. 3.2. The correspondence is displayed in Table 3.1. It is rough in the sense that the column on $\mathcal{A}^{\mathcal{P}_{fix}^{\omega}}$ refers to a single structure and a general meta-theory parameter T', whereas the column on the theory view employs syntactic parlance and takes the position of the overall theory (but not of a single model of it).

PROBLEM OF CLASSICAL SEMANTICS AND A WAY OF ESCAPE

That overall choice basically arises from a property of classical semantics which is now visible from two different angles: it is the fact that there is a difference between A and $\mathcal{A}^{\mathcal{P}_{fix}^{\omega}}$ or between T and $\tau^+(T)$, resp. Avoiding that difference suggests that no choice must be made, but rather that one option is the only remaining then. "Avoiding that difference" can be rendered more precise in the theory view. For this purpose, we first abstract from the particular object- to meta-level translation τ (and τ^+) a corresponding parameter $\hat{\tau}$ (and $\hat{\tau}^+$) that is supposed to play the same rôle, in a hypothetical semantic approach, like τ in the theory view of classical semantics. That means, $\hat{\tau}$ yields a translation of object level theories into the language of the meta level. Further it might be necessary to augment $\hat{\tau}$ by adding a fixed presupposed theory Ω that applies at the meta level (resulting in $\hat{\tau}^+$). In terms of this abstraction, a truely / ideally ontologically neutral account of semantics should be one where an object level theory stands for itself and nothing else, possibly modulo equivalent formulation.

Looking at this step more closely and from the perspective of aiming at theories that capture ontological commitments, we distinguish two aspects, in parallel broadening the scope of Table 3.1 from existential claims to arbitrary sentences, understood as (representations of) ontological commitments. The first aspect of comparing a theory T with $\hat{\tau}^+(T)$ concerns the (non-)assumption of additional (kinds of) entities, which

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[400]This train of thoughts may be extended even further to formal approaches in general. Notably, formalization in terms of a logical language, i.e., by writing formulas in a logical syntax (and hence in syntactically restricted form), must be conducted with the set-theoretic semantics in mind. Put differently, the logical syntax is "just" a representation of a set-theoretic model (in the sense of a reflection / abstraction of, e.g., real-world conditions; not in the sense of an interpretation structure), where the latter remains implicit. In contrast, formalization in terms of "standard" mathematics and corresponding semi-formal notation is more liberal in the sense that sets (and arbitrary established mathematical machinery) can be directly worked with.

Stepping back a little further, note that this view of logical formalization is one which we perceive as prevalent in computer science. From a philosophical point of view, logical considerations need not be tied with model-theoretic / set-theoretic concerns.

starts with (dis-)allowing differing signatures. For neutrality, no novel signature elements should be introduced beyond those assumed for T. The second aspect emerges from the difference between $\hat{\tau}^+(T)$ and T in terms of their commitments / sentences. Remembering the examples in connection with ontology-based consequence and equivalence in sect. 2.2.2.2 (above $\mathcal{Q}_5$ on p. 67), their working cases are based on the addition of ontological commitments. Such ontological "enrichment" may thus be useful to consider – against the background of assuming a common language (for semantic entities) at object and meta level. Extending such additional commitments to allowing for novel kinds of entities and an extended (or different) language constitutes a further weakening. What remains as possibly preservable in the latter situation is that object level commitments concluded at the meta level (in translated form) are already justified by the object level theory itself, as well as object-level conclusions should be preserved.

These reflections yield an attempt to capture notions of ontological neutrality by the following definitions, with a common starting point in Cond. 3.19 and a canonical extension of Def. 3.20 in Def. 3.21.

3.19 Condition
Let $\mathfrak{S}$ a semantic approach for an object-level language L, s.t. a theory view corresponding to $\mathfrak{S}$ is defined based on object- to meta-level translations $\hat{\tau} : L \to L'$, extended to theories by $\hat{\tau}(T) := \{\hat{\tau}(\phi) \mid \phi \in T\}$, and letting $\hat{\tau}^+$ be defined by $\hat{\tau}^+(T) := \Omega \cup \hat{\tau}(T)$ for every theory $T \subseteq L$ and a fixed theory $\Omega \subseteq L'$. ❑

3.20 Definition (versions of ontological neutrality wrt theories)
Assume Cond. 3.19 and let $T \subseteq L$ a theory. Then $\mathfrak{S}$ is termed

- *ideally ontologically neutral wrt T*   :iff $L = L'$, $\hat{\tau}(T) \equiv T$, and $\hat{\tau}^+(T) \equiv \hat{\tau}(T)$
- *partially ontologically neutral wrt T*   :iff $L = L'$ and $\hat{\tau}(T) \equiv_\Omega T$
- *weakly ontologically neutral wrt T*   :iff $\hat{\tau}^+(T) \models \hat{\tau}(\phi)$ iff $T \models \phi$ for all $\phi \in Sen(L)$ ❑

3.21 Definition (ideal, partial, and weak ontological neutrality of semantic accounts)
Given Cond. 3.19, call the semantic approach $\mathfrak{S}$ *ideally* (*partially*, *weakly*) *ontologically neutral*   :iff $\mathfrak{S}$ is ideally (partially, weakly) ontologically neutral wrt arbitrary theories $T \subseteq L$. ❑

Let us first provide additional support for the definition of partial ontological neutrality. 'Partial' is appropriate on the one hand, because the semantic translation is enriched by Ω and the equivalence between T and $\hat{\tau}(T)$ is examined against Ω as a background theory. On the other hand, we see neutrality being reasonably (and partially) given not only because the same language is required, but further since faithfulness / ideal ontological neutrality of the semantic translation wrt the commitments in Ω entails that every theory is "only a single step away" from a theory for which the semantic approach is ideally ontologically neutral.

3.22 Observation (transfer of semantic fixpoint characteristic wrt partial ontological neutrality)
Assume Cond. 3.19 and let $\mathfrak{S}$ partially ontologically neutral. If $\mathfrak{S}$ is ideally ontologically neutral wrt Ω, then it is ideally ontologically neutral wrt all images of $\hat{\tau}^+$, i.e., $\hat{\tau}^+(\hat{\tau}^+(T)) \equiv \hat{\tau}^+(T)$ for all theories $T \subseteq L$. ❑

Proof. For any $T \subseteq L$, the following chain can be derived: $\hat{\tau}^+(\hat{\tau}^+(T))$ $=(1)$ $\Omega \cup \hat{\tau}(\hat{\tau}^+(T))$ $=(2)$ $\Omega \cup \hat{\tau}(\Omega \cup \hat{\tau}(T))$ $=(3)$ $\Omega \cup \hat{\tau}(\Omega) \cup \hat{\tau}(\hat{\tau}(T))$ $\equiv(4)$ $\Omega \cup \hat{\tau}(\hat{\tau}(T))$ $\equiv(5)$ $\Omega \cup \hat{\tau}(T)$ $=(6)$ $\hat{\tau}^+(T)$, with these justifications: (1, 2, 6) by def. of $\hat{\tau}^+$ in Cond. 3.19; (3) by def. of $\hat{\tau}$ for theories, which entails $\hat{\tau}(A \cup B) = \hat{\tau}(A) \cup \hat{\tau}(B)$, applied here to $\Omega \cup \hat{\tau}(T)$; (4) due to the precondition of ideal ontological neutrality of $\mathfrak{S}$ wrt Ω, which by Def. 3.20 entails $\hat{\tau}(\Omega) \equiv \Omega$; (5) due to the precondition of partial ontological neutrality of $\mathfrak{S}$ (wrt every theory) applied ot $\hat{\tau}(T)$, yielding $\hat{\tau}(\hat{\tau}(T)) \equiv_\Omega \hat{\tau}(T)$, which by definition of $\equiv_\Omega$ is equivalent with $\Omega \cup \hat{\tau}(\hat{\tau}(T)) \equiv \Omega \cup \hat{\tau}(T)$. ❑

'ONTOLOGICAL NEUTRALITY' CONNECTS TO EARLIER CRITICISM, NOW WEAKENED

The reader might wonder why the term 'ontological neutrality' is so heavily utilized here. The reason for this wording is that the criteria esp. for ideally and partially ontologically neutral semantic accounts and the implications of these criteria suggest that a number of the points mentioned in sect. 1.2 and some criticism in ch. 2 would not occur or be decently reduced in such approaches. Many of those points and criticisms

are also linked to the claim of the ontological neutrality of, e.g., FOL, which we rejected on an initial, pre-analytic / -theoretic view, cf. the discussion in sect. 2.2.2.2.

Now, as another effect of the definitions, we concede that the rejection of set-theoretic semantics as being ontologically neutral may be too harsh, at least in the light of the definitions. Clearly, Prop. 3.18 has an immediate consequence.

3.23 Observation (status of set-theoretic semantics)
Set-theoretic semantics is weakly ontologically neutral by Def. 3.21.   ❏

Overall, if $\hat{\tau}$ is a faithful theory interpretation, this results in weak ontological neutrality, independently of whether the theory into which one interprets is actually intended to capture ontological commitments or how entities in that theory relate to those of object-level theories. Insofar the definition of weak ontological neutrality is rather a qualified / conditional proposal, whereas we are most interested in the ideal variant.

In this regard, remembering the above statement that an (ontological) object-level theory should "stand for itself and nothing else" finds attribution in the fact that a semantic account satisfying $\hat{\tau}^+(T) = T$ appears as *the* clearest case of ideal ontological neutrality. However, just demanding this strict property fulfills the definition only abstractly, so to say. But it does not answer the question of what that means, e.g. in terms of semantic structures for a referential semantic account, and in relation to established semantics in general. This observation provokes studying whether one can do better than to adhere to a weakly ontologically neutral account. The next chapter is devoted to this project, developing 'ontological semantics'.

QUALIFICATION ON CONCEIVABLE NOTIONS OF EQUIVALENCE

We remark that the above Def. 3.20 and 3.21 is a proposal and still close in character to a working hypothesis, especially regarding the "right" form of equivalence, namely formal semantic equivalence (from a metameta level perspective, if $\hat{\tau}^+(T)$ and $\hat{\tau}(T)$ are seen as meta level theories). At the present stage we conceive of logical equivalence as the best possible alternative that is immediately available for formulation. Equality might be an alternative wrt availability, but it is inappropriately narrow under our reading of 'theory' as 'set of sentences', without requiring their deductive closure. E.g., if for any semantic account $\hat{\tau}(T)$ were an extension of an axiomatization T by one of its consequences, this should not raise a problem for calling the semantic approach ontologically neutral. This leaves us with "standard" equivalence. Albeit admitting more liberality basically by theory interpretations between T, $\hat{\tau}(T)$, and / or $\hat{\tau}^+(T)$ in the case of weak ontological neutrality, it is open for us how delicate this is, as stated just above wrt Obs. 3.23.

Another alternative is met by anticipating ontological semantics from the next chapter. This is a distinct semantic account, which likewise offers a notion of equivalence, especially in two of its instances for FOL, which we elaborate and which are based on the ontology of categories and relations discussed so far in sect. 2.4. We see this, as well, as a candidate notion for Def. 3.21, but it is still close to the classical case. Beyond that, novel accounts of intensional notions of equivalence may turn out to be more appropriate in the future. Altogether, there is surely room for reconsidering the above way of defining an ontologically neutral account. In its present form, it allows to explain why we perceive the set-theoretic semantics of FOL as (at most) weakly ontologically neutral.

SAMPLE ADVANTAGES

In addition to the last sentence, some examples of potential advantages of semantics that are ideally or partially ontologically neutral are in order. First of all, an object-level theory is not interpreted (along the lines of Prop. 3.18) into a meta level theory of *other* entities. An ideal ontologically neutral semantics does not introduce additional / its own entities at the meta level. Thus no decision has to be made which of these additional entities to adopt as being existent from the point of view of an ontology formalized in the corresponding language. Put differently, the (semantic / referent)[401] entities at object and meta level would not differ. As a consequence, there is no need / coercion to encode entities that one has in mind in terms of other entities. For example, the semantics of predicates, which may be used for naming (intensional) categories, would not be encoded inevitably into sets (which, in turn, prevents undesired meta-level reasoning about them, e.g., thinking of the principle of extensionality). It might further turn out that a

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[401]The metalanguage comprises further entities that are not covered of $\hat{\tau}^+$, e.g., symbols, relations such as entailment, etc.

corresponding semantic approach can consolidate the distinction between formal and intensional semantics (cf. sect. 2.2.1). Eventually, we expect that it suggests itself as vantage point for revising the notion of ontology-based equivalence as necessary.

TRANSITIONING TO ONTOLOGICAL SEMANTICS

These thoughts lead directly to the next chapter, in which a novel semantic account (which we already touched upon) is developed – *ontological semantics* – among others with the aim of avoiding encodings and thus of being ontologically neutral according to Def. 3.21. In its development, the views on classical semantics of this chapter shall prove useful. Moreover, the foundational considerations of ch. 2 play their rôle, not at least the ontology $\mathcal{CR}$ of categories and relations in sect. 2.4.

Chapter 4

Ontological Semantics

The foundational analyses in ch. 2 motivate our consideration and development of novel semantic foundations for the representation of ontologies. This is the contribution of this chapter, further intended to pave the way, in the long run, for novel accounts of intensional equivalence.

BASIC SINGLE 'LANGUAGE USER' PERSPECTIVE

Prior to developing actual semantic definitions, the assumed application perspective should be made explicit. Similarly to our understanding of classical semantics, the semantics to be developed presupposes that statements in the syntax of the language shall represent a single and integral perspective, e.g., of a person or of a group of people who agree on the contents that is represented. The latter agent(s) will be referred to below as 'the language user'. We see that this background of assumptions constitutes quite an idealization, e.g. in times where ontology development has become a matter of teams (if not large and distributed groups) more than the matter of a single developer. Nevertheless we advocate this as a reasonable basic case / form of representation. The perspective is further well-suited to the intended application of resulting ontological theories for the overall purpose of (conceptual) semantic integration. Addressing issues such as working with multiple conceptualizations of the same domain or integrating multiple ontologies are (largely) left for future work.

GENERIC VS FOL-SPECIFIC VIEWS

A final note preparing the exposition concerns the scope of the chapter regarding languages. FOL is clearly the prototypical language according to the classical semantics of which we develop ontological semantics, with effects on the chapter structure, its terminology, etc. However, we strongly believe that this factor is weak and can be abstracted from especially in the first sections, 4.1–4.3, which deal with ontological structures and their relationship to the signature, i.e. basically, to atomic constant symbols in languages. A very generic, yet admittedly vague, characterization of ontological semantics for arbitrary syntax is provided with Def. 4.3 against this background. The subsequent sections on syntactic compounds, sect. 4.4 on predication and sect. 4.5 on connectives and quantification, are much more attuned to FOL. From a more generic perspective, this may be seen as a case description of one particular instance of ontological semantics, applied to FOL syntax.

4.1 Definition of Ontological Structures by Analogy to the Set-Theoretic Approach

AIM AND REMINDER OF TARSKIAN SEMANTICS IN SECT. 3.1

Our approach to define ontological semantics for predicate logic syntax follows directly the classical Tarskian semantics as a paradigm example, but by exchanging set-theoretic structures by *ontological structures*. The motivation for introducing a different kind of interpretation structure is to avoid the encoding aspect that is built into the classical notions, cf. sect. 2.2.1. In ch. 3, Tarskian semantics is reconsidered in two ways, (1) by taking the set-theoretic superstructure into account and (2) by viewing semantics as theory interpretation. The first reconsideration, discussed in sect. 3.1, starts from the classical presentation with structures of the form $\mathcal{A} = (A, R_1, \ldots, R_l, f_1, \ldots f_m, c_1, \ldots, c_n)$, with universe / carrier set (or class) A, (mathematical) relations R_i, (mathematical) functions f_j, and (logical) individuals / objects c_k. On the basis of $\mathcal{A}$, a less common view on Tarskian semantics is expounded around Def. 3.3 in sect. 3.1.3, namely in terms of the structure $\mathcal{A}_{\in}^{\mathcal{P}_{\text{fix}}^{\omega}} = \left(A^{\mathcal{P}_{\text{fix}}^{\omega}}, \bar{\in}, R_1, \ldots, R_l, f_1, \ldots, f_m, c_1, \ldots, c_n \right)$. The main differences between $\mathcal{A}$ and $\mathcal{A}_{\in}^{\mathcal{P}_{\text{fix}}^{\omega}}$ are that (a) the hidden set theoretic assumptions are explicated via the new universe $A^{\mathcal{P}_{\text{fix}}^{\omega}}$ and the membership relation[402], and thus (b) all relations and functions over A are elements of $A^{\mathcal{P}_{\text{fix}}^{\omega}}$, just like the logical individiduals c_k.

ONTOLOGICAL STRUCTURES

We argue that the $\mathcal{A}_{\in}^{\mathcal{P}_{\text{fix}}^{\omega}}$ view on set theoretic model structures is more suited than $\mathcal{A}$ for the introduction of ontological structures by analogy to the set theoretic construction, because the domain $A^{\mathcal{P}_{\text{fix}}^{\omega}}$ comprises,

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[402] $\bar{\in}$ is a restriction of the overall (meta-language) membership relation, namely to those $\in$ connections that can be made "visible" among $A^{\mathcal{P}_{\text{fix}}^{\omega}}$ elements in conformance with the adopted set-theoretic assumptions, cf. the comments after formula (3.2) on p. 106.

a.o., *all* entities required for interpreting the presumed vocabulary, together with their $\in$ interrelationships. The encoding aspects are to be prevented by two deviations. Firstly, no classifying assumptions are made about the nature of the entity that serves to provide referents for all signature elements in an interpretation. Secondly, no assumptions about the "internal" interrelations "within" that entity / structure shall be held. This yields a first attempt of a definition.

4.1 Definition (ontological structure (preliminary variant))
An ontological structure $\mathcal{O}$ can be described as $\mathcal{O} = (O, \bar{\rho}, e_1, e_2, \ldots)$ where O and the e_i are arbitrary entities, such that all e_i are "in" O (in terms of ρ, i.e., $\rho(e_i, O)$ in common notation[403]).[404] ❑

O AND ρ IN GENERAL – 1ST ATTEMPT

Note first, remembering sect. 1.1.5.1 and FN 383, that the notion of 'entity' is understood very broadly herein, allowing for entities in different modes of existence,[405] including actual as well as non-actual, ideal, and fictitious entities, for example. Turning to the "referent provider" O, this may comprise additional entities beyond the e_i, as usual. More importantly, it should be emphasized that O in $\mathcal{O}$ need not be a set, although it can be a set. But a major point of introducing ontological structures is to generalize the classical notion of (mathematical) structure. Albeit, rendering O as being "anything" may sound problematic. In other attempts to characterize O one may state that it is a "structure of intended semantics" (intended by the user of the language), or that O is a "part of the world"[406]. In its origin from the classical case, one would expect that O is an entity of such type that comes with an "internal" relation, e.g. sets / classes with the membership relation $\in$ or situations / configurations with their constituent parts (cpart), cf. [398, sect. 11.2, 12.2, and 17.5.3] or, less specifically to GFO, e.g. situation theory [56, 202].[407] However, this is not a necessity, even if the issue of clarifying the notion of 'internal relation' is ignored.[408] Arbitrary relations could be used to "reach" entities from the given O. Therefore, the most general view appears to be one which refers to arbitrary relatedness for ρ, i.e., arbitrary relationships between O and the e_i (and further entities that might exist in "the scope of" O / $\mathcal{O}$).

OBSERVE: ONTOLOGICAL STRUCTURE RELATIONAL, ρ A RELATION

Two observations result from these preliminary considerations. First, speaking of potential entities that may serve as O and ρ suggests that the underlying notions are relational. Secondly, the discussion is not yet free of ontological assumptions, because ρ is referred to as a relationship. That confines ρ to an ontological category that need not exist in every ontological theory.[409] Both aspects shall be considered in order to derive a revised definition of ontological structure.

RELATIONAL VIEW ON STRUCTURES: QUANTIFICATION & DOMAIN OF DISCOURSE

The claim of the relational nature of $\mathcal{O}$, O, and ρ can be supported by considering their rôles wrt the further definition of classical FOL semantics, cf. sect. 1.5.2. The primary task of O (and of A in the well-known relational structure) in the context of quantificational logic and presupposing objectual quantification (see sect. 3.1) is to "deliver" / connect to all entities that are subject to quantification, and such that the

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[403]Notably, $\rho(e_i, O)$ is a matter of interpretation. One may understand this in the metatheory of this text on a set-theoretic basis. However, taking ontological semantics seriously (once fully introduced below) would mean to find an ontological interpretation for $\rho(e_i, O)$ itself, where one could have the $\mathcal{CR}$ ontology in mind.

[404]The relationship between $\bar{\rho}$ and ρ is in the same spirit as that of $\bar{\in}$ and $\in$ in the classical case.

[405]In particular, see respective analyses by Roman Ingarden [444], cf. also [153].

[406]In the widest sense of that phrase, e.g., it must allow for fictitious stories, as well. Moreover, "part of the world" must be considered in connection with a reflecting observer, see the remarks near the end of sect. 4.2.

[407]Indeed, our intentions wrt O appear close to (some) early accounts of situation theory / semantics, in the light of Barbara H. Partee's assessment "In early situation semantics as developed in Barwise (1981) [54] and in Barwise and Perry (1983) [56], the ontological status of situations and 'situation types' was a matter of some controversy, especially with respect to those authors' avoidance of possible worlds or possible situations. Subsequent work by Kratzer and by some of her students has developed the possibility of letting situations, construed as parts of worlds, function both as individuals [. . .] and as 'world-like' in that propositions are reinterpreted as sets of possible situations and expressions are evaluated at situations rather than at world-time pairs." [657, sect. 4.1]. The aspect of ontological structures as actual vs possible "parts of the world" is discussed in sect. 4.2.

[408]The examples of inherence or part-of relations may initiate a vague feeling for internal relations, whereas the status of the membership relation as an internal relation is unresolved for us. The interested reader may consider, a.o., works of Kevin Mulligan on the subject of internal relations [606, 607].

[409]The ontological status of relations is heavily debated in philosophy, cf. [121].

semantics of quantifiers can be defined on the basis of A or O, respectively. In the classical case, for $\mathcal{A} = (A, R_1, \ldots, R_l, f_1, \ldots f_m, c_1, \ldots, c_n)$ (in combination with a variable assignment β) this is visible in the evaluation of quantified formulas.

$$(4.1) \quad \langle \mathcal{A}, \beta \rangle \models \forall x \,.\, \phi \quad \text{iff} \quad \text{for every } a \in A : \langle \mathcal{A}, \beta_a^x \rangle \models \phi$$

$$(4.2) \quad \langle \mathcal{A}, \beta \rangle \models \exists x \,.\, \phi \quad \text{iff} \quad \text{there is an } a \in A : \langle \mathcal{A}, \beta_a^x \rangle \models \phi$$

A generalized form for $\mathcal{O} = (O, \bar{\rho}, e_1, e_2, \ldots)$ would be the following. The discussion of effects on quantification wrt the notion of order in predicate logic is deferred to sect. 4.5.1.

$$(4.3) \quad \langle \mathcal{O}, \beta \rangle \models \forall x \,.\, \phi \quad \text{iff} \quad \text{for every } o \text{ with } \rho(o, O) : \langle \mathcal{O}, \beta_o^x \rangle \models \phi$$

$$(4.4) \quad \langle \mathcal{O}, \beta \rangle \models \exists x \,.\, \phi \quad \text{iff} \quad \text{there is an } o \text{ with } \rho(o, O) : \langle \mathcal{O}, \beta_o^x \rangle \models \phi$$

Such definitions yield the rôles of $\mathcal{O}$, O, and ρ wrt the overall definition of a formal semantics, and this is an aspect that is "internal" to defining the formal semantics. In addition, there is a "pragmatic" rôle of the O-analog[410] A in the context of utilizing logic for axiomatizations of intended structures. It is the delimitation of the domain, in terms of its objects / entities (often specified by one or more types). Enderton's textbook [221, e.g. ch. 2] highlights this nicely by treating the universal quantifier $\forall$ as a parameter when specifying a first-order language. In formalizations focusing on a certain domain of reality / knowledge, it is not only predicate symbols which have an associated intended meaning (e.g. a range of entities in the case of unary predicates), but likewise $\forall$. For a more concrete example, the axiomatization PA of Peano arithmetic, cf. A.2.2, bears the assumption that its quantifiers ranging over logical individuals (from the formal point of view) cover quantification over exactly all natural numbers. In this sense the set $\mathbb{N}$ of natural numbers forms the universe (of logical individuals) of the intended model, i.e., $\mathbb{N}$ provides a collection of entities together with a background assumption for the corresponding theory, namely that quantification is implicitly bound to those entities. That assumption can be explicated by relativizing the theory in terms of a novel unary predicate, cf. Def. A.1 in sect. A.1.1 of the App., [693, sect. 6.6, p. 258] or [213, sect. VIII.2.D], e.g. in this case by a unary predicate Nat. Regarding the terminology from the beginning of this section, the label 'carrier set' for A exhibits the stronger correspondence with the first, semantics-internal aspect, whereas 'domain of discourse' is more strongly reminiscent of the second, "pragmatic" aspect.

ONTOLOGY IS TO "GOVERN" SEMANTICS, FAITHFULNESS OF THE LATTER TO THE FORMER

Yet another issue arises from the initial attempt of defining ontological structures, regarding the mutual relationship between ontology and semantic definitions.[411] On the hypothesis that top-level ontologies (TLOs) – ontologies potentially covering literally *every* entity (even if only "indirectly", due to highly abstract categories) – can sensibly be studied,[412] it appears reasonable that such an ontology should "govern" the range of entities available for constructing / specifying semantic accounts.[413] We find this a desirable and defendable position, even if a clear distinction between object level and meta level wrt languages is to be maintained. Hence, defining ontological structures in outmost generality must take this into account, i.e., it must be achieved that ontological semantics can be faithful to the ontological accounts that are to be stated in its respect. For instance, if a process ontology is to be defended as a TLO,[414] the definition of the semantics should not utilize relations (unless an analysis of relations in terms of processes has been provided). Put differently, ontological semantics must be able to take the syntactically expressed ontologies seriously and, consequently, it should embody as few commitments as possible. Besides the question of what kind of entity ρ is in Def. 4.1, the same question applies to O, which functions pragmatically as domain of discourse. However, an ontology may not commit to the existence of an entity that, in some sense or another (as discussed above), "covers" / "captures" all entities.

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[410]Although O is derived from $A^{\mathcal{P}_{\text{fix}}^\omega}$, the distinction between A and $A^{\mathcal{P}_{\text{fix}}^\omega}$ is no longer visible "within" O, such that analogies for O arise for both, A and $A^{\mathcal{P}_{\text{fix}}^\omega}$.

[411]Where so far semantics is defined only for atomic / non-composed symbols.

[412]We admit that the subsequent argument is tailored to top-level ontologies. Domain ontologies, on the flipside, need not bear a strong tie with this aspect of defining ontological semantics.

[413]This applies likewise to the syntactic "half" of the underlying language, which should lend itself to an ontological analysis based on that very (top-level) ontology or an extension of it, as well. But this issue is secondary for our purposes and remains to be tackled.

[414]I.e., an ontology that classifies every entity by a top-most category of process and analyzes all entities accordingly. For example, see the work of Johanna Seibt [744, 745].

In the light of these reflections, the initial variant of ontological structure can be moderately further generalized. The only remaining ontological assumption in the resulting definition is the existence of symbol referents, at least for all elements of the (extralogical) signature. We keep a "component" O as the universe / domain of discourse, but omit the relation ρ, to be justified below. An ontological interpretation is a pair of an ontological structure and a variable assignment. Before entering the detailed discussion of ontological structures, a generic definition / characterization of 'ontological semantics' is included, which can be applied wrt arbitrary syntaxes and thus languages.

4.2 Definition (ontological structure and ontological interpretation)

An *ontological structure* $\mathcal{O}$ is a single entity or a plurality of entities which is or are considered *for* the rôle of referents in the context of (a language user's) interpreting referring symbols of a signature Σ.

Formally, an ontological structure $\mathcal{O}$ (*suitable* for Σ) is captured as $\mathcal{O} = (O, I)$, where O stands for the entity or plurality of entities under consideration, and $I : \Sigma \to O$ is a (total) function capturing the interpretation / denotation of referring symbols. Alternatively, I can be replaced by an enumeration of its values, $\mathcal{O} = (O, e_1, e_2, \ldots)$, if the relation to the symbols in Σ is clear or irrelevant.

Let *Var* the set of variables used in combination with Σ. Any pair $\langle \mathcal{O}, \beta \rangle$ of an ontological structure $\mathcal{O} = (O, I)$ and a variable assignment $\beta : Var \to O$ is an *ontological interpretation*. ❑

4.3 Definition (ontological semantics)

An *ontological semantics* for an arbitrary language / syntax is any semantic definition that is substantially based on ontological structures. ❑

ANALYSIS OF O AND $\wr\; \int$ NOTATION

What one may refer to as the universe / domain O in this definition should primarily be understood as a symbol (at the metameta level) that refers to / denotes multiple entities (those being "in the semantics", at the meta level) without the assumption that (at least at the meta level) we necessarily find an entity that "binds together", by any certain relation ρ, all those entities that O refers to. Cases are *possible*, though, where there is such an entity, e.g., if one commits to the existence of sets, finite sets of entities would be obvious candidates. Yet to indicate the character of O also by notation, e.g. when providing examples, we may use $\wr\; \int$ as a new type of parentheses. This yields $O = \wr a, b \int$ in the case that a and b form the domain O, for example.

LACK OF ρ IN THE DEFINITION

The potential unavailability of a "semantic" counterpart / entity for O itself is one of the reasons that ρ is lacking in the definition of 'ontological structure'. ρ can be analyzed as denotation at the metameta level, which provides the background for correctly understanding certain phrases that we use below to relate (the symbol) O with the corresponding entities. Sample phrases are "entities in O", "entities covered by O", and "entities captured by O". The second reason for omitting ρ in Def. 4.2 is that, in contrast to $\mathcal{A}^{\mathcal{P}_{\text{fix}}^{\omega}}$ and set membership, no background theory on ρ could be assumed, i.e., no specific structuring of the entities in O based on any ρ should be expected by the semantics of the language.

MISC. FURTHER COMMENTS ON THE GENERAL DEFINITION

Some further comments on Def. 4.2 may aid its comprehension. Intentionally, this definition allows everything that exists (everything whose existence anyone wishes to claim within an ontological theory) to be termed an ontological structure. 'Existence', again, must here be understood in the broadest conceivable sense, including fictitious and mental entities, for instance. Sect. 4.2 discusses this aspect further. We assume that a 'plurality' of entities refers to multiple entities, i.e., at least two. Nevertheless, the case of a single referent x is admitted in Def. 4.2, leading to $\wr x \int$. The latter is equivalent to writing x plainly, resulting in $\wr x \int = x$ (at the meta level), since no novel entity needs to be expected "next to" x, just because x is an entity under consideration – the only one, in this case. In effect of "a single entity or a plurality of entities" in Def. 4.2, there is no ontological structure that captures nothing.[415]

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[415]This is in line with the assumption of a non-empty universe of classical set-theoretic structures. Moreover, observe that empty "parts" / "subpluralities" of an ontological structure can be addressed, e.g. by means of predicates that do not apply to any entity (e.g.

Moreover, the definition carves out the relational aspect of the term 'ontological structure' in the context of language interpretation. More precisely, this concerns signature / vocabulary interpretation (against the background of an informal ontology / a conceptualization). The word 'for' in the phrase "for the rôle of referents" is used instead of 'in' in order to accommodate ontological structures $\mathcal{O}$ that comprise more entities apart from those assigned to constants and variables in any specific $\mathcal{O}$. (In terms of $\mathcal{CR}$, see sect. 2.4, esp. sect. 2.4.3.3, it suffices for the entities in the ontological structure to be potential players / playerables of the referent role in the interpretation, instead of being actual players). This also links to the meaningfulness of multiple ontological structures interpreting the same theory, see sect. 4.3.

STATUS OF INTERPRETATION FUNCTION I

The mathematical capturing of $\mathcal{O}$ by means of O and a function $I : \Sigma \to O$ follows a common style of defining structures (yet of mathematical nature / a set-theoretic basis usually), cf. [214, sect. III.1] for the closest analog, but also [221, 691]. That capturing is affected by the view of O as a symbol, as well. Since no entity for O is guaranteed / presupposed by the semantics under development, in general that function I can be understood by means of an intuitive notion of (mathematical) function,[416] whose range is the plurality of entities under consideration. Understanding that function I via the usual grounding in set-theoretic terms remains open at this point. It leads to the question of which semantics is or can be applied to this overall account of ontological semantics itself, which cannot be in our focus before that semantics is actually proposed.

STATUS OF $\mathcal{O}$ VS. O

Let us therefore return to the constituents of the semantics / semantic basis themselves. While $\mathcal{O}$ and O are clearly differently captured formally, they are actually very similar: in a non-mathematical reading, $\mathcal{O}$ itself (as a symbol) refers to / denotes the actual ontological structure / entity / entities that are intentionally referred to or deemed possible by the language user, just as is the case for O. From that point of view, both are rather (metalanguage) names / means of reference for the potential plurality of entities, with only a minor difference in connotation. Assuming that somebody intends to state an ontological theory claiming that there are exactly four distinct entities, using O in the above definition does not entail that there is a fifth entity that corresponds to the totality of those four entities or that is a whole or collection of the other four.[417] We see the case of $\mathcal{O}$ to be very similar, at least. It is meant to refer to exactly the same entity / entities as O, yet there is the connotation that those entities are under consideration by the language users, for making statements about them by means of symbols in Σ. Since this connotation is formally reflected by I, it may have higher presence for $\mathcal{O}$ than for O, where the connotation could be dropped – although not truly completely, since there is the, in our case implicit and in Enderton's approach of defining structures [221, sect. 2.2] explicit, linkage between quantification / the quantifier symbol $\forall$ and O.

RESULTING MODIFICATION OF QUANTIFICATION WITH INITIAL ASSESSMENT

Naturally, a revised definition of ontological structures necessitates a modification of quantifier semantics.

(4.5) $\langle \mathcal{O}, \beta \rangle \models \forall x . \phi$ iff   for every o in O / under consideration in discourse : $\langle \mathcal{O}, \beta_o^x \rangle \models \phi$

(4.6) $\langle \mathcal{O}, \beta \rangle \models \exists x . \phi$ iff   there is an o in O / under consideration in discourse : $\langle \mathcal{O}, \beta_o^x \rangle \models \phi$

Note first that the "pragmatic" aspect of a limited domain of discourse applies in those definitions (based on ontological structures as defined in Def. 4.2) just as in the set-theoretic account of quantification. This appears reasonable for formalizing ontologies, because most theories will be of limited scope, e.g. domain

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by axiomatic enforcement within a theory). Therefore, from the application point of view of formalizing ontological theories, the limitation of not considering 'nothing' as the basis for interpretations of such theories is not seen as severe, because it mainly excludes the theory that there does not exist anything, in an absolute / universal sense.

In general, however, allowing for the empty universe is a philosophical issue in itself already for standard semantics, cf. e.g. [152], referring further to Quine [682, p. 161–162], [681].

[416]I.e., as a primitive / non-reduced mathematical notion such as, e.g., underlying the early development of (type-free) lambda calculus, at the time designed by Alonzo Church to serve as a foundation of mathematics, cf. [49, sect. 0]. Notably, later expositions of lambda calculus are compatible with or presuppose a set-theoretic view on functions, cf. e.g. the view on functions in [403, see preface].

[417]In contrast, in a standard formal account, one would speak of *the set* of those four entities and use that set as the universe of a standard structure, which yields a (possibly) novel entity not considered among the four initially.

ontologies. The latter had to be formalized less succinctly in a language with ontological semantics that were based on purely unrestricted quantification.[418] Yet the approach may insofar count as an unconventional form of quantification as there is no implicit commitment to entities of certain kinds due to the semantics (since O (as a symbol) need not refer to any new, single entity, other than those intended as the domain of discourse). On the other hand, the definitions may appear as a step backwards from what has been achieved by model-theoretic semantics in general, in terms of using a clarified notion of sets instead of a natural language phrase like "every o under consideration", and the overall definition of the notion of truth (in a structure) in set-theoretic terms, as originally developed by Tarski.[419] Yet again, the background for this move includes, firstly, to avoid presuppositions of any specific ontological kind by the semantics to be defined. Secondly, formalizations of top-level ontologies with an unrestricted domain of discourse (covering literally every entity) shall be supported, even at the meta level. Therefore, a notion of sets (like that of ZFC) that does not offer urelements and, e.g., a universal set should not be adopted for this purpose.[420] Ad interim, this discussion leaves the topic of quantification, which therefore is interrupted here, despite further major issues arising e.g. from subsequent points. Considering quantification is continued in sect. 4.5.1, after a specialized and more complete version of overall ontological semantics.

4.2 Properties and Further Background for Ontological Structures in General

Some properties as well as some remarks on the further background of the general account of ontological structures shall be provided next. Remember that this approach to specifying semantics for languages is intended for the coherent interpretation of using those languages for the specification of ontologies.

DIRECT INTERPRETATION IN UNSTRUCTURED $\mathcal{O}$

Accordingly, it is a central property of ontological structures in their general form that all entities are tantamount in the semantics, i.e., they are treated equally within / by the semantics. Entities of all kinds which are taken into account within a language, e.g., through the glasses of $\mathcal{CR}$ or GFO for a moment, categories and individuals, including relations and relators, can occur in an ontological structure $\mathcal{O}$. From the perspective of interpretations, symbols of the signature of a language, either constants or variables, are interpreted into the same unstructured "pool" of entities. There is no built-in mechanism to reduce any type of entity to other – presumed – types of entity, in order to leave as much "freedom" as possible to the ontologist. To emphasize again, that "pool" of entities is not a set / class, and it need not even be assumed to exist itself, as *one* entity that in any form aggregates those "within it".

UNDERLYING PHILOSOPHY CAN ONLY BE OUTLINED, BUT NOT ELABORATED HERE

'Existence' is an important concept for ontological structures as well as for the projected account of quantification. Above, our use / intended interpretation of 'existence' is already indicated, stating that it should be as wide as possible. Our views can only be outlined in the sequel, as far as it is necessary for later parts of this work. The in-depth elaboration of this sketch of ideas must be deferred in favor of continuing the development of ontological semantics, in order to more directly proceed to some of applications that we are aiming at and, similarly, to work towards a revised account of intensional equivalence. By these remarks, we (must) deliberately abandon further discussion (a) of philosophical theories of pure existence (or non-existence [700], often connected with the work of Alexius Meinong, cf. [554]), (b) of accounts of truthmakers and truthbearers [180, 545] as well as of notions to which the latter are potentially applied,

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[418]For example, relativations of axiomatizations may then be required, cf. sect. A.1.1 in the App. for relativations.

[419]For instance, Gila Sher in [749, p. 342] quotes Robert Vaught from [845] with "[Tarski's] major contribution was to show that the notion 'σ is true in $\mathcal{A}$' can simply be *defined* inside of ordinary mathematics, for example, ZF." [parenthetical modification and emphasis as in [749]].

[420]While the subsequent exposition of ontological semantics has been developed without attempting to recourse to another set theory, we note that there are (i) various such systems available, e.g. as listed by Randall Holmes [@113], and (ii) it appears worthwhile to consider such system in the future (e.g., if sets are to be covered by a top-level ontology anyway). In particular, from initial glimpses into [425], Quine's New Foundations (NF) [680] extended with urelements (NFU) appears promising, cf. previous remarks in sect. 2.4.2.3. It is further under consideration in connection with GFO.

like beliefs, thoughts, judgments, etc. [180, esp. sect. 2], and (c) of modal logic and possible world semantics, especially wrt the actualism–possibilism debate [575].[421] Clearly, much of the work in these areas is relevant to our proposals, in particular in this chapter. However, a deepened and profound philosophical grounding in those respects must remain for the future.

OBJECTIVE AND SUBJECTIVE ENTITIES

First of all, it is assumed that reality comprises all of the following: objective, i.e., subject-independent entities/"parts of the world", subjects, and subject-created entities (like mental entities). For instance, above fictitious stories and "their" entities are mentioned to be possibly contained in an ontological structure. Such entities are subject-created entities, certainly existing in a different mode of existence as compared to entities studied in the natural sciences, for example. Moreover, we expect a certain grounding of subject-created entities in concepts derived from observing e.g. physical reality. It is important to note that not each ontological structure must capture / reflect all those kinds of entities. But the option is not be excluded a priori.

VIEWING ALL ENTITIES AS AN OBSERVER-REFLECTION, WITH EXAMPLE

Of course, in an ontology covering "objective" and fictitious contents, a corresponding distinction should be drawn. But even entities that in principle do not require subjects for their existence should be understood, with meticulous precision, as a "reflection"[422] of an observer (e.g., the language user U) in the context of ontological structures.[423] This view is important in order to explain alternative ontological structures that are not directly observable or derivable from objective parts of reality. Consider an example of a cat C which is a female. Let C be directly perceived by a language user U, who recognizes that C is a cat, but who is unaware of C's being a female. Naturally as well as implemented in classical logical semantics in terms of the relationship / distinction between structures and models, it is conceivable for U that C is a female, but equally that C is a male. A very restrictive account of ontological structures would be to allow only for objective parts of the world to serve as ontological structures that a language user would refer to. However, this would cause all structures to support only actual circumstances. In particular, every structure that covers C and the notions of female and/or male could only support the fact that C is a female, according to our assumption above and if constants for C, 'female' and 'male' are each interpreted by exactly those entities.[424]

MOTIVATING A MORE LIBERAL APPROACH

Primarily two arguments suggest to follow a more liberal approach than the last one. Firstly, we argue that nobody has any "direct access" to objective entities,[425] not even considering the idea of acquiring absolute knowledge of / truth about an entity. Secondly, following that restrictive approach would mean to deviate tremendously from classical semantics, which would be problematic in the further definition of ontological semantics as well as for the aim of reusing (in appropriate ways) set-theoretic semantics in terms of classical results and implementations, to the greatest possible extent. A far-reaching deviation of this kind could only be justified by very good reasons (if it can be sensibly construed further, at all). However, we do not see such good reasons to hazard these problems and their consequences.

ONTOLOGICAL STRUCTURES FOR ACTUALITY AND POSSIBILITY

Accordingly, a more liberal view on ontological structures is required. Besides ontological structures that are / correspond to actual circumstances, further ontological structures need to be admitted that correspond to what remains *possible*. Possibility should here be understood as logical possibility, without any hidden restrictions. This allows for "logical" variability of ontological structures, i.e., there will be structures that differ from reality. In the above example, besides the case where C is a female cat, this approach allows for an ontological structure with C being a tomcat. If one aims at being very correct, both structures in

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[421]At least, comments on the nature of ontological structures wrt actuality vs possibility appear below in this sect. 4.2.

[422]The distinction between subject-independent entities and their "reflections" corresponds closely to that between reality ("Realität") and "Wirklichkeit" in [785, ch. 12], there adopted from [716]. In the context of this distinction, cf. also the introductory notes to DOLCE, esp. those on their " 'cognitive' metaphysics", as opposed to a "referentialist metaphysics" [558, sect. 3.1, p. 13].

[423]The question as to what extent the observation exhibits "constructive aspects" must remain open here.

[424]Such requirements play a certain rôle below, see sect. 4.3.1 for 'signature satisfaction', esp. Def. 4.4 on p. 133.

[425]with some reminiscience to the 'Ding an sich' as associated with Immanuel Kant [464]

the example and all ontological structures in general should be viewed as "reflections" of an observer. Some ontological structures therefore correspond to reality while others do not.[426] Admittedly, that way of proceeding introduces an epistemological aspect into ontological semantics. Reusing the argument of the unavailability of any direct access to objective reality from above, however, that step should cause no harmful difference compared to the case of referring directly to objective entities.

TRUTH CLAIMS IN THEORIES AND MODELS

If U states a formal theory with the intention to describe U's ontological intuitions / theory, this usually involves the implicit claim of the truth of the stated theory.[427] "Truth" – in a rather naïve understanding, not in the Tarskian reading that already utilizes structures for its definition[428] – is here understood in the sense of corresponding to reality, but it must be viewed to be limited to *valid* sentences of the theory only, i.e., to sentences that are satisfied in all structures that satisfy the overall theory (in the formal sense). In the cat example from above, if U states as theory merely that C is a cat, then this sentence is to be postulated as true while no claim of truth is involved about 'C is female'. Focusing on a single structure, that structure cannot generally be understood to claim the (naïve) truth of all sentences that are satisfied within a structure. For continuing our account of ontological semantics, these views allow for equating objective entities with their observer "reflections". If the language user makes new observations, e.g., by empirical means (where possible), those should be added to the theory and thereby exclude previous ontological structures that, with hindsight, could only have been logical variations of ("reflections" of) actual ontological structures. Eventually[429], Def. 4.2 of the notion of ontological structure is retained in the form given on p. 128.

SLIGHTLY MORE COMMENTS ON EPISTEMOLOGICAL ISSUES

Two restrictions in relation to epistemiological issues of languages for ontology representation and of ontological structures shall be mentioned in addition. Let L a language and U a user of L. Firstly, U's (e.g., mental) capabilities are assumed to be restricted. Hence, U can intentionally refer to a limited number of entities only, in the sense that U's accessibility of those entities rests on a finite basis. While potentially infinitely many entities may be covered, take natural numbers as an example, we expect a constructive way of description / access that arises from a starting point of finite extent, such as 0 and the successor operation. L should be viewed to be limited accordingly, especially in terms of the constants that it can provide. Ontological structures, as interpretation codomains for L, need only comprise referents for those constant symbols.[430] Secondly, U's knowledge constrains what U can represent. Accordingly, there will be, e.g., relations between entities that U does not / cannot reliably explicate on a factual basis.

4.3 Ontological Models & Signature Aspects

The considerations in the previous sect. 4.2 are tightly linked with the definition of ontological *models* on the basis of ontological *structures*. Compared to common logical definitions, ontological models shall satisfy additional requirements in connection with the signature of a language as well as their behavior wrt meta-theoretical properties.

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[426] Still, there is not just a single ontological structure that corresponds to / is taken out of reality, because one may select extracts with different extensions / "sizes".

[427] Although some authors (including ourselves) may prefer the weaker claim of merely proposing a theory, e.g. as a working hypothesis or as foundation for further development, without alleging to being aware of any "ultimate truth". But that subtlety is deliberately disregarded here.

[428] We refer, everywhere in this work, to a modern Tarskian reading. Notably, Tarski's original notion of 'model' as developed in [811] originates from a substitutional approach that is not immediately tied to the notion of truth, see [63].

[429] Much more could be discussed in this context. For instance, we see no restrictions of entities that "act as" possibilia, apart from logical exclusions due to the theory under consideration. Nevertheless, once that one were about to explicate possible models, we would expect a kind of derivation relation / chains of fictitious entities starting from "reflections" of actual objective entities. A completely neglected aspect thus far is to lift these considerations from the perspective of a single language user to the case of a number of communicating language users, i.e., to a social level. But all of that remains subject to future work.

[430] As usual, equal referents for distinct symbols are admitted in general; but cf. the notion and discussion of 'signature satisfaction', Def. 4.4 in the next section.

4.3.1 Epistemological Standard Models and Signature Satisfaction

ONTOLOGICAL MODELS ARE 'EPISTEMOLOGICAL STANDARD MODELS'

An addition to the usual model notion of classical logic is to pursue the goal that the signature / (non-logical) constants of a language L are interpreted just as the language user U intends them to be interpreted. That means, with ontological models we actually aim at (in analogy) so-called *intended* or *standard models* or, as we would phrase more precisely and explain below, 'epistemological standard models'. However, classical semantics is much more liberal. Regard a very simple example in terms of natural numbers, say, the atomic sentence $0 < 1$. A classical logical structure $\mathcal{A}$ can have arbitrary[431] assignments for the symbols '0', '1', and '<'. $\mathcal{A}$ is a model of $0 < 1$ iff $(0^{\mathcal{A}}, 1^{\mathcal{A}}) \in <^{\mathcal{A}}$. An appropriate interpretation by character strings over $\{a, b\}$ is,[432] for instance, $0^{\mathcal{A}} = a$, $1^{\mathcal{A}} = ab$, and $<^{\mathcal{A}} = \{(x, y) \mid x, y \in \{a, b\}^* \text{ and } x \text{ is a proper prefix of } y\}$. Obviously, this interpretation (with a suitable domain D; $D = \{a, b\}$ suffices) is a classical model of the sentence $0 < 1$. By transferring the classical liberty to an analogous notion of ontological models based on Def. 4.2 of ontological structures, arbitrary entities could be assigned as interpretations to the signature symbols and still yield a model if the conditions are satisfied that are linked to, e.g., atomic sentences, which are defined in sect. 4.4. Eventually, we make allowance for that freedom also for ontological models, with the perspective of *theory comparison* in mind, remembering sect. 2.3.1. Yet by default and related to the complementary view of *analyis explication*, we stipulate for ontological models that '0', '1', and '<' shall be interpreted by the numbers 0, 1, and by <, the less-than relation (from the point of view of intended reference). We write $iref_U(S)$ to refer to *the* intended referent of the symbol S by language user U.[433]

SYMBOL AND SIGNATURE SATISFACTION

In order to distinguish between both types of models, the notion of 'signature satisfaction' is introduced, which plays an important rôle for some applications of ontological semantics. While the term 'ontological structure' (and thus 'ontological model') is opaque regarding signature satisfaction, we refer to structures that interpret at least one symbol not as intended as 'signature ignorant'.

4.4 Definition (signature / vocabulary satisfaction and ignorance)
Let U a user of L, a language over a signature $\Sigma = Sig(L)$, $S \in \Sigma$ a symbol, and $\mathcal{O} = (O, I)$ an ontological structure suitable for Σ.

  $\mathcal{O}$ *satisfies* a symbol and a signature, resp.,   :iff   it interprets those with U's intended referents:

- $\mathcal{O} \approx S$:iff $I(S) = iref_U(S)$.
- $\mathcal{O} \approx \Sigma$:iff $\mathcal{O} \approx X$ for every symbol $X \in \Sigma$.

Otherwise, $\mathcal{O}$ *ignores* S :iff $\mathcal{O} \not\approx S$ and $\mathcal{O}$ *ignores* Σ :iff $\mathcal{O} \not\approx \Sigma$. ❏

GENERAL DEFINITION OF ONTOLOGICAL MODELS, ORIENTED AT FOL

Although ontological semantics for specific syntactic constructions in the case of FOL, i.e., predication, sentential connectives, and quantified formulas are not treated until sect. 4.4 and 4.5, let us formally introduce the notion of 'ontological model', which is continually referred to subsequently (and follows common practice). Moreover, signature satisfaction allows for distinguishing two kinds of ontological models. Note that Def. 4.5 leaves the genericity wrt languages.[434]

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[431]"Arbitrary" modulo respecting that $0^{\mathcal{A}}$ and $1^{\mathcal{A}}$ must be elements of the universe of $\mathcal{A}$, $<^{\mathcal{A}}$ a binary mathematical relation over the latter.

[432]Regarding formal languages, $\{a, b\}^*$ denotes the set of finite character strings of arbitrary length.

[433]Notably, this presupposes that our object languages use only symbols with a unique intended referent, in contrast to, e.g., O in Def. 4.2, p. 128. While some remarks below concern object and meta level, extending $iref_U$ to allow for multiple denoted entities remains a consideration for the future.

[434]Although one may argue that analogous definitions can be derived for other languages, if sufficient parallels are available or can be established, the proposal of ontological usage schemes in sect. 5.2 may be a more convenient way of indirectly assigning ontological semantics to a syntax, which takes advantage of the definitions that are provided from here on.

4.5 Definition (ontological model, free, signature-satisfying, and -ignorant)

Assume Cond. 4.8, where especially $L := Lg(\Sigma)$ is a FOL language over an arbitrary non-empty signature Σ, $\mathcal{O} = (O, I)$ is any ontological structure suitable for Σ, and $\beta : Var(L) \to O$ is any variable assignment. Let $\phi \in L$ a formula.

$\langle \mathcal{O}, \beta \rangle$ is a (*free*) *ontological model* of ϕ :iff it satisfies ϕ according to Def. 4.10, 4.23, or 4.24, denoted by $\langle \mathcal{O}, \beta \rangle \approx \phi$. If ϕ is a sentence, $\mathcal{O} \approx \phi$ abbreviates that $\langle \mathcal{O}, \beta \rangle \approx \phi$ for all variable assignments $\beta : Var(L) \to O$.

The *model* property is canonically extended to any set of formulas $T \subseteq L$ by $\langle \mathcal{O}, \beta \rangle \approx T$:iff $\langle \mathcal{O}, \beta \rangle \approx \psi$ for all $\psi \in T$. If T is a theory, $\mathcal{O} \approx T$ abbreviates $\langle \mathcal{O}, \beta \rangle \approx T$ for all variable assignments $\beta : Var(L) \to O$.

An ontological model $\langle \mathcal{O}, \beta \rangle$ of a formula or a set of formulas is a *signature-satisfying ontological model* :iff $\mathcal{O} \approx \Sigma$. Otherwise, an ontological model $\langle \mathcal{O}, \beta \rangle$ is a *signature-ignorant ontological model*. ❑

The attributes 'free', 'signature-satisfying', and 'signature-ignorant' are introduced in order to allow for clear expression / distinctions, where needed. As stated above and where in doubt below, our default reading of plainly using the term 'ontological model' shall be taken as a synonym for 'signature-satisfying ontological model'.

TERMINOLOGY PURPOSE BEHIND EPISTEMOLOGICAL STANDARD MODELS

This default assumption is clearly a difference wrt the classical blueprint, esp. from the point of view of mathematical logic.[435] Its rationale arises from different aims of logic and ontology, and to some degree from the epistemological issues touched upon at the end of the previous sect. 4.2. Laws of logical truths are among the subjects of logic, concerning the "formal" truth of sentences that arises from its components, regardless of any specific interpretation. This vindicates the view of allowing for arbitrary interpretations of signature elements. The rôle of formalized ontologies in computer science and information systems is quite different. Ch. 2, esp. sect. 2.2 points out that their purpose is to provide categories / entities as a common basis for accomplishing (conceptual) semantic interoperability. In essence this is at least a part of the purpose of terminologies[436], namely to achieve a form of language standardization, with unambiguous reference to concepts – to the best possible extent – as a central idea. Despite not being enforced in general, we contend that this reference to entities[437] should be utilized wrt the semantics of ontology representation by default and as much as possible.

EPISTEMOLOGICAL ASPECT ON SENTENCES

It is important to note that we consider "standard models" / "intended models" to be "epistemologically limited". Ontological models correspond to what a language user U refers to knowingly – and must be open to what U deems / leaves possible beyond stated knowledge. In particular, ontological models do not reflect unlimited objectivity / "absolute knowledge", neither universally, nor wrt a domain. For instance, "the" frequently discussed non-standard models of natural numbers that comprise elements that are separated from 0 by infinitely many others, cf. e.g. [693, sect. 3.3, esp. p. 106ff.], [221, sect. 3.1, esp. p. 179], are not per se excluded from the collection of epistemological standard models. The only restriction that ontological models shall impose over classical models by means of signature satisfaction is that entities referred to by explicit names / by non-logical constants are interpreted according to U's intention.[438] For example, if

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[435]Intended models have been considered earlier, of course. For instance, see Sher's summary [749, p. 363–365, in sect. V] of Quine's view of ontological commitment by regarding the intended models of first-order formalizations, cf. e.g. [685]. Another discussion from a logical-philosophical perspective is available in [187, esp. sect. 7]. 'Intended models' or 'interpretations' are also familiar in AI, cf. e.g. the mentioning in [722, p. 246]. Further links to philosophy are given by the *Direct Reference* theory of names, e.g. associated with works of Ruth Barcan Marcus [47] and Saul A. Kripke [487] in [702, sect. 2], and the related notion of *rigid designators* in modal logic, see [503].

[436]This occurrence refers to terminologies / terminological systems as information artifacts, in contrast to terminology as a discipline. For references (rather) to the field in general, cf. [884] for terminology and terminology management. Moreover, there is a huge body of work related to terminology in the biomedical domain, cf. e.g. [154, 155, 188, 189]. The latter context has also seen controversial debates on the relationship between terminology and ontology, and on the use of "the" notion, actually several notions of 'concept', cf. e.g. the criticism of Eugen Wüster's work in terminology, esp. of (the first edition of) [885], in [764].

[437]including categories (not only, but possibly and even usually)

[438]In a sense, with signature satisfaction all constants are to be viewed as *rigid designators* [503], if models are equated with possible worlds.

U sets up a vocabulary with constants 0, 1, and $<$ under the above (and conventional) interpretation, a *signature-satisfying* ontological *model* satisfying the sentence $1 < 0$ remains possible. The motive for this view is that *referring* to 0, 1, and the less-than relation *as entities* does not automatically yield / entail an a priori theory about those entities.[439]

(IN-)EQUALITY-GENERATED TAUTOLOGIES DUE TO SIGNATURE-SATISFACTION

Noticeably, a certain a priori theory seems to emerge through fixing the reference of constants, namely concerning their equality and co-reference, resp. Since all ontological models with signature-satisfaction interpret 0 and 1 conventionally, they all interpret them as distinct entities. Hence, the sentence $0 \neq 1$ is satisfied in every signature-satisfying ontological model. Constrained to this class of models, that sentence follows even from an empty theory, i.e., it is a tautology (based on ontological semantics under signature-satisfaction). [46, 47, 487] are mentioned in [702, sect. 2] for analogous observations in a philosophical context concerned with modal logic, including a pointer to a formal proof in [46]. Generalizing the argumentation for other vocabularies, requiring signature satisfaction at the side of semantics seemingly leads to (in-)equalities as "new" tautologies that arise in addition to FOL tautologies under classical semantics (based on the assumption that those remain valid under ontological semantics).[440]

EPISTEMOLOGICAL ISSUES APPLY EVEN TO REFERENCE

However, epistemological limitations might apply even in connection with intended reference, to the extent that co-reference can be indeterminate for the language user. Even without recourse to more intricate classical puzzles of co-reference like that of morning star and evening star, cf. e.g. [178, 243, 702], it is hard to defend that all questions of co-reference are as clear as that of the natural numbers 0 and 1. This potentially leads to a gap between reality and the knowledge of a language user U, which is to be explicated in a formalized ontology. There the following case appears peculiar to us. If two symbols S and S' are used by U without any knowledge of their (discoverable) co-reference, i.e., while actually there is only one and the same entity that U refers to by means of S and S', it is unclear how language semantics should account for $S = S'$ as a tautology.

EXPLICATE (IN-)EQUALITIES LEADING TO NEW "TAUTOLOGIES" FOR THREE REASONS

The more general question, also in connection with considering signature-ignorant ontological models in some occasions, is: How should signature satisfaction be related to the notion of tautologies in ontological semantics? For three reasons we believe that theories should be augmented with *explicit* declarations of (in-)equalities of all constants (as far as known). Firstly, (in-)equality-generated tautologies are signature-dependent, i.e., a formula resulting from substituting certain vocabulary elements by others of the very same type (and arity) does not necessarily present a tautology, as well. The reason is that (in-)equality-generated tautologies arise from signature satisfaction of certain constants. Secondly, yet less significantly,

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[439]This claim could be supported by a more elaborate discussion of cognitive aspects. For instance, a mental image (including the relevant cognitive processes) of a directly perceived object mediates such an intended referent which the observer (and language user) can refer to immediately in terms of a symbol. Certainly, every object is perceived in the context of a situation. But we would not equate this with the case that the observer has already got a theory of the perceived object (although perception might be a step towards such a theory). For abstract entities, e.g. notions like category (in an ontological reading) or set, that kind of argument cannot be applied straightly. Indeed, our view on the history of set theory indicates a stronger relation of reference and an underlying theory.

Moreover, fictional entities, in particular individuals pose further issues, because one cannot assume any mind-independent reference for them. Further dealing with this overall complex of questions, however, must remain a worthwhile task for future research, taking into account the varied body of philosophical literature on the topic. As a working hypothesis, we regard reference in general to involve (at least) two facets: (a) one of mind-independent existence and (b) reflection (or generation) in the mind (of the language user). For reference to abstract entities, the position is adopted that it can be performed in a manner largely detached from the theory about that entity. A brief passage by Chris Patridge [658, sect. 3.3] seems to broadly refer to ideas similar to the approach of determined reference without a theory. In terms of earlier theories and terminology, he refers to, in Frege's terminology, reference without sense, cf. [248], (which must be taken modulo the cognitive layer of reference discussed above) and, following Mill, cf. [585], denotation without connotation, further linking to Ruth Barcan Marcus' notion of tagging in [48]. Cf. further the next footnote.

[440]Notably, this could have further effects in combination with the notion of predication definitions introduced in sect. 4.4. According to their overall rôle in ontological semantics, theories just comprising predication definitions may be compared with empty theories in the classical case, to the extent that the axiomatization of undefined entities is empty. Equality tautologies and predication definitions can then have consequences that arise in all signature-satisfying ontological structures. For example, consider introducing definitions like $\forall x . P(x) \leftrightarrow x = 0$ and $\forall x . Q(x) \leftrightarrow x = \mathsf{zero}$. If co-reference of 0 and zero is intended and signature-satisfaction employed, then $0 = \mathsf{zero}$ is a tautological equality (in the language of the respective signature). This immediately yields $\forall x . P(x) \leftrightarrow Q(x)$, clearly no (in-)equality, as another non-classical "tautology", i.e., merely due to the predication definitions.

explicit (in-)equalities that are added to a theory T, yielding T', exclude some free ontological structures as models (compared to those of T itself), in favor of slightly greater proximity to their signature-satisfying counterparts of T.[441] Thirdly, the explication of constant (in-)equalities supports the reuse of set-theoretic semantics and reasoners, cf. sect. 4.6.

4.3.2 Constants for Direct Reference in Analysis Explication

PLURALITY OF ONTOLOGICAL MODELS

Let us return to "unintended" structures that may arise as models of a theory, such as a structure of natural numbers satisfying $1 < 0$. In order to exclude / avoid those, the axiomatic method is still required. In general, in the approach of ontological models that are 'epistemological standard models', many such models are possible. This is contrary to a single, objective standard model of a given set of entities, because only the constants of a language are fixed in the interpretation.[442] Rephrased, *reference* to entities must meet user intentions in ontological models, but the *intension* of that user for those entities needs to be captured axiomatically, i.e., characterized exactly in terms of their logical interrelationships with other entities. Consequently, the Galois connection between theories and models [770][443] will apply to ontological models, as well. The weaker / smaller a theory is, the more models exist / are possible.

KEY ARGUMENT: SEMANTICS MUST SUPPORT WHAT IS TO BE REPRESENTED

The key argument for demanding signature satisfaction for ontological semantics by default is the following. Language semantics defines the meaning of syntax. An ontology representation as a syntactic object is therefore to be understood in terms of the semantics of the underlying representation language. If ontologies are to provide unambiguous / unique / definite reference to entities, the semantics of the representation language must support that feature itself in the first place.[444] However, there remains a gap between the need for unambiguous reference at the one hand and purely axiomatic characterizations at the other, because the latter will remain principally incomplete wrt achieving a unique characterization of all entities.

EXAMPLES AT DIFFERENT LEVELS OF GENERALITY

The last claim is visible in a number of "pragmatic" circumstances / examples. Generally, in most subject domains, e.g. as in biology or medicine, one must expect that an enormous fraction of U's (the language user's) intension / knowledge of an entity does not enter / has not yet entered axiomatic theories formalizing U's knowledge. However, from a pragmatic point of view, even rather simple[445] theories like taxonomies prove useful in practice. Indeed, systems like SNOMED [@115] are employed for the purpose of unambiguous reference.[446] The limited / partial capture of knowledge, under a classical semantic approach, allows for numerous unintended models and leads to equating theories and entities within those theories that should be kept distinct for the pragmatic reasons just mentioned. As stated above, unintended models in terms of unintended sentences that a theory may not prevent / prove false will not be avoided by epistemological standard models. But establishing the intended reference to entities helps to discriminate models and theories. Recall first structures that are isomorphic to an intended model. Formally, these are models, but they

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[441] Where T and T' should have the same signature-satisfying ontological models, because T' is an extension of T by tautologies.

[442] Indeed, until here we are still considering the case of mere constants, i.e., not yet covering any syntactic constructions. Even in this context multiple models arise from the – unknown / unstated / not entailed – existence of further entities.

[443] Herein, this view is attributed to F. William Lawvere's [510], which seems to be in direct and central connection with the theory of institutions [283, cf. esp. sect. 2.1 for this claim], initiated by Rod Burstall and Joseph Goguen. On institution theory, cf. sect. 2.2.2.2, near FN 245, p. 68.

[444] Admittedly, this does not provide any better account of unambiguous reference to entities than is given by, e.g., URIs in the Semantic Web. Mere constant symbols convey neither any meaning / intension, nor intended referents, hence the problem of, e.g., how to share/communicate those referents remains. Nevertheless, the imputation of unambiguous reference by the semantics is a theoretical assumption or postulate, i.e., we expect a language user to believe that using a vocabulary element refers to a clear referent (clear at least for that user). It appears reasonable to us to incorporate this assumption into the language semantics in order to support / justify, from the user's perspective, e.g., unconventional translations between languages.

[445] "Simple" in terms of the type(s) of axioms contained, not wrt the number of axioms or other theory characteristics. For instance, SNOMED [@115] is simple in this sense [31, p. 325] (referring to [774, 775]), if all theories in the description logic $\mathcal{EL}$ [31, 32] are considered simple due to the restricted language constructs of $\mathcal{EL}$.

[446] This applies in spite of the weaknesses and many ideas for enhancements that "real-world" systems (intensified by a size like that of SNOMED) may exhibit, cf. e.g. [740].

are unintended models in terms of the definite reference of the entities named by constants in the language. They cannot be eliminated by any increase of the logical expressiveness of the language, e.g., by sentence constructors.

Another case arises from the purely formal comparison of theories.[447] Consider a very simplified example in terms of two classical theories T_1 and T_2.

(4.7) $\quad T_1 = \{\forall xyz \,.\, \mathsf{above}(x, y) \wedge \mathsf{above}(y, z) \rightarrow \mathsf{above}(x, z)\}$

(4.8) $\quad T_2 = \{\forall xyz \,.\, \mathsf{below}(x, y) \wedge \mathsf{below}(y, z) \rightarrow \mathsf{below}(x, z)\}$

All that differs between T_1 and T_2 is the use of different predicate symbols. From a classical point of view, the freedom of interpreting signature elements leads to the same class of structures satisfying T_1 and T_2, if the semantic entities / structures are detached from the mapping aspect of the interpretations (which corresponds to the mere assignment of terms). In effect, that means that T_1 and T_2 have the same models, i.e., they are semantically indistinguishable. The formal perspective suggests the same indistinction for the symbols that occur in both theories, namely that they refer to one and the same entity. Analogous cases can be made for larger signatures, and even if the theories are complete.[448]

ONTOLOGICAL SEMANTICS PRIMARILY FOR ANALYSIS EXPLICATION

The above perspective is perfectly sensible logically, but not for fixing ontological entities as referents. Importantly, this and the previous arguments shall not diminish the role of formal comparison or the comparative perspective in sect. 2.3.1, e.g. for matching ontologies that originate from different sources. For these cases it may be very valuable to detect that certain theories – e.g. modulo a mapping between their signatures – are shared among the two ontologies. However, we also aim at an appropriate semantic account for the *analytic / analysis explication* perspective. From this perspective, T_1 and T_2 may have been provided by the very same ontology engineer U, and for U above and below are clearly distinguished by different intended referents. Enforcing signature satisfaction based on Def. 4.4 and 4.5 for ontological models allows U to argue that T_1 and T_2 have different models. Another case of application, although that would require a more detailed look, is conceivable wrt distinct versions of an ontology / conceptualization, e.g. during its development, or due to contemporaneous formalizations in differently expressive logical languages (cf. sect. 1.2.1).

OBJECT AND META LEVEL AND FAITHFULNESS OF ONTOLOGICAL SEMANTICS AND THEORIES

Admittedly, the above statement across distinct formalized ontologies is meta-theoretical from the point of view of the two ontologies. But the tenability / plausibility of arguing across models of different theories is based on another position that is adopted for ontological semantics. Due to the fundamental nature of ontological analyses, ontological semantics and ontological theories – as explications of conceptualizations and syntactic "reflections" of ontological models – shall each be *faithful* to themselves. On the one hand, this can be understood as a mere paraphrase of requiring/aiming at ontological neutrality for the semantics to be developed, cf. Def. 3.21 in sect. 3.4. But that focuses on the aspect of avoiding "hidden" entities due to the language semantics. On the other hand, there is the distinction between object level and meta level as such, as introduced by Alfred Tarski, see e.g. [803, esp. p. 347–352] or the brief outline in sect. 1.2.3.2.[449] By faithfulness of ontological semantics to itself we mean that the same semantics (and resulting meta-logical notions, such as consistency or entailment) is, or at least can be, applied at the meta level. Faithfulness of ontological theories to themselves only arises if an ontology covers entities that are relevant for meta-level considerations. For example, this is the case for the notion of symbol and, more generally, for top-level and abstract core ontologies. Accordingly, faithfulness then refers to adopting the same theory (or at least a weakening of it, akin to the study of inner models [212, sect. XI.1, p. 179 ff.] in set theory) as part of the meta-level theory.

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[447]Remember the perspective of theory comparison in sect. 2.3.1 and, for a further moment, let us ignore that syntactic constructions under ontological semantics have not been treated yet.

[448]For instance, cf. certain theories of time, e.g., in [849, sect. 1.3 and 1.4].

[449]Tarski was mainly driven by the antinomy of the liar [802, sect. 1, esp. p. 270 ff. and p. 278–279] [803, p. 347 ff.], further mentioning the antinomies of definability (by Richard) and of heterological terms (by Grelling-Nelson), cf. also [67].

NB on 'object level' and 'meta level': in [803], he argues against "semantically closed languages" and introduces / uses the notions of "object-language" and "meta-language" [803, sect. I.9, p. 349 ff.].

THEORY EXTENSIONS: IMPROVEMENTS, BUT NO OVERALL SOLUTION

Returning and concluding the example above, referring to two distinct ontologies wrt above and below should be understood as components / sub-theories of a broader conceptualization defended by U, probably not (yet) expounded as a unified theory.[450] The union of T_1 and T_2, possibly extended, might have appeared as an immediate "solution" of the above problem to one reader or another. For example, one could adopt the theory $T_1 \cup T_2 \cup \{\forall xy \,.\, \mathsf{above}(x,y) \rightarrow \neg\mathsf{below}(x,y)\}$, wrt which above and below can no longer be interpreted equally. In this sense a theory extension is a solution to that specific problem. Of course, the argument of isomorphic models (under classical semantics) shows that theory extensions cannot provide a comprehensive, general solution to definite entity reference.[451] Pausing for a moment, much more discussion of further logical properties looms ahead. However, this is better realized after the semantics is fully defined.

CLOSING THE STANDARD MODEL DISCUSSION & FINAL EXAMPLE OF NATURAL NUMBERS

In summary of the discussion of epistemological standard models, ontological semantics in its default reading is associated with the particular type of intended models outlined above. To round this off, let us again illustrate epistemological standard models in terms of natural numbers, and the comparably elaborated / well-understood notions of standard and non-standard models wrt this domain. Postulating the existence of numbers ontologically, the set of all natural numbers is typically felt to be clearly understood as the smallest set of entities created by a successor function applied to the (ontologically postulated) natural number zero. Hence, an axiomatization like Peano's, cf. sect. A.2.2 in the appendix, or e.g. [691, p. 83 ff.], [221, ch. 3], provides a specification of interdependencies among natural numbers. It is common to find phrases about "the standard model of natural numbers".[452] This should embody the aspect of standard reference, i.e., that constants like '0' and '1' are interpreted by 0 and 1, respectively, as well as the aspect of absolute knowledge within the domain, e.g., the standard model of natural numbers satisfies the sentence $\forall x \,.\, 0 \leq x$, as well as "all other true sentences about natural numbers" (in the respective signature) – independently of any axiomatization. That second aspect is not available from epistemological standard models as required for ontological semantics. Only the reference aspect is maintained, such that constants are interpreted by intended referents, whereas multiple epistemological standard models remain possible, depending on the strength of the axiomatization. According to the aspect of reference, common reductions of the natural numbers to, e.g., set-theoretic constructions like in [212, ch. V] cannot be considered ontological in this sense, because of the non-intended referents. Those models rather present *encodings*, cf. sect. 2.2.1.

4.3.3 Constants and Expressiveness

COLLECTIONS OF CONSTANTS SUFFICE FOR REPRESENTING SITUATIONS

Having settled the issues of epistemological standard models and reference for the present context, it is due to return to the still very constrained stage in defining ontological semantics. Up to this point, ontological structures are only employed for interpreting atomic symbols. Syntactically, this is a severe restriction, and the usual first-order syntactic compounds will be considered in the next section. However, let us conclude this section with brief considerations on the expressiveness of just using constants, i.e., the following issue.

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[450]One should not lose sight, though, of the seemingly exponential growth of the quantity of ontological content of different domains that is being produced nowadays. For a (highly incomplete) impression, consider solely the NCBO BioPortal [@15] [636] with, as of January 27, 2014, 376 ontologies covering almost 6 million classes / categories (precisely: 5,986,820). This justifies that it appears unrealistic to consider only a "linear" development of a single ontological theory. Cf. also the next FN.

[451]Not even the idea of a universal, complete theory of the world could account for this purpose, because permutations of the mapping of constants to referent entities were conceivable. But making this argument shall not hide the fact that such theory of absolute knowledge, of each and every entity and all of its connections to every other entity is impossible on principle. The necessity of a physical grounding of symbols in combination with the limitations of physical resources is one simple reason for that impossibility.

And this being stated without even touching the philosophical debate over a, hardly less ambitious, "universal ontology" that seems to be pursued in some works, e.g. [1], but is widely considered impossible. [499, esp. sect. 1 and 5] offers several convincing arguments for logical and ontological pluralism, declining a universal ontology as well as a universal ontology language. Cf. also FN 273, p. 74.

[452]I.e., using singular wrt "model", which we will adopt below. The common mathematical non-distinction between isomorphic structures should be kept in mind, however.

$\mathcal{Q}_{10}$ Can complex interrelations between the entities e_i in an ontological
structure be formalized by the mere use of constants in a language?

Uttering / stating a number of constant symbols is taken as claims of existence for their referents, (some of the) e_i. Indeed, claiming the existence of several entities (of appropriate kinds) can already model situations. Propositional logic accounts for well-known evidence: a single propositional variable may be used to denote an overall situation, like 'Chair e_1 has the seat e_3 as part', loosely speaking. We clearly simplify matters here by not drawing a distinction between, in particular, propositions and situations / states of affairs. Sentences and facts are further notions that may likewise be considered if one were to enter corresponding analyses. Moreover, we adhere to the use of 'propositional *variable*', mainly because the term 'propositional constant' is commonly interpreted by syntactic reflections of truth values, e.g. $\top$ and $\bot$. From the point of view of formalizing natural language by means of propositional logic, however, one may assume an intended interpretation for propositional variables. In this sense propositional variables are utilized in connection with discussing constants, in general.[453]

A PLURALITY OF CONSTANTS CAN REPRESENT SITUATIONS

However, more fine-grained representations are possible if apposite entities are ontologically assumed. For instance and in parlance of the $\mathcal{CR}$ ontology, see sect. 2.4, assume that e_1 is a chair, e_3 is its seat and e_2 a part-of relator which connects e_1 as the whole to e_3 as the part. If we use constants c_1, c_2, c_3 to denote those entities (interpreted by e_1, e_2, and e_3, resp.), the sequence / set just stated may be understood to represent that e_3 is a part of e_1. Despite the use of distinct symbols which may suggest "isolated" entities, those entities bear further "internal interrelations" summarized in the descriptive sentence before, which arise from their mere existence. In particular, e_2 as a relator has mediating capacities, as described in sect. 2.4.3.2–2.4.3.3. Likewise, if c_2' stands for an instantiation relator e_2' which connects e_1 and $e_3' = $ 'the category chair', denoted by c_3', three corresponding constants state that e_1 is an instance of chair: c_1, c_2', c_3' (where order is irrelevant).

CATEGORIES REQUIRED FOR COMPREHENSION (ONLY)

Note that, at least wrt the $\mathcal{CR}$ ontology availed here, this applies in spite of the comprehension[454] / understanding deficiencies of constants that denote individuals. *Comprehension* of a fact like 'Chair e_1 has the seat e_3 as part' necessitates the categories 'chair' and 'seat', and clearly requires the connection of e_2 to the 'part-whole' relation. Being only aware of e_1–e_3 as individuals would not convey the situation to a language user. However, this does not affect the idea that exactly the relator e_2 delivers the mediating relational connection between e_1 and e_3. Again and as stated in sect. 2.4.3.2, we consider relators to connect their relata immediately, independently of further possibilities of analyzing a relator in question more closely (or entities from which that relator derives).

CONCLUSION: CONSTANTS SUFFICIENT FOR FACTS, BUT INEFFICIENT

Let us conclude this intermediary commentary on ontological structures / models and their potential. Concerning the expression of simple facts, it may well be sufficient to introduce only constants in a language, as the most basic and unbiased form of representation. However, such form of representation is neither succinct nor versatile, and logical connections are still lacking at this stage. Accordingly, the introduction of syntactic constructions over atomic symbols is due next.

4.4   Semantics of Predication

GENERAL COMPOSITION PERSPECTIVE WITH GOALS & LIMITATIONS FOR THE NEXT TWO SECTIONS

In general, our main analytic assumption on symbol composition is that arbitrary compound syntactic constructs are analyzable as related atomic symbols. For instance, in the case of written languages or, more

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[453]Cf. further Enderton's [221, sect. 1.1, p. 18, remark 3] on the terminology of "sentential logic" vs "propositional logic", and the examples provided in [ibid., p.19]. In another line of thinking, matters relate to the question of the differences between the ability to understand a natural language and prelinguistic capabilities, cf. [743, esp. sect. VIII, p. 15 ff.].

[454]The term is to be read in an intuitive sense in this paragraph, instead of referring to the technical / logical notion, e.g. as in "comprehension axioms"; cf. [221, p. 270 ff.] for the latter.

generally, languages for visual reception, appropriate relations are commonly of spatial character (e.g. proximity of symbols) or similarity relations wrt shape, color, etc. In particular, this view is meant to include diagrammatic languages like UML [95, 720].[455] Nevertheless and in accordance with our chief application context, further discussion is conducted wrt FOL syntax, with the aim to provide ontological semantics for the well-known first-order constructs in this and the next section.

Functions (with one or more arguments) and thus non-atomic terms are widely neglected for a simpler exposition, also in the light of viewing functional expressions solely as convenient abbreviating notation of relational interconnections[456] and corresponding results under standard set-theoretic semantics by means of which functions can be eliminated by means of relational "rewriting", cf. [214, sect. VIII.1], and may even be considered partial instead of total, cf. [214, sect. III.7.1]. The applicability of these arguments to ontological semantics is justified with hindsight on the basis of its connection to classical semantics, as established in sect. 4.6. Apart from functions, another category of symbols and its appropriate treatment is silently taken for granted, namely technical auxiliary symbols such as parentheses or separators.

4.4.1 Analysis of Predication and Naïve Definition

PREDICATION ANALYSIS

Predication is the most basic syntactic composition in the context of FOL that leads to formulas. Concerning ontological semantics, it provides another significant difference compared to the set-theoretic approach. Consider examples of unary predication like $\mathsf{Apple}(x)$ and $\mathsf{Red}(x)$. For the language user U the same syntactic way of composition bears an informal / conceptual meaning that *can differ* in terms of the relationship between the unary predicate and its argument. It is easily conceivable that U reads $\mathsf{Apple}(x)$ to represent that x is an (instance of the[457] category) apple, whereas U might understand $\mathsf{Red}(x)$ to mean that x has the color red (or more elaborate descriptions like x exhibits red among its surface coloring).[458] This is certainly not a new observation, as Nicola Guarino's sect. "2. Reds and apples" in [326] demonstrates, providing greater detail and further potential understandings of $\mathsf{Red}(x)$ [ibid., Fig. 2 on p. 447].[459] Moreover, numerous philosophical debates and approaches to properties (and the various senses discussed for the term 'property' in philosophy) [795], cf. also sect. 2.4.2.1 above, indicate this "overloading" of the syntactic composition.[460] The same argument applies to predicates of arbitrary arity. Certainly many cases of, say, binary and ternary predicates can be analyzed in terms of (ontological) relations. But even there different interpretation schemes can be applied. Furthermore, a predicate $\mathsf{walk}(x, y, z)$ may stand immediately for a *process* of walking in which x and y are involved and which ends at z, instead of a *relation* that one may think of to exist on the basis of such process. Generally, in order to keep languages manageable for human users, the number of basic construct types and their compositions must be limited.[461]

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[455] It may be harder to defend the same position for images / pictures, e.g. due to indefinite atomic symbols in that case, but this subject will not be enlarged upon here. Notably, a broad, valuable overview on pictures and diagrams is provided by Alexander Heußner in [400, esp. ch. 4-6].

[456] We distinguish between functional expressions (from a logical point of view) and functions (from a broader mathematical point of view). In particular, the latter lend themselves to a genuine ontological analysis, e.g. remembering lambda calculus in FN 416. We see no immediate connection between these two views.

[457] I.e., a specific category / understanding of apple that U refers to, although we admit that this is a simplifying assumption. It remains future work to study the consequences of utilizing different types of categories, cf. sect. 2.4.2, in cases like this one.

[458] For readers aware of GFO, the example corresponds to distinguishing 'category' and 'property' in GFO, cf. [392, sect. 14.4.5.1].

[459] Overall, the matter connects closely to work of Guarino et al. on metaproperties, see first comments on that herein at the end of sect. 2.4.2.1. Sect. 7.2.1 contains a brief closer examination of the relations between ontological semantics and metaproperties.

[460] This is clearly no problem that is specific to FOL or even logical languages. According to our experience, very few languages or information systems are so appropriately specialized, documented, and can be employed in such a standardized manner that no overloading or "misuse" / encoding of unsupported states of affairs occurs. As an example from a specific domain, think of the plurality of existing bibliography tools and formats. Editors of collective works like event proceedings are commonly supported by means of a corresponding field or data element, whereas the distinction between editors (in a literal sense) and the program chairs of an event is rarely reflected (syntactically). Hence, some users might "misuse" the editor field for specifying program chairs.

[461] Even though some languages provide an already sophisticated set of constructions, e.g., UML as a language for a broad spectrum of purposes. Languages focusing on specific purposes seek the required limitation in a different way. In particular, domain-specific languages (DSLs), as en vogue anew nowadays and traced back in computer science even to the late 1950s in [578, esp. sect. 1], are designed for domains / tasks of such size that abstract syntax categories (sect. 2.1) of the DSL reflect a larger fraction of the conceptual distinctions relevant in that domain / task. Cf. also domain-specific modeling languages (DSMLs) in this context, e.g. [190].

RE-ANALYSIS OF THE CLASSICAL APPROACH / FORMALIZATIONS

Classical semantics does not take such distinctions into account. A uniform interpretation of predicates in terms of set membership in (mathematical) relations (of the same arity like the predicate) is defined instead. In a sense, syntactic indifferences are therefore carried over into the semantics. Of course, it is an admissible reply to the argument of uncovered distinctions to require a different formalization. But this reply also points out the difference between what ontological semantics is to provide and what formal semantics is capable of. Descriptions in terms of mathematical entities are formalizations, but they are not immediately / for themselves analytic in an ontological sense. Usual formalizations establish a certain congruence with the modeled domain, according to an – often enough undocumented – conceptualization. But as such, mathematical formalizations must remain limited, in the sense of not being further extensible on some occasions (without a fairly complete re-design / re-formalization). In contrast, ontological analysis should be extensible in an unlimited sense, i.e., we expect that an ontological theory can be extended to as yet uncovered domains of reality/knowledge (and thus entities) without the need for revision wrt covered domains.[462] Similarly as in the previous sect. 4.3.2, this is *not* to be understood as a general criticism of formal methods. Those can be employed very beneficially wrt many problems, benefitting from their conjoined theoretical tools which often lead to back-effects wrt the analytic view of a domain. However, given the analyses in sect. 2.2 we doubt that formalization *alone* is appropriate for large scale data and knowledge representation and integration. Not to be mistaken also in this regard, it is only the "semantic anchoring" that appears impossible if entities (at least some entities) are transformed into / modeled as / reflected mathematical entities.

NAÏVE SOLUTION

In the context of defining the semantics for predication, what could solutions for a differentiating and non-encoding account of predication be? A naïve approach is to fix the individual meanings of predicates in sole recourse to informal descriptions – similarly to the introduction of the semantics of logical connectives and quantifiers (see the next section).[463] In the case of Apple(x) and Red(x) above this could be implemented as follows.

(4.9)   Apple(x) stands for "x is an apple."

(4.10)   Red(x) stands for "x exhibits red."

Predicates introduced in this way do not rely on a formal encoding of the underlying entities, but on those entities themselves (in a direct / their ontological understanding), here the category 'apple' and the color 'red'.[464] Clearly, logical relationships can be stated / axiomatically captured among predicates "defined" / explained in such a way, and respective conclusions can be drawn on that basis[465].

On the other hand, this approach of defining predication semantics remains highly domain-specific. In order to understand and work with such semantics, an understanding of the defining natural-language phrases is required.[466] One would have to expect then that predication semantics would differ for distinct predicates, and thus across signatures. In this respect, there is clearly much less of a problem with set-theoretic semantics, where basic set-theoretic knowledge, e.g. of membership and (mathematical) relations, suffices to understand predication semantics – in a *signature-independent* manner[467] and backed up by axiomatically established and theoretically analyzed systems of set theories. This raises the question for an alternative "between" the naïve, direct approach and the established uniform set-theoretic encoding.

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[462] We see no contradiction to allowing for different views on the same domain of reality / knowledge. For those, as well, we would expect that they can be integrated into an "overarching picture".

[463] One may argue whether this amounts to connectives and quantifiers to the same degree, on the basis of introducing connectives in terms of Boolean functions.

[464] "Color" is used without further analysis here. In terms of GFO, 'red' refers to a property value [392, sect. 4.5.1], [398, sect. 9.1–9.2].

[465] I.e., on the basis of an adopted logical calculus, ignoring the questions of correctness and completeness of that calculus wrt such a semantics for the moment

[466] In addition, no limitations on such phrases were introduced so far, such that even extensive texts could be utilized – taking the approach to precarious extremes.

[467] Although this should not mislead one over the remaining problem of the conceptually correct application of predicates with an intended interpretation. In this connection, notice the latent link to the symbol grounding problem [372], briefly mentioned in sect. 2.2.2.2, p. 69.

4.4.2 Formalizing Predication Analysis and Abstract Core Ontologies

INTERMEZZO: PREDICATION ANALYSIS / DEFINITION TYPES

Two types of analysis shall be distinguished before turning to an alternative proposal. The exemplary phrases (4.9) and (4.10) for $\mathsf{Apple}(x)$ and $\mathsf{Red}(x)$ above define the *relationship* between the two atomic symbols that occur in each predication. It would be a very different enterprise to define the *entities* themselves instead, e.g. to give a characterization of apple á la "$\mathsf{Apple}(x)$ applies to such x that are a roundish, usually green, yellow, or red fruit that grows from trees or shrubs ...". Such definitions, which we call *analytic definitions*, correspond to the common understanding in providing natural language definitions for the constituents of an ontology, e.g., as required for OBO Foundry ontologies [763, p. R46.2] and more generally for bio-ontologies [458, sect. 3.V]. We hold that these are not immediately required for predicate introduction, although they should be explicated additionally (as far as possible) in the course of the axiomatization of a domain. For the understanding of *symbol composition* it suffices to provide "semantic transcriptions" of the relation between the syntactically composed symbols, like those given above. The resulting type of definitions is named, in the general case, *composition definition* herein. In connection with predicates we also use the term *predication definition*. Admittedly, it depends on the conceptual background of the language user U, on the choice of admissible phrases, and their meaning wrt U where or how clearly the borderline between both types of definitions can be drawn. For instance, the following would be an explication[468] of the relation between a(n ontological) property and an entity, applied to $\mathsf{Red}(x)$.

(4.11) $\mathsf{Red}(x)$ stands for "There is an instance of (the property) red that inheres in x."

The general guideline for composition definitions is to accept everything that explains the relation between Red and x, whereas statements that characterize 'red' (expectedly in relation to other named categories / entities) fall under the auspices of an analytic definition / characterization.

2ND OPTION: FORMALIZED PHRASES BASED ON AN ONTOLOGY

Without deeper experience in applying the naïve option from the previous sect. 4.4.1, we expect that most readers agree with the view that that solution remains unsatisfactory.[469] Fortunately, it seems that one can do better, in terms of more methodological rigor. The example of the ontology-based / -inspired explication (4.11) of $\mathsf{Red}(x)$ indicates how a given ontology may be used for this purpose. Because no *analytic* "reduction" of red is required, it appears as an entity in the phrase explaining the composition of Red and x. In addition, relations (instantiation and inherence in the example) and possibly further entities are required. If a corresponding ontology is formally available, the explanation itself can be captured formally. This yields $\exists y \,.\, y :: \mathsf{red} \wedge \mathsf{inh}(y, x)$ in the last case[470] and more simply $x :: \mathsf{apple}$ for $\mathsf{Apple}(x)$. Importantly, the constants red and apple in these formulas are constants that are supposed to have the very same intended referent like the "original" predicate Red, yet they belong to a different abstract syntax category. In the case of the variable x, the same assumption is taken and thus the very same x is employed in the explanatory formula. In general[471], we suggest to use a previously established ontology for describing the semantics of non-atomic[472] syntactic constructs as abbreviations of claims of existence for the very entities involved by signature elements of the construct, together with further, usually hidden existential claims. With the syntactic means of FOL, those explanatory formulas can serve as definientes in proper definitions, e.g.

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[468]The explication is close to GFO and DOLCE, but simplified here by neglecting the distinction of quality and quale in DOLCE, and property and property value in GFO, cf. also FN 299.

[469]One reason is that further well-known issues that are solved / widely accepted in classical semantics and its mathematical foundations must be reconsidered. For instance, how would / should the naïve account behave wrt predicate comprehension? Relying on informal phrases for every predicate would certainly impede such treatments.

[470]The classification of red as a property (parenthesized in (4.11) above) is omitted in the formula in order to link more directly with the phrase structure, and to keep the latter slightly simpler.

[471]The proposal in this sentence can be transferred beyond FOL, thereby applied to other basic syntactic categories of "predicative flavor".

[472]Notice that atomic formulas are compound symbols, as well, composed of a predicate symbol and a sequence of argument symbols. Those formulas are *included* under 'non-atomic syntactic constructs', because 'atomic' therein does not refer to parthood among formulas, but to parthood among symbols. For example, the predicate symbol is one atomic part of the compound symbol that the atomic formula is.

$\forall x$. $\mathsf{Apple}(x) \leftrightarrow x$ :: apple. Again, we use the term *predication definition* as a synonym for 'composition definition' in the context of FOL.

CIRCULARITY OF LANGUAGE AND ONTOLOGY

The approach just started appears to run into an argumentative circularity. In the overall context of the present ch. 4, we aim at providing an ontological semantics for languages / syntax in order to formalize ontologies without the encoding aspect involved in existing semantic approaches. In the course of defining ontological semantics for complex syntactic expressions it is now proposed to rely on an ontology, which is itself to be formalized already. Naturally the question arises in which language that pre-established ontology should be formalized. A conventional language with an extensional semantics must be expected to run counter to the overall aim of an ontological semantics. Accordingly, ontological semantics is required already for that given formalized ontology.

ENDING ITERATED COMPOSITIONAL ANALYSIS

If not a complete resolution, at least an acceptable diminution of this circularity is in prospect. The first main idea is that the iteration of compositional analysis in explanatory phrases eventually leads to analyses involving relations, in line with the arguments presented in sect. 2.4. The second major aspect is that hidden entities and relations that are elicited by composition definitions are of greater generality than the entities already referred to by the signature elements that occur in the syntactic construct in question. In the previous examples, this is clearly the case for the analysis of $\mathsf{Apple}(x)$ as x :: apple, where instantiation is certainly a relation applicable at top-most generality[473]. Similarly, in the formalized phrase $\exists y$. y :: red $\wedge$ inh(y, x), instantiation and inherence occur, which both are relations "situated in" top-level ontologies.[474] We see no way for strictly proving these claims on iterated composition analyses, namely to eventually arrive at relations, and to move to more general categories and relations during this process. One argument may be that in order to analyze / explicate the semantic relation between signature element referents / entities, one is forced to classify / categorize those entities first, so as to determine their relationship subsequently.

TLOs AND ACOs AS FIXED POINT

Following the preceding claims, one should arrive at notions of top-level ontologies (TLOs). Independently of a particular TLO, does that mean that all top-level categories and relations are bound to be introduced only informally, in an ontological semantics for a language for formalizing TLOs? That could still be a rather large number and as such not attractive. But of course, also entities within TLOs can be categorized and eventually are "only" related, which refers back to the notion of abstract core ontologies (ACOs) in sect. 2.3.3. Moreover, ACOs that are capable of analyzing themselves, such as the $\mathcal{CR}$ ontology, cf. sect. 2.4.4.3 above, yield a kind of "fixed point" in the foregoing iteration.[475] For example, instantiation between two entities, which may syntactically be captured by x :: y, can be explained in terms of $\mathcal{CR}$ by the phrase "there is an instantiation relator between x and y, such that x plays the instance-role and y the instantiated-role of that relator". This phrase draws several assumptions from the $\mathcal{CR}$ ontology, e.g. the existence of instantiation and its corresponding role base, see sect. 2.4.3.3. It is to some extent hidden in the phrase that it refers to instantiation itself, by postulating a relator that instantiates the relation of instantiation. It is interesting to note that a syntactic construct like x :: y is very close to representing a situation / fact through a collection of constants, as discussed at the end of sect. 4.3.3. Here the difference is the reference to a relation, i.e., a relator category, denoted by ::, instead of an individual relator. Albeit x :: y cannot be explained directly / only as an abbreviation or form of notation of a set of constants, the remaining gap to the collection of constants that involves the :: relator in the above phrase is reasonably narrowed.

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[473]More precisely, the range of arguments that instantiation (potentially) applies to is only very broadly constrained. In $\mathcal{CR}$, for example, the notion of Categorizable (entities potentially subject to playing the InstanceRL role) extensionally coincides with Entity, while Instantiable is coextensional with Category, cf. Tab. 2.1 in sect. 2.4.3.3. NB: wrt actual relation subsumption, instantiation appears not outstandingly generic, e.g. compared to 'relatedness'.

[474]As an aside, this strengthens the application perspectives for foundational ontologies, i.e., it adds to the value of embedding / ontologically founding domain-specific categories and relations into foundational ontologies. See sect. 2.3.1 for the notions of ontological foundation and analysis.

[475]This contributes to the faithfulness aspect in the previous sect. 4.3.2.

4.4.3 Definition and Examples

4.4.3.1 Predication System and Predication Definitions

BOOTSTRAPPING PREDICATION BASED ON AN ACO AS FUNDAMENTAL THEORY
As a consequence of the last reflections, we take the following approach to defining predication for onto-logical semantics. Firstly, an ontology / conceptualization is adopted as a kind of background theory, in order to provide natural language phrases for defining the satisfaction of FOL atoms with so-called *fundamental predicate symbols* as predicates.[476] Secondly, all other, "normal" predicate symbols are assigned a formula in the overall language itself, to which the satisfaction of their application can be reduced (without circularities). This is captured formally by the notion of a *predication system* in Def. 4.6.

Technically, this definition uses (a variant of) λ-notation / expressions, adopted from / inspired by λ-calculus, cf. e.g. [49, 403] for the latter. However, all that we utilize is the idea of λ-expressions as a means of precisely specifying functions and addressing their arguments, in combination with argument substitutions by applying λ-expressions to others (corresponding to β-conversion in λ-calculus). For instance, $(\lambda x \,.\, P(x))(t) = P(t)$, i.e., the term t applied to $\lambda x \,.\, P(x)$ yields $P(t)$. As the example already demonstrates, the λ-expressions that we use involve either FOL formulas (of the object language) or natural language phrases (with variables). It is further important to note that we employ the means of λ-notation only at the meta level and only in order to define the semantics, whereas the object-level syntax remains plain FOL syntax.[477] On this basis we specify the notion of a predication system. An example of how the predication semantics so defined works follows after Def. 4.10 on the satisfaction of atomic formulas. As a last note, in this section we employ $\dot{=}$ for the (object-level) binary equality predicate, which is an element of the logical signature, in order to distinguish it from $=$ at the meta level.

4.6 Definition (predication system)
Let Σ a FOL signature, *Var* the set of variables for $Lg(\Sigma)$, and $\mathcal{X}$ a set of (metameta-level) variables for (meta-level) entities[478].
A *predication system* $\mathfrak{P} := (\Sigma, \Sigma_F, \Sigma_\Pi, fp, \pi)$ for Σ is defined[479] by these conditions:

- Σ_F, Σ_Π are FOL signatures s.t. $\Sigma = \Sigma_F \,\dot{\cup}\, \Sigma_\Pi$, where Σ_F is called the *fundamental signature* of $\mathfrak{P}$, Σ_Π its *normal signature*, and *fundamental* and *normal predicate symbols* as those in $Pred(\Sigma_F)$ and $Pred(\Sigma_\Pi)$, resp., are analogously distinguished.
- fp is a (total) function that maps each $P \in Pred(\Sigma_F)$ to a λ-expression based on a well-behaved[480] declarative natural language phrase η_P, named the *fundamental phrase* of P, i.e.,

$$fp_P := \lambda x_1 \ldots x_{ar(P)} \,.\, \lambda \bar{\chi} \,.\, \eta_P$$

  where η_P contains only variables from $\{x_1, \ldots, x_{ar(P)}\} \subseteq Var$ or from $\bar{\chi}$, which is a tuple of mutually distinct variables in $\mathcal{X}$. The length of $\bar{\chi}$ is denoted by $ar^{\mathfrak{P}}(P) := |\bar{\chi}|$.[481]
- π is a (total) function that maps each $P \in Pred(\Sigma_\Pi)$ to a λ-expression based on a formula $\delta_P \in Lg(\Sigma)$, named the *predication definiens* of P, i.e., with $fVar(\delta_P) \subseteq \{x_1, \ldots, x_{ar(P)}\} \subseteq Var$,

$$\pi_P := \lambda x_1 \ldots x_{ar(P)} \,.\, \delta_P$$

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[476]The attributive term 'fundamental' is preferred over 'basic' or 'primitive' in order not to interfere with the notion of *primitive relation* as established in the context of the axiomatic method, see sect. 1.1.5.3.

[477]We have noticed a similar notation in [574], although there λ-expressions are constituents of logical formulas in the object language. Moreover, they are intended to be interpreted as relations, instead of as functions.

[478]That means, the range of objects for a (metameta-level) assignment to these variables includes especially semantic entities, such as ontological structures $\mathcal{O}$, pluralities of entities O as their universes, interpretation functions I, (meta-level) variable assignments β, and ontological interpretations $\langle \mathcal{O}, \beta \rangle$; as well as possibly syntactic entities (such as signatures Σ, sets of variables *Var*, individual terms and tuples thereof, etc.).

[479]Note that F and Π act merely as indexes here such that we could likewise have used 1 and 2 instead, although F and Π have a mnemonic character which clarifies with Def. 4.14, Prop. 4.20, and Obs. 4.22. To give a rough idea, F hints at a "fundamental" conceptualization (as a background theory) that is intended to account for natural language definitions of predications of fundamental character, and Π hints at a definitional theory for normal predications.

[480]This notion of well-behavedness can only be defined after Def. 4.10. More precisely, this aspect is covered after Ex. 4.12.

[481]In case of $ar^{\mathfrak{P}}(P) = 0$, $fp_P = \lambda x_1 \ldots x_{ar(P)} \,.\, \eta_P$.

- The *predication definition graph* $\mathcal{G}_{\mathfrak{P}} := (Pred^=(\Sigma), Occ(\pi))$ is a directed acyclic graph, where $Occ(\pi) := \{ (P, Q) \in Pred(\Sigma_\Pi) \times Pred^=(\Sigma) \mid Q \text{ occurs in } \pi_P \}$ and there is no infinite path in $\mathcal{G}_{\mathfrak{P}}$, i.e., there is no sequence $P_0, P_1, P_2, \ldots$ s.t. $(P_i, P_{i+1}) \in Occ(\pi)$ for all $i \in \mathbb{N}$. ❏

4.7 Observation (predication definition recursion terminates in $Lg(\Sigma_F \cup \{=\})$)
For any predication system $\mathfrak{P}$ with fundamental signature Σ_F and predication definition graph $\mathcal{G}_{\mathfrak{P}}$, all paths in $Occ(\pi)$ are finite and the sinks / successor free elements in $\mathcal{G}_{\mathfrak{P}}$ are exactly the elements of $Pred^=(\Sigma_F)$. ❏

Proof. Let $\mathfrak{P} = (\Sigma, \Sigma_F, \Sigma_\Pi, fp, \pi)$ a predication system with corresponding predication definition graph $\mathcal{G}_{\mathfrak{P}} = (Pred^=(\Sigma), Occ(\pi))$. The exclusion of infinite paths[482] in the definition of $\mathcal{G}_{\mathfrak{P}}$ yields immediately that all maximal paths terminate in a sink in $\mathcal{G}_{\mathfrak{P}}$, which is any $P \in Pred^=(\Sigma)$ such that there is no Q with $(P, Q) \in Occ(\pi)$. Due to $Pred^=(\Sigma) = Pred^=(\Sigma_F) \mathbin{\dot\cup} Pred(\Sigma_\Pi)$, it suffices to show for every $P \in Pred^=(\Sigma)$: $P \in \Sigma_\Pi$ iff there is a $Q \in Pred^=(\Sigma)$ s.t. $(P, Q) \in Occ(\pi)$. If $P \in Pred(\Sigma_\Pi)$, it has a predication definition π_P which must contain at least one atom, i.e., there is a $Q \in Pred^=(\Sigma)$ that occurs in π_P. For $P \notin Pred(\Sigma_\Pi)$, there is no such Q immediately due to $dom(Occ(\pi)) = Pred(\Sigma_\Pi)$. ❏

Obs. 4.7 ensures that predication semantics is well-defined, eventually on the basis of the conceptualization for the fundamental predicate symbols. As proposed in the preceding analysis, ACOs should be suited and utilized to filling this rôle, thereby bootstrapping the semantics of predication satisfaction for ontological structures.

ONTOLOGICAL SEMANTICS FOR ATOMIC FORMULAS

For the definition of an ontological structure satisfying an atomic formula $P(\bar{t})$, as well as for subsequent considerations, we stipulate the following conditions for recurring setups. We proceed in analogy to the definition of classical semantics provided in the preliminaries in sect. 1.5.2. Then everything is prepared for the central Def. 4.10 of predication satisfaction for ontological semantics.

4.8 Condition (Λ, Σ, *Var*, $\mathfrak{P}$, L and an interpretation $\langle \mathcal{O}, \beta \rangle$)
Assume the following language setup.

- $\Lambda = \{\neg, \wedge, \vee, \dot\vee, \rightarrow, \leftrightarrow, \forall, \exists, \dot=\} \cup \{(,),.\}$ is the logical signature
- Σ and *Var* are two disjoint, non-empty sets of symbols, where *Var* contains variables and the signature Σ comprises only predicates and individual constants.
- $\mathfrak{P} = (\Sigma, \Sigma_F, \Sigma_\Pi, fp, \pi)$ is a predication system for Σ
- $L := Lg(\Sigma)$

On this basis, let further $\langle \mathcal{O}, \beta \rangle$ an arbitrary ontological interpretation, where $\mathcal{O} = (O, I)$ has the universe O and is suitable for Σ (i.e., $dom(I) = \Sigma$), and let $\beta : Var \rightarrow O$ a variable assignment. ❏

4.9 Condition ($P(\bar{t})$, P, and $\bar{t}$)
In addition to and based on Cond. 4.8, we determine:

- $P(\bar{t}) \in At(L)$ is an arbitrary atomic formula, hence
- $P \in Pred^=(\Sigma)$ is an arbitrary predicate symbol (incl. equality), and
- $\bar{t} \in Tm(L)^{ar(P)}$ is an arbitrary tuple of terms over Σ and *Var* of an arity equal to that of P. ❏

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[482]Excluding infinite paths actually excludes cycles already, hence acyclicity may be omitted from the definition of $\mathcal{G}_{\mathfrak{P}}$. In the case of finite signatures, $\mathcal{G}_{\mathfrak{P}}$ is finite and acyclic. If the signature is infinite, it must be proved that $\mathcal{G}_{\mathfrak{P}}$ contains no infinite paths. In addition, note that an infinite $\mathcal{G}_{\mathfrak{P}}$ can contain infinite sequences in the reverse direction of $Occ(\pi)$, allowing for infinite schemes of normal predicates.

4.10 Definition (predication satisfaction / atomic formulas)

Assume Cond. 4.8–4.9. An ontological interpretation $\langle \mathcal{O}, \beta \rangle$ *satisfies* the atomic formula $P(\bar{t})$ (wrt $\mathfrak{P}$), $\langle \mathcal{O}, \beta \rangle \approx^{\mathfrak{P}} P(\bar{t})$:iff   exactly one of the subsequent conditions applies.

- $P(\bar{t}) = t_1 \doteq t_2$ and $t_1^{\langle \mathcal{O}, \beta \rangle} = t_2^{\langle \mathcal{O}, \beta \rangle}$ (i.e., assuming equality and $\bar{t} = (t_1, t_2)$ with any $t_i \in Tm(L)$).
- $P \in Pred(\Sigma_F)$ and $fp_P(\bar{t}, \langle \mathcal{O}, \beta \rangle)$ obtains.[483]
- $P \in Pred(\Sigma_\Pi)$ and $\langle \mathcal{O}, \beta \rangle \approx^{\mathfrak{P}} \pi_P(\bar{t})$.

If $\mathfrak{P}$ is clear from context, it is omitted as index such that $\langle \mathcal{O}, \beta \rangle \approx P(\bar{t})$ is used.   ❏

ILLUSTRATING PREDICATION SATISFACTION

In order to foster understanding of both, Def. 4.6 and Def. 4.10, and to demonstrate how they function in interaction, consider the following examples. Example 4.11 shows the introduction of instantiation (::) as a binary, fundamental predicate. Thereafter, the category entity is introduced as a unary normal predicate Ent on the basis of instantiation and a constant ·Ent that is intended to denote that category, as well.

4.11 Example (of fundamental predicate satisfaction)

In a predication system $\mathfrak{P}$, let $:: \in Pred(\Sigma_F)$ a binary fundamental predicate. In order to characterize its intended reading / meaning, one may state informally that $x :: y$ stands for "x instantiates y", cf. the naïve definitions for Apple(x) in (4.9) and Red(x) in (4.10) on p. 141. In order to define this meaning wrt ontological structures, let $\eta_{::} :=$ "u^α instantiates v^α" and $fp_{::} := \lambda uv \,.\, \lambda \alpha \,.\, \eta_{::}$ in $\mathfrak{P}$.

Turning to Def. 4.10 and the question of whether an arbitrary ontological interpretation $\langle \mathcal{O}, \beta \rangle$ satisfies a formula involving ::, e.g., $x :: y$, application of the semantics (via $\eta_{::}$ and $fp_{::}$) yields the intended effect based on the informal phrase above as follows.[484]

$\langle \mathcal{O}, \beta \rangle \approx x :: y$ iff $fp_{::}(x, y, \langle \mathcal{O}, \beta \rangle)$ obtains   iff $(\lambda uv \,.\, \lambda \alpha \,.\, \eta_{::})(x, y, \langle \mathcal{O}, \beta \rangle)$ obtains
   iff   "$x^{\langle \mathcal{O}, \beta \rangle}$ instantiates $y^{\langle \mathcal{O}, \beta \rangle}$".   ❏

4.12 Example (of normal predicate satisfaction)

Continuing on the same predication system $\mathfrak{P}$ of the previous example, one may wish to utilize a unary predicate Ent such that Ent(x) stands for "x is an instance of entity". By introducing a logical individual constant ·Ent with entity as its intended referent, the available fundamental predicate :: allows for introducing Ent as a unary normal predicate via $\delta_{\mathsf{Ent}} := u :: \cdot\mathsf{Ent}$ and $\pi_{\mathsf{Ent}} := \lambda u \,.\, \delta_{\mathsf{Ent}}$ in $\mathfrak{P}$.
Anew turning to Def. 4.10, satisfaction $\langle \mathcal{O}, \beta \rangle \approx \mathsf{Ent}(x)$ unfolds as follows.

$\langle \mathcal{O}, \beta \rangle \approx \mathsf{Ent}(x)$ iff $\langle \mathcal{O}, \beta \rangle \approx \pi_{\mathsf{Ent}}(x)$ iff $\langle \mathcal{O}, \beta \rangle \approx (\lambda u \,.\, \delta_{\mathsf{Ent}})(x)$
   iff $\langle \mathcal{O}, \beta \rangle \approx x :: \cdot\mathsf{Ent}$ iff   "$x^{\langle \mathcal{O}, \beta \rangle}$ instantiates $\cdot\mathsf{Ent}^{\langle \mathcal{O}, \beta \rangle}$".

The last step is based on the definition of ::, applied in a single step, but is refinable, cf. Ex. 4.11.   ❏

These examples should illuminate the connections to the analysis prior to this sect. 4.4.3, e.g. the reliance of all normal predicates on, eventually, fundamental predicates. We remark that Def. 4.6 is deliberately laid out generically in terms of the variable tuple $\bar{\chi}$, since alternative formulations of Def. 4.10 can be considered (and might be employed in parallel for different kinds of predicates in future extensions). In spite of this, for the remainder of this work the definition of the satisfaction of atomic formulas is settled by Def. 4.10.

WELL-BEHAVEDNESS / -DEFINITION OF $\approx$

Another aspect that is hidden in requiring the fundamental phrases in Def. 4.6 to be "well-behaved" arises from the use of unrestricted metameta-level variables and (otherwise) unrestricted natural language phrases. This is not only very general, but also very close to examples criticized by and motivating Alfred Tarski in his truth definition(s), see, e.g., [805, esp. §1], [412, sect. 1.3]. Indeed, without any restriction, one can produce a variant of the liar paradox [67] for a unary predicate P by, e.g., the fundamental phrase

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[483]We use "$fp_P(\bar{t}, \langle \mathcal{O}, \beta \rangle)$ obtains" as a (pseudo-)formal shorthand for the fact that the fundamental phrase assigned to P applies with appropriate substitutions of its variables with components of $\bar{t}$ and $\langle \mathcal{O}, \beta \rangle$, resp.
[484]We silently suppose $x :: y$ as an infix notational variant of $::(x, y)$.

$\lambda u \,.\, \lambda \alpha \alpha' \rho \,.\,$ "not $\alpha \rho \alpha'$"$(x, \langle \mathcal{O}, \beta \rangle, P(x), \approx)$ which yields with Def. 4.10 that $\langle \mathcal{O}, \beta \rangle \not\approx P(\bar{t})$ iff "not $\langle \mathcal{O}, \beta \rangle \approx P(\bar{t})$".

Therefore, we require the following Cond. 4.13 for any predication system of any ontological semantics, characterizing well-behavedness of fundamental phrases, to rely on a well-defined satisfaction relation. The condition is implicitly assumed in the sequel and is satisfied by the subsequent examples of predication systems. Regarding the form of their fundamental phrases, they are held in close analogy to the set-theoretic definition and do not use meta-level notions "inappropriately" in fundamental phrases, as can be verified in Def. 4.16 and 4.17.

4.13 Condition (fundamental phrases of $\mathfrak{P}$ yield well-defined satisfaction of atomic formulas)
The considered predication system $\mathfrak{P}$ for signature Σ has only well-behaved fundamental phrases, i.e., for every ontological interpretation $\langle \mathcal{O}, \beta \rangle$ and atomic formula $P(\bar{t})$ with $P \in Pred(\Sigma_F)$ and $\bar{t} \in Tm(\Sigma)^{ar(P)}$, $\langle \mathcal{O}, \beta \rangle \approx^{\mathfrak{P}} P(\bar{t})$ is defined and either true or false (at the meta-level). ❑

CAPTURING PREDICATION DEFINITIONS FORMALLY

Ex. 4.12 is already suggestive of the fact that the notion of a predication system allows for recasting the notion of a *predication definition* in Def. 4.14, introduced only informally in sect. 4.4.2.[485]

4.14 Definition (predication definitions, $PDef(\cdot)$ and Π)
Given Cond. 4.8 with predication system $\mathfrak{P}$ for signature Σ, let S any signature. The set of *predication definitions* induced by S, i.e., of all normal predicate symbols (wrt $\mathfrak{P}$) in S, is this set of explicit definitions.

$$PDef^{\mathfrak{P}}(S) := \{ \forall \bar{x} \,.\, P(\bar{x}) \leftrightarrow \pi_P(\bar{x}) \mid P \in Pred(\Sigma_\Pi \cap S) \}$$

By convention, the set of predication definitions of the overall signature is denoted by Π, i.e., $\Pi^{\mathfrak{P}} := PDef^{\mathfrak{P}}(\Sigma)$. If Σ and $\mathfrak{P}$ are clear from context, we may just write $PDef(S)$ and Π. ❑

USE OF SETS OF DEFINITIONS FOR SPECIFYING π

Hereinafter we utilize sets of predication definitions in the obvious way for specifying the assignment of a predication definiens to a predicate symbol, i.e., component π in Def. 4.6 of a predication system $\mathfrak{P}$. For any definition $\forall \bar{x} \,.\, P(\bar{x}) \leftrightarrow \delta(\bar{x})$ in a set utilized for this purpose, $\pi_P = \lambda \bar{x} \,.\, \delta(\bar{x})$. Inevitably, the remaining conditions of Def. 4.6 regarding π must be fulfilled, as well, in particular those on non-circularity.

4.4.3.2 Sample Predication Systems Based on $\mathcal{CR}$

ILLUSTRATION INTRODUCTION

After these generic definitions of the notions of predication system, Def. 4.6, and of the satisfaction of an atomic formula, Def. 4.10, let us next illustrate components of a predication system on the basis of utilizing the $\mathcal{CR}$ ontology as an abstract core ontology that is underlying the fundamental signature. Necessarily, this results in making some ontological commitments, but only within the context of illustration. Note again that Def. 4.6 and 4.10 are parametric in general, while the subsequent definitions are merely sample cases.

FUNDAMENTAL PREDICATES IN $\mathcal{CR}$: $::$, $\leadsto$, $\multimap$

The first question for establishing a predication system is the question for what its fundamental predicate symbols and associated fundamental phrases will be. Sect. 2.4 presents a number of notions / categories for the theory of categories and relations, among them *category* and *relation* themselves, but further *entity*, *individual* (in an ontological sense), *relator* (instances of relations, entities capable of mediating between others), etc. All those kinds of entities need not be introduced as fundamental unary predicates, with specific fundamental phrases. Instead, as "usual" categories they can be introduced (also) as (logical individual) constants into the language. Then, predication with these categories can be captured in terms of predication definitions whose definiens comprises those constants and the first fundamental notion: the *instantiation*

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[485] In languages like FOL that offer the logical connective of equivalence and explicit variables, predication definitions / composition definitions can be stated completely in the object language, using the equivalence connective (a.o.). In languages without these means, such definitions may still be given at the meta level (either in natural language or a different formal language adopted for that level), to the extent that expressions of the object language cannot be used.

relation. Indeed, it is essential for the establishment of fundamental predicates that *relators* as instances of relations are those entities in $\mathcal{CR}$ that "blend" / "link" / mediate between entities of arbitrary kind (which leads to facts and more complex configurations.) Therefore, the two relations for analyzing structure and mediation of relators, are also included: the relations of *role-playing* / plays and *role-having* / role-of.

FUNDAMENTAL PHRASES IN TWO FLAVORS

Regarding the fundamental phrases for the core signature $\Sigma_{\mathcal{CR}^{core}} := \{::, \rightsquigarrow, \multimap\} \subseteq \Sigma_{\mathcal{CR}}$ we distinguish two variants, captured in two separate definitions sharing their preconditions in Cond. 4.15. The two forms originate from taking or not taking *a single step* in terms of the self-analyzing capabilities of $\mathcal{CR}$, cf. sect. 2.4.4.3. Without that analysis, the phrases in Def. 4.16 an ontological structure must be inspected without any further hints / means, to decide about the satisfaction of the corresponding atomic formulas in that structure. Taking the analytical step in Def. 4.17 that also instantiation, role-playing, and role-having are mediated between entities by relators allows for identifying such relators to account for the satisfaction of an atomic formula, in the sense of truthmakers. Postponing further discussion, we just remark that both definitions are compatible with each other, in particular, the first can be seen as an abridged version of the second. Note further that we use infix notation, as introduced earlier.

4.15 Condition

Assume Cond. 4.8 with signature Σ, predication system $\mathfrak{P}$, and ontological interpretation $\langle \mathcal{O}, \beta \rangle$. Moreover, $x, y \in Tm(\Sigma)$ and α a meta-language variable for $\mathfrak{P}$.

Adopt further $\mathcal{CR}$ as background conceptualization and let $\Sigma_{\mathcal{CR}^{core}} := \{::, \rightsquigarrow, \multimap\} \subseteq \Sigma$ a fundamental signature of three binary predicate symbols, i.e., $\Sigma_{\mathcal{CR}^{core}} = \Sigma_F$ of $\mathfrak{P}$. ❏

4.16 Definition (fundamental predicate satisfaction of $\mathcal{CR}^{core}$, unanalyzed variant)

Based on Cond. 4.15, relabeling $\mathfrak{P}$ to $\mathfrak{P}^0_{\mathcal{CR}}$, *fundamental predicate satisfaction* of $\Sigma_{\mathcal{CR}^{core}}$ in its *unanalyzed* form is defined by the function fp^0 for $\mathfrak{P}^0_{\mathcal{CR}}$ in the last "column" of (4.12)–(4.14). In addition, we name the relation and specify the resulting semantic phrases.

	formula	relation	resulting semantic phrase	definition of fp^0
(4.12)	$x :: y$	instantiation	$x^{\langle \mathcal{O}, \beta \rangle}$ instantiates $y^{\langle \mathcal{O}, \beta \rangle}$	$\lambda xy . \lambda \alpha .$ "x^α instantiates y^α"
(4.13)	$x \rightsquigarrow y$	role-playing	$x^{\langle \mathcal{O}, \beta \rangle}$ plays $y^{\langle \mathcal{O}, \beta \rangle}$	$\lambda xy . \lambda \alpha .$ "x^α plays y^α"
(4.14)	$x \multimap y$	role-having	$x^{\langle \mathcal{O}, \beta \rangle}$ is a role of $y^{\langle \mathcal{O}, \beta \rangle}$	$\lambda xy . \lambda \alpha .$ "x^α is a role of y^α" ❏

4.17 Definition (fundamental predicate satisfaction based on $\mathcal{CR}^{core}$, analyzed variant)

Based on Cond. 4.15, relabeling $\mathfrak{P}$ to $\mathfrak{P}^1_{\mathcal{CR}}$, *fundamental predicate satisfaction* of $\Sigma_{\mathcal{CR}^{core}}$ in its *analyzed* version is defined by the function fp^1 for $\mathfrak{P}^1_{\mathcal{CR}}$, which arises from the schematic definition

$$fp^1_E := \lambda xy . \lambda \alpha . \textit{"there is an R relator between } x^\alpha \textit{ and } y^\alpha, \textit{ such that } x^\alpha \textit{ plays the}$$
$$\textit{Q}_1\textit{-role and } y^\alpha \textit{ the Q}_2\textit{-role of that relator, and these roles are distinct and}$$
$$\textit{the only roles of that relator, with } x^\alpha \textit{ and } y^\alpha, \textit{ resp., as their only players"}.$$

and rows (4.15)–(4.17) by substituting for E a predicate symbol occurring in column E and for R, Q_1, and Q_2 the words of the same row and the equally labeled columns.

The resulting phrases of Def. 4.16 form abbreviated phrases wrt the analyzed version.

	E	abbreviated phrase	R	Q_1	Q_2
(4.15)	$x :: y$	$x^{\langle \mathcal{O}, \beta \rangle}$ instantiates $y^{\langle \mathcal{O}, \beta \rangle}$	instantiation	instance	instantiated
(4.16)	$x \rightsquigarrow y$	$x^{\langle \mathcal{O}, \beta \rangle}$ plays $y^{\langle \mathcal{O}, \beta \rangle}$	role-playing	player	played
(4.17)	$x \multimap y$	$x^{\langle \mathcal{O}, \beta \rangle}$ is a role of $y^{\langle \mathcal{O}, \beta \rangle}$	role-having	role	context ❏

For convenience we define according predication expressions in infix notation in both orders / arrangements of the corresponding role bases. In particular, predications in inverse order can be provided by means of predication definitions.

4.18 Definition (inverse notation for $\mathcal{CR}^{core}$)

These predication definitions determine the assignment of predication definientes for $\underline{::}, \leftarrowtail, \circ\!\!-\, \in \Sigma_{\mathcal{CR}}$.

$$(4.18) \quad \forall xy \,.\ x \underline{::} y \leftrightarrow y :: x$$

$$(4.19) \quad \forall xy \,.\ x \leftarrowtail y \leftrightarrow y \rightsquigarrow x$$

$$(4.20) \quad \forall xy \,.\ x \circ\!\!- y \leftrightarrow y -\!\!\circ x \qquad\qquad\qquad\qquad\qquad \square$$

INTRODUCTION OF FURTHER PREDICATES & REFLECTION CONSTANTS

Besides a mere reordering of arguments, arbitrary further predicate symbols can be introduced by predication definitions on the (eventual) basis of fundamental predicates. Indeed, ontological semantics *requires* predication definitions for each normal predicate symbol. These definitions can solely be based on predicate symbols previously introduced, which is the case in Def. 4.18, or constants are utilized in addition. With recourse to the notion of intended referent of any symbol S by the language user U in sect. 4.3.1, denoted by $iref_U(S)$ and related to Def. 4.4 of signature satisfaction, we emphasize the notion of *reflection constants* for predicates, which agree with the latter on their intended referents (in signature-satisfying ontological models) as already used in Ex. 4.12.

4.19 Definition (reflection constant)

Let U a language user and Σ a signature including a predicate symbol $P \in Pred(\Sigma)$. A (logical individual) constant symbol $c \in Const(\Sigma)$ is called a *reflection constant* of P :iff $iref_U(P) = iref_U(c)$.

As a naming convention, reflection constants are written as the predicate symbol P with a leading dot $\cdot$, e.g., $\cdot P$ is a reflection constant for P. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \square$

EXAMPLES OF PREDICATION DEFINITIONS

One sample case of utilizing constants in predication definitions concerns the kinds of entities distinguished in $\mathcal{CR}$, as indicated above. With a new reflection constant $\cdot\mathsf{Cat}$, formula (4.21) exemplifies the introduction of a unary predicate for the notion of *category*, in analogy to the Apple example discussed in sect. 4.4.1 and 4.4.2, and closely associated with Ex. 4.12.

$$(4.21) \quad \forall x \,.\ \mathsf{Cat}(x) \leftrightarrow x :: \cdot\mathsf{Cat}$$

Recalling the motivating thoughts at the beginning of sect. 4.4, other schemes of predication definitions are conceivable for unary predicates already, as well as "iterated" definitions are admitted by Def. 4.6 of a predication system as long as this is non-circular. The analysis of a predicate Red as in (4.11) is formally captured in (4.22) and is justified[486] after a predicate inh for inherence has been introduced.

$$(4.22) \quad \forall x \,.\ \mathsf{Red}(x) \leftrightarrow \exists y(y :: \cdot\mathsf{Red} \wedge \mathsf{inh}(y,x))$$

For possibly greater intuitions than in the case of inherence, a[487] part-of relation shall serve as an example of a(n ontological) relation, with a predication definition pattern that is typical in the context of $\mathcal{CR}$ for the introduction of relations based on reflection constants (and closely related to the defining phrase in Def. 4.17). For this pattern a role base for the part-of relation must be fixed in advance, which shall comprise the role categories that should be intuitively graspable from the names of the (logical) individual constants $\cdot\mathsf{Part}$ and $\cdot\mathsf{Whole}$. The pattern itself states that $\mathsf{partOf}(x, y)$ stands for the existence of a part-of relator (r) with exactly two role individuals (q_p, q_w), s.t. x plays the $\cdot\mathsf{Part}$ role and y is in the role of the $\cdot\mathsf{Whole}$. Note the analogy with fp_E^1 in Def. 4.17 of the analyzed variant.

$$(4.23) \quad
\begin{aligned}
\forall xy \,.\ \mathsf{partOf}(x,y) \leftrightarrow \exists r q_p q_w (\ & q_p \neq q_w \wedge r :: \cdot\mathsf{partOf} \wedge \\
& x \rightsquigarrow q_p \wedge q_p -\!\!\circ r \wedge q_p :: \cdot\mathsf{Part} \wedge \\
& y \rightsquigarrow q_w \wedge q_w -\!\!\circ r \wedge q_w :: \cdot\mathsf{Whole} \wedge \\
& \forall q'(q' -\!\!\circ r \rightarrow (q' = q_p \vee q' = q_w))\ )\ )
\end{aligned}$$

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[486]That is, only then the ontological semantics of Red is at all defined.

[487]"a" is used here instead of "the" because there are numerous variants of part-of relations, cf. [394, 707]. Any particular choice does not matter for the example to be given, however, as long as the relation is construed to have a role base of two roles as described in the paragraph. In the sequel we therefore use '*the* part-of relation' for readability.

OPTIONS OF INTRODUCING REFLECTION CONSTANTS

Eventually, we stress that reflection constants should be considered / introduced for all predicates, at least if a theory shall satisfy the criterion that all symbol referents are covered by its ontological structures. This applies in particular to normal predicates, for which reflection constants appear to us as a natural means to explicate the character of predication. Nevertheless, the option of introducing reflection constants applies to fundamental predicates, as well. In their case formulas akin to predication definitions (which need to be stipulated as axioms of a theory) can be expected not to be proper definitions, but to create circularities.[488] From a purely formal / technical point of view, the semantics of fundamental predicates is well-defined without any reflection constants. The same applies even to normal predicates, as long as any definition is available as predication definition for each predicate (and supposing acyclicity among those definitions). Before the continuation of these thoughts forestalls the main observation of sect. 4.4.4.2, let us clarify the mnemonic character of the index Π in Σ_Π, the latter denoting the normal signature in Def. 4.6.

4.4.4 Special Kinds of Tautologies

4.4.4.1 Theory of Predication Definitions Π

SIGNATURE-DEPENDENCE OF ONTOLOGICAL SEMANTICS AND ASPECTS OF A PREDICATION SYSTEM

Let us drop the presupposition of the $\mathcal{CR}$ ontology and return to the generic level at which the Def. 4.6 of predication system and Cond. 4.8–4.9 were given. It is worth noting that the definition of the satisfaction of atoms under ontological semantics is signature-dependent, in some sense to a greater extent than this is the case for classical set-theoretic semantics. In particular, certain sentences of the *same* signature turn out to be tautological or non-tautological, depending on the choice of the predication system. Insofar a predication system as defined above involves two aspects, one of determining a (structured) signature, and one providing an influential interpretation of the predicate symbols of a language.

PREDICATION DEFINITIONS ARE TAUTOLOGICAL UNDER ONTOLOGICAL SEMANTICS

This influence is visible in that the predication definitions Π of the overall signature Σ turn out to be tautological. To corroborate this claim we need to anticipate some definitions that, regarding satisfaction being defined thus far only for atomic formulas, actually go beyond that level. However, the definitions of satisfying an equivalence, (4.37) in Def. 4.23, of universal quantification in Def. 4.24 (which is adopted from earlier discussion as (4.5), p. 129 above), as well as the notion of a tautology under ontological semantics (denoted by $\emptyset \approx X$ for any tautological sentence or theory X) are all highly analogous to the set-theoretic precedent. Therefore, we state and prove the next proposition in the present context.

4.20 Proposition (predication assignments entail validity of predication definitions)
Let $\mathfrak{P} = (\Sigma, \Sigma_F, \Sigma_\Pi, fp, \pi)$ a predication system for any signature Σ, and $\Pi = PDef(\Sigma)$.
All predication definitions arising from $\mathfrak{P}$ are tautologies wrt $\approx$, $\quad \emptyset \approx \Pi$. $\qquad\qquad \square$

Proof (by definitions). Assume Cond. 4.8 (ignoring β thereof), s.t. esp. $\mathcal{O} = (O, I)$ is an arbitrary, but suitable ontological structure. For any $P \in Pred(\Sigma_\Pi)$, let '$\forall \bar{x} \,.\, P(\bar{x}) \leftrightarrow \pi_P(\bar{x})$' $\in PDef(\Sigma)$. Then Def. 4.23 and 4.24 yield $\mathcal{O} \approx \forall \bar{x} \,.\, P(\bar{x}) \leftrightarrow \pi_P(\bar{x}) \quad$ iff $\quad$ every $\bar{o} \in O^{ar(P)}$ and every $\beta : Var \to O$ satisfy: $\langle \mathcal{O}, \beta_{\bar{o}}^{\bar{x}} \rangle \approx P(\bar{x})$ iff $\langle \mathcal{O}, \beta_{\bar{o}}^{\bar{x}} \rangle \approx \pi_P(\bar{x})$. Due to $P \in Pred(\Sigma_\Pi)$ the latter is clear from Def. 4.10. $\qquad\qquad \square$

4.4.4.2 Referent Interpretation vs. Predication Interpretation

INDEPENDENCE OF PREDICATION ASSIGNMENTS FROM REFERENT INTERPRETATIONS

Inspecting the proof of Prop. 4.20 reveals that the validity of predication definitions is a consequence of the fact that the definition of predication satisfaction is directly based on the predicate *symbols*, to each of

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[488] Although there is no necessity for such circularities. For example, let P a normal predicate in one predication system $\mathfrak{P}_1$ and let the same P be a fundamental predicate in another predication system $\mathfrak{P}_2$ that equals $\mathfrak{P}_1$ except for the case of P. If then the predication definition $\phi_P \in PDef^{\mathfrak{P}_1}(\{P\})$ emerging from $\mathfrak{P}_1$ is maintained as an axiom in a theory based on $\mathfrak{P}_2$, there is no circularity created by ϕ_P. However, note that this yields an equivalence between the fundamental phrase of P in $\mathfrak{P}^2$ and the definiens of ϕ_P in $\mathfrak{P}^2$, which should be compatible in that case.

which a definiens is assigned by π, instead of the referents of those symbols within a particular ontological structure $\mathcal{O} = (O, I)$. Rephrased from the perspective of the latter, the *referent* of any predicate symbol P, assigned to it in terms of the interpretation function I of $\mathcal{O}$, is *independent* of / *irrelevant* for the question of whether or not an atomic formula involving P applies to a certain argument tuple. Terminologically, we name I the *referent interpretation*, π (and fp) the *predication interpretation* of a predicate symbol.[489]

A DEVIATION FROM THE CLASSICAL SAMPLE CASES

This independence may render $\not\approx$ for atomic formulas inadept, especially if the effect on the relationship between co-reference and equivalence for predicates is compared to its classical prototype(s). Assume any set-theoretic structure $\mathcal{A} = (A, I)$ for a signature that includes predicate symbols P and Q. In FOL with set-theoretic semantics (4.24) turns out to be true.

(4.24)   If $I(P) = I(Q)$, then $\mathcal{A} \models \forall \bar{x} \, . \, P(\bar{x}) \leftrightarrow Q(\bar{x})$

Similarly in second-order logic (SOL) under standard semantics, cf. [223, sect. 2], by letting P, Q now be predicate variables and $\bar{x}$ a tuple of (logical) individual variables, (4.25) is a tautology wrt SOL.

(4.25) $\forall PQ \, . \, P = Q \rightarrow \forall \bar{x}(P(\bar{x}) \leftrightarrow Q(\bar{x}))$

In both cases, the eventual use of the same referent $I(P) = I(Q)$ of both predicate symbols is essential for these observations, i.e., for any variable assignment $\beta \in A^{Var}$, $\beta(x) \in I(P)$ iff $\beta(x) \in I(Q)$ is immediate from that equality, hence $\langle \mathcal{A}, \beta \rangle \models P(\bar{x})$ iff $\langle \mathcal{A}, \beta \rangle \models Q(\bar{x})$ for all $\beta \in A^{Var}$.

   Analogous preconditions "behave" differently under ontological semantics. Assume any predication system with normal signature Σ_Π and any suitable ontological structure $\mathcal{O}$ and let $\langle \mathcal{O}, \beta \rangle \not\approx P(\bar{x})$ iff $\langle \mathcal{O}, \beta \rangle \not\approx Q(\bar{x})$ for all variable assignments $\beta : Var \to O$. By Def. 4.10 and in the sample case of two normal predicates $P, Q \in Pred(\Sigma_\Pi)$, this yields that $\langle \mathcal{O}, \beta \rangle \not\approx \pi_P(\bar{x})$ iff $\langle \mathcal{O}, \beta \rangle \not\approx \pi_Q(\bar{x})$ for all β. Obviously, π_P and π_Q are involved instead of $I(P)$ and $I(Q)$, underlining the interpretative aspect of predication systems themselves. Further applications of Def. 4.10 may lead to an equivalence that involves the referents of P and Q in $\mathcal{O}$ – yet not necessarily, but depending on the set of predication definitions.

ACCEPTING THE DEVIATION NEVERTHELESS

Questionable as it may seem not to incorporate a "behavior" analogous to that of FOL or SOL into ontological semantics, we see some arguments and use cases of the kind that two predicates share their referent, but behave differently when used in predication / atomic formulas. The use cases that we find for this emerge primarily from (ontological) relations, as discussed in sect. 2.4, therefore resting on the $\mathcal{CR}$ ontology.

RATIONALE 1: ONTOLOGICAL VS. MATHEMATICAL RELATIONS (AND TUPLE ARRANGEMENT)

The first case concerns the arrangement problem of (mathematical) relations: consider the case of the ordering relation over numbers, e.g. the fact usually formalized as $0 < 1$. The relation between 0 and 1 (regarding their order) does not seem to differ ontologically, irrespectively of considering that relationship either "from the side" of 0 or 1. That is, the $\mathcal{CR}$ ontology allows for the view that there is one and the same fact that 0 is less than 1 as well as that 1 is greater than 0. However, the mathematical relations $<, >$ (say, over $\mathbb{N}$), are clearly distinct relations, even disjoint. Ontological semantics based on predication systems allows for introducing $<, >$ as predicate symbols[490] that both refer to one and the same ordering relation (ontologically), whereas the symbol $<$ when applied to an argument tuple assumes the reading 'less than', $>$ 'greater than'. The interleave $\forall xy \, . \, x < y \leftrightarrow y > x$ already follows from an appropriate

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[489]On the one hand, the sole distinction between referent interpretation and predication interpretation is highly reminiscient of the two (mathematical) functions int_I and rel_I in the semantics of Common Logic (CL) [452, sect. 6.2, p. 13]. The function names are suggestive of an *intension* / denotation interpretation of the predicate symbol P (applied first) and a *relational* interpretation / extension of (the intensional interpretation of) P, cf. "The truth value of an atomic sentence in an interpretation is determined by the relational extension of the denotation of the predicate together with the denotations of the remaining components." [452, p. 4, clause 3.17]. On the other hand and as suggested in the quoted sentence, those two functions form "consecutive stages" in the evaluation of atomic sentences. More precisely (and adapting the notation slightly), any atomic sentence $P(\bar{x})$ is true in a CL interpretation I :iff $I(\bar{x}) \in rel_I(int_I(P))$ [452, p. 15, clause E7]. This remains in contrast to the independence of referent and predication interpretation just observed. See sect. 7.2.2.2 for relations between CL and ontological semantics with hindsight.

[490]For simplicity and since further not needed, we use the same symbols as for the intended referents in the sentence before.

formal predication definition for each reading[491] because the same role categories are merely tied to different argument positions. Another example is available from the introduction of the fundamental predicates ::, ⤳, and ⊸ in Def. 4.16 and Def. 4.17, resp., and the establishment of inverse reading variants by the predication definitions in 4.18.

RATIONALE 2: FORMALIZING ANADIC RELATIONS BY MULTIPLE PREDICATE SYMBOLS

A related use case originates from anadic relations, i.e., (ontological) relations whose relators vary in terms of the number of individual roles. An example may be the notion of connection (e.g., in a technical sense), based on a connector role category and a connectee role category. Individual relators may be constrained to have only one connector role, but at least two connectee roles. The limitation to a single, fixed arity of each predicate symbol in FOL requires the introduction of several distinct predicate symbols that shall capture an anadic relation. Then it appears just natural that all referent interpretations of those predicate symbols have one and the same referent, namely that ontological relation. On the other hand, the predication definitions will vary, already due to their distinct arities.

RATIONALE 3: FORMALIZING ENTITIES IN DIFFERENT SYNTACTIC CATEGORIES

From predicates of different arities it is only a small step to entities that are formalized multiply in a FOL syntax. The intended referents of reflection constants, cf. Def. 4.19, constitute a prime example by, on the one hand, having a predicate intended to formalize that entity, and having a constant intended to refer to the very same entity, on the other hand, cf. formulas (4.21) and (4.22) on p. 149. The effective interpretation[492] of symbols in distinct syntax categories can sensibly be expected to differ, for their should be a reason of having distinct syntax categories. In FOL this is clearly the case. Thus, if it is acceptable that entities are referents of symbols in different syntactic categories, it appears similarly acceptable to have different symbols within a single syntax category – predicate symbols in our case – that *refer* to the same entity, without allowing that fact to determine the overall interpretation of each symbol completely.

RATIONALE 4: DIFFERENCE OF FORMULAS AND PREDICATE SYMBOLS

The last argument builds further on the difference in syntactic categories. Obviously, predicate symbols are atomic signs, while atomic formulas are composed signs already. From that point of view, it appears well admissible to allow for a distinct treatment of these two types of signs in the semantics, taken by itself. We admit that the independence of the predication interpretation of a predicate symbol P of the referent interpretation of P might be considered a renunciation from the Principle of Compositionality [271]. Then again, both, referent interpretation and predication interpretation are assigned to P. Compositionality should thus be sustainable by viewing both as constituents of an "overall interpretation" of P.[493]

RELATION OF CO-REFERENCE AND EQUIVALENCE AS A PROPERTY

The relationship between co-reference of two predicate symbols and the validity of their equivalence is not generally determined under ontological semantics, in contrast to (4.24), for instance. Thus, on the one hand, an analog of (4.24) is not true for all ontological models and predication systems. On the other hand, it is not excluded a priori. We define the following relationship as a property of a predication system for a pair of predicates, and additionally in general.

4.21 Definition (enforcing equivalence based on co-reference)

Let $\mathfrak{P}$ a predication system for any signature Σ, and let $P, Q \in Pred(\Sigma)$.

$\mathfrak{P}$ is said to *enforce equivalence based on co-reference* wrt P and Q :iff   for every ontological structure $\mathcal{O} = (O, I)$ suitable for Σ: $I(P) = I(Q)$ implies $\mathcal{O} \approx\!\!\!\mid \forall \bar{x} . P(\bar{x}) \leftrightarrow Q(\bar{x})$.

$\mathfrak{P}$ *generally enforces equivalence based on co-reference*   :iff   it does so for each pair of predicate symbols in $Pred(\Sigma)$.   ❏

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[491]Thinking of specifications in close analogy to the part-of example (4.23) in the previous sect. 4.4.3.2.

[492]'Interpretation' here not in the sense of a function that yields a referent for the symbol, but rather in the sense of having effects in the semantics of expressions of which the symbol is a part.

[493]We think that the results of reconsidering the relation between ontological semantics and set-theoretic semantics in sect. 4.6 support this view further.

EXAMPLE WITH REFLECTION CONSTANTS

In order to provide an example, consider a FO language over a signature involving two unary predicates P and Q and two *reflection constants* for these predicate symbols, $\cdot P$ and $\cdot Q$ according to the naming convention in Def. 4.19 of reflection constant. Let the predication system establish (4.26) and (4.27) as predication definitions for P and Q, resp.

$$(4.26) \quad \forall x \, . \, P(x) \leftrightarrow x :: \cdot P \qquad\qquad (4.27) \quad \forall x \, . \, Q(x) \leftrightarrow x :: \cdot Q$$

Then it is easy to see that the assumption of $I(P) = I(Q)$ within any ontological model $\mathcal{O} = (O, I)$ or the addition of "this" equality in the language itself (4.28) yield the desired equivalence in (4.29), i.e., such a predication system enforces the equivalence of P and Q based on their co-reference.

$$(4.28) \quad \cdot P = \cdot Q \qquad\qquad (4.29) \quad \forall x \, . \, P(x) \leftrightarrow Q(x)$$

We remark that the way of capturing the relationship between a predicate symbol / predication interpretation and its reflection constant in (4.26) and (4.27) is considered a typical case (against the background of the $\mathcal{CR}$ ontology).

4.4.4.3 Fundamental Theory F and (In-)Equality Theory E

INTERACTION / REFLECTION OF ACO NOTIONS: FUNDAMENTAL THEORY F

There is a similar case of tautologies besides predication definitions that is constituted by merely determining a predication system for the ontological semantics that a language user is going to assume. Namely, there is the natural and immediate question of what impact on the overall semantics emerges from the abstract core ontology (ACO) that is adopted / presupposed as a conceptualization for fundamental predicate symbols / signature and fundamental phrases of a predication system. In our understanding, an ACO "discards" certain ontological structures a priori, which seem logically possible (at the object level) without assuming that ACO (at the meta level).

4.22 Observation (tautological fundamental theory, with axiomatization F)
Let $\mathfrak{P} = (\Sigma, \Sigma_F, \Sigma_\Pi, fp, \pi)$ a predication system for any signature Σ. Further consider an arbitrary (meta-level) interrelation among the intended referents of fundamental predicates in $Pred(\Sigma_F)$ that is expressible / reflectable by a(n object-level) FOL sentence $\phi \in Lg(\Sigma)$ in accordance with the (predicative application of) those fundamental predicate symbols and their fundamental phrases, resp.

Then ϕ is a tautology wrt $\approx$, $\quad \emptyset \approx \phi$.

By convention, $F^{\mathfrak{P}}$ denotes an axiomatization[494] of such interrelations / tautologies, F where $\mathfrak{P}$ is clear. ❏

EXAMPLES BASED ON $\mathcal{CR}$

To accept this observation while lacking a formally available meta-level theory at this point, let us look at a particular case first, merely illustrating the justification that we see here. Assume the ontology of categories and relations $\mathcal{CR}$, cf. sect. 2.4, as an ACO for a predication system with the fundamental predicates $\Sigma_{\mathcal{CR}^{core}} = \{::, \rightsquigarrow, \multimap\}$. $\mathcal{CR}$ demands, for example, that all relators are (ontological) individuals and that individuals do not have any instances. Therefore relators cannot be instantiated. With the fundamental phrases for $\Sigma_{\mathcal{CR}^{core}}$ of Def. 4.17 this yields the fact that no ontological structure can satisfy $x :: y$ in combination with $x \multimap y$, because – given the (informal) theory on categories and relations – the former entails that y is a category, while the latter entails that y is an ontological invidivual, and these two conditions are mutually exclusive.[495] Put differently, for a predication system based on $\mathcal{CR}$ (4.30) applies in every ontological

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[494]Of course, the axiomatization $\emptyset$ of all tautologies wrt $\approx^{\mathfrak{P}}$ is not meant here, but, similarly to the case of (in-)equalities in sect. 4.3.1, an axiomatization formed by making those meta-level presuppositions explicit in the object language. From another angle, this amounts to neglecting those presuppositions at the meta-level, but adding them as a theory (component, possibly) at the object level. The issue is discussed further in the main body of the text shortly below.

[495]In greater detail, an instantiation relator with y playing the InstantiatedRL (read 'RL' as role) entails that y itself instantiates Cat, cf. e.g. Fig. 2.4 on p. 84, whereas a role-of relator with y in the RoleRL entails that y itself is an instance of Ind. But $\mathcal{CR}$ postulates

structure a priori and is therefore a tautology.

$$(4.30) \quad \forall xy . \ x :: y \rightarrow \neg \ x \multimap y \qquad\qquad (4.31) \quad \forall xy . \ x :: y \rightarrow \neg \exists z(z \rightsquigarrow y)$$

Formula (4.31) is an analogous example, minimally generalized in the consequent of the implication. Sect. 6.1 presents a FOL axiomatization for the ontology of categories and relations $\mathcal{CR}$, based on an enlarged fundamental signature $\Sigma_{\mathcal{CR}}$. For all of its sentences, the same argument could be made. More precisely, this applies to all sentences in $\Sigma_{\mathcal{CR}^{core}}$ that are consequences of that axiomatization of $\mathcal{CR}$. The formulation "for all of its sentences" builds on a relaxation that we admit on the understanding of 'fundamental theory' compared to F in Obs. 4.22. Besides the three fundamental predicates ::, $\rightsquigarrow$, and $\multimap$ in $\Sigma_{\mathcal{CR}^{core}}$, reflection and other constants are involved in sect. 6.1, as well as further predicates are introduced by means of predication definitions. This yields indirectly specified interrelations between the actual fundamental predicates in $\Sigma_{\mathcal{CR}^{core}}$. Nevertheless, these are justifiable by the equally tautological character as that of predication definitions by Prop. 4.20 and by a similar effect wrt (in-)equalities in signature-satisfying structures discussed in sect. 4.3.1 and recapitulated in the next but one paragraph.

ACO AS "BACKGROUND THEORY" AND COMPARED TO THE SET-THEORETIC CASE

Overall, we expect that the ACO is a "background" theory attached with the notions employed in fundamental phrases, even if that theory may exist only implicitly and informally. This is clearly reminiscent of set theory in classical FOL semantics, described in sect. 3.1 as 'background set theory' in the discussion of the structure $\mathcal{A}^{\mathcal{P}^{\omega}_{fix}}$ derived from the usual view of a set-theoretic structure $\mathcal{A}$, and met as the formal theory ST in the theory view of set-theoretic semantics discussed in sect. 3.3. Since the membership relation is not accessible in the object-level syntax in the classical case, one prominent deviation in the present approach consists in the availability / containment of the fundamental predicate symbols within the signature. They can thereby be used in (object-level) expressions/formulas. Eventually, it shall be preferable to build on this feature and, to the extent possible, expound the ACO itself/the background theory of the notions underlying the fundamental phrases explicitly at the object level. This yields an axiomatization F (the one introduced in Obs. 4.22) – despite the fact that F is tautologous for the selected predication system. The latter is itself, so to say, based on F / a conceptualization that F formalizes.

INTERMEDIATE RECAP: (IN-)EQUALITIES E VALID IN SIGNATURE-SATISFYING STRUCTURES

On the move to defending this requirement, we look back on the discussion of the status of (in-)equalities when considering signature-satisfying structures in sect. 4.3.1. There, the arguments for explicating (in-)equalities (as far as known to the language user) were (1) the signature-dependence of such (in-)equalities, (2) the exclusion of some signature-ignorant ontological structures as models, and (3) the prospective reuse of reasoners. Of course, Σ_F may comprise constants, as well. In analogy to F and Π, we reserve the symbol E for the set of (in-)equalities postulated by the language user to be valid in signature-satisfying structures. The first and third of these arguments for explicating E likewise pertain to the cases of F and Π.

FUNDAMENTAL THEORY EXPLICATION DESPITE TAUTOLOGICAL IMPRINT

Not only regarding E, but overall capturing as much as possible of the underlying fundamental theory explicitly and formally within the object-level language itself is desirable,[496] despite the tautological character from the point of view of ontological semantics given a predication system based on that ACO (and possibly assuming signature-satisfaction). One additional benefit is that such explications enforce the stated assumptions to be shared across possibly diverging conceptualizations that, e.g., different language users may have for the same fundamental signature.[497] Another welcome effect is the (at least partial) "synchronization" between object level and meta level, in line with demanding faithfulness of ontological theories

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that there is no entity which instantiates both, Cat and Ind.

[496] We admit that utilizing the object-level language itself may not suffice regarding the available means of expression, especially if the overall approach of ontological semantics is transferred from the sample case of FOL (syntax) to other languages, which may be more restricted, thinking of description logics as an example. Ontological usage schemes in sect. 5.2 offer an alternative "access" to ontological semantics.

[497] For one moment, that argument dismisses our basic assumption of a single and integral perspective for the language user(s), see the beginning of this chapter, p. 124. The related issue of letting the underlying fundamental theory / the ACO vary itself is further analyzed in sect. 4.6.

to themselves in sect. 4.3.2. Thinking of ontological semantics for languages other than FOL, how far this "synchronization" can be pushed depends on the question of which language and logic is adopted at the meta level. For the particular case of FOL, one may go as far as to demand that no additional assumptions are to be exploited at the meta level than those that are formalized within a respective ontological theory in object-level syntax[498] – further keeping half an eye on Def. 3.21 of ontologically neutral semantics.

4.4.5 Summary on Predication in Ontological Semantics

PREDICATION SYSTEMS FOR BOOTSTRAPPING PREDICATION SEMANTICS

Let us condense some quintessences from the development of this sect. 4.4 on predication. It is an important deviation from the classical blueprint that ontological semantics does not offer a unique and uniform account of predication,[499] based on the analysis conducted in the beginning, in sect. 4.4.1. Aiming at a rigorous and preferably formal account, the notion of a *predication system* is introduced in Def. 4.6. Together with the complementary Def. 4.10 of the satisfaction of an atomic formula it accommodates a bootstrapping approach to predication semantics. Initially, the semantics of a desirably small set of *fundamental predicates* is determined by associated natural language phrases, which are expected to be applied against the background of an abstract core ontology that the language user accepts. The introduction of *normal predicates* is completely based on defining first-order formulas which may rely on other predicates in any way that ensures non-circularity within these dependencies. We see this kind of bootstrapping as an appropriate form of addressing / taming the problem of the mutual foundation between language semantics and ontologies coherently, allowing for the truthfulness / faithfulness of ontologies to themselves.

PREDICATION INTERPRETATION IS RELAXED WRT REFERENTS, BUT INTENTIONALLY

We emphasize that the predication definitions of normal predicates are to characterize solely the interconnection of the referents of the predicate and the symbols to that it is applied that is to be stated by an atomic formula involving that predicate and a number of arguments. Nevertheless, on the one hand and likewise for their fundamental counterparts, the linkage between the referent interpretation of a predicate and the interpretation of an atomic formula involving that predicate is less constrained than in standard set-theoretic semantics, where equal referent interpretation guarantees equivalence in predication, cf. the discussion around (4.24) on p. 151. This *can* be a property that a predication system evokes, but it is not enforced in general for the reasons discussed in the previous sect. 4.4.4. On the other hand, predication in ontological semantics is more restrictive than considered in usual FO structures in the set-theoretic case, if the room for varying interpretations of atomic formulas / predicate applications is considered.[500] The tautological character of predication definitions and ACO interrelations observed in Prop. 4.20 and Obs. 4.22 elicits that the predication system selected for ontological semantics of a particular signature manifests largely an *intended* interpretation of *all* atomic formulas, even for free, i.e., possibly signature-ignorant ontological structures.

POSTPONING A MORE LOGICAL STANCE UNTIL SECT. 4.6

Not at least, this is a result of the rather ontological perspective on predication thus far. Taking up a more logical stance, the natural and immediate next step would be to investigate modifications of, e.g., predication systems, in order to allow for "free" predication interpretations, potentially including fundamental predicates. Although we grant the immediacy, it turns out that this leads to an even closer connection with classical semantics and is therefore and beneficially addressed in sect. 4.6, in which the relations between the two semantic accounts are studied. We turn to propositional connections and quantified formulas in order to complete the overall definition of ontological semantics beforehand.

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[498]This view is in need of future clarifications, e.g. regarding dealing with syntax / syntactic objects ontologically, their potential reflection / inclusion among semantic entities (in a broadened sense then), etc. – but we shall not dwell on that at this stage.

[499]We hold that this would also apply in the context of other languages and their complex syntactic constructions of a similar kind, i.e., forming atomic, sentence-like statements / claims.

[500]Although in classical semantics an analogous treatment to that of predication under ontological semantics could be claimed for the background membership relation at the meta level.

4.5 Semantics of Connectives and Quantifiers & Semantic Notions

4.5.1 Sentential Connectives and Quantifiers

NO REFERENTS FOR LOGICAL CONSTANTS

Having settled the ways ontological semantics treats constants, variables, and the construct of predication / atomic formulas, it remains to introduce / re-consider interpretations for the syntactic constructs that involve logical constants. A first, possibly trivial observation in the classical context is that logical constants are not assigned any referents in interpretation structures, which might even be adopted as a general criterion for the notion of 'logical constant'.[501] Rather, they manipulate / determine the "selection" of satisfying interpretations based on those interpretations satisfying their subformulas. This assumption appears equally adequate for ontological semantics. Indeed, the classical definitions are transferred one to one for connectives, and with minor necessary modifications for quantifiers, due to relying on ontological structures. In recollection of construct "overloading" as discussed for predicate interpretation in sect. 4.4.1, the natural language use of "logical" conjuncts[502] is overloaded as well. For instance, the natural language term 'and' may be read in a "classical" flavor[503] or in an additionally temporal one[504]. But these overloadings are already disregarded in the context of classical logic and in this respect we subscribe to the same attitude herein.

SEMANTICS OF SENTENTIAL CONNECTIVES

We strictly follow the classical definitions of the semantics of sentential connectives, see Def. 1.5 in sect. 1.5.2 for the definition adopted herein, and cf. e.g. [826, p. 52], [691, p. 50], [221, p. 82]. We specify the six common connectives that are used in FOL formulas in this overall work.

4.23 Definition (satisfaction of combinations by connectives)

Suppose Cond. 4.8, where $\langle \mathcal{O}, \beta \rangle$ is an arbitrary ontological interpretation wrt a signature Σ. Let further $\phi, \psi \in Lg(\Sigma)$. Then $\langle \mathcal{O}, \beta \rangle$ *satisfies* the negation of ϕ and combinations of ϕ and ψ by each of the remaining connectives as follows.

$$(4.32) \quad \langle \mathcal{O}, \beta \rangle \approx \neg\phi \qquad :\text{iff} \qquad \langle \mathcal{O}, \beta \rangle \not\approx \phi$$

$$(4.33) \quad \langle \mathcal{O}, \beta \rangle \approx \phi \wedge \psi \qquad :\text{iff} \qquad \langle \mathcal{O}, \beta \rangle \approx \phi \text{ and } \langle \mathcal{O}, \beta \rangle \approx \psi$$

$$(4.34) \quad \langle \mathcal{O}, \beta \rangle \approx \phi \vee \psi \qquad :\text{iff} \qquad \langle \mathcal{O}, \beta \rangle \approx \phi \text{ or } \langle \mathcal{O}, \beta \rangle \approx \psi$$

$$(4.35) \quad \langle \mathcal{O}, \beta \rangle \approx \phi \dot{\vee} \psi \qquad :\text{iff} \qquad \text{either } \langle \mathcal{O}, \beta \rangle \approx \phi \text{ or } \langle \mathcal{O}, \beta \rangle \approx \psi$$

$$(4.36) \quad \langle \mathcal{O}, \beta \rangle \approx \phi \rightarrow \psi \qquad :\text{iff} \qquad \langle \mathcal{O}, \beta \rangle \not\approx \phi \text{ or } \langle \mathcal{O}, \beta \rangle \approx \psi$$

$$(4.37) \quad \langle \mathcal{O}, \beta \rangle \approx \phi \leftrightarrow \psi \qquad :\text{iff} \qquad \langle \mathcal{O}, \beta \rangle \approx \phi \text{ if and only if } \langle \mathcal{O}, \beta \rangle \approx \psi \qquad \square$$

PROPOSITIONAL EQUIVALENCES PERTAIN

There should be no surprising effects of these definitions due to using ontological structures instead of classical set-theoretic structures for interpretations, because the definitions themselves rely on formulas only and are therefore transparent wrt the underlying structures. Accordingly, the well-established equivalences of propositional logic, see e.g. [826, p. 16], transfer directly to ontological semantics, cf. also [826, Satz 2.8, p. 59] on the relation between tautologies in (classical) predicate and propositional logic. The underlying structures mainly influence predication and quantification, so let us turn to the latter to complete the general definition of ontological semantics for FOL syntax.

SEMANTICS OF QUANTIFICATION

Basically, the definition of quantification is already given in the course of developing the notion of ontological structure in sect. 4.1, formulas (4.5) and (4.6). Remember that ontological structures in their most

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[501] This is not to claim that an ontological analysis of logical interconnections, e.g., among propositions were not possible. This would be especially relevant if propositions or sentences were among the intended referents of signature elements and were thus covered by signature-satisfying models. However, such considerations are not in the present scope.

[502] 'Conjunct' in a linguistic sense.

[503] Like in "The book is red and lies on your desk."

[504] E.g. "John buys the book and gives it to you." Similar examples can be found in [214, p. 34].

generalized form can be arbitrary (pluralities of) entities and all entities considered together as an ontological structure (in the context of an interpretation) are tantamount. Already in sect. 4.1 that leads us to the following definition of quantification. We stress again that, in analogy to set-theoretic structures, ontological structures single out parts of the world, i.e., their "range" of entities is not necessarily unlimited.[505] In contrast to the classical case, however, there is no assumption of any superstructure nor any collection that frames the assumed entity / plurality of entities, remembering the discussion of the understanding of O in an ontological structure $\mathcal{O} = (O, I)$ after Def. 4.2, p. 128.

4.24 Definition (satisfaction of quantified formulas)

Given Cond. 4.8 with its language L and its arbitrary ontological interpretation $\langle \mathcal{O}, \beta \rangle$, and ontological structure $\mathcal{O} = (O, I)$ therein, let further $\phi \in L$ an arbitrary formula.

The universal and existential quantifications of ϕ are *satisfied* according to these conditions.

(4.38) $\quad \langle \mathcal{O}, \beta \rangle \approx \forall x \,.\, \phi \quad$:iff $\quad$ for every o in O : $\langle \mathcal{O}, \beta_o^x \rangle \models \phi$

(4.39) $\quad \langle \mathcal{O}, \beta \rangle \approx \exists x \,.\, \phi \quad$:iff $\quad$ there is an o in O : $\langle \mathcal{O}, \beta_o^x \rangle \models \phi \qquad$ ❏

Again and similarly to the sentential connectives, the well-known equivalences for quantifier sentences transfer to ontological semantics. In particular, the duality of the quantifiers wrt negation is maintained by Def. 4.24 in combination with the previous definitions on satisfaction and with the Def. 4.26 (below) of logical equivalence under ontological semantics, which defines two formulas as equivalent ($\approx$) :iff they have the same ontological models.

4.25 Observation (quantifier duality)

The following equivalences hold for arbitrary formulas $\phi, \psi \in Lg(\Sigma)$, where Σ is any signature, and a predication system for Σ is assumed for ontological semantics.

(4.40) $\quad \exists x \,.\, \phi \approx \neg \forall x \,.\, \neg \phi$

(4.41) $\quad \forall x \,.\, \phi \approx \neg \exists x \,.\, \neg \phi \qquad$ ❏

Proof (by definitions). The proof proceeds in close analogy in both cases, therefore only the case of (4.41) is presented. It is developed through equivalences from right to left and assumes the propositional tautology $\phi \approx \neg\neg\phi$. Let $\langle \mathcal{O}, \beta \rangle$ an arbitrary ontological interpretation, where $\mathcal{O} = (O, I)$.

$\langle \mathcal{O}, \beta \rangle \approx \neg \exists x \,.\, \neg \phi \quad$ iff $\quad \langle \mathcal{O}, \beta \rangle \not\approx \exists x \,.\, \neg \phi \quad$ iff $\quad$ it is not the case that there is an o covered by O with $\langle \mathcal{O}, \beta_o^x \rangle \not\approx \neg \phi \quad$ iff $\quad$ for every o' in O: $\langle \mathcal{O}, \beta_{o'}^x \rangle \not\approx \neg \phi \quad$ iff $\quad$ for every o' in O: $\langle \mathcal{O}, \beta_{o'}^x \rangle \approx \neg\neg\phi$ iff $\quad \langle \mathcal{O}, \beta \rangle \approx \forall x \,.\, \neg\neg\phi, \quad$ all by Def. 4.23 and 4.24, and hence $\langle \mathcal{O}, \beta \rangle \approx \forall x \,.\, \phi$ by the propositional tautology, which yields the equivalence. $\qquad$ ❏

This observation and its proof may be taken as an indication of how classical logical results for FOL that do not require "descendence" to the level of atomic formulas transfer straightforwardly for ontological semantics. In anticipation of studying the relationship between the two semantics in sect. 4.6 the reproduction of further classical results for ontological semantics is discontinued. Instead, we briefly (re-)turn to a distinctive aspect concerning ontological semantics.

QUANTIFICATION AND THE NOTION OF ORDER

In connection with quantification, it is of immediate interest to comment on the question of "which order" the language given by FOL syntax and ontological semantics is and which further effects arise from quantifying wrt ontological structures instead of a classical, set-theoretic structure. First of all, reconsider the standard formulation for universal quantification wrt set-theoretical interpretations.

(4.42) $\quad \langle \mathcal{A}, \beta \rangle \models \forall x \,.\, \phi \quad$:iff $\quad \langle \mathcal{A}, \beta_a^x \rangle \models \phi$ for every $a \in A$

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[505]Even when / for formalizing a top-level ontology, this need not (and cannot, by the definition of ontological structure) be assumed. Depending on the axiomatization, an ontological structure could mainly involve the categories of the ontology and other entities to satisfy its axioms, but may not account for, say, all (ontological) individuals or for various more specific categories.

Especially in comparison to the structure $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$ (see (3.1) at p. 106) and recalling the analysis expounded in sect. 3.1, the nowadays commonly understood notion of order in logical languages reflects the levels in the tuple-powerset hierarchy (Def. 3.3, p. 106),[506] which in turn links with the (background) membership relation wrt $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$. From the point of view of $\mathcal{A}^{\mathcal{P}^{\omega}_{\text{fix}}}$, the classical definitions attribute a special status to a single element, namely the "original" set / universe of discourse A above. Quantification over A constitutes first-order quantification.

GENERAL ONTOLOGICAL SEMANTICS CANNOT DEFINE AN ORDER

In contrast, for ontological semantics it is not immediately possible to define a comparable notion of order. The reasons are the choices (a) to view all entities on a par with each other and (b) to avoid pre-established distinctions in the semantics. That is, especially without any fixed predication system there is nothing that could be used to derive an order. Everything in an ontological structure is merely an entity and no determinate way[507] lends itself to selecting a specific entity or a subcollection / plurality of entities to serve as a basis of any restricted form of quantification (that may then count as "first-order" quantification, but would still require a way to reach "higher orders").

ACO-BASED VERSION WOULD ALLOW FOR THAT, YET IT IS UNCLEAR IN CASE OF $\mathcal{CR}$

However, a full-fledged definition of ontological semantics for FOL syntax requires a predication system, which in turn should rest on an abstract core ontology (ACO) to provide ultimate, fundamental notions for the methodological introduction of predicates, as explained in sect. 4.4. Accordingly, entities postulated by that ACO may be adduced for revising the definition of quantification and / or for novel notions of order.

However, taking the case of the $\mathcal{CR}$ ontology / conceptualization from sect. 2.4, it is discussed in sect. 2.4.4.3 above that there is no natural / immediately convincing approach for an order from the *logical* or *quantificational* point of view, despite that one may utilize instantiation to distinguish first-order and higher-order categories, for example. But relators (ontological individuals) may connect categories of any (categorial) order, and role-nesting / chaining of the plays relation suggests alternatives without any outstanding definition candidate and without straightforward integration with instantiation-based / categorial order. Congruously,[508] no ACO-based variant of order is adopted for the remainder of this thesis, and we adhere to the general Def. 4.24 for quantification.

4.5.2 Common Semantic Notions

FOUNDATION COMPLETED AND FURTHER STANDARD SEMANTIC NOTIONS WITH DISTINCT NOTATION

The usual foundation of a Tarskian semantic approach is established with Def. 4.2 of *ontological structure* and *interpretation* (p. 128) and Def. 4.5 of an ontological *model*/an ontological structure *satisfying* a formula (p. 134), itself being based on *predication systems* for the case of atomic formulas in Def. 4.10 (p. 146), and on treatments of connectives in Def. 4.23 (p. 156) and quantifiers in Def. 4.24 (p. 157) that parallel the set-theoretic blueprint. Accordingly and despite the additional element / parameter of a predication system for a signature, this can be taken to yield an instance of the abstract notion of logic as characterized in sect. 1.5.1. Beyond the changes elaborated in detail up to this point in the present chapter, no deviations concerning further semantic notions from the "standard" setup in that section are pursued.

Nevertheless, we need some distinguished notation when discussing ontological and set-theoretic semantics in parallel. Hence and for clarity, we specify the definitions of major notions that occur elsewhere in this work. It is noteworthy again that all those semantic notions are governed / to be understood to be parametrized by an underlying predication system that the language user needs to fix in advance, similarly and in addition to the signature. Regarding verbal expressions, for disambiguation from their classical analogs we append "wrt ontological semantics" (or a similar phrase) where necessary.[509]

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[506] Although other variants of understanding 'order' are available, esp. in earlier writings, cf. [160, sect. 4, p. 128–129].

[507] One may consider a random selection to serve as an A analog, but we see no point in this, in comparison with classical semantics.

[508] Admittedly, one may certainly attempt to reconstruct the classical notion of order, e.g., in terms of the instance-based extension of categories and defining tuple extensions for relations. However, wrt categories and relations as ontological entities in the $\mathcal{CR}$ context, this appears to us partially artificial.

[509] But we shall avoid inflating the subsequent Def. 4.26 by that.

4.26 Definition (model class, consequence, equivalence, satisfiability, tautology)
Let Σ an arbitrary FOL signature and let $\mathfrak{P}$ a corresponding predication system (which parametrizes all defined notions). Let X, Y any two sentences or theories over Σ.

- The *model class* / class[510] of all ontological models of X is denoted by $OMod(X)$.

- A formula $\phi \in Lg(\Sigma)$ is a *consequence of* / *entailed by* X :iff   it is satisfied in all models of X,
$$X \approx\!\!\!| \phi \quad \text{:iff} \quad OMod(X) \subseteq OMod(\phi).$$

- X and Y are *equivalent*   :iff   they have the same ontological models,
$$X \approx Y \quad \text{:iff} \quad OMod(X) = OMod(Y).$$

- X and Y are *equivalent modulo* theory T :iff   they have the same ontological T models,
$$X \approx_T Y \quad \text{:iff} \quad OMod(T) \cap OMod(X) = OMod(T) \cap OMod(Y).$$

- X is *satisfiable* / *semantically consistent*   :iff   there is an ontological model of X, i.e., $OMod(X) \neq \emptyset$. Otherwise, X is *unsatisfiable* / *semantically inconsistent*.

- X is a *tautology* / *tautological*   :iff   every ontological structure suitable for $Sig(X)$ satisfies X, denoted by and equivalent with $\emptyset \approx\!\!\!| X$.
The set of all tautologies wrt Σ and $\mathfrak{P}$ is $OTaut(\Sigma, \mathfrak{P}) := \left\{ \phi \in Lg(\Sigma) \mid \emptyset \approx\!\!\!|^{\mathfrak{P}} \phi \right\}$.[511]   ❑

Note that by Def. 4.26 the usual connection between entailment and unsatisfiability remains intact, as well as other meta-logical properties (remain expected to) transfer, e.g. the monotonicity of the consequence relation. We capture only the first case in preparation of the next sect. 4.6, keeping the straightforward proof short. Potential additional meta-logical analyses are commented on below.

4.27 Observation (link between consequence and unsatisfiability)
Let Σ an arbitrary FOL signature and let $\mathfrak{P}$ a predication system for Σ. Further let $T \subseteq Lg(\Sigma)$ a theory, $\phi \in Lg(\Sigma)$ a sentence.
$$T \approx\!\!\!| \phi \quad \text{iff} \quad T \cup \{\neg\phi\} \text{ is unsatisfiable (wrt ontological semantics).}$$ ❑

Proof (by definitions). From the preconditions in the Obs. 4.27 and respective definitions follow these equivalences: $T \approx\!\!\!| \phi$ iff $OMod(T) \subseteq OMod(\phi)$ iff   there is no ontological structure $\mathcal{O}$ s.t. $\mathcal{O} \approx\!\!\!| T$ and $\mathcal{O} \not\approx\!\!\!| \phi$, equally s.t. $\mathcal{O} \approx\!\!\!| T$ and $\mathcal{O} \approx\!\!\!| \neg\phi$ iff $T \cup \{\neg\phi\}$ is unsatisfiable (wrt ontological semantics).   ❑

ADDITIONAL "NON-STANDARD" NOTIONS

A few additional notions arise from aspects of ontological semantics that diverge from or augment classical FOL semantics. Firstly, the notion of signature satisfaction captured in Def. 4.4 (p. 133) allows for a more restricted form of equivalence. Secondly, the parametrization of the concepts captured in Def. 4.26 by the underlying predication system can be transcended, which is implemented for tautologies in Def. 4.29.

4.28 Definition (equivalence under signature satisfaction)
Let Σ an arbitrary FOL signature and let $\mathfrak{P}$ a corresponding predication system (which parametrizes all defined notions). Let X, Y any two sentences or theories over Σ.

The class of all signature-satisfying models of X wrt Σ is denoted by $O^{\Sigma}Mod(X)$.

X and Y are *equivalent under signature satisfaction*   :iff they have the same signature-satisfying models,

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[510]We use the (mathematical) term 'class' here for commonality with "set"-theoretic notation. There is the issue that an ontological structure, due to not being constrained, may itself be a (mathematical) class. In that case it cannot be a member of a 'model *class*', because classes have only sets as members. One may therefore need to utilize another notion of collection or possibly a formulation in terms of categories instead of model classes (paying attention to facts such as that the principle of extensionality is assumed in the definition of equivalence by $OMod(X) = OMod(Y)$). From the point of view of the definitions and observations in this sect. 4.5.2 this should not lead to any problematic effects, because only class / set membership ($\in$), the subclass / subset relation ($\subseteq$) and $\emptyset$ are employed. For $\in$ we would expect a suitable "equivalent" in any such theory of sets / classes, collections, or categories, whereas $\subseteq$ and $\emptyset$ (or analogs) should be available in "the same interrelation" (thinking of theory interpretation) with $\in$ as used from set / class theory. Due to this assessment, further exploration of these issues remains future work.

[511]$\mathfrak{P}$ is made explicit at $\approx\!\!\!|^{\mathfrak{P}}$ because it appears as an argument in $OTaut(\Sigma, \mathfrak{P})$, whereas $\approx\!\!\!|$ in the sentence before is just another notation for $\approx\!\!\!|^{\mathfrak{P}}$, referring implicitly to $\mathfrak{P}$ in the context of this definition.

$$X \approx^{\Sigma} Y \quad :\text{iff} \quad O^{\Sigma}Mod(X) = O^{\Sigma}Mod(Y). \qquad \qquad \square$$

4.29 Definition (predication-invariant tautologies)
Let Σ an arbitrary FOL signature and X any sentence or theory over Σ.

X is a *predication-invariant tautology/tautological* :iff it is satisfied by every ontological structure, for all predication systems for Σ. This is denoted by $\emptyset \models^{*} X$. $\square$

TOWARDS THE RELATION TO THE SET-THEORETIC CASE

It is one option to continue examining the metalogical aspects of ontological semantics genuinely, as just started to a minimal extent with Obs. 4.27. However, it shall turn out as a viable alternative to first research the relationship of ontological to set-theoretic semantics on the basis of the former now being introduced, and draw further conclusions thereafter.

4.6   Relations between Ontological and Set-Theoretic Semantics

4.6.1   Preliminaries & Ontological Status of Sets

MOTIVATION: REUSE REASONERS

One of our secondary aims adopted early for formalizing conceptualizations is to reuse existing theorem proving and reasoning machinery in order to benefit from the tremendous body of theoretical and practical / implementation work that has already been established, cf. sect. 1.2.2. The proposal of ontological semantics seems to run counter to this goal. Indeed, proposing "yet another" semantics did originally not seem plausible/sensible, but eventually the analyses presented in ch. 2 and during the development of ontological semantics in the previous sections of this chapter have led to the current proposal. Now it remains to be seen whether and to what extent set-theoretic Tarskian semantics can be related to ontological semantics, what that entails for established, classical results and ultimately in which sense reasoning software can be utilized on its established foundations. After establishing formal axiomatizations of conceptualizations, the meta-theoretical tasks that we are primarily interested in comprise producing and verifying deductions as well as analyzing the consistency of theories.

TWO "PRAGMATIC" OPTIONS, YET WITH SNAGS IN THEM

Before starting the actual analysis, two more "pragmatic" options of dealing with the above development of ontological semantics shall be listed. They may appear appropriate to readers that remain generally skeptical about ontological structures and models, although those options have some disadvantages from our point of view. The discussion of predication / introducing predicates with varying meaning of the syntactic construction in sect. 4.4 mainly results in adopting an abstract core ontology as a theory for a number of fundamental predicates, whereas all further predicates are to be equipped with a definitional formula based on previously established predicates and possibly fresh logical individual constants (that name the intended referents of the predicate). Nevertheless and after all, the resulting theories are syntactically plain FOL theories. The first pragmatic option is purely proof-oriented and disregards the semantic side, either classical or ontological. It would mean to reason over theories established in this way by any calculus whose derivation rules are deemed acceptable, e.g. rules of natural deduction, cf. [275, 276, 673]. We rate the disconnection from formal semantics problematic, though.

The second pragmatic option is more attuned to formal semantic approaches. One may simply consider such theories under classical formal semantics. Again, syntactically they are FOL theories, although reference to categories therein and potentially to set membership (if considered in an ontological theory) link this view also with Henkin's general structures for higher-order logic, cf. [223, sect. 3] or sect. 3.1. This view relates to sect. 4.6.4 from a reasoning perspective, but the problem that arises here from an ontological point of view, especially when adopting a firm top-level approach, is "the" meta-level problem: one must then accept the existence of a meta-level membership relation and meta-level sets/entities that are not accessible within the ontological theory that is formalized.

PREPARING FOR DEALING WITH SETS

The observations in and the overall remainder of this sect. 4.6 depend on the ontological status of sets, also regarding the conceptualization to be formalized, and to some extent the adopted theory of sets. In order to conduct this discussion actually and meaningfully, we postulate the following condition. Notably, this view conforms with GFO, where sets are likewise acknowledged as entities.

4.30 Condition

Sets are granted a genuine ontological status, i.e. in particular, they are entities / they exist.[512]   ❑

For referring to sets in terms of ontological semantics based on a predication system $\mathfrak{P}$, the membership relation is added to $\mathfrak{P}$.[513] In the sequel and for better readability / comprehensibility, we assume tacitly the ontology of categories and relations $\mathcal{CR}$ as abstract core ontology in combination with the predication system $\mathfrak{P}_{\mathcal{CR}}$ for $\mathcal{CR}$ and, in particular, with fundamental predicates in $\mathcal{CR}^{core}$ as introduced in Def. 4.17.[514] However, the choice of $\mathcal{CR}$ is not crucial for the overall analysis, i.e., another theory could likewise be applied as a parameter.

There are two options under ontological semantics for adding a new binary predicate $\Subset$ with the membership relation as its intended referent[515]: either as a normal predicate, then equipped with a predication definition, or as a new fundamental predicate with a fundamental phrase for defining satisfaction of an atomic formula with $\Subset$ in predicate position. For simplicity and due to its well-established character, the following definition adheres to the second approach and augments the fundamental predicates in Def. 4.17 with a fundamental phrase for $\Subset$.

4.31 Definition ($\mathcal{CR}$ extended with set membership)

Assume Cond. 4.8 with signature Σ, predication system $\mathfrak{P}$, ontological structure $\mathcal{O} = (O, I)$, and variable assignment $\beta : Var \to O$. Let $x, y \in Tm(\Sigma)$ be terms. Further let $\Sigma_{\mathcal{CR}\Subset} := \Sigma_{\mathcal{CR}} \cup \{\Subset\}$ s.t. $\Sigma_{\mathcal{CR}\Subset} \subseteq \Sigma$ the signature of an abstract core ontology with $\Sigma_{\mathcal{CR}^{core}} \cup \{\Subset\} = \Sigma_F$ in $\mathfrak{P}$.

In analogy to Def. 4.17, the fundamental phrase of $\Subset$ is defined to result in the following semantics.[516]

$\langle \mathcal{O}, \beta \rangle \approxeq x \Subset y$ iff $x^{\langle \mathcal{O}, \beta \rangle}$ is a member of $y^{\langle \mathcal{O}, \beta \rangle}$, i.e., there is a membership relator between $x^{\langle \mathcal{O}, \beta \rangle}$ and $y^{\langle \mathcal{O}, \beta \rangle}$, such that $x^{\langle \mathcal{O}, \beta \rangle}$ plays the member-role and $y^{\langle \mathcal{O}, \beta \rangle}$ the collection-role of that relator, and these roles are the only roles of that relator.   ❑

With this version of Def. 4.31, an explanation / declaration of set membership predication in terms of $\mathcal{CR}$ is provided. Indeed, this could be extended / converted into a proper predication definition, assuming the introduction of further constants. We would defend the given explanation as a proper ontological understanding of applying the membership predicate to two arguments (in the usual order), given the background of $\mathcal{CR}$ and our view of set membership as a(n ontological) relation. On the other hand, there is no necessity to adhere to this view for what follows, so that only the part before the "i.e." (without the $\mathcal{CR}$-based reading) can be adopted, which would parallel the unanalyzed variant of Def. 4.16. In any case, the major point is to bind the semantics of the atom '$x \Subset y$' to $x^{\langle \mathcal{O}, \beta \rangle} \in y^{\langle \mathcal{O}, \beta \rangle}$ at the meta level.

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[512]Remember, 'existence' refers to arbitrary modes of existence, cf. the comments after Def. 4.1 (p. 126) or FN 383 (p. 110).

[513]This is certainly a minimal requirement. Further additions may be considered.

[514]Below we shall not distinguish between $\mathcal{CR}$ and $\mathcal{CR}^{core}$, except for cases where the restriction to $\mathcal{CR}^{core}$ is necessary.

[515]The symbol $\Subset$ is chosen instead of $\in$ in order to avoid confusion / overlap with the use of $\in$ in formal parts of this text as "standard" membership relation. Yet the intended reading for $\Subset$ is exactly that of "the" "standard" membership relation, which should coincide with the background membership relation for classical models.

[516]With the aim of a more readable version, in the specified resulting phrase we condense the steps of defining two variants (unanalyzed and analyzed according to Def. 4.16 and 4.17, resp.) for two predication systems and specifying the λ-expressions as the actual fundamental phrases followed by those resulting phrases that are contained in the present definition. Since we rely on these thoughts, however, the "defining" clause contains 'iff' instead of ':iff'.

4.6.2 Generalization/Specialization Relations on Structures and Models

SET-THEORETIC STRUCTURES ARE ONTOLOGICAL STRUCTURES (GENERALLY)

Let us start establishing relations between classical set-theoretic and ontological semantics with the question of how the *structures* of both semantics relate, e.g., whether one notion of structure is more general than the other. First of all, it is immediate from Cond. 4.30 and Def. 4.2 of ontological structures that any set-theoretic structure can also be considered as, and thus *is* an ontological structure. The converse is not possible since, for instance, an ontological structure need not comprise any sets. For a more general argument, ontological structures should be broader by definition because they are derived from set-theoretic structures with the intent to avoid certain ontological presuppositions.

NO SPECIALIZATION WITH EQUAL SIGNATURE ASSUMPTION

The generic specialization relation needs to be qualified, however. If we take signatures into account, which must be assumed for both types of structure, then viewing a set-theoretic structure $\mathcal{A}$ as an ontological structure $\mathcal{O}$ can only be maintained if two differing signatures are admitted to be underlying $\mathcal{A}$ and $\mathcal{O}$, resp. Allowing for such differences, one may refer to a set-theoretic structure $\mathcal{A}$ (as a single, whole entity) as an ontological structure that covers exactly one entity. Likewise, $\mathcal{A}$ taken as a single whole, but together with its components, their interrelations as well as relations to the whole (all in the form of relators) form a plurality of entities, i.e., another ontological structure. Both of these ontological structures, however, must assume an extended signature compared to that of $\mathcal{A}$ in order to account for the set-theoretic interrelations. A more common way of posing the question for the relationship between set-theoretic and ontological structures (and models) would be to consider both wrt the *same* signature, however.

ILLUSTRATION BY A SIMPLE THEORY T_{apple}

In order to address the latter question and to showcase effects of both semantic definitions more clearly, we resort to the "apple example" in sect. 4.4. For brevity in notation we use predicate A instead of Apple for the category apple / A [517] and ·A for its reflection constant (Def. 4.19), i.e., ·A stands for the same A/category apple, but is a constant symbol. a is another constant for denoting a particular apple a.[517] Hence, the respective signature for this example is $\Sigma_{\mathsf{apple}} := \Sigma_{\mathcal{CRE}} \cup \{\mathsf{A}, \cdot\mathsf{A}, \mathsf{a}\}$.[518] In the predication system $\mathfrak{P}_{\mathsf{apple}}$ that is assumed for interpreting $Lg(\Sigma_{\mathsf{apple}})$ under ontological semantics A is a normal predicate and is mapped to the predication definiens $x :: \cdot\mathsf{A}$. (4.43) is therefore a tautology under ontological semantics (with $\mathfrak{P}_{\mathsf{apple}}$), cf. Prop. 4.20 in sect. 4.4.4. Since this is not the case under classical semantics, but likewise by following earlier considerations on such tautologies, the axiomatization T_{apple} includes (4.43) explicitly. The only other sentence of T_{apple} declares a to be an apple / an instance of A.

(4.43) $\quad \forall x \,.\, \mathsf{A}(x) \leftrightarrow x :: \cdot\mathsf{A}$

(4.44) $\quad \mathsf{A}(\mathsf{a})$

SEMANTIC STRUCTURES FOR T_{apple}

Now consider the four structures in Table 4.1 under the two semantic regimes. $\mathcal{O}$ is a signature-satisfying ontological model based on $\mathcal{CR}$, $\mathcal{A}$ similarly interprets a and ·A with their intended referents, and :: such that T_{apple} is satisfied classically. $\mathcal{A}'$ is another, arbitrary set-theoretic model of T_{apple}, parts of which are viewed as ontological structure $\mathcal{O}'$ in the fourth column.[519] The two rows at the bottom display the predication extensions of both predicates, forestalling this notion in ontological semantics from Def. 4.38 below. Let us ignore those last two rows for a moment.

The set-theoretic structures demand no further explanations, but some comments on the ontological structures and their entities are in order. First of all, we stress that the constructs $\mathsf{a} \xrightarrow{::} \mathsf{A}$ and $0 \xrightarrow{\in} \{0, 1\}$ each denote a single, invidual relator, namely an instantiation relator from a to A ("a is an instance of A") and a membership relator from 0 to the set $\{0, 1\}$. Accordingly, the universe of $\mathcal{O}$ comprises exactly four entities,

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[517]The symbols A and a as well as others in this font denote entities at the meta level.

[518]As usual and is justified / justifiable for both semantics, we consider only such signature elements below that are relevant for evaluating a formula or a theory.

[519]$\mathcal{O}'$ is designed to mimick $\mathcal{A}'$ (though not completely, cf. omitting 1 and 3 in the universe of $\mathcal{O}'$), in a sense that becomes clear below.

Σ_{apple}	$\mathcal{O} \mathrel{\approx\!\!\!\mid}^{\mathcal{CR}} T_{\text{apple}}$	$\mathcal{A} \models T_{\text{apple}}$	$\mathcal{A}' \models T_{\text{apple}}$	$\mathcal{O}' \mathrel{\not\approx\!\!\!\mid}^{\mathcal{CR}\in} T_{\text{apple}}$
(universe)	$\wr ::, \mathtt{A}, \mathtt{a}, \mathtt{a}\xrightarrow{::}\mathtt{A}\wr$	$\{\mathtt{a}, \mathtt{A}\}$	$\{0,1,2,3\}$	$\wr \in, \{0,1\}, 0, 2, 0\xrightarrow{\in}\{0,1\}\wr$
::	::	$\{(\mathtt{a},\mathtt{A})\}$	$\{(0,2),(1,2)\}$	$\in$
A	$\mathtt{A}$	$\{\mathtt{a}\}$	$\{0,1\}$	$\{0,1\}$
·A	$\mathtt{A}$	$\mathtt{A}$	2	2
a	$\mathtt{a}$	$\mathtt{a}$	0	0
(:: extension)	$\{(\mathtt{a},\mathtt{A})\}$	$\{(\mathtt{a},\mathtt{A})\}$	$\{(0,2),(1,2)\}$	$\emptyset$
(A extension)	$\{\mathtt{a}\}$	$\{\mathtt{a}\}$	$\{0,1\}$	$\emptyset$

Table 4.1: Comparison of structures for Σ_{apple} and their relationship to theory T_{apple} according to the contrasted semantic approaches, with indicated predication systems for ontological semantics. The structure named in the expression in column head n, $2 \leq n \leq 5$, is given by its respective universe in row 1, column n. In the case of ontological structures, remember the notation of $\wr$ $\wr$ that is introduced below Def. 4.3, p. 128, and see the text body for relator symbols. The (referent) interpretation maps the entry in row m, $2 \leq n \leq 5$, column 1 to that of row m, column n.

$U(\mathcal{O}')$ has exactly five. It is already discussed above that (4.43) is a tautology (wrt $\approx\!\!\!\mid^{\mathcal{CR}}$ as well as $\approx\!\!\!\mid^{\mathcal{CR}\in}$) – it is satisfied by $\mathcal{O}$ plainly by Def. 4.10 of satisfaction on the basis of a predication system. Then wrt (4.44), we have $\mathcal{O} \approx\!\!\!\mid^{\mathcal{CR}} \mathtt{A}(\mathtt{a})$ since $\mathcal{O} \approx\!\!\!\mid^{\mathcal{CR}} \mathtt{a} :: \mathtt{A}$, which itself is eventually justified by the existence of $\mathtt{a}$, $\mathtt{A}$, and $\mathtt{a}\xrightarrow{::}\mathtt{A}$.

The case of $\mathcal{O}'$ may appear less clear than $\mathcal{O}$, in particular, why it is not a model of T_{apple}. Since $\mathcal{O}'$ satisfies the tautology (4.43) (also wrt $\mathcal{CR}\in$) in any case, there must be reasons that $\mathcal{O}' \not\approx\!\!\!\mid^{\mathcal{CR}\in} \mathtt{a} :: \mathtt{A}$ – despite the analogy to $\mathcal{O}$ in that there are 0, $\{0,1\}$, and the mediating relator $0\xrightarrow{\in}\{0,1\}$.[520]

OBSERVE INTENDED INTERPRETATION OF FUNDAMENTAL PREDICATES

We identify two main reasons in these regards. The first is the irrelevance / independence of predication interpretation and referent interpretation, as defended in sect. 4.4.4.2. The second reason emerges from the treatment of fundamental predicates and their predication semantics.

4.32 Observation

Resorting to natural-language definitions for the fundamental predicates in a predication system constrains their predication interpretation to being the *intended* interpretation. ❏

In combination, that means that atomic formulas with a fundamental predicate are satisfied only if the referent interpretations of the arguments "behave" as intended / associated with the natural language phrase for that predicate – but independently of the referent interpretation of the predicate (and thus whether a structure is signature-satisfying for the predicate or not). In the case of $\mathcal{O}'$, $\mathtt{A}(\mathtt{a})$ were satisfied iff $\mathtt{a} :: \mathtt{A}$ were, i.e., in turn, iff 0 were an instance of $\{0,1\}$ (in the short paraphrase), to be witnessed by the existence of an instantiation relator between the two that must be *present in* $U(\mathcal{O}')$ (in the analyzed version), cf. the case of $U(\mathcal{O})$. Hence, the referent interpretation of :: to set membership does not render the membership relator in $U(\mathcal{O}')$ relevant for evaluating $\mathcal{O}' \approx\!\!\!\mid^{\mathcal{CR}\in} \mathtt{a} :: \mathtt{A}$.

4.6.3 Extending Ontological Structures for Varying Predication

PREDICATION SYSTEMS EXHIBIT TWO FLAVORS

To some extent and due to the influential role of fundamental predicates / predication, the effects just observed convey to normal predicates (with predication definitions), because these are traced back to atoms with fundamental predicates, eventually. Less strictly / directly than predication definitions were proved

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[520]Moreover, these three work together in justifying $\mathcal{A}' \models \mathtt{A}(\mathtt{a})$. This holds albeit $0\xrightarrow{\in}\{0,1\}$ is not a part of $U(\mathcal{A}')$, but it occurs in connection with the background membership relation of classical semantics (and, from a $\mathcal{CR}$-based perspective on that, as a relator).

in Prop. 4.20 to be tautologies under ontological semantics (wrt a predication system), the intended interpretation of predication by means of fundamental predicates is apparently assumed for the argumentation following the analogous Prop. 4.22, that there is a tautological, a priori theory that depends on the abstract core ontology assumed for / "behind" a predication system and its natural language phrases for fundamental predicates.

With hindsight, this reveals two distinct flavors in predication systems that result from their design following ontological considerations in sect. 4.4.1 and 4.4.2. On the one hand and like the signature, the predication system is fixed prior to (referent) interpretation within ontological structures, because it is required for determining predication semantics. Exactly due to having that purpose, on the other hand, it provides an interpretation to predication – one specific interpretation for each predicate. Consequently, that predication interpretation may be changed by utilizing a different predication system.

ALLOWING FOR VARIATION IN PREDICATION INTERPRETATIONS

Taking a more logical stance, one may be interested in allowing for variation in connection with predication interpretation, as is the case with referent interpretation in free ontological structures. This suggests revising the notions of predication system and, consequently, interpretation and satisfaction. In particular, let us follow the idea that the rôle of the predication system would be adopted by a new part in an extended interpretation within an ontological structure. An ontological structure could then be formalized as $\tilde{\mathcal{O}} = (O, I_{ref}, I_{ext})$, where $I_{ref} : \Sigma \to O$ is the usual interpretation of signature elements by entities. One option that occurs to us for capturing I_{ext} in connection with the original idea of predication systems is to remain within their "framework" of Def. 4.6, but allow for arbitrary selections in its components. That could mean to make an arbitrary choice on which predicates are fundamental vs normal, and accordingly just let arbitrary fundamental phrases as well as arbitrary definientes be assigned, one per predicate. However, this approach appears cumbersome to pursue,[521] whereas a second one leads closer to set-theoretic semantics and can be grasped, also formally, much more clearly.

2ND APPROACH WITH PRELIMINARIES

Given that atomic formulas consist of a predicate and a sequence of arguments in FOL syntax, the second consideration amounts to assigning arbitrary sequences of entities (argument referent interpretations) to a predicate, as that second component I_{ext} from above. Instead of a engaging in an ontological analysis of 'sequence', we rely on the notion of tuples on its usual set-theoretic basis, also with the goal in mind to relate to set-theoretic semantics eventually. Some groundwork is required for following this approach on a set-theoretic basis. In the sequel, we shall limit considerations to ontological structures that are compatible with a set-theoretic superstructure, i.e., they should not comprise entities that cannot be members of sets, in principle. For instance, this excludes all (mathematical) classes and (the extensions of) the membership relation, the subset relation, etc. More cautiously, one may have excluded all set-theoretic entities, but this is not required subsequently.[522] The notion of set-compatibility and Cond. 4.34 capture that restriction more formally.

4.33 Definition (set-compatibility of ontological structures and *SOMod*)

Let $\mathcal{S}$ a set-theoretic conceptualization, possibly formalized by a FO theory ST, that supports mathematical relations over sets. Moreover, assume Cond. 4.8 with its signature Σ, predication system $\mathfrak{P}$ for Σ, and ontological structure $\mathcal{O} = (O, I)$ suitable for Σ.

$\mathcal{O}$ is *$\mathcal{S}$-compatible* (or *ST-compatible*)   :iff $\mathcal{S}$ allows for a set $ext(\mathcal{O})$ that comprises exactly all entities covered by O as its elements. If such a set exists, it is denoted and defined by

$$ext(\mathcal{O}) := ext(O) := \{o \mid o \text{ is covered by } O\} \ .$$

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[521] Issues to consider were: What is the range of "arbitrary fundamental phrases"? How to ensure acyclicity of the predication definition graph (cf. Def. 4.6) or deal with cyclic cases?

[522] For clarity, the term 'set-theoretic entities' refers to entities that are considered in set or class theories like ZFC or NBG and covers basic notions / entities (a) that are purely axiomatically captured as well as (b) such that are defined. In particular, the membership relation, all sets and classes, as well as defined entities like the subset relation are all set-theoretic entities.

Definitions that *set-theoretically encode* other entities and their interrelationships are not meant by "defined entities". For instance, the encoding of natural numbers in terms of sets, cf. [212, sect. V.1], does not account for viewing natural numbers as set-theoretic entities.

We use the term *set-compatible* for $\mathcal{O}$ for the conception of sets in ZFC (but with urelements).[523] The class of all *set-compatible* ontological models of a given formula or theory X in $Lg(\Sigma)$ is denoted by $SOMod^{\mathfrak{P}}(X)$, or $SOMod(X)$ if $\mathfrak{P}$ is clear from context. ❑

4.34 Condition (Σ, $\mathfrak{P}$, $\langle \mathcal{O}, \beta \rangle$ with set-compatible $\mathcal{O}$)

Given Cond. 4.8, which covers an arbitrary signature Σ, predication system $\mathfrak{P}$ for Σ, and ontological interpretation $\langle \mathcal{O}, \beta \rangle$, where $\mathcal{O}$ is suitable for Σ. Assume in addition that $\mathcal{O}$ is set-compatible. ❑

Limiting the satisfaction relation (under ontological semantics) to set-compatible structures leads to a constrained notion of entailment that is a part of formulating Th. 4.44 below.

4.35 Definition (set-compatible consequence $\mathrel{\vDash_{sc}}$)

Let Σ an arbitrary FOL signature, $\mathfrak{P}$ a predication system for Σ, X an arbitrary sentence or theory over Σ, and $\phi \in Lg(\Sigma)$ a sentence.

$X \mathrel{\vDash_{sc}} \phi$:iff $SOMod(X) \subseteq SOMod(\phi)$, where $X \mathrel{\vDash_{sc}} \phi$ reads as ϕ is a *consequence of X on set-compatible models*. ❑

Note that set-compatibility is intrinsic to ontological structures, rather than being dependent on any relationship between an ontological structure $\mathcal{O}$ and a theory or sentence X, the satisfaction of which wrt $\mathcal{O}$ is in question.[524] It is mainly that fact, together with Def. 4.35 and other definitions, which the next observations result from immediately – this concludes the preluding groundwork.

4.36 Observation ($\mathrel{\vDash_{sc}}$ subsumes $\mathrel{\vDash}$)

Given Cond. 4.34, let in addition $T \subseteq Lg(\Sigma)$ an arbitrary theory, and $\phi \in Lg(\Sigma)$ a sentence.

Then (i) $T \mathrel{\vDash_{sc}} \phi$ iff $SOMod(T) \subseteq OMod(\phi)$ and (ii) $T \mathrel{\vDash} \phi$ entails $T \mathrel{\vDash_{sc}} \phi$. ❑

4.37 Observation (link between set-compatible consequence and unsatisfiability)

Let Σ an arbitrary FOL signature, $\mathfrak{P}$ a corresponding predication system, and further let $T \subseteq Lg(\Sigma)$ a theory, $\phi \in Lg(\Sigma)$ a sentence. Then $T \mathrel{\vDash_{sc}} \phi$ iff $SOMod(T \cup \{\neg\phi\}) = \emptyset$. ❑

The ingredients for the proof of Obs. 4.36 are named in the text right above it. Proving Obs. 4.37 proceeds in exactly the same way as that of the analogous Obs. 4.27 for $\mathrel{\vDash}$, merely with ontological structure, model, and semantics all restricted to set-compatible models, and using $SOMod(\cdot)$ at every occurrence of $OMod(\cdot)$.

VARYING PREDICATION SEMANTICS IN THE 2ND APPROACH: ASSIGNING PREDICATION EXTENSIONS

For set-compatible ontological structures it is safely possible to consider those sets of tuples that reflect predicate applications, i.e., *predication extensions*.[525]

4.38 Definition (predication extension, $ext.(\cdot)$)

Assume Cond. 4.34, where $\mathcal{O} = (O, I)$ is any set-compatible ontological structure suitable for Σ, and let $P \in Pred^=(\Sigma)$ a predicate symbol in Σ. The *predication extension* of P in $\mathcal{O}$ (wrt $\mathfrak{P}$) is

$$ext_{\mathcal{O}}(P) := \left\{ \bar{o} \in O^{ar(P)} \mid \text{there is a } \beta : Var \to O \text{ s.t. } \langle \mathcal{O}, \beta_{\bar{o}}^{\bar{x}} \rangle \mathrel{\vDash} P(\bar{x}) \right\} .$$ ❑

The predication extension of any predicate P is obviously determined by the chosen predication system, except for equality, where $ext_{\mathcal{O}}(P) = id_{\mathcal{O}}$ results from Def. 4.10 and 4.38. Returning to the thought above to let predication semantics vary by means of assigning arbitrary sets of tuples of entities can now be cast into an extended notion of ontological structure, that assigns predicates a referent interpretation (in the form

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[523] Respectively and to explain the general name, in several circumstances the recourse to a specific set-theoretic conceptualization can be understood more flexibly (e.g., corresponding to weakenings of ZFC or to forms of naïve set theory), e.g., when hardly any particular conditions are required. In that sense, though ZFC (with urelements) should be seen as a default assumption where nothing else or nothing more specific is determined, one can rely on alternative set-theoretic conceptions where they agree with ZFC (with urelements) on those set-theoretic issues / matters that are *required* in a specific regard.

[524] Even the predication system is only relevant for the question of $\mathcal{O} \mathrel{\vDash} X$, but not for the set-compatibility of $\mathcal{O}$ itself.

[525] Note that we use "predication" instead of "predicate" to remind of the disconnection between predication and referent interpretation of the predicate.

of a member of the universe) as well as a predication interpretation (a set of tuples acting as its predication extension[526]). Note that a similar idea on model-theoretic semantics that utilizes interpretations with referents in the universe as well as tuple sets as extensions has been followed throughout the development of Common Logic [452], cf. also [439, 577] and [⊚18], as well as it is present in the semantics of RDF [380, 384], cf. also [406, ch. 3]. A comparison especially with Common Logic is presented in sect. 7.2.2.2.

4.39 Definition (ontological structure with assigned predication extensions)
Let Σ an arbitrary FOL signature. Any triple $\hat{\mathcal{O}} = (\hat{O}, I_{ref}, I_{ext})$ is referred to as (free) *ontological structure with assigned predication extensions*   :iff

$\quad (\hat{O}, I_{ref})$ is a set-compatible (free) ontological structure suitable for Σ and

the function $I_{ext} : Pred^=(\Sigma) \to (ext(\hat{O}))^{\mathcal{P}_{\text{fix}}^1}$ satisfies $I_{ext}(=) = id_{ext(\hat{O})}$ and $I_{ext}(P) \subseteq (ext(\hat{O}))^{ar(P)}$ for every $P \in Pred(\Sigma)$.   ❏

SEMANTICS ON THE BASIS OF ENHANCED STRUCTURES IS NOT REQUIRED

The straightforward continuation from ontological structures with predication extensions is to revise the notion of predication satisfaction according to $\hat{O}$, i.e., as would seem natural, to $\hat{\mathcal{O}} \mathrel{|\!\approx} P(\bar{t})$ iff $\bar{t}^{\langle \hat{\mathcal{O}}, \beta \rangle} \in I_{ext}(P)$. This could further be completed into a second kind of ontological semantics, with the subsequent question of how the latter would relate to ontological semantics with predication systems and "usual" ontological structures. However, great proximity between ontological structures with assigned predication extensions and classical, set-theoretic structures can already be observed. The completion of a second ontological semantics is therefore abandoned. Instead, we return to the relationship between ontological semantics and set-thereotic semantics, mediated by ontological structures with assigned predication extensions.

4.6.4 Inconsistency and Reasoning Transfer from Set-Theoretic Semantics

EXTENSIONALLY ENHANCED AND ASSOCIATED SET-THEORETIC STRUCTURES

The considerations of the previous section pave the way for a main result, Th. 4.44, that allows for transferring classical reasoning under set-theoretic semantics, more precisely, the notion of entailment, to entailment under ontological semantics. In preparation of Th. 4.44, two operators shall associate with an ontological structure a specific ontological structure with assigned predication extensions and a specific set-theoretic structure, resp. The *extensionally enhanced structure* of $\mathcal{O}$ is just that ontological structure with assigned predication extensions that mirrors the effects of the predication system.

4.40 Definition (extensionally enhanced structure, $eStr(\cdot)$)
Assume Cond. 4.34, where $\mathcal{O} = (O, I)$ is set-compatible. The *extensionally enhanced structure* of $\mathcal{O}$ is the ontological structure with assigned predication extensions $eStr(\mathcal{O}) := (\hat{O}, I_{ref}, I_{ext})$ that is defined by $\hat{O} = O, I_{ref} = I, I_{ext} : Pred^=(\Sigma) \to (ext(\mathcal{O}))^{\mathcal{P}_{\text{fix}}^1}$ s.t. for every $P \in Pred^=(\Sigma)$: $I_{ext}(P) = ext_{\mathcal{O}}(P)$.   ❏

The extensionally enhanced structure of any ontological structure $\mathcal{O}$ can be converted trivially into a set-theoretic structure by reference to a set as universe, reordering the interpretation components, and restricting the ontological referent interpretation to only those mappings from logical individual constants, excluding predicates.

4.41 Definition (associated set-theoretic structure, $sStr(\cdot)$)
Assume Cond. 4.34 and let $eStr(\mathcal{O}) = (O, I, I_{ext})$. The *associated set-theoretic structure* of $\mathcal{O}$ is

$$sStr(\mathcal{O}) := \big(ext(O), I_{ext}, I\big|_{Const(\Sigma)}\big) \ .$$ ❏

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[526]Of course, this set is explicitly assigned then instead of being derived from the predication system that this assignment replaces.

Obs. 4.42 on interpreting terms equally and Prop. 4.43, stating the equal valuation of arbitrary formulas between a (set-compatible) ontological structure and its associated set-theoretic structure, are two lemmata for the inconsistency transfer of Th. 4.44. For simpler notation, in the sequel we identify the variable assignments $\beta : Var \to O$ and $\beta' : Var \to ext(O)$, where $\beta'(x) = \beta(x)$ for every $x \in Var$.

4.42 Observation (identical term interpretation in associated set-theoretic structure)
Assume Cond. 4.34 and let $\mathcal{A} = sStr(\mathcal{O})$. For every tuple of terms $\bar{t} \in (Tm(\Sigma))^{\mathcal{P}^1_{\text{fix}}}$: $\bar{t}^{\langle \mathcal{A}, \beta \rangle} = \bar{t}^{\langle \mathcal{O}, \beta \rangle}$. □

Proof. Without functions, every term is either a variable or an individual constant. Due to referring to β in both cases, $x^{\langle \mathcal{A}, \beta \rangle} = \beta(x) = x^{\langle \mathcal{O}, \beta \rangle}$ is immediate for all $x \in Var$. For any constant $c \in Const(\Sigma)$, mainly $\mathcal{A} = sStr(\mathcal{O})$ justifies $c^{\mathcal{A}} = (I|_{Const(\Sigma)})(c) = I(c) = c^{\mathcal{O}}$. $\bar{t}^{\langle \mathcal{A}, \beta \rangle} = \bar{t}^{\langle \mathcal{O}, \beta \rangle}$ is immediate from the analogous componentwise definitions of $\cdot^{\langle \mathcal{A}, \beta \rangle}$ and $\cdot^{\langle \mathcal{O}, \beta \rangle}$ for tuples $\bar{t} \in (Tm(\Sigma))^{\mathcal{P}^1_{\text{fix}}}$. □

4.43 Proposition (associated set-theoretic structures agree on formula satisfaction)
Assume Cond. 4.34 and let $\mathcal{A} = sStr(\mathcal{O})$.

$$\langle \mathcal{A}, \beta \rangle \models \phi \text{ iff } \langle \mathcal{O}, \beta \rangle \approx\!\!\!\mid \phi \text{ for every formula } \phi \in Lg(\Sigma). \qquad \square$$

Proof (structural induction on formulas). Assume Cond. 4.34 with arbitrary signature Σ, predication system $\mathfrak{P}$, and set-compatible ontological structure $\mathcal{O} = (O, I)$, moreover let $\mathcal{A} = sStr(\mathcal{O}) = (A, I_{ext}, I|_{Const(\Sigma)})$, hence $A = ext(\mathcal{O})$.

For the base case of atomic formulas, suppose further Cond. 4.9 with its arbitrary atom $P(\bar{t})$ and let $\bar{o} = \bar{t}^{\langle \mathcal{O}, \beta \rangle}$. Then, the following chain of equivalences applies: $\langle \mathcal{O}, \beta \rangle \approx\!\!\!\mid P(\bar{t})$ iff(1) $\langle \mathcal{O}, \beta^{\bar{x}}_{\bar{o}} \rangle \approx\!\!\!\mid P(\bar{x})$ iff $\bar{o} \in ext_{\mathcal{O}}(P)$ iff(2) $\bar{o} \in I_{ext}(P)$ iff $\langle \mathcal{A}, \beta^{\bar{x}}_{\bar{o}} \rangle \models P(\bar{x})$ iff(3) $\langle \mathcal{A}, \beta \rangle \models P(\bar{t})$. These equivalences can be seen straightforwardly from the respective definitions. Apart from those, (1) is satisfied due to $\bar{o} = \bar{t}^{\langle \mathcal{O}, \beta \rangle}$, which with Obs. 4.42 and $\mathcal{A} = sStr(\mathcal{O})$ further yields $\bar{o} = \bar{t}^{\langle \mathcal{A}, \beta \rangle}$ and hence (3). (2) is justified by $\mathcal{A} = sStr(\mathcal{O})$.

Complex formulas are next. For any formulas that are satisfied equivalently by $\langle \mathcal{O}, \beta \rangle$ and $\langle \mathcal{A}, \beta \rangle$ we write $E(\cdot)$, i.e., $E(\phi)$ iff $\langle \mathcal{O}, \beta \rangle \approx\!\!\!\mid \phi$ iff $\langle \mathcal{A}, \beta \rangle \models \phi$. The induction hypothesis now is $E(\phi)$ and $E(\psi)$. Then $E(\neg\phi)$ as well as $E(\phi \circ \psi)$ with $\circ \in \{\wedge, \vee, \dot{\vee}, \to, \leftrightarrow\}$ are immediate due to the exactly analogous definitions for $\approx\!\!\!\mid$ (Def. 4.23) and $\models$ (Def. 1.5) in these cases. The same is true for quantified formulas $Qx \, . \, \phi$, with $Q \in \{\forall, \exists\}$, according to Def. 4.24 and 1.6 in combination with $A = ext(\mathcal{O})$. □

Prop. 4.43 allows us to conclude next that ontological semantics restricted to set-compatible structures has a close and practically exploitable relation with classical set-theoretic semantics.

4.44 Theorem ((in)consistency and reasoning transfer)
Let Σ a FOL signature, $\mathfrak{P}$ a predication system for Σ, $T \subseteq Lg(\Sigma)$ a theory, and $\phi \in Lg(\Sigma)$ a sentence.

1. If T has a set-compatible ontological model, then T is set-theoretically satisfiable.
   Equivalently, if T is set-theoretically unsatisfiable, T is not satisfiable under ontological semantics restricted to set-compatible models.

   $$Mod(T) = \emptyset \text{ entails } SOMod(T) = \emptyset \, .$$

2. Classical implies ontological entailment wrt set-compatibility: If $T \models \phi$, then $T \approx\!\!\!\mid_{sc} \phi$. □

Proof. To draw the first conclusion (of model existence transfer) directly, assume Cond. 4.34. Since its ontological structure $\mathcal{O}$ is set-compatible and suitable for Σ, Def. 4.40 and 4.41 yield the existence of the set-theoretic structure $sStr(\mathcal{O})$, likewise suitable for Σ. Now suppose further that $\mathcal{O} \in SOMod(T)$, thus $\mathcal{O} \approx\!\!\!\mid_{sc} T$. Prop. 4.43 immediately yields agreement of respective satisfaction relations among the two structures for any formula and any variable assignment, and therefore, in particular, $sStr(\mathcal{O}) \in Mod(T)$.

The second item in Th. 4.44 is equivalent to the first by $T \models \phi$ iff $Mod(T \cup \{\neg\phi\}) = \emptyset$ and by the analogous Obs. 4.37 for ontological semantics wrt set-compatible structures. ❏

RELEVANCE AND INTERPRETATION OF THE THEOREM

One may find the transfer of reasoning under set-theoretic semantics to reasoning under ontological semantics with set-compatible structures interesting from a theoretical point of view, for example, due to the fact that it is shown for arbitrary predication systems. Of course, and from a more practice-oriented perspective, there are nearby questions such as the subsequent two. Is this a useful result for engineering ontologies in connection with ontological semantics? To what extent does it really justify the reuse of standard reasoners, given the limitation to set-compatible ontological consequence?

We find and argue that it is beneficial for ontology engineering, at least for most ontologies under consideration today, despite its limitation to set-compatible consequence. A first point is that set-compatible models are at least not "worse" than what is available today under the regime of classical semantics, where all structures are of set-theoretic nature. Accordingly, even if a set theory with urelements is assumed for set-theoretic semantics, all set-theoretic structures have only set-compatible universes, by definition.

Another argument in favor of Th. 4.44 is based on the relationship between signature-satisfying and set-compatible ontological models. As far as we can see, most / a wide range of formalized ontologies are intended to capture conceptualizations of domains of reality / knowledge that do *not* involve entities which cannot be members of sets in the respective domain of entities. We call such a domain set-compatible[527]. Then let T be such an ontology. Thinking of the default case of signature-satisfying structures for ontological semantics, one may be misled, by presuming the intended domain of entities for T, that every signature-satisfying model were a set-compatible model. This need not be the case, because – in analogy to universes of set-theoretic structures – ontological structures may comprise "additional" entities, i.e., entities that are neither denoted by any signature element nor that they are among those entities in the intended domain. However, we contend the subsequent Cond. 4.45, as a special case of the more general expectation that for signature-satisfying models with such additional entities there should be signature-satisfying models without them – otherwise, if they are *necessary* in all signature-satisfying models "for" the intended domain, they actually seem to be part of the intended domain.

4.45 Condition (set-compatible model existence)

An ontological theory $T \subseteq Lg(\Sigma)$ for an intended, set-compatible domain of entities has a signature-satisfying ontological model   iff   it has a set-compatible ontological model, i.e.,

$$O^\Sigma Mod(T) = \emptyset \quad \text{iff} \quad SOMod(T) = \emptyset.$$ ❏

Accordingly, e.g. the inconsistency transfer from set-theoretic to ontological semantics wrt set-compatibility by Th. 4.44 continues the non-existence of a signature-satisfying (but not necessarily set-compatible) ontological structure, based on Cond. 4.45. Hence, e.g., refutation-complete provers can be exploited to detect inconsistency wrt signature-satisfying ontological models. Notice that this remains a sufficient condition only, due to the one-directional entailment in item 1 of Th. 4.44.

TOWARD A GENERALIZATION FOR $\not\approx$

Another route to justifying an even more general version of reasoning transfer can be found by involving "heavier machinery", namely the completeness of FOL and (a) proof theory.

4.46 Observation (approach for general inconsistency- and reasoning transfer)

Let Σ a FOL signature, $\mathfrak{P}$ a predication system for Σ, $L := Lg(\Sigma)$, $T \subseteq L$ a theory, and $\phi \in Lg(\Sigma)$ a sentence. Further let $\vdash \subseteq \mathcal{P}(L) \to L$ a derivability relation for FOL.

If $\vdash$ is complete wrt classical FOL semantics and correct wrt ontological semantics, then

$$T \models \phi \quad \text{entails} \quad T \approx\!\!\!\!\approx \phi .$$ ❏

Proof. The main idea behind this observation is to rely on a complete proof calculus for FOL. By its completeness, $T \models \phi$ yields $T \vdash \phi$, such that the assumption of correctness of $\vdash$ wrt ontological semantics

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[527] A more precise version would be analogous to Def. 4.33, p. 164.

results directly in $T \not\approx \phi$. ❏

Limiting ourselves to semantic considerations herein, we leave treatments for particular proof theories in combination with particular or classes of predication systems for future work. Nevertheless, we conjecture that predication systems of the kind of those based on $\mathcal{CR}$ together with common FO proof theories satisfy the preconditions of Obs. 4.46. Consequently, we count the observation as another argument (if only indicative so far) for the use of established theorem provers and reasoners in combination with adopting ontological semantics for a FOL theory.

4.6.5 On Consistency Transfer from Set-Theoretic Semantics

SEMANTIC CONSISTENCY IN THE REVERSE DIRECTION IS HAMPERED IN GENERAL

After the last section, it is natural to investigate the converse direction of the reasoning transfer established so far. It is tempting to think of "inverting" the definitions from any set-theoretic structure $\mathcal{A}$ to construct a corresponding ontological structure. An immediate idea is the following: build an ontological structure $\mathcal{O}$ by starting from all members of the universe of $\mathcal{A}$ and possibly augment that universe according to "translating" the tuples in the interpretation / extension of each predicate that is a fundamental predicate according to the predication system.

However, one finds that the move from set-theoretic to ontological models in any strict analogy to Def. 4.41 is not generally possible. Not surprisingly, this depends on the predication system, where it is instructive to study an example of a theory T that has a set-theoretic model and two predication systems s.t. only one of them allows for a (free) ontological model of T.

EXAMPLE OF CONSISTENCY DIFFERENCES, DERIVED FROM $\mathcal{CR}$ AND T_{apple}

The case of $\mathcal{CR}$ and, indeed, both of the predication systems that are defined on its basis in sect. 4.4.3.2, namely $\mathfrak{P}^0_{\mathcal{CR}}$ of Def. 4.16, p. 148 and $\mathfrak{P}^1_{\mathcal{CR}}$ of Def. 4.17, p. 148, lend themselves for the intended illustration, further borrowing from the former apple example, cf. sect. 4.6.2.

4.47 Example (of a classically satisfiable theory that lacks some or any ontological models)
Let us merely adopt $\Sigma_F = \{::\}$ from $\Sigma_{\mathcal{CR}}$ as the relevant fundamental predicate and $\Sigma_\Pi = \{\mathsf{a}, \cdot\mathsf{A}\}$ as two logical individual constants which form the normal signature. On that basis, consider the following theory $T = \{\forall x . x = \mathsf{a} \vee x = \cdot\mathsf{A} , \neg \mathsf{a} = \cdot\mathsf{A} , \mathsf{a} :: \cdot\mathsf{A}\}$. Following the notation in sect. 4.6.2, where a denotes the intended referent of a and A / the category of apple is the intended referent of $\cdot$A, T may be intended to formalize the facts that (1) (in the domain under consideration) there are exactly the two entities a and A and (2) a instantiates A. Taking (2) as a truth on the meta level leads to the following observations.

1. T is set-theoretically satisfiable, e.g. by[528] $\mathcal{A} = (\{\mathsf{a}, \mathsf{A}\} , ::^{\mathcal{A}}, \mathsf{a}, \mathsf{A})$ with $::^{\mathcal{A}} = \{(\mathsf{a}, \mathsf{A})\}$.
2. T is satisfiable under ontological semantics with the clause in (4.12) in Def. 4.16 as predication interpretation of $::$, e.g. by $\mathcal{O} = (\wr\mathsf{a}, \mathsf{A}\wr, ::^{\mathcal{O}}, \mathsf{a}, \mathsf{A})$ with $::^{\mathcal{O}} = \mathsf{a}$ (as an arbitrary selection between a and A). 'a instantiates A' obtains (at the meta level) wrt $\mathcal{O}$ by the supposed truth of (2) above. Notably, however, there is no signature-satisfying model for this predication system, because three symbols with three pairwise distinct intended referents require those three referents in the universe of any signature-satisfying structure. Consequently, there is no signature-satisfying model with a universe of two entities.
3. However, T is semantically inconsistent wrt ontological semantics based on the defining fundamental phrase $fp^1_{::}$, i.e., the clause filled by values from (4.15), in Def. 4.17 as predication interpretation of $::$. The reason is the existential quantification in $fp^1_{::}$, which (loosely speaking) requires an instantiation relator in the ontological structure that mediates between a and A. This cannot be present in any (free) ontological structure with a universe of two entities, because it is a distinct entity. Instead, any signature-satisfying model would require at least four entities, or more precisely, its universe must comprise $\wr ::, \mathsf{A}, \mathsf{a}, \mathsf{a}\overset{::}{\longrightarrow}\mathsf{A}\wr$. ❏

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[528]We tacitly presume the order of universe, fundamental predicate interpretation, and normal constant interpretations (the latter ordered as in the signature specification above) to clarify the overall assignments by the interpretation function, s.t. only ::

REASON 1: META-LEVEL DISTINCTIONS WITHOUT OBJECT-LEVEL REFLECTION

Ex. 4.47 reveals two reasons for inconsistency of a theory under ontological semantics despite being consistent wrt classical semantics. In its second case, where there is no signature-satisfying model of T wrt the predication interpretation of :: in the unanalyzed variant (Def. 4.16), there are the meta-level observations that[529] :: $\neq$ a and :: $\neq$ A which serve as argument for the non-existence of a signature-satisfying structure. From another angle, that means that all signature-satisfying structures (which are of main interest by default) of that signature fulfill those inequalities in addition to a $\neq$ A. However, only the latter has an object-level reflection as $\neg$ a $= \cdot$A, whereas the cardinality constraint on the number of domain entities forces the violation of at least one of the remaining two.

REASON 2: FUNDAMENTAL PHRASES WITH EXISTENTIAL CLAIMS

In the third item of Ex. 4.47, the reason for inconsistency wrt ontological semantics based on the predication interpretation of :: in its analyzed variant (Def. 4.17) lies in the fundamental phrase that involves an existential quantification that leads to another entity that (from the meta-level perspective) is distinct from the two entities postulated by T itself.[530] This is the instantiation relator a$\xrightarrow{::}$A, which serves as a "truthmaker" of a :: $\cdot$A. From the lack of that relator in an ontological structure $\mathcal{O} = (O, I)$ we conclude that $\mathcal{O}$ does not satisfy a :: $\cdot$A.

Implicitly, this conclusion involves another presupposition, namely to expect that relator in the *same* domain of entities (of $\mathcal{O}$ under consideration wrt satisfaction). We see this as justified by the faithfulness assumption between object and meta level that we argued for in sect. 4.3.2. A more liberal stance appears possible as a variant of ontological semantics, then allowing for semantic entities "outside of" $\mathcal{O}$ / O. The latter is clearly the case in set-theoretic semantics, where $A^{\mathcal{P}^{\omega}_{\mathrm{fix}}} \setminus A$ comprises only entities that are outside of the universe of a set-theoretic structure. It further reminds us of the distinction of the 'domain of reference' and the 'domain of discourse' in Common Logic, cf. [452, clauses 3.25–26, p. 5]. We remark that there is a definitional variant of the fundamental phrases in Def. 4.17 that involves a second meta-level variable[531] α' for $\mathcal{O}$ (requiring a further modification of predication satisfaction, cf. Def. 4.10) and starts with "there is an R relator between x^α and y^α covered by α', s.t. [. . .]", which would make this requirement explicit in the fundamental phrase. At the present stage and from the point of view that ontologies should commit to themselves, we maintain the general implicit assumption for Def. 4.17.

FURTHER REASONS: PREDICATION-SYSTEM TAUTOLOGIES AND (IN-)EQUALITIES

Besides unreflected distinctions of entities and effects from fundamental phrases, there are more sources that can result in finding that an ontological structure does *not* satisfy a theory, in spite of the latter having a set-theoretic model. These sources are the ACO adopted as the background theory and the predication definitions, "acting through" their corresponding tautologies, cf. sect. 4.4.4. In this connection and as a reminder of different (axiomatic) sets of tautologies occurring above, notice first that all tautologies under ontological semantics are satisfied in the associated set-theoretic structures from the previous sect. 4.6.4.

4.48 Observation (associated set-theoretic structures satisfy tautologies of ontological semantics)
Assume Cond. 4.34, where $\mathcal{O}$ is any set-compatible ontological structure suitable for signature Σ and $\mathfrak{P}$ is a predication system for Σ. Let $\Pi = PDef(\Sigma)$, F an (object-level) axiomatization of the abstract core ontology assumed for $\mathfrak{P}$, and E a set of (in-)equalities postulated for signature-satisfying structures. Moreover, let $\mathcal{A} = sStr(\mathcal{O})$ the associated set-theoretic Σ structure of $\mathcal{O}$. Then:

- $\mathcal{A} \models \phi$ if $\emptyset \approx^{\mathfrak{P}} \phi$, i.e., for every tautology $\phi \in OTaut(\Sigma, \mathfrak{P})$, and
- $\mathcal{A} \models F \cup \Pi$ in particular.
- $\mathcal{A} \models E$ if $\mathcal{O}$ is signature-satisfying.   ❏

These observations are immediate implications of Prop. 4.43 on the agreement of $\mathcal{O}$ and its associated set-theoretic structure $\mathcal{A}$ on arbitrary formulas, given the definition of tautologies under ontological semantics,

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[529] We use '::' here both as object-level predicate symbol and as its intended referent, an (ontological) relation at the meta level.

[530] Actually, already by the signature of T through the introduction of constant symbols, in view of being setup for analysis explication (cf. sect. 2.3.2).

[531] At this point, the genericity of the definition of predication system (Def. 4.6) is beneficial, in contrast to one that were relying directly and only on the interpretation $\langle \mathcal{O}, \beta \rangle$ (as a whole).

see Def. 4.26, as well as Prop. 4.20 and Obs. 4.22 in sect. 4.4.4 on the tautological character of F and Π, and in the case of E corresponding arguments briefly listed there and originally introduced in sect. 4.3.1.

The related Obs. 4.49, which is likewise based on the nature of tautologies under ontological semantics (wrt some predication system) and of (in-)equalities wrt signature-satisfying models, clarifies that their inconsistency with a theory under classical semantics excludes the existence of set-compatible ontological models of that theory. This reinforces the former idea of adopting F, Π, and E as axiomatic parts of theories. Obs. 4.49 is another direct corollary of Th. 4.44 (with Def. 4.26 and 4.28).

4.49 Observation (consistency requirement wrt ontological tautologies an (in-)equalities)
Let Σ any FOL signature, $\mathfrak{P}$ a predication system for Σ, $T \subseteq Lg(\Sigma)$ a theory on a set-compatible domain of entities, assuming Cond. 4.45. Further let E a set of (in-)equalities that are postulated for signature-satisfying structures. Then:

- $T \approx T \cup OTaut(\Sigma, \mathfrak{P})$
- $T \approx^{\Sigma} T \cup OTaut(\Sigma, \mathfrak{P}) \cup E$
- If $Mod(\, T \cup OTaut(\Sigma, \mathfrak{P}) \cup E\,) = \emptyset$, then T has no signature-satisfying, set-compatible model under ontological semantics (wrt $\mathfrak{P}$). ❏

All of the above reasons reoccur wrt the question in which this chapter shall culminate next.

4.7 Ontological Neutrality

ASKING FOR THE GOAL OF ONTOLOGICAL NEUTRALITY

The final sect. 3.4 of the previous chapter introduced notions of *ideal* and *partial ontological neutrality* wrt semantic approaches in Def. 3.21, p. 121. Hence, the remaining issue after ontological semantics is established by all foregoing sections of this chapter is, whether it can meet its design goal. The exposition of ontological semantics is pursued above at two levels of specificity, where the first presents ontological semantics as a generic semantic approach that is parametrized by an abstract core ontology, while the second instantiates the first level in particular by the two predication systems that are based on the ontology of categories and relations, namely $\mathfrak{P}^0_{\mathcal{CR}}$ in Def. 4.16, p. 148, and $\mathfrak{P}^1_{\mathcal{CR}}$ in Def. 4.16, following the former.

$\mathcal{Q}_{11}$ Can ontological semantics be understood to result in ontologically neutral semantic accounts?

$\mathcal{Q}_{12}$ Can ontological semantics, instantiated by the predication systems $\mathfrak{P}^{\mathcal{CR}}_0$ and $\mathfrak{P}^{\mathcal{CR}}_1$, be understood to be ontologically neutral?

4.7.1 Object- to Meta-Level Translation

COMMON PARTS OF THE OBJECT- TO META-LEVEL TRANSLATIONS

We start with both questions in parallel and investigate these in terms of an analogous object- to meta-level translation as is performed for set-theoretic semantics in sect. 3.3.3, see esp. Def. 3.11, p. 114. A similar notation will be employed, in particular, we reuse the level indices $^{(\cdot)}$ introduced in sect. 3.3.2 to distinguish between syntactic and semantic entities at object and meta levels. In the present section, equal syntactic entities with different level indices should be understood as names that denote symbols that are distinct, e.g., in their level assignment, but that are to be of the same shape if the level assignment or similar properties are ignored.

The two highly related definitions of the $\mathfrak{P}^{\mathcal{CR}}_i$ exhibit a difference in the definition of the phrases for the fundamental predicates, whereas by Def. 4.10, p. 146, of satisfaction in ontological semantics, a generic treatment of normal predicates is possible. Hence, we provide a specification skeleton of the targeted translation function $\tau : L \to L$ first. It specifies τ except for the translation result in the case of fundamental predicates. The latter are added by Def. 4.51 after some explanatory remarks after Def. 4.50.

4.50 Definition (skeleton of translation τ of object-level to meta-level syntax wrt ontological semantics)
Let Σ any FOL signature and $\mathfrak{P} = (\Sigma, \Sigma_F, \Sigma_\Pi, fp, \pi)$ a predication system for Σ. Set $L := Lg(\Sigma)$ and
$\mathcal{E} := \Sigma \,\dot\cup\, Var(L)$. We define the translation function $\rho : \mathcal{E} \to \mathcal{E}$ and partially specify the definition of a
translation function $\tau : L \to L$ by:

1. $\rho(x) := x$ for every variable $x \in Var(L)$
2. $\rho(c) := c$ for every constant $c \in Const(L)$
3. $\rho(P) := P$ for every predicate $P \in Pred(L)$

for every $\phi \in L$

4. if $\phi = {}$ '$x =^{(0)} y$', $\tau(\phi) := {}$ '$\rho(x) =^{(1)} \rho(y)$'   for any $x, y \in Tm(L)$
5. if $\phi = P(\bar{x})$, $\tau(\phi)$ ***is to be defined***   for any $P \in Pred(\Sigma_F)$ and $\bar{x} \in (Tm(L))^{ar(P)}$
6. if $\phi = P(\bar{x})$, $\tau(\phi) := \tau(\pi_P(\bar{x}))$ for any $P \in Pred(\Sigma_\Pi)$ and $\bar{x} \in (Tm(L))^{ar(P)}$
7. if $\phi = \neg\psi$, $\tau(\phi) := \neg\tau(\psi)$
8. if $\phi = \psi \wedge \psi'$, $\tau(\phi) := \tau(\psi) \wedge \tau(\psi')$
9. if $\phi = \exists x . \psi$, $\tau(\phi) := \exists \rho(x) . \tau(\psi)$

τ is naturally extended to sets of formulas: $\tau(T) := \{\tau(\phi) \mid \phi \in T\}$ for any $T \subseteq L$.   ❑

DISTINGUISHING ρ AND τ, AND COMMENTS ON LEVEL INDICES

A first deviation from Def. 3.11 is an explicit distinction between function τ that maps formulas to each
other, and a function ρ with domain and range $\Sigma \,\dot\cup\, Var(L)$. Besides a formally more precise picture, a
certain correspondence between ρ and the referent interpretation of an entity may be seen.[532] Although
$\rho = id_\mathcal{E}$, with level indices the definitions have the form $\rho(e^{(0)}) := e^{(1)}$ for all $e \in \mathcal{E}$.

JUSTIFICATION OF FORMULA TRANSLATIONS (AS FAR AS DEFINED)

Similarly, we indicate the level difference wrt equality directly in clause 4. $\tau(\pi_P(\bar{x}))$ of clause 6 reads
as $\tau(\pi_P^{(2)}(\bar{x}^{(0)}))$ with level indices. That means, we adopt the λ-term from Def. 4.6 of predication system
at / for the metameta level to specify that the result of $\tau(\phi)$ in the case of normal predicates is the definiens
formula of P in $\mathfrak{P}$ (δ_P in Def. 4.6) with the object-level variables of $\bar{x}$ substituted for free variables in δ_P
in accordance with the full λ-term π_P.[533] Put more shortly, for atomic formulas with a normal predicate,
τ maps them to their definiens formula with appropriate variable renaming. We consider this formula to
remain in object-level syntax. Hence, recursive application of τ is possible and well-defined due to Obs.
4.7, because of which this recursion bottoms out in all cases, namely at fundamental predicates or equality.
Clauses 7 and 8 should be clear from Def. 4.23, p. 156, with the recursion terminating in atomic formulas,
which applies likewise to the quantifier clause.

This final clause on existential quantification may require more support, esp. compared to clause 8
in Def. 3.11, which resulted in $\exists \tau(x) . \tau(x) \,\dot\in\, \cdot U \wedge \tau(\psi)$. The latter binds / "relativizes" all entities
under quantification to the universe $\cdot U$ of the set-theoretic structure. However, for ontological semantics
we deliberately and explicitly avoid the assumption that O, which is referred to as 'the universe' of an
ontological structure in analogy to the classical parlance, exists as a (semantic) entity in its own right, cf.
the discussion in sect. 4.1. Eventually, O is seen there as a symbol, utilized in a certain role in defining the
semantics of a language. However, neither the set-theoretic object- to meta-level translation takes symbols
or the syntax-semantic relationship *at the side of semantics / semantic entities* into account. On the grounds
of these views, we argue that a non-relativizing translation of quantification is appropriate for clause 9.

TRANSLATION OF FUNDAMENTAL PREDICATES

The missing step / element in Def. 4.50 is a defining clause for fundamental predicates. However[534], we
see no generic account for specifying a translation on the basis of an (obtaining) natural language phrase in

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[532] Although not every occurrence of $\rho(x)$ can be understood as an "application of the referent interpretation", cf. e.g. the use in
clause 9, where ρ is merely used for a technical transition from a variable of the object-level language to one of the meta-level
language (which can be distinct symbols in similar translations). The case of predicates is discussed after Def. 4.51.

[533] As in the semantics, $\pi_P^{(2)}(\bar{x}^{(0)})$ is a metameta level expression that results in an object level / [(0)] formula, s.t. $\tau(\pi_P^{(2)}(\bar{x}^{(0)}))$
yields a meta-level / [(1)] formula.

[534] And without making any further assumptions at this point, in contrast to further below.

Def. 4.10. For example, the translation should account for the structure of such a phrase, which is unknown in the general formulation of a predication system (Def. 4.6).

Hence, let us introduce corresponding clauses that complete Def. 4.50, yielding specific translations τ_0 and τ_1 for the cases of $\mathfrak{P}_0^{\mathcal{CR}}$ and $\mathfrak{P}_1^{\mathcal{CR}}$, which are available as predication systems from sect. 4.4.3.2.

4.51 Definition (τ_i for $\mathfrak{P}_i^{\mathcal{CR}}$ with fundamental predicate clauses)
For $i \in \{0, 1\}$, adopt the translation skeleton of Def. 4.50 (relabeling τ to τ_i) with its preconditions, but refining Σ s.t. $\Sigma_{\mathcal{CR}} \subseteq \Sigma$, and presume $\mathfrak{P}_i^{\mathcal{CR}}$. Infix notation is used for the fundamental predicates ::, $\rightsquigarrow$, $\multimap$, and for their images wrt ρ.

The specifications of the translation functions $\tau_i : L \to L$ wrt $\mathfrak{P}_i^{\mathcal{CR}}$ are completed by

- for $\tau_0(\phi)$

  5. If $\phi = x \, P \, y$, where $P \in \{::, \rightsquigarrow, \multimap\}$ and $x, y \in Tm(L)$ are arbitrary,   then

  $$\tau_0(\phi) \quad := \quad \rho(x) \, \rho(P) \, \rho(y)$$

- for $\tau_1(\phi)$, with E as in (4.15)–(4.17) in Def. 4.17 (p. 148), $\cdot$R, $\cdot$Q$_1$, $\cdot$Q$_2 \in \Sigma_{\mathcal{CR}}$ as reflection constants of respective values in columns R, Q_1, Q_2 ibid., and reflection constants $\cdot$Relator, $\cdot$Role $\in \Sigma_{\mathcal{CR}}$

  5. If $\phi = E$,   read as a(n infix) atom of a predicate $R \in \{::, \rightsquigarrow, \multimap\}$ and any $x, y \in Tm(L)$,   then

  $$\begin{aligned}
  \tau_1(\phi) \quad := \quad \exists r q_1 q_2 (\ & r \, \rho(::) \, \cdot\mathsf{Relator} \ \wedge \ q_1 \, \rho(::) \, \cdot\mathsf{Role} \ \wedge \ q_2 \, \rho(::) \, \cdot\mathsf{Role} \ \wedge \\
  & r \, \rho(::) \, \cdot\mathsf{R} \ \wedge \ \neg(q_1 = q_2) \ \wedge \\
  & \rho(x) \, \rho(\rightsquigarrow) \, q_1 \ \wedge \ q_1 \, \rho(\multimap) \, r \ \wedge \ q_1 \, \rho(::) \, \cdot\mathsf{Q}_1 \\
  & \rho(y) \, \rho(\rightsquigarrow) \, q_2 \ \wedge \ q_2 \, \rho(\multimap) \, r \ \wedge \ q_2 \, \rho(::) \, \cdot\mathsf{Q}_2 \\
  & \forall q' (\, q' \, \rho(\multimap) \, r \ \rightarrow \ (q' = q_1 \vee q' = q_2) \,) \ \wedge \\
  & \forall p \, (\, p \, \rho(\rightsquigarrow) \, q_1 \ \rightarrow \ p = \rho(x) \,) \ \wedge \ \forall p \, (\, p \, \rho(\rightsquigarrow) \, q_2 \ \rightarrow \ p = \rho(y) \,) \ ) \quad \square
  \end{aligned}$$

Clause 5 of τ_0 is motivated by the problem that natural language phrases are not available in L. In contrast to extending the range of τ_0,[535] the assumption is made that at the meta-level the same formal expression can be utilized for that obtaining semantic phrase that is used for representing that on the object level (possibly modulo "renaming" via ρ). We argue that this should be an acceptable and minimal commitment to the same formal notation for fundamental phrases at object and meta level. We see here no commitment to the overall abstract core ontology wrt which these phrases are formed, which will become clearer in the next sect. 4.7.2.

Clause 5 of τ_1 follows a similar assumption in using ::, $\rightsquigarrow$, and $\multimap$ in their ρ-translations in the definiens of the definition, in addition to formalizing the logical structure of the schema fp_E^1 in Def. 4.17, the analyzed variant of fundamental $\mathcal{CR}$ predicates. In contrast to τ_0, the explication of those interconnections produces some commitments for τ_1, which are also dealt with below.

STATUS OF $\rho(P)$
Eventually and in this regard, the role of $\rho(P)$ in Def. 4.50 and 4.51 deserves more attention / explanation. Clearly, $\rho(P) = P$ in 4.50 is defined in a way that allows for establishing Def. 4.51 as it is above, in particular, such that $\rho(P)$ can be employed in predicate position. This is a deviation from the analogous set-theoretic object- to meta-level translation in Def. 3.11, p. 114, where predicates are mapped to constants. However, we argue that the choice of keeping $\rho(P)$ as a predicate is rather a tribute to the syntactic categories of FOL, while semantic adequacy is still justified by the construction of ontological structures.

In the alternative case of defining $\rho(P)$ as a mapping to a constant, in order to be semantically adequate from the (metameta-level) point of view of the language user, by which we here mean co-reference of the predicate symbol and that constant, should thus establish a reflection constant for P as the result of $\rho(P)$.[536] If this is accepted, $P^{(0)}$, $P^{(1)}$ and a reflection constant $\cdot P^{(1)}$ have literally the same referent (from the metameta-level perspective) and there should be no problem in exchanging that constant $\cdot P^{(1)}$

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[535]where it is unclear how natural language phrases, including meta-level variables, should be treated formally
[536]Cf. the similarity in the set-theoretic case, where the same relation between P and $\cdot P = \tau(P)$ in Def. 3.11 is implicitly assumed.

by the predicate symbol $P^{(1)}$ as value of $\rho(P^{(0)})$. Note further that the perspective of "just" having three symbols (the values of the ρ applications) in the definition of $\tau_0(xPy)$ is reminiscent of the consideration of representation by constants only, see sect. 4.3.3.

The second line of argument supports the choice on different grounds, namely due to the separation of predication interpretation and predicate referents in ontological semantics. I.e., even if a constant were assigned as value of $\rho(P)$, the question remains what an appropriate translation of an atomic sentence $P(\bar{x})$ of a fundamental predicate would be, and whether or not that should depend on that constant.[537] Indeed, given that the predication interpretation in ontological semantics does not depend on the referent interpretation of the predicate symbol, we argue to simply adopt an atomic sentence involving the fundamental predicate of the object level for the interpretation at the meta level, for the same arguments as provided for clause 5 of τ_0 three paragraphs above. Altogether, we consider ρ and the τ_i as justified and adequate object- to meta-level translations.

4.7.2 Examination of Ontological Neutrality

FIRST ANALYSIS REGARDING DEF. 3.21

The functions τ_0 and τ_1 allow for examining the status of the two $\mathcal{CR}$-based ontological semantics wrt ontological neutrality. Starting with τ_0, clause 6 in combination with the requirement of non-circular definitions, cf. Def. 4.6 and Obs. 4.7, lead directly to the following Obs. 4.52. Predication definition (4.21), i.e., $\forall x . \mathsf{Cat}(x) \leftrightarrow x :: \cdot\mathsf{Cat}$, is a concrete witness for it by the fact that this formula is no tautology of FOL under classical semantics. More precisely, $\tau_0(\mathsf{Cat}(x)) = x :: \cdot\mathsf{Cat}$, but $\mathsf{Cat}(x) \not\equiv x :: \cdot\mathsf{Cat}$. Although a fundamental predicate occurs, this example does not depend on the definition of the fundamental predicates in view of (classical) logical equivalence. Hence the same analysis applies to $\mathfrak{P}_1^{\mathcal{CR}}$.

4.52 Observation (no ideal ontological neutrality wrt $\mathfrak{P}_i^{\mathcal{CR}}$ and $\equiv$-based Def. 3.21)
Ontological semantics based on $\mathfrak{P}_0^{\mathcal{CR}}$ or $\mathfrak{P}_1^{\mathcal{CR}}$ is not ideally ontologically neutral according to Def. 3.21. ❏

ONTOLOGICAL NEUTRALITY BASED ON ONTOLOGICAL SEMANTIC EQUIVALENCE

This may seem discouraging. But before considering the weaker notions of ontological neutrality or limitations to theories, we examine the impact of the notion of equivalence that is utilized. As already indicated after the introduction of Def. 3.21, p. 121, it may be appropriate to consider other kinds of equivalences for the definition of ontological neutrality. Hence one may consider a variation of ontological neutrality, defined via the notion of semantic equivalence wrt ontological semantics in Def. 4.53. Indeed, this is more promising, as can be seen by Prop. 4.54 thereafter.

4.53 Definition (ideal and partial ontological neutrality of semantic accounts (ontological semantics variant))
Assume Cond. 3.19 with its semantic approach $\mathfrak{S}$, object- to meta-level translation $\hat{\tau} : L \to L'$, extended to theories $T \subseteq L$ and to $\hat{\tau}^+(T) := \Omega \cup \hat{\tau}(T)$ for a fixed theory Ω.

$\mathfrak{S}$ is termed *ideally ontologically neutral*   :iff
$\qquad L = L'$, $\hat{\tau}(T) \approx T$, and $\hat{\tau}^+(T) \approx \hat{\tau}(T)$ for every object-level theory $T \subseteq L$.
$\mathfrak{S}$ is called *partially ontologically neutral*   :iff
$\qquad L = L'$ and $\hat{\tau}(T) \approx_\Omega T$ for every object-level theory $T \subseteq L$. ❏

4.54 Proposition (T and $\tau_0(T)$ are equivalent wrt equal $\mathfrak{P}_0^{\mathcal{CR}}$ semantics)
Accepting ontological semantics at the meta level wrt a $\mathfrak{P}_0^{\mathcal{CR}}$ predication system for a FOL language L, ontological semantics wrt that predication system satisfies

$$\tau_0(T) \approx T \quad \text{for all theories } T \subseteq L .$$ ❏

Proof (nested inductions). Let Σ a FOL signature s.t. $\Sigma_{\mathcal{CR}} \subseteq \Sigma$, $L := Lg(\Sigma)$, assume a predication system $\mathfrak{P}_0^{\mathcal{CR}} = (\Sigma, \Sigma_F, \Sigma_\Pi, fp, \pi)$ for Σ. Further (*) accept ontological semantics at the meta level wrt $\mathfrak{P}_0^{\mathcal{CR}}$.

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[537]Remember the examples in Table 4.1 as well as the discussion in sect. 4.4.4.2.

We show that $\tau_0(\phi) \approx \phi$, for arbitrary $\phi \in L$, from which $\tau_0(T) \approx T$ for all $T \subseteq L$ is clear by Def. 4.50 and Def. 4.26. Although the proof is only complete for τ_0, for later considerations we use τ_0 only were there is specific reference to clause 5 for τ_0 in Def. 4.51, otherwise we write just τ (also easing readability).

The proof proceeds by induction on the definition nesting of formulas, with "inner" inductions on the structure of formulas. For the nesting-based induction, we use upper indices $\cdot^n$, $n \in \mathbb{N}$, to mark the definition nesting level of predicates in accordance with the following signature and associated language levels (with $L = \bigcup_{n \in \mathbb{N}} L^n$):

$$L^n := Lg(\Sigma^n) \quad \text{for all } n \in \mathbb{N}, \text{ where}$$
$$\Sigma^0 := Pred^=(\Sigma_F) \cup Const(L)$$
$$\Sigma^{n+1} := \Sigma^n \cup \{P \in \Sigma_\Pi \mid \pi_P = \lambda\bar{x}.\delta \text{ and } \delta \in L^n\}$$

The base case (L0) of the outer induction starts with the first inner induction on formulas in L^0, for which we prove the stricter $\tau(\phi) = \phi$, entailing $\tau(\phi) \approx \phi$ by Def. 4.26 of $\approx$.

The base case (L0-B) of the latter concerns atoms $\phi \in At(L^0)$. Σ^0 contains no normal predicates. Hence and by clause 4 (and those for ρ) of Def. 4.50 and clause 5 for τ_0 of Def. 4.51 we have $\tau_0(\phi) = \phi$, concluding (L0-B). For the induction step (L0-I), let $\phi, \psi \in L^0$ and $\tau(\phi) = \phi, \tau(\psi) = \psi$. Def. 4.50 together with the induction hypothesis justifies

- $\tau(\neg\phi) = \neg(\tau(\phi)) = \neg\phi$
- $\tau(\phi \wedge \psi) = \tau(\phi) \wedge \tau(\psi) = \phi \wedge \psi$
- $\tau(\exists x \,.\, \phi) = \exists \rho(x) \,.\, \tau(\phi) = \exists x \,.\, \phi$

and thus $\tau(\phi) = \phi$ for all $\phi \in L^0$. This ends (L0-I) and (L0).

For the outer induction and its induction step (Ln), assume that $\tau(\psi) \approx \psi$ for all $\psi \in L^n$. We must prove that $\tau(\phi) \approx \phi$ for all $\phi \in L^{n+1}$. Turning to the base case (Ln-B) of the inner induction (Ln), consider any $\phi \in At(L^{n+1})$. If the predicate of ϕ is equality or a fundamental predicate, $\phi \in L^0$ and covered by the induction hypothesis for (Ln). The same applies to normal predicates in Σ^i for $1 \leq i \leq n$. The interesting case is that of $\phi = P(\bar{x})$ for a normal predicate P^{n+1}. In that case, by clause 6 in Def. 4.50 and supposing $\pi_P = \lambda\bar{x}.\delta(\bar{x})$,[538] $\tau(\phi) = \tau(\pi_P(\bar{x})) = \tau(\delta(\bar{x}))$, where $\delta(\bar{x}) \in L^n$ due to the requirements on the predication definition graph of predication systems in general, cf. Def. 4.6. The induction hypothesis for (Ln) yields $\tau(\delta(\bar{x})) \approx \delta(\bar{x})$, and from Prop. 4.20 on the tautological character of predication definitions (and Def. 4.26), now applied at the meta level from presupposition (*), we have[539] $\delta(\bar{x}) \approx P(\bar{x})$ and thus arrive at $\tau(\pi_P(\bar{x})) \approx P(\bar{x})$. This completes the base case (Ln-B).

The induction step (Ln-I) is analogous to (L0-I). Let $\phi, \psi \in L^{n+1}$ and assume $\tau(\phi) \approx \phi$ and $\tau(\psi) \approx \psi$ as induction hypothesis of the structural induction step (Ln-I). This and Def. 4.50 entail

- $\tau(\neg\phi) = \neg(\tau(\phi))$ and $\neg(\tau(\phi)) \approx \neg\phi$, hence $\tau(\neg\phi) \approx \neg\phi$
- $\tau(\phi \wedge \psi) = \tau(\phi) \wedge \tau(\psi)$ and $\tau(\phi) \wedge \tau(\psi) \approx \phi \wedge \psi$, thus $\tau(\phi \wedge \psi) \approx \phi \wedge \psi$
- $\tau(\exists x \,.\, \phi) = \exists \rho(x) \,.\, \tau(\phi)$ and $\exists x \,.\, \tau(\phi) \approx \exists x \,.\, \phi$, therefore $\tau(\exists x \,.\, \phi) \approx \exists x \,.\, \phi$

overall resulting in $\tau(\phi) = \phi$ for all $\phi \in L^{n+1}$, concluding (Ln-I), (Ln), and the proof altogether. ❑

Before proceeding, we remark and strongly conjecture that the proof can be strengthened to the extent that a statement $\tau_0'(T) = T$ for all theories T can be defended. The argument for this would be, if ontological semantics with the same predication system is accepted at the meta-level, then a modified translation $\tau_0' : L \to L$ appears justified that changes clause 6 of Def. 4.50 to $\tau_0'(P) := \rho(P)(\rho(\bar{x}))$. The justification is that, then at the meta level, the same equivalence between $P(\bar{x})$ and its definiens applies tautologically, s.t. $P(\bar{x})$ can be used as a "short-hand version" of the definiens. Accordingly, the proof for normal predicates can follow the τ_0 applications to fundamental predicates in case (L0-B).

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[538] We tacitly assume a possible selection of variables in moving from $P(\bar{x})$ to $\delta(\bar{x})$, as possible via π_P in Def. 4.6.

[539] Again, modulo variable changes, cf. the previous FN.

Thinking of the second equivalence of Def. 4.53, the question arises what the augmented theory $\tau_0^+(T)$ must comprise in addition to $\tau_0(T)$, in analogy to $\tau^+(T)$ in the case of set-theoretic semantics. There, the addition consists in the set-theoretic axiomatization ST, including the definition of a set-theoretic structure $\Delta_{\Sigma Str}$ and binding the theory to one such structure that can be addressed in terms of logical individual constants.

A rather clear analog at the side of ontological semantics can be seen by the axiomatization of the fundamental theory F, derived as an axiomatic reflection of the abstract core ontology adopted at the meta level. We argue in sect. 4.4.4.3 that these form tautologies under ontological semantics with / due to asserting a predication system based on an abstract core ontology. An analogous argument applies to the set of predication definitions Π. Due to the status of tautologies, choosing $\Omega = \emptyset$ is equivalent to $\Omega = OTaut(\Sigma, \mathfrak{P}_0^{\mathcal{CR}})$ as well as to $\Omega = F \cup \Pi$ due to $F \cup \Pi \subseteq OTaut(\Sigma, \mathfrak{P}_0^{\mathcal{CR}})$, and thus $\hat{\tau}^+(T) = \hat{\tau}(T)$ or at least $\hat{\tau}^+(T) \approx \hat{\tau}(T)$. Note further that the equality theory E is similar, but not completely analogous, because it applies only in signature-satisfying ontological models and is not tautologous. Altogether, this argumentation already verifies Obs. 4.56 for which the following condition sets the scene formally.

4.55 Condition ($\mathfrak{P}_0^{\mathcal{CR}}$ assumptions for questioning ontological neutrality)

Suppose ontological semantics wrt a $\mathfrak{P}_0^{\mathcal{CR}}$ predication system for a signature Σ and an object-level FOL language $L := Lg(\Sigma)$. Consider $\tau_0 : L \to L$ as object- to meta-level translation. ❏

4.56 Observation (ontological neutrality wrt $\mathfrak{P}_0^{\mathcal{CR}}$ and $\approx$-based Def. 4.53)

Assume Cond. 4.55 and let the same $\mathfrak{P}_0^{\mathcal{CR}}$ system act on the meta level. Let E a set of equalities over $Const(L)$ that are satisfied in signature-satisfying ontological structures.

Assuming $\Omega = \emptyset$, s.t. $\tau_0^+(T) := \tau_0(T)$, then ontological semantics wrt the $\mathfrak{P}_0^{\mathcal{CR}}$ predication system is ideally ontologically neutral according to Def. 4.53.

Assuming $\Omega = E$, s.t. $\tau_0^+(T) := E \cup \tau_0(T)$, it is partially ontologically neutral, i.e., $\tau_0(T) \approx_{\Omega^E} T$. ❏

On this basis and presupposing the axiomatizability of the tautologies under ontological semantics, we merely indicate the return to the $\equiv$-based Def. 3.21 in form of the following conjectures, which we find strongly supported by the above reflections, further relying on Th. 4.44 and the indication of Obs. 4.46.

4.57 Conjecture (ideal ontological neutrality wrt $\mathfrak{P}_0^{\mathcal{CR}}$ and Def. 3.21)

Assume Cond. 4.55. Let F an axiomatic fundamental theory and Π the set of predication definitions wrt the $\mathfrak{P}_0^{\mathcal{CR}}$ system, set $\Omega := F \cup \Pi$ for τ_0^+ and consider an arbitrary extension $T \supseteq F \cup \Pi$ at the object level.

If $F \cup \Pi \equiv OTaut(\Sigma, \mathfrak{P}_0^{\mathcal{CR}})$, then ontological semantics wrt the $\mathfrak{P}_0^{\mathcal{CR}}$ predication system is ideally ontologically neutral wrt T according to Def. 3.21. ❏

System $\mathfrak{P}_1^{\mathcal{CR}}$ and its related translation τ_1 brings in a novel aspect. Basically, it can be treated by an analogous approach as that expounded above for $\mathfrak{P}_0^{\mathcal{CR}}$. However, there is one important difference: $\tau_1(\phi)$ for atoms with fundamental predicates results in complex formulas instead of yielding the original formula as in the case of τ_0. Importantly, an application of τ_1 to those complex formulas must be avoided, otherwise an infinite regress results *for the definition* of τ_1. But if this is paid attention to (as is the case in Def. 4.51), the translation of a fundamental predicate just leads to a new formula. Remembering Obs. 3.22 on the fixpoint characteristic of partially ontologically neutral accounts, see p. 121, it is interesting to observe that repeated application of the semantics, i.e., of τ_1, yields increasingly complex sentences. Indeed, this shows that the regress is *required* then at the object level, in order to account for the necessary equivalence between an atom of fundamental predication and the defining formula, cf. the (so-called) reflection axioms in sect. 6.1.2, only then achieving faithfulness. These thoughts are carried over via Conj. 4.57 to the last expectation of the section.

4.58 Conjecture (partial ontological neutrality wrt $\mathfrak{P}_1^{\mathcal{CR}}$ and Def. 3.21)
Assume Cond. 4.55 modulo $\mathfrak{P}_1^{\mathcal{CR}}$ and τ_1, resp. Let $\Omega := OTaut(\Sigma, \mathfrak{P}_1^{\mathcal{CR}})$, note the axiomatic fragment $\mathcal{CR}_0^{\mathrm{FOL}}$, described in sect. 6.1.2, and let exactly (6.11)–(6.13), ibid., be the members of the set RA.

If $\Omega \models \mathcal{CR}_0^{\mathrm{FOL}} \cup RA$, then ontological semantics wrt the $\mathfrak{P}_1^{\mathcal{CR}}$ predication system is partially ontologically neutral according to Def. 3.21. ❏

The last circle within this chapter closes with a short (re)connection to Obs. 4.49 of sect. 4.6.5. In the light of Prop. 4.54 and Conj. 4.57, note that each of the three issues that demonstrate how a set-theoretic model exists despite there being an ontological model can be "cured" by an ontologically more complete object-level theory, which prevents those set-theoretic models by, in a sense, accounting for the meta-level arguments at the object level already.

Chapter 5

Ontological Engineering and Applications

DEFERRING FURTHER ANALYSIS AND COMPARISON IN FAVOR OF GOING TOWARD APPLICATIONS

Having achieved a semantics definition that is / can be considered to be ontologically neutral, as expounded in the last sect. 4.7 of the previous chapter, further discussion of ontological semantics itself and relating it to other work would be a natural step at this point. Nevertheless, taking this step is withheld until sect. 6.1 on the formalization of the $\mathcal{CR}$ ontology and beyond that, especially in comparison with other approaches, until ch. 7, which concludes the overall thesis. The main reason for this strategy is that it turns out to be revealing to see some further development on the basis of the concepts of ontological semantics, as defined thus far. Moreover, this allows us to explore its utility and effects in combination with initial applications. Indeed, these have largely been pursued in parallel to the development of ontological semantics, such that influences from them can be expected.

CHAPTER STRUCTURE

Sect. 5.1 proposes a formalization method for ontologies that is inspired by ontological semantics and maintains strong ties with it. Moreover and related to the former, we shall develop the idea of *ontological usage schemes* as an approach of assigning ontological semantics to any language *in a particular usage context*, in parallel to a possibly available formal semantics of that language, and implemented by translation

into an ontological theory under ontological semantics, constructed following the formalization method. Then everything is prepared for presenting some applications based on elements of ontological semantics and of $\mathcal{CR}$, mainly exploiting the conceptualization of sect. 2.4.

5.1 Formalization Method for Ontology Representation in FOL

PURPOSE OF A FORMALIZATION METHOD DERIVED FROM ONTOLOGICAL SEMANTICS

Readers reaching this point after processing the possibly lengthy ch. 4 on ontological semantics may welcome a short section for a start of the present ch. 5. This is sect. 5.1, which, more importantly and together with the subsequent sect. 5.2, originates from the question of how the considerations of ontological semantics may be exploited for ontological engineering. In accordance with our previous bias to FOL as a representation formalism, the present section is specifically tailored to it.

We admit the following contrast. On the one hand, FOL under classical semantics is very well established and acknowledged, not to speak of the huge body of available theoretical work. On the other hand, ontological semantics as developed in ch. 4 is a novel proposal, potentially based on debatable positions that were subscribed to in the course of its development. This section aims at a formalization approach that yields a compromise wrt classical and ontological semantics when using FOL for ontology representation, in the sense that classical reasoning in theories resulting from adherence to the method transfers to / can be seen as reasoning under ontological semantics.

5.1.1 Basic Method $\mathcal{OS}$

GATHERING ESSENTIAL ELEMENTS FROM ONTOLOGICAL SEMANTICS

For the purpose just laid out, let us recollect some core ingredients from ontological semantics and its relationship to set-theoretic semantics. Looking at the definitions that govern formula satisfaction under ontological semantics, one observes that the major significant change compared to the classical case is the treatment of predication. The primary insight, inspired from earlier work such as Nicola Guarino's on the ontological level, cf. a.o. [326, 328, 333], was to abandon any uniform, fixed interpretation of predication (in terms of set-membership in the classical case), leading on to introducing a predication system $\mathfrak{P}$, see Def. 4.6 on p. 185, that needs to be tailored to the signature that is setup for representing an ontological theory.[540] Remember that a predication system distinguishes *fundamental* and *normal* predicates. The *intended* interpretation of predicate symbols by the predication system reflects itself in terms of two sets of tautologies (under ontological semantics), which correspond to the two types of predicates. First, predication definitions Π emerge from normal predicates, which must be equipped with a definiens formula in $\mathfrak{P}$ that characterizes the (ontological) relationship between the argument referents and, typically, also the predicate referent in predication expressions with this predicate. In this connection, note that a reflection constant of a predicate P, by Def. 4.19 on p. 149, is a logical individual constant that has the same intended referent as P. Secondly, meta-level interconnections among the *intended* referents of fundamental predicates affect the existence of ontological structures and correspond to tautologies that depend on the predication system, more precisely, on any assumed "background theory" about fundamental predicate referents. F is used in order to denote an axiomatization of those tautologies.

There is a third component, arising from the view that those ontological models of a theory are of primary interest that satisfy the signature, i.e., in which all symbols, esp. logical individual constants are interpreted by *intended referents*. Sect. 4.2 and 4.3.1 explain in which sense there are epistemological aspects wrt the language user. A theory of (in-)equalities E is held to account for signature satisfaction, restricted to the awareness of the language user. Besides and associated with their status of validity in all[541] ontological models, the particular rôle of F, Π, and E is observed in the consideration of ontological neutrality in Obs. 4.56 (and Conj. 4.57), based on Prop. 4.54.

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[540]The next section 5.2 extends the idea to representations for other purposes, but covering any "conceptual content".
[541]or all signature-satisfying ontological models, in the case of E

On this overall basis, we propose the following method $\mathcal{OS}$, the name / abbreviation of which is inspired by 'ontological semantics', because the theories arising from this method can be perceived under both, classical as well as ontological semantics. Note that a very early sketch of the method is described in [534, sect. 3.3].

[534] 📖

5.1 Definition (Basic Method $\mathcal{OS}$)

Let Σ a FOL signature. Constructing (or converting) a theory $T \subseteq Lg(\Sigma)$ according to the *basic method* $\mathcal{OS}$ proceeds in these steps.

1. *reflection constants for all predicates*

   Define a (total) function $\rho : Pred(\Sigma) \rightarrow Const(\Sigma)$ that maps every predicate P to a reflection constant $\rho(P)$. If not available, new constants need to be added.

2. *(in-)equality theory*

   Add (in-)equalities, to the extent known or assumed, for all pairs over $Const(\Sigma)$.

3. *fundamental and normal signatures*

   Split Σ s.t. $\Sigma = \Sigma_F \,\dot\cup\, \Sigma_\Pi$, where Σ_F is considered as the fundamental signature and Σ_Π as the normal signature (wrt a predication system of ontological semantics).

4. *predication definitions*

   Add a set Π of predication definitions to T s.t. there is at least one definition and there are at most finitely many for each $P \in Pred(\Sigma_\Pi)$, where it is ensured that there are no infinite chains or cycles of dependencies among predicates, as follows.
   Define $\Delta_P := \{\delta(\bar{z}) \mid \text{'}\forall\bar{x} \,.\, P(\bar{x}) \leftrightarrow \delta(\bar{z})\text{'} \in \Pi \text{ with } Var(\bar{z}) \subseteq Var(\bar{x})\}$ for every $P \in Pred(\Sigma_\Pi)$ and furthermore a function π with $dom(\pi) = Pred(\Sigma_\Pi)$ that assigns to every P a λ-expression $\pi_P := \lambda\bar{x} \,.\, \big(\bigwedge_{\Delta_P} \delta(\bar{z})\big)$. The graph $\mathcal{G} := (Pred^=(\Sigma), Occ(\pi))$ must be a predication definition graph as defined in Def. 4.6, i.e., $Occ(\cdot)$ is defined as there, applied to parameter π as defined here.

5. *fundamental axiomatization*

   Add an axiomatization $F \subseteq Lg(\Sigma_F)$ as a fundamental theory characterizing predications involving fundamental predicates.

6. *domain[542] axiomatization*

   Add to (or maintain in) T arbitrary further axioms $\phi \in Lg(\Sigma)$ that characterize the domain of reality / knowledge under consideration in terms of Σ.

It should be fairly straightforward to see that the adoption of method $\mathcal{OS}$ yields theories, call them $\mathcal{OS}$ theories, that, unsurprisingly since by design, can be understood to be under an ontological semantics for FOL wrt a predication system $\mathfrak{P}$ that each $\mathcal{OS}$ theory "establishes itself", at least for the purpose of reasoning in that theory. Although there is one missing element yet, namely that fundamental predicates are / can be equipped with phrases that match the constraints in the fundamental axiomatization, step 5 of $\mathcal{OS}$. Then, Th. 4.44 allows for reasoning transfer from classical semantics in general, i.e., for arbitrary theories, and thus for each theory itself as a specific instance.

5.2 Observation (classical reasoning in an $\mathcal{OS}$ theory is correct wrt its ontological semantics)

Let $T \subseteq Lg(\Sigma)$ an $\mathcal{OS}$ theory, i.e., a FOL theory that is constructed (or transformed) according to method $\mathcal{OS}$, and let Σ_F, Σ_Π, Π, π, and F relate to Σ and T as in Def. 5.1. Suppose further that a function fp with $dom(fp) = Pred(\Sigma_F)$ can be specified that assigns declarative natural language phrases to all fundamental predicates, s.t. F is accepted as a meta-level / fundamental theory underlying those phrases. Then:

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[542] 'Domain' is not restrictive here, i.e., it is not meant in the sense of domain-specific ontologies. Arbitrary domains of reality / knowledge can be considered, including abstract core ontologies and top-level ontological notions.

- $\mathfrak{P} := (\Sigma, \Sigma_F, \Sigma_\Pi, fp, \pi)$ is a predication system that constitutes an ontological semantics, incl. $\approx_{sc}^{\mathfrak{P}}$.

- Classical entailment in T is correct wrt entailment under ontological semantics regarding $\mathfrak{P}$ and set-compatible structures, i.e., $\qquad$ for all $\phi \in Lg(\Sigma)$: if $T \models \phi$, then $T \approx_{sc}^{\mathfrak{P}} \phi$. $\qquad$ ❑

ONTOLOGICAL NEUTRALITY AND FOCUS ON CLASSICAL ENTAILMENT

We merely but strongly conjecture that the predication system induced by an OS theory is ontologically neutral in the sense of Obs. 4.56, the proof of which would be based on a translation function completely analogous to τ_0, Def. 4.50 and 4.51, p. 172f., in particular, translating atoms with fundamental predicates into themselves (for the same argument). This is the key property in the proof of Prop. 4.54 for the base case of formulas that involve only fundamental predicates (and equality). Since the argument there does not depend on anything more specific in $\mathfrak{P}_0^{CR}$, a generic proof of ontological neutrality for logically non-decomposed fundamental phrases is in prospect.

Based on Obs. 5.2 (although bearing Cond. 4.34 and 4.45 of set-compatibility in mind), concerning entailment we shall focus on and often utilize classical entailment from now on, which in this setting is guaranteed to be correct for ontological semantics. Notably, on the basis of the results established above, we are not entitled to the converse transfer, nor to conclude the existence of an ontological model from the consistency of a theory under classical semantics. In spite of this fact, we argue that the formal analysis of theories is an important task, esp. regarding consistency and since the establishment of an even closer relation to ontological semantics is not yet excluded.

AIMING AT (LOGICAL INDIVIDUAL) CONSTANTS AND SIGNATURE-SATISFYING MODELS

The first two steps in Def. 5.1 of OS are not exploited for Obs. 5.2. They are nevertheless introduced in connection with the aim of considering / "describing" signature-satisfying models of ontological semantics. The first step ensures the existence of "suitable" referents for all predicates, which is actually required for signature-satisfying ontological structures.[543] The second step accounts for the co-reference relations that the language user is aware of. Note that for this purpose the Unique Name Assumption (UNA) [109, sect. 11.2.5, p. 215][544], can be considered as a useful presupposition, which asserts that for every pair c_1, c_2 of constant symbols their inequality $\neg\, c_1 = c_2$ is in the theory. Of course, that needs to be appropriate for the signature under consideration.

JUSTIFICATION OF MULTIPLE PREDICATION DEFINITIONS

Reconsidering step 4, there is a seeming difference between allowing for finitely many predication definitions in that step for a single predicate P, but being restricted to a single definition formula in Def. 4.6 of predication system. This resolves on the basis of (5.1) below, which justifies the definition of π_P in step 4 of OS. Moreover, (5.2) illuminates why, in Def. 4.6, the assignment of (a conjunction of) multiple definientes to a predication system does not have the same effect as adding several predication definitions to a theory, which we can allow for in OS and from which the "actual" predication definiens of P is derived, i.e., the one that affects the predication system for ontological semantics. Accordingly and anew, a set of predication definitions for P cannot be replaced solely by a single predication definition that joins all "former" definiens conjunctively.

$$(5.1) \quad \text{for } n \in \mathbb{N}, n \geq 1: \quad \{\, \forall \bar{x}\,.\, P(\bar{x}) \leftrightarrow \phi_i(\bar{x}) \mid 1 \leq i \leq n \,\} \quad \models \quad \forall \bar{x}\,.\, P(\bar{x}) \leftrightarrow \bigwedge_{1 \leq i \leq n} \phi_i(\bar{x})$$

$$(5.2) \quad \text{for } n \in \mathbb{N}, n \geq 2: \quad \{\, \forall \bar{x}\,.\, P(\bar{x}) \leftrightarrow \phi_i(\bar{x}) \mid 1 \leq i \leq n \,\} \quad \not\models \quad \forall \bar{x}\,.\, P(\bar{x}) \leftrightarrow \bigwedge_{1 \leq i \leq n} \phi_i(\bar{x})$$

Proof. To see (5.1), consider any model $\langle \mathcal{A}, \beta \rangle$ of the left-hand side and with universe A. Let $\bar{a} \in A^{ar(P)}$. If $\bar{a} \in P^{\langle \mathcal{A}, \beta \rangle}$, it is true that $\langle \mathcal{A}, \beta_{\bar{a}}^{\bar{x}} \rangle \models \phi_i(\bar{x})$ for all $1 \leq i \leq n$, thus $\langle \mathcal{A}, \beta_{\bar{a}}^{\bar{x}} \rangle \models \bigwedge_{1 \leq i \leq n} \phi_i(\bar{x})$. Conversely,

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[543] Yet, not introducing reflection constants for all predicates does not entail a problem for having free ontological structures for Σ. The interpretation function can map each predicate to any entity in the universe, where the latter must exhibit at least one entity. By definition, it is problematic for finding signature-satisfying structures, in that these do not exist then.

[544] The unique name assumption is traced back in [109, ch. 11.7, p. 233] to work of Raymond Reiter, where it is introduced in connection with databases (logically perceived), as a property of a database called 'E-saturated' [704, sect. 7, p. 247].

let $\langle \mathcal{A}, \beta_{\bar{a}}^{\bar{x}} \rangle \models \bigwedge_{1 \le i \le n} \phi_i(\bar{x})$. Then esp. $\langle \mathcal{A}, \beta_{\bar{a}}^{\bar{x}} \rangle \models \phi_1(\bar{x})$ and, by $\langle \mathcal{A}, \beta \rangle \models \forall \bar{x} \, . \, P(\bar{x}) \leftrightarrow \phi_1(\bar{x})$, it is clear that $\bar{a} \in P^{\langle \mathcal{A}, \beta \rangle}$. This proves (5.1).

For (5.2) and a counterexample of $\models$|, note that for $n \ge 2$ this is true: $\{ \, \forall \bar{x} \, . \, P(\bar{x}) \leftrightarrow \phi_i(\bar{x}) \mid 1 \le i \le n \, \}$ $\models \bigwedge_{1 \le i \le n-1} \forall \bar{x} \, (\phi_i(\bar{x}) \leftrightarrow \phi_{i+1}(\bar{x}))$,[545] whereas this is not entailed by (5.2). Hence, consider (*) $\forall x \, . \, P(x) \leftrightarrow Q(x) \wedge R(x)$ and a model $\langle \mathcal{A}, \beta \rangle$ of it s.t. $P^{\langle \mathcal{A}, \beta \rangle} = \emptyset$, $Q^{\langle \mathcal{A}, \beta \rangle} = \{a\}$, $R^{\langle \mathcal{A}, \beta \rangle} = \emptyset$. $\langle \mathcal{A}, \beta \rangle \models$ (*), but $\langle \mathcal{A}, \beta \rangle \not\models \forall x \, . \, P(x) \leftrightarrow Q(x)$. $\qquad \qquad \square$

5.1.2 Additional Considerations and Guidelines

RESTRICTIONS ON THE INTERPLAY OF FUNDAMENTAL THEORY AND DOMAIN AXIOMATIZATION?

Requiring a "fundamental" axiomatization might let appear the proposed method particularly useful for the formalization of ACOs and possibly TLOs. While we do not see any restriction here in principle, let us postpone this issue to reconsideration in the next sect. 5.2. However, the comparison of rather basic notions and much more domain-specific ones reminds us of two issues from the literature. First, one can remember criticism of RDF semantics to the extent that it involves axiomatic triples in its semantic definition and "allows for (and even employs in its own specification) reflection on its own syntax." [433, sect. 4.1], followed by a position that basically rejects capturing language semantics in axiomatic form. The second issue is the search for appropriate notions of *module* for logical theories (primarily concerning DLs, but also FOL) as intensely conducted in the late 2000s, cf. e.g. [787, esp. ch. 2 and 6], [172, 482, 529, 532]. There, the notion of *conservative extension* plays a major rôle, a.o., in connection with achieving a relationship between modules and larger theories that contain / embed them, such that the module, having a specific signature, is exactly that part of the theory that determines which sentences *in the language of the module* are entailed by the overall theory.[547] Notably, extending a theory by a proper definition yields a conservative extension wrt the theory without the defined symbol in its language, cf. [693, Th. 6.1, p. 85]. Based on these thoughts, one may consider restrictions such that the domain axiomatization of the last step in the $\mathcal{OS}$ method should be a conservative extension over $Lg(\Sigma_F)$.

PROVISION OF A UNIVERSAL PREDICATE AND AWARENESS OF "NON-INDIVIDUALS"

We shall not immediately pursue the previous interjection, but consider additional, more basic methodic constraints, although keeping the combination with reusing theories within others and (some form of) a modular approach in mind. One aspect that applies to FOL theories under classical as well as ontological semantics is the unrestricted / relative domain of entities / range of quantification. We propose to introduce a *universal predicate* symbol / predication formula[548] whose extension is bound to the universe of interpreting structures. From the theory comparison point of view, see sect. 2.3.2, this may not be necessary, e.g., for providing theory interpretations. However, for analysis explication, it appears useful to clarify the scope of the theory (also) by a universal predicate. Moreover, this should be useful for the next item of relativizing quantification.

But before that item, let us highlight that it is a significant difference between the classical semantics with its set-theoretic superstructure *outside* of the universe of discourse and ontological semantics with all of its semantic entities *inside* that universe, everything being in the same domain of entities. Thereby all entities are "accessible" through the language. But likewise, all those entities must be taken into account when developing an ontological theory, e.g., in that they are in the range of quantification. With training in classical logic, this can be expected to be challenging, e.g., when writing axioms.

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[545] Indeed, though without a proof, for $n \ge 2$ $\{ \, \forall \bar{x} \, . \, P(\bar{x}) \leftrightarrow \phi_i(\bar{x}) \mid 1 \le i \le n \, \}$ is equivalent with the conjunction of this consequence and that in (5.1).

[546] The contributions [529, 532] are briefly discussed in sect. 5.3.

[547] More precisely, here we think of the notion of (deductive) conservative extension: A theory $T \supseteq S$ with $Lg(T) \supseteq Lg(S)$ is a (*deductive*) *conservative extension* of S :iff $S \models \phi$ iff $T \models \phi$ for all $\phi \in Sen(Lg(S))$. Cf. e.g. [693, sect. 2.6, p. 85] in the context of a FOL introduction. [320, Def. 3, p. 171] is also based on FOL, with the overall work being deployed in applied ontology. Notably, hierarchies of theories therein deal also / primarily with non-conservative extensions. Deductive and other notions of conservativity referring to DLs, but also FOL and SOL, are discussed in [787, ch. 2, e.g., Def. 2, p. 33].

[548] i.e., a formula with a single free variable that is not an atom with a unary predicate symbol, to the effect of a universal predicate as described in this sentence, e.g., in terms of $\mathcal{CR}$ utilizing the instantiation relation as in $x :: \cdot \mathsf{Ent}$

RELATIVIZING QUANTIFICATION (FURTHER)

Indeed, allowing for predicate referents in the domain of quantification adds support to another guideline that is proposed in connection with $\mathcal{OS}$: All axioms in an axiomatization should be written in an already *relativized form*, where every quantified variable is "immediately" categorized / classified. By that we mean that for each variable bound by a quantifier there is a unary predicate or a subformula with a single free variable that applies to the quantified variable in the scope of that quantifier. More precisely and more formally, instead of arbitrary quantifications of the form $Qx \cdot \phi$ with $Q \in \{\forall, \exists\}$, only quantifications of the form $\exists x \cdot \varrho(x) \wedge \phi$ and $\forall x \cdot \varrho(x) \rightarrow \phi$ should be used, where $\varrho(x)$ is the relativizing formula. The proximity of those forms to the notion of relativation, cf. sect. A.1.1 (adapted from [693, sect. 6.6, p. 258]), accounts for the choice of the name.[549]

The question of whether this can be achieved has a trivial answer, in particular in connection with the foregoing requirement of a universal predicate[550], because any theory involving such a predicate can be written such that all occurrences of quantification are relativized to the universal predicate, because this does not yield any restriction. Even without a universal unary predicate, FOL with equality allows for relativation to $x = x$, to the same effect. Whereas one might speculate over some utility / value of quantifiers that are relativized to a unary universal predicate for theory comparisons, it is not really helpful from the perspective of a single theory. Instead, one would strive for a categorization that is more specific. We adopt the background and terminology of $\mathcal{CR}$ for a moment in order to refine this statement. In view of the distinction between (1) *relators* and *roles* as relating entities and (2) *non-relating entities*, e.g., lions and apples, it appears desirable in many cases to specify a category of non-relating entities that extensionally subsumes the potential players[551] of relations that the variable is involved in within the scope of the quantifier under consideration. The mentioning of categories of potential players of certain relations allows for the argument that $\mathcal{CR}$ offers means to implement the guideline of relativizing quantification even if no clear-cut categories of non-relating entities are determined. One widely studied example are mereological systems, cf. e.g. [394, 707], where commonly a single binary relation of part-of is considered, without assumptions on the nature of its arguments, in the sense of non-relating categories.[552]

EXCEPTION OF DOMAIN AND RANGE CONSTRAINTS, YET MOTIVATING RELATIVATION FURTHER

The qualification in the above phrase "desirable in many cases" is contained due to some exceptions. A prominent kind of such exceptions are formulas that are known as domain and range constraints, esp. wrt binary relations, cf. e.g. the case of constraining the temporal part-of relation (tpart) of the time theory $\mathcal{BT}^\mathcal{C}$, see esp. sect. 6.3.5.1–6.3.8 below, where (for the domain of $\mathcal{BT}^\mathcal{C}$) axiom A6 is[553] (5.3).

(5.3) $\forall xy \cdot \mathsf{tpart}(x, y) \rightarrow \mathsf{Chron}(x) \wedge \mathsf{Chron}(y)$

Of course, (5.3) would be trivialized by relativizing it by $\mathsf{Chron}(x) \wedge \mathsf{Chron}(y)$, whereas the axiom is exactly intended to declare (non-relating entity) categories as potential players of tpart.[554]

We remark that wrt other axioms, domain and range constraints indirectly support the proposal of relativized quantification, in the following sense. Establishing these constraints does not have the same effect as using a typed / many-sorted version of FOL. Indeed, the latter may be seen as a more adequate choice of language, if quantifiers are to be relativized (or "typed") in any case. This is a possibility, although one must pay attention to "hidden" assumptions, for instance whether all types are supposed to be non-empty, which excludes the consideration of certain structures. Returning to the untyped case, domain and range constraints do *not* have the (completely) same effect as introducing typed relations in typed FOL. In particular, they do not imply that every use of a predicate, e.g. of tpart, has the domain and range predicates

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[549]Further remarks on associated notions follow below.

[550]We simplify to that case here only, referring to other unary predication formulas only implicitly.

[551]Remember the notion of base playerable categories from a role base, cf. BasePlayerableC in Fig. 2.8 and sect. 2.4.3.3.

[552]Admittedly, if there is only a single relation in the signature, it does not appear very fruitful to restrict its arguments to its potential players. At least, the latter may refer to different roles, s.t. awareness of those roles is raised. If definitions are introduced, esp. of relations, new roles appear. Moreover, one must bear in mind that (a referent for) part-of itself is covered by quantification in any $\mathcal{OS}$ theory, which may not be among potential role players of itself. The matter of predicate referents in general is discussed below, at the beginning of the next sect. 5.2.

[553]modulo omitting leading universal quantification

[554]The axiom is a usual approach to capture that, yet without making explicit further details from the point of view of $\mathcal{CR}$.

as *pre*conditions, which would correspond to the case that the variables are relativized / classified to domain and range, resp. But such uses do imply the domain and range as *post*conditions – which may have unintended effects. In the simple example of adding a (non-relativized) reflexivity statement for tpart, i.e., (*) $\forall x$. tpart(x, x) , this formula and (5.3) entail $\forall x$. Chron(x), in contrast to a possibly intended equivalent to (*) of $\forall x$. Chron$(x) \rightarrow$ tpart(x, x), given (5.3). A general default of relativizing quantification has the benefit of paying attention to such and similar issues during developing an axiomatization. A potential drawback results from reusing axioms in a theory for a broader domain of entities, which then require the revision of relativizing conditions. Otherwise, inspecting these conditions can support detecting axioms which cannot be transferred by simple adoption in the new domain of entities.

A final remark on relativized quantification concerns *guarded fragments* of FOL, cf. e.g. [16, 296, 837].[555] "The" original *guarded fragment* (GF) comprises all FO formulas such "that all quantifiers must be relatived (or 'guarded') by atomic formulae." [296, p. 1]. This sounds very close to the restriction formulated above, but guarding is a narrower constraint than relativized quantification. Following [296, p. 1], GF is defined by induction such that (1) all relational atomic formulas are in GF, (2) it is closed under propositional connectives, and (3) "If $\bar{x}$, $\bar{y}$ are tuples of variables, $\alpha(\bar{x}, \bar{y})$ is atomic and $\psi(\bar{x}, \bar{y})$ is a formula in GF, such that $fVar(\psi) \subseteq fVar(\alpha) = \{\bar{x}, \bar{y}\}$, then the formulae $\exists \bar{y}(\alpha(\bar{x}, \bar{y}) \wedge \psi(\bar{x}, \bar{y}))$ and $\forall \bar{y}(\alpha(\bar{x}, \bar{y}) \rightarrow \psi(\bar{x}, \bar{y}))$ belong to GF." [296, p. 1, notation adapted to the one in this thesis]. The definition shows that all variables must occur in a *single*, and therefore polyadic atom as the guard, whereas the relativation in the above sense just classifies each variable on its own. A similar difference remains for the *loosely guarded fragment* (LGF) [836, esp. p. 9], where quantified variables must co-occur at least pairwise in atoms, which can form a conjunctive guard. As far as we can see wrt guarded fragments, on the one hand, the above guideline of relativized quantification does not lead to theories in the guarded fragment and therefore, unfortunately, cannot exploit its meta-logical properties, such as the satisfiability problem of GF and LGF being decidable and GF having the finite model property [296, p. 2]. On the other hand, having those properties means limitations in expressiveness, which are not present in the case of relativized quantification, at least if it is taken to its extremes, remembering relativation with $x = x$.

CONCLUSION ON METHOD $\mathcal{OS}$

Overall, we defend the position that the basic method $\mathcal{OS}$ (also together with the extensions discussed) puts further emphasis and amplification on the broader axiomatic method as introduced in sect. 1.1.5.3. This applies likewise to ontological semantics. It should therefore integrate itself well with the development approach that is already being followed in the context of the General Formal Ontology.

We underline anew at this point that, despite the distinction of fundamental and normal predicates in *the definition of* ontological semantics and their rôle in that semantics, we see no semantic distinctions among referring symbols, neither between predicates nor logical individual constants. Similarly, we do not find support for the view that distinguishing primitive and defined predicates in an axiomatization should give rise to any ontological dependence among the referents of those predicate symbols. If there are such dependencies, they should be captured in corresponding axioms. Various choices of primitive notions and definitions may result in theories that are equivalent with each other, under classical and / or ontological semantics.

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[555]Originally, "the" guarded fragment has emerged from diverse motivations and, e.g., considerations in algebraic and modal logic, cf. [837, sect. 5 and App.], with a first report [17] by Hajnal Andréka, Johan van Benthem and István Németi in 1996, cf. also [16], published in 1998. [837] treats also guarded fragments of SOL. [296] is an abstract to an invited talk, which links to DL.

5.2 Ontological Usage Schemes

5.2.1 Two Lines of Motivation

A PRAGMATIC REFLECTION ON ONTOLOGICAL SEMANTICS AND METHOD $\mathcal{OS}$

We approach the notion of *ontological usage schemes* (OUS) in terms of two major motivating considerations, the first of which connects directly to those of the previous sect. 5.1.2. That started with the question of whether ontological semantics[556] and the formalization method $\mathcal{OS}$ are particularly attuned to the formalization of ACOs and TLOs. On the one hand, as stated above, this is not the case, e.g., because fundamental predicates in a predication system can refer to arbitrary levels of specificity. For example, a fundamental phrase à la (4.9), "x is an apple" can be defined for a predicate. On the other hand, ontological semantics and $\mathcal{OS}$ theories do not only enable, e.g., that referents for predicate symbols are part of the semantics of a language, they actually enforce that. From our point of view, in principle, this attitude is adequate in order to ensure a full ontological characterization of a (FO) syntax at hand. Nevertheless and particularly taking into account that many ontologies have been developed with a clear distinction between entities in the domain of discourse and predicate interpretations that are not in the scope of entities that quantifiers range over. Insofar, one can pragmatically expect an interest in a form of ontological semantics without predicate referents.

A WEAKENING: PREDICATES WITHOUT REFERENTS

In this connection it is interesting to observe that the definitions of ontological semantics in ch. 4 do actually work as soon as a predication system is established. In particular, as motivated and elaborated in sect. 4.4.4.2, predication interpretation / the interpretation of atomic formulas is independent from the referent interpretation of predicates, where only reflection constants establish a link. But if it is not of interest to predicate and quantify over those intended referent entities of predicates, one need not introduce reflection constants. Accordingly, step 1 of method $\mathcal{OS}$ would not apply. Actually even independent of that,[557] a well-defined semantics is given by ignoring / dropping the referent interpretation of predicates from ontological structures.[558] That means that any theory that is developed "only" according to the (standard) axiomatic method of introducing primitive predicates and definitions on the (eventual) basis of primitive predicates allows for viewing such a theory as a *weakened $\mathcal{OS}$ theory*, if the remaining steps / conditions of $\mathcal{OS}$ are fulfilled and primitive predicates are assigned the rôle of fundamental predicates.[559]

Mixed forms are equally conceivable, i.e., theories for which it is assumed that some predicates are covered in the domain of discourse whereas others are not.[560] Forsooth, this could be completely adequate from an ontological point of view, if a predicate is introduced as a mere abbreviation of another formula, without assuming that there were a referent entity for that predicate itself. Accordingly, the two views on introducing definitions in a theory over some language L, namely of (1) introducing a formula as a mere (syntactic) abbreviation of an L formula vs (2) considering the theory within an extended language L', where newly defined predicates are/must be interpreted, is mirrored in ontological semantics by predication definitions for whose predicates referents are assumed, corresponding to case (2), or not, in case (1).[561]

This point is further well-suited to mention the special rôle of equality in ontological semantics, which under set-theoretic semantics has a referent in the form of the identity relation over the universe. With

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[556]In the overall section, the term 'ontological semantics' refers to the FOL variant defined esp. from sect. 4.4 on, based on predication systems. Not all considerations must apply to other kinds of ontological semantics, in the general sense of Def. 4.3.

[557]Even if reflection constants are introduced, in free ontological models there is no guarantee that a reflection constant has the same referent like its predicate.

[558]Ignoring comes with the detriment that, if the intended domain of quantification comprises only (ontological) individuals, but the interpretation function must map also predicates, assuming that these / some of them refer to categories, then there remain no signature-satisfying structures (from the meta-level perspective). This does not lead to technical problems, but produces more of a discrepancy between object-level and meta-level theory than merely assuming predicate referents outside the domain of quantification.

[559]In the, from an analysis point of view, worst case, all predicates are declared to be fundamental and are equipped with a fundamental phrase. Then ontological semantics covers the case of the naïve (predication) semantics discussed in sect. 4.4.1.

[560]Just to mention it again, these issues have close links to the semantics of Common Logic [@18] [452], but those are discussed not until sect. 7.2.2.2.

[561]Thinking of Beth's definability theorem in classical logic, cf., e.g., [410, sect. 6.6, Th. 6.6.4, p. 301], [53, sect. A.7.8, Th. 8.6, p. 274] or [362, sect. 2.5], depending on comprehension principles in the ontological theory, however, it may be the case that a referent is entailed.

evidence in Def. 4.2 of ontological structure, p. 128, and Def. 4.10 of predication satisfaction, p. 146, we decided not to enforce the existence of a referent for the equality predicate as an entity in the semantic domain. This would constitute an ontological assumption. Instead, equality is primarily understood as co-reference, but of course, if identity is defended as an ontological entity, it can be introduced, e.g., as a relation wrt $\mathcal{CR}$.

ACCEPTANCE VIA UNSTATED (META-)ONTOLOGICAL BACKGROUND THEORY

Natural questions after these considerations are whether and why such a theory can be considered to be ontogically adequate. "Such a" here refers to a theory with predicates that have no counterparts in the semantic domain, but of which the language user may think of having referents "actually", just not in the (intended) universe of entities that the theory addresses. We argue that ontological adequacy can be attributed, if an ontological/meta-ontological background theory can be assumed[562] that accounts for both, the stated theory and that assumed background theory, as a joint theory under ontological semantics. While it becomes clear more exactly below what is meant by "a joint theory", in general, we expect that the domain-specific theory can be transferred/translated into a full-fledged $\mathcal{OS}$ theory. Capturing this idea is a first rationale for ontological usage schemes (OUS).

THE POINT OF 'USAGE' FOR DIFFERENT ONTOLOGICAL INTERPRETATIONS OF ONE LANGUAGE

This rationale does not well explain the term 'usage' in OUS, however. Indeed, that part results rather from the second motivating aspect, which combines those translational aspects just seen (and also part of Def. 2.3, p. 64), with the insight for *signature-specific* predication semantics from sect. 4.4.1. There it is formulated as: "For the language user U the same syntactic way of composition bears an informal/conceptual meaning that *can differ* in terms of the relationship between the unary predicate and its argument." We strongly believe that this view generalizes to basically all languages, i.e., we argue that, regarding the conceptual content that is (meant to be) expressed within any language, involving syntactic composition according to its abstract syntax[563], can differ for the same language constructs. In addition, there may be room for enriching the content of syntactic expressions made in the language by additional assumptions that a user of the respective language may have held at the conceptual/ontological level, similar to the first motivation of OUS above.

Following this conviction means for us that, at the greatest level of detail, each usage of a language in terms of employing a certain set of identifiers (e.g., a particular signature for a logical language) and additional language constructs for any representation, could be assigned a translation – an ontological usage scheme – into an ontological theory that accounts for the conceptual content of what is represented in the source language. Insofar we are in full agreement with the notion of ontological reduction, cf. sect. 2.3.1, as well as the preconditions of Def. 2.3 in sect. 2.2.2.1, cf. Fig. 2.2, p. 64, which are abstracted from ideas in [156, 733].

Importantly, such translations/OUS can differ for distinct sets of identifiers/signatures, despite using the same language (and thus language constructs). Examples are not far below. As a consequence of that we argue further that it is reasonable to consider the (potentially available) formal semantics of a language in parallel with multiple OUS for the same language, because the formal semantics is typically uniquely defined based on the abstract syntax, but not on particular identifiers.[564] But if multiple ontological interpretations of the same language are possible and adequate for different cases of use, strictly requiring full formal semantic integration for OUS, i.e., loosely speaking, expecting adherence of the translation result to the formal "behavior" of the semantics of the source language can be questioned, as similarly argued in sect. 2.2.2.2. As stated there, this is not to say that it is not desirable! But this seems to be a question of appropriate formal modeling, e.g., of having chosen the right formalism and an appropriate encoding for the underlying ontological issues, rather than one of adherence of those underlying circumstances from an ontological point of view to the formal behavior of an encoding.

Altogether, these two lines of motivation prepare the ground for a translational way of assigning ontological semantics to languages.

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[562]which may possibly be implicit/unconscious at the side of the language user

[563]Sect. 2.1 sets our terminology on languages, including 'abstract syntax' and 'identifier'.

[564]More or less by definition, looking at the difference between identifiers and keywords in sect. 2.1.1.

5.2.2 Generic Definition of Ontological Usage Schemes

DEFINITION BACKGROUND

An initial definition capturing the general idea of ontological usage schemes is presented as Def. 4 in [534, sect. 3.4, p. 56], where they are basically characterized as a translation into a language of a formalized ontology T_Ω (constructed by a very early variant of method $\mathcal{OS}$ herein). In connection with the OUS that applies to certain description logic formalizations of ontologies, sketched in sect. 5.2.4 below and published in [533], we have provided a more specialized variant, cf. [533, Def. 1], which utilizes three specific translation functions in the definition of that "overall translation" meant in [534, Def. 4]. [533, 534]

From the perspective of today, it appears rather difficult to provide a detailed, but generally applicable definition for the notion of ontological usage scheme. Therefore, we specify a generic variant below and shall discuss what we find to be typical elements in actually specifying such OUS below that definition. For the generic variant(s), it is another choice to refer abstractly to a theory T_Ω that (1) is expressed in a language under an(y) ontological semantics, cf. Def. 4.3, p. 128,[565] and (2) is ontologically neutral. We leave this fully abstract variant unstated and provide only that version that is utilized in the sequel and employs the notion of $\mathcal{OS}$ theories[566].

5.3 Definition (ontological usage scheme (generic $\mathcal{OS}$ variant))
Let $Lg(\Sigma)$ an arbitrary language that is used with a particular identifier set / signature Σ. Let Σ_Ω^+ a FO signature for an $\mathcal{OS}$ theory $T_\Omega \subseteq Lg(\Sigma_\Omega^+)$ that formalizes a conceptualization Ω.

Let (σ, τ) a pair of functions and on their basis define a translation function ω such that

- $\sigma : \Sigma \cup \{\Sigma\} \to \mathcal{P}(Lg(\Sigma_\Omega^+))$ is a *signature analysis* function for ontological analysis of Σ, wrt which $\sigma_\Sigma := \sigma(\Sigma) \cup \left(\bigcup_{s \in \Sigma} \sigma(s) \right)$

- $\tau : Lg(\Sigma) \to Lg(\Sigma_\Omega^+)$ is a *translation of expressions* in $Lg(\Sigma)$, extended to sets of formulas $S \subseteq Lg(\Sigma)$ by $\tau(S) := \{\tau(e) \mid e \in S\}$

- $\omega : \mathcal{P}(Lg(\Sigma)) \to \mathcal{P}(Lg(\Sigma_\Omega^+))$ is defined by $\omega(S) := T_\Omega \cup \sigma_\Sigma \cup \tau(S)$

(σ, τ) is an *ontological usage scheme* of Σ and $Lg(\Sigma)$:iff   for every set of expressions $S \in Lg(\Sigma)$, $\omega(S)$ makes explicit ontological / conceptual / intensional meaning of Σ and $Lg(\Sigma)$ wrt T_Ω, i.e., $\omega(S)$ is an ontological translation or foundation of S wrt T_Ω (in the sense of sect. 2.3.1).

$\omega(S)$ is called the *ontological image* of S under (σ, τ). $\tau(e)$ is the ontological image of an expression $e \in Lg(\Sigma)$ under (σ, τ). In abuse of notation, we may write $\omega(e)$ for $\tau(e)$. $\omega^{(\sigma, \tau)}$ may be used for clarity.□

A few comments are in order. The '+' sign in Σ_Ω^+ is merely an indicator for the fact that the signature Σ_Ω of T_Ω commonly requires an extension in terms of (logical individual) constants and / or predicates. Moreover, a safe variant is chosen with $\bigcup_{s \in \Sigma} \sigma(s)$ in σ_Σ compared to the alternative, from the point of view of $\omega(S)$, that takes only those symbols into account that occur in S itself, i.e., $\bigcup_{s \in S} \sigma(s)$. E.g., in the case that σ maps to predication definitions, leaving some of them out may clash with dependencies among them. Finally and importantly, the ontological image $\omega(S)$ is not deductively closed in the definition. Certainly, the deductive closure of $\omega(S)$ appears more relevant in many occasions, whereas the definition allows for (literally) distinct, but logically equivalent[567] ontological images. For the reason that the statement of equivalence can be made easily for unequal ontological images, the deductive closure is not directly incorporated in Def. 5.3, maintaining more fine-grained distinctions of ontological images resulting from different functions σ and τ.

PROXIMITY TO INTERPRETATION FUNCTIONS AND COMMON AUXILIARY FUNCTIONS

Readers who wonder, for a FO source language $Lg(\Sigma)$, about a close connection between OUS and translation functions that are theory interpretations are not mistaken, as the examples in the next sect. 5.2.3 demonstrate. In general, a FO theory interpretation[568] $\alpha : Lg(\Sigma) \to Lg(\Sigma_\Omega^+)$ in which the universe of the

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[565]i.e., a semantics substantially based on ontological structures
[566]which are understood to establish at least partially ontologically neutral semantics
[567]under classical or ontological semantics
[568]in the sense of Enderton's definition [221, p. 157f.], see App. A.1.2.3

source language is relativized to the unary predicate P can be modeled by an ontological usage scheme (σ, α) where σ comprises the closure axioms wrt P that ensure the existence of substructures in models of $\omega(S)$, cf. e.g. [693, sect. 6.6]. However, obviously and as argued at the end of sect. 5.2.1, OUS do not (necessarily) account for the formal semantic transfer of entailment from a theory S to $\omega(S)$, i.e., for a proper theory interpretation, it remains to be shown that $S \models \phi$ entails $\omega(S) \models \omega(\phi)$.

This proximity to FO theory interpretations suggests some common auxiliary functions in order to specify an OUS, at least if a translation between FO languages is concerned. In our own experience, we have utilized what may be referred to as *entity functions*, which map (referential) signature elements to (logical individual) constants in the target language. Depending on the translation τ esp. of atomic formulas, multiple entity functions may be required, if the (formalized) ontological analysis of a predication involves several constants. The *domain relativation*, i.e., a unary predicate or formula in one free variable usually affects τ in quantifier translation and σ by providing closure axioms. The actual rôle of σ is to account for an ontological analysis of signature elements wrt the ontological theory T_Ω. For an example in terms of $\mathcal{CR}$, a unary predicate symbol A, more precisely, a reflection constant that an entity function yields, may be classified as an instance of IndividualCategory, providing additional information compared to the implicit classification by Category that would result from $\tau(A(x)) := x :: \cdot A$ in a $\mathcal{CR}$ formalization that constrains the second argument to Cat. But apparently we almost enter the discussion of a first extended example, which is actually looked at next.

5.2.3 Signature-Specific OUS in FOL

SINGLE THEORY EXAMPLE FOR A START

A simple example in which we are interested in (more) ontological grounding of a single theory connects to the first line of motivation of OUS in sect. 5.2.1 as well as to the much-stressed domain of apples. As a starting point, we presume this condition.

5.4 Condition (signature Σ and theory T for Ex. 5.5 and 5.6)
Let $\Sigma = \{A, R, a\}$ a FOL signature, where (intendedly) A stands for apple, R stands for red, and a denotes a particular apple. *Var* is the standard variable set in all FOL languages under consideration.
Define theory $T \subseteq Lg(\Sigma)$ as the set of these three sentences.

(5.4) $\quad \forall x \,.\, A(x)$

(5.5) $\quad \exists x \,.\, R(x)$

(5.6) $\quad A(a)$ ❑

Note that T may be an $\mathcal{OS}$ theory of the weakened kind discussed at the beginning of sect. 5.2.1 (without predication referents for both predicates), e.g., if we assume the naïve readings of (4.9) "x is an apple" and (4.10) "x exhibits red" as fundamental phrases. Providing further ontological grounding, following sect. 4.4.2 on this example and the later formula (4.22), in terms of an ontological usage scheme can proceed by the following translation functions. Let $\Sigma_{\mathcal{CR}}^{+_0} = \Sigma_{\mathcal{CR}} \cup \{\text{inh}, \cdot A, \cdot R, a\}$, providing a (fundamental) binary predicate inh for inherence and one constant for each signature element of Σ in addition to notions of $\mathcal{CR}$. Due to (5.4) we consider $\tau_0(A(x))$ an appropriate relativation formula.[569]

5.5 Example (first OUS (σ_0, τ_0) for T)
Adopting Cond. 5.4, define an entity function ε, translation function τ_0, and signature analysis function σ_0.
$\varepsilon : \Sigma \cup Var \to Tm(\Sigma_{\mathcal{CR}}^{+_0})$ s.t. $\varepsilon(x) = x$ for every $x \in Var$, $\varepsilon(a) = a$, $\varepsilon(A) = \cdot A$, $\varepsilon(R) = \cdot R$.

$$\text{for all } x \in Tm(\Sigma) \;:\quad \tau_0(A(x)) := \varepsilon(x) :: \varepsilon(A)$$
$$\text{for all } x \in Tm(\Sigma) \;:\quad \tau_0(R(x)) := \exists \varepsilon(y) \,.\, \neg\tau_0(A(y)) \wedge \varepsilon(y) :: \varepsilon(R) \wedge \text{inh}(\varepsilon(y), \varepsilon(x))$$

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[569]This is not to say that every arbitrary universal predicate (or formula in one free variable) should be viewed as an appropriate characterizing predicate for the domain of entities under consideration.

for $\phi, \psi \in Lg(\Sigma)$:[570] $\tau_0(\neg\phi) := \neg\tau_0(\phi)$, $\tau_0(\phi\wedge\psi) := \tau_0(\phi)\wedge\tau_0(\psi)$, $\tau_0(\exists x.\phi) := \exists\varepsilon(x).\tau_0(\mathsf{A}(x))\wedge\tau_0(\phi)$.

$$\sigma_0(\Sigma) := \{\exists\varepsilon(x) \,.\, \tau_0(\mathsf{A}(x))\}$$
$$\sigma_0(\mathsf{A}) := \{\varepsilon(\mathsf{A}) :: \cdot\mathsf{Cat}, \quad \varepsilon(\mathsf{A}) \mathrel{\rightarrow\!\!\!\!\triangleright} \cdot\mathsf{Ind}\}$$
$$\sigma_0(\mathsf{R}) := \{\varepsilon(\mathsf{R}) :: \cdot\mathsf{Cat}, \quad \varepsilon(\mathsf{R}) \mathrel{\rightarrow\!\!\!\!\triangleright} \cdot\mathsf{Ind}\}$$
$$\sigma_0(\mathsf{a}) := \{\varepsilon(\mathsf{a}) :: \varepsilon(\mathsf{A}), \quad \tau_0(\mathsf{A}(\mathsf{a}))\} \qquad\qquad \Box$$

Three points are noteworthy in this example. (1) the existential quantifier in the ontologically analyzed predication of $\mathsf{R}(x)$ takes care of the fact that properties (y in that case) were not part of the domain under consideration for T. (2) The non-emptiness of the domain of apples is stated in $\sigma_0(\Sigma)$. Similarly, (3) $\tau_0(\mathsf{A}(\mathsf{a}))$ originates from a being a constant in Σ, whereas $\varepsilon(\mathsf{a}) :: \varepsilon(\mathsf{A})$ is added as a classifying statement for a. Both result in the same formula, eventually. (σ_0, τ_0) yields the ontological image $\omega_0(T) = F_{\mathcal{CR}} \cup \{\exists x \,.\, x :: \cdot\mathsf{A}\} \cup \{\cdot\mathsf{A} :: \cdot\mathsf{Cat}, \,\cdot\mathsf{A} \mathrel{\rightarrow\!\!\!\!\triangleright} \cdot\mathsf{Ind}, \,\cdot\mathsf{R} :: \cdot\mathsf{Cat}, \,\cdot\mathsf{R} \mathrel{\rightarrow\!\!\!\!\triangleright} \cdot\mathsf{Ind}, \mathsf{a} :: \cdot\mathsf{A}\} \cup \tau_0(T)$ where $\tau_0(T) = \{\forall x \,.\, x :: \cdot\mathsf{A} \rightarrow x :: \cdot\mathsf{A}, \quad \exists x \,.\, x :: \cdot\mathsf{A} \wedge \exists y \,.\, \neg\, y :: \cdot\mathsf{A} \wedge y :: \cdot\mathsf{R} \wedge \mathsf{inh}(y, x), \quad \mathsf{a} :: \cdot\mathsf{A}\}$ and $F_{\mathcal{CR}}$ is an axiomatization of $\mathcal{CR}$. If $\omega_0(T)$ should be inconsistent, reconciliation of the (meta-)ontological interpretation is required. If $\omega_0(T)$ is a consistent theory, we have reached an enriched ontological representation of theory T (itself an ontology) that can be taken to be faithful to itself due to ontological semantics.

DEFINITIONS FOR A 2ND OUS FOR T, LEADING TO A FAITHFUL THEORY INTERPRETATION

Although there is the question of whether T entailments are preserved in $\omega_0(T)$,[571] we skip proving directly that τ_0 is a FO theory interpretation from T into $\omega_0(T)$. The example of (σ_0, τ_0) can be taken as a demonstration of translating from another language, in that the ontological image does not contain expressions of the source language. However, in the particular case of FOL, there are two suggestions which attract notice to definitions. Firstly, definitions may be introduced for easier comprehensibility of $\omega_0(T)$. Secondly and due to starting from a FOL language itself, we may simply reuse the predicates of the source language (each with the same arity).[572] This approach results in Ex. 5.6, for which $\Sigma_{CR}^{+1} := \Sigma_{CR}^{+0} \cup \{\mathsf{A}, \mathsf{R}\}$. On its basis, Obs. 5.7 reports that it yields relativizations, and with Prop. 5.8 a theory interpretation of the original apple theory into its ontological image.

5.6 Example (second OUS (σ_1, τ_1) for T)

Assume Cond. 5.4 and define an entity function ε, translation function τ_1, and signature analysis function σ_1.

$\varepsilon : \Sigma \cup Var \rightarrow Tm(\Sigma_{CR}^+)$ s.t. $\varepsilon(x) = x$ for every $x \in Var$, $\varepsilon(\mathsf{a}) = \mathsf{a}$, $\varepsilon(\mathsf{A}) = \cdot\mathsf{A}$, $\varepsilon(\mathsf{R}) = \cdot\mathsf{R}$.

$$\text{for all } x \in Tm(\Sigma) : \quad \tau_1(\mathsf{A}(x)) := \mathsf{A}(\varepsilon(x))$$
$$\text{for all } x \in Tm(\Sigma) : \quad \tau_1(\mathsf{R}(x)) := \mathsf{R}(\varepsilon(x))$$

for $\phi, \psi \in Lg(\Sigma)$: $\tau_1(\neg\phi) := \neg\tau_1(\phi)$, $\tau_1(\phi\wedge\psi) := \tau_1(\phi)\wedge\tau_1(\psi)$, $\tau_1(\exists x.\phi) := \exists\varepsilon(x).\tau_1(\mathsf{A}(x))\wedge\tau_1(\phi)$.

$$\sigma_1(\Sigma) := \{\exists\varepsilon(x) \,.\, \tau_1(\mathsf{A}(x))\}$$
$$\sigma_1(\mathsf{A}) := \{\varepsilon(\mathsf{A}) :: \cdot\mathsf{Cat}, \varepsilon(\mathsf{A}) \mathrel{\rightarrow\!\!\!\!\triangleright} \cdot\mathsf{Ind},$$
$$\qquad\qquad \forall\varepsilon(x) \,.\, \mathsf{A}(\varepsilon(x)) \leftrightarrow \varepsilon(x) :: \varepsilon(\mathsf{A})\}$$
$$\sigma_1(\mathsf{R}) := \{\varepsilon(\mathsf{R}) :: \cdot\mathsf{Cat}, \varepsilon(\mathsf{R}) \mathrel{\rightarrow\!\!\!\!\triangleright} \cdot\mathsf{Ind},$$
$$\qquad\qquad \forall\varepsilon(x) \,.\, \mathsf{R}(\varepsilon(x)) \leftrightarrow \exists\varepsilon(y) \,.\, \neg\tau_1(\mathsf{A}(y)) \wedge \varepsilon(y) :: \varepsilon(\mathsf{R}) \wedge \mathsf{inh}(\varepsilon(y), \varepsilon(x))\}$$
$$\sigma_1(\mathsf{a}) := \{\varepsilon(\mathsf{a}) :: \varepsilon(\mathsf{A}), \tau_1(\mathsf{A}(\mathsf{a}))\}$$

These definitions, in accordance with Def. 5.3, yield the following results.

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[570] We limit the consideration to a minimal system s.t. all remaining formulas can be defined. Well-known equivalences may be applied below, nevertheless.

[571] We strongly conjecture that they are, which receives confirmation below.

[572] Assume for simplicity that the sets of predicates of both signatures are disjoint, otherwise renaming or more care is required.

$$\sigma_{1_\Sigma} = \{\exists x \,.\, \mathsf{A}(x), \ \cdot\mathsf{A} :: \cdot\mathsf{Cat}, \ \cdot\mathsf{A} \dashrightarrow \cdot\mathsf{Ind}, \ \cdot\mathsf{R} :: \cdot\mathsf{Cat}, \ \cdot\mathsf{R} \dashrightarrow \cdot\mathsf{Ind}, \ \mathsf{a} :: \cdot\mathsf{A}, \ \mathsf{A}(\mathsf{a}),$$
$$\forall x \,.\, \mathsf{A}(x) \leftrightarrow x :: \cdot\mathsf{A}, \ \forall x \,.\, \mathsf{R}(x) \leftrightarrow \exists y \,.\, \neg\mathsf{A}(y) \wedge y :: \cdot\mathsf{R} \wedge \mathsf{inh}(y, x)\}$$
$$\tau_1(T) = \{\forall x \,.\, \mathsf{A}(x) \rightarrow \mathsf{A}(x), \ \exists x \,.\, \mathsf{A}(x) \wedge \mathsf{R}(x), \ \mathsf{A}(\mathsf{a})\}$$
$$\omega_1(T) = \ F_{\mathcal{CR}} \cup \sigma_{1_\Sigma} \cup \tau_1(T) \hspace{6cm} \square$$

5.7 Observation (τ_1 yields relativations)

Presuppose Cond. 5.4 with its particular theory $T \subseteq Lg(\Sigma)$, set $\Sigma_{\mathcal{CR}}^{+_1} := \Sigma_{\mathcal{CR}}^{+_0} \cup \{\mathsf{A}, \mathsf{R}\}$ and assume Ex. 5.6 with translation function $\tau_1 : Lg(\Sigma) \rightarrow Lg(\Sigma_{\mathcal{CR}}^{+_1})$. In addition, let $S \subseteq Lg(\Sigma)$ an arbitrary theory. Then:

$$\tau_1(S) \text{ is the relativation of } S \text{ to predicate } \mathsf{A}, \quad \tau_1(S) = S^{|\mathsf{A}}. \hspace{2cm} \square$$

Proof. First show $\tau_1(\phi) = \phi^{|\mathsf{A}}$ for every $\phi \in Lg(\Sigma)$, cf. sect. A.1.1 for relativations and the notation $\phi^{|\mathsf{A}}$, based on [693, sect. 6.6] and [214, sect. VIII.2]. With $\varepsilon(x) = x$ for all $x \in Tm(\Sigma)$, this is immediate from the "exact same definition" of τ_1 and $\cdot^{|\mathsf{A}}$ (and can be fully written out by induction on the structure of formulas). By the same argument, this transfers to theories S, cf. Def. 5.3 for $\tau_1(S)$ (there '$\tau(S)$'). $\square$

Obs. 5.7 on τ_1 turns out to be useful to realize the character of τ_1 as theory interpretation in Obs. 5.9, in terms of the subsequent, general proposition / corollary, which is similar to and whose proof actually utilizes the relativation lemma in the version of Rautenberg [693, Lemma 6.1, p. 259], see Prop. A.7 in sect. A.1.2.2, cf. also Prop. A.3 in sect. A.1.1. The former section of the Appendix further comprises Def. A.6 on closure axioms.

5.8 Proposition (relativations and theory interpretations)

For an arbitrary FO language L and theory $T \subseteq L$, let U a unary predicate s.t. $U \notin Sig(L)$ and let τ a translation function with $dom(\tau) = L$. Further, let $\tau(T) \equiv T^{|U}$ and $\tau^+(T) = \tau(T) \cup CA_U$, where CA_U are the closure axioms wrt L and U. Then:

- τ is a faithful theory interpretation of T into $\tau^+(T)$, $\quad T \models \phi \quad$ iff $\quad \tau^+(T) \models \tau(\phi) \quad$ for all $\phi \in L$.
- τ is a theory interpretation of T into every extension $T' \supseteq \tau^+(T)$. $\hspace{2cm} \square$

Proof. The second item in Prop. 5.8 is a direct consequence of the first – if $T \models \phi$, then $\tau^+(T) \models \tau(\phi)$ must be shown for all $\phi \in L$, and follows by monotonicity of entailment in FOL from the first item.

In showing $T \models \phi$ iff $\tau^+(T) \models \tau(\phi)$ for every $\phi \in L$, we mainly "lift" the relativation lemma, cf. Prop. A.3, sect. A.1.1, to the levels of theories, by utilizing the compactness theorem of FOL (wrt entailment). Due to the precondition $\tau(T) \equiv T^{|U}$, it suffices to show $T \models \phi$ iff $CA_U \cup T^{|U} \models \phi^{|U}$ for all $\phi \in Lg(\Sigma)$. A.o. thanks to the compactness theorem of FOL wrt entailment, cf. e.g. [214, Prop. 2.1(a), p. 95], this chain of equivalences applies: $\quad T \models \phi \quad$ iff(1) $\quad$ there is a finite subset $T_{fin} \subseteq T$ s.t. $T_{fin} \models \phi \quad$ iff(2) $\bigwedge_{\alpha \in T_{fin}} \alpha \models \phi \quad$ iff(3) $\quad CA_U \cup \{\bigwedge_{\alpha \in T_{fin}} \alpha^{|U}\} \models \phi^{|U} \quad$ iff(4) $\quad$ there is a finite set $T_{fin}^{|U} \subseteq T^{|U}$ s.t. $CA_U \cup T_{fin}^{|U} \models \phi^{|U} \quad$ iff(5) $\quad CA_U \cup T^{|U} \models \phi^{|U}$.

The compactness theorem justifies (1,5). (2) is clear with basic definitions of $\models$ and conjunction, as is (4) partially, which requires in addition that $T' \subseteq T''$ iff $T'^{|P} \subseteq T''^{|P}$ for any $T', T'' \subseteq Lg(\Sigma)$ and unary predicate P. The latter is true due to the bijective character of relativation on formulas and theories.

For (3), let us derive more generally that, for all $\phi, \psi \in Lg(\Sigma)$, $\quad \phi \models \psi \quad$ iff $\quad CA_U \cup \phi^{|U} \models \psi^{|U}$. Let $\mathcal{A}$ an arbitrary Σ structure and $\mathcal{B}$ an arbitrary $\Sigma \cup \{U\}$ structure s.t. $\mathcal{A} = [U^{\mathcal{B}}]^{\mathcal{B}^-}$, where $\mathcal{B}^-$ is the Σ reduct of $\mathcal{B}$, i.e., $\mathcal{A}$ is the substructure generated by the interpretation function of $\mathcal{B}$ applied to U. Due to $U \notin \Sigma$, each of $\mathcal{A}$ and $\mathcal{B}$ can be considered to be arbitrary, in which case there exists a corresponding structure determined by the above condition to the one chosen first. Then relying on an equivalent view on Rautenberg's version of the relativation lemma, Prop. A.7 in sect. A.1.2.2, where its precondition $\mathcal{B} \models CA_U$ is "shifted" into the right-hand side of its equivalence yields immediately $\mathcal{B} \models CA_U \cup \{\phi^{|U}\}$ iff $\mathcal{A} \models \phi$. Accordingly, one can be substituted for the other, resulting in equivalent implications for any $\phi, \psi \in Lg(\Sigma)$:

(6) $\mathcal{B} \models CA_U \cup \left\{\phi^{|U}\right\}$ implies $\mathcal{B} \models \psi^{|U}$ iff   (7) $\mathcal{B} \models CA_U \cup \left\{\phi^{|U}\right\}$ implies $\mathcal{B} \models CA_U \cup \left\{\psi^{|U}\right\}$ iff
(8) $\mathcal{A} \models \phi$ implies $\mathcal{A} \models \psi$, s.t. the equivalence of (8) and (6) verifies $\phi \models \psi$ iff $CA_U \cup \left\{\phi^{|U}\right\} \models \psi^{|U}$.   ❑

As announced before, Prop. 5.8 in connection with Obs. 5.7 allow for a direct conclusion on the nature of τ_1, the translation of the ontological usage scheme (σ_1, τ_1) for the example theory T on apples in Cond. 5.4.

5.9 Observation (τ_1 is a theory interpretation into $\omega_1(T)$)
Presuppose Cond. 5.4 with its theory T and Ex. 5.6 with translation function τ_1. τ_1 is a theory interpretation of T into its ontological image $\omega_1(T)$.   ❑

COMPARISON WITH FORMER OUS OF EX. 5.5

This observation contributes in addition to the understanding of the first OUS (σ_0, τ_0), because it is easy to verify by comparison of the individual members of ω_0 and ω_1 that, in effect, in terms of the set $\Delta_1(\Sigma) := \{\forall x . \mathsf{A}(x) \leftrightarrow x :: \cdot\mathsf{A}, \ \forall x . \mathsf{R}(x) \leftrightarrow \exists y . \neg\mathsf{A}(y) \wedge y :: \cdot\mathsf{R} \wedge \mathsf{inh}(y, x)\} \subset \sigma_{1_\Sigma}$, the relationship $\omega_1(T) \equiv \omega_0(T) \cup \Delta_1(\Sigma)$ is fulfilled. The definitions in $\Delta_1(\Sigma)$ (modified to comply with τ_1) are in σ_1 the only additions to σ_0, cf. the separate lines in Ex. 5.6, whereas τ_1 results from τ_0 by rewriting the direct specification of the definiens there to using the same atomic formulas in conjunction with those new definitions. Hence, Obs. 5.9 transfers to (σ_0, τ_0), where τ_0 equally is a theory interpretation of T into $\omega_0(T)$. Moreover, the view of $\Delta_1(\Sigma)$ complies with Rautenberg's approach of defining/specifying theory interpretations, which can also be found in [320, sect. 6.1].[573] It further allows for considering an alternative specification (σ, U, Δ) of an OUS, where σ is a signature analysis function as above, U is a relativation predicate, and Δ a set of definitions for all predicates in Σ. On that basis, define $\tau(\phi) := \phi^{|U}$ and redefine the ontological image $\tilde{\omega}(T) := \omega(T) \cup CA_U \cup \Delta$. With Prop. 5.8 each OUS specified in this way for a theory T yields a theory interpretation wrt the ontological image of T. As argued earlier, this is desirable, on the one hand, on the other hand it is limiting to cases where such formal semantic integration / integration of language dynamics can be achieved. In a concrete case inspired by the first two examples, $(\sigma_0, \mathsf{A}, \Delta_1)$ should yield an ontological image $\tilde{\omega}(T)$ which is equivalent with $\omega_0(T)$ and $\omega_1(T)$.

DEFINITIONS AND THEIR STATUS AS PREDICATION DEFINITIONS

From the perspective of ontological semantics, it is appealing to view such definitions, e.g., those in Δ_1, as predication definitions for the predicates in Σ. In this regard, some related issues result from the aspect of (possibly) relativizing formulas in $Lg(\Sigma)$. One of those is silently taken for granted above, namely the "relativization" of the existentially quantified variable y by $\neg\mathsf{A}(y)$ in Δ_1. One may argue that the R definition without that conjunct is ontologically inadequate, given the domain relativation to apples (A) in T.[574] On the other hand, the domain of entities of T's ontological image $\omega(T)$ is broader and must obviously accommodate entities for y. Insofar and in general, we advocate the position that the choice on constraining new variables in the definiens of a translation definition must be decided case by case.

A similar issue concerns the universally quantified variable(s) that occur in the definiendum of a predication definition. Imagine the case that an $\mathcal{OS}$ theory T is to be enhanced by additional ontological analysis in terms of an ontological usage scheme for T, providing additional signature analysis.[575] An OUS of the last kind, say, (σ, U, Δ), has the effect that predication definitions of T, e.g., $\forall x . P(x) \leftrightarrow \delta_P(x)$, are translated into relativized definitions of the form $\forall x . U(x) \rightarrow (P(x) \leftrightarrow \delta_P(x))$.[576] Then the question is how to regain a predication definition. Considering propositional logic with propositional variables P (reusing P for an atom $P(x)$ in FOL), C (for a condition), and D (for the "actual" definiens of P) is instructive in terms of the equivalence of $P \leftrightarrow C \wedge D \equiv (C \rightarrow (P \leftrightarrow D)) \wedge (P \rightarrow C)$. Accordingly for FOL, if the

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[573][320, sect. 6.1] proves the equivalence relationship between Rautenberg's and Enderton's definition of theory interpretation. Moreover, Rautenberg's version is there traced back to [799]. We remark that the latter introduces an even more general notion of interpretation / interpretability based on a *code* with a so-called *exponent*, i.e., a natural number, s.t. a "translation" function between structures is defined in which elements of the source structures correspond to tuples of elements of the target structures of the length of the exponent of the code.

[574]As long as A has no axiomatic relations to inherence and its potential role players, constraints on the arguments of inherence, e.g., in the assumed fundamental theory, cannot account for this issue.

[575]E.g., there could be an $\mathcal{OS}$ domain ontology without predication referents, which are now to be made explicit.

[576]For simplicity, let $\delta_P(x)$ be free of quantifiers and additional variables.

predicate under consideration is meant to be restricted to the domain of T, i.e., if $\forall x . P(x) \to U(x)$ is an adequate assumption *also in the target domain* of the ontological image, then a new predication definition for R is produced by adding the relativizing predicate to its definiens, as in (5.7).

$$(5.7) \quad \forall x . P(x) \leftrightarrow U(x) \wedge \delta_P(x)$$
$$(5.8) \quad \forall x . P(x) \leftrightarrow \delta_P(x)$$

The second variant (5.8) can also be adequate in the target domain, and is consistent with the relativized of the original definition, more precisely, the latter follows from (5.8). The third case is one which does not accept either revision immediately, i.e., where $\forall x . U(x) \to (P(x) \leftrightarrow \delta_P(x))$, but on the other hand $\neg \forall x . P(x) \to U(x)$ is true in the (intended) target domain. One solution to that views P as a general category (incl. relations) and introduces a specialization P' by domain-restriction of P to U, s.t. the P' analog of (5.7) can be validly adopted as a predication definition maintaining the meaning for entities in the source domain. Admittedly, the task of a predication definition for P itself remains (and may only follow a more generic scheme).

5.2.4 Sample Generic OUS for FOL and Description Logic

OUS BEYOND PARTICULAR SIGNATURES IN FOL

Although the previous sect. 5.2.3 is devoted to OUS that are defined wrt specific theories / signatures, the last comments there have a more generic flavor already. Moreover, looking at Ex. 5.6 of (σ_1, τ_1), there is not much that appears signature-specific in its definition. The most specific case appears to be the ontological interpretation of $\mathsf{R}(x)$ in the form of the predication definition utilizing inherence. In contrast, the ontological interpretation of the unary predicate A by, eventually, $x :: \cdot\mathsf{A}$, appears to be a highly frequent case.

These thoughts lead straightforwardly to the idea of describing OUS that apply to multiple signatures, in the context of FOL as well as for other (kinds of) languages. That means, the notion of ontological usage schemes may be extended and allow for, e.g., parametric descriptions of OUS in the sense of Def. 5.3. As a shorthand form, we refer to these as pOUS for *parametrized OUS*. The almost exclusive task of this section is to illustrate this idea by means of one such parametrized scheme for FOL and another for description logic, which prove useful in sect. 5.4.1.

A GENERIC SCHEME FOR FO SIGNATURES, WITH PRELIMINARIES

Drawing inspiration from Ex. 5.6 and maybe more from Def. 4.17 on p. 148 in sect. 4.4.3.2 and the conceptualization of categories and relations $\mathcal{CR}$ in sect. 2.4 as its background, Def. 5.10 specifies a pOUS for FO theories with a signature Σ, supposed to be disjoint with $\Sigma_{\mathcal{CR}}$. Beforehand we present three predication definitions / definition schemes wrt $\mathcal{CR}$ for a better comprehensible presentation of Def. 5.10. As a convention on variable notation, $\hat{x}$ is used for variables that instantiate any category category in the formula under consideration, whereas "usual" variables like x or x_i can refer to arbitrary entities. Clearly, x and $\hat{x}$ are distinct, completely independent variables from the logical point of view. Moreover, we apply the same condensed notation for binary $\mathcal{CR}$ predicates as introduced in sect. 2.4.2.3. All free variables are to be considered as universally quantified.

$\mathsf{IndNonRGRN}$ characterizes <u>ind</u>ividual–<u>non</u>-relating–<u>relations</u>, i.e., relations whose arguments / role players are all individuals and non-relating entities (which excludes roles and relators). $\mathsf{RoleBase}_0^n$ applies to an n-ary relation and its n role categories, all of which are assumed to be disjoint and exhaustive for the role base of x.[577] rel_0^n has an abbreviating function to the effect of "$\hat{r}(x_1, x_2, \ldots, x_n)$", i.e., it reflects

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[577]Disjointness refers to their role individuals, whereas the same players can fill different individuals of those roles within a relator.

the fact that a relation $\hat{r}$ applies to a number of arguments, in the ontological sense of / via role-playing.

(5.9) $\quad$ IndNonRGRN$(\hat{r})$ $\quad \leftrightarrow_{df}$

$\qquad \hat{r} :: \cdot\text{Relation} \quad \wedge \quad \forall rx \,.\, (r :: \hat{r} \wedge \exists q(x \rightsquigarrow q \multimap r)) \to x :: \cdot\text{Ind} \wedge x :: \cdot\text{NonRelating}$

(5.10) $\quad$ RoleBase$_0^n(x, y_1, \ldots, y_n)$ $\quad \leftrightarrow_{df}$

$\qquad x :: \cdot\text{Relation} \quad \wedge \quad \bigwedge_i y_i :: \cdot\text{RoleCat} \quad \wedge \quad \bigwedge_{j<k} \neg\exists z(z :: y_j \wedge z :: y_k) \wedge$

$\qquad \bigwedge_l \text{hasBaseRole}_0(x, y_l) \quad \wedge \quad \forall z \,.\, \text{hasBaseRole}_0(x, z) \to \bigvee_m z = y_m$

(5.11) $\quad$ rel$_0^n(\hat{r}, \hat{q}_1, x_1, \hat{q}_2, x_2, \ldots, \hat{q}_n, x_n)$ $\quad \leftrightarrow_{df} \quad$ RoleBase$_0^n(\hat{r}, \hat{q}_1, \hat{q}_2, \ldots, \hat{q}_n)$ $\wedge$

$\qquad \exists r q_1 q_2 \ldots q_n \quad . \quad r :: \hat{r} \quad \wedge \quad \bigwedge_i q_i :: \hat{q}_i \quad \wedge \quad \bigwedge_j x_j \rightsquigarrow q_j \multimap r \quad \wedge \quad \forall q' \,.\, q' \multimap r \to \bigvee_k q' = q_k$

Further implications arise depending on the axiomatization of $\mathcal{CR}$, e.g., $\forall \hat{x}\hat{y} \,.\, \text{hasBaseRole}_0(\hat{x}, \hat{y}) \to \forall r \,.\, r :: \hat{x} \to \exists q \,.\, q :: \hat{y} \wedge q \multimap r$. Altogether, these three predicates are convenient not only in the following definition, which introduces a pOUS for FOL. It is oriented at a categorial view on unary predicates and the position-based treatment of n-ary predicates as known from FOL. Note that 'q_i^P' is just an i-th constant name that is derived / generated for linking relations to their role categories. Accordingly, the set of terms still contains only variables and logical individual constants.

5.10 Definition (sample FO pOUS $(\sigma^\Sigma, \tau^\Sigma)$ with categories and relations)
Assume $\mathcal{CR}$ as a background theory, specified by a fundamental theory $F_{\mathcal{CR}} \subseteq Lg(\Sigma_{\mathcal{CR}})$. Let Σ a FOL signature s.t. $\Sigma_{\mathcal{CR}} \cap \Sigma = \emptyset$, let U a new unary predicate symbol and $\cdot U$ a new (logical individual) constant. $\Sigma_{\mathcal{CR}}^+ := \Sigma_{\mathcal{CR}} \cup \Sigma \cup \{U, \cdot U\} \cup \{q_i^R \mid R \in Pred(\Sigma), 2 \leq i \leq ar(R)\}$, where the q_i^R are new logical individual constants. *Var* denotes the standard set of variables for $Lg(\Sigma)$ and $Lg(\Sigma_{\mathcal{CR}}^+)$.

Define an entity function $\varepsilon^\Sigma : \Sigma \cup Var \to Tm(\Sigma_{\mathcal{CR}}^+)$ s.t. $\varepsilon^\Sigma(x) = x$ for every $x \in Tm(\Sigma)$, $\varepsilon^\Sigma(P) = \cdot P$. The translation function τ^Σ maps atomic formulas by $\tau^\Sigma(P(\bar{x})) := P(\varepsilon(\bar{x}))$ for all $x \in Tm(\Sigma)$, and for all $\phi, \psi \in Lg(\Sigma)$: $\tau_1(\neg\phi) := \neg\tau^\Sigma(\phi)$, $\tau^\Sigma(\phi \wedge \psi) := \tau^\Sigma(\phi) \wedge \tau^\Sigma(\psi)$, $\tau^\Sigma(\exists x \,.\, \phi) := \exists \varepsilon(x) \,.\, U(\varepsilon(x)) \wedge \tau^\Sigma(\phi)$.

The signature analysis function σ^Σ first adopts closure axioms CA_U wrt U and Σ. Moreover, it corresponds to analyzing unary predicates as categories and, for $n \geq 2$, n-ary predicates as relations with distinct roles in all places, expressed through both, predication definitions and additional analysis facts.

$$\sigma^\Sigma(\Sigma) := \{\cdot U \rightarrow\hspace{-0.3em}|\, \cdot\text{Ind}, \quad \forall \varepsilon(x) \,.\, U(\varepsilon(x)) \leftrightarrow \varepsilon(x) :: \cdot U\} \cup CA_U$$

$$\sigma^\Sigma(P) := \{\varepsilon(P) :: \cdot\text{Cat}, \quad \varepsilon(P) \rightarrow\hspace{-0.3em}|\, \cdot\text{Ind},$$
$$\forall \varepsilon(x) \,.\, P(\varepsilon(x)) \leftrightarrow \varepsilon(x) :: \varepsilon(P)\} \qquad \text{for all } P \in Pred(\Sigma), ar(P) = 1$$

$$\sigma^\Sigma(R) := \{\text{IndNonRGRN}(\varepsilon(R)), \quad \text{RoleBase}_0^n(\varepsilon(R), q_1^R, \ldots, q_n^R)$$
$$\forall \varepsilon(x_1) \ldots \varepsilon(x_n) \,.\, R(\varepsilon(x_1), \ldots, \varepsilon(x_n)) \leftrightarrow \text{rel}_0^n(\varepsilon(R), q_1^R, \varepsilon(x_1), \ldots, q_n^R, \varepsilon(x_n))\}$$
$$\text{for all } R \in Pred(\Sigma), n = ar(R) > 1$$

$$\sigma^\Sigma(c) := \{\tau^\Sigma(U(c))\} \qquad\qquad\qquad \text{for all } c \in Const(\Sigma) \qquad \square$$

On the one hand, based on experiences from various biomedical ontologies, for example, we argue that this scheme has a potential of wide applicability. On the other hand, its presupposition on disjoint role categories as role bases for *all* relations may be too strong, because there are also natural cases in which the scheme is ontologically inadequate. One kind of example are relations with symmetry aspects due to multiple roles of the same role category, cf. [526, sect. 3.3.4], in conflict with RoleBase$_0^n$, see (5.10).

Realizing that, similarly to the above case of τ_1, τ^Σ is defined, again, in "exactly the same" way as the relativation of Σ formulas to U, i.e., $\tau^\Sigma(T) = T^{|U}$ for all $T \subseteq Lg(\Sigma)$, in combination with ensuring $CA_U \subseteq \omega^\Sigma(T)$ for every $T \subseteq Lg(\Sigma)$, where $\omega^\Sigma(T) := F_{\mathcal{CR}} \cup \sigma_\Sigma^\Sigma \cup \tau^\Sigma(T)$ is determined by Def. 5.3, allows for drawing a concluding observation by means of Prop. 5.8. Of course, the consistency of each such $\omega^\Sigma(T)$ is another and crucial issue which, at present, remains undetermined in general.

ALC SYNTAX	POUS $(\sigma_0^{\mathrm{DL}}, \tau_0^{\mathrm{DL}})$ WITH AUXILIARY FUNCTIONS $\varepsilon, \varepsilon_1, \varepsilon_2, \hat{\tau}_x^V$
Vocabulary	
$C \in N_{\mathsf{C}}$	$\varepsilon(C) := \; \cdot C$
	$\sigma(C) := \{ \cdot C :: \cdot\mathsf{Cat}, \; \cdot C \twoheadrightarrow \cdot\mathsf{Ind}, \; \cdot C \twoheadrightarrow \cdot U \}$
$R \in N_{\mathsf{R}}$	$\varepsilon(R) := \; \cdot R \qquad \varepsilon_1(R) := q_1^R \qquad \varepsilon_2(R) := q_2^R$
	$\sigma(R) := \{ \mathsf{IndNonRGRN}(\cdot R), \mathsf{RoleBase}_0^2(\cdot R, q_1^R, q_2^R) \}$
$a \in N_{\mathsf{I}}$	$\varepsilon(a) := \; a$
	$\sigma(a) := \{ a :: \cdot\mathsf{Ind}, \; a :: \cdot U \}$
Concepts	
$C \in N_{\mathsf{C}}$	$\hat{\tau}_x^V(C) := x :: \cdot C$
$C \sqcap D$	$\hat{\tau}_x^V(C \sqcap D) := \hat{\tau}_x^V(C) \wedge \hat{\tau}_x^V(D)$
$\neg C$	$\hat{\tau}_x^V(\neg C) := \neg \hat{\tau}_x^V(C)$
$\exists R.C$	$\hat{\tau}_x^V(\exists R.C) := \exists y \,.\, y :: \cdot U \wedge \mathsf{rel}_0^2(\cdot R, q_1^R, x, q_2^R, y) \wedge \hat{\tau}_y^{V \cup \{y\}}(C)$
	(for any new variable $y \notin V$)
TBox and ABox axioms (where C, D are ALC concepts, $a, b \in N_{\mathsf{I}}$)	
$C \sqsubseteq D$	$\tau(C \sqsubseteq D) := \forall x \,.\, x :: \cdot U \wedge \hat{\tau}_x^{\{x\}}(C) \to \hat{\tau}_x^{\{x\}}(D)$
$C = D$	$\tau(C = D) := \forall x \,.\, x :: \cdot U \wedge \hat{\tau}_x^{\{x\}}(C) \leftrightarrow \hat{\tau}_x^{\{x\}}(D)$
$C(a)$	$\tau(C(a)) := a :: \cdot C$
$R(a,b)$	$\tau(R(a,b)) := \mathsf{rel}^2(\cdot R, q_1^R, a, q_2^R, b)$

Table 5.1: An exemplary parametrized ontological usage scheme $(\sigma_0^{\mathrm{DL}}, \tau_0^{\mathrm{DL}})$ based on $\mathcal{CR}$ for (a subset of) ALC syntax [33, ch. 2], with sets of concept names N_{C}, (DL) role names N_{R}, and (logical) individual names N_{I}. $\cdot U$ is a constant for a category wrt which the translations of DL formulas are relativized and for which closure axioms must be assumed in addition, in analogy to the (defined) predicate U in the pOUS $(\sigma^\Sigma, \tau^\Sigma)$. For readability, σ_0^{DL} and τ_0^{DL} are written without their indices in the definitions.

5.11 Observation (τ^Σ yields theory interpretations)

Let Σ a FO signature and consider the pOUS of Def. 5.10 instantiated by Σ to $(\sigma^\Sigma, \tau^\Sigma)$.

For every theory $T \subseteq Lg(\Sigma)$, τ^Σ is a theory interpretation of T into its ontological image $\omega^\Sigma(T)$. ❑

A GENERIC SCHEME FOR ALC

Without leaving the context of FOL essentially, a final OUS for a syntactically deviating language shall be illustrated with the case of ALC, the / a fundamental description logic, cf. FN 53. Table 5.1 presents a pOUS which serves the ontological foundation of certain uses of ALC. This is a revised version of the pOUS presented in [533, Table 1], where we maintain the structural layout of the presentation for a clearer relationship. The overall publication advocates the view of distinguishing and pursuing in parallel formal and ontological semantics and it contributes an intermediate formal version of the notions of ontological usage scheme and ontological image between the initial, general ideas in [534, sect. 3.4] and the revised account presented in the first sections of this chapter.

The pOUS $(\sigma_0^{\mathrm{DL}}, \tau_0^{\mathrm{DL}})$ in Table 5.1 follows rather the style of (σ_0, τ_0) to the extent that no predicates are introduced for the $C \in N_{\mathsf{C}}$ and $R \in N_{\mathsf{R}}$, which would be an alternative option and in even closer analogy to (σ_1, τ_1) and $(\sigma^\Sigma, \tau^\Sigma)$. Nevertheless, the latter is clearly also given. First, there is a strong proximity in the values of σ_0^{DL} and σ^Σ. Secondly, τ_0^{DL} is inspired by the common translation function from DL to FOL, cf. e.g. [405, sect. 6.1.1, esp. Fig. 6.1, p. 168], on the basis of which their formal semantics are interrelated.

Regarding the ontological image of $(\sigma_0^{\mathrm{DL}}, \tau_0^{\mathrm{DL}})$, we do not engage in a formal proof here, but it is intended by the construction and we strongly conjecture that, modulo the introduction of predication definitions, an ALC theory T^{DL} and its formal semantics preserving translation into FOL T^{FOL} result in equivalent ontological images with $(\sigma_0^{\mathrm{DL}}, \tau_0^{\mathrm{DL}})$ applied to T^{DL} and $(\sigma^\Sigma, \tau^\Sigma)$ applied to T^{FOL}. Moreover and admit-

tedly, it is still[578] an open issue to return to the question and notion(s) of intensional/conceptual equivalence. Yet, having ontological semantics available and remembering earlier criticism of the standard account of ontology-based semantic integration in sect. 2.2.2.2, which is primarily affecting formulas within ontological theories instead of those theories as a whole, we now see support for the claim – at the level of whole theories – that the conjectured relation between $(\sigma_0^{\mathrm{DL}}, \tau_0^{\mathrm{DL}})$ and $(\sigma^\Sigma, \tau^\Sigma)$ renders the formal semantic translation from ALC to FOL ontologically adequate *under these ontological usage schemes*.

As a last remark in these regards, we find that the possibility of specifying pOUS does not conflict our conviction that intensional meaning is (primarily) signature-specific. While a pOUS may add some level of ontological analysis to a language, a dedicated OUS for a particular signature / use of a language where the OUS relies on a domain ontology at the "right" level of specialization should result in a much higher degree of ontological foundation of language expressions, up to their complete ontological translation in the sense of sect. 2.3.1.

5.3 Glimpse on Characterizing Modular Representation

MODULARIZATION AS AN IMMEDIATE DESIDERATUM IN ONTOLOGY ENGINEERING

While the end of the last section has a straightforward connection to the application of OUS in connection with formalizing a biological core ontology,[579] the route from engineering considerations to applications suggests a short interlude to another branch of work that we have pursued during the development of ontological semantics, which has affected its background and views. Forsooth, *modularity / modularization* is another engineering aspect that the development of GFO and most other ontology projects, in any case those of large scale, pose almost immediately, and is therefore included among our motivations in sect. 1.2.4.

SURVEY CHAPTER [532] AND ITS CORE NOTIONS

In [532] we define an abstract framework for ontology modules, based on analogies to software engineering as we suggested in an early requirements paper for modules in ontologies [529]. Its core concepts are *modules* equipped with *interfaces* and forming a *system* via some *composition operation*, a distinction between *explicit* and *implicit* information in modules, as well as a naïve notion of *information flow*[580]. The framework is formally captured by conventional mathematical / set-theoretic concepts and notation, whereas other abstract accounts like the one of Oliver Kutz, Till Mossakowski et al. [496, 497, 499] rely on the theory of institutions [283], cf. also [596], as introduced by Joseph Goguen and Rod Burstall, which itself is stated in terms of (mathematical) category theory, see e.g. [3, 52, 518, 544] for introductions.

MODULE CRITERIA GATHERED FROM THE LITERATURE

On the basis of the proposed framework, in [532, sect. 3.5] we gather / abstract and present 4 informal and 9 formal module characteritics / criteria, most of which have appeared (scattered) in the literature on modules / modularity of DL and FOL theories. We rephrase them abridged here to provide some intuition.

- informal characteristics

  CI-1 *comprehensibility* of modules / module content
  CI-2 *stability* wrt side-effects of changes to single modules
  CI-3 *compositionality*, i.e., maintenance / derivability of system properties from module properties, e.g., consistency transfer
  CI-4 *directionality* of information flow

- formal criteria

  CF-1 *basic and identity interface* stand for no changes of im- or exported formulas / theories, apart from selection by language restrictions

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[578]and, beyond the remarks in sect. 7.4, will remain
[579]That application case is eventually expounded in sect. 5.4.1.
[580]I.e., *not* in the sense of Jon Barwise and Jerry Seligmann, see [57]. Notably, the latter has been adopted in works associated with semantic integration, incl. [470–472, 734], [668, ch. 23].

CF-2 *black-box interfaces* accept arbitrary consistent input consistently in the module[581]
CF-3 *module language overlap* (with a variety of options)
CF-4 *classically closed composition* refers to whether the system is deductively closed
CF-5 *inclusion of explicit information* of modules by the system
CF-6 *inclusion of complete module information* by the system, incl. implicit information[582]
CF-7 *deductive conservativity* of the system over its modules[583]
CF-8 *deductive conservativity over sublanguages* of the system over its modules
CF-9 *transitive information flow among modules*

LINES OF PROPOSALS DISCUSSED

Naturally, those criteria can be employed in systematic comparisons of available approaches. Five major lines of proposals for modules are surveyed and analyzed in [532, sect. 3.6], the first of which is split in two sublines. Those six lines are labeled as follows, incl. a selection of major references:

(1) disjoint signatures [670–672]
(2) basic modules with conservativity [171–173, 279, 480, 481, 542, 543]
(3) partition-based reasoning [12, 13, 546]
(4) semantic encapsulation [170, 175–177]
(5) package-based description logic [43–45]
(6) distributed logics [97, 102, 277, 278, 746]

CURRENT SITUATION

According to our impression in early 2015, especially in the field of DLs the resulting main interest appears to have been approach (2), i.e., basically the acceptance of modules as subtheories s.t. a larger theory is a deductive conservative extension. Therefore we skip recapitulating the evaluation from [532], which would further require a more detailed introduction of the framework.

Regarding current trends, we are aware of two advanced (and partially related) efforts, in a wider sense both linked to ontology repositories. One of them is Hets (from 'Heterogeneous Toolset'), as a major part of the foundation of the Ontohub system [@81] [598], cf. a respective paragraph in sect. 1.2.1. Hets with its history in algebraic specification[584] may be said to exhibit a "natural imprint" of modularity, not at least due to the proposed heterogeneous ontologies [598], the components of which can even be specified in distinct logics. The other effort is pursued by Michael Grüninger and colleagues, e.g., by their proposals of ontology development in terms of theory hierarchies, cf. e.g. [320, 321], which support highly modularized ontologies. This effort connects to the Common Logic Ontology Repository (COLORE) [@20] [320, sect. 8], in which ontologies are organized in theory hierarchies.

SINGLE CATEGORY PERSPECTIVE LEADS TO APPLICATION FIELDS

Initial parts of [532] are devoted to another perspective that is influenced by the structure of ontologies. Whereas the brief summary above assumes a view of an "ontology as a whole" and corresponding questions, e.g., of structuring that, one may focus on the relationship between invidiual categories and "their place" within an ontology. This relates to questions of the compositionality of categorial descriptions[585] as well as the navigability and searchability within large ontologies, which in turn can lead to modularization and organization strategies as they have emerged in the context of medical terminological systems[586]. We actually see further potential of transferring ideas from those fields to ontologies formalized in logic, but must escape the temptation to explore this further in the context of this thesis. Rather we take the arrival at medical terminologies as a welcome opportunity to switch to our main field of applications.

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[581]Abstracted from [171].

[582]This corresponds to *local correctness* in [177, p. 198], traced back to [266].

[583]*Local completeness* in [177, p. 198].

[584]related to software systems and thereby exposed to a demand for modularization early

[585]The composition / formation of categorial descriptions by complex expressions of languages may appear to be a general precondition against the logical background of ontologies which we largely adhere to herein. However, e.g., taxonomies, glossaries, etc. may lack such features. Moreover, the approaches of pre- vs post-coordinated systems gave rise to much debate in the area of medical terminologies, cf. e.g. [694, 741], see also there for the notions of pre- and post-coordination.

[586]like the Systematized Nomenclature of Medicine (SNOMED) [@115] [167, 204, 442, 675] or the Unified Medical Language System (UMLS) [@127] [440]

5.4   Applications in the Biomedical Domain

WORKING IN PARALLEL IN APPLICATION PROJECTS

The development of the concepts described thus far proceeded in parallel to working on projects usually involving colleagues from the group Ontologies in Medicine (Onto-Med) [@86] on the continued development of the General Formal Ontology (GFO), see sect. 1.1.5, as well as on applications in the field of biomedical ontology and knowlege representation. This has helped us shaping further thoughts on ontological semantics and usage schemes, and finding cases in which one can see benefits of the latter in the form of having an additional theoretical justification and underpinning in their terms, which can contribute to guiding further steps in the application projects.

SECTION OUTLINE AND FOCUS ON RELATIONS TO THE THEORY HEREIN

The purpose of this section is to introduce two application cases in a moderate level of detail that allows us to bring to bear the connection with OUS, ontological semantics, and the ontology of categories and relations $\mathcal{CR}$. More precisely, the first case of GFO-Bio in sect. 5.4.1, utilizes OUS and the perspective of ontological semantics to justify the way in which an ontology is represented in OWL in two different ways. In the second case, primarily the concepts of $\mathcal{CR}$, cf. sect. 2.4, likewise with OUS in mind, are applied to the problem of representing phenotypes in biomedical ontologies.

The actual contributions are already contained in corresponding publications which are cited in the respective section and, as stated, which are the product of the groups of their authors, including ourselves. Accordingly, we limit the case descriptions to short summaries and focus on the connections to other parts of this thesis. For a more complete picture of the issues in the biomedical domain and respective proposals in reply to them, the reader is referred to those publications.

5.4.1   GFO-Bio

THE PROJECT, ITS CONTEXT AND OUR CONTRIBUTION

The advent of many biomedical ontologies in the early 2000s caused a broader interest in core / mid-level / upper-domain ontologies for the life sciences domain(s) and in response to that a number of candidate proposals, incl. BioTop [@16] [71, 736–738], GFO-Bio [@47] [415, 417, 418], [668, sect. 15.4.3], [413, sect. 5.1], and the Simple Bio Upper Ontology (SBUO) [697]. GFO-Bio among them, more precisely its development, was initiated in 2005 by Robert Hoehndorf, who performed the greatest part of a team effort of constructing / conceptualizing the ontology, implementing it in OWL, SWRL (see sect. 1.1.3.1), and DLVHEX [@29] [217], and conceptualizing and demonstrating applications in the integration of bio-ontologies, cf. [416]. Besides participating in the general conceptualization of GFO-Bio on the basis of GFO, our own contribution in this project further fed on early ideas on ontological semantics and usage schemes, which, as a result, influenced the approach of implementing GFO-Bio in the three languages.

GFO-BIO IN BRIEF: DOMAIN STRUCTURING AND MID-LEVEL CATEGORIES

As its name suggests, GFO-Bio is developed on the basis of the General Formal Ontology (GFO), see sect. 1.1.5. GFO-Bio is designed as a core ontology, in a stricter sense, originating from [831] in our context, than the one which accepts upper-domain or mid-level ontology as synonyms, as allowed for in sect. 1.1.4.2. In addition to the latter notions, such a core ontology does *not only* aim at providing categories at a high level of generality in the context of the domain / field under consideration (i.e., mid-level categories), but in addition it identifies *principal categories*, see [392, sect. 14.7.1, p. 339], and accounts for the / a thematic structure of the domain of reality / knowledge. Fig. 5.1 shows a fragment of the latter, further explained next, according to version 1.0 of GFO-Bio. Principal categories are principal / fundamental for the domain to the extent that other categories "derive from" them by "important" relations. For example, with *organism* and *cell*, two principal categories for the biological domain were identified.[588] They are focal in the domain in the (certainly still rather rough) sense that, e.g., considering the structure / mereological relations starting

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[587] We are co-author of [417, 418] and recommend [418] as the latest main publication on GFO-Bio. [416] is reasonable to consider in combination and addresses GFO-Bio partially.

[588] In particular, the category of organism is associated with the notion of autopoiesis, cf. [564, 842]. NB: The latter has also been studied wrt the social stratum, see, e.g., [893].

from the category organism unfolds (a major fraction of) the subdomain of anatomy, and starting from cell yields cell structure (as an area). Similarly, considering processes in connection with autopoiesis may lead to physiology. Another aspect is the classification of, e.g., organisms, i.e., the study of its subcategories and their organization, involving the relation of subsumption and connecting to subdomain of taxonomy.

These reflections on principal categories are further integrated with the theory of levels of reality / ontological levels [664, 665] briefly indicated in sect. 1.1.5.1. In particular and wrt the biological level, version 1.0 of GFO-Bio distinguishes two layers in accordance with the identified principal categories, namely an organismal and a cellular layer. As a first step of utilizing levels in GFO and GFO-Bio, these are understood as higher-order categories, whose instances are categories of individuals. For example, the categories Animal and Mouse instantiate the biological level and its organism layer.[589] Further, the design of GFO-Bio and the approach of "following relations" is inspired by facet analysis/faceted classification. The latter has been studied in the library and information sciences[590], which also produced an offshoot in terms of faceted browsing and search, cf. e.g. [723], incl. in the Semantic Web [402]. Facet analysis led to the approach of "following"

Figure 5.1: A fragment on biological subdomains in GFO-Bio.

certain relations in identifying kinds of entities that are relevant for the domain, starting from principal categories.

BRANCHES OF INDIVIDUAL-CATEGORIES AND CATEGORY CATEGORIES

In parallel to the category categories (incl. levels) just outlined in connection with domain structuring, GFO-Bio provides general individual-categories for the biological domain, which include, a.o., categories of material objects, e.g., atom, molecule, cell, tissue, and organism, as well as categories of biological processes such as development stage. According to GFO as its foundation, all of these categories of first or higher orders of instantiation chaining coexist in the conceptualization. However, for easier comprehensibility of the overall ontology as well as to support different forms of reasoning, the two branches of categories have been separated. In particular, GFO-Bio is implemented in two OWL files, gfo-bio.owl and gfo-bio-meta.owl [@47], which constitute a formalized version of the overall ontology. In the sequel, let us refer to the formalized version of GFO-Bio as $\mathcal{GB}$, and to its components as $\mathcal{GB}^{\mathcal{I}}$, for gfo-bio.owl / the individual-categories branch, and $\mathcal{GB}^{\mathcal{C}}$ for gfo-bio-meta.owl / the category categories branch.

APPLICATION OF OUS FOR A UNIFIED VIEW

The aspect that is of interest from an application perspective for this thesis is that both files are designed wrt different ontological usage schemes (OUS). We shall restrict considerations to the taxonomy subdomain, for a clear and / though simple example of OUS mediating a unified picture between the two ontology components.

The individual-categories in $\mathcal{GB}^{\mathcal{I}}$ basically follow the OUS $(\sigma_0^{\mathrm{DL}}, \tau_0^{\mathrm{DL}})$ specified in Table 5.1, although we must implicitly assume exchanging / enhancing the formula translation into one that can account for all employed constructs of the DL.[591] Yet the main point is that individual-categories like Mouse are formalized as OWL classes / DL concepts in $\mathcal{GB}^{\mathcal{I}}$. In contrast, the subdomain categories in $\mathcal{GB}^{\mathcal{C}}$ require a different analysis, which is not available from the schemes discussed in previous sections, namely (and not surprisingly)

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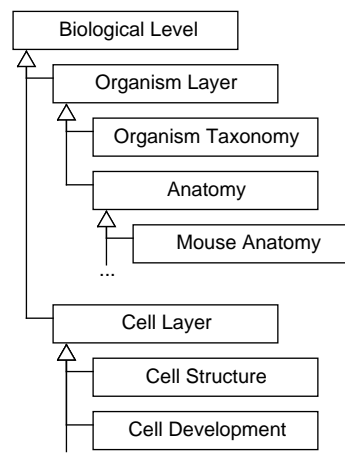
[589]Admittedly, we believe that behind this still simplified view on levels there are much more complex relationships that can and should be elaborated by further ontological analysis of these matters.

[590]The origin of facet analysis in library science and of the *colon classification* as a scheme designed on the grounds of that theory is clearly attributed, e.g., in [494, p. 45], [282, sect. 5, p. 153], to Shiyali Ramamrita Ranganathan with publications starting from 1931 on what is today know as the 'Five Laws of Library Science', even if some criticism has been raised wrt making the sources of his ideas more explicit [494, p. 46]. [494, p. 45] names Ranganathan's 'Prolegomena to library classification' [690] as his major work, see also [689]. Cf. further [104, 778, 779] on the subject primarily in library science, [676] links to knowledge representation.

[591]Protégé [@97] (version 4.3) determines $\mathcal{SHIN}$ as the DL expressivity of gfo-bio.owl.

as category categories.[592] Moreover, in order to classify individual-categories by categories in $\mathcal{GB}^{\mathcal{C}}$ via the DL reasoner, individual-categories are represented as DL individuals in $\mathcal{GB}^{\mathcal{C}}$. Hence, instantiation relators between individual-categories and category categories parallel the set-membership in the DL semantics, thereby allowing for classification according to axioms stated for DL concepts / category categories in $\mathcal{GB}^{\mathcal{C}}$. In addition to that, subcategory relations among individual-categories can be imported / expressed in $\mathcal{GB}^{\mathcal{C}}$ in terms of a DL role elsA for the (extensional) is-a relation, while the latter is reflected by DL subsumption for category categories.

The question to which OUS and ontological semantics provide a solution that we find at least improved compared to what is available without them is to account for a precise argument, a proof, that the subsumption relations among DL concepts in $\mathcal{GB}^{\mathcal{I}}$ express the same content as those that can be found in $\mathcal{GB}^{\mathcal{C}}$.

5.12 Example (a case of conceptually equivalent statements)
Here is a minimal example, for which we assume (5.12) and (5.13) incl. the assignments to the resp. theories, with M and m referring to the category Mouse, and A and a denoting the category Animal.[593]

(5.12)   in $\mathcal{GB}^{\mathcal{I}}$: $\mathsf{M} \sqsubseteq \mathsf{A}$

(5.13)   in $\mathcal{GB}^{\mathcal{C}}$: $\mathsf{elsA(m, a)}$

Why should these DL sentences account for the same conceptual content? To discuss that by means of OUS, or better, their resulting ontological images, we first accept (5.14)–(5.16) by application of $(\sigma_0^{\mathrm{DL}}, \tau_0^{\mathrm{DL}})$ to $\mathcal{GB}^{\mathcal{I}}$. Secondly, we assume that an appropriate OUS $(\sigma^{\mathcal{GB}^{\mathcal{C}}}, \tau^{\mathcal{GB}^{\mathcal{I}}})$ for $\mathcal{GB}^{\mathcal{C}}$ yields (5.17) and (5.18), with $\varepsilon^{\mathcal{GB}^{\mathcal{C}}}$ denoting an auxiliary entity function. We adopt a mapping to the same constants $\cdot M$ and $\cdot A$ in both OUS against the background of developing GFO-Bio, i.e., we know that the symbols in those pairs are intended to refer to the same category. (4.21) and (2.8) remind of (predication) definitions of $\mathcal{CR}$ that occurred above, namely of the category predicate in sect. 4.4.3.2 on p. 149 and of extensional is-a in sect. 2.4.2.2 on p. 84.

(5.14) $\sigma_0^{\mathrm{DL}}(\mathsf{A}) = \{\cdot\mathsf{A} :: \cdot\mathsf{Cat},\ \cdot\mathsf{A} \twoheadrightarrow \cdot\mathsf{Ind},\ \cdot\mathsf{A} \twoheadrightarrow \cdot U\}$

(5.15) $\sigma_0^{\mathrm{DL}}(\mathsf{M}) = \{\cdot\mathsf{M} :: \cdot\mathsf{Cat},\ \cdot\mathsf{M} \twoheadrightarrow \cdot\mathsf{Ind},\ \cdot\mathsf{M} \twoheadrightarrow \cdot U\}$

(5.16) $\tau_0^{\mathrm{DL}}(\mathsf{M} \sqsubseteq \mathsf{A}) = \forall x\,.\, x :: \cdot U \wedge x :: \cdot\mathsf{M} \to x :: \cdot\mathsf{A}$

(5.17) $\varepsilon^{\mathcal{GB}^{\mathcal{C}}}(\mathsf{a}) = \cdot\mathsf{A}$ and $\varepsilon^{\mathcal{GB}^{\mathcal{C}}}(\mathsf{m}) = \cdot\mathsf{M}$

(5.18) $\tau^{\mathcal{GB}^{\mathcal{C}}}(\mathsf{elsA(m, a)}) = \cdot\mathsf{M} \twoheadrightarrow \cdot\mathsf{A}$

(4.21) $\forall x\,.\, \mathsf{Cat}(x) \leftrightarrow_{df} x :: \cdot\mathsf{Cat}$

(2.8) $\forall xy\,.\, x \twoheadrightarrow y \leftrightarrow_{df} \mathsf{Cat}(x) \wedge \mathsf{Cat}(y) \wedge \forall z\,.\, z :: x \to z :: y$

The claim that the DL sentences in (5.12) and (5.13) are conceptually equivalent statements is based on the ontological semantics assigned by the resp. OUS, where we consider the union of both ontological images as the established ontology GFO-Bio. More precisely, let $\mathfrak{J}$ the set of all formulas occurring in (5.14), (5.15), (4.21), and (2.8). Then $\tau_0^{\mathrm{DL}}(\mathsf{M} \sqsubseteq \mathsf{A}) \equiv_{\mathfrak{J}} \tau^{\mathcal{GB}^{\mathcal{C}}}(\mathsf{elsA(m, a)})$,[594] which we advocate as the justification of the equivalence of both DL expressions.   ❑

COMMENTS ON THE EXAMPLE
First of all, we defend this consideration as a justification of the conceptual equivalence of (5.12) and (5.13), but acknowledge that it must appear as an ad hoc argument / approach here. Declaredly, the route to more general accounts of conceptual equivalence on the basis of the theory developed thus far can only be outlined as future work herein, see sect. 7.4. Moreover, further discussion in sect. 7.3 shall not be forestalled

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[592]In conflict with the atoms $\cdot C \twoheadrightarrow \cdot\mathsf{Ind}$ in σ_0^{DL} in Table 5.1, for example.

[593]Just explaining the mnemonics of the symbols, yet without particular effects on the argument technicalities.

[594]With $\cdot\mathsf{M} \twoheadrightarrow \cdot U$, $\tau_0^{\mathrm{DL}}(\mathsf{M} \sqsubseteq \mathsf{A})$ entails (*) $\forall x\,.\, x :: \cdot\mathsf{M} \to x :: \cdot\mathsf{A}$, and this together with $\cdot\mathsf{A} :: \cdot\mathsf{Cat}$, $\cdot\mathsf{M} :: \cdot\mathsf{Cat}$, (4.21) and (2.8) yields $\cdot\mathsf{M} \twoheadrightarrow \cdot\mathsf{A}$. Conversely, the latter entails (*) modulo the definitions, and (*) can be weakened to $\tau_0^{\mathrm{DL}}(\mathsf{M} \sqsubseteq \mathsf{A})$ by $\cdot\mathsf{M} \twoheadrightarrow \cdot U$.

too much. We only briefly mention that this clearly looks very close to the standard account of ontology-based equivalence, cf. Def. 2.3, p. 64, because Ex. 5.12 utilizes equivalence modulo a theory. However, the overall theory / ontology is the union of the ontological images, modulo which especially all of its members are equivalent by Obs. 2.5, p. 65,[595] e.g., also $\mathsf{Cat}(\cdot M)$ and $\cdot M \twoheadrightarrow \cdot A$. In contrast, the theory $\mathfrak{J}$ modulo which equivalence is considered in the example is much more limited. In particular, it does not include the right-hand sides / ontological images in (5.16) and (5.18). In addition, the formal argument in terms of the ontological image can be assumed to involve no more encoding behind the formalized ontology, on the basis of ontological semantics.

We see two potential caveats / points of criticism wrt this demonstrated application of OUS. (1) The specified translation could also be achieved by "purely formal" means, i.e., one may simply define a function for translating $C \sqsubseteq D$ from $\mathcal{GB}^{\mathcal{I}}$ to $\mathsf{elsA}(c, d)$ in $\mathcal{GB}^{\mathcal{C}}$ and claim that they are "stating the same". However, and relating to our key motivating questions, how would such a claim be justified? As far as we can see, an available kind of response could refer to the conceptualization behind $\mathcal{GB}^{\mathcal{I}}$ and $\mathcal{GB}^{\mathcal{C}}$ in the form of some documentation, similar to our description prior to Ex. 5.12. Compared to that, OUS allow for formalized and exact statements, such that formal proofs of conceptual equivalence (once defined within a more general theory) can be formulated (and possibly be found by theorem provers). (2) Considering the two component DL theories, reasoning in each of them remains incomplete compared to the joint ontological images in FOL. This incompleteness is not only due to the more restricted character of DLs, but depends further on the strength of additional axiomatization that can be achieved (or automatically generated) from one component for the other. Full integration is available by simple means in the FOL theory of the ontological images, yet reasoning can face tractability problems. [418, Table 1, p. 223] shows some examples of equivalence axioms in a SWRL-style notation, which was an initial account of integrating both components via another language available in the Semantic Web.

HINT ON COVERAGE OF NONMONOTONIC ASPECTS

In addition, the approach of reflecting complex interrelations via atomic formulas has been taken further to default logic, which links back to Table 2.2 in sect. 2.4.4.1 with some schematic definitions of relations starting from relations between (ontological) individuals (marked by an ll- prefix). Without dwelling much further on this topic, e.g., due to remaining in the FO context herein, we merely note for illustration the following answer-set programming (ASP) [117] rule from [416, p. 377.5], where (by intention) $\mathtt{ind(X)}$ classifies X as an individual, $\mathtt{class(X)}$ as a category, $\mathtt{inst(X, Z)}$ expresses instantiation.

[416]

```
IC-has-part(X, Y)  :− ind(X), class(Y), class(Z), inst(X, Z),
                        CC-canonical-has-part(Z, Y), not IC-lacks-part(X, Y).
```

ASP is used in order to account for nonmonotonic aspects of biological knowledge. The `CC-canonical-has-part` relation, following the schemes in Table 2.2, has a default character and allows for stating that, e.g., by default, humans have an appendix. Nevertheless, this is only concluded for $(\mathtt{X}, \mathtt{Y})$ via this rule unless `IC-lacks-part`$(\mathtt{X}, \mathtt{Y})$ is derivable from the ASP program. By rules of this kind, Robert Hoehndorf implemented (also) interconnections among the GFO-Bio branches in the ASP system DLVHEX [@29] [217], which integrates OWL / DL reasoning and ASP according to dl-programs, see e.g. [218] for the latter. We remark that the formal semantic integration between DL theories and answer set programs in the semantics of dl-programs [218, sect. 4.2.5 and 4.2.6] constitutes a case where, besides the formal semantic integration, it appears worthwhile to ensure that the linkage between the ontological semantics / OUS in particular use cases is in harmony between the two formal approaches with their formal semantic integration. Briefly returning to the two OWL components of GFO-Bio, it appears worthwhile today to seek additional formal semantic integration in terms of the approach of heterogeneous ontologies, cf. analogous motivating comments in [599, p. 337].

A concluding note on GFO-Bio is that also the concepts of relations and roles have been employed in the ontological analysis of developing GFO-Bio, e.g. the case of chemical reactions, see [413, e.g., p. 93–95]. Relations and roles (and thus $\mathcal{CR}$) move closer into our application focus in the next section.

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[595]This links to effects of equivalence under ontological semantics, which are discussed in sect. 7.3.
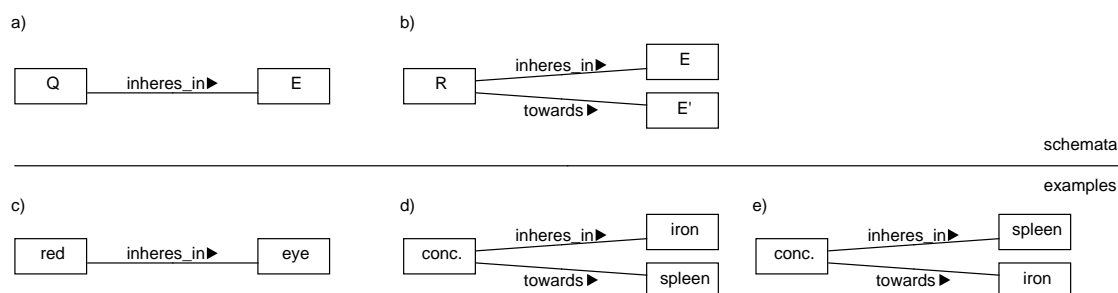
Figure 5.2: Entity-Quality (EQ) model of phenotype representation. E stands for entity, Q for quality, and R for relational quality in the schema part. Schema a) accounts for a simple (unary or non-relational) quality, while b) refers to relational qualities. In examples d) and e) 'conc.' abbreviates concentration of. The interchanged use of inheres in and towards in d) and e) illustrates one problem in the approach.

5.4.2 *CR*-based Phenotype Representation in OWL

5.4.2.1 Introduction and Problems

PHENOTYPE REPRESENTATION WITH THE EQ MODEL

Another, weakly related topic in biomedical ontology that we have engaged in concerns the representation of phenotypes[535, 536], targeted at OWL. Phenotype ontologies are employed for the annotation of muta- genesis experiments, primarily in species-specific databases, as well as for characterizing human diseases. In [69, 608, 865], a syntax for phenotype decompositions has been devised for the purpose of translating phenotypes across species, which has become an important and widespread approach for capturing pheno- type information [42]. This *EQ model*, cf. Fig. 5.2[597], expresses phenotypes by a combination of a quality category and one or more entity categories. The figure provides first examples. Subfigure c) might be suggestive of 'red eye' in natural language, but since EQ descriptions are typically understood as quality specializations, 'red that inheres in an eye' is more appropriate [608, p. 3], [609], utilizing the ontological relation of inherence. Subfigures d) and e) refer to 'concentration of iron in the spleen'. For such relational qualities (in the terminology of bio-ontologies), the EQ model allows for linking the quality category to a second entity category by means of a relation named towards. In general, EQ descriptions have been de- veloped for many phenotype ontologies, including the Human [710] and Mammalian Phenotype ontologies [86, 125, 768].

ISSUES IN THE EQ MODEL

In [536] we identify the three main problems in connection with phenotype descriptions, namely (I) the ontological foundation of complex phenotypes, (II) the representation of phenotypes in formal languages, and (III) the ontological foundation of phenotype annotations. Concentrating on the second problem and aiming at an EQ compatible OWL representation led us to the five issues listed next.

1. ontological adequacy / coherence of ontological interpretation
2. invalid permutations / ambiguities
3. relational expressiveness
4. consistency of domain modeling
5. formal reflection of annotations

Let us sketch these problems briefly. The first is closely related to the question of which ontological interpretation is given to phenotype description, incl. EQ expressions. In line with established theories of phenotypes [608, 609], these are considered as qualities in [536]. However, a direct representation

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[596] [535, 536] are joint works with, primarily, Frank Stumpf, with problem motivation by Robert Hoehndorf, and support by him and Heinrich Herre. [536] is a revised and extended journal version of [535], s.t. we refer only to the former in the sequel. Major parts of this section are summarized from these publications, yet with a focus on how *CR* is applied.

[597] reproduced from [536, Fig. 1, p. S5.2] under Creative Commons license Attribution 2.0 (CC BY 2.0) [@17].
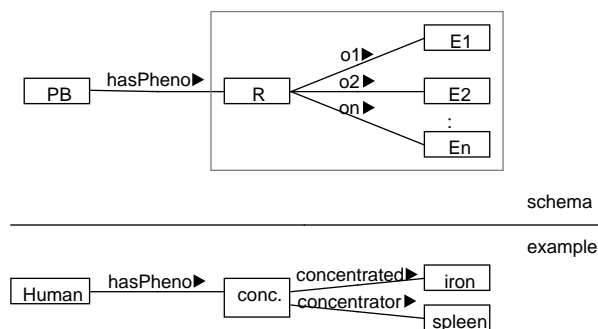
Figure 5.3: (Relational) Roles as properties in OWL. The "variables" in the schematic part stand for a property bearer category PB, a relation R, role categories o_j in its role base, and entity categories E_i.

in the form of Red $\sqcap$ $\exists$inheresIn . Eye faces the problem that plain conjunction cannot be employed in assigning several such representations to an annotated object, e.g., because reds and lengths and weights are all disjoint quality categories. On the other hand, conjunction is a known and welcome operation for the purpose of multiple annotations. The second problem refers to the commutativity and associativity of conjunction, which can produce undesired mixes of phenotype descriptions and ambiguities. Third, the EQ model admits only for binary relations, whereas relations of higher arity are requested for biomedical KR, cf. [784, esp. sect. 5.1], [307], and for EQ-based phenotypes in [609]. Subfigures d) and e) in Fig. 5.2 indicate the problem of inter-modeler consistency, cf. also [307], especially with technically motivated features like towards. I.e., the question is how to find unambiguous ways of linking a relation with its arguments in a phenotype representation. The final fifth item asks for clarifying and formally capturing the aspect that phenotype descriptions are used as annotations.

5.4.2.2 Overview on Representation Patterns and Annotation Aspect

OVERVIEW ON RESULTS: OWL REPRESENTATIONS ON THE BASIS OF A $\mathcal{CR}$-BASED ANALYSIS
Against this background, we present several options of formalizations in OWL that are intended to represent phenotypes in a way that avoids those problems *and* that has a common, underlying ontological background. More precisely, those representations have been developed with corresponding ontological usage schemes in mind, whose underlying ontological semantics is based on (earlier stages of) the $\mathcal{CR}$ conceptualization described in sect. 2.4. Overall, this should contribute to improvements in the consistency/coherence as well as the expressiveness of formal phenotype descriptions. Moreover, [536] discusses and compares these proposals wrt implementation aspects for phenotype annotation in databases. We omit these parts here because they are not immediately relevant wrt seeing applications of $\mathcal{CR}$ and OUS as developed herein and they have primarily been examined by Frank Stumpf. Accordingly, we focus on the OWL representations and their interconnection by means of OUS.

Prior to our own proposals, observe that the EQ model itself (in its native variant as well as its direct OWL variant) is an instance of / comparable to generic encodings that have been considered in order to circumvent the limitation of DLs to binary relations, already touched upon in sect. 2.2.1.[598] Recourse to inherence, however, renders an ontological aspect in EQ models, whereas we argue that the towards relation has a merely technical character.

OWL REPRESENTATION PATTERNS PURELY BASED ON RELATIONS AND ROLES
We propose three formalization patterns [536, p. S5.6–10] on the basis of $\mathcal{CR}$, which are referred to as 'roles as properties' (RP), see Fig. 5.3, 'roles as classes' (RC), Fig. 5.4, and 'relator-based quality' (RQ), Fig. 5.5.[599] The schematic parts of those UML-style diagrams [@126] should be understood as models / patterns

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[598]In the bio-ontology and Semantic Web context, see, e.g., [307, 631], [784, sect. 5.1] and [@66] before [608] and the more recent [260]. Comments on theoretical treatments of reification in description logics, e.g., in [139, sect. 6.5] are mentioned in FN 206.

[599]All three reproduced from [536, Fig. 2–4] under Creative Commons license Attribution 2.0 (CC BY 2.0) [@17].
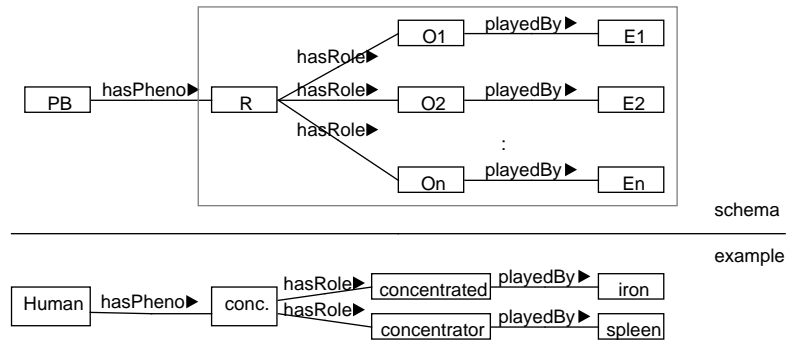
Figure 5.4: (Relational) Roles as classes in OWL. This pattern uses the relations of role-playing and role-having of $\mathcal{CR}$ explicitly.

that are instantiated by the example parts, where the latter are (akin to) UML diagrams. The surrounding boxes mark which pattern fragment eventually corresponds to attributing a relational quality to a single set of arguments. Note that the third pattern RQ actually regards a separate aspect that can be combined with roles as properties (as indicated in the example) or roles as classes.

The general approach underlying all three patterns is based on the understanding of annotations of entries held in biological databases by such phenotype descriptions[600]. While the annotation relation appears basically unrestricted, in the sense that it can be analyzed in various ways, depending on what the database entries are about and on the annotation / annotated content, we assume that at least an organism – or more generically, a phenotype bearer – is involved in that complex relationship, and it is this phenotype bearer which is characterized by phenotype representations. Compatible with conjunctive combinations of phenotypic descriptions, the latter are seen as categories that the phenotype bearer instantiates. Maintaining in addition that phenotype descriptions[601] themselves are basically seen as property categories and drawing inspiration from the notion of *phenes* in [419], an abstract relation labeled hasPheno is introduced to link the "annotated property bearer" with those phenotypic properties.

(5.19) $\exists$hasPheno(Red $\sqcap$ $\exists$inheresIn . Eye) $\sqcap$ $\exists$hasPheno(Short $\sqcap$ $\exists$inheresIn . Tail)

The DL concept in (5.19) is an example of a DL concept that reflects the annotation of a database entry with two (unary) EQ phenotype descriptions, which is meant to apply to an organism / property bearer that can be hypothesized in the complex interrelation of the annotated database entry and the phenotype description. In addition and in $\mathcal{CR}$ terminology, each hasPheno relator between a property bearer (cf. PB in Fig. 5.3–5.5) and a quality / property in the pheno role must be "justified" by a single or a chain of basic ontological relations like inheres-in, part-of, has-function, participates-in, etc. Independently of relations, this approach resolves the problem item 2 and contributes to a coherent ontological view, item 1, incl. on the link between database entries and their annotations (with phenotypes), problem 5.

5.4.2.3 On $\mathcal{CR}$-based OUS for the Representation Patterns

TOWARD EQUIVALENT ONTOLOGICAL IMAGES OF **RP** AND **RC**

Next, let us examine the three proposals in this setup exemplarily and against the background of ontological usage schemes, which is not covered in [536]. Indeed, roles as properties (RP) and roles as classes (RC) are established by aiming at, eventually, ontological images that are equivalent (under a $\mathcal{CR}$-based ontological semantics). In all pattern figures, R stands for the relation / relational quality of a (simple) phenotype description, PB for the phenotype bearer, O_i / o_i for role categories that form a role base for R, and E_j for entity categories (of arguments of R). Now two OUS need to be specified (albeit we refrain from

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[600]Names for complex descriptions may be established by definitions / equivalence axioms between a DL concept name / atomic concept and the phenotype description.

[601]more precisely / in particular, phenotypes described by the EQ approach

implementing this herein), such that the sentences (5.20) and (5.21) as well as the formula (5.22) (free in x) are part of the ontological image of the DL concept (scheme) expressed in the diagrams. Note that $\cdot R$, PB, the $\cdot O_i$ and E_j are parameters in the scheme, but in the ontological image of an *applied/instantiated* scheme, these are substituted by constant names. Accordingly, these symbols should be regarded as (parameters of) constants in the subsequent formulas, s.t. they are equally interpreted across all formulas.

(5.20) $\quad \cdot R :: \cdot \mathsf{Relation} \;\; \wedge \;\; \bigwedge_{1 \leq i \leq n} (\cdot O_i :: \cdot \mathsf{RoleCat} \;\wedge\; \mathsf{hasBaseRole}(\cdot R, \cdot O_i))$

(5.21) $\quad \forall x \,.\, \mathsf{hasBaseRole}(\cdot R, x) \rightarrow \bigvee_{1 \leq i \leq n} x = \cdot O_i$

(5.22) $\quad PB(x) \quad \wedge$

$\qquad \exists r \;.\; \mathsf{hasPheno}(x, r) \;\wedge\; r :: \cdot R \;\wedge$

$\qquad\qquad \exists q_1 \ldots q_n \;.\; \bigwedge_i q_i :: \cdot O_i \;\wedge\; \bigwedge_{j<k} \neg\, q_j = q_k \;\wedge\; \forall x \;(\; r \multimapdotinv x \rightarrow \bigvee_m x = q_m \;) \;\wedge$

$\qquad\qquad\qquad \exists e_1 \ldots e_n \;.\; \bigwedge_l r \multimapdotinv q_l \leftrightsquigarrow e_l \;\wedge\; e_l :: \cdot E_l$

Though we do not fully specify the relevant OUSes, further indication may be given to allow for minimal support of the conjecture that OWL formalizations following RP or RC are consistent under an ontological semantics, mediated by OUS.[602] As (5.20)–(5.22) show, the ontological image of the DL languages is supposed to be an extension of $\mathcal{CR}$. The closer relationship to $\mathcal{CR}$ is given for RC, so let us start from there. We can basically adopt the OUS $(\sigma_0^{\mathrm{DL}}, \tau_0^{\mathrm{DL}})$, i.e., DL concepts are translated into individual-categories, and DL roles into binary relations of individuals[603], except for the roles hasRole and , which are mapped into their $\mathcal{CR}$ counterparts $\multimapdotinv$ and $\leftrightsquigarrow$, resp. This constitutes (an initial sketch of) an applicable OUS for RC.

OUS FOR RP AND BRIEF DISCUSSION

The OUS for RP is overall the same as that for RC, except for two deviations. Firstly, in most cases we can assume that role-playing and role-having are not among the object properties, hence these do / need not occur in the domain of the translation function. Secondly and more importantly, the DL roles o_i, whose names refer to ontological roles, are translated by themselves in the translations of role restrictions and role assertions, and they are assigned the following predication definition, on the assumption that (from the meta-level perspective) $o_i = O_i$, i.e., both symbols denote exactly the same role category.

(5.23) $\quad \forall xy \,.\, o_i(x, y) \leftrightarrow_{df} x :: \cdot \mathsf{Relator} \wedge y :: \cdot \mathsf{Ent} \wedge \exists q \,.\, q :: \cdot O_i \wedge x \multimapdotinv q \leftrightsquigarrow y$

This establishes a first connection of RP to $\mathcal{CR}$, which is in harmony with that of RC. Both should at least account for the transfer of ABox statements, whereas it may be more involved to ensure appropriate closure conditions on relators under both representation schemes. Hence, we expect that additional constraints must be added in order to eventually account for equivalent ontological images. However, this is left for future work, concluding the connection to OUS as another(, here partial) proof of concept. For the purposes pursued in [536], OUS are not even explicitly mentioned and the integration of OWL theories under both schemes is not discussed. Instead, the approaches are offered conceptually, arguing that the resulting OWL formalizations (which correlate to / have the ontological image (5.22)) cure the remaining problems raised in sect. 5.4.2.1. In particular, no limitations apply wrt the arity of relations, and fully role-based relations as in the RC model offer some distinctions that we do not see in an extensional, tuple-based approach to relations, cf. 2.4.3. We see the fourth problem item of inter-modeler consistency also improved by the use of dedicated rolenames. This is not a new observation, e.g., cf. [784, Fig. 6, p. 589] or [@66], where "rolenames"[604] are also used, in great similarity with RP. We claim an increase ontological grounding due to the considerations above even for RP, though.

NEW ONTOLOGICAL ANALYSIS LEADING TO (A.O.) RELATOR-BASED QUALITY PATTERN

Obviously, the third scheme is not yet considered above. Again, an OUS similar to the basic $(\sigma_0^{\mathrm{DL}}, \tau_0^{\mathrm{DL}})$ forms the starting point for its grounding in ontological semantics. Yet the more interesting aspect, as we

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[602]Ceteris paribus, i.e. (here), if the sentences corresponding to RP or RC instances are ignored, the remaining theories should be logically equivalent.

[603]A relaxation of the $\mathsf{RoleBase}_0^2$ requirements may be appropriate in some case, because these enforce disjoint role categories in the role base.

[604]at least, variable, relation-specific names
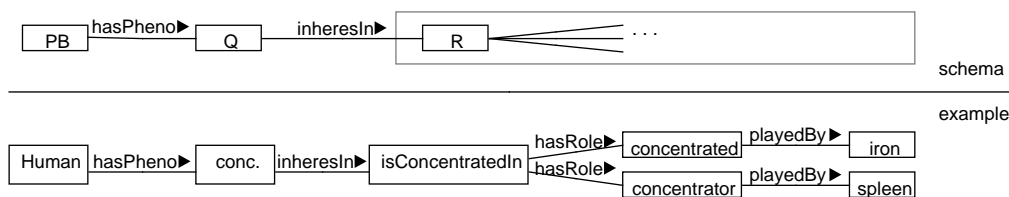
Figure 5.5: Relator-based quality allows for attributing properties to relators.

find, of the 'relator-based quality' pattern in Fig. 5.5 is a novel ontological analysis that can be applied to relational qualities, e.g. 'concentration of iron in the spleen', although we mainly just refer to the detailed exposition in [536, p. S5.9–10]. Just capturing a minimal core, that analysis variant arose from dealing with the approaches above, reaching a distinction between a mere propositional circumstance of, e.g., "(a particular amount of) iron I is concentrated in a (particular) spleen S", and a measurable (or computable) quality, as in phrases "increased concentration" or the direct specification of value as in "the concentration of X in Y is 0.5 g/l". Examining the question of which entity such a quality inheres in led to several variants (leaving aside those where it is attributed to either X or Y), one of which is the attribution to a relator of the former, propositional kind. Note the effect that 'relational quality' becomes subject to ontological reinterpretation, is a "standard" property that inheres in a single entity (just that this is a relator), and therefore can benefit from, e.g., any general account of measurements that is formulated for "unary properties".

BRIEF OUTLOOK

Concluding this case of application, we see many lines of potential further work spawn from this starting point. For example, completing the ontological usage schemes should not facilitate the integration of phenotype descriptions across distinct theories (which may use only one pattern), as well as it can clarify the combined use of those patterns in the same theory. At least in the case of RP and RC, no conflicts should arise if the same ontological image is accomplished and formal semantic integration between DL and the $\mathcal{CR}$-based FOL theory into which it is interpreted is maintained. Another branch of continuation is to deal with modified qualities like 'increased concentration' and / or comparative conditions / relations like 'X is heavier than Y'.

5.4.3 Further Applications in Interaction with $\mathcal{CR}$ and OUS

CONCLUDING NOTES

Possibly not justifying a separate section based on its length, but likewise hard to accommodate in the previous one, the overall part on applications of ontological semantics and / through ontological usage schemes as well as of $\mathcal{CR}$, the ontology of categories and relations conceptualized in sect. 2.4 is finished by a few notes and pointers on further applications in or underlying prior publications. As the overall section is concerned with self-citations, except for this comment the self-citation markers are omitted until the next chapter. Moreover, our contributions to all of the related projects have been pursued in parallel to (primarily early) phases on the work on ontological semantics and the continued development of theories of categories, relations, and roles, a.o. We count / view these as "applications" of the current theories because they lend themselves as early "testbeds" and can be understood by the theory of today, even if most publications do not refer to, e.g., ontological semantics.

ROLES IN BOWIKI DEVELOPMENT AND HL7 ANALYSIS

We start with $\mathcal{CR}$ and the ('Bio-Ontology Wiki') BOWiki [37, 420], a semantic wiki actively developed and supported[605] between 2005 and 2008 with the original intention of gene function annotation, soon extended to cover ontology curation in general, with a remaining focus on bio-ontologies. While we were not involved in the implementation of the system, the concepts of relations with instances that are composed

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[605]neither by ourselves, except for occasional conceptual hints on relations and roles

of roles inspired the first data model of the BOWiki [420, see Fig. 1]. Despite later modification of the data model for reasons of performance, that approach to relations remains conceptually present [37, see Fig. 2].

Moreover, in a brief study we have employed the broader theory of roles, which is pursued in continuation of [526] and of which [530] is our latest publication, to an ontological analysis of some main concepts of the HL7 [@55] Reference Information Model (RIM) [407]. The major outcome was the presence of all top-level role kinds of the role theory, i.e., relational, processual, and social roles, among the classes of HL7 RIM, which can be regarded as mutual support between the utility of notions in the role theory and a general, covering information model; cf. [530, sect. 4.1], [528].

OUS BEHIND METADATA AND FORMAL CONCEPT ANALYSIS

We participated in the conceptualization of a metadata platform [375, 376] for the German D-Grid initiative [@27] that supports the evolution of semantic grid metadata. In this context our contribution mainly refers to the three-layered architecture for metadata representation [376, sect. 2], which we helped to shape. This comes with two types of categories not only with different effects in the system, but – with the terminology herein – which also require different ontological usage schemes for an integrated understanding of the system's content.

[399] is our early publication on meta-ontological architecture, parts of which are covered in sect. 2.3 already. Further methodological contributions were made to [622], which comprises extended considerations in a four-layered approach that includes an implementation level and has been inspired from and applied in modeling surgical interventions. The applications section of [399] relates even more closely to OUS, by briefly presenting elements of first application cases of what herein are ontological usage schemes, by considering uses of Formal Concept Analysis (FCA) [261].

Chapter 6

Contributions to Ontologies

START ON $\mathcal{CR}$ AND SAMPLE METHODOLOGY CASE OF TIME

We have been conducting ontological analyses and developing ontologies in the course of as well as in parallel to writing this thesis. This includes work of more conceptual nature as well as formalizations of such conceptualizations, and their implementation for automated reasoning. This chapter, in its first item, delivers the state of our work on formalizing an ontology of categories and relations. Our conceptual analysis to date of these matters, largely reported in sect. 2.4, led us to the position that any ontology for $\mathcal{CR}$ that we propose remains preliminary and subject to substantial further research and extension. While we cannot pursue this immediately, the next section introduces formal fragments of $\mathcal{CR}$ and discusses the relation with ontological semantics. In addition to $\mathcal{CR}$, we move to the subject of time and include two time ontologies in a self-contained sect. 6.3, recapitulating [62], that were developed together with Ringo Baumann and Heinrich Herre. This section covers a sample case of the basic methodology for ontology development as understood in the overall context of the General Formal Ontology (GFO) project [@46], cf. sect. 1.1.5.3, which is further in line with the previous results herein. The latter aspect is explained in sect. 6.2, which additionally indicates other contributions besides $\mathcal{CR}$ and time for GFO.

6.1 Formalizations of Categories and Relations – $\mathcal{CR}$

TWO MAIN ASPECTS: FUNDAMENTAL RELATIONS AND TAXONOMIC STRUCTURE

The ontology of categories and relations pervades the chapters of this thesis at least since sect. 2.4. That section introduces our (general-level) conceptualization $\mathcal{CR}$ of that specific domain of reality / knowledge, against a brief background from other fields. Aiming at the formalization of that conceptualization, oriented on the content of that sect, observe first that, on the one hand, it deals with the three fundamental relations of instantiation (::), role-playing ($\rightsquigarrow$), and role-having ($\multimap$), which are employed as referents of fundamental predicates in the predication systems $\mathfrak{P}_0^{\mathcal{CR}}$ and $\mathfrak{P}_1^{\mathcal{CR}}$. On the other hand, sect. 2.4 introduces mainly a number of categories with subcategories, in different subgroups, cf. Fig. 2.4–2.9. This raises the question of whether those different parts form a consistent integrated theory, even as a taxonomy, i.e., investigating the is-a structure among the categories discussed as $\mathcal{CR}$ constituents in sect. 2.4.2–2.4.4.

PROCEEDING FROM TAXONOMY ANALYSIS TO FOL FRAGMENT

Therefore, we adopt the following formalization approach. Description Logics (DLs), introduced in sect. 1.1.3.2, lend themselves to taxonomic analysis and can clearly be beneficially applied, despite their restrictions in expressiveness compared to FOL. Thus we first formalize the $\mathcal{CR}$ taxonomy in DL, which is performed by an implementation in the Web Ontology Language (OWL) [@91], see sect. 1.1.3.1 for the latter. This DL theory and its implementation are described in sect. 6.1.1.

The subsequent sect. 6.1.2 tackles the first issue above and considers axioms that should and that cannot (reasonably) be imposed on the three relations ::, $\rightsquigarrow$, and $\multimap$. This remains a proposed fragment for $\mathcal{CR}$ in

FOL. But even without or with very weak axioms, ontological semantics is defined as a semantics – then with less embedded ontological assumptions.

6.1.1 Taxonomy in OWL

The taxonomy $\mathcal{CR}_0^{\mathrm{DL}}$ comprises the set of DL/OWL axioms that are specified in detail in the Appendix ch. B. Let us note at the beginning that description logic is employed with its standard translation[606] into FOL in mind, cf. e.g. [33, sect. 4.2], i.e., all axioms can equally be considered as valid / true in the domain after that standard translation into FOL. From the ontological semantics point of view, all DL concepts / OWL classes are categories, without being able to name a more specific category in terms of $\mathcal{CR}$ that applies to all of them.[607] Accordingly, the assumed ontological usage scheme $(\sigma_{\mathcal{CR}}^{\mathrm{DL}}, \tau_{\mathcal{CR}}^{\mathrm{DL}})$ for $\mathcal{CR}_0^{\mathrm{DL}}$ adopts the standard translation into FOL as $\tau_{\mathcal{CR}}^{\mathrm{DL}}$, which is similar to $(\sigma_0^{\mathrm{DL}}, \tau_0^{\mathrm{DL}})$ in Table 5.1 on p. 194 which involves an auxiliary function $\hat{\tau}_x^V$, but which for $(\sigma_{\mathcal{CR}}^{\mathrm{DL}}, \tau_{\mathcal{CR}}^{\mathrm{DL}})$ translates concept names C via (6.1), and – appropriately without domain relativation – role restrictions $\exists R.C$ via (6.2). The translation of assertions corresponds to that in (6.3) and (6.4).

(6.1) $\qquad \hat{\tau}_x^V(C) := C(x)$

(6.2) $\qquad \hat{\tau}_x^V(\exists R.C) := \exists y \,.\, R(x,y) \wedge \hat{\tau}_x^{V \cup \{y\}}(C)$

(6.3) $\qquad \tau_{\mathcal{CR}}^{\mathrm{DL}}(C(a)) := C(a)$

(6.4) $\qquad \tau_{\mathcal{CR}}^{\mathrm{DL}}(R(a)) := R(a,b)$

Predication definitions are established via $\sigma_{\mathcal{CR}}^{\mathrm{DL}}$, which in addition captures the classification of everything represented by a DL concept name as a category, and analogously of referents of DL role names as binary (ontological) relations. The role bases for each $\mathcal{CR}$ relation are hidden / implicitly assumed in the present exposition, cf. the definition of rel^n in (5.11) on p. 193 (as the scheme from which rel^2 derives), which explains the use of $:\supseteq$ in (6.6). Logical individual constants for all role categories in the resp. role bases must be considered in lieu of the q_1^R and q_2^R, used here for a schematic description in analogy to Def. 5.10, p. 193. The three fundamental relations are excluded in (6.6) in their "forward" ($::$, $\leadsto$, $\multimap$) and inverted notational variants ($\underline{::}$, $\leftsquigarrow$, $\circ\!-$),[608] because the scheme does not lead to predication *definitions*, but instead to (what we call) reflection axioms. Accepting the latter is debatable and briefly discussed in sect. 6.1.2. At least, wrt the latter cases, the role bases are displayed explicitly in Table 2.1,[609] p. 95 in sect. 2.4.3.3, cf. also the part of the lower left-hand side of Fig. 6.1 on p. 210 for the respective role categories.

(6.5) $\qquad \sigma_{\mathcal{CR}}^{\mathrm{DL}}(C) := \{\forall x \,.\, C(x) \leftrightarrow x :: \cdot C \,,\ \cdot C :: \cdot \mathsf{Cat}\}$

(6.6) $\qquad \sigma_{\mathcal{CR}}^{\mathrm{DL}}(R) :\supseteq \{\forall xy \,.\, R(x,y) \leftrightarrow \mathsf{rel}^2(\cdot R, q_1^R, x, q_2^R, y) \,,\ \cdot R :: \cdot \mathsf{Relation}\}$ for $R \notin \{::, \underline{::}, \leadsto, \leftsquigarrow, \multimap, \circ\!-\}$

The axioms of $\mathcal{CR}_0^{\mathrm{DL}}$ have been developed on the basis of sect. 2.4, including all categories and relations in Fig. 2.4–2.9 and Table 2.1. These asserted axioms capture our presuppositions on those categories and relations, which may partially be derived from other axioms postulated for the FO axiomatization. Regarding types of axioms, these are primarily (extensional) subcategory / is-a relations among the categories (including some General Concept Inclusions (GCIs) [33, sect. 2.2.2.1][610] in order to capture sufficient conditions), disjointness axioms, sporadic cardinality restrictions, and some relational properties.

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[606]preserving formal semantics

[607]E.g., IndividualCategory is itself an individual-category, hence, not all $\mathcal{CR}$ categories fall under CategoryCategory, for example.

[608]The exlusion set $\{::, \underline{::}, \leadsto, \leftsquigarrow, \multimap, \circ\!-\}$ uses the FOL notation of the fundamental relations at all for brevity, compared to the names employed in OWL. The latter should be self-explanatory wrt the relations. They are listed shortly below.

[609]More precisely, in the second column of that table.

[610][33, sect. 2.2.2.1] does not contain the term 'general concept inclusion', but describes them, starting with "In the most general case, terminological axioms have the form $C \sqsubseteq D$ [. . .] where C, D are concepts [. . .]. Axioms of the first kind are called *inclusions*, [. . .]" [33, p. 51].
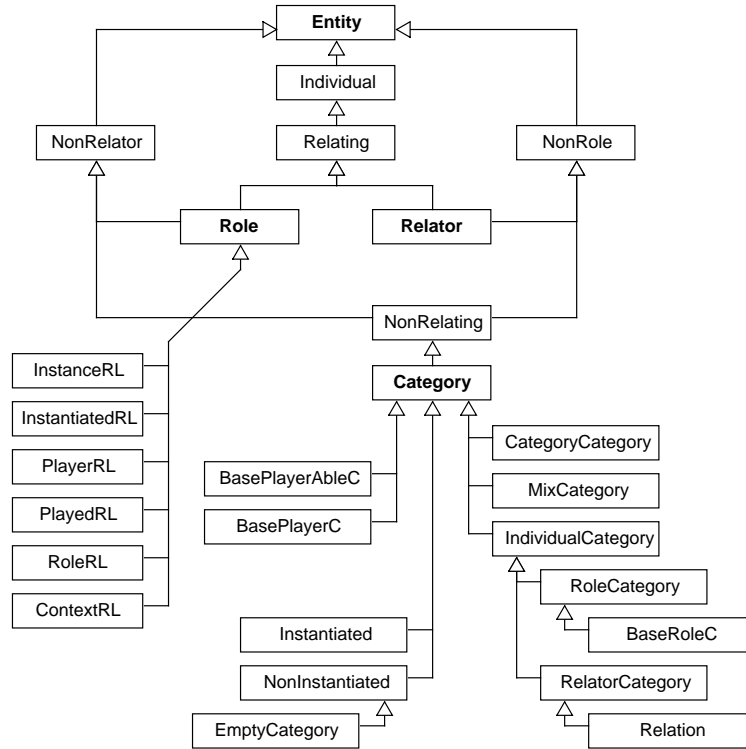
Figure 6.1: Classified taxonomy of $\mathcal{CR}$. Bold-face categories are representatives of groups of equivalent named categories.

IMPLEMENTATION, CLASSIFICATION RESULT, AND RELATIONS

We used the Protégé [@97] OWL Editor in version 4.3.0, together with the Fact++ [@35] [824] (reasoner) plugin in version 1.6.2 and the HermiT [@53] [604] reasoner in version 1.3.8 to implement and classify the file `cr-dl.owl`, resp., which is currently[611] available at [@21].

6.1 Observation (consistency and coherence of $\mathcal{CR}_0^{\mathsf{DL}}$)
According to HermiT (v1.3.8) and Fact++ (v1.6.2), $\mathcal{CR}_0^{\mathsf{DL}}$ as implemented in the OWL file `cr-dl.owl` [@21] is consistent and coherent, i.e., no named class is necessarily empty.   ❏

Both reasoners classify the ontology file and detect no inconsistency and no incoherence[612],[613] i.e., all named classes are satisfiable concepts, in DL terminology. The resulting category hierarchy is displayed in Fig. 6.1[614] and the following DL roles are utilized for relations.

- for fundamental relations and the (FOL-)defined extensional is-a relation:
  eIsA, hasRole, instanceOf, instantiatedBy, playedBy, plays, roleOf
- in connection with role bases, cf. sect. 2.4.3.3:
  baseEIsA, basePlayedBy, basePlayerAble, hasBaseRole

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[611]There is no (long-term) permanent link to the source file. If no longer available from [@21], it can be shared on request.

[612]The use of 'incoherence' follows the terminology in [729, see the definitions in sect. 2.1.1, p. 321].

[613]Notably, when using Fact++ the displayed inferred class hierarchy shows unintended effects, in particular, the two classes BaseRoleCategory and Relation are presented as direct subclasses of Entity, although by declaration they are subclasses of RoleCategory and RelatorCategory, resp.

[614]In contrast to Fact++ (cf. the previous FN), HermiT classifies the hierarchy in full correspondence with the expectations based on our conceptualization of the matter. Its result is adopted for Fig. 6.1.

All DL roles are equipped with domain and range constraints, and the pairs of inverse relations are captured in accordance with Def. 4.18 on p. 149. We could capture several further restrictions on DL roles that correspond to the conceptualization. However, the goal to use OWL 2 reasoners for classification, which is subject to limitations for decidability, forced us to avoid a few statements, such as declaring disjointness axioms for extensional is-a (eIsA). While several such axioms follow from other constraints, e.g., disjointness of eIsA with plays and role-of from the respective domain and range constraints,[615] we can only admit that this weakens / limits the reasoner-approved / -approvable meta-theoretic property of consistency. Still, the classification leads to interesting results discussed in the next paragraph. Other formulas are not accepted as axioms for the domain (in the $\mathcal{CR}$ theory), e.g., eIsA cannot be declared to be reflexive, because it is only reflexive on categories,[616] but we also consider individuals in the domain and the OWL reflexivity property applies in the overall domain of discourse. On the aspect of specialization among OWL properties, there the only cases are that eIsA extensionally subsumes baseEIsA and basePlayerAble. Importantly, specialization as well as disjointness are actually assessed wrt the tuple extensions of the relations, cf. Def. 2.7 on p. 93, not the relator extensions, i.e., sets of all relators / instances of a relation.

EQUIVALENCES AND DISTINGUISHING EXTENSION AND INTENSION

Turning to category subsumption and Fig. 6.1, it shows only 27 instead of all 38 categories in sect. B.1 due to inferred equivalences. We selected the left-most category in each of the subsequent equivalence chains as a representative for any item in such a group of equivalents, which appears in bold-face in Fig. 6.1.

(6.7)   Entity $\equiv$ Categorizable $\equiv$ Instance $\equiv$ PlayerAble $\equiv$ Player

(6.8)   Category $\equiv$ Instantiable

(6.9)   Relator $\equiv$ Context $\equiv$ RoleHaveAble

(6.10) Role $\equiv$ Playable $\equiv$ Played $\equiv$ RoleRole $\equiv$ RoleBeAble

Those many equivalences may seem awkward compared to "usual" ontologies, for which we expect almost no two *named* classes to be equivalent. However, first, there are reasons for accepting these equivalences. Second, they are nice illustrations of intensionally distinct categories that apply to the same entities / instances[617]. Take the case of Entity. In $\mathcal{CR}$ we claim that everything is an entity, more precisely, everything instantiates the category entity. Hence, everything is an (instance of) Instance, because this category is declared equivalent to $\exists$instanceOf Entity. Similarly, Player is defined by $\exists$plays Role. Then either (1) the assumption that every entity is related to at least one other entity by a non-fundamental relator or (2) accepting the reflection axioms in sect. 6.1.2 below, which yields role-playing immediatedly due to the instantiation relator to entity, one can conclude that everything is a player. Voilà, both being an instance and being a player require being categorizable and being capable of being a player, resp., cf. the characterizing GCIs[618] of Categorizable and PlayerAble. Similarly for Role, where one may even consider its four equivalent categories as the characterizing categorial parts[619] of being a role.

EMPTY AND NON-INSTANTIATED CATEGORIES IN RELATION TO OTHER KINDS

In recourse to our treatment of *empty categories* in sect. 2.4.4.2, a final aspect on empty and non-instantiated categories is to be mentioned. As a brief reminder of the analysis of sect. 2.4.4.2, categories are non-instantiated :iff they do not have any instance (in an interpretation or as a consequence of a theory). Empty categories like the round square are held to be instantiable / subject to the instantiated role (category, as base role category) of the relation of instantiation in $\mathcal{CR}$, but cannot possibly be instantiated in any model due to inconsistent conditions that their instances had to satisfy, were they existing. In $\mathcal{CR}_0^{\mathrm{DL}}$, empty and

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[615]More precisely, plays and role-of have at least one role / argument position that is constrained to (ontological) individuals, whereas eIsA is a relation on categories. Declaring hasBaseRole disjoint with eIsA would furnish a real strengthening of the axiomatization (although only due to possibly empty relator categories and role categories, see below).

[616]Cf. its definition in 2.8 as well as more general considerations around (5.7) on p. 192.

[617]Some care may be required with the phrase "categories with the same extension", depending on whether the categories of $\mathcal{CR}$ are considered to possibly be members of sets or not.

[618]The respective formulas express sufficient conditions, in standard logical terminology.

[619]Whereas only a conjecture at this stage, we believe, in a sense of 'categorial part' compatible to that looked for in [399] by the relation catp.

non-instantiated categories are not declared to be disjoint with any of individual-category or mix or category category, in order to leave the possibility of specifying different types of empty and non-instantiated categories, such as the empty individual-category / empty category of individuals[620]. This decision is mirrored in some of the GCIs, see e.g. the last axioms of CategoryCategory and IndividualCategory in sect. B.1, instead of establishing equivalences for those categories.[621] Another effect is in the same spirit, noting that all non-instantiated categories *extensionally* subsume each other, i.e., they are related by eIsA. Accordingly, although relator and role are disjoint individual-categories, their "induced" category categories of relator category and role category may have non-instantiated categories as instances, and if so, these are in mutual eIsA relation, i.e., extensionally equal. If two such non-instantiated categories are related by hasBaseRole, the latter itself cannot be disjoint from eIsA. Hence, and independently of the OWL restriction for disjointness axioms of eIsA, these two relations are not claimed to be disjoint.

6.1.2 Towards Axiomatization in FOL

$\mathcal{CR}_0^{\mathbf{FOL}}$ AND TWO OBVIOUS WAYS OF EXTENDING AXIOMATIZATIONS

The previous section presents the (primarily) taxonomic theory $\mathcal{CR}_0^{\mathrm{DL}}$ of $\mathcal{CR}$. The result of applying the OUS for $\mathcal{CR}_0^{\mathrm{DL}}$ described in sect. 6.1.1, which employs the DL translation into FOL following the usual approach based on formal semantics, cf. e.g. [33, sect. 4.2], can straightforwardly be accepted as an initial FOL formalization $\mathcal{CR}_0^{\mathrm{FOL}}$. Moreover, it should be taken as an indirect approach of establishing the signature $\Sigma_0^{\mathcal{CR}} := Sig(\mathcal{CR}_0^{\mathrm{FOL}})$. Due to the limitations of DLs / OWL, there is a natural interest in finding extensions that can be expressed in FOL. We see two kinds of such extensions. (1) One can consider definitions of new predicates, potentially in combination with new logical individual constants in order to provide predication definitions for those predicates. On that basis, further axioms may be formulated. We have come across some such cases[622] already in the foregoing ch. 5, cf. e.g. "auxiliary" predicates such as $\mathrm{RoleBase}_0^n$ in (5.10) and rel_0^n in (5.11) on p. 193. Moreover, due to the OUS for $\mathcal{CR}_0^{\mathrm{DL}}$, most axioms of $\mathcal{CR}_0^{\mathrm{FOL}}$ arise in an analogous way, starting from the fundamental signature $\Sigma_{\mathcal{CR}^{core}} := \{::, \rightsquigarrow, \multimap\}$ for $\mathcal{CR}$, i.e. / the one for the predication systems considered in sect. 4.4.3.2. Accordingly and relying on the Replacement Theorem [693, Theorem 4.1, p. 74] applied to logical equivalence, all axioms of $\mathcal{CR}_0^{\mathrm{FOL}}$ can be viewed as formulas whose only predicates are those in $\Sigma_{\mathcal{CR}^{core}}$ (but, notably, which rest on the logical individual constants in addition).

This nicely leads to (2), the other and even more immediate way of extending axiomatizations, which proceeds within a fixed signature and thus language. Ideally, additional axioms can be found on a way moving towards completeness of the theory under development. Albeit, in the case of $\mathcal{CR}$, we expect that no complete axiomatization can be achieved.

DEALING WITH $\Sigma_{\mathcal{CR}^{core}}^+$ IN THIS SECTION

A genuine FOL axiomatization of $\mathcal{CR}$ remains in a very early phase at the present stage. We defend the position that this does not cause harm to ontological semantics in ch. 4 or to the considerations in ch. 5, because there the fundamental theory is basically only treated as a parameter or, where $\mathcal{CR}$ is used in examples, it does not rely on any strong assumptions on those fundamental predicates. Moreover, sect. 6.1.1 adds some confidence that the categories referred to and several of their weaker presuppositions like on disjointness and functionality mate consistently.

As a first step towards a genuine FOL axiomatization, we actually focus on the fundamental signature $\Sigma_{\mathcal{CR}^{core}}$ and consider formulas for their acceptability as axioms, largely in separation between the three relations, except for considering them to be disjoint. However, we allow for some of the unary predicates in $\Sigma_0^{\mathcal{CR}}$ and adopt some axioms of $\mathcal{CR}_0^{\mathrm{FOL}}$, which turn out to be beneficial for shorter arguments and notation in

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[620]which we would accept as a category that (intensionally (despite this attribute requires elaboration)) subsumes round squares, if square is subsumed by individual

[621]Of course, subcategories of those can be introduced, e.g., instantiated individual-category, and would be disjoint with non-instantiated categories "by definition".

[622]though without explicating their predication definitions

Binary Predicates		Unary Predicates		Logical Individual Constants	
$x :: y$	x instantiates y	$\mathsf{Cat}(x)$	x is a category	$\cdot\mathsf{Cat}$	category
$x \rightsquigarrow y$	x plays y	$\mathsf{Ent}(x)$	x is an entity	$\cdot\mathsf{Ent}$	entity
$x \multimap y$	x is a role of y	$\mathsf{Ind}(x)$	x is an individual	$\cdot\mathsf{Ind}$	individual
$x ::\!\!\cdot\, y$	x has instance y	$\mathsf{Relation}(x)$	x is a relation	$\cdot\mathsf{Relation}$	relation
$x \leftarrowtail y$	x is played by y	$\mathsf{Relator}(x)$	x is a relator	$\cdot\mathsf{Relator}$	relator
$x \circ\!\!- y$	x has role y	$\mathsf{Role}(x)$	x is a role	$\cdot\mathsf{Role}$	role

Table 6.1: Signature $\Sigma^+_{\mathcal{CR}^{core}}$ for considering $\mathcal{CR}$ in FOL.

the sequel. The verification of consistent integration with all of $\Sigma^{\mathcal{CR}}_0$ remains a task for the future,[623] after the set of axioms appears to have settled in a(n intuitively) more complete state. Of course, all formulas that are identified to be acceptable as axioms below are intended to become integrated with $\Sigma^{\mathcal{CR}}_0$ as a whole.

SIGNATURE AND SELECTED BASIC AXIOMS

The presentation / discussion roughly follows the organization of sect. 2.4. Beforehand we setup a group of axioms from $\mathcal{CR}^{\mathrm{FOL}}_0$ which are of major relevance for further considerations. Nevertheless, we assume basically all of $\mathcal{CR}^{\mathrm{FOL}}_0$ and may refer to one or another of the remaining notions therein, e.g., for justifying claims of existence. The signature of the selected group of axioms is contained in Table 6.1.

Note that from here on $x \neq y$ is used for any terms x, y to abbreviate $\neg\, x = y$, also in object-level syntax. Any variables occurring free in an axiom, consequence, or definition are implicitly universally quantified. Occasionally a single formula identifier is associated with a set of axioms for brevity. All of those axioms are assumed / postulated in that case.

Predication Definitions

D1. $\left\{ P(x) \leftrightarrow_{df} x :: \cdot P \mid P \in Pred(\Sigma^+_{\mathcal{CR}^{core}}),\, ar(P) = 1 \right\}$

(predication definitions for all unary predicates)

D2. $x ::\!\!\cdot\, y \leftrightarrow_{df} y :: x$ ($::\!\!\cdot$ is inverse notation of $::$, cf. Def. 4.18)

D3. $x \leftarrowtail y \leftrightarrow_{df} y \rightsquigarrow x$ ($\leftarrowtail$ is inverse notation of $\rightsquigarrow$, cf. Def. 4.18)

D4. $x \circ\!\!- y \leftrightarrow_{df} y \multimap x$ ($\circ\!\!-$ is inverse notation of $\multimap$, cf. Def. 4.18)

(Selected) Analytic Definition

D5. $\mathsf{Ent}(x) \leftrightarrow \exists y\,.\, y = x$ (x is an entity :iff x exists)

Categorization and Inequality Axioms

A1. $\left\{ \mathsf{Cat}(\cdot P) \mid \cdot P \in Const(\Sigma^+_{\mathcal{CR}^{core}}) \right\}$ (classification of all constants as categories)

A2. $\left\{ \cdot P \neq \cdot Q \mid \cdot P, \cdot Q \in Const(\Sigma^+_{\mathcal{CR}^{core}}) \right\}$ (any two constants refer to distinct entities)

(Selected) Axioms

A3. $\exists x\,.\, \mathsf{Ind}(x)$ (there is an individual)

A4. $\neg\exists x\,.\, \mathsf{Cat}(x) \wedge \mathsf{Ind}(x)$ (category and individual are disjoint categories)

A5. $\exists y\,.\, y :: x \rightarrow \mathsf{Cat}(x)$ (if an entity is instantiated, it is a category)

A6. $\neg\exists x\,.\, \mathsf{Role}(x) \wedge \mathsf{Relator}(x)$ (role and relator are disjoint categories)

A7. $x :: y \rightarrow \mathsf{Ent}(x) \wedge \mathsf{Cat}(y)$ (instantiation relates entities with categories)

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[623]Though only partially addressing (mainly) the axiomatization step of the axiomatic method, see sect. 1.1.5.3, we argue that a set of proposed axioms with their discussion can be of interest in terms of the domain knowledge / assumptions captured therein.

A8. $x \rightsquigarrow y \rightarrow \mathsf{Ent}(x) \wedge \mathsf{Role}(y)$ (plays relates entities with roles)

A9. $x \multimap y \rightarrow \mathsf{Role}(x) \wedge \mathsf{Relator}(y)$ (role-of relates roles with relators)

A10. $x \multimap y \wedge x \multimap z \rightarrow y = z$ (role-of is functional)

ON THE THEORY OF INSTANTIATION

Now we are prepared to consider sentences, mainly in the signature only involving $::,$[624] for their status as axioms for a theory of instantiation. Alternatively, they may already follow from $\mathcal{CR}_0^{\mathrm{FOL}}$. Inspired by the logical properties available for DL roles, we first consider the relationship to formulas as known for ordering relations. Indeed, $\mathcal{CR}_0^{\mathrm{FOL}}$ determines already a number of such sentences, though unfortunately, usually confirming their negation, as this list of consequences of $\mathcal{CR}_0^{\mathrm{FOL}}$ shows.

Consequences for instantiation wrt general properties related to orders

C1. $\exists x \, . \, x :: x$ (not irreflexive, e.g., $x = \cdot\mathsf{Ent}$)

C2. $\exists x \, . \, \neg x :: x$ (not reflexive, e.g., $x = \cdot\mathsf{Ind}$)

C3. $\exists xy \, . \, x :: y \wedge \neg y :: x$ (not symmetric, e.g., $x = \cdot\mathsf{Ind}, y = \cdot\mathsf{Cat}$)

C4. $\exists xy \, . \, x :: y \wedge y :: x$ (not asymmetric, e.g., $x = y = \cdot\mathsf{Ent}$)

C5. $\exists xy \, . \, x \neq y \wedge x :: y \wedge y :: x$ (not antisymmetric, e.g., $x = \cdot\mathsf{Ent}, y = \cdot\mathsf{Cat}$)

C6. $\exists xyz \, . \, x :: y \wedge y :: z \wedge \neg x :: z$ (intransitive, e.g., $x = \cdot\mathsf{Ent}, y = \cdot\mathsf{Cat}, z = \cdot\mathsf{CategoryCategory}$)

C7. $\exists xyz \, . \, x \neq y \wedge x \neq z \wedge y \neq z \wedge x :: y \wedge y :: z \wedge \neg x :: z$
(strengthening of C6, witnessed by its example already)

C8. $\exists xyz \, . \, x \neq y \wedge x \neq z \wedge y \neq z \wedge x :: y \wedge y :: z \wedge x :: z$
("transitive" counterpart of C7, e.g., $x = \cdot\mathsf{Ind}, y = \cdot\mathsf{Cat}, z = \cdot\mathsf{Ent}$)

C9. $\exists x \forall y \, . \, y :: x$ (there is a "maximum" entity, e.g., $x = \cdot\mathsf{Ent}$)

C10. $\forall x \exists y \, . \, x :: y$ (everything instantiates something, direct consequence of C9)

C11. $\exists x \forall y \, . \, \neg y :: x$ (there is a non-instantiated entity, due to A3–A5)

C12. $\forall x \exists y \, . \, \neg x :: y$ (there is no "minimum" entity, direct consequence of C11)

C13. $\forall x \exists y \, . \, x \neq y \wedge x :: y$ (there are no "maximal" entities,[625] e.g., $\cdot\mathsf{Ent}$ and $\cdot\mathsf{Player}$ are "maxima")

C14. $\exists x \forall y \, . \, y :: x \rightarrow y = x$ (there are "minimal" entities, an immediate consequence of C11)

C15. $\exists xy \, . \, x \neq y \wedge \neg x :: y \wedge \neg y :: x$ (not connex, e.g., $x = \cdot\mathsf{Role}, y = \cdot\mathsf{Relator}$)

Admittedly, (most of) these consequences do not pose strong restrictions, in particular, as most of them are already witnessed plainly by constants in the theory itself (and assumptions on them). The sentences may just show that instantiation is no partial ordering. Further properties such as constraining chains in length, constraining instantiation cycles, or branching are by no means obvious to us.

HESITATIONS REGARDING SET-THEORETIC ANALOGIES

Moreover, beyond orderings it appears likewise difficult to identify appropriate constraints at the general level, e.g., of the form $\forall xy \, . \, x :: y \rightarrow \ldots$ (or possibly with multiple instantiation antecedents and some quantifier nesting). Taking recourse to set theory and, in particular, ZFC, seems to require more training in this field. Considering the axioms of ZFC in App. sect. A.2.1, we are skeptical on most axioms as to whether they should be "rewritten" for a general theory of instantiation that is meant to be applicable to

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[624]equality is available as usual herein

[625]Despite using the terminology of orders here, $::$ is no partial order, as the first consequences show. Accordingly, all terms in quotation marks cannot be assumed to have the same implications as for partial orders. We adopt them based on their usual defining condition, which the axiom reflects (possibly in negated form) where they are used. This explains why there is a "maximum" entity (actually, several, in the light of 6.7 above), which all entities instantiate, but there are no "maximal" entities, which would be entities instantiating at most themselves. Without partial order axioms, a "maximum" is no longer a special case of being "maximal", and no longer uniquely determined.

the various subtypes that we have considered in sect. 2.4.2.1. Regarding specific axioms, Existence (Ex), rewritten to the existence of a category can be "trivially" adopted. Extensionality (Ext) must clearly be rejected, as this is one of our motivating factors in this thesis. The remaining axioms have an existential import the adequacy of which for categories in general is hard to assess for us. We hesitate to adopt axioms such as the Axiom of Infinity (Inf) or the Axiom of Union ($\bigcup$-Ax), and had discussed the same caution wrt comprehension axioms in sect. 2.4.2.3 already. A next step would be to consider the powerset axiom (Pow) and effects of adding it to $\mathcal{CR}_0^{\text{FOL}}$ (without further axiom equivalents from set theory). More extensively, other set theories should be studied and possibly related, e.g., set theories with a universal set, cf. sect. 2.4.2.3.

Of course, another source for axioms may be found in existing work, and in ontologies in particular. We have inspected the KIF [273] axiomatization of DOLCE [@31] presented in [558, p. 129–164] as well as the SUMO [@119] [626, 662] source file [@120], focusing on its sections "structural ontology" and "base ontology".[626] As another rather closely related approach, we have further examined axiomatic triples of RDF(S) in [384, sect. 8–9]. All of these formalizations rely on distinct formal semantic frameworks.[627] But the more important point for the present project is that we were not able to draw on an established theory of instantiation itself. Without going into further detail, several axioms, e.g., domain and range constraints, are comparable in nature to those in $\mathcal{CR}_0^{\text{DL}}$, though usually relating to a different meta-ontology.[628] Yet again, we are not aware of a structural theory of instantiation. On the other hand, there may be work e.g. in philosophy, which is left for future exploration.

ROLES "ORBIT" RELATORS VIA ROLE-OF

Leaving instantiation behind for a moment, let us briefly look at role-playing and role-having, starting with the latter. The role-of relationship is fairly well constrained by being a functional, "single step" relation mediating between entities in two disjoint categories. In effect (and on the one hand), roles "crystallize" at relators (of any kind), i.e., the latter act as focal points to which role-of connects roles. $\mathcal{CR}_0^{\text{FOL}}$ contains the axioms A6 and A10, which have these entailments.

C16. $x \multimap y \rightarrow \neg\exists z \,.\, z \multimap x$ (single-step left; x had to be role and relator otherwise)

C17. $x \multimap y \rightarrow \neg\exists z \,.\, y \multimap z$ (single-step right; analogously)

These consequences, again together with the domain and range constraints of role-having, further yield that there are only chains (but no cycles) of length two, if the direction of reading in the atomic formulas is "ignored". In the following list, we leave out the analogous cases that reject role-having connections at z, which are completely analogous to the case of x.

C18. $x \multimap y \wedge y \circ\!\!-\, z \rightarrow \neg\exists u \,.\, u \multimap x$ (due to the single-step characteristic)

C19. $x \multimap y \wedge y \circ\!\!-\, z \rightarrow \neg\exists u \,.\, u \neq y \wedge u \circ\!\!-\, x$ (x cannot be role and relator *or* by functionality)

C20. $x \multimap y \wedge y \circ\!\!-\, z \rightarrow \neg\exists u \,.\, u \circ\!\!-\, y$ (y cannot be role and relator *or* by functionality)

It follows that chains and cycles of any other length are excluded for role-of. This includes the entailment of irreflexivity and asymmetry, for example. Altogether and figuratively speaking, roles "orbit"/"surround"

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[626] A comment that is unrelated to the present discussion shall highlight a certain unease about relations labeled 'immediateSubclass' and 'immediateInstance' in [@119], but which we have found elsewhere, as well, e.g. as 'direct instance' or 'direct subclass'. Just to stress, this is no SUMO specific criticism, but rather a general comment on an issue. Direct instance is commonly defined à la 'x is a *direct instance of* y :iff x is an instance of y and there is no proper subclass z of y such that x is an instance of z'.

While we can imagine utility of those relations, e.g., in connection with reasoning and querying, we find these ontologically cumbersome because they are (more) relative to an established theory than, e.g., subclass / is-a itself. We refer to the fact that those relations may cause nonmonotonic effects (which is, in general, worthwhile considering, remembering sect. 5.4.1), in that the addition of knowledge my invalidate the conclusion that a class is a direct subclass of another between two versions of an ontology. To us, this appears to be different compared to, e.g., retracting / repairing an axiom due to noticing its inappropriateness, e.g., by missed cases. From another angle, even though we do not yet include comprehension axioms for $\mathcal{CR}$, the only stable assertion (stable under theory extensions) of direct instance that we see is that of being a direct instance of a singleton category, as the extensionally smallest category that can comprise an entity. A proposed consideration for further use would be to relativize those relations explicitly to a certain theory or maybe a group of categories within an ontological theory.

[627] At least, by declaration; making no claim about potential formal semantics preserving translations here.

[628] For example, SUMO accepts relations as sets or classes of tuples.

relators, all attached by role-having. More precisely, one might say "by role-having relators", depending on the question of whether the reflection axioms are adopted that follow nearby below.[629]

COMPLEX STRUCTURE OF PLAYS, YET ANALOGY BY RESTRICTION TO NON-RELATING PLAYERS

In combination with the latter, the plays relationship is structurally much more complex than role-of, because we assume (independently of reflection) that there are no (finite) cycles of plays by this axiom schema, which utilizes shorthand notation analogous to the one presented for :: in sect. 2.4.2.3.

A11. $\neg\exists x_1 x_2 \ldots x_n \;.\; x_1 \rightsquigarrow x_2 \rightsquigarrow \ldots \rightsquigarrow x_n \rightsquigarrow x_1$ for all $n \in \mathbb{N}_+$ (no plays cycles)

For relational[630] roles, we reject in particular that a role can be played by itself, corresponding to the scheme applied to $n = 1$. We further argue that a role of a relator r cannot play any of the roles of that relator, see A12, – regarding a reason, if only for a simpler theory at this stage. Albeit there is even less intuition for larger values of n, we propose the full scheme for $\mathcal{CR}$ for the same reason. Yet with A11 available in the overall theory, the structure of the plays relation is complex, especially if reflection axioms are accepted. The latter lead to infinite chains of plays relators, and to infinitely many such chains for each single entity that is related (in any way) to another – again, *if* reflection is accepted.

    Nevertheless, there is a strong similarity between plays and role-of in the following sense. If plays is examined with the additional restriction that the player must be a NonRelating entity, a very similar picture emerges as in the case of role-of. In effect (and on the other hand)[631], roles "orbit"/"surround" their players in the same way of a single (restricted) plays step and chains of order-ignoring (restricted) plays steps of length at most two.

INTERACTION AMONG ::, $\rightsquigarrow$, AND $\multimap$

Turning to axioms that involve several fundamental predicates of $\mathcal{CR}$, let us highlight the disjointness axioms in $\mathcal{CR}_0^{\mathrm{DL}}$ that make explicit the (tuple-extensional) disjointness of the three basic relations. Indeed, it turns out that these are already consequences of their domain and range constraints, in combination with the postulated taxonomy.

C21. $\neg\exists xy \;.\; x :: y \land x \rightsquigarrow y$ (A4 excludes $\mathsf{Cat}(y) \land \mathsf{Ind}(y)$)

C22. $\neg\exists xy \;.\; x :: y \land x \multimap y$ (A4 excludes $\mathsf{Cat}(y) \land \mathsf{Ind}(y)$)

C23. $\neg\exists xy \;.\; x \rightsquigarrow y \land x \multimap y$ (A6 excludes $\mathsf{Role}(y) \land \mathsf{Relator}(y)$)

Note that an analogous argument applies to extensional is-a, a relation among categories, whereas plays and role-of connect to at least one individual among their arguments. Hence, the omission of these two disjointness claims (due to limitations for decidable reasoning problems) is no severe issue.

    Apart from disjointness considerations, at the present stage we do not see many additions that would not also result from the domain and range restrictions and further axioms assumed in $\mathcal{CR}_0^{\mathrm{DL}}$. One exception, already mentioned above, is axiom A12. It entails, together with the requirement that each role must be a role of a relator, the case $n = 1$ of the scheme A11, by assuming $x = y$ in A12. Another exception is the subsequent and similar A13. We argue that both contribute to a clear separation of entities and relators that mediate between them. This matches the view that relators existentially depend on their relata, which we accept as a justification for the exclusion of "self-supporting" relators.[632]

A12. $\neg\exists xyz \;.\; x \multimap z \land y \multimap z \land x \rightsquigarrow y$ (roles of a relator play no roles in it)

A13. $\neg\exists xy \;.\; x \rightsquigarrow y \land y \multimap x$ (relators play no roles of themselves)

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[629] As a final note, despite viewing role-having as a kind of mereological relation, we believe that these properties lead to deviations from classical mereological systems, cf. [394, sect. 3.2], [707].

[630] potentially in contrast to social roles, cf. [530, sect. 3.6]

[631] see "In effect (and on the one hand)" near the last paragraph heading for the complementary case

[632] At first glance, a tempting case for a relator that plays one of its roles may be seen in the relation 'is argument of', thinking of a definition $\forall xy \;.\; \mathsf{argumentOf}(x, y) \leftrightarrow \exists z \;.\; x \rightsquigarrow z \land z \multimap y$. However, thinking of the role base of the argument-of relation, there should be two role categories, one of the argument, the other of the relator / mediating entity – obviously independently of the roles of the "actual" relator of which an argument is considered. A relator r that mediates between, say, a and b (where the latter must therefore play the roles that constitute r), is a different entity than one that had in addition a third role of the kind relator / mediating, played by itself.

Last, but not least, a – long deferred – "add-on" consists in what we call the reflection axioms of $::$, $\rightsquigarrow$, and $\multimap$, which have an exceptional status wrt the overall theory / conceptualization. Since they / their addition to $\mathcal{CR}_0^{\text{FOL}}$ remains a matter of debate, we present them not in the form of the axiom listings above, but like other formulas / formal expressions in the text. All three formulas follow a common underlying scheme, differing only in the targeted relation and its base role categories. Compared with Def. 4.51, we remark that the axioms stated here assume the remaining axiomatization of $\mathcal{CR}_0^{\text{FOL}}$, whereas $\tau_1(\phi)$ in Def. 4.51 comprises further conditions explicitly.

$$(6.11) \quad \forall xy \, . \, x :: y \;\leftrightarrow\; \exists r q_1 q_2 \, (\, r :: ::: \wedge q_1 \neq q_2 \wedge$$
$$x \rightsquigarrow q_1 \wedge q_1 \multimap r \wedge q_1 :: \cdot \mathsf{InstanceRL}$$
$$y \rightsquigarrow q_2 \wedge q_2 \multimap r \wedge q_2 :: \cdot \mathsf{InstantiatedRL}$$
$$\forall q' \, (\, q' \multimap r \rightarrow (q' = q_1 \vee q' = q_2) \,) \quad)$$

$$(6.12) \quad \forall xy \, . \, x \rightsquigarrow y \;\leftrightarrow\; \exists r q_1 q_2 \, (\, r :: \cdot \rightsquigarrow \wedge q_1 \neq q_2 \wedge$$
$$x \rightsquigarrow q_1 \wedge q_1 \multimap r \wedge q_1 :: \cdot \mathsf{PlayerRL}$$
$$y \rightsquigarrow q_2 \wedge q_2 \multimap r \wedge q_2 :: \cdot \mathsf{PlayedRL}$$
$$\forall q' \, (\, q' \multimap r \rightarrow (q' = q_1 \vee q' = q_2) \,) \quad)$$

$$(6.13) \quad \forall xy \, . \, x \multimap y \;\leftrightarrow\; \exists r q_1 q_2 \, (\, r :: \cdot \multimap \wedge q_1 \neq q_2 \wedge$$
$$x \rightsquigarrow q_1 \wedge q_1 \multimap r \wedge q_1 :: \cdot \mathsf{RoleRL}$$
$$y \rightsquigarrow q_2 \wedge q_2 \multimap r \wedge q_2 :: \cdot \mathsf{ContextRL}$$
$$\forall q' \, (\, q' \multimap r \rightarrow (q' = q_1 \vee q' = q_2) \,) \quad)$$

First consider some technical properties and implications. Clearly, these reflection axioms are equivalences, but no definitions, in the sense that the predicate symbol in the left-hand side of each equivalence occurs immediately in the resp. right-hand side, as well. For arguing in more detail for another observation, let T_{refl} the theory that comprises (6.11)–(6.13) in addition to $\mathcal{CR}_0^{\text{FOL}}$ and all other axioms discussed up to this point. On this basis, another, often criticized property results, a.o. due to the uniqueness conditions in (6.11)–(6.13).

6.2 Observation (infinite regress caused by reflection axioms)
T_{refl} can only have infinite set-theoretic or ontological models. ❑

To indicate the reason for that briefly, note first that every relator is uniquely associated with a set of arguments, namely the role players of its roles, due to the functionality conditions for role-of and played-by, resp. Since by A12 and A13 the roles and that relator itself are distinct from the argument entities, the former constitute "new" entities, due to the reflection axioms related to their arguments and among each other in terms of "further new" relators, etc., without the possibility of "reusing" entities considered before, because those are either non-relating, or they are relating, but already bound to others.

The question of whether this regress is of the vicious kind is already addressed in sect. 2.4.4.3 and basically rejected. The main argument corresponds to the 'externalist' version mentioned in [796, sect. 7.7], which we find reflected in the axioms, e.g., in A12 and A13. Of course, the possibility of repeated explanation of relation formation should not suggest to conduct that indefinitely. From our point of view, it suffices to have reached the role base of the three involved relations, because they "carry the meaning" of those relators (which become infinitely many "only" at the level of individuals), in a certain sense, which still needs to be related to notion(s) of intension. From a more practical point of view, e.g., of reasoning, we expect the reflection axioms to highly complicate proof search, which is part of the reason of not including them in the $\mathcal{CR}$ formalization straightaway, despite agreement from an analytical point of view.

As a next step of continuing this line of work, we plan to investigate (1) variants derived from the fundamental relations such that the regress can only be followed once or twice, starting from non-relating entities; and (2) a proof of the conjecture that such repeated "explanations" / "applications" by recourse

to the reflection axioms do at least not contribute to theories about non-relating entities that are related by other relations, even if those other relations are analyzed by means of $\mathcal{CR}$.

So far as to our proposal of a theory of categories and relations as emerging from and in this thesis. Further additions and metalogical analysis beyond $\mathcal{CR}_0^{\mathrm{DL}}$ is our goal for future work. Sect. 6.3 offers two related, much more comprehensive cases of ontological analysis that result in a conceptualization, its formalization, and corresponding metalogical analyis. In between, some pointers to other projects adumbrate further involvement wrt ontological analysis and ontology development.

6.2 Remarks on Further Contributions

An initial comment on writing technicalities is apposite here: In the first subsection we primarily refer to co-authored publications that are weakly related to the topics of the present thesis, while the second subsection prepares the inclusion of a co-authored publication as sect. 6.3. Therefore, in analogy to sect. 5.4.3, we omit further self-citation markers until the end of this section.

6.2.1 GFO Taxonomy and Fragments

IMPLEMENTATION OF EARLY TAXONOMIC VARIANTS OF GFO

Just as the $\mathcal{CR}$ ontology is a proposed component in the context of the project of developing the General Formal Ontology (GFO) [@46], most other ontology engineering efforts that we had a chance to participate in while working on this thesis are associated with GFO in one way or another. Proceeding from the whole to some of its parts, we begin with initial OWL implementations of the "taxonomic backbone" of GFO, conducted together with Robert Hoehndorf. The resulting files `gfo-basic.owl` and `gfo.owl` are available from [@48], where the latter named file precedes the former. `gfo-basic.owl` is a narrower selection of GFO categories and relations, designed for stability and developed in loose connection with the biological core ontology GFO-Bio, which is based on it. Considering developments of the GFO conceptualization and terminology, an update of `gfo-basic.owl` is foreseen.

From the perspective of ontological semantics and usage schemes (OUS), the same OUS ($\sigma_{\mathcal{CR}}^{\mathrm{DL}}, \tau_{\mathcal{CR}}^{\mathrm{DL}}$) as sketched for the taxonomy of $\mathcal{CR}$ applies in particular to `gfo-basic.owl`.[633] That is, GFO categories are represented as OWL classes, binary relations of GFO are expressed as OWL object properties, the standard translation from DL to FOL yields a first-order axiomatization that must only be extended by predication definitions, for categories along the lines of (6.5) and for relations by (6.6) in the previous sect. 6.1. GFO-Bio, the biological core ontology on the basis of GFO, has already been dealt with, including the relevant OUS, in sect. 5.4.1.

INCREMENTS TO THE THEORY OF ROLES

The notion of roles was the first that we have examined, first and foremost conceptually, starting and with first results in [526] prior to this thesis. During the time of the latter, this theory has been (moderately) further developed in [527, 530, 531], among which [530] is a comprehensive article of the present state of that theory of roles. In a nutshell, roles – in outmost generality of the term – are analyzed as entities that are filled / played by other entities and that are constituted in yet another kind of entity, called a context (which is a relative term from the point of view of the role). [526] distinguished three kinds of roles based on subdomains in which role examples could / can be observed, namely *relational*, *processual*, and *social roles*, in line with different entities serving as contexts, namely relations, processes, and social entities such as social institutions, organizations, or groups. The later [530] refines the theory by aggregating/abstracting the role kinds of relational and processual into the notion of *abstract roles*, for the reasons set forth in [530, esp. sect. 2.3]. Moreover, the notion of role-holder as presented in [788], associated with the YAMATO top-level ontology [@137] [591] and its related ontology editor Hozo [@56] [485, 591], is adopted for the conceptualization in [530] and extended by the notion of *role closure*, basically referring to a role player in

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[633]Most parts of `gfo.owl` are covered by this scheme, as well, but for the top-most distinction therein into sets and items, associated with Fig. 2.3 in sect. 2.3.3, we suggest a revised treatment on the basis of $\mathcal{CR}$, following `gfo-basic.owl`.

combination with all of its roles. [530, sect. 3] forms another major novel part, in which 12 features and problems[634] that are associated with the notion of role are examined wrt the different types of roles.

As clarified in the resp. sect. 1.1.5.2 and 2.4.3.2 above, the term 'role' in this thesis refers by default to relational role in the broader theory, which must be taken into account when combining the present theory with that in [530]. Indeed, sect. 2.4 and 6.1 establish a significant extension of the theory of roles by proposing a theory of relational roles in combination with (a very abstract view / theory on) categories and instantiation, with a partial formalization.

TOPICS IN THE GFO REPORT AND FUNCTIONS

Further contributions arose in the framework of describing and detailing collaboratively (in a group of 5-6 co-authors) the GFO conceptualization and formal fragments for it in a series of report versions [387, 397, 398], originally started and supervised by Barbara Heller and Heinrich Herre. In particular and besides assuming responsibility for roles (and meta-ontological architecture, cf. sect. 2.3), we participated in (initial) analyses of topics of processes[635], properties, relations (cf. $\mathcal{CR}$ herein), and time, to be discussed below. The close relationship between functions and roles led to an involvement with functions and collaboration on two publications [129, 131]. Yet, Patryk Burek and Robert Hoehndorf are the main authors of [131], and so is Patryk Burek of [129], overall, as he shouldered the elaboration of an ontology of functions in his PhD thesis [128].

Apart from the conceptual / analytic aspects of that engagement, working towards formalizations in connection with those efforts had a clear impact on shaping (1) our motivation for this thesis and (2) its theory, further providing us with early testbeds for initial ideas on ontological semantics and ontological usage schemes.

6.2.2   Preface to Ontologies of Time

IMPORTANCE OF THE TOPIC FOR THIS THESIS AND HISTORY IN THE GFO CONTEXT

The last paragraph of the previous section applies all the more to the matter of *time*, which is only listed there as one topic among others. It plays an important rôle for our work, notwithstanding its "invisibility" to this point. Indeed, first ideas on a theory of time were present when we started our work in the context of GFO. From those "early days" on and despite extended interruptions, the theory / ies of time and its / their axiomatization versions yield an interesting, sufficiently complex, but also "manageable" domain of reality / knowledge and good opportunities for formalization and metalogical analysis, as well as for methodic reflections. Nevertheless, the impression might arise that the next section were disconnected from the remainder of the work. From our point of view, however, this is at most insofar the case that references to ontological semantics and usage schemes cannot be found explicitly, indeed.

On the other hand, the same methodological foundation will become clear from sect. 6.3.2, which we kept as is – also to corroborate the last claim as a fact – despite slight overlap with sect. 1.1.5.3 and 1.5. Moreover, although the work on the two axiomatizations started prior to the final passage of writing the sections on ontological semantics and usage schemes, it should be possible (if not easy) to observe that those ontologies are developed in a manner that is compatible with method $\mathcal{OS}$. More precisely, the theories $\mathcal{BT}^{\mathcal{C}}$ and $\mathcal{BT}^{\mathcal{R}}$ in sect. 6.3.7 and 6.3.10, resp., are developed with the ontological usage scheme $(\sigma^{\Sigma}, \tau^{\Sigma})$ of Def. 5.10, p. 193, in mind, where the new unary predicate for the universe can be identified with the universal predicate TE in $\mathcal{BT}^{\mathcal{C}}$ and $\mathcal{BT}^{\mathcal{R}}$.[636] In this sense we find methodological elements as discussed wrt method $\mathcal{OS}$ in sect. 5.1, primarily as add-ons in sect. 5.1.2, and we claim those theories to be amenable of being perceived as use cases of that OUS. Likewise, they account for their own meaning by being interpreted (language-wise) by ontological semantics, either including the extensions through the OUS, or by adopting the basic relations as fundamental predicates.

To round off the historical aspect, in 2012, a first ontology of time [61],[637] covering time intervals and

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[634]gathered from the literature, drawing on earlier compilations such as in [782, sect. 2] by Friedrich Steimann or in [561, sect. 2] by Claudio Masolo et al.

[635]Cf. also [622, e.g., sect. 3.2], where the overall article discusses the description of surgical interventions.

[636]Nevertheless, the relationship between $\mathcal{BT}^{\mathcal{C}}$ and $\mathcal{BT}^{\mathcal{R}}$ is more involved than being a mere subset relation, cf. sect. 6.3.11.5.

[637]a preceding version of $\mathcal{BT}^{\mathcal{C}}$ in the next sect. 6.3

their boundaries (as a kind of time points) could be presented successfully to the audience of the FOIS conference, cf. [@42]. Equipped with a second, extended theory – $\mathcal{BT}^{\mathcal{R}}$ below – which captures time regions, i.e., mereological sums of time intervals, in addition, a substantially revised version [62] is in press at the time that this thesis is submitted.

PROPOSAL AS A METHODOLOGICAL SAMPLE

This unavailability of [62] at this time is an additional, though minor incentive for including the theories of time for GFO in the next and last section of this chapter, before the overall thesis turns to its end. Much more relevant besides the relationship to ontological semantics and usage schemes sketched above is the fact that the development of those theories is suggested as a model at least for developing further component theories in the context of GFO, covering the range from developing an actual conceptualization to its reliable formalization in terms of (one or more) axiomatic systems that are metalogically analyzed.

THE MANUSCRIPT OF [**62**] AS SECT. **6.3**

For the reasons just discussed we include the final manuscript "Axiomatic theories of the ontology of time in GFO" [62], which is accepted and in press for publication by Applied Ontology [@6]. The work on the results and the text in the next section was evenly distributed among its three co-authors, i.e., Ringo Baumann, Heinrich Herre, and ourselves. The two former co-authors have (verbally) agreed to the inclusion of the manuscript as a part of this thesis, as well as the publisher has granted the permission to reproduce the content. Thanks to these permissions, we appreciate the approach of an (almost) perfectly literal inclusion except for reformatting to the layout of the present document. This appears to us as a clearer and better procedure than modifying some amount of text (or other elements), which makes is hard for readers of both publications to determine whether and how many differences there are. The only exceptions to this selfimposed rule are (1) the omission of the hint on the related prior publication [61] on the first page of the original article, (2) the removal of the associated keywords "Ontology of Time, Time Point, Time Interval, Time Region, Coincidence, Brentano" between the abstract and the beginning of sect. 6.3.1, (3) the removal of a line of acknowledgements to the reviewers of the publication (placed after the last section in the original article), and (4) the substitutions of (4a) the terms 'article' and 'paper' by 'section', occasionally together with a section number, and of (4b) $\mathbb{N}$ for the positive natural numbers in [62] by $\mathbb{N}_+$ herein for the same meaning and of three resp. occurrences of the term 'natural number' by 'positive natural number' in sect. 6.3.11. In addition, layout changes and renumberings of, e.g., propositions are present. The numbers acting as axiom identifiers, though being formatted differently, are equal to those in the original article, e.g., referring to axiom A30 of theory $\mathcal{BT}^{\mathcal{R}}$ identifies the same axiom on the least contained chronoid of a time region and their boundaries in [62] and herein. This foretaste of the terminology of time entities in GFO might spark additional interest in the matter, such that we shall no longer detain / distract "sequential readers" from being exposed to it.

⁶³⁸ 📖 6.3 Ontologies of Time

SECTION ABSTRACT

Time is a pervasive notion of high impact in information systems and computer science altogether. Respective understandings of the domain of time are fundamental for numerous areas, frequently in combination with closely related entities such as events, changes, and processes. The conception and representation of time entities and reasoning about temporal data and knowledge are thus significant research areas. Each representation of temporal knowledge bears ontological commitments concerning time. Thus it is important to base temporal representations on a foundational ontology that covers general categories of time entities.

In this section we introduce and discuss two consecutive ontologies of time that have been developed for the top-level ontology General Formal Ontology (GFO). The first covers intervals, named *chronoids*, and *time boundaries* of chronoids, as a kind of time points. One important specialty of time boundaries is

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their ability to *coincide* with other time boundaries. The second theory extends the first one by additionally addressing *time regions*, i.e., mereological sums of chronoids.

Both ontologies are partially inspired by ideas of Franz Brentano, especially from his writings about the continuum. In particular, we view continuous time intervals as a genuine phenomenon which should not be identified with intervals (sets) of real numbers. On these grounds the resulting ontologies allow for proposing novel contributions to several problematic issues in temporal representation and reasoning, among others, the Dividing Instant Problem and the problem of persistence and change.

Following our general approach to ontology development, both ontologies are axiomatized as formal theories in first-order logic and are analyzed metalogically. We prove the consistency of both ontologies, and completeness and decidability for one. Moreover, standard time theories with points and intervals are covered by both theories.

6.3.1 Introduction

Space and time are basic categories of any top-level ontology. They are fundamental assumptions for the mode of existence of those individuals that are said to be in space and time. In this section we expound the ontology of time which is adopted by the General Formal Ontology (GFO) [392], a top-level ontology being developed by the research group Ontologies in Medicine and Life Sciences[639] (Onto-Med) at the University of Leipzig. The time ontology together with the space ontology of GFO [60] forms the basis for an ontology of material individuals.

There is an ongoing debate about whether time is ideal and subject-dependent or whether it is a real entity being independent of the mind. We defend the thesis that time exhibits two aspects. On the one hand, humans perceive time in relation to material entities through phenomena of duration, persistence, happening, non-simultaneity, order, past, present and future, change and the passage of time. We adopt the position that these phenomena are mind-dependent. On the other hand, we assume that material entities possess mind-independent dispositions to generate these temporal phenomena. We call these dispositions *temporality* and claim that they unfold in the mind/subject as a manifold of temporal phenomena. This distinction between the temporality of material entities and the temporal phenomena (called *phenomenal time*) corresponds to the distinction between temporality and time as considered by Nicolai Hartmann [373]. A satisfactory ontology of time and its formal representation should treat the various temporal phenomena in a uniform and consistent manner. We suggest to address the following basic tasks.

1. The development of an ontology of time itself, abstracted from real phenomena. We call this time *abstract phenomenal time*.
2. The development of an ontology, describing precisely how material entities (objects, processes) are related to abstract phenomenal time and how temporal phenomena are represented.
3. The establishment of a truth-relation between temporal propositions and spatio-temporal reality, also termed the problem of *temporal incidence* [849].
4. The elaboration of a unifying formal axiomatization of the ontology of time and of material entities of the spatio-temporal reality, the *formalization problem*.

The present section 6.3 is a contribution to solving these problems. While our results mainly pertain to the first and fourth issues, the second and third provide corresponding motivations and application cases.

We propose and analyze two ontologies of time as axiomatic theories, aiming at a solid foundation for several applications. Among the latter are truth assignments to temporal propositions and a capable and consistent solution to the Dividing Instant Problem (DIP), cf. [6]. The first theory of abstract phenomenal time, labeled $\mathcal{BT}^{\mathcal{C}}$, focuses on time intervals perceived as genuine entities, from an ontological point of view. To avoid confusion with accounts of time that are based on intervals (sets) of real numbers (and that may even identify these with time intervals), we refer to such genuine intervals as *chronoids*. A clear distinction between the mathematical rendering of continuous phenomena, e.g. in terms of the real numbers, and the actual phenomena themselves is an important inspiration to this work, primarily following corresponding

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[639]http://www.onto-med.de

criticism purported by Franz Brentano [114]. Chronoids have two extremal *time boundaries*, which is the second kind of entities in the domain of $\mathcal{BT}^C$. No time boundary can exist without a chronoid that it is the boundary of, hence they are existentially dependent on chronoids. Roughly speaking, time boundaries may be called and seen as time points. Yet there is an uncommon but powerful feature of time boundaries, expounded in section 6.3.5.1, that is their ability to *coincide*. The "existential priority" of chronoids over time boundaries and the notion of coincidence constitute major differences to usual dual time theories of points and intervals, such as $\mathcal{IP}$ [848], [849, sect. 1.7.2].[640] The second theory developed here, $\mathcal{BT}^R$, addresses chronoids, time boundaries, and in addition mereological sums of the entities of each kind, called *time regions* and *time boundary regions*, respectively. Due to technical reasons, $\mathcal{BT}^C$ is not literally a sub-theory of $\mathcal{BT}^R$, but there is a natural interpretation into $\mathcal{BT}^R$. Initial metalogical analysis completes the introduction of each ontology, in particular establishing the consistency of the axiomatizations.

The section 6.3 is organized as follows. First, we outline the Onto-Axiomatic Method, i.e., general principles underlying our approach of ontological analysis and the development of formal axiomatizations of a domain, in section 6.3.2, which further includes some formal preliminaries. In section 6.3.3 we present a focused state of the art on time, concentrating on axiomatic theories in temporal representation and reasoning and summarizing how other top-level ontologies deal with time. Section 6.3.4 introduces selected problems of temporal qualification and incidence and proposes desiderata for an ontology of abstract phenomenal time. An informal introduction to time in GFO and to related kinds of entities in section 6.3.5 allows us in section 6.3.6 to illustrate new approaches to the problems previously introduced. Section 6.3.7 introduces $\mathcal{BT}^C$, the first axiomatization of abstract phenomenal time for GFO in first-order logic (FOL), which focuses on chronoids (time intervals) and time boundaries (time points). This is accompanied by investigating the metalogical properties of the theory $\mathcal{BT}^C$ in section 6.3.8. Thereafter, sections 6.3.9–6.3.11 follow the same structure of conceptual outline, axiomatization, and metalogical analysis to present the extended theory $\mathcal{BT}^R$. Section 6.3.12 completes this part by briefly discussing temporal abstraction as a problem for which time regions are directly relevant. In section 6.3.13 we summarize our results, draw some conclusions and outline several problems and tasks for future research.

6.3.2 Principles of the Onto-Axiomatic Method

A few methodological and technical clarifications are beneficial before we focus on the specific domain of time. The understanding of *Formal Ontology* as presupposed in this section has its roots in formal logic, philosophical ontology, and artificial intelligence. Formal Ontology is an evolving science which aims at the development of formal theories describing forms, modes, and views of being of the world at different levels of abstraction and granularity, cf. also [159]. An essential characteristic of formal ontology is the *axiomatic method* which comprises principles for the development of theories, aiming at the foundation, systematization, and formalization of a field of knowledge about a domain of the world. If knowledge of a domain is assembled in a systematic way, a set of categories is stipulated as primitive or basic. Primitive categories are not defined by explicit definitions, but by axioms that define their meaning implicitly [401].

The considered axioms exhibit various degrees of generality. At the most general level of abstraction these theories are called top-level ontologies, the axioms and categories of which can be applied to most domains of the world. The *Onto-Axiomatic Method* combines the axiomatic method with the inclusion of a top-level ontology which is used to establish more specialized ontologies which can be classified into *core ontologies* and *domain specific ontologies*.

Top-level ontologies are intended to play an analogous role for the world in general as set theory for mathematics[641]. GFO adopts set theory, in contrast to other top-level ontologies such as the Basic Formal Ontology (BFO) [306] and the Descriptive Ontology for Linguistic and Cognitive Engineering (DOLCE) [101], as the top-level ontology for mathematics that exhibits a part of the ontological region of abstract

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[640]$\mathcal{IP}$ is contained in section 6.3.8.3, where an interpretation into $\mathcal{BT}^C$ is shown.

[641]Most mathematicians accept the fundamental status of set theory. To be more explicit, set theory plays the role of a core ontology for mathematics. This does not mean that any mathematical discipline is a part of set theory, but only that arbitrary mathematical notions can be reconstructed in the framework of set theory. Furthermore, we note that there are competing core ontologies for mathematics, for example, mathematical category theory.

entities [399].[642] Abstract objects as introduced by Zalta [890] correspond to another example in the region of abstract entities in GFO. The other ontological regions that are addressed in GFO are classified into the psychological, the material, and the social ontological region.[643]

    [399]

    The most difficult methodological problem concerning the introduction of axioms is their justification. In general, four basic problems are related to an axiomatization of the knowledge of a domain, cf. also [60, esp. sect. 2].

1. What are adequate / appropriate concepts and relations of a domain?   (*conceptualization problem*)
2. How may we find axioms?   (*axiomatization problem*)
3. How can we know that our axioms are true in the considered domain?   (*truth problem*)
4. How can we prove that our theory is consistent?   (*consistency problem*)

In the remainder of this section 6.3.2 we summarize basic notions and theorems from model theory, logic and set theory which are relevant for section 6.3, in particular for the metalogical analyses. These notions are presented in standard text books, such as those by Barwise and Feferman [55], Chang and Keisler [147], Devlin [200], Enderton [221], Hodges [410], and Rautenberg [693].

    A logical language $\mathcal{L}$ is determined by a syntax specifying its formulas and by a semantics. Throughout this section 6.3 we use first-order logic (FOL) as a framework. The semantics of FOL is presented by relational structures, called σ-structures, which are interpretations of a signature σ consisting of relational and functional symbols. We use the term model-theoretic structure to denote first-order relational structures. For a model-theoretic structure $\mathcal{M}$ and a formula ϕ we use the expression $\mathcal{M} \models \phi$ to mean that the formula ϕ is true in $\mathcal{M}$. A structure $\mathcal{M}$ is called a model of a theory $\mathcal{T}$, being a set of formulas, if for every formula $\phi \in \mathcal{T}$ the condition $\mathcal{M} \models \phi$ is satisfied. Let $Mod(\mathcal{T})$ be the class of all models of $\mathcal{T}$. Conversely, we define for a class $\mathcal{K}$ of σ-structures the theory of $\mathcal{K}$, denoted by $Th(\mathcal{K})$: $Th(\mathcal{K}) = \{\phi \mid \mathcal{A} \models \phi$ for all $\mathcal{A} \in \mathcal{K}\}$. The logical consequence relation, likewise denoted by $\models$, is defined by the condition: $\mathcal{T} \models \phi$ if and only if $Mod(\mathcal{T}) \subseteq Mod(\{\phi\})$. In that case ϕ is called a consequence or a theorem of $\mathcal{T}$. If a theory T is specified axiomatically, i.e., by a set of postulated formulas, called axioms, the notion of theorem is usually restricted to formulas ϕ satisfying $T \models \phi$, but which are not axioms.

    For first-order logic the completeness theorem holds: $\mathcal{T} \models \phi$ if and only if $\mathcal{T} \vdash \phi$, where the relation $\vdash$ is a suitable formal derivability relation. The operation $Cn(\mathcal{T})$ is the classical closure operation which is defined by: $Cn(\mathcal{T}) = \{\phi \mid \mathcal{T} \models \phi\}$. A theory $\mathcal{T}$ is said to be *decidable* if there is an algorithm Alg (with two output values 0 and 1) that stops for every input sentence and satisfies the condition: For every sentence ϕ in the language of $\mathcal{T}$: $\mathcal{T} \models \phi$ if and only if $\mathsf{Alg}(\phi) = 1$. An extension $\mathcal{S}$ of a theory $\mathcal{T}$ is said to be *complete* if for every sentence ϕ: $\mathcal{S} \models \phi$ or $\mathcal{S} \models \neg\phi$. A complete and consistent extension of $\mathcal{T}$ is called an *elementary type* of $\mathcal{T}$ (type of $\mathcal{T}$, for short). Assuming that the language $\mathcal{L}$ is countable then there exists a countable set X of types of $\mathcal{T}$ such that every sentence ϕ which is consistent with $\mathcal{T}$ is consistent with a type from X. In this case we say that the set X is dense in the set of all types of $\mathcal{T}$. Determining such a set of types defines the *classification problem* for $\mathcal{T}$, which is solved if a reasonable description of a countable dense set of types is presented.

6.3.3 Time in Logic, Artificial Intelligence, and Ontologies

The literature on time and temporal representation and reasoning is vast. We merely point to some of the corresponding surveys [241, 251, 255, 835] and strictly limit the scope of what follows to works closely related to the present section 6.3.

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[642]The ontological character of abstract entities, presented in set theory, is uncovered by the mysterious phenomenon of the practical applicability of mathematics in the field of material objects, in particular in physics [877].

[643]There is the problem of how these four ontological regions are connected. At present, we adhere to the view that there is the following more precise classification: (1) spatio-temporal reality, subdivided in spatio-temporal material entities, and temporal mental-psychological and sociological entities, (2) entities being independent of space and time, but dependent on the mind, e.g., concepts, (3) ideal entities which are independent of space and time, and exhibit an objective world of their own, independently of the mind. A similar classification is discussed by Roman Ingarden [443].

6.3.3.1 Axiomatic Theories of Time

Axiomatizations of time have been studied primarily in artificial intelligence. Lluís Vila surveys time theories according to three major branches: purely point-based and purely interval-based theories, plus theories that combine points and intervals [849]. An earlier catalog of temporal theories is provided by Patrick J. Hayes [379]. Recently, several of these and further theories have been implemented, verified and further formally analyzed regarding their metatheoretic relationships by Michael Grüninger et al., cf. [324].

The claimed focus of Vila's survey is on "the most relevant theories of time proposed in Artificial Intelligence according to various representational issues [...]" [849, p. 1]. Additionally, its introduction links to many, frequently more specific works on time. For pure point-theories, Vila mainly presents typical axioms for a single relation *before* (time point x is properly before time point y): those of a linear order, unboundedness/infinity in both directions and the mutually exclusive axioms of discreteness (every time point has an immediate successor and predecessor) and density (between any two distinct time points there is a third one). This allows for interesting completeness results: if *before* is axiomatized as an unbounded, strict linear ordering and satisfies either discreteness or density, that yields a syntactically complete theory already [78].

In the case of purely interval-based theories, there is a similar result on an extended version of the interval theory of James F. Allen and Patrick J. Hayes [9, 10], referred to as $\mathcal{AH}$ in [849]. In [6], Allen had introduced a temporal interval algebra, cf. also [241, ch. 8] that is based on disjunctive combinations of the 13 well-known simple qualitative interval relations, including *equal*, *meets*, *before/after*, *starts/ends*, *overlap*, etc. Allen and Hayes then provided a FOL axiomatization of *meets* consisting of five axioms, and showed that the remaining 12 relations can be defined solely based on *meets*.[644] Together with an additional axiom enforcing a kind of density of the meeting points of time intervals, Peter Ladkin presents an extension to $\mathcal{AH}$ which he proved to be complete [500].

Despite the valuable results on pure interval theories, they are frequently considered to be insufficient unless time points are reintroduced or reconstructed [41, 849, sect. 1.6–1.7]. Reconstruction is typically mathematical in nature, e.g. by defining points as maximal sets of intervals that share a common intersection. In contrast, theories have been proposed that genuinely consider time points and intervals on a par. One example is the theory $\mathcal{IP}$ by Lluís Vila [848], [849, sect. 1.7.2], which extends the axioms of the pure point theory (without fixing density or discreteness) with seven axioms that naturally link points and intervals using the point-interval relations *begin* and *end* (time point x is the begin/end of interval y). $\mathcal{IP}$ enjoys interesting metalogical properties: firstly, every model of $\mathcal{IP}$ is completely characterized by an infinite set with an unbounded strict linear order on it; secondly, the theory $\mathcal{IP}_{\text{dense}}$, obtained by adding density of time points to $\mathcal{IP}$, is logically equivalent to Ladkin's complete extension of $\mathcal{AH}$ [849, p. 7, 16–17].

In summary, these are well-established and important theories to which each new proposal should be related (cf. section 6.3.8). Notably, there are various further theories which consider additional, frequently more controversial and/or purpose-driven views on time, e.g. directedness of time intervals [379, sect. 5.3], branching time approaches, etc., which cannot be covered here.

Regarding time regions, we are not aware of any pure first-order axiomatization in the domain of time that considers points and/or intervals as well as mereological sums or, more generically, aggregates of them. Parts of the work on OWL-Time [409], cf. more specifically [653, especially ch. 4], which is a proposal of a time theory for the Semantic Web and thus primarily represented in the Web Ontology Language (OWL) [856], may be most closely related. OWL-Time is a continuation of [408]. It is based on a combined point and interval theory axiomatized in FOL that includes Allen's interval relations, and considers additionally time durations, descriptions of calendar dates and times, as well as temporal aggregates [408, sect. 6], all of which appears to be covered and linked to proposals for the Semantic Web in [653, treating temporal aggregates in ch. 4]. However, those axiomatizations are claimed to "make moderate use of second-order formulations" [653, sect. 4.1, p. 42], and they presuppose some set theory [408, sect. 6.1] due to introducing the aggregate notion of "temporal sequence" as referring to a set of temporal entities with a temporal ordering over their elements, in contrast to conceiving of "temporal sequence" as a mereological sum.

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[644]Although the axiomatization uses FOL with equality, it is stated that the *equal* relation can be defined by a simple adaptation of one axiom [10, p. 228].

These are clear deviations from the theory $\mathcal{BT}^{\mathcal{R}}$ as introduced in sections 6.3.9 and 6.3.10 below. Finally, some spatio-temporal theories comprise mereotopological axioms, e.g., cf. [164], but they have a different domain than solely time entities (and may rely on very different foundations). Hence, spatio-temporal theories are out of the scope of this section. Nevertheless they are of interest in future work, not at least in connection with the spatial theory of GFO [60].

6.3.3.2 Time in Top-Level Ontologies

In the context of top-level ontologies, time entities like time points or intervals are usually classified within the corresponding taxonomic structure, but specific theories of time are developed in rare cases (and if so, they often adopt the theories just introduced). Due to spatial limitations, we restrict a closer look to DOLCE [558, mainly ch. 3–4], [101], BFO [777], and PSL [317, 450].[645]

Time is included in DOLCE through the category *temporal region*, which is subsumed by the category *abstract* [entity]. Temporal regions are subject to a general, atemporal parthood relation. Temporal localization is a special case of quality assignment in DOLCE, involving *temporal qualities* whose values are temporal regions, themselves part of the temporal space. Deliberately, no further assumptions about the temporal space are made in order to remain neutral about ontological commitments on time [101, p. 287].

BFO [558, ch. 7–8], [777] includes the disjoint categories *temporal interval* and *instant* as subcategories of *temporal region* and *occurrent*. Any temporal region is part of *time* (the whole of time). A few initial axioms for time in BFO are available [558, ch. 8], [823]. Axiomatic interval theories do not seem to be included or adapted. Some axioms in [823, sect. 5.1] suggest that time instants are linearly ordered, mereology is applied to temporal regions, and there is a link with the notion of boundaries, as introduced e.g. in [766], by requiring that instants can only exist at the boundary of temporal intervals [823, sect. 5.1].

PSL [317, 450] aims at process modeling and is thus related with time. This ontology is highly modularized and completely presented as a FOL axiomatization, with verified consistency for some modules. The core module of the PSL ontology distinguishes as its four most general, pairwise disjoint categories *activity*, *activity occurrence*, *object*, and *time point*; further there is a module for time duration. The relation *before* on time points forms an infinite linear order, with "auxiliary" bounds $+\infty$ and $-\infty$. Density and discreteness are not assumed in the core of PSL, but may be added as an extension. Eventually, we are not aware of any extensions covering intervals and/or interval relations within PSL. However, there are a number of modules axiomatizing mereological, ordering, and duration relations for activities/activity occurrences.

6.3.4 Motivating Problems and Requirements for Time in GFO

First of all, an ontology of time should establish a rich basis for analyzing temporal phenomena. In this section we describe four time-related problems with open issues that lead to further demands on a time ontology. These *motivating* problems are reconsidered in section 6.3.6, necessarily in connection with additional categories of entities such as processes or continuants.

6.3.4.1 The Holding Problem of Temporal Propositions

The holding problem of temporal propositions (called temporal incidence, e.g. by Reichgelt and Vila [701], Vila [849]) is concerned with domain-independent conditions that determine the truth-value of propositions through and at times. One aspect of temporal incidence theories is to account for interrelations of propositions holding at different time entities, e.g. *homogeneity*, cf. [7, sect. 2]. Such conditions can be expressed using relations like $holds(\phi, t)$, referred to as temporal incidence predicates in [849, p. 19]. Their intended meaning is that the proposition ϕ is true at time entity t. We argue that propositions can hold at time intervals and/or at time points. Considering the tossing of a ball (see Figure 6.2), for example, *the*

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[645]The acronyms stand for Descriptive Ontology for Linguistic and Cognitive Engineering (DOLCE), Basic Formal Ontology (BFO), and Process Specification Language (PSL); the respective websites are:
http://www.loa.istc.cnr.it/DOLCE.html ,
http://www.ifomis.org/bfo ,
http://www.mel.nist.gov/psl .
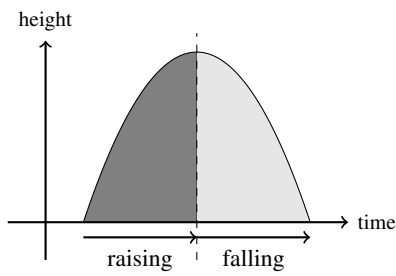
Figure 6.2: Tossing a ball into the air.
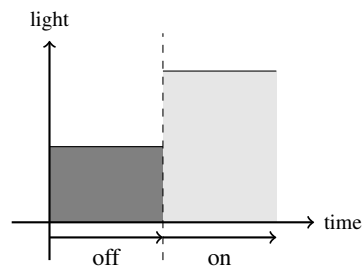
Figure 6.3: Switching on the light.

velocity of the ball is zero holds at one time point, *the ball is raising* holds at an interval (and maybe at many points), *the tossing takes 10 seconds* applies only to an interval.

At the center of these problems is the development of a truth relation between propositions and spatio-temporal reality. The ontology of time appears as a constituent of an ontology of concrete material entities. We must develop an ontology of parts of reality / parts of the world which can serve as truth-makers for propositions (eventually as expressed by natural language sentences). In these regards, the relation $holds(\phi, t)$ with the meaning that the proposition ϕ holds at time t is insufficiently specified. A further argument s is required, and a ternary relation, denoted by $holds(\phi, t, s)$ with the meaning: ϕ holds at time t in s, where s is a part of the world that includes a time component. According to GFO, the entity s can be considered as a temporally extended situation, also called *situoid*, cf. [392, sect. 14.4.6], [398, sect. 12]. A situoid s provides a temporal frame of reference through its time component. There are propositions without any reference time, for example, $2 + 2 = 4$. Others have a time aspect, for example, the proposition *the tossing of this ball takes 10 seconds*. Let us briefly go into some detail in this case: One may imagine a temporally extended situation s which contains several entities, for example, a ball, a throwing person, and the throw itself. The throw is a process, actuated by this person and having the ball as participant. This process has a duration of at least 10 seconds. It is embedded into the situoid s, hence this situoid must possess a time frame with a duration of at least 10 seconds. Intuitively, we may say that the proposition is satisfied in s (alternatively, that ϕ is true in s, or that s is a truth-maker for ϕ). Some corresponding requirements are formulated in the context of an initial outline of an *ontological semantics*, see [534, esp. sect. 3]. There are related further issues at the side of languages. We need an explication of elementary sentences (or atomic propositions), and of rules for combinations of elementary sentences to more complex expressions. Even the notion of elementary sentence itself is a non-trivial and unsolved problem, cf. e.g. criticism by Ludwig Wittgenstein [881].

In summary, the current theory of temporal incidence reveals many open problems. Relevant basic notions are insufficiently founded, for example, the incidence relation $holds$ and the notion of an atomic proposition (or elementary sentence). Moreover, the holding of negation, conjunction and disjunction of temporal propositions is a non-trivial problem, cf. [849, p. 5]. These matters clearly deserve further treatment in future work, while an ontology of time should support the natural expression of temporal incidence conditions and/or the translation of a proposition of a language into an ontologically founded formal sentence.

6.3.4.2 The Dividing Instant Problem

One famous problem involving the holding of propositions shall be given special attention: it is termed Dividing Instant Problem (DIP), e.g. in [6], or the problem of the Moment of Change (MOC) [786] or Instant of Change [254]. Allen illustrates it by *switching on the light* [6], see Figure 6.3. The central question (in this exemplary case) is whether the light is off or on at the switching point, assuming instantaneous changing from off to on. One might claim that the light is both, off and on, or it is neither off, nor on. Logically, the former leads to an inconsistency, the latter violates the law of the excluded middle. The two remaining basic options are, firstly, to argue for a specific choice of either off or on to apply at the dividing instant, and

secondly, to let exactly one of off or on apply, without defending any particular choice, i.e., allowing for an arbitrary selection. More complexity can be added if several types of dividing instants are distinguished and treated differently according to the basic options just mentioned, e.g. cf. "Neutral Instant Analysis" surveyed in [786].

There are several proposals to solve the DIP/MOC. Niko Strobach has devoted a whole book to the treatment of the problem in philosophy, including a novel contribution by himself [786]. His "systematic history" of the MOC starts with the views of Plato and Aristotle and is further covering medieval approaches and the twentieth century. As summarized by Jansen [457], there are representatives in the twentieth century for each of the four basic options introduced above. Antony Galton has authored another survey on the DIP [254] that, besides philosophy, captures mathematical views as well as accounts in artificial intelligence. Focusing on the latter, we note that Allen's solution excludes instants from the time ontology, and claims that propositions do not satisfy the condition to be true at a time point. We reject this as a general approach due to the implicit reduction of propositions, in the light of the previous section. Another approach stipulates that all intervals are semi-open, e.g. left-closed and right-open [553]. Then, if a proposition is true throughout a time interval and is false throughout a subsequent interval, the truth-value at the dividing instant is false. A weakness of this approach is the arbitrary choice between employing left-closed and right-open vs. left-open and right-closed intervals.

As the case of switching on the light illustrates, the following conditions describe a common situation that needs to be captured:

1. There are two processes following one another immediately, i.e., without any gaps (the process *light off* meets *light on*)
2. There is a last point t_l in time where the first process ends and there is a first point t_f in time where the second process starts.
3. The points t_l and t_f are distinct.

An adequate solution to the DIP can be achieved through an ontology of time points and intervals that allows for satisfying these conditions, due to supporting the consecutiveness of processes without any overlap; cf. section 6.3.6.2. Note that these conditions cannot be satisfied if we represent time by the ordering of real numbers, the usual understanding of the continuum since the work of Richard Dedekind [193] and Karl Weierstrass, cf. [293].

6.3.4.3 The Continuum

The continuum has two sources, phenomenal time and phenomenal space, and it can be accessed through introspection. Phenomenal time is a subjective assumption of any movement, therefore, one origin of the notion of continuum is closely related to the movement of a body. We adopt the position that the notion of continuum is abstracted from subjective temporal (and spatial) phenomena. The result of this abstraction – according to our approach – can be accessed through a particular kind of introspection, which Immanuel Kant calls *reine Anschauung* [464, I.§1].[646]

Franz Brentano [114] criticizes the approach taken by Dedekind and Weierstrass to construct the continuum by using numbers, more precisely by means of the rational numbers and infinite sequences of rationals. The problems associated with the classical mathematical approach to the continuum can be demonstrated by one of Zeno's paradoxes, the Paradox of the Arrow, cf. [437, esp. sect. 3.3]. Consider an arrow a flying from location l_1 to another location l_2 during the time interval $I = [t_1, t_2]$. For the subsequent argument, we assume that the interval I equals the set of time points of I, and in particular, that these time points can be represented by real numbers. It is immediately clear that a cannot move at any time point, because time points have no temporal extension. If the interval I *equals* the set of time points of I, and a does not

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[646]In general, introspection means to look into one's own mind, to find what one thinks and feels. In our opinion, the notion of continuum can be grasped by a particular form of introspection, following Kant. There is a related debate about the a priori nature of time and space. According to Kant [464], time is an a priori notion, accessible without any experience of reality. In contrast, Franz Brentano believes that all notions are based on experience, hence there are no a priori notions, cf. [113, 115]. We advocate the thesis that both theories can be reconciled in the framework of *integrative realism* [392, esp. sect. 14.2.6], [395, esp. sect. 6]. (Section 6.3.6.4 below comprises more details on this form of realism.)

move at any time point (neither at t_1, t_2, or any in between), then this implies that a does not move at all, in particular, it cannot move from location l_1 to location l_2. From these considerations, independently of and in addition to those at the end of the previous section 6.3.4.2, one may conclude that the idea of identifying an interval with its set of points cannot be further maintained.

An exhaustive discussion of the continuum problem, from the historical perspective and the view of contemporary mathematics is presented in [72].

6.3.4.4 Persistence and Change

Another problem, which lacks a comprehensive solution and rests upon a tailored ontology of time, concerns entities that persist through time, though, exhibit different properties at different times. What does it mean that an entity is identical through time, and concurrently changes its properties?

Possibly to an even greater extent than in the case of the DIP, there are several approaches to cope with the problem of persistence and change. David Lewis [522] classifies entities into *perdurants* and *endurants* (also termed *continuants* in the literature). An entity perdures if it persists by having different temporal parts, or stages, at different times, whereas an entity endures if it persists by being wholly present at any time of its existence. Persistence by endurance is paradoxical and leads to inconsistencies [51]. Furthermore, the stage approach exhibits serious weaknesses [858]. Both of these interpretations and explanations of the phenomenon of persistence are criticized by various arguments, while further alternatives, e.g. as discussed by Sally Haslanger [377], reveal shortcomings, as well. Consequently, the development of a satisfactory, widely acceptable theory of persistence remains an open problem.

Let us demonstrate some of the problems related to persistence in greater detail. A (material) continuant persists through time and is wholly present at every point of its life time, which is a temporally extended interval. This is obviously paradoxical, because if a continuant is wholly present at one time point, then the same entity, being a spatio-temporal, material individual, cannot be wholly present at another time point. This argument assumes the condition that a spatio-temporal individual cannot be multi-located as a whole throughout a 4D region.

On the other hand, the doctrine of endurantism claims the existence of spatio-temporal individuals that are multi-located in space and time, hence they are wholly present and located at distinct space-time regions. Stephen Barker and Phil Dowe [51] show that endurantism leads to various paradoxes, which are reduced to the problem of how the individuals that are located at distinct spatio-temporal regions are related to the continuant / endurant as such. First, one may distinguish the continuant c in itself and individuals $c(t)$, each of which is determined by the location of c at spatio-temporal regions t. The definition of endurantism yields $c = c(t)$, for every region t at which c is located (ibid., p. 69).

On this basis, one of the paradoxes (ibid., p. 69–70) derives from the assumptions that (i) c is also identical to the mereological fusion of all $c(t)$ and (ii) all regions t are assumed to have distinct time intervals as their temporal coordinates. Now a problem arises for the temporal extent of the mereological fusion of all $c(t)$. On the one hand, selecting any t, the fusion should have a temporal extent as is the case for t, because the fusion is identical to c and thus with $c(t)$. On the other hand, the mereological fusion has all the $c(t)$ as its parts, hence the temporal extent of the fusion should be the sum of all their corresponding time intervals. This yields a paradox if at least two $c(t)$ with distinct regions are assumed.

In the case that the temporal coordinates of the regions t are assumed to be time points (instead of extended intervals), another paradox is derived in (ibid., p. 70). The time point assumption implies that c itself has no temporal extension, due to $c = c(t)$, i.e., c is a 3D object – contrary to the presupposition that c, as a continuant, has a non-zero life time. These paradoxes can be avoided if we assume that $c \neq c(t)$, that $c(t)$ are entities of their own, and that the relation between c and the $c(t)$ is not the part-of relation.

A solution to the mentioned problems may be found in the stage approach, which states that an entity persists by perdurance because it is temporally extended by having stages at different times. Stages, in this sense, are small processual parts, whereas the whole perdurant is a mereological sum of these stages [753]. However, this approach to persistence leads to new problems, if we attempt to formalize stage structures. The following questions must be answered: Are the stages linearly ordered? They should be, because we assume that a stage s occurs before (or after) another stage t. But if they are linearly ordered, one may ask
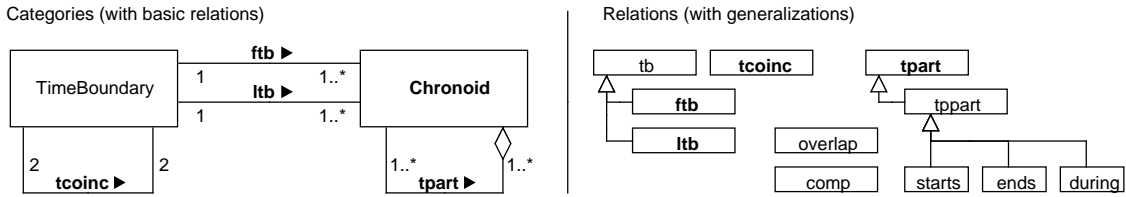
Figure 6.4: Categories and relations of the time theory $\mathcal{BT}^{\mathcal{C}}$ of GFO. Boldface signifies primitives.

which order-type these structures possess. Is there another stage between any two different stages? If the ordering is a dense ordering there may be problems with the measurements of the duration of intervals, due to stages being temporally extended [68].

6.3.5 Time in GFO and Basics of the Material Stratum

6.3.5.1 $\mathcal{BT}^{\mathcal{C}}$ – An Ontology of Chronoids, Boundaries and Coincidence

The basic theory of phenomenal time in GFO is abstracted from real-world entities and is inspired by ideas of Franz Brentano [114]; we refer to it as $\mathcal{BT}^{\mathcal{C}}$. Figure 6.4 provides an overview of its relevant categories and relations, using for relations the mnemonic predicate names introduced in section 6.3.7. Abstract phenomenal time consists of intervals, named *chronoids*, and of *time boundaries*, i.e., time points (roughly speaking). Both are genuine types of entities, where time boundaries depend existentially on chronoids. Only chronoids are subject to *temporal parthood*, and Allen's interval relations (see section 6.3.3) apply to them. Every chronoid has exactly two extremal boundaries, which can be understood as the *first* and *last* time point of it. Further, chronoids are truly extended and have infinitely many *inner time boundaries* that arise from proper subchronoids.

An outstanding and beneficial feature of $\mathcal{BT}^{\mathcal{C}}$ is the relation of *temporal coincidence* between time boundaries, adopted from Brentano. Intuitively, two coinciding time boundaries have temporal distance zero, although they may be distinct entities. By their means, if a chronoid c_1 *meets* a chronoid c_2, both have distinct extremal boundaries without any overlap or gaps between the last boundary of c_1 and the first of c_2. Time boundaries coincide pairwise. Section 6.3.7 captures these and further details of $\mathcal{BT}^{\mathcal{C}}$ axiomatically, from which it is derivable, among others, that time is linear and unbounded, see section 6.3.8.

6.3.5.2 Basics of GFO Related to the Material Stratum

A few further remarks on GFO are required before revisiting the motivating problems from above in section 6.3.6. First of all, GFO adopts the theory of levels of reality, as expounded by Nicolai Hartmann [373] and Roberto Poli [664]. We restrict the exposition to the *material stratum*, the realm of entities including individuals that are in space and time.

According to their relations to time, individuals are classified into *continuants* (being material endurants), material *presentials* and material *processes*. Processes happen in time and are said to have a temporal extension. Continuants persist through time and have a lifetime, which is a chronoid. A continuant exhibits at any time point of its lifetime a uniquely determined entity, called presential, which is wholly present at the (unique) time boundary of its existence.[647] Examples of continuants are this ball and this tree, being persisting entities with a lifetime. Examples of presentials are this ball and this tree, any of them being wholly present at a certain time boundary t. Hence, the specification of a presential additionally requires the declaration of a time boundary.

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[647]Despite employing the widely used term 'continuant', this notion is very specific in GFO. For instance, continuants are not wholly present at time boundaries. Note that earlier accounts of this approach made use of different terms, e.g. *abstract substance* [386] and *perpetuant* [392].

In contrast to a presential, a process cannot be wholly present at a time boundary. Examples of processes are particular cases of the tossing of a ball, a 100 m run as well as a surgical intervention, the conduction of a clinical trial, etc. For any process p having the chronoid c as its *temporal extension*, each temporal part[648] of p is determined by taking a temporal part of c and restricting p to this subchronoid. Similarly, p can be restricted to a time boundary t if the latter is a time boundary or an inner boundary of c. The resulting entity is called a *process boundary*, which does not fall into the category of processes.

6.3.6 New Modeling Contributions to Temporal Phenomena

Based on the previous section, we outline new approaches or contributions to tackling the motivating problems from section 6.3.4.

6.3.6.1 The Holding Problem of Temporal Propositions

Altogether, we propose the ontology $\mathcal{BT}^{\mathcal{C}}$ as a solid foundation for the analysis of temporal phenomena. It appears immediate from section 6.3.5.1 that it provides at least the conceptual means that are known from point-interval theories. The existence of a formal theory interpretation of the theory $\mathcal{IP}$ (and thus $\mathcal{AH}$, see section 6.3.3) into $\mathcal{BT}^{\mathcal{C}}$ provides a formal underpinning to this intuition, see section 6.3.8.3. Beyond common features, a major novel aspect of $\mathcal{BT}^{\mathcal{C}}$ is the notion of coincidence. Coincident time boundaries are important regarding the holding of temporal propositions. They allow for the case that a proposition holds at a time boundary, but it does not hold at its coincident time boundary.

6.3.6.2 The Dividing Instant Problem (DIP)

The DIP finds clear and conclusive solutions based on $\mathcal{BT}^{\mathcal{C}}$, to the extent that the latter provides an expressive foundation for capturing circumstances that "instantiate" the DIP in analysis and applications. At least three of the four basic replies to the DIP can be implemented on this basis.

First, let us return to the example of switching on the light (Figure 6.3). The corresponding analysis yields two processes p and q, where p is extended over a chronoid with last boundary t_l and q stretches over a chronoid with first boundary t_f. We may consistently stipulate that all process boundaries of p exhibit the property *light-off*, whereas *light-on* applies to all of q, and t_l and t_f are distinct, but coincide. This exactly satisfies the requirements set forth at the end of section 6.3.4.2.

In this situation there are two properties, contradicting each other, and holding at two different time points having temporal distance zero. This analysis is adequate because abstract phenomenal time, exhibiting the phenomenon of coincidence, is accessible introspectively without any metrics, whereas the notion of distance is a result of measuring by using an abstract scale of numbers.

The analysis just provided corresponds to the solution of the DIP that the light is "both, on and off" at the dividing "instant" where p and q meet, as which one may consider the pair of coincident boundaries t_l and t_f. Note that the logical framework remains purely classical, including the law of non-contradiction (applicable to each time boundary, whereas pairs of coincident boundaries are not under consideration). The cases of making a specific choice for the dividing instant or allowing for an arbitrary selection both correlate with the temporal assignment of one and the same property or proposition to both time boundaries.

The remaining approach to the DIP was to assume neither on nor off as applicable to the dividing "instant". One interpretation of this view may just be the one above, where each time boundary is assigned on or off, while no uniform statement can be aggregated from the individual time boundaries to the pair of coincident time boundaries. Alternatively and taking "neither on nor off" more literally, we reject logical modifications such as revoking the law of the excluded middle as well as "gaps" in assigning propositions to time boundaries, cf. section 6.3.4.1. For instance, regarding "(not) to be at maximum height" in the example of tossing a ball into the air (Figure 6.2), its positive manifestation applies solely at the last time

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[648]Notably, there are other dimensions by means of which parts of processes can be considered, cf. layers of processes in [398, sect. 8.2.4].

boundary of raising and the first time boundary of falling, which coincide. Moreover, there is no last time boundary in raising to which the negative manifestation applies, nor any such first time boundary in falling.

6.3.6.3 The Continuum

Regarding the continuum itself, actually no *additional* contribution arises from the time ontology adopted in section 6.3.5. However, we stress the inspiration from Brentano's work [114]. His criticism of defining the continuum by the set of real numbers is based on the conviction that the continuum – which is considered by him as a notion that is abstracted from experience – cannot be constructed from numbers by a transfinite inductive procedure. We adopt and defend the corresponding position that the real numbers cannot be identified with the structure of phenomenal time, in contrast to wide-spread "mathematicist's" views, cf. [254, sect. 2–3]. Notably, we do not generally deny the applicability and utility of the typical mathematical modeling of time. Rejecting the said identification rather amounts to a change in the foundation and interpretation of such modeling. Finally, the interpretation of the continuum by means of real numbers provides a metric for time as a basis for measurements.

6.3.6.4 Persistence and Change

A further application of the ontology of time just presented is given by being among the sources of the GFO approach to persistence. Yet the basic assumption of that approach is grounded on the idea of *integrative realism*, originally introduced in [392] and further pursued in [60] and especially [395]. We remark that a similar approach was pursued by the Chinese philosopher Zhang Dongsun, who established the philosophy of epistemological pluralism in his ground-breaking work [894]. The term *integrative realism* denotes a doctrine of realism which postulates that there exists a world, being existentially independent of the mind, but which can be accessed by the mind through mental constructions, called concepts. These mental constructions establish a correspondence relation between the mind (the subject) and the entities of the independent reality. This relation can be understood as *unfolding the real world disposition d in the mind's medium m, resulting in the phenomenon p.* The mind plays an active role in this relation.[649] This kind of realism fits well into the framework of levels and strata of reality [373, 664].

One approach to persistence by endurance refers to a conception of immanence, for which a particular class of universals is utilized. On the one hand, universals are independent of space and time and, consequently, they may be said to persist, in some sense. On the other hand, they have a relation to space and time via their spatio-temporal instances. The idea of accounting for persistence in terms of immanence via universals was pursued with the notion of *persistant* in an earlier version of GFO, cf. [392, esp. sect. 14.4.2], [398, esp. sect. 6, 10]. However, universals are not individuals, and we prefer to understand and accept continuants as individuals. 📖 [398]

The latest and stable solution to the persistence problem in GFO rests on the following ideas. There is a new[650] class of individuals, called *presentials*. Each presential is wholly present at a single, fixed time boundary. *Continuants* constitute another class of individuals. They are cognitive creations of the mind, constructed on the basis of presentials. The idea of the mental creation of continuants from presentials is expounded in more detail in [395, esp. sect. 7.2], and it is claimed that this cognitive construction is grounded on, eventually, personal identity. According to this account, continuants possess features of a universal, occurring as the phenomenon of persistence, but also of spatio-temporal individuals, by being grounded in presentials. We say that a continuant c *exhibits* a presential p, if p exists at a time boundary t and p corresponds to conceiving of c at t (or as viewing p as a "snapshot" of c at t). At every time boundary within its life time a material continuant exhibits a presentic material structure. Continuants may change, because (1) they persist through time and (2) they exhibit different properties at distinct time boundaries of their lifetime (due to exhibiting different presentials). We argue that only persisting individuals may change. Presentials are not subject to change since they exist at unique times / time boundaries. A process

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[649]The investigation of this type of relation is in its initial stages. It can be associated with the mind-body problem [709, 813].

[650]"new" compared to common accounts distinguishing between the two categories of continuants / endurants and perdurants.

as a whole cannot change either, but it may comprise changes or it may be a change. Hence, *to change* and *to have a change* or *to be a change* are different notions.

In GFO, each process has processual boundaries, which can be understood as the restrictions of the process to the time boundaries of its temporal extension. A process is neither the mereological sum of its boundaries, nor can it be identified with the set of its boundaries.[651] There is no way to construct a process from process boundaries, because processes are the more fundamental kind of entity in GFO. Indeed, and similarly to the relation of time boundaries and chronoids, process boundaries are (specifically) existentially dependent on their processes. Presentials participate in processes. More precisely, a presential p participates in a process if and only if there is a boundary of the process of which p is a part (which includes the case that p itself is the process boundary).

The following principle is stipulated for GFO, linking the notions of continuant, presential, and process. It is a core feature that is unique to GFO compared to the top-level ontologies in section 6.3.3.2, but also others. A more formal version in a more elaborated setting is expounded in [395, esp. sect. 7.1].

> **Principle of Object-Process Integration** Let c be a material continuant. Then there exists a uniquely determined material process, denoted by $\mathsf{Proc}(c)$, such that the presentials exhibited by c at the time boundaries of c's lifetime correspond exactly to the process boundaries of $\mathsf{Proc}(c)$; cf. [392, 395, 398].

We adopt the position that a continuant c depends on that process, on the one hand, and, on the other hand, on the mind, since c is supposed to be cognitively created in the framework of GFO. By this principle GFO integrates aspects of 3D and 4D ontologies into one coherent framework.

Let us emphasize that this approach differs from the stage theory as discussed by Theodore Sider [753] and as in David Lewis' approach [522], cf. also [386, 395]. For example, in stage theory processes are considered as mereological sums of stages, temporally extended entities, cf. also sect. 6.3.4.4. In contrast, processes in GFO do not have such stages as smallest parts. As discussed above, a process has process boundaries which are temporally located at time boundaries (and thus not extended) and which are not parts of their processes.

Finally, we find support for the GFO approach to continuants as cognitive constructions in results of cognitive psychology, notably in Gestalt theory [870–872], but also others. For example, there are experiments in cognitive psychology that show that an observer identifies a moving entity through time (say, a running dog), even if this entity is occluded in the visual space during a suitably small amount of time, see [133, 584]. We believe that this phenomenon, known as the tunnel effect, supports the thesis of continuants as cognitive constructions.

6.3.7 Axiomatization of the Ontology $\mathcal{BT}^{\mathcal{C}}$

Section 6.3.5.1 introduces the major notions of the GFO time ontology of chronoids and their boundaries conceptually (e.g. recall Figure 6.4). The present section contains the corresponding axiomatic system $\mathcal{BT}^{\mathcal{C}}$ in first-order predicate logic with equality (FOL), followed by a metalogical analysis in section 6.3.8. The set of axioms reflects important properties of chronoids and time boundaries, assuming the domain of discourse is limited to these entities only.[652] The axiom set is not minimal, e.g. axiom A20 is entailed by others. $\mathcal{BT}^{\mathcal{C}}$ is available[653] in the syntax of the SPASS theorem prover[654] [866], by means of which entailments were checked, including several further consequences, see section 6.3.7.3. We judge almost every axiom on its own to be easily comprehensible. Therefore, the axioms are presented compactly in the style of a catalog[655] with only short explanatory phrases.

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[651] Of course, this set exists, but it is distinct from the process itself.

[652] Thus one cannot simply form the union of the present theory with other formalized components of GFO, but a corresponding relativation or even a theory interpretation will be required for combinations, e.g. cf. section 6.3.11.5.

[653] http://www.onto-med.de/ontologies/gfo-time.dfg

[654] http://www.spass-prover.org

[655] The actual axioms are arranged by adopting the complexity of formulas and mutual relations regarding content as guiding aspects, which also leads to the grouping into three types. Finally, note that all formulas are implicitly universally quantified.

6.3.7.1 Signature and Definitions

The signature splits into basic and defined symbols. The five basic predicates, highlighted through boldface in Figure 6.4, are introduced in Table 6.2 by means of atomic formulas in order to specify their intended informal reading with reference to argument positions through variables. Defined symbols are introduced in the context of their corresponding definition in D1–D12 below. For the purpose of reference, the list of definitions is sorted by the arity of symbols and alphabetically for the same arity.

The use of basic and defined symbols in those definitions is displayed in Table 6.3. Horizontal lines indicate boundaries between levels of dependence, where the definiens symbols of a certain level in the table use only basic symbols or such from the level(s) strictly above. The rows within each level are organized by the number of symbols in the definiens and, if that is equal, by definition number. Considering the vertical direction, the first column states the defined symbol and the associated definition number.[656] The second column on definiens symbols enumerates basic and defined symbols in the corresponding definiens by referring to the numbers of basic symbols and the numbers of definitions for defined symbols, respectively. The third column clusters symbol names into sets that contain exactly those symbols required in the definitions on the overall level. Altogether, Table 6.3 clarifies the interdependencies among defined symbols, in particular demonstrating their acyclicity through the three linearly ordered levels. Moreover, it yields orderings for reading the definitions consecutively and such that all symbols have been introduced before.

Atomic Formula	Intended Reading
B1. Chron (x)	x is a chronoid
B2. lb (x,y)	x is the first boundary of y
B3. rb (x,y)	x is the last boundary of y
B4. tcoinc (x,y)	x and y are coincident
B5. tpart (x,y)	x is a temporal part of y

Table 6.2: Basic signature of $\mathcal{BT}^\mathcal{C}$.

Definition		Definiens Symbols	Clustered
tov	D9	B5	
tppart	D10	B5	(only basic
ft	D11	B2	signature
lt	D12	B3	elements in
comp	D3	B1, B5	definiens, see
tb	D8	B2, B3	Table 6.2)
meets	D6	B1, B2, B3, B4	
before*	D13	B1, B2, B3, B4	
Tb	D1	D8	{Chron,
ends	D5	B1, B3, D10	ftb, ltb,
starts	D7	B1, B2, D10	tb, tppart}
TE	D2	B1, D1	{Chron, Tb,
during	D4	B1, D5, D7, D10	starts, ends,
innerb*	D14	B1, D1, D8, D10	tb, tppart}

Table 6.3: Defined signature with dependencies in $\mathcal{BT}^\mathcal{C}$.

Relations

D1. $\mathsf{Tb}(x) =_{df} \exists y \, \mathsf{tb}(x,y)$ (x is a time boundary)

D2. $\mathsf{TE}(x) =_{df} \mathsf{Chron}(x) \vee \mathsf{Tb}(x)$ (x is a time entity)

D3. $\mathsf{comp}(x,y) =_{df} \mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \exists z(\mathsf{Chron}(z) \wedge \mathsf{tpart}(x,z) \wedge \mathsf{tpart}(y,z))$
                                          (x and y are compatible chronoids)

D4. $\mathsf{during}(x,y) =_{df} \mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \mathsf{tppart}(x,y) \wedge \neg\mathsf{starts}(x,y) \wedge \neg\mathsf{ends}(x,y)$
                                              (Allen's "during" relation [6])

D5. $\mathsf{ends}(x,y) =_{df} \mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \mathsf{tppart}(x,y) \wedge \exists u(\mathsf{rb}(u,x) \wedge \mathsf{rb}(u,y))$
                                              (Allen's "ends" relation [6])

D6. $\mathsf{meets}(x,y) =_{df} \mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \exists uv(\mathsf{rb}(u,x) \wedge \mathsf{lb}(v,y) \wedge \mathsf{tcoinc}(u,v))$
                                              (Allen's "meets" relation [6])

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[656]Note that the definitions D13 and D14 of the two starred symbols are only given in section 6.3.7.3. They are not used in any of the axioms A1–A34, but are included in Table 6.3 to show their dependencies, as well.

D7. $\mathsf{starts}(x,y) =_{df} \mathsf{Chron}(x) \land \mathsf{Chron}(y) \land \mathsf{tppart}(x,y) \land \exists u(\mathsf{lb}(u,x) \land \mathsf{lb}(u,y))$

<div align="right">(Allen's "starts" relation [6])</div>

D8. $\mathsf{tb}(x,y) =_{df} \mathsf{lb}(x,y) \lor \mathsf{rb}(x,y)$ $\qquad\qquad\qquad\qquad$ (x is a time boundary of y)

D9. $\mathsf{tov}(x,y) =_{df} \exists z(\mathsf{tpart}(z,x) \land \mathsf{tpart}(z,y))$ $\qquad\quad$ (temporal overlap of chronoids)

D10. $\mathsf{tppart}(x,y) =_{df} \mathsf{tpart}(x,y) \land x \neq y$ $\qquad\qquad\qquad$ (proper temporal part-of)

Functions

D11. $\mathsf{l}(x) = y \leftrightarrow_{df} \mathsf{lb}(y,x)$ $\qquad\qquad$ (functional lb: the first time boundary of x is y)

D12. $\mathsf{r}(x) = y \leftrightarrow_{df} \mathsf{rb}(y,x)$ $\qquad\qquad$ (functional rb: the last time boundary of x is y)

6.3.7.2 Axioms

Taxonomic Axioms

A1. $\mathsf{TE}(x)$ $\qquad\qquad\qquad$ (the domain of discourse covers only time entities)

A2. $\neg\exists x(\mathsf{Chron}(x) \land \mathsf{Tb}(x))$ $\qquad$ (chronoid and time boundary are disjoint categories)

A3. $\mathsf{Chron}(x) \land \mathsf{Chron}(y) \rightarrow \mathsf{comp}(x,y)$ $\qquad\quad$ (every two chronoids are compatible)

A4. $\mathsf{tb}(x,y) \rightarrow \mathsf{Tb}(x) \land \mathsf{Chron}(y)$ $\qquad$ (tb relates time boundaries with chronoids)

A5. $\mathsf{tcoinc}(x,y) \rightarrow \mathsf{Tb}(x) \land \mathsf{Tb}(y)$ $\qquad$ (coincidence is a relation on time boundaries)

A6. $\mathsf{tpart}(x,y) \rightarrow \mathsf{Chron}(x) \land \mathsf{Chron}(y)$ $\qquad$ (temporal part-of is a relation on chronoids)

Structure of single relations

A7. $\mathsf{Chron}(x) \rightarrow \mathsf{tpart}(x,x)$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (reflexivity)

A8. $\mathsf{tpart}(x,y) \land \mathsf{tpart}(y,x) \rightarrow x = y$ $\qquad\qquad\qquad\qquad$ (antisymmetry)

A9. $\mathsf{tpart}(x,y) \land \mathsf{tpart}(y,z) \rightarrow \mathsf{tpart}(x,z)$ $\qquad\qquad\qquad\qquad$ (transitivity)

A10. $\mathsf{Chron}(x) \rightarrow \exists y(\mathsf{starts}(x,y))$ $\qquad$ (every chronoid has a future extension)

A11. $\mathsf{Chron}(x) \rightarrow \exists y(\mathsf{ends}(x,y))$ $\qquad\qquad$ (every chronoid has a past extension)

A12. $\mathsf{Chron}(x) \rightarrow \exists y(\mathsf{during}(y,x))$ $\qquad$ (during every chronoid there is another one)

A13. $\mathsf{Chron}(x) \rightarrow \exists y(\mathsf{lb}(y,x))$ $\qquad\qquad$ (every chronoid has a first boundary)

A14. $\mathsf{Chron}(x) \rightarrow \exists y(\mathsf{rb}(y,x))$ $\qquad\qquad$ (every chronoid has a last boundary)

A15. $\mathsf{Chron}(x) \land \mathsf{lb}(y,x) \land \mathsf{lb}(z,x) \rightarrow y = z$ $\qquad$ (the first boundary of chronoids is unique)

A16. $\mathsf{Chron}(x) \land \mathsf{rb}(y,x) \land \mathsf{rb}(z,x) \rightarrow y = z$ $\qquad$ (the last boundary of chronoids is unique)

A17. $\mathsf{Tb}(x) \rightarrow \exists y(\mathsf{tb}(x,y))$ $\qquad$ (every time boundary is a boundary of a chronoid)

A18. $\mathsf{Tb}(x) \rightarrow \mathsf{tcoinc}(x,x)$ $\qquad\qquad\qquad\qquad\qquad\qquad$ (reflexivity)

A19. $\mathsf{tcoinc}(x,y) \rightarrow \mathsf{tcoinc}(y,x)$ $\qquad\qquad\qquad\qquad\qquad\quad$ (symmetry)

A20. $\mathsf{tcoinc}(x,y) \land \mathsf{tcoinc}(y,z) \rightarrow \mathsf{tcoinc}(x,z)$ $\qquad\qquad\qquad$ (transitivity)

A21. $\mathsf{Tb}(x) \rightarrow \exists y(x \neq y \land \mathsf{tcoinc}(x,y))$ $\qquad$ (every time boundary coincides with another one)

A22. $\mathsf{tcoinc}(x,y) \land \mathsf{tcoinc}(x,z) \rightarrow x = y \lor x = z \lor y = z$

<div align="right">(at most two distinct time boundaries coincide)</div>

Interaction axioms

A23. $\mathsf{tov}(x,y) \to \exists z(\mathsf{tpart}(z,x) \land \mathsf{tpart}(z,y) \land \forall u(\mathsf{tpart}(u,x) \land \mathsf{tpart}(u,y) \to \mathsf{tpart}(u,z)))$

(two overlapping chronoids have an intersection)

A24. $\mathsf{Chron}(x) \land \mathsf{Chron}(y) \land \neg\mathsf{tpart}(x,y) \to \exists z(\mathsf{tpart}(z,x) \land \neg\mathsf{tov}(z,y))$

(where one chronoid is not a part of another one, there exists a non-overlapping part)

A25. $\mathsf{Chron}(x) \land \mathsf{Chron}(y) \land \mathsf{tcoinc}(\mathsf{l}(x),\mathsf{l}(y)) \land \mathsf{tcoinc}(\mathsf{r}(x),\mathsf{r}(y)) \to x = y$

(there are no distinct chronoids with coincident boundaries)

A26. $\mathsf{tcoinc}(x,y) \to \neg\exists w((\mathsf{lb}(x,w) \land \mathsf{rb}(y,w)) \lor (\mathsf{rb}(x,w) \land \mathsf{lb}(y,w)))$

(coincident boundaries are boundaries of distinct chronoids)

A27. $\mathsf{Tb}(x) \land \mathsf{Tb}(y) \land \neg\mathsf{tcoinc}(x,y) \to$
$\qquad \exists z(\mathsf{Chron}(z) \land ((\mathsf{tcoinc}(x,\mathsf{l}(z)) \land \mathsf{tcoinc}(y,\mathsf{r}(z))) \lor (\mathsf{tcoinc}(x,\mathsf{r}(z)) \land \mathsf{tcoinc}(y,\mathsf{l}(z)))))$

(between any two non-coincident time boundaries there is a corresponding chronoid)

A28. $\mathsf{tcoinc}(x,y) \land \mathsf{lb}(x,u) \land \mathsf{lb}(y,v) \to (\mathsf{tpart}(u,v) \lor \mathsf{tpart}(v,u))$

(coincident first boundaries entail parthood)

A29. $\mathsf{tcoinc}(x,y) \land \mathsf{rb}(x,u) \land \mathsf{rb}(y,v) \to (\mathsf{tpart}(u,v) \lor \mathsf{tpart}(v,u))$

(coincident last boundaries entail parthood)

A30. $\mathsf{Chron}(x) \land \mathsf{Chron}(y) \land \exists u(\mathsf{Chron}(u) \land \mathsf{l}(u) = \mathsf{l}(y) \land \mathsf{tcoinc}(\mathsf{r}(u),\mathsf{l}(x))) \land$
$\qquad \exists v(\mathsf{Chron}(v) \land \mathsf{tcoinc}(\mathsf{l}(v),\mathsf{r}(x)) \land \mathsf{r}(v) = \mathsf{r}(y)) \quad \to \quad \mathsf{during}(x,y)$

(x is during y if embedded between two chronoids with appropriate boundaries)

A31. $\mathsf{tpart}(x,y) \land \mathsf{l}(x) \neq \mathsf{l}(y) \to \exists z(\mathsf{starts}(z,y) \land \mathsf{tcoinc}(\mathsf{r}(z),\mathsf{l}(x)))$

(for every part with distinct first boundaries there is a corresponding starts-fragment)

A32. $\mathsf{tpart}(x,y) \land \mathsf{r}(x) \neq \mathsf{r}(y) \to \exists z(\mathsf{ends}(z,y) \land \mathsf{tcoinc}(\mathsf{l}(z),\mathsf{r}(x)))$

(for every part with distinct end boundaries there is a corresponding ends-fragment)

A33. $\mathsf{Chron}(x) \land \mathsf{Chron}(y) \land \mathsf{meets}(x,y) \to$
$\qquad \exists z(\mathsf{Chron}(z) \land \mathsf{l}(x) = \mathsf{l}(z) \land \mathsf{r}(y) = \mathsf{r}(z) \land \neg\exists u(\mathsf{tpart}(u,z) \land \neg\mathsf{tov}(u,x) \land \neg\mathsf{tov}(u,y)))$

(the sum of meeting chronoids is a chronoid)

A34. $\mathsf{tov}(x,y) \land \exists w(\mathsf{starts}(w,x) \land \neg\mathsf{tov}(w,y)) \to$
$\qquad \exists z(\mathsf{Chron}(z) \land \mathsf{l}(x) = \mathsf{l}(z) \land \mathsf{r}(y) = \mathsf{r}(z) \land \neg\exists u(\mathsf{tpart}(u,z) \land \neg\mathsf{tov}(u,x) \land \neg\mathsf{tov}(u,y)))$

(the sum of overlapping chronoids is a chronoid)

6.3.7.3 Entailments

As stated at the beginning of the current section 6.3.7, $\mathcal{BT}^{\mathcal{C}}$ as specified above is not minimal, i.e., not all axioms are independent of each other. For instance, axiom A20 is derivable (see Table 6.4 below), but it is kept as one of the axioms enforcing coincidence to be an equivalence relation.

Moreover, during the development of the formalization and the proofs of the metalogical results, two further definitions and a number of formulas were considered, the latter as earlier variants of axioms or as useful intermediate steps for proofs.

Table 6.4 lists interrelations within the overall set of axioms and entailments, additionally verified with SPASS [866].

D13. $\mathsf{before}(x,y) =_{df} \exists uvz(\mathsf{Chron}(z) \land \mathsf{lb}(u,z) \land \mathsf{rb}(v,z) \land \mathsf{tcoinc}(u,x) \land \mathsf{tcoinc}(v,y))$

(time boundary x is strictly before time boundary y)

D14. $\mathsf{innerb}(x,y) =_{df} \mathsf{Tb}(x) \land \mathsf{Chron}(y) \land \exists u(\mathsf{tppart}(u,y) \land \mathsf{tb}(x,u)) \land \neg\mathsf{tb}(x,y)$

(x is an inner boundary of y)

Sub theory	Consequence
D1	A17
A5, A18, A19, A22	A20
A26	C5
D1, D8, A18, C5	C1
D7, D8, A4, A15, A19, A20, A21, A22, A26, A28, A31, C1	C3
D5, D8, A4, A15, A19, A20, A21, A22, A26, A29, A32, C1	C4
D1, D8, A5, A21, C3, C4	C2
D1, D7, D5, D8, D10, A4, A14, A18, A19, A22, A25, A32	C6
D1, D7, D5, D8, D10, A4, A13, A18, A19, A22, A25, A31	C7

Table 6.4: Verified entailment relations in system $\mathcal{BT}^{\mathcal{C}}$. The basic order of the rows is by increasing axiom numbers in column "Consequence", but such that formulas used in the proof of a consequence are comprised above that consequence (cf. C5 above C1).

C1. $\mathsf{Chron}(x) \wedge \mathsf{lb}(u,x) \wedge \mathsf{rb}(v,x) \rightarrow u \neq v$ (the boundaries of a chronoid are different)

C2. $\neg\exists xyz(\mathsf{ftb}(x,y) \wedge \mathsf{ltb}(x,z))$ (boundaries are either only first boundaries or last boundaries)

C3. $\mathsf{tcoinc}(x,y) \wedge \mathsf{lb}(x,u) \wedge \mathsf{lb}(y,v) \rightarrow x = y$

                             (coincident first boundaries of two chronoids are equal)

C4. $\mathsf{tcoinc}(x,y) \wedge \mathsf{rb}(x,u) \wedge \mathsf{rb}(y,v) \rightarrow x = y$

                             (coincident last boundaries of two chronoids are equal)

C5. $\mathsf{Chron}(x) \wedge \mathsf{lb}(u,x) \wedge \mathsf{rb}(v,x) \rightarrow \neg\mathsf{tcoinc}(u,v)$ (chronoid boundaries do not coincide)

C6. $\mathsf{starts}(x,y) \rightarrow \exists z(\mathsf{tcoinc}(\mathsf{l}(z),\mathsf{r}(x)) \wedge \mathsf{tcoinc}(\mathsf{r}(z),\mathsf{r}(y)))$ (starts-complement)

C7. $\mathsf{ends}(x,y) \rightarrow \exists z(\mathsf{tcoinc}(\mathsf{l}(z),\mathsf{l}(y)) \wedge \mathsf{tcoinc}(\mathsf{r}(z),\mathsf{l}(x)))$ (ends-complement)

6.3.8 Metalogical Analyses of $\mathcal{BT}^{\mathcal{C}}$

The development of $\mathcal{BT}^{\mathcal{C}}$ is grounded on the axiomatic method, established by David Hilbert [401], and systematically introduced as a basic task of formal ontology, e.g. in [60, sect. 2], remember also section 6.3.2 herein. This method additionally includes the metalogical investigation of the considered theory. Consistency is central in this respect, but not the only issue. In this section we present our metalogical results for $\mathcal{BT}^{\mathcal{C}}$.

6.3.8.1 Linearity of Time

It appears to be intuitive that time is associated with a linear ordering that should be established on the basis of the theory $\mathcal{BT}^{\mathcal{C}}$. Such a linear ordering can be defined on the set of equivalence classes of time boundaries. According to the axioms (A18–A22), the coincidence relation as defined on time boundaries is an equivalence relation whose equivalence classes contain exactly two elements. Let $[x]$ be the equivalence class corresponding to the time boundary x. We introduce a relation $\prec$ ('before') on coincidence classes of time boundaries that is defined by the following condition: $[x] \prec [y]$ iff there exists a chronoid whose first boundary belongs to $[x]$ and whose last boundary belongs to $[y]$.

6.3 Proposition
Let $\mathcal{A}$ be a model of $\mathcal{BT}^{\mathcal{C}}$, and let $[\mathsf{Tb}]$ the set of coincidence classes of time boundaries. Then the relation $\prec$ defines a dense linear ordering (without least and without greatest element) on $[\mathsf{Tb}]$.   ❑

Proof. The following conditions must be proved.

1. Irreflexivity: For any $[a] \in [Tb], [a] \not\prec [a]$.
2. Asymmetry: For any $[a], [b] \in [Tb]$, if $[a] \prec [b]$, then $[b] \not\prec [a]$.
3. Transitivity: For any $[a], [b], [c] \in [Tb]$, if $[a] \prec [b]$ and $[b] \prec [c]$, then $[a] \prec [c]$.
4. Comparison: For any $[a], [b] \in [Tb]$ exactly one of the following three conditions has to hold: $[a] \prec [b], [b] \prec [a]$ or $[a] = [b]$.
5. Density: For any $[a], [b] \in [Tb]$ such that $[a] \prec [b]$ there exists a $[c] \in [Tb]$ satisfying the condition $[a] \prec [c]$ and $[c] \prec [b]$.
6. Unboundedness: For any $[a] \in [Tb]$ there are $[b], [c] \in [Tb]$ such that $[b] \prec [a]$ and $[a] \prec [c]$.

(Irreflexivity) In order to show $[a] \not\prec [a]$, assume the contrary, $[a] \prec [a]$. Then by definition of $\prec$ there exists a chronoid c such that $\mathsf{l}(c) \in [a]$, and $\mathsf{r}(c) \in [a]$, contradicting entailment C5.

(Transitivity) Let $[a] \prec [b]$ and $[b] \prec [c]$. We must show that $[a] \prec [c]$. The assumptions imply the existence of two chronoids u, v such that $\mathsf{l}(u) \in [a], \mathsf{r}(u) \in [b]$, and $\mathsf{l}(v) \in [b], \mathsf{r}(v) \in [c]$. From this follows that $[\mathsf{r}(u)] = [\mathsf{l}(v)]$. Hence, the condition meets(u, v) is satisfied, which by axiom A33 implies the existence of a chronoid w, such that $\mathsf{l}(w) \in [a]$ and $\mathsf{r}(w) \in [c]$, hence $[a] \prec [c]$.

(Asymmetry) Is implied by irreflexivity and transitivity of $\prec$.

(Comparison) Let $[a], [b] \in [\mathsf{Tb}]$, and assume that $[a] \neq [b]$. We need to prove $[b] \prec [a]$ or $[b] \prec [a]$. $[a] \neq [b]$ entails that for every $x \in [a], y \in [b]$ we have $\neg\mathsf{tcoinc}(x, y)$. By axiom A27 there exists a chronoid w such that either $\mathsf{l}(w) \in [x]$ and $\mathsf{r}(w) \in [y]$, which yields the condition $[x] \prec [y]$, and hence $[a] \prec [b]$, or $\mathsf{l}(w) \in [y]$ and $\mathsf{r}(w) \in [x]$, which yields $[y] \prec [x]$, and hence $[b] \prec [a]$.

(Density) Let $[a] \prec [b]$. There exists a chronoid c such that $\mathsf{l}(c) \in [a], \mathsf{r}(c) \in [b]$. By axiom A12 there exists a chronoid u such that the condition during(u, c) holds. Definition D4 together with D5 and D7 entails $\mathsf{ft}(u) \neq \mathsf{ft}(c)$ and $\mathsf{lt}(u) \neq \mathsf{lt}(c)$. With axioms A31 and A32 there are a starts- and an ends-fragment of c that meet u and thus justify the conditions $[a] \prec [\mathsf{l}(u)]$ and $[\mathsf{l}(u)] \prec [b]$.

(Unboundedness) Finally, we prove that the linear ordering $([\mathsf{Tb}], \prec)$ has neither any smallest nor any greatest element. First observe that each chronoid v has future and past extensions due to axioms A10 and A11. Then the existence of complements for starts- and end-fragments according to entailments C6 and C7 in combination with the definition of $\prec$ ensures that the boundaries of those extensions are appropriately positioned, i.e., for every future extension w of v follows $[\mathsf{lt}(v)] \prec [\mathsf{lt}(w)]$, analogously, for every past extension u of v holds $[\mathsf{ft}(u)] \prec [\mathsf{ft}(v)]$. For the reasons that (i) this argumentation applies to every chronoid (including the extensions mentioned), (ii) all chronoids are compatible by A3, and (iii) time boundaries depend on chronoids by A17, there cannot be any smallest or greatest element of $\prec$. ❑

6.4 Corollary

If $[\mathsf{Tb}]$ is a countable set, then the structure $([\mathsf{Tb}], \prec)$ is isomorphic to the linear ordering of the rational numbers. ❑

Proof. By a theorem of Georg Cantor any two countable dense linear orderings without smallest and greatest element are isomorphic [221, p. 149]. ❑

6.3.8.2 Consistency, Completeness and Decidability of the Theory $\mathcal{BT}^{\mathcal{C}}$

A proof of (relative[657]) consistency of $\mathcal{BT}^{\mathcal{C}}$ can be achieved by reduction to the monadic second order theory of linear orderings. Let λ be the order type of the real numbers $\mathbb{R}$. We construct a new linear ordering $\mathcal{M} = (M, \leq)$, the type of which is denoted by $\lambda(2)$, by taking the linear ordered sum over $\mathbb{R}$ of linear orderings of type $\mathbf{2}$.

For an intuition on that sum, imagine the line of the reals on which each number α has been cut into two halves, which themselves are ordered (i.e., one half precedes the other, which is an order of type $\mathbf{2}$). Following the usual "less than" order of the reals (which has type λ), all those distinct orders of two halves each are combined into a single ordering (of type $\lambda(2)$) over the set of all those halves. More precisely, the linear ordering $\mathcal{M} = (M, \leq)$ is constructed as follows. Let $\mathcal{D}_\alpha = (D_\alpha, \leq_\alpha)$ be a linear ordering for every

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[657]as will be clear from below, see footnote 659.

$\alpha \in \mathbb{R}$, such that $\mathcal{D}_\alpha$ is isomorphic to $(\{0, 1\}, \leq^{\mathbb{R}})$,[658] and furthermore, $\mathcal{D}_\alpha \cap \mathcal{D}_\beta = \emptyset$ for every $\alpha \neq \beta$. Then $\mathcal{M} = (M, \leq)$ is defined as follows: $M = \bigcup_{\alpha \in \mathbb{R}} \mathcal{D}_\alpha$, and for any $a, b \in M : a \leq b$ if and only if (i) there exists an $\alpha \in \mathbb{R}$ such that $a, b \in D_\alpha$ and $a \leq_\alpha b$ or (ii) there exist $\alpha, \beta \in \mathbb{R}$ such that $\alpha <^{\mathbb{R}} \beta$ and $a \in D_\alpha, b \in D_\beta$.

Within the monadic second order theory of $\mathcal{M}$ we define the relations $\mathsf{Chron}(x)$, $\mathsf{lb}(x, y)$, $\mathsf{rb}(x, y)$, $\mathsf{tcoinc}(x, y)$, and $\mathsf{tpart}(x, y)$ as follows. $\mathsf{Chron}(x)$ if and only if x is an interval $[a, b]$ over M whose first element a has an immediate predecessor, and whose last element b has an immediate successor. The first and the last boundary of a chronoid is determined by the corresponding elements of the associated interval. Any boundary coincides with itself and any two distinct boundaries coincide if one is an immediate successor of the other. Eventually, temporal part-of corresponds to the subinterval relation, restricted to intervals that satisfy the conditions of a chronoid. In this way we get a new structure $\mathcal{A}$ for which it follows that $\mathcal{A}$ satisfies all axioms of $\mathcal{BT}^\mathcal{C}$.[659]

6.5 Proposition

The theory $\mathcal{BT}^\mathcal{C}$ is consistent. ❑

Proof. Given the considerations above the proposition, it remains to be shown that the axioms A1–A34 are true in the structure $\mathcal{A}$. For most of the axioms these proofs are straightforward, based on general mathematical properties of intervals and linear orderings. Therefore, here we present the interesting cases and only some of the simpler ones.

C2 states that no time boundary is a first boundary of one chronoid and a last time boundary of the same or another chronoid. Although C2 is actually a consequence of a large set of axioms, cf. Table 6.4, it is useful to observe that the following reformulation is satisfied in $\mathcal{A}$: the set of time boundaries, i.e., M in $\mathcal{A}$, is partitioned into a set of first time boundaries and a set of last time boundaries. This follows directly from the chosen linear ordered sum, where each element either has an immediate predecessor or an immediate successor. The former are first time boundaries (only), the latter are last time boundaries (only).

A3 requires mutual compatibility of all chronoids. If u, v are two chronoids in $\mathcal{A}$, then there are elements a, b, c, d such that $u = [a, b]$ and $v = [c, d]$. There is an interval w such that $[a, b] \subseteq w$ and $[c, d] \subseteq w$, namely $w = [\min(\{a, c\}), \max(\{b, d\})]$. This is a chronoid because a and c are first time boundaries (of u and v, resp., and thus in general, cf. the observation w.r.t. C2 above), whereas b and d are last time boundaries. u and v are subintervals of w, because $\min(\{a, c\}) \leq a < b \leq \max(\{b, d\})$ and $\min(\{a, c\}) \leq c < d \leq \max(\{b, d\})$.

Due to the fact that the subinterval relation is a partial order on intervals we deduce that tpart is a partial order on Chron, i.e., axioms A7, A8 and A9 are satisfied in $\mathcal{A}$.

Unboundedness and density of the reals are the main reasons for the fulfillment of axioms A10, A11, and A12 in $\mathcal{A}$, i.e., any chronoid has a future/past extension as well as proper subintervals.

A15 declares the first boundary of each chronoid to be uniquely determined. Due to $[a, b] = [c, d]$ iff $a = c$ and $b = d$, uniqueness of the first boundary is an immediate consequence of the definitions of chronoids as intervals $[a, b]$ over M and of the relation ftb as above. (A16 is proved analogously.)

Due to the construction of $\mathcal{A}$, any element possesses either exactly one predecessor or exactly one successor, and tcoinc of two boundaries is given if they are identical or one is an immediate successor of the other. Consequently, the interpretation of tcoinc in $\mathcal{A}$ is an equivalence relation on Tb in $\mathcal{A}$, i.e., axioms A18, A19, and A20 are satisfied.

A21 says that for every time boundary x there exists a time boundary y such that x and y are distinct and coincide. Let x be a time boundary. By definition of $\mathcal{A}$ there exists a chronoid u such that, w.l.o.g., $x = \mathsf{l}(u)$. Since u satisfies the definition of the predicate Chron, x must possess an immediate predecessor y, hence $x \neq y$, while by definition x and y coincide.

A22 limits the number of coinciding time boundaries to at most two. By the formation of the linear ordered sum, especially using type **2**, there are exactly only pairs of elements that are immediate successors of each

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[658] $\leq^{\mathbb{R}}$ denotes the usual "less than" ordering on the reals, $<^{\mathbb{R}}$ below its strict variant.

[659] The consistency proof is not an absolute one. We construct a structure $\mathcal{A}$ the existence of which must be assured by another theory. It is obvious that the existence of $\mathcal{A}$ can be established on the basis of set theory, say ZFC (Zermelo-Fraenkel with the axiom of choice). Hence, the theory $\mathcal{BT}^\mathcal{C}$ is relatively consistent with respect to ZFC.

other. Since the latter is the definition for the predicate tcoinc, this directly leads to coincidence classes of exactly two elements. (At once this proves the existence of a distinct, but coincident time boundary for each time boundary, required in A21.)

A25 states, in contraposition, that two distinct chronoids do not possess coincident pairs of first and last time boundaries. Accordingly, let $[a, b]$ and $[c, d]$ be chronoids in $\mathcal{A}$ and assume tcoinc(a, c). It follows that $a = c$, because both must have an immediate $\leq$-predecessor by definition of Chron, and as seen for A22, coincidence classes have exactly two elements in $\mathcal{A}$. Analogously $b = d$ holds. Therefore, $[a, b] = [c, d]$ due to the same equivalence for intervals used for A15 above, i.e., there are no distinct chronoids with coincident boundaries.   ❏

Further metalogical results on $\mathcal{BT}^{\mathcal{C}}$ concern its completeness and decidability. The latter means that there is an effective method to decide whether a sentence is a consequence of $\mathcal{BT}^{\mathcal{C}}$, cf. e.g. [221, sect. 2, for these notions].

6.6 Proposition
The theory $\mathcal{BT}^{\mathcal{C}}$ is complete and decidable.   ❏

The proof relies on showing that $\mathcal{BT}^{\mathcal{C}}$ is an ω-categorical theory, i.e., any two countably infinite models of $\mathcal{BT}^{\mathcal{C}}$ are isomorphic. We define a corresponding isomorphism between two arbitrary countably infinite models of $\mathcal{BT}^{\mathcal{C}}$, starting from an isomorphism between their sets of coincidence classes. The latter is justified by Proposition 6.3 and Corollary 6.4, where the latter basically refers to Cantor's theorem that any two countable dense linear orderings are isomorphic [221, p. 149]. ω-categoricity of $\mathcal{BT}^{\mathcal{C}}$ entails its completeness. Eventually, completeness and axiomatizability of a theory yield its decidability [221, p. 147].

Proof. We show that the theory $\mathcal{BT}^{\mathcal{C}}$ is ω-categorical, which entails completeness and, given axiomatizability, decidability.

Let $\mathcal{A}$, $\mathcal{B}$ two arbitrary countably infinite models of $\mathcal{BT}^{\mathcal{C}}$. We consider two structures $([Tb^{\mathcal{A}}], \prec)$ and $([Tb^{\mathcal{B}}], \prec)$,[660] where $\prec$ refers to the 'before' relation among coincidence classes of time boundaries introduced in section 6.3.8.1. By Corollary 6.4 these structures are isomorphic and we fix an isomorphism δ from $([Tb^{\mathcal{A}}], \prec)$ onto $([Tb^{\mathcal{B}}], \prec)$. δ is used to construct an isomorphism α from $\mathcal{A} = (U^{\mathcal{A}}, Tb^{\mathcal{A}}, Chron^{\mathcal{A}}, ftb, ltb, tcoinc, tpart)$ onto $\mathcal{B} = (U^{\mathcal{B}}, Tb^{\mathcal{B}}, Chron^{\mathcal{B}}, ftb, ltb, tcoinc, tpart)$.[660] Note that the axioms of $\mathcal{BT}^{\mathcal{C}}$ ensure that the universes $U^{\mathcal{A}}$ and $U^{\mathcal{B}}$, respectively, are disjoint unions of the corresponding pairs of predicates Tb· and Chron·.

Firstly, we introduce the *characteristic* of a boundary x within $\mathcal{A}$ resp. $\mathcal{B}$ as follows: $ch_{\mathcal{A}}(x) = f$, if x is the first boundary of a chronoid, and $ch_{\mathcal{A}}(x) = l$ if x is the last boundary of a chronoid. Note that the characteristic of a time boundary is uniquely determined, cf. entailment C2 in section 6.3.7.3.

As the next step we specify a function β between the sets $Tb^{\mathcal{A}}$ and $Tb^{\mathcal{B}}$ by $\beta(x) = y$ iff $\delta([x]) = [y]$ and $ch_{\mathcal{A}}(x) = ch_{\mathcal{B}}(y)$. It is easy to show that β is a bijection between the sets $Tb^{\mathcal{A}}$, $Tb^{\mathcal{B}}$ which preserves the characteristics of the boundaries.

The function β is extended to a bijective function $\alpha : U^{\mathcal{A}} \to U^{\mathcal{B}}$, where it remains to define the function α for the set Chron$^{\mathcal{A}}$. Let be $c \in$ Chron$^{\mathcal{A}}$ and for $x, y \in Tb^{\mathcal{A}}$: ftb(x, c) and ltb(y, c), then by the axioms of $\mathcal{BT}^{\mathcal{C}}$ it follows that $[x] \prec [y]$, and hence $\delta([x]) \prec \delta([y])$. Let $\delta([x]) = U$, and $\delta([y]) = V$. $\mathcal{BT}^{\mathcal{C}}$ entails the uniquely determined existence of a chronoid d and time boundaries u, v in $\mathcal{B}$ such that $u \in U$ with ftb(u, d) and $v \in V$ with ltb(v, d), cf. e.g. A15, A16, and A25. Accordingly, we stipulate $\alpha(c) = d$.

It remains to be shown that α is an isomorphism from $\mathcal{A}$ onto $\mathcal{B}$. Obviously, α is an injective function from $Tb^{\mathcal{A}} \cup$ Chron$^{\mathcal{A}}$ into $Tb^{\mathcal{B}} \cup$ Chron$^{\mathcal{B}}$. To see that α is surjective, note first that β is bijective from $Tb^{\mathcal{A}}$ onto $Tb^{\mathcal{B}}$, based on its definition, cf. above. We prove that α is surjective for the set Chron$^{\mathcal{B}}$. Let d be a chronoid of $\mathcal{B}$, for which there must be $u, v \in Tb^{\mathcal{B}}$ with ftb(u, d) and ltb(v, d). We form the classes $\delta^{-1}([u]) = X$ and $\delta^{-1}([v]) = Y$. The axioms of $\mathcal{BT}^{\mathcal{C}}$ imply the existence of a uniquely determined chronoid c and time boundaries x, y such that $x \in X$ with ftb$(x, c) \in X$ and $y \in Y$ with ltb$(y, c) \in Y$. By definition of α this implies $\alpha(c) = d$.

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[660]For readability, we omit structure indices at binary relations since these will be clear from the typing of their arguments by means of indexed unary predicates.

Finally, α must preserve the relations $\mathsf{Chron}(x)$, $\mathsf{Tb}(x)$, $\mathsf{ftb}(x,y)$, $\mathsf{ltb}(x,y)$, $\mathsf{tcoinc}(x,y)$, and $\mathsf{tpart}(x,y)$. The proof of these homomorphy conditions is mainly straightforward, though cumbersome. It is immediately clear that α is compatible with the relations $\mathsf{Chron}(x)$ and $\mathsf{Tb}(x)$, as well as tcoinc. In the sequel we restrict the proof to the relation $\mathsf{ftb}(x,y)$; the compatibility condition for the relation $\mathsf{ltb}(x,y)$ is proved analogously. The case of $\mathsf{tpart}(x,y)$ is omitted.

Homomorphy in the case of ftb means $\mathsf{ftb}(x,c) \leftrightarrow \mathsf{ftb}(\alpha(x),\alpha(c))$ for all $x \in \mathsf{Tb}^{\mathcal{A}}$ and $c \in \mathsf{Chron}^{\mathcal{A}}$.

($\rightarrow$). Let $\mathsf{ftb}(x,c)$, then $ch_{\mathcal{A}}(x) = f$, and hence $ch_{\mathcal{B}}(\alpha(x)) = f$, because of $\alpha(x) = \beta(x)$ (for time boundaries) and of $ch_{\mathcal{A}}(x) = ch_{\mathcal{B}}(\beta(x))$. By the definition of α the chronoid $\alpha(c) = d$ with its first time boundary u, $\mathsf{ftb}(u,d)$, has the property $u \in \delta([x])$. This implies $u = \beta(x)$, because x and $\beta(x)$ have the same characteristic.

($\leftarrow$). This direction is similar to the previous one. ❑

6.3.8.3 Relationship with Established Time Theories

Remembering the available work on time, cf. section 6.3.3, it is of interest to relate $\mathcal{BT}^{\mathcal{C}}$ to other axiomatizations. It is rather straightforward to show that $\mathcal{BT}^{\mathcal{C}}$ covers $\mathcal{IP}_{\text{dense}}$ [848–850] (and thus $\mathcal{AH}$, the well-known theory of Allen and Hayes, as well, see section 6.3.3) in the following sense.

6.7 Proposition
The point-interval theory $\mathcal{IP}_{\text{dense}}$ is interpretable in $\mathcal{BT}^{\mathcal{C}}$. ❑

Theory Interpretation in Logic and Central Proof Ideas   Prior to introducing $\mathcal{IP}_{\text{dense}}$ more precisely in order to prove Proposition 6.7 in some detail, a coarse-grained view on this result and its logical setup shall lay out some intuition.

In mathematical logic, interpreting a theory S into another theory T requires a translation function τ of (roughly[661]) the signature of S into the language of T, where T must satisfy a set of so-called closure axioms CA_τ (as determined by τ), $T \models CA_\tau$. S is called *interpretable* in T, if such τ exists and the result of translating S is already entailed by T, as well, i.e., $T \models CA_\tau \cup \tau(S)$. Put differently, S is interpretable in T if it is "preserved" within T by means of an appropriate "understanding" or "reflection" of the signature of S in terms of the language of T. We generally refer to [693, sect. 6.6] and [221, sect. 2.7] for the mathematical background, which follow slightly different, yet equivalent approaches of capturing those translations formally. Notably, [320, sect. 3.2] recapitulates the definition of (an earlier edition of) [221] and employs it in connection with establishing ontology repositories. Below we basically adopt the definition in [693, sect. 6.6], simplified to the specific case where it suffices to specify a set Δ of explicit definitions for symbols in S and to prove $T \cup \Delta \models S$.

The proof of Proposition 6.7 relies on a quite natural translation of $\mathcal{IP}_{\text{dense}}$ vocabulary into (the language of) $\mathcal{BT}^{\mathcal{C}}$. $\mathcal{IP}_{\text{dense}}$ is based on a distinction between *instants* and *periods*, and contains three primitive relations: *before* is a strict linear order on instants, whereas *begin* and *end* relate instants to those periods of which the former are their initial and terminal instants. The interpretation of this vocabulary into $\mathcal{BT}^{\mathcal{C}}$ associates instants$^{\mathcal{IP}}$ with time boundaries$^{\mathcal{BT}^{\mathcal{C}}}$, and periods$^{\mathcal{IP}}$ with chronoids$^{\mathcal{BT}^{\mathcal{C}}}$. This linkage is in agreement with mapping *before*$^{\mathcal{IP}}(x,y)$ between two instants x and y to the formula $\exists uvz(\mathsf{Chron}(z) \wedge \mathsf{lb}(u,z) \wedge \mathsf{rb}(v,z) \wedge \mathsf{tcoinc}(u,x) \wedge \mathsf{tcoinc}(v,y))$ (cf. the tantamount informal definition of the 'before' relation $\prec$ in section 6.3.8.1), and *begin*$^{\mathcal{IP}}(x,y)$ and *end*$^{\mathcal{IP}}(x,y)$ to the formulas $\exists z(\mathsf{tcoinc}(x,z) \wedge \mathsf{lb}(z,y))$ and $\exists z(\mathsf{tcoinc}(x,z) \wedge \mathsf{rb}(z,y))$, respectively. Observing the occurrence of temporal coincidence in all formulas reflecting these three $\mathcal{IP}_{\text{dense}}$ relations indicates that coincidence requires some care in the interpretation. In particular and unconventionally, equality of instants$^{\mathcal{IP}}$ must be interpreted by $\mathsf{tcoinc}^{\mathcal{BT}^{\mathcal{C}}}$. This interpretation ensures that each instant$^{\mathcal{IP}}$ is aligned with a unique time boundary$^{\mathcal{BT}^{\mathcal{C}}}$, but such that distinct instants$^{\mathcal{IP}}$ yield non-coincident time boundaries$^{\mathcal{BT}^{\mathcal{C}}}$. Importantly and although all of this nicely matches the intuitions behind the two theories, theory interpretation is a formal tool for analyzing theory

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[661] In addition to the non-logical symbols of S, quantification in S needs to be relativized to quantification within T, cf. e.g. [693, p. 258]. This can be realized by a formula with one free variable or by defining an additional unary predicate, neither occurring in S nor T.

interrelations. It cannot necessarily be understood to provide ontological insights across the theories under consideration.

Proof of Interpretability of $\mathcal{IP}$ **into the Theory** $\mathcal{BT}^{\mathcal{C}}$ In preparation of the proof of Proposition 6.7, let us first introduce the theory $\mathcal{IP}_{\text{dense}}$ formally, based on [849, sect. 1.7.2]. Therein, this theory is formulated in two-sorted FOL with one sort for *instants* ($\mathcal{I}$) and one for *periods* ($\mathcal{P}$). Thus, we have the three primitive binary relations of $\mathcal{IP}_{\text{dense}}$ (besides identity): a strict linear order $< \subseteq \mathcal{I} \times \mathcal{I}$ as *before*$^{\mathcal{IP}}$ and two cross-sortal relations $begin, end \subseteq \mathcal{I} \times \mathcal{P}$ as explained above.

Aiming at a clear separation of transitioning from sorted to unsorted FOL on the one hand, and the actual interpretation function in the proof on the other hand, we specify the axioms of $\mathcal{IP}_{\text{dense}}$ in terms of classical FOL. For this purpose, we replace sorts with unary predicates, namely $I(x)$ for instants and $P(x)$ for periods, relativize the formulas in [849, sect. 1.7.2] correspondingly, and augment the theory with axiom IP0 to account for the sort constraints, namely that both sorts form a partitioning of the domain of discourse and are not empty. The symbol $=_{\text{IP}}$ denotes standard identity in the context of $\mathcal{IP}_{\text{dense}}$. To maintain a clearer connection with [849, sect. 1.7.2], we follow its presentation by using variable names i, i', i'' for instants and p, p' for periods on all occurrences of a single sort. The ordering of axioms is maintained, as well, except for density (IP7) being inserted in the block of axioms concerned with instants only. All of that yields the following as theory $\mathcal{IP}_{\text{dense}}$ in FOL (without sorts).

IP0.	$(I(x) \vee P(x)) \wedge \neg \exists x\, (I(x) \wedge P(x)) \wedge \exists x\, I(x) \wedge \exists x\, P(x)$	(sort constraints)
IP1.	$I(i) \rightarrow \neg(i < i)$	(irreflexivity)
IP2.	$I(i) \wedge I(i') \wedge i < i' \rightarrow \neg(i' < i)$	(asymmetry)
IP3.	$I(i) \wedge I(i') \wedge I(i'') \wedge i < i' \wedge i' < i'' \rightarrow i < i''$	(transitivity)
IP4.	$I(i) \wedge I(i') \rightarrow (i < i' \vee i' < i \vee i =_{\text{IP}} i')$	(linearity)
IP5.	$I(i) \rightarrow \exists i'\, (I(i') \wedge i' < i)$	(unboundedness, in left argument)
IP6.	$I(i) \rightarrow \exists i'\, (I(i') \wedge i < i')$	(unboundedness, in right argument)
IP7.	$I(i) \wedge I(i') \wedge i < i' \rightarrow \exists i''\, (I(i'') \wedge i < i'' < i')$	(density)
IP8.	$P(p) \wedge I(i) \wedge I(i') \wedge begin(i, p) \wedge end(i', p) \rightarrow i < i'$	(ordering period endpoints)
IP9.	$P(p) \rightarrow \exists i\, (I(i) \wedge begin(i, p))$	(existence of begin)
IP10.	$P(p) \rightarrow \exists i\, (I(i) \wedge end(i, p))$	(existence of end)
IP11.	$P(p) \wedge I(i) \wedge I(i') \wedge begin(i, p) \wedge begin(i', p) \rightarrow i =_{\text{IP}} i'$	(uniqueness of begin)
IP12.	$P(p) \wedge I(i) \wedge I(i') \wedge end(i, p) \wedge end(i', p) \rightarrow i =_{\text{IP}} i'$	(uniqueness of end)
IP13.	$I(i) \wedge I(i') \wedge i < i' \rightarrow \exists p\, (P(p) \wedge begin(i, p) \wedge end(i', p))$	
	(existence of a period between two ordered instants)	
IP14.	$P(p) \wedge P(p') \wedge I(i) \wedge I(i') \wedge begin(i, p) \wedge end(i', p) \wedge begin(i, p') \wedge end(i', p') \rightarrow p =_{\text{IP}} p'$	
	(uniqueness of the period between two instants)	

Proof. In accordance with [693, sect. 6.6], cf. also [324, sect. II], we will now specify an interpretation of $\mathcal{IP}_{\text{dense}}$ into $\mathcal{BT}^{\mathcal{C}}$ by providing the set Δ of explicit definitions ID1-ID6 for all $\mathcal{IP}_{\text{dense}}$ predicates, where the definientes are limited to the language of $\mathcal{BT}^{\mathcal{C}}$. Note that the interpretation of identity accommodates a special case for instants, relaxing their identity in $\mathcal{IP}_{\text{dense}}$ to temporal coincidence in $\mathcal{BT}^{\mathcal{C}}$.

ID1.	$I(x)$	$=_{df} \mathsf{Tb}(x)$
ID2.	$P(x)$	$=_{df} \mathsf{Chron}(x)$
ID3.	$x < y$	$=_{df} \exists uvz(\mathsf{Chron}(z) \wedge \mathsf{ftb}(u, z) \wedge \mathsf{ltb}(v, z) \wedge \mathsf{tcoinc}(u, x) \wedge \mathsf{tcoinc}(v, y))$
ID4.	$begin(x, y)$	$=_{df} \exists z\, (\mathsf{tcoinc}(x, z) \wedge \mathsf{ftb}(z, y))$

ID5. $end(x, y)$ $=_{df} \exists z \,(\mathsf{tcoinc}(x, z) \wedge \mathsf{ltb}(z, y))$

ID6. $x =_{\text{IP}} y$ $=_{df} x = y \vee (\mathsf{Tb}(x) \wedge \mathsf{Tb}(y) \wedge \mathsf{tcoinc}(x, y))$

In order to prove proposition 6.7, now it suffices to show $\mathcal{BT}^{\mathcal{C}} \cup \Delta \models \mathcal{IP}_{\text{dense}}$. Observe that the interpretation equates the domains of discourse of $\mathcal{IP}_{\text{dense}}$ and $\mathcal{BT}^{\mathcal{C}}$ models, since it dispenses with any domain restriction. Moreover, there are neither function nor individual constant symbols in $\mathcal{IP}_{\text{dense}}$. Accordingly, no additional closure axioms [693, p. 258–259] need to be proved from $\mathcal{BT}^{\mathcal{C}} \cup \Delta$.[662] Subsequently, we provide coarse-grained arguments for the entailment of $\mathcal{IP}_{\text{dense}}$ and only refer to axioms of major relevance, e.g. without resolving all definitions and argument type constraints of relations that are required for verifying these proofs in a theorem prover.

1. The reformulation of the sort constraints in IP0 via Δ into

$$(\mathsf{Tb}(x) \vee \mathsf{Chron}(x)) \wedge \neg \exists x \,(\mathsf{Tb}(x) \wedge \mathsf{Chron}(x)) \wedge \exists x \, \mathsf{Tb}(x) \wedge \exists x \, \mathsf{Chron}(x)$$

    can be easily derived from the "partitioning axioms" for chronoids and time boundaries in $\mathcal{BT}^{\mathcal{C}}$ (A1, A2) and, starting from a non-empty universe, axioms that require mutual existence of chronoids and time boundaries such as A13 and A17.
2. The properties of $<$ as an unbounded dense linear order (IP1-IP7) should be accepted in the light of proposition 6.3 and its proof in section 6.3.8.1, in combination with basically equivalent definitions of the relation $\prec$ ('before') in that section and $<$ in the present context.
3. Assuming the preconditions in IP8 together with the corresponding definitions, its consequence results from a mere application of ID3, the definition of $<$.
4. Existence of begin and end of a period (IP9, IP10) corresponds directly to the existence of first and last time boundaries (A13, A14), with reflexivity of temporal coincidence (A18) closing the technical gap in definitions ID4 and ID5.
5. "Uniqueness" of begin and end of a period (IP11, IP12) derives directly from the uniqueness of first and last time boundaries of chronoids, axioms A15, A16. Temporal coincidence in ID4 and ID5 is not problematic, since the consequents of IP11 and IP12 under the interpretation do no longer enforce identity (within $\mathcal{BT}^{\mathcal{C}}$), but only temporal coincidence in the case of time boundaries. Notably, uniqueness applies to equivalence classes of tcoinc (not available within $\mathcal{BT}^{\mathcal{C}}$, but from the metatheoretic perspective, cf. section 6.3.8.1).
6. Just the definitions of $<$, $begin$, and end (ID3-ID5) entail the existence of a period between two instants that are ordered by $<$.
7. Uniqueness of a period between two instants (IP14) is a consequence of excluding distinct chronoids with coinciding pairs of first and last time boundaries, A25 in $\mathcal{BT}^{\mathcal{C}}$.   ❑

6.3.9 $\mathcal{BT}^{\mathcal{R}}$ – Towards an Ontology of Time Regions

$\mathcal{BT}^{\mathcal{C}}$ as presented in the previous sections focuses on chronoids and time boundaries. Notice that all chronoids are understood to be connected wholes, i.e., without any temporal gaps. In this section we consider the domain of time regions, where a time region is not necessarily connected, in contrast to a chronoid.

6.3.9.1   Motivating Scenarios

There are situations in reality the modeling of which may use time regions. Let us consider the course of a disease, for example, of malaria. One may be interested only in those time intervals in the course of malaria of a patient during which something happens. In general, such a process exhibits a certain pattern, typically including active phases and periods of rest. For instance, in the case of malaria, one may observe hot stages (intervals of raising and then remittent fever) alternating with stationary phases of no fever.

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[662]But note that the sort constraints in IP0 are similar in spirit to some of the closure axioms [693, p. 258–259].
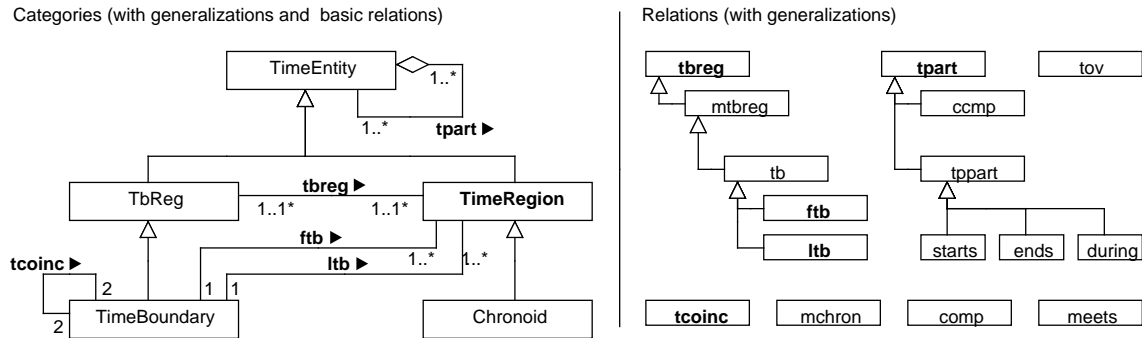
Figure 6.5: Categories and relations of the time theory $\mathcal{BT}^{\mathcal{R}}$ of GFO. Boldface signifies primitives.

Moreover, at the social level many processes proceed with "temporal gaps", such as court cases and lecture series. The representation of corresponding information refers to underlying sequences of non-overlapping chronoids during which the active phases of a process occur. Such sequences of non-overlapping chronoids are subsequently called time regions. They play a significant role in methods of temporal abstraction, i.e., abstracting higher-level concepts from time-stamped data, see [747].

6.3.9.2 Conceptual Outline

In this and the next two sections we sketch an ontology of time regions, termed $\mathcal{BT}^{\mathcal{R}}$, which is intended as an extension of the domain of chronoids axiomatized in $\mathcal{BT}^{\mathcal{C}}$. Analogously to Figure 6.4 in section 6.3.5.1, Figure 6.5 presents the categories and relations for this enhanced ontology of time.

Each *time region* is understood as the mereological sum of a number of *chronoids* (as known from $\mathcal{BT}^{\mathcal{C}}$) which are called its constituting components. Vice versa, chronoids can now be understood as specific time regions, namely exactly as the connected time regions. As in $\mathcal{BT}^{\mathcal{C}}$, any chronoid (and thus any constituent component of a time region) possesses two *time boundaries*, its respective first and last one, each uniquely determined. *Time boundary regions* parallel the notion of time boundaries, but at the level of time regions. They arise as mereological sums of the time boundaries of constituting components of time regions. Accordingly, each time region R has a greatest time boundary region (the sum of all time boundaries of all of R's components), and any time boundary region of R is a part of that greatest time boundary region, where time boundaries constitute minimal parts (or atoms).

A number of basic relations connect instances of time regions and time boundary regions, either within the same category or across the two. *Parthood* in $\mathcal{BT}^{\mathcal{R}}$ has its intuitive meaning, but more generally than in $\mathcal{BT}^{\mathcal{C}}$, it applies separately to time regions and time boundary regions, i.e., both arguments must be of the same kind. The *coincidence relation* tcoinc continues to be applicable only to time boundaries. A notion of generalized temporal coincidence can be defined on this basis such that its arguments can be arbitrary time boundary regions. For linking time regions with their time boundary regions, a general relation of being a *time boundary region* of a time region, tbreg, is introduced. In addition, the relational notions of being the *first* and *last boundary* are extended to time regions and likewise serve as primitive relations, based on the following intuitions. We assume that any time region r is a temporal part of a chronoid, and that there is a (mereologically) least chronoid c containing r as its part. The first boundary of c is then taken to be the first boundary of r (analogously for the last boundary). An alternative view for the first and last boundary of r is the adoption of the first boundary of the first constituting component for the latter. While both intuitions can guide axiom development, it is subject to using $\mathcal{BT}^{\mathcal{R}}$ after its establishment below to prove the equivalence of these connections.

6.3.10 Axiomatization of the Ontology $\mathcal{BT}^{\mathcal{R}}$

In general, the current axiomatization is mainly derived from $\mathcal{BT}^{\mathcal{C}}$ in section 6.3.7, by adapting its defi-nitions and axioms to the more general categories of time regions and time boundary regions, where this appeared appropriate. New axioms that were not derived in this way are marked with $^+$ in section 6.3.10.2. Moreover, we highlight interesting deviations and additions in the respective subsections.

6.3.10.1 Signature and Definitions

The formal ontology of time regions is based on the category of time regions and the five relations discussed in section 6.3.9.2, here introduced as predicates in Table 6.5. These notions are taken as primitives in $\mathcal{BT}^{\mathcal{R}}$, whereas the categories of chronoids (via the notion of connectedness), time boundary regions and time boundaries become definable. A number of further predicates (and functions, for the purpose of readability) are defined in addition in D1–D19, D20 is defined in section 6.3.10.3.[663] Regarding defined symbols and their definitions, we follow the same approach as in the case of $\mathcal{BT}^{\mathcal{C}}$, explained in detail in section 6.3.7.1. In summary, defined symbols are introduced together with their definitions below and Table 6.6 shows their dependencies.[664]

Atomic Formula	Intended Reading
B1. TReg (x)	x is a time region
B2. ftb (x, y)	x is the first boundary of y
B3. ltb (x, y)	x is the last boundary of y
B4. tbreg (x, y)	x is time boundary region of y
B5. tcoinc (x, y)	x and y are coincident
B6. tpart (x, y)	x is a temporal part of y

Table 6.5: Basic signature of $\mathcal{BT}^{\mathcal{R}}$.

Definition		Definiens Symbols	Clustered
TbReg	D4	B4	
tov	D15	B6	(only basic
tppart	D16	B6	signature
ft	D17	B2	elements in
lt	D18	B3	definiens, see
tb	D14	B2, B3	Table 6.5)
meets	D11	B1, B2, B3, B5	
TE	D5	B1, D4	{TReg, TbReg,
mtbreg	D12	B4, D16	tbreg, tppart}
Tb	D3	D12	{TReg, tcoinc,
Conn	D2	B1, B5, D12, D15	mtbreg, tov}
Chron	D1	D2	{Conn}
before*	D20	B2, B3, B5, D1, D3	{TReg, Tb,
comp	D7	B1, B6, D1	Chron, Conn,
mchron	D10	B1, B6, D1	ftb, ltb, tcoinc,
ccmp	D6	B1, B6, D1, D2	tpart}
mchr	D19	D10	{mchron}
ends	D9	B1, B3, B6, D16, D19	{TReg, ftb, ltb,
starts	D13	B1, B2, B6, D16, D19	tpart, tppart,
			mchr}
during	D8	B1, D9, D13, D15, D16	{TReg, tppart,
			tov, starts, ends}

Table 6.6: Defined signature with dependencies in $\mathcal{BT}^{\mathcal{R}}$.

The following signature elements are added to those available in $\mathcal{BT}^{\mathcal{C}}$: Conn for connected time re-gions (without any "gaps" between their components), TbReg capturing time boundary regions as such, ccmp linking maximal connected components to time regions, mchron (as function: mchr), which relates a time region to its minimal covering chronoid, and mtbreg, relating mereologically minimal time boundary regions with the regions they are a boundary of. Furthermore, Chron is now equipped with a definition and most other definitions are adapted, except for l, r, meets, tb, tov, tppart.

Note that the relations starts, ends, and during are originally defined for chronoids (cf. Allen's relations

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[663]Therefore before is starred in Table 6.6.

[664]Definitions in lower levels in Table 6.6 use only symbols from levels strictly above, i.e., the dependencies are acyclic.

[6]). One option of adapting these to time regions would be to consider those relations componentwise, i.e., for all chronoid components of two time regions. Since time regions can possess many connected components this leads to a very large number of conditions expressing how these chronoid components are associated in terms of starts, ends, and during. More precisely, consider two time regions with m and n chronoid components, respectively. Each of the m components of one time region may be related by any of those relations with any of the n components of the other time region. All such combinations can be explicitly defined, which results in a set $DF^{sed}(m,n)$ of definitions. For describing the possible combinations of associations which may hold between two time regions in general, we then get an infinite set of definitions $Comb^{sed} = \bigcup_{m,n\in\mathbb{N}} DF(m,n)$, i.e., for arbitrary combinations of numbers of components. For our purposes it is inappropriate to introduce a large number of specific predicates for those definitions. Another option is to consider generalizations of starts, ends, and during that can be formulated independently of the specific number of chronoid components of the arguments. Again, there is a variety of possibilities. The one adopted below is motivated by maintaining the following interplay between the three: for each time region R, any pair of a starts fragment S and an ends fragment E of R (that do not meet) determines a during fragment D, such that the mereological sum of S, D, and E yields R – and vice versa. For chronoids, i.e., connected time regions, this corresponds to Allen's relations.

Notably, the relations ftb, ltb and mchron are functional in the respective argument, either by stipulation as axioms or as a consequence of the axioms below. Hence we may introduce functional expressions.

Relations

D1. $\mathsf{Chron}(x) =_{df} \mathsf{Conn}(x)$ 　　　　　　　　　　　　　　　　　　(x is a chronoid)

D2. $\mathsf{Conn}(x) =_{df} \mathsf{TReg}(x) \wedge$ 　　　　　　　　　　　　(x is a connected time region)
　　　　$\forall yz(\ \mathsf{TReg}(y) \wedge \mathsf{TReg}(z) \wedge \neg\mathsf{tov}(y,z) \wedge \forall w(\mathsf{tov}(w,y) \vee \mathsf{tov}(w,z) \leftrightarrow \mathsf{tov}(w,x))\ \rightarrow$
　　　　　　$\exists uv(\mathsf{mtbreg}(u,y) \wedge \mathsf{mtbreg}(v,z) \wedge \mathsf{tcoinc}(u,v))\ )$

D3. $\mathsf{Tb}(x) =_{df} \exists y\ \mathsf{mtbreg}(x,y)$ 　　　　　　　　　　　　　　　　(x is a time boundary)

D4. $\mathsf{TbReg}(x) =_{df} \exists y\ \mathsf{tbreg}(x,y)$ 　　　　　　　　　　　　　　(x is a time boundary region)

D5. $\mathsf{TE}(x) =_{df} \mathsf{TReg}(x) \vee \mathsf{TbReg}(x)$ 　　　　　　　　　　　　　　(x is a time entity)

D6. $\mathsf{ccmp}(x,y) =_{df} \mathsf{Chron}(x) \wedge \mathsf{TReg}(y) \wedge$ 　　　　(x is a maximal connected component of y)
　　　　$\mathsf{tpart}(x,y) \wedge \forall z(\mathsf{Conn}(z) \wedge \mathsf{tpart}(x,z) \wedge \mathsf{tpart}(z,y) \rightarrow \mathsf{tpart}(z,x))$

D7. $\mathsf{comp}(x,y) =_{df} \mathsf{TReg}(x) \wedge \mathsf{TReg}(y) \wedge \exists z(\mathsf{Chron}(z) \wedge \mathsf{tpart}(x,z) \wedge \mathsf{tpart}(y,z))$
　　　　　　　　　　　　　　　　　　　　　　　　　　(x and y are compatible time regions)

D8. $\mathsf{during}(x,y) =_{df} \mathsf{TReg}(x) \wedge \mathsf{TReg}(y) \wedge \mathsf{tppart}(x,y) \wedge$ 　　(generalized "during" relation)
　　　　$\exists uv(\mathsf{starts}(u,y) \wedge \mathsf{ends}(v,y) \wedge \neg\mathsf{tov}(u,x) \wedge \neg\mathsf{tov}(x,v) \wedge \neg\mathsf{tov}(u,v) \wedge$
　　　　$\exists w(\forall s(\mathsf{tov}(s,u) \vee \mathsf{tov}(s,x) \leftrightarrow \mathsf{tov}(s,w)) \wedge \forall t(\mathsf{tov}(t,w) \vee \mathsf{tov}(t,v) \leftrightarrow \mathsf{tov}(t,y))))$ [665]

D9. $\mathsf{ends}(x,y) =_{df} \mathsf{TReg}(x) \wedge \mathsf{TReg}(y) \wedge \mathsf{tppart}(x,y) \wedge \mathsf{tppart}(\mathsf{mchr}(x),\mathsf{mchr}(y)) \wedge$
　　　　$\exists z(\mathsf{rb}(z,x) \wedge \mathsf{rb}(z,y)) \wedge \forall u(\mathsf{tpart}(u,\mathsf{mchr}(x)) \wedge \mathsf{tpart}(u,y) \rightarrow \mathsf{tpart}(u,x))$
　　　　　　　　　　　　　　　　　　　　　　　　　　　　(generalized "ends" relation)

D10. $\mathsf{mchron}(x,y) =_{df} \mathsf{Chron}(x) \wedge \mathsf{TReg}(y) \wedge$ 　　　(x is the minimal chronoid containing y)
　　　　$\mathsf{tpart}(y,x) \wedge \forall z(\mathsf{Chron}(z) \wedge \mathsf{tpart}(y,z) \rightarrow \mathsf{tpart}(x,z))$

D11. $\mathsf{meets}(x,y) =_{df} \mathsf{TReg}(x) \wedge \mathsf{TReg}(y) \wedge \exists uv(\mathsf{ftb}(u,y) \wedge \mathsf{ltb}(v,x) \wedge \mathsf{tcoinc}(u,v))$
　　　　　　　　　　　　　　　　　　　　　　　　　　　　(generalized "meets" relation)

D12. $\mathsf{mtbreg}(x,y) =_{df} \mathsf{tbreg}(x,y) \wedge \neg\exists z(\mathsf{tppart}(z,x))$ 　　(x is a minimal time boundary region of y)

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[665] The definition of x during y requires a complementary starts-fragment u of y and an ends-fragment v of y for each x during y, such that the mereological sum of u, x, and v yields exactly y. With a defined notion of *temporal sum* according to $\mathsf{tsum}(x,y,z) =_{df} \forall u(\mathsf{tov}(u,x) \vee \mathsf{tov}(u,y) \leftrightarrow \mathsf{tov}(u,z))$ for z being the sum of x and y, the last line of D8 could be simplified to $\exists w(\mathsf{tsum}(u,x,w) \wedge \mathsf{tsum}(w,v,y))$. However, we do not introduce temporal sum, intersection and complement explicitly in $\mathcal{BT}^\mathcal{R}$ at this stage.

D13. $\text{starts}(x, y) =_{df} \text{TReg}(x) \wedge \text{TReg}(y) \wedge \text{tppart}(x, y) \wedge \text{tppart}(\text{mchr}(x), \text{mchr}(y)) \wedge$
$\exists z (\text{ftb}(z, x) \wedge \text{ftb}(z, y)) \wedge \forall u (\text{tpart}(u, \text{mchr}(x)) \wedge \text{tpart}(u, y) \rightarrow \text{tpart}(u, x))$
(generalized "starts" relation)

D14. $\text{tb}(x, y) =_{df} \text{ftb}(x, y) \vee \text{ltb}(x, y)$ (x is a first or last time boundary of y)

D15. $\text{tov}(x, y) =_{df} \exists z (\text{tpart}(z, x) \wedge \text{tpart}(z, y))$ (temporal overlap)

D16. $\text{tppart}(x, y) =_{df} \text{tpart}(x, y) \wedge x \neq y$ (proper temporal part-of)

Functions

D17. $\text{ft}(x) = y \leftrightarrow_{df} \text{ftb}(y, x)$ (the first time boundary of x is y)

D18. $\text{lt}(x) = y \leftrightarrow_{df} \text{ltb}(y, x)$ (the last time boundary of x is y)

D19. $\text{mchr}(x) = y \leftrightarrow_{df} \text{mchron}(y, x)$ (the minimal chronoid containing x is y)

6.3.10.2 Axioms

Taxonomic Axioms

A1. $\text{TE}(x)$ (the domain of discourse covers only time entities)

A2. $\neg \exists x (\text{TReg}(x) \wedge \text{TbReg}(x))$ (time region and time boundary region are disjoint categories)

A3. $\text{TReg}(x) \wedge \text{TReg}(y) \rightarrow \text{comp}(x, y)$ (every two time regions are compatible)

A4. $\text{tb}(x, y) \rightarrow \text{Tb}(x) \wedge \text{TReg}(y)$ (tb relates time boundaries with time regions)

A5.$^{+}$ $\text{tb}(x, y) \rightarrow \text{mtbreg}(x, y)$ (time boundaries of a time region are minimal boundary regions of it)

A6.$^{+}$ $\text{tbreg}(x, y) \rightarrow \text{TbReg}(x) \wedge \text{TReg}(y)$ (tbreg relates time boundary regions with time regions)

A7. $\text{tcoinc}(x, y) \rightarrow \text{Tb}(x) \wedge \text{Tb}(y)$ (coincidence is a relation on time boundaries)

A8. $\text{tpart}(x, y) \rightarrow (\text{TReg}(x) \wedge \text{TReg}(y)) \vee (\text{TbReg}(x) \wedge \text{TbReg}(y))$
(tpart is a relation on either time regions or time boundary regions)

Structure of single relations

A9. $\text{TE}(x) \rightarrow \text{tpart}(x, x)$ (reflexivity)

A10. $\text{tpart}(x, y) \wedge \text{tpart}(y, x) \rightarrow x = y$ (antisymmetry)

A11. $\text{tpart}(x, y) \wedge \text{tpart}(y, z) \rightarrow \text{tpart}(x, z)$ (transitivity)

A12.$^{+}$ $\text{TReg}(x) \rightarrow \exists y (\text{Chron}(y) \wedge \text{tpart}(y, x))$ (every time region has a chronoid as its part)

A13. $\text{TReg}(x) \rightarrow \exists y (\text{Chron}(y) \wedge \text{starts}(\text{mchr}(x), y))$ (every time region has a future extension)

A14. $\text{TReg}(x) \rightarrow \exists y (\text{Chron}(y) \wedge \text{ends}(\text{mchr}(x), y))$ (every time region has a past extension)

A15. $\text{TReg}(x) \rightarrow \exists y (\text{during}(y, x))$ (during every time region there is another one)

A16. $\text{TReg}(x) \rightarrow \exists y (\text{ftb}(y, x))$ (every time region has a first time boundary)

A17. $\text{TReg}(x) \rightarrow \exists y (\text{ltb}(y, x))$ (every time region has a last time boundary)

A18. $\text{TReg}(x) \wedge \text{ftb}(y, x) \wedge \text{ftb}(z, x) \rightarrow y = z$ (the first boundary of time regions is unique)

A19. $\text{TReg}(x) \wedge \text{ltb}(y, x) \wedge \text{ltb}(z, x) \rightarrow y = z$ (the last boundary of time regions is unique)

A20.$^{+}$ $\text{TReg}(x) \wedge \text{TReg}(y) \rightarrow \exists z (\forall w (\text{tov}(w, x) \vee \text{tov}(w, y) \leftrightarrow \text{tov}(w, z)))$
(for every pair of time regions x and y, their mereological union exists)

A21. $\text{TbReg}(x) \rightarrow \exists y (\text{tbreg}(x, y))$ (time boundary regions are boundary regions of time regions)

A22. $\text{Tb}(x) \rightarrow \text{tcoinc}(x, x)$ (reflexivity)

A23. $\mathsf{tcoinc}(x, y) \rightarrow \mathsf{tcoinc}(y, x)$ (symmetry)

A24. $\mathsf{tcoinc}(x, y) \wedge \mathsf{tcoinc}(y, z) \rightarrow \mathsf{tcoinc}(x, z)$ (transitivity)

A25. $\mathsf{Tb}(x) \rightarrow \exists y (x \neq y \wedge \mathsf{tcoinc}(x, y))$ (every time boundary coincides with another one)

A26. $\mathsf{tcoinc}(x, y) \wedge \mathsf{tcoinc}(x, z) \rightarrow x = y \vee x = z \vee y = z$
(at most two distinct time boundaries coincide)

Interaction axioms

A27. $\mathsf{tov}(x, y) \rightarrow \exists z (\mathsf{tpart}(z, x) \wedge \mathsf{tpart}(z, y) \wedge \forall u (\mathsf{tpart}(u, x) \wedge \mathsf{tpart}(u, y) \leftrightarrow \mathsf{tpart}(u, z)))$
(two overlapping time regions have an intersection)

A28.⁺ $\mathsf{TReg}(x) \wedge \mathsf{TReg}(y) \wedge x \neq y \rightarrow \exists z (\forall u (\mathsf{tpart}(u, z) \leftrightarrow \mathsf{tpart}(u, x) \wedge \neg \mathsf{tov}(u, y)))$
(for distinct time regions, there is the relative complement)

A29. $\mathsf{TReg}(x) \wedge \mathsf{TReg}(y) \wedge \neg \mathsf{tpart}(x, y) \rightarrow \exists z (\mathsf{tpart}(z, x) \wedge \neg \mathsf{tov}(z, y))$
(where one time region is not a part of another one, there exists a non-overlapping part)

A30.⁺ $\mathsf{TReg}(x) \rightarrow \forall y (\mathsf{mchron}(y, x) \leftrightarrow \mathsf{Chron}(y) \wedge \mathsf{ft}(x) = \mathsf{ft}(y) \wedge \mathsf{lt}(x) = \mathsf{lt}(y))$
(the least containing chronoid of a time region is the one between its boundaries)

A31. $\mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \mathsf{tcoinc}(\mathsf{l}(x), \mathsf{l}(y)) \wedge \mathsf{tcoinc}(\mathsf{r}(x), \mathsf{r}(y)) \rightarrow x = y$
(there are no distinct chronoids with coincident boundaries)

A32. $\mathsf{tcoinc}(x, y) \rightarrow \neg \exists z ((\mathsf{lb}(x, z) \wedge \mathsf{rb}(y, z)) \vee (\mathsf{rb}(x, z) \wedge \mathsf{lb}(y, z)))$
(coincident boundaries are boundaries of distinct time regions)

A33. $\mathsf{Tb}(x) \wedge \mathsf{Tb}(y) \wedge \neg \mathsf{tcoinc}(x, y) \rightarrow$
$\exists z (\mathsf{Chron}(z) \wedge ((\mathsf{tcoinc}(x, \mathsf{l}(z)) \wedge \mathsf{tcoinc}(y, \mathsf{r}(z))) \vee (\mathsf{tcoinc}(x, \mathsf{r}(z)) \wedge \mathsf{tcoinc}(y, \mathsf{l}(z)))))$
(between any two non-coincident time boundaries there is a corresponding chronoid)

A34. $\mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \mathsf{lb}(u, x) \wedge \mathsf{lb}(v, y) \wedge \mathsf{tcoinc}(u, v) \rightarrow (\mathsf{tpart}(x, y) \vee \mathsf{tpart}(y, x))$
(for chronoids, coincident first boundaries entail parthood of the chronoids)

A35. $\mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \mathsf{rb}(u, x) \wedge \mathsf{rb}(v, y) \wedge \mathsf{tcoinc}(u, v) \rightarrow (\mathsf{tpart}(x, y) \vee \mathsf{tpart}(y, x))$
(for chronoids, coincident last boundaries entail parthood of the chronoids)

A36. $\mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \exists u (\mathsf{Chron}(u) \wedge \mathsf{l}(u) = \mathsf{l}(y) \wedge \mathsf{tcoinc}(\mathsf{r}(u), \mathsf{l}(x))) \wedge$
$\exists v (\mathsf{Chron}(v) \wedge \mathsf{tcoinc}(\mathsf{l}(v), \mathsf{r}(x)) \wedge \mathsf{r}(v) = \mathsf{r}(y)) \quad \rightarrow \quad \mathsf{during}(x, y)$
(chronoid x is during y if x is embedded between two chronoids with appropriate boundaries)

A37. $\mathsf{TReg}(x) \wedge \mathsf{TReg}(y) \wedge \mathsf{tpart}(x, y) \wedge \mathsf{l}(x) \neq \mathsf{l}(y) \rightarrow \exists z (\mathsf{starts}(z, y) \wedge \mathsf{tcoinc}(\mathsf{r}(z), \mathsf{l}(x)))$
(for every part with distinct first boundaries there is a corresponding starts-fragment)

A38. $\mathsf{TReg}(x) \wedge \mathsf{TReg}(y) \wedge \mathsf{tpart}(x, y) \wedge \mathsf{r}(x) \neq \mathsf{r}(y) \rightarrow \exists z (\mathsf{ends}(z, y) \wedge \mathsf{tcoinc}(\mathsf{l}(z), \mathsf{r}(x)))$
(for every part with distinct end boundaries there is a corresponding ends-fragment)

A39. $\mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \mathsf{meets}(x, y) \rightarrow$ (the sum of meeting chronoids is a chronoid)
$\exists z (\mathsf{Chron}(z) \wedge \mathsf{l}(x) = \mathsf{l}(z) \wedge \mathsf{r}(y) = \mathsf{r}(z) \wedge \neg \exists u (\mathsf{tpart}(u, z) \wedge \neg \mathsf{tov}(u, x) \wedge \neg \mathsf{tov}(u, y)))$

A40. $\mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \mathsf{tov}(x, y) \wedge \exists w (\mathsf{starts}(w, x) \wedge \neg \mathsf{tov}(w, y)) \rightarrow$
$\exists z (\mathsf{Chron}(z) \wedge \mathsf{l}(x) = \mathsf{l}(z) \wedge \mathsf{r}(y) = \mathsf{r}(z) \wedge \neg \exists u (\mathsf{tpart}(u, z) \wedge \neg \mathsf{tov}(u, x) \wedge \neg \mathsf{tov}(u, y)))$
(the sum of overlapping chronoids is a chronoid)

6.3.10.3 Entailments

In this section we have gathered consequences of the axiomatization $\mathcal{BT}^{\mathcal{R}}$. They are utilized in proofs below or constitute intermediate steps to such entailments that are used. Table 6.7 lists them together with the axioms from which they can be proved. The row with consequence A29 illustrates that $\mathcal{BT}^{\mathcal{R}}$ is not constituted by a set of independent axioms, as is the case for $\mathcal{BT}^{\mathcal{C}}$.

Sub theory	Consequence
D8, D16, A15, A28	A29
D1, D2	C1
D17, D14, A4	C2
D18, D14, A4	C3
D1, D2, D5, D6, A9, A10	C4
D10, D19	C5
D10, A10	C6
A16, A17, A30, A31, A32, A33	C7
D8, D16, A10, A11, A12, A15	C8
D15, A11	C9
D17, D20, A22, A37, C2	C10
D18, D20, A22, A38, C3	C11
D17, D18, D20, A1, A9, A24, A31, A34, A35, C2, C3, C11, C2$^{\mathcal{BT}^{\mathcal{C}}}$	C13
D14, D11, D17, D18, D20, A4, A16, A17, A22, A24, A39, C1, C2, C3, C10, C11, C13, C3$^{\mathcal{BT}^{\mathcal{C}}}$, C4$^{\mathcal{BT}^{\mathcal{C}}}$	C12

Table 6.7: Entailment relations in system $\mathcal{BT}^{\mathcal{R}}$.

We anticipate the theory interpretation of $\mathcal{BT}^{\mathcal{C}}$ into $\mathcal{BT}^{\mathcal{R}}$ that is presented in section 6.3.11.5 below, which yields a way to transfer $\mathcal{BT}^{\mathcal{C}}$ entailments to $\mathcal{BT}^{\mathcal{R}}$. A formula reference equipped with the superscript $^{\mathcal{BT}^{\mathcal{C}}}$ indicates that an entailment of $\mathcal{BT}^{\mathcal{C}}$ in section 6.3.7.3 is reused in its interpreted version, i.e., with the definitions in the proof of Proposition 6.15 in section 6.3.11.5 applied. The reintroduction of before in D20 is an example of such a "rewriting" that we employ later, another one is contained in the proof of Lemma 6.8 below the entailments.

D20. $\text{before}(x,y) =_{df} \exists uvz(\text{Chron}(z) \wedge \text{Tb}(u) \wedge \text{Tb}(v) \wedge \text{ftb}(u,z) \wedge \text{ltb}(v,z) \wedge$
$\quad\quad\quad\quad\quad\quad\quad\quad \text{tcoinc}(u,x) \wedge \text{tcoinc}(v,y))$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ (time boundary x is strictly before time boundary y)

C1. $\text{Chron}(x) \rightarrow \text{TReg}(x)$ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ (chronoids are time regions)

C2. $\text{ftb}(x,y) \rightarrow \text{Tb}(x)$ $\quad\quad\quad\quad\quad\quad\quad\quad$ (first time boundaries are time boundaries)

C3. $\text{ltb}(x,y) \rightarrow \text{Tb}(x)$ $\quad\quad\quad\quad\quad\quad\quad\quad$ (last time boundaries are time boundaries)

C4. $\text{Chron}(x) \leftrightarrow \text{ccmp}(x,x)$
$\quad\quad\quad\quad$ (chronoids are exactly those time regions that are their own connected component)

C5. $\text{Chron}(x) \leftrightarrow \text{mchr}(x) = x$
$\quad\quad\quad\quad$ (chronoids are exactly those time regions that are their own least containing chronoids)

C6. $\text{TReg}(x) \wedge \text{mchron}(y,x) \wedge \text{mchron}(z,x) \rightarrow y = z$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ (minimal chronoids of a time region are uniquely determined)

C7. $\text{TReg}(x) \rightarrow \exists y(\text{mchron}(y,x))$ $\quad\quad\quad\quad\quad$ (each time region is framed by a least chronoid)

C8. $\text{TReg}(x) \rightarrow \exists y(\text{Chron}(y) \wedge \text{tppart}(y,x))$ $\quad\quad$ (every time region has a chronoid as proper part)

C9. $\text{Chron}(x) \wedge \text{Chron}(y) \wedge \text{Chron}(z) \wedge \text{tpart}(x,y) \wedge \text{tov}(x,z) \rightarrow \text{tov}(y,z)$
$\quad\quad\quad\quad\quad\quad$ (a chronoid that overlaps a part of another chronoid overlaps the latter itself)

C10. $\text{Chron}(x) \wedge \text{Chron}(y) \wedge \text{tpart}(x,y) \rightarrow \text{before}(\text{ft}(y), \text{ft}(x)) \vee \text{tcoinc}(\text{ft}(y), \text{ft}(x))$
$\quad\quad\quad\quad\quad\quad$ (the first boundary of a chronoid is before or coincides with those of its parts)

C11. $\text{Chron}(x) \wedge \text{Chron}(y) \wedge \text{tpart}(x,y) \rightarrow \text{before}(\text{lt}(x), \text{lt}(y)) \vee \text{tcoinc}(\text{lt}(x), \text{lt}(y))$
$\quad\quad\quad\quad\quad\quad$ (the last boundary of a chronoid is after or coincides with those of its parts)

C12. $\mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \mathsf{Chron}(z) \wedge \mathsf{tpart}(x,y) \wedge \mathsf{meets}(x,z) \rightarrow \mathsf{meets}(y,z) \vee \mathsf{tov}(y,z)$
　　　　　　(a chronoid, a part of which meets another chronoid, meets or overlaps the latter)

C13. $\mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \mathsf{before}(\mathsf{ft}(x), \mathsf{lt}(y)) \wedge \mathsf{before}(\mathsf{ft}(y), \mathsf{lt}(x)) \rightarrow \mathsf{tov}(x,y)$
　　　　　　(chronoids with mutually interlaced "first boundary-last boundary" pairs overlap)

Entailments C9 and C12 are important for the proofs of Propositions 6.9 and 6.10. Moreover, especially C12 is based on an extensive set of prerequisites. For these reasons and in order to illustrate the kind of proofs underlying Table 6.7, we show the subsequent lemma.

6.8 Lemma
In the situation that a part x of a chronoid y meets or overlaps another chronoid z, then the whole y itself meets or overlaps z. C9 and C12 capture this connection formally[666]:

　　C9.　　$\mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \mathsf{Chron}(z) \wedge \mathsf{tpart}(x,y) \wedge \mathsf{tov}(x,z) \rightarrow \mathsf{tov}(y,z)$
　　C12.　$\mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \mathsf{Chron}(z) \wedge \mathsf{tpart}(x,y) \wedge \mathsf{meets}(x,z) \rightarrow \mathsf{meets}(y,z) \vee \mathsf{tov}(y,z)$ ❏

Proof. Starting with C9, let c, d, and e be chronoids such that c is a part of d and overlaps e. By D15, overlap between c and e yields a shared part, which is also shared between d and e, because tpart is transitive (A11). Hence, e overlaps d, as well, concluding the proof of C9.

For C12, let a chronoid c be part of a chronoid d and $\mathsf{meets}(c,e)$ for a chronoid e. We must show that d meets e or that d and e overlap. In the remainder of this proof we rely on the fact that c, d, and e have first and last time boundaries based on axioms A16–17 (aux.[667] C1), e.g., when using the function symbols ft and lt. Moreover, for the latter symbols, we mention only once here that D17 and D18 are required in several proof steps to switch between ft and ftb, on the one hand, and lt and ltb on the other hand.

Due to $\mathsf{tpart}(c,d)$ and C11, (1) $\mathsf{lt}(c)$ is before $\mathsf{lt}(d)$ or (2) the two boundaries coincide. In the latter case, employing $\mathsf{C4}^{\mathcal{BT}^{\mathcal{C}}}$: $\mathsf{Tb}(x) \wedge \mathsf{Tb}(y) \wedge \mathsf{Chron}(u) \wedge \mathsf{Chron}(v) \wedge \mathsf{tcoinc}(x,y) \wedge \mathsf{ltb}(x,u) \wedge \mathsf{ltb}(y,v) \rightarrow x = y$, the reflection of C4 from $\mathcal{BT}^{\mathcal{C}}$ via the interpretation in section 6.3.11.5, we deduce (aux. D14, A4) that c and d have the same last time boundary. Given the premise $\mathsf{meets}(c,e)$, D11 yields that $\mathsf{lt}(c)$ and $\mathsf{ft}(e)$ must coincide, thus with $\mathsf{lt}(c) = \mathsf{lt}(d)$ also $\mathsf{tcoinc}(\mathsf{lt}(d), \mathsf{ft}(e))$. But then d meets e, as a consequence of D11 (aux. C1), concluding case (2) successfully.

In case (1) we have $\mathsf{before}(\mathsf{lt}(c), \mathsf{lt}(d))$ and, as just seen, the premise $\mathsf{meets}(c,e)$ entails that $\mathsf{lt}(c)$ and $\mathsf{ft}(e)$ coincide. The order between $\mathsf{lt}(c)$ and $\mathsf{lt}(d)$, the latter coincidence and the definition of before (D20, aux. A24) imply $\mathsf{before}(\mathsf{ft}(e), \mathsf{lt}(d))$. A second fact relating certain boundaries of e and d derives from the initial premises $\mathsf{tpart}(c,d)$ and $\mathsf{meets}(c,e)$. Due to C10 (aux. c, d being chronoids), the first boundary of d must coincide or be before the first time boundary of c. In both sub cases, axiom A39 on the existence of a chronoid as the mereological sum of two meeting chronoids and the initial premise that c and e meet (and aux. D11, D20, A22, C2–3, $\mathsf{C3}^{\mathcal{BT}^{\mathcal{C}}}$) ensure that there is a chronoid that justifies $\mathsf{before}(\mathsf{ft}(d), \mathsf{lt}(e))$: if the first boundaries of d and c coincide, A39 is utilized once regarding c and e; if the first boundary of d is before that of c, A39 can be applied once to the chronoid d' between the first boundaries of d and c that exists due to definition D20 of before, and once more to the sum of d' and c, forming another sum with e. That means we have derived that the boundaries of d and e are interlaced: $\mathsf{before}(\mathsf{ft}(e), \mathsf{lt}(d))$ and $\mathsf{before}(\mathsf{ft}(d), \mathsf{lt}(e))$, satisfying the precondition in C13 (aux. d and e being chronoids, and C2–3). Consequently, overlap between d and e is verified for case (1), and from both cases we conclude with $\mathsf{meets}(d,e) \vee \mathsf{tov}(d,e)$. ❏

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[666]C9 and C12 are a bit stricter than the natural language formulation, by differentiating the two cases in the antecedent and C9 having a stricter consequent.

[667]"aux." stands for "auxiliary", referring to definitions, formulas, and phrases that are needed in a proof step, yet are less decisive for understanding the proof, but are included here to conform exactly to the list of required axioms and definitions in Table 6.7 and point out their use.

6.3.11   Metalogical Analyses of $\mathcal{BT}^{\mathcal{R}}$

This section provides first observations and results concerning the metalogical properties of $\mathcal{BT}^{\mathcal{R}}$. We start by considering the numbers of maximal components that can occur within time regions. Consistency can be proved in a way which is very similar to the case of $\mathcal{BT}^{\mathcal{C}}$. Moreover, we discuss the incompleteness of the current theory and an option for a complete extension. The remaining considerations are concerned with a systematic approach to the classification of time regions on the basis of $\mathcal{BT}^{\mathcal{R}}$ as a first-order theory, and we briefly look at the relationship with $\mathcal{BT}^{\mathcal{C}}$.

6.3.11.1   Cardinality Considerations on the Components of Time Regions

Here we focus on the aspect of how many maximal connected components are guaranteed to be comprised in set of all time regions in models of $\mathcal{BT}^{\mathcal{R}}$. For this purpose, we refer to a time region with *exactly* n maximal connected components as an *n-time region*, and use the following abbreviating notation, defined for each positive natural number $n \in \mathbb{N}_+$.

$$\mathsf{treg}_n(x) := \mathsf{TReg}(x) \wedge \exists x_1 \ldots x_n . \bigwedge_{1 \leq i \leq n} \mathsf{ccmp}(x_i, x) \wedge \bigwedge_{1 \leq i < j \leq n} x_i \neq x_j \wedge \forall y (\mathsf{ccmp}(y, x) \rightarrow \bigvee_{1 \leq i \leq n} y = x_i)$$

This allows for a clear specification of the first observation, namely that time regions within $\mathcal{BT}^{\mathcal{R}}$ are not limited in terms of the number of their components.

6.9 Proposition
$\mathcal{BT}^{\mathcal{R}}$ ensures that there is an n-time region for every $n \in \mathbb{N}_+$, i.e., $\mathcal{BT}^{\mathcal{R}} \models \exists x . \mathsf{treg}_n(x)$.

Proof. The proof is by induction on n. We first show that $\mathcal{BT}^{\mathcal{R}} \models \exists x . \mathsf{treg}_1(x)$. This can be seen as follows: Definition D5 and axiom A1 imply $\exists x . \mathsf{TReg}(x) \vee \mathsf{TbReg}(x)$. Furthermore, by definition D4 and axiom A6 we have $\exists x . \mathsf{TReg}(x)$. Given consequence C7 and definition D10 we deduce $\exists x . \mathsf{Chron}(x)$. Using consequence C4 we have $\exists x . \mathsf{treg}_1(x)$, concluding the base case.

Now assume $\mathcal{BT}^{\mathcal{R}} \models \exists x . \mathsf{treg}_n(x)$ for an arbitrary $n \in \mathbb{N}_+$. Let r an n-time region and select a single chronoid component c of r, i.e., $\mathsf{treg}_n(r) \wedge \mathsf{ccmp}(c, r)$. With axiom A15 we obtain a new time region s such that $\mathsf{during}(s, c)$. Using definition D8 we deduce the existence of two additional time regions s' and s'', complementary to c within r, more precisely $\mathsf{starts}(s', c)$ and $\mathsf{ends}(s'', c)$, which do neither overlap nor meet. Applying A12 we derive the existence of two chronoids t' and t'' being temporal part of s' and s'', respectively. Due to this parthood t' and t'' can neither meet nor overlap, because otherwise, by Lemma 6.8, s' and s'' would meet or overlap, contradicting the fact that they do not. Similarly, t' and t'' cannot overlap nor meet with any of the remaining $n - 1$ connected components of r, say d (if $n > 1$), because otherwise s' and s'', and thus c would do so with another component d of r, by Lemma 1 and the given parthood relations among c, s', s'', t', and t''. But then A39 or A40 in combination with the transitivity of parthood A11 would yield a chronoid component of r with c and d as parts, contradicting the maximality of c from $\mathsf{ccmp}(c, r)$. Now, forming the mereological sum, which can be done via axiom A20, of those remaining $n - 1$ components with t' and t'' immediately yields an $n + 1$-time region, which shows $\mathcal{BT}^{\mathcal{R}} \models \exists x . \mathsf{treg}_{n+1}(x)$. ❑

This observation can be extended immediately. For each $m, n \in \mathbb{N}_+$, let $\mathsf{treg}_n^{\geq m}$ abbreviate the sentence that at least m n-time regions exist.

$$\mathsf{treg}_n^{\geq m} := \exists x_1 \ldots x_m . \bigwedge_{1 \leq i < j \leq m} x_i \neq x_j \wedge \bigwedge_{1 \leq i \leq m} \mathsf{treg}_n(x_i)$$

6.10 Proposition
For any positive natural numbers m, n it is the case that $\mathcal{BT}^{\mathcal{R}} \models \mathsf{treg}_n^{\geq m}$. ❑

Proof. The proof idea is to "shorten" one chronoid component of a given n-time region due to C8, which yields a distinct n-time region via mereological union of the shortened component with the remaining ones, using A20. This can be considered multiple times at once.

For any $n \in \mathbb{N}_+$, Proposition 6.9 guarantees the existence of an n-time region, i.e., $\mathcal{BT}^{\mathcal{R}} \models \mathsf{treg}_n^{\geq 1}$. Consider a fixed n-time region t and select a single chronoid component c of it. For any $m \in \mathbb{N}_+$, applying consequence C8 m times yields a tppart-chain of chronoids. More precisely, we obtain $\mathsf{tppart}(c_1, c)$, $\mathsf{tppart}(c_2, c_1), \ldots, \mathsf{tppart}(c_m, c_{m-1})$ with $\mathsf{Chron}(c_i)$ for all $1 \leq i \leq m$. By definition D16 of tppart as well as the partial order axioms of tpart (A9, A10, and A11), $c_i \neq c_j$ results for any $1 \leq i < j \leq m$. Moreover, by transitivity of temporal part A11 and Lemma 6.8 we deduce that the constructed c_is cannot overlap nor meet with any of the remaining $n - 1$ connected components of t. For each $1 \leq i \leq m$, let r_i be the mereological sum of c_i and the remaining connected components of t, which exists by A20. These temporal entities are all distinct and obviously n-time regions by construction. Consequently, $\mathcal{BT}^{\mathcal{R}} \models \mathsf{treg}_n^{\geq m}$ is shown, concluding the proof. ❑

Results of this kind turn out to be useful in considering the completeness of $\mathcal{BT}^{\mathcal{R}}$ as well as ways of classifying time regions. But prior to both, the consistency of the theory should be established.

6.3.11.2 Consistency of $\mathcal{BT}^{\mathcal{R}}$

A proof of the (relative[668]) consistency of $\mathcal{BT}^{\mathcal{R}}$ can be achieved by a reduction of this theory to the monadic second order theory of linear orderings that is very similar to the one used in the proof of $\mathcal{BT}^{\mathcal{C}}$'s consistency in section 6.3.8.2. The major addition is to capture the intuitive understanding of time regions as mereological sums of finitely many chronoids.

6.11 Proposition
The theory $\mathcal{BT}^{\mathcal{R}}$ is consistent. ❑

Proof. Let λ be the order type of the real numbers $\mathbb{R}$. Then take the linear ordered sum over $\mathbb{R}$ of linear orderings of type **2** to define a new linear ordering $\mathcal{M} = (M, \leq)$ of type $\lambda(2)$. Within the monadic second order theory of $\mathcal{M}$, we must provide definitions for the basic signature elements of $\mathcal{BT}^{\mathcal{R}}$, TReg, ftb, ltb, tbreg, tcoinc, and tpart.

Defining time regions requires some preparation, where we aim at identifying a time region R with a union of a finite number of pairwise disjoint chronoids, the latter being captured as in the consistency proof of $\mathcal{BT}^{\mathcal{C}}$. Accordingly and as an auxiliary means, we define $\mathsf{Chron}(C)$ such that C is a chronoid if and only if C is an interval $[a, b]$ over M whose first element a has an immediate predecessor and whose last element b has an immediate successor. Consequently, chronoids are convex subsets of M. Furthermore, the first and the last boundary of a chronoid are determined by the corresponding elements of the associated interval, i.e., we set $\mathsf{ftb}(a, C)$ and $\mathsf{ltb}(b, C)$ for $C = [a, b]$. Notice that disjoint chronoids / intervals can be ordered by their first time boundaries / least elements. More precisely, a set $\mathcal{C}$ of pairwise disjoint chronoids has an associated linear order $(\mathcal{C}, \lhd)$ that derives from $\mathcal{M}$ by $C \lhd D$ iff $\mathsf{ftb}(x, C)$ and $\mathsf{ftb}(y, D)$ and $x < y$.

Now time regions are defined as any subset $R \subseteq M$ of M that satisfies the conditions

1. Regarding $\leq$, R has a least element s and a greatest element t such that s has an immediate predecessor and t has an immediate successor.
2. Every maximal convex subset of R satisfies the conditions of a chronoid.
3. The linear ordering between the maximal convex subsets of R, determined by the linear ordering of their first elements, does not contain any subset of order type ω or $\omega^\star$.

All of these conditions can be expressed by a monadic second order formula for defining $\mathsf{TReg}(R)$, which is satisfied by those subsets of M representing time regions. The finiteness of the number of components follows from the non-existence of suborderings of type ω or $\omega^\star$.

The remaining predicates are more straightforward. Also in the general case of time regions R, $\mathsf{ftb}(x, R)$ applies if x is the least element of R, and $\mathsf{ltb}(x, R)$ if x is the greatest element of R. The set $B(R)$ of all

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[668]Cf. footnote 659 in section 6.3.8.2.

time boundaries of R is defined as the set of all least and greatest elements of the chronoids included in R. The relation $\mathsf{tbreg}(X, R)$ is satisfied if X is a non-empty subset of $B(R)$. Two distinct boundaries *coincide* if one is an immediate successor of the other. Eventually, *temporal part-of* corresponds to the subinterval relation.

The definitions of the $\mathcal{BT}^{\mathcal{R}}$ primitives allow for extracting a new structure $\mathcal{A}$ with $\mathcal{A} \models \mathcal{BT}^{\mathcal{R}}$. We omit the technical proof steps that show that all axioms of $\mathcal{BT}^{\mathcal{R}}$ are satisfied in $\mathcal{A}$. ❑

6.3.11.3 $\mathcal{BT}^{\mathcal{R}}$ and the Problem of Completeness

Following the case of $\mathcal{BT}^{\mathcal{C}}$ anew, further metalogical considerations on $\mathcal{BT}^{\mathcal{R}}$ concern its completeness. Given the existence of an axiomatization, completeness of a theory further yields decidability [221, sect. 2.6, p. 147]. The theory $\mathcal{BT}^{\mathcal{C}}$ was proved to be complete in section 6.3.8.2. The situation for $\mathcal{BT}^{\mathcal{R}}$ turns out to be different.

6.12 Proposition
The theory $\mathcal{BT}^{\mathcal{R}}$ is incomplete. ❑

In order to see this we construct a sentence ϕ such that both of the sentences ϕ and $\neg\phi$ are consistent with $\mathcal{BT}^{\mathcal{R}}$. Section 6.3.11.1 proves that every $\mathcal{BT}^{\mathcal{R}}$ model contains time regions with n different components for each natural number n. But nothing can be proved about the structure of time regions with infinitely many components. By the completeness (compactness) theorem of first-order logic, cf. e.g. [221, sect. 2.5], one can derive that there are models of $\mathcal{BT}^{\mathcal{R}}$ in which there exist time regions with infinitely many components. This suggests a direction to search for a sentence that is undetermined.

Proof. We aim at a sentence ϕ such that $\mathcal{BT}^{\mathcal{R}} \cup \{\phi\}$ as well as $\mathcal{BT}^{\mathcal{R}} \cup \{\neg\phi\}$ are consistent. The basic idea for ϕ rests on the observation that the chronoid components of any time region form a pairwise non-overlapping set of chronoids and thus can be linearly ordered by adopting, e.g., their first time boundaries' ordering (as via ◁ in the consistency proof in the previous section 6.3.11.2). Then the sentence ϕ shall state that there is a time region r where this ordering of its chronoid components is dense and, therefore, r has infinitely many components.

Approaching ϕ formally, we introduce the predicate $\mathsf{cbf}(x, y)$ and, by definition, let it reflect the mentioned ordering ◁ of non-overlapping chronoids. Note that this definition reuses the definition D20 of before, the strict linear ordering among time boundaries (originating from D13 in $\mathcal{BT}^{\mathcal{C}}$ in section 6.3.7.3, cf. also the proof of linearity in section 6.3.8.1). Thus linearity of cbf follows from the definition and additional considerations that non-overlap behaves accordingly.

D21. $\mathsf{cbf}(x, y) =_{df} \mathsf{Chron}(x) \wedge \mathsf{Chron}(y) \wedge \neg\mathsf{tov}(x, y) \wedge \mathsf{before}(\mathsf{ft}(x), \mathsf{ft}(y))$
$$\text{(chronoid } x \text{ is before or meets } y)$$

Now let us specify ϕ, following the ideas above. While $\mathsf{cbf}(x, y)$ itself does not exclude that its arguments meet instead of being properly before one another, this is irrelevant in ϕ because only maximal connected components are considered therein.

$$\phi \; : \;\; \exists x(\mathsf{TReg}(x) \wedge \forall uw(\mathsf{ccmp}(u, x) \wedge \mathsf{ccmp}(w, x) \wedge \mathsf{cbf}(u, w) \to \exists v(\mathsf{ccmp}(v, x) \wedge \mathsf{cbf}(u, v) \wedge \mathsf{cbf}(v, w))))$$

Based on the consistency proof it is immediate that $\mathcal{BT}^{\mathcal{R}} \cup \{\neg\phi\}$ is consistent, because all time regions in the corresponding structures of the proof are restricted to a finite number of components. On the other hand, condition 3 in section 6.3.11.2 may be omitted from the definition of time regions. It is then straightforward to extend it in order to demonstrate also the consistency of $\mathcal{BT}^{\mathcal{R}} \cup \{\phi\}$. ❑

$\mathcal{BT}^{\mathcal{R}}$ can be extended such that the sentence ϕ is no longer consistent with that extension. One option for that is certainly $\neg\phi$. Another is to add a sentence ψ saying that, for every time region r, every component of r that is neither the greatest nor the least element with respect to cbf has an immediate successor and an immediate predecessor.

Besides the sentence ϕ, we are currently examining further properties of time regions that seem to be independent, even of $\mathcal{BT}^{\mathcal{R}} \cup \{\neg\phi\}$ or $\mathcal{BT}^{\mathcal{R}} \cup \{\psi\}$. Nevertheless, let us conclude our considerations on the completeness of a theory for time regions with an, admittedly speculative, hypothesis.

6.13 Conjecture
The theory $\mathcal{BT}^{\mathcal{R}}$ can be completed by a finite (and readily comprehensible) number of axioms, such that the corresponding extension yields a decidable theory of time regions.   ❑

6.3.11.4 Classification of Time Regions

Classifying the entities of a domain is a fundamental task for formal ontology, where we are especially interested in systematic approaches to classification. With formalized ontologies available, the objects of a domain can be classified with respect to elementary properties (in the formal-logical sense) that are expressible in the language of the ontology, cf. [60, sect. 5].

Let us outline this approach in a little more detail with respect to time regions. The basic idea is to consider a time region r within any model $\mathcal{A}$ of $\mathcal{BT}^{\mathcal{R}}$, $r \in \mathsf{TReg}^{\mathcal{A}}$, and associate with it a relational structure $\mathcal{S}(r)$ that can be derived from $\mathcal{A}$. For instance, one may assign a relational structure to such an r by $\mathcal{S}(r) := (Parts(r), \mathsf{TbReg}, \mathsf{tpart}, \mathsf{ftb}, \mathsf{ltb}, \mathsf{tcoinc})$, where $Parts(r) = \{p \mid \mathsf{tpart}^{\mathcal{A}}(p, r)\}$ and further relations are restricted corresponding to members in $Parts(r)$ and elements in $\mathcal{A}$ related to them, e.g. $\mathsf{ftb} = \{(x, y) \in \mathsf{ftb}^{\mathcal{A}} \mid y \in Parts(r)\}$. The elementary type of r is defined by the theory $Th(\mathcal{S}(r))$, and any two time regions are said to be elementary equivalent if their theories coincide.

This general approach allows for studying novel connections between metalogical properties and classification. In the context of the present theory, we can provide the following example. A time region is said to be *standard* if it contains only a finite number of connected components. Otherwise, a time region is called *non-standard*. Now we can formulate the following hypothesis for $\mathcal{BT}^{\mathcal{R}}$.

6.14 Conjecture
Two standard time regions are elementary equivalent if they contain the same number of connected components.   ❑

The classification of the *non-standard* time regions is more complicated and is among our future investigations, together with a rigorous proof of the conjecture.

6.3.11.5 Relationship with $\mathcal{BT}^{\mathcal{C}}$

In relating $\mathcal{BT}^{\mathcal{R}}$ with other theories of time it is natural to first consider the relation with the ontology that has served as its foundation, $\mathcal{BT}^{\mathcal{C}}$. This is the main relationship to be discussed in this section 6.3, while analyses of connections to other time theories remain for the future.

Looking "back" from $\mathcal{BT}^{\mathcal{R}}$ to $\mathcal{BT}^{\mathcal{C}}$, it is tempting to expect that the latter has just become a subtheory of the former. However, already the combination of the domain coverage axiom $\forall x \,.\, \mathsf{TE}(x)$ (itself contained in both theories as A1) with distinct definitions of the predicate $\mathsf{TE}(x)$ for "time entities" in both theories suggests a more complex relationship.

As a next step, one may consider the relativation of each $\mathcal{BT}^{\mathcal{C}}$ axiom to the unary predicates $\mathsf{Chron}(x)$ and $\mathsf{Tb}(x)$, i.e., basically, guarding quantification by one of these predicates, cf. [693, p. 258].[669] But even this approach fails, because further basic extensions have lead to stronger deviations. An example already visible in the diagrammatic surveys in Figures 6.4 and 6.5 is the "lifting" of temporal part-of from being regarded in connection with chronoids only in $\mathcal{BT}^{\mathcal{C}}$ to becoming applicable to time boundary regions in $\mathcal{BT}^{\mathcal{R}}$, and in particular, to time boundaries, as well. Let φ be the sentence $\forall x \,.\, \mathsf{Tb}(x) \to \neg\mathsf{tpart}(x, x)$ and observe that this formula has already the form of a relativation. Because of $\mathcal{BT}^{\mathcal{C}} \models \varphi$, but $\mathcal{BT}^{\mathcal{R}}$ being inconsistent with φ (mainly) due to its axiom A9, it is not in general the case that all $\mathcal{BT}^{\mathcal{C}}$ relativations are consequences of the presented theory of time regions.

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[669] If this approach could be pursued successfully, the domain coverage axiom A1 just mentioned would require a special treatment.

Eventually, looking for an interpretation from the theory of chronoids into the theory of time regions is a fairly general means, but it turns out to be successful.

6.15 Proposition
The theory $\mathcal{BT}^{\mathcal{C}}$ is interpretable in $\mathcal{BT}^{\mathcal{R}}$. ❑

We merely outline the proof by specifying definitions of the $\mathcal{BT}^{\mathcal{C}}$ primitives against the background of $\mathcal{BT}^{\mathcal{R}}$, while it is straightforward, yet technically cumbersome, that all (relativized) $\mathcal{BT}^{\mathcal{C}}$ axioms must then be proved within $\mathcal{BT}^{\mathcal{R}}$.

Proof. (outline) Analogously to providing the interpretation of $\mathcal{IP}_{\text{dense}}$ in $\mathcal{BT}^{\mathcal{C}}$ in section 6.3.8.3, a set of explicit definitions Δ is specified such that the five basic signature elements of $\mathcal{BT}^{\mathcal{C}}$ are defined by formulas in the language of $\mathcal{BT}^{\mathcal{R}}$, cf. [693, sect. 6.6]. Clearly, on that basis the defined signature elements of $\mathcal{BT}^{\mathcal{C}}$ are uniquely determined, as well. We use the upper indices $^{\mathcal{C}}$ and $^{\mathcal{R}}$ to distinguish between predicate names in $\mathcal{BT}^{\mathcal{C}}$ and $\mathcal{BT}^{\mathcal{R}}$, respectively.

CD1. $\mathsf{Chron}^{\mathcal{C}}(x) \quad =_{df} \mathsf{Chron}^{\mathcal{R}}(x)$

CD2. $\mathsf{ftb}^{\mathcal{C}}(x,y) \quad =_{df} \mathsf{Tb}^{\mathcal{R}}(x) \wedge \mathsf{Chron}^{\mathcal{R}}(y) \wedge \mathsf{ftb}^{\mathcal{R}}(x,y)$

CD3. $\mathsf{ltb}^{\mathcal{C}}(x,y) \quad =_{df} \mathsf{Tb}^{\mathcal{R}}(x) \wedge \mathsf{Chron}^{\mathcal{R}}(y) \wedge \mathsf{ltb}^{\mathcal{R}}(x,y)$

CD4. $\mathsf{tcoinc}^{\mathcal{C}}(x,y) =_{df} \mathsf{tcoinc}^{\mathcal{R}}(x,y)$

CD5. $\mathsf{tpart}^{\mathcal{C}}(x,y) \quad =_{df} \mathsf{Chron}^{\mathcal{R}}(x) \wedge \mathsf{Chron}^{\mathcal{R}}(y) \wedge \mathsf{tpart}^{\mathcal{R}}(x,y)$

Quantification regarding $\mathcal{BT}^{\mathcal{C}}$ is strictly limited compared to $\mathcal{BT}^{\mathcal{R}}$, to the extent that the intended domain of $\mathcal{BT}^{\mathcal{C}}$ covers only chronoids and time boundaries, while $\mathcal{BT}^{\mathcal{R}}$ comprises in addition time regions and time boundary regions. Accordingly, the axioms of $\mathcal{BT}^{\mathcal{C}}$ must be relativized to chronoids and time boundaries only, before their validity in $\mathcal{BT}^{\mathcal{R}}$ is considered. Remembering definition D2 and axiom A1 in $\mathcal{BT}^{\mathcal{C}}$, predicate $\mathsf{TE}^{\mathcal{C}}$ can serve as relativization predicate. The respective closure axioms, see [693, p. 258–259], in this case require $\exists x(\mathsf{TE}^{\mathcal{C}}(x))$, which $\mathcal{BT}^{\mathcal{R}}$ entails based on its axioms, e.g., the subset of D14, A1, A4, A6, A13, A16, and A21.

Now, for any $\mathcal{BT}^{\mathcal{C}}$ formula φ, let $\hat{\varphi}$ denote the relativization of φ to $\mathsf{TE}^{\mathcal{C}}$. The overall proof is completed by showing $\mathcal{BT}^{\mathcal{R}} \cup \Delta \models \hat{\varphi}$ for each axiom $\varphi \in \mathcal{BT}^{\mathcal{C}}$. ❑

Despite a theory interpretation being used to establish the link between the two main theories in this section 6.3, obviously there is a strong formal tie between them, not only one by construction. The high proximity of the definitions CD1–CD5 to the case of relativization is one indication. Another may be drawn from the consistency proofs of both theories, where the predicates shared (intuitively) among the theories are defined in (almost) exactly the same way.

6.3.12 Time Regions in Applications

Temporal regions are ubiquitous in many applications. Let us conclude the overall proposal of an ontology that provides for time regions in addition to chronoids and time boundaries with a discussion of its applications. In particular, we focus on one type of applications related to the notion of temporal abstraction, cf. [747], demonstrated by some examples.

The basic problem of temporal abstraction consists in interpreting time-indexed data by temporal propositions. This problem pertains to an aspect of the inverse problem of the incidence problem in temporal logic. The incidence problem consists in the establishment of a truth relation between temporal propositions and spatio-temporal reality, cf. section 6.3.4.1. Temporal abstraction is focused on the task of constructing temporal propositions from partial information about spatio-temporal reality such that these propositions are considered to be true in reality. The constructed temporal propositions exhibit an interpretation and understanding of the data, anchored in reality. A solution of this task must solve several sub-tasks, among them the following three most basic ones.

1. The development of an ontology of parts of the real world that can serve as truth-makers for temporal propositions.
2. The development of a theory of temporal propositions together with a truth relation, which links propositions and truth-makers.
3. The development of methods for constructing true temporal propositions from partial information about the real world.

There is no general solution to these problems, although there are various partial results, notably in practical applications related to medical decision support systems.

Furthermore, one observes that there are several levels of abstraction. In this connection, time-indexed data can be abstracted into a temporal region, such that certain properties are attached to the connected components. An example is the course of disease of malaria, where there are phases of high fever with interruptions in between. Put differently, the property of high fever is distributed over the connected components of a time region. Usually the temporal structure of the first abstraction level in temporal abstraction is a time region, but not a chronoid (a time interval). On levels of higher abstraction several separate connected components from the lower level(s) may be united, resulting in a more coarse-grained temporal region. In many cases, after a number of steps of abstraction, one reaches an abstraction that links one property to a connected time region.

6.3.13 Conclusions and Future Work

In this section 6.3 we present a new approach to phenomenal time, which is adopted by the top-level ontology GFO [392]. The approach is elaborated in two consecutive theories, $\mathcal{BT}^{\mathcal{C}}$ and $\mathcal{BT}^{\mathcal{R}}$, both being inspired in their foundations by ideas of Franz Brentano [114]. The basic concepts of $\mathcal{BT}^{\mathcal{C}}$ are *chronoids* (time intervals) and *time boundaries* (time points). $\mathcal{BT}^{\mathcal{R}}$ is based on generalizations of these concepts that originate from forming mereological sums, namely *time regions* and *time boundary regions*, respectively. The basic relations are *temporal part-of* either among time regions or among time boundary regions; temporal *coincidence of time boundaries*; and relations that link a time boundary (region) and a time region (including chronoids), declaring the former as a boundary of the latter, such as being the *first* or the *last time boundary* of a region.

Our approach in developing the presented ontologies adheres to the Onto-Axiomatic Method, outlined in section 6.3.2, in the context of which four basic problems are raised in axiomatizing knowledge of a domain: (1) determining an adequate conceptualization, (2) finding axioms, (3) assessing the truthfulness of the axioms, and (4) proving consistency of the theory. Accordingly, resulting ontologies should be evaluated against criteria such as conceptual adequacy and completeness, truth, consistency, and deductive completeness.

The invention of axioms, issue (2), is usually inspired by considering and observing real world phenomena, of temporal character in our case, and by the mind's ability to create abstractions. Axioms about a domain can be understood as beliefs about, e.g., the structures, processes, and causal laws of this domain. We see the provision of explanations of real world phenomena as a function of an ontology / theory. In accordance with this view, the axiomatizations of $\mathcal{BT}^{\mathcal{C}}$ and $\mathcal{BT}^{\mathcal{R}}$ serve as the basis for the development of a comprehensive ontology of material entities, including continuants and processes. Both ontologies are formalized by a set of axioms, specifying logical interrelationships between the categories and the relations. Concerning issue (4), we find investigations of consistency and, more generally, metalogical analysis of formalized ontologies to be an important task. Moreover, this activity can be another valuable source in finding new axioms. Corresponding investigations in this section 6.3 concern the consistency of both systems, their completeness and decidability, and a few more theory-specific considerations in both cases. Furthermore, the relation between $\mathcal{BT}^{\mathcal{R}}$ and $\mathcal{BT}^{\mathcal{C}}$ is discussed and the latter is formally related to $\mathcal{IP}_{\text{dense}}$ [849], which has close links to the well-established interval theory of time of Allen and Hayes [9, 10]. These metalogical analyses result in, firstly, proving both presented theories to be consistent. Only $\mathcal{BT}^{\mathcal{C}}$ can be shown to be complete and thus decidable, whereas $\mathcal{BT}^{\mathcal{R}}$ is provably incomplete, leaving room for hypotheses of potential complete extensions. The theory $\mathcal{IP}_{\text{dense}}$ is interpretable in $\mathcal{BT}^{\mathcal{C}}$, and the latter is interpretable in $\mathcal{BT}^{\mathcal{R}}$.

We see a number of potential benefits and applications for the overall approach to time proposed in this work, which is also associated with issues (1) and (3) above. Regarding ontological adequacy, firstly, we believe that the temporal continuum can be introspectively accessed without any metrics, and that it cannot be understood and grasped by sets of points. Accordingly, our approach does not rely on such a reduction. Secondly, an ontology of time is conceptually complete if the underlying conceptualization provides means to describe all essential phenomena of temporal domains. We are convinced that a conceptually complete ontology of time must include both intervals and time points, and moreover, the coincidence relation between points. One supporting argument is that $\mathcal{BT}^{\mathcal{C}}$ (as well as $\mathcal{BT}^{\mathcal{R}}$) establishes a basis that allows for expressing various solutions to the Dividing Instant Problem (DIP), cf. [6, 786] consistently (and in the classical framework of first-order logic). We remark that the DIP is further generally related to the theory of discontinuous changes, cf. [398, esp. sect. 8.3]. Several other contributions are discussed in the section, among them the potential use of time regions in connection with the problem of temporal abstraction. Overall, our presentation of motivating problems in section 6.3.4 with their reconsideration in section 6.3.6 after the main conceptualization has been established in section 6.3.5 is meant to promote the adequacy of the proposed conceptualization. It might further become useful regarding issue (3) of truthfulness, e.g., in terms of applications that are based on the proposed ontologies. Indeed, we expect a series of new applications. In other contexts, we have already verified the usefulness of our ontology as a tool for modeling cellular genealogies [130] and surgical interventions [622], for instance.

Concerning future work, there is a large number of open problems and tasks that can be classified into logical, ontological, and semantical problems. Logically and ontologically, both theories can be further analyzed and extended in several directions. $\mathcal{BT}^{\mathcal{R}}$, in particular, is proposed as a first version of an axiom system to specify the theory of the domain of time regions. Naturally and in general, modifications of the axioms may be required based on further analysis and use. Of particular importance, likewise regarding ontological aspects, is the (continued) connection and combination of the theories of time with those of other domains, e.g. of objects and processes in the material stratum. An open logical problem is to find minimal sets of axioms, such that each axiom is independent of the remaining theory. The mereology of time regions in $\mathcal{BT}^{\mathcal{R}}$ is to be studied further, e.g. in relation to the notion of relatively complemented distributive lattices. The continued study of relations to other theories in the domain of time is another issue. For example, we are interested in connections with PSL [317], selected theories in [379], and the FOL variant [408] of OWL-Time [409]. The main semantical problem concerns the temporal incidence problem for propositions, already touched upon in sections 6.3.4.1, 6.3.6.1, and 6.3.12. We expect that an expressive ontology of truthmakers must be established as a semantic basis for the interpretation of temporal propositions. Initial steps towards this end can be found in [534, 622]. The approach is inspired by several sources, among them Ludwig Wittgenstein's Tractatus Logicus [880] and mainly the situation theory of Jon Barwise and John Perry [56], cf. also [202]. Any such solution to truthmakers will depend on an underlying ontology of time.

[534, 622]

Chapter 7

Conclusion and Continuation

CHAPTER OVERVIEW

As sect. 6.3.13 closes "its" sect. 6.3, this chapter continues on concluding, here the overall thesis and as follows. The first section reviews the major contributions and statements of the work, preparing further discussion of the motivating questions as well as of ontological semantics, both in sect. 7.3. In between and with similar purpose, we relate, in particular, ontological semantics in some detail to primarily two selected approaches, namely Common Logic [@18] [452] and Naive Predicate Logic [619]. Eventually, it is briefly indicated which route we intend to take in dealing with problems that remain open up to this point, in particular, starting to study notions of conceptual / intensional equivalence.

7.1   Résumé

GFO DEVELOPMENT AND OBSTACLES IN ONTOLOGY REPRESENTATION

The work on this thesis is inspired by the development of the top-level ontology General Formal Ontology (GFO) [@46] [392, 398], for which – as for many other ontologies – formal languages shall be used, e.g., to precisely capture the ontological commitments that constitute the ontology and to make it amenable to reasoning over its content. Natural requirements arise from that goal of representing the ontology by formal languages, for example, at all being able to express the content / the ontological commitments that are conceptualized for the domain of reality / knowledge in a chosen formal language, as well as to offer the ontology in distinct languages, e.g., for different communities of (potential) users. These major motivations, detailed in sect. 1.2, and first steps attempting to follow them by studying available languages,

their interrelations, and approaches of bridging between languages, cf. a rough summary in sect. 1.1.3, left us with what we feel to be major obstacles of plainly using, for instance, first-order logic (FOL) with its set-theoretic semantics for representing ontological commitments, e.g., commitments wrt categories, due to a seemingly strong binding to the syntactic category of predicate in FOL. Similarly, approaches of language translations that preserve the formal semantics from one formalism (or theory) to another are very fruitful in some respects, such as the transfer of reasoning problems. However, we do not see how they explain the link between an ontological commitment that is to be expressed, like the specialization of the category animal by the category lion, and formal semantic entities (and their interrelations) that are typically involved in a formal language.

ENTANGLEMENT BETWEEN SEMANTICS AND ONTOLOGY LEADS TO DEFINING ONTOLOGICAL SEMANTICS

Indeed, viewing ontologies as an intended means to address actually that problem (of hidden / implicit encoding in formal semantics) – e.g., by allowing one to refer to ontological entities and commitments "behind" certain symbols – produces the impression of a problematic entanglement between this goal and representing ontologies by the very same means that are to be enhanced on their basis. This is investigated in depth in ch. 2, including studying the available approaches of ontology-based consequence and equivalence, e.g., in [156, 733], see esp. sect. 2.2. This analysis assures us to investigate the question of whether a semantic approach may be defined that overcomes the built-in encoding problem. In addition, we argue that, although the available notion of ontology-based *consequence* works[670], its nearby extension to ontology-based *equivalence* cannot be seen as an appropriate account of an intuitively perceived notion of conceptual / intensional equivalence. The latter is another motivation to search for a novel semantic approach beforehand – *ontological semantics*. From the beginning of this enterprise, there is the antagonism of (1) the existence of a large variety of semantic proposals, as well as (2) the (potential) loss of various "results" for the formalism in use, ranging from theoretical properties of logics over their proof systems to implemented reasoning software. Accordingly, keeping proximity to an established formalism is another driving factor, hoping for the reutilization of as much as possible of what is available from it. FOL is the formalism of our choice and sect. 4.6 allows for its use with established reasoners / theorem provers for deriving consequences compatibly with ontological semantics.

THEORY VIEW OF CLASSICAL SEMANTICS AND ONTOLOGICAL NEUTRALITY

Prior to actually developing ontological semantics, ch. 3 reviews the standard semantics of FOL from different angles, in order to "localize" where encodings are enforced in the semantics, as well as to determine points for analogy. One result of this is what we call the 'theory view of classical semantics', which reveals[671] an understanding of the assignment of a semantics as a theory interpretation into an extension of set theory. The way in which this is derived lends itself to abstracting and capturing, in the first place, precise options of defining *ontological neutrality* in sect. 3.4. Notably and in contrast to not accepting classical semantics as being ontologically neutral on intuitive grounds, one of the detailed options characterizes set-theoretic semantics of FOL as weakly ontologically neutral.

ONTOLOGICAL SEMANTICS AND ITS MAJOR DEVIATION IN PREDICATION SEMANTICS

The main ch. 4 follows tightly the usual, modern definition of FOL semantics. The overall development is accompanied by a thorough analysis of our motivations, weighing different options and, to some extent, their effects. Albeit orientation at the classical blueprint, in the course of establishing ontological semantics we attempt to avoid as many ontological assumptions as possible at the side of *ontological structures*, the analog to mathematical structures in set-theoretic semantics. When moving to the semantics of complex symbols in the language, the semantic definitions of logical connectives and quantification parallel that of the standard precedent almost exactly.

The major deviation emerges for the semantics of predication. Observations and ideas that led to the characterization of the ontological level [326, 333] in works of Nicola Guarino are adopted and let us defend the position that no uniform account of predication should be assumed in the general approach,[672] due to

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[670]With hindsight from sect. 5.1 and 5.2, we agree with this position.

[671]It "revealed" at least to us, whereas this may be a known connection in other communities. We are not aware of elaborations at the level of detail adhered to in ch. 3.

[672]Set-theoretic semantics treats predication uniformly as membership of argument tuples in a mathematical relation, thereby assuming set theory as available ontology.

the aim of ontological neutrality. Nevertheless and of course, predication requires a semantics in order to achieve the definition of a satisfaction relation between ontological structures and formulas. From the point of view of a "language user" with the goal of representing ontologies, the position of *analysis explication* as characterized in sect. 2.3.2 is adopted, which justifies the assumption of an intended semantics "behind" the FOL signature, esp. if that is specified in order to capture ontological commitments.

The solution to defining predication semantics for this purpose lies in tieing this intended semantics individually / specifically to each predicate, which turns 'ontological semantics' into a (parametrized) family of semantics. In order to capture this approach in general, *predication systems* are introduced, involving a distinction of predicates into some being called *fundamental*, others *normal*. A fundamental predicate is assigned a semantics, for atomic formulas that involve the predicate, in terms of natural language phrases that must obtain if an ontological structure is to satisfy such an atomic formula. The semantics of atomic formulas of a normal predicate is basically established by a definition in the FO language, called *predication definition*, whose definiens may utilize predicates that have been assigned a semantics already. An example of the latter is (4.21) $\forall x$. $\mathsf{Cat}(x) \leftrightarrow x :: \cdot\mathsf{Cat}$, where Cat stands for 'category' and $::$ for 'instantiates'[673]. Due to being part of the semantics of the language (though specifically for a selected signature), predication definitions are tautologies. Similarly, additional tautologies arise from any interrelationships that the language user accepts / expects for the fundamental phrases, to the extent that they reflect into object-level formulas.

Importantly, predication definitions do *not* capture the intension of the language user wrt the entity that the predicate symbol is meant to denote, but instead they shall account for the ontological relationship of the intended referent of the predicate and the entities that are referents of the argument symbols. This is based on the idea (and actually requirement) that it is possible to formulate this ontological relationship, eventually, by means of the fundamental predicates together with constant symbols of the signature. Any intensional characterization of the intended referent of predicate symbols remains to be specified by postulating further axioms.

RECOURSE TO SET-THEORETIC SEMANTICS AND ONTOLOGICAL NEUTRALITY

Once that ontological semantics is established, sect. 4.6 investigates the possibility to "approximate" it in terms of set-theoretic semantics. This results in the proof of the transferability of inconsistency checking and of deriving consequences under classical semantics to ontological semantics. This applies constructively to a (weakly restricted) class of ontological structures by Th. 4.44, p. 167 in sect. 4.6.4, with a, though non-constructive, outlook to a fully general version in Obs. 4.46. Moreover, sect. 4.7 reconsiders the question of ontological neutrality, where essentially it turns out that sample instances of ontological semantics are *ideally ontologically neutral*, as defined in sect. 3.4, wrt theories that are faithful to / include all "additional" tautologies under ontological semantics. This is a welcome result insofar that such theories include *all* presuppositions / ontological commitments on their signature, establishing "their own" semantics if viewed under a corresponding ontological semantics.

ONTOLOGY FORMALIZATION AND ONTOLOGICAL USAGE SCHEMES

This property / relationship of ontological semantics wrt ontological neutrality inspires the proposal of $\mathcal{OS}$, a formalization method for ontologies, in sect. 5.1, perhaps mainly[674] for top-level ontologies. $\mathcal{OS}$ theories can easily be comprehended as theories under an ontological semantics. For other ontologies, but – as we believe – likewise generalizable to arbitrary forms of representation in order to elucidate their ontological commitments, we propose *ontological usage schemes* (OUS) as a translational approach of assigning ontological semantics to any language *in a particular usage context*, in parallel to a possibly available formal semantics of that language. With hindsight, this suggestion is very connatural to ontology-based semantics [156, 733] and to ontological reduction [396, prim. sect. 5], [388, prim. p. 58–59], [397, 2.4.2]. Yet at least, besides the relation of OUS with ontological semantics, they deviate from the former

[397]

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[673]More precisely, $::$ stands for the relationship of instantiation, but the symbol in the context of an atomic formula in infix notation reads most appropriately as 'instantiates', given the link of the first argument position to the instance, and of the second to the instantiated entity.

[674]I.e., $\mathcal{OS}$ is not limited to top-level ontologies, but less general ontologies may already profit from ontological usage schemes mentioned next.

approaches[675] by being liberated from the *necessity* that the translation must preserve the semantics of the original language, while in many cases that remains a desirable property.

APPLICATIONS

Examples of OUS are first contained in sect. 5.2.3 and 5.2.4. Demonstrating their capability of providing theoretical and ontological grounding, they are further occasionally met in ch. 6 on ontologies and in sect. 5.4 on applications in the biomedical domain.

As we mention applications, the authoring process of this thesis proceeded in parallel to work on developing and employing ontologies, primarily in the context of GFO or of fragments of it. There are two main "application projects" that we present in moderate detail in this thesis. Firstly, by means of OUS the biomedical core ontology GFO-Bio in sect. 5.4.1 receives additional clarification of its implementation / formalization in distinct ways in OWL, which are incompatible from the perspective of formal semantics. Secondly, sect. 5.4.2 reports on providing several ontologically founded options of representing phenotypes in OWL, where OUS may further serve the integration of information captured by different options. The ontological foundation itself rests on / constitutes an application of $\mathcal{CR}$.

ONTOLOGY $\mathcal{CR}$ AND ONTOLOGIES OF TIME

$\mathcal{CR}$, abbreviating (ontology of) c̲ategories and r̲elations, is the first of two final major keywords for the present summarizing section 7.1. Indeed, to this point ontologies and $\mathcal{CR}$, in particular, have not received the attention that correlates with their rôle /importance in the overall thesis. The development of ontological semantics and OUS is continuously accompanied by the case of $\mathcal{CR}$. We see the proposal of this ontology of categories and relations as another major contribution of this thesis. In order to refer to notions of $\mathcal{CR}$ and utilize them, e.g., for examples of defining predication systems for ontological semantics in sect. 4.4.3.2 or sample OUS in sect. 5.2.3 and 5.2.4, its conceptualization is introduced already in sect. 2.4 among the foundations for the overall work in ch. 2. Actual formalizations of $\mathcal{CR}$ are split into a mainly taxonomic fragment $\mathcal{CR}_0^{\text{DL}}$ and further extensions. The taxonomic fragment is formalized in description logics through implementation in OWL [@21], cf. App. B. This allows us to claim (classical) consistency of $\mathcal{CR}_0^{\text{DL}}$. On its basis, transferred by the "standard", formal semantics preserving translation into FOL, we conduct a few further considerations for an extended FOL formalization of $\mathcal{CR}$ in sect. 6.1.2.

As another domain that has been ontologically analyzed and for which first formalizations were developed in the time span of writing this work, sect. 6.3 presents $\mathcal{BT}^{\mathcal{C}}$ and $\mathcal{BT}^{\mathcal{R}}$, two ontologies devoted to time, jointly developed in equal parts with Ringo Baumann and Heinrich Herre. First, there is clearly a contribution in the conceptualization of time in terms of chronoids / time intervals which have boundaries that can coincide, based on work by Franz Brentano [114]. In addition, also methodologically, the development of both theories yields an exemplary case of conducting the steps of conceptualization, formalization, and meta-theoretic analysis, as considered in the establishment of modules of GFO. Both theories are presented as axiomatizations in FOL, where an ontological usage scheme sketched earlier bridges to the former parts in the thesis.

In the remainder of this concluding chapter, next we return briefly to the literature, now equipped with a comprehensive perspective wrt the contributions just summarized.

7.2   Related Work

TREATMENT OF RELATED WORK IN THIS THESIS AND THIS SECTION

First of all and as can be expected, where we rely on existing publications we make efforts to cite these sources throughout this work, up to mentioning such that we have become aware of and that are closely related to an aspect under consideration. The structure of this sect. 7.2 follows roughly the grouping of Table 1.3 of fields related to this thesis. Accordingly, a section on applied ontology is followed by relationships of, primarily, ontological semantics to approaches in knowledge representation (KR) and (mathematical)

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[675]Possibly, the deviation is even less wrt ontology-based semantics, depending on the question of how much freedom the "logical rendering function" [156] admits.

logic. Weakly related fields can only be covered by indicating a few "scattered points" in the landscape of publications.

7.2.1 Applied Ontology

META PROPERTIES IN WORKS OF GUARINO ET AL. AND GUIZZARDI ET AL.

Let us start with related work in applied ontology (AO), as the main field where we situate this thesis. A larger fraction of works (co-)authored by Nicola Guarino, e.g. [325, 326, 332, 341–343, 345, 867], adopted and continued by Giancarlo Guizzardi et al., cf. e.g. [348, 349], rests on the idea of distinguishing various kinds of categories[676], a.o. sortals, roles[677], mixins, etc. First, the $\mathcal{CR}$ ontology may be extended by these / such meta categories / category categories. Moreover and not surprisingly, due to inspiration from, e.g., [326], if its foundations are considered, ontological semantics at the general level is very close to this thinking wrt its "free" / undetermined interpretation of predication. In principle, arbitrary constructions (expressible as a FOL formula) can be utilized in defining predication even for normal predicates, and separately for each predicate. Such constructions may be interpreted as kinds of categories / metacategories. Hence, one may further study the embedding / use of meta properties in predication definitions, or in the form of additional axioms, in both cases making them amenable to reasoning within the formalism. It links further to an aspect that has not been exploited here, namely the theory behind / of those meta properties.

ONTOLOGY HIERARCHIES AND REPOSITORIES

There are many works (co-)authored by Michael Grüninger that relate to aspects in this thesis. Thematically, most of these are situated in a shared sector of AO, knowledge representation, and mathematical logic.[678] We focus on recent approaches and results in esp. [320, 321], but cf. also [319, 324], with respect to theory hierarchies and repositories. A *theory hierarchy* is a set of theories over equal signature that are partially ordered by (conservative or non-conservative) extension relations [320, Def. 4, p. 172] or, put simpler, ordered by entailment, based on [320, Lemma 1, p. 174][679]. Without going into full detail of [320, Def. 20, p. 189], a *repository* is a partially ordered set of hierarchies, whose ordering relation is based on a notion of reduction among theories, with connections to the notion of faithful theory interpretation. From the point of view and in the terminology of this thesis, these works are conducted from the perspective of theory comparison, cf. sect. 2.3.2. We see them as valuable contributions that can be utilized for ontology development (which for us refers primarily to the complementary analysis explication perspective), as intended for their modular structuring, and for the meta-theoretic analysis of ontologies. For example, small, well-understood theory fragments might be useful for model construction / consistency proofs.

Being developed wrt classical semantics and given a proximity between ontological usage schemes and theory interpretations, it appears interesting to examine potential benefits of combining these ideas. This might lead to structured / modularized ontological usage schemes in one direction. Conversely, ontological usage schemes may contribute novel aspects wrt relations among theories in hierarchies and repositories, although this remains to be analyzed in future work. The idea is based on seeing the PSL ontology, cf. sect. 6.3.3.2, in examples of, e.g., [320] and observing that the notions of activity and activity occurrence, through the lenses of $\mathcal{CR}$ (and additional fragments of GFO on processes, in addition) require a "nonstandard" ontological usage scheme, which establishes instantiation links between activity occurrences and activities.

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[676]'categories' and 'kinds' as understood herein, whereas these terms name specific kinds of categories in the work of Guarino et al. and Guizzardi et al.

[677]in a different, though related reading compared to the technical term 'role' herein

[678]This nicely allows for a smooth transition to the subsequent sect. 7.2.2.

[679]and with reminiscence of Lindenbaum-Tarski algebras, cf. e.g. [613].

7.2.2 Knowledge Representation and Mathematical Logic

7.2.2.1 Ontology-Based Semantics

RECAPITULATION OF ONTOLOGY-BASED SEMANTICS IN [**156**]

After the possibly severely sounding criticism of the definition of ontology-based equivalence in sect. 2.2.2 and some relativization of that in connection with ontological usage schemes, e.g., also in sect. 7.1, our proposals do involve some parallels with ontology-based semantics as proposed by Mihai Ciocoiu and Dana S. Nau in [156], taken further in [157, 322, 323] (connecting to the efforts on ontology / theory hierarchies and repositories discussed in the previous sect. 7.2.1). A quick recapitulation is in order at this point.

Our work shares much of the motivation and goals strictly with that of [156], and we agree with most of the analysis in its sections 1 and 2. In order to define *ontology-based models* for a set of expressions S in an arbitrary declarative language L, [156] comprises four steps based on FOL. (1) S is transformed into a first order theory $\Sigma \subseteq \mathcal{L}$, called *logical render* of S. (2) An ontology Ω for L is specified in a first order language $\mathcal{L}_\Omega$, and Σ is interpreted (via theory interpretation in the sense of [220, sect. 2.7]) with respect to $\mathcal{L}_\Omega$, resulting in Σ^π. An $\mathcal{L}_\Omega$ *explanation* of the original set S is any theory $T \subseteq \mathcal{L}_\Omega$ s.t. $T \models \Omega$ and $T \models \Sigma^\pi$. (3) T has $\mathcal{L}_\Omega$-structures as models. (4) Out of these, models for Σ are extracted according to [220, p. 159], i.e., reversing the interpretation. Such structures form *ontology-based models* for S because they obey the additional constraints of the ontology Ω.

COMPARING STRUCTURES, TRANSLATIONS, AND THEIR IMAGES

Compared to the motivation of using a semantic approach for ontology representation that is ontologically neutral, ontology-based models do still require set-theory, in contrast to ontological structures. On the other hand, we conjecture that a close connection between ontology-based models and 'associated set-theoretic structures' as derived from ontological structures, see Def. 4.41, can be established.

Similarly, ontological usage schemes (OUS) are very similar to / parallel the combination of logical render and subsequent theory interpretation, and thus ontological images may actually coincide with explanations in the sense of [156]. By definition, OUS are/seem less restricted, in that no full semantic integration is required. However, this "loss" may be realized by the logical render of [156], such that it remains to be studied in detail whether ontological usage schemes could be completely reduced to a splitting into logical render and a theory interpretation (e.g., by the identity mapping for the latter). A related question in this regard is whether theory interpretations alone can accommodate distinct forms of using FOL, e.g., in cases as required for GFO-Bio, see sect. 5.4.1, where predicates are translated into logical individual constants. Theory interpretations as defined in [220] consider only interpretation formulas that maintain the number of free variables in interpreted predicates. Yet again, this step might be strengthened by other notions, such as Szczerba's interpretability [799].

In any case, in both approaches it remains fairly open how to translate from an arbitrary source language into (eventually) a language with an underlying ontology. This arbitrariness should be countered by methodological support, which remains an aspect of future work herein. Yet insofar, we wonder whether a single step of finding a translation $\tau : L \to L_\Omega$ as in the case of OUS may be easier to apply than first developing a logical render, for which then a theory interpretation must be found. "Easier" does not only refer to having one step instead of two, but also to potential methods of establishing those translations. It is not clear to us how much guidance can be given to developing a logical render, as a mapping into a FOL language. In contrast, the idea that an OUS expresses an ontological analysis[680] of the original representation together with the makeup of $\mathcal{OS}$ theories with an emphasis of (logical individual) constants for all entities of relevance may provide more points of contact for linking actual methodic steps.

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[680]Clearly, this aspect of ontological analysis is present in the theory interpretation in [156], but / where it is coupled with formal semantic integration between the logical render and the ontological theory.

7.2.2.2 Common Logic

Common Logic (CL) [@18] [452], standardized by ISO [@61] in 2007,[681] is an approach aimed at accommodating "more expressiveness" and "metamodeling aspects" into logical languages without a classical higher-order semantics, cf. also [577].[682] Insofar some of our motivations in sect. 1.2 correspond to those of CL. Indeed, we have more or less followed the development of CL via its mailing list [@18] during writing this thesis and have certainly been affected by that, but without being able to point to specific messages there.

Syntactically, CL defines a relieved version of FOL syntax in that there are exactly two abstract syntax categories of *names* and of *sequence markers* for a signature / vocabulary. Since with the latter CL cannot be understood as a notational variant of FOL,[683] we ignore the availability of sequence markers in the sequel and concentrate on a fragment of CL, which we call CL(FO).[684] More precisely, it comprises the abstract syntax categories [452, p. 8–9] (6.1.1.6)-(6.1.1.14) (sentence, quantified sentence, Boolean sentence, atom, term, functional term, term sequence, vocabulary, name) under omission of sequence markers. Semantically, CL(FO) is defined by [452, p. 15], conditions E1, E3, E5-14, E16.

An interpretation structure for CL(FO), see [452, sect. 6.2], $I = (U_R, U_D, rel_I, fun_I, int_I)$ comprises two sets $U_D \subseteq U_R$ (universes of discourse, and of reference, resp.) and three mappings which assign to every member of U_R a set of tuples of arbitrary arity over U_D (via rel_I) and a partition of total functions of every arity over U_D (via fun_I), whereas each name is mapped to a member of U_R (via int_I).[685] Lacking a syntactic distinction between predicates, functions, and constants, every symbol in the signature is interpreted via a function int_I by a member of U_R (and thus in parallel with a polyadic relation, and with functions of every arity). The interesting case (for the present comparison) of the satisfaction relation [452, Table 1, rows E6–E14, p. 15] is that of predication in (7.1).[686] Quantification in CL(FO) remains first order, cf. [577, e.g. p. 6–7].

$$(7.1) \quad I \models P(\bar{x}) \quad \text{iff} \quad \bar{x}^I \in rel_I(P^I).$$

There are clear parallels between CL semantics and ontological semantics, as well as some distinctions. First of all, both approaches strive for semantic type-freedom [577, p. 3], while they differ syntactically in that we rely on "usual" FOL syntax, whereas CL avoids syntactic (sub)categories of its names by design. However, we believe / conjecture that the latter is a marginal difference, i.e., ontological semantics should be easily extensible / adaptable to CL syntax by relying on an analogous approach of distinguishing symbol occurrences based on "contextually defined [syntactic] roles" [577, p. 4].[687] Indeed, this would be a very reasonable extension, allowing for the use of the same symbol as a predicate symbol and its reflection constant.[688]

Conceptually, the interpretation of a symbol / name by a member of U_R by means of int_I is understood in CL as an intensional step,[689] whereas extensions in the form of relations and functions are assigned to

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[681] CL is currently under revision, cf. the latest working draft [454] of 2015.

[682] Readers capable of German may also confer Axel Hummel's Master's Thesis [439] on CL, RDF(S) [@105], [119, 735], and Extended RDF [14, 15].

[683] "The addition of sequence markers adds considerable expressive power to CL but, of course, at a cost – one can, for example, express finitude, but that immediately implies that compactness fails and, hence, that validity for the framework is not axiomatizable." [576, FN 5].

[684] This coincides with CL as considered in [577].

[685] A further distinction between discourse and non-discourse names may apply in a CL dialect, which int_I must respect, but we refrain from going into further details.

[686] (7.1) is largely adapted to our syntactic style herein.

[687] This would require some adaptation of the definitions, which do depend on arities of predicates.

[688] Although this requires some care, as will be clear from differences in predication below. I.e., one must remain aware that using exactly the same symbol with distinct interpretations in distinct syntactic positions might lead to misconceptions.

[689] This is our understanding from early CL mailing list [@18] messages. But beyond the symbol name 'int_I' in [452] it is hard to trace in [452] or [577]. Against the background of ontological semantics, the attribute term 'referential' appears more appropriate to us, as should become clear from sect. 7.4.

the members of U_R. The intensional step corresponds to the interpretation function J in an ontological structure $\mathcal{O} = (O, J)$.

The step of interpreting a name in predicate position[690] by means of rel_I in (7.1) yields a distinction to ontological semantics, because in CL the predication application is evaluated wrt the relational extension of the *referent* of the predicate symbol, whereas ontological semantics has no built-in connection between interpreting an atomic formula $P(\cdot)$ via the predication system and the referent interpretation of an ontological structure, cf. sect. 4.4.4.2 for the rationale. As a result, (4.25) on p. 151, i.e., $\forall PQ . P = Q \rightarrow \forall \bar{x}(P(\bar{x}) \leftrightarrow Q(\bar{x}))$, is a tautology in second-order logic as well as in Common Logic.[691] [692] Not being a first-order sentence, this cannot be a tautology of FOL under ontological semantics, but Def. 4.21 of 'enforcing equivalence based on co-reference' singles out those predication systems which satisfy the corresponding meta-level property, requiring for every $\mathcal{O} = (O, I)$ "$I(P) = I(Q)$ implies $\mathcal{O} \models \forall \bar{x} . P(\bar{x}) \leftrightarrow Q(\bar{x})$". But this is not generally satisfied by every predication system.

Both semantics share their attitude towards comprehension principles by not assuming any of those, see for CL [577, p. 7], thus being joint in the deviation from second-order logic. For example, $\exists x \forall P . P(x)$ is satisfiable in CL as well as under a suitable ontological semantics, but it is unsatisfiable in second-order logic. In the latter, comprehension ensures for every set S the existence of $\overline{S} := U \setminus S$, the relative complement of S wrt the universe (of logical individuals) in any interpretation structure. Hence, no member of U can satisfy both, P interpreted by S and P interpreted by $\overline{S}$.

Against the background of ontological semantics with a "fixed" predication system in its instances, the assignment of relations of *every* arity to each name appears as a technical overloading, although this may be acceptable from a logical point of view. Moreover, the methodological aspects of ontological semantics, i.e., of setting up the predication system for a language, remain of major importance, also in connection with method $\mathcal{OS}$ in sect. 5.1. Nevertheless and as already mentioned, we expect that an ontological semantics may be derived from the case of FOL, such that ontological semantics can provide another way to look at CL, as in the case of FOL.

FURTHER, SIMILAR ACCOUNTS TO CL

As a final note on CL and further approaches similar to and / or preceding it, we remark the a number of logics and representation approaches with similar features can be found in the literature, sometimes under the label of 'logics with *punning with names*', explained as using symbols in different syntactic roles with distinct interpretations according to those roles, cf. [600, p. 626]. The latter presents a similar DL semantics for metamodeling. HiLog from the context of logic programming is even more "expressive" (syntactically), because arbitrary terms can appear in predicate position, which seems fairly useful in some cases. [151, sect. 3] elaborately discusses the relation of HiLog to FOL and links to comprehension formulas (referring to [220]), see [151, p. 206]. We see further similarities with RDF and its semantics [380], the non-standardness of which is criticized in [433]. A slightly other view is discussed in [123], referring further to [185]. Chris Menzel does not only present a proof theory for CL in [577], but also earlier work of his has parallels with CL, cf. e.g. [574]. Of course, one / the very own predecessor of CL, still visible through the CLIF dialect of CL, is the Knowledge Interchange Format (KIF) [272–274], remember also FN 45 on p. 9.

7.2.2.3 Naive Predicate Logic

SUMMARY OF THE APPROACH

Fabian Neuhaus in [619] (in German) presents a logic named "Naive Prädikatenlogik" / Naive Predicate Logic [own transl.] (NPL), i.e., a logical syntax (a specific standard FO syntax) equipped with a proof theory and a semantics. Neuhaus points out that, technically, NPL is (almost)[693] a standard many-sorted

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[690]i.e., in the syntactic role of a predicate application

[691]Note that CL allows for co-reference and non-equivalence wrt multiple relation interpretations for different arities. But for fixed arity, there is only one possible relation extension, i.e. (and as it is stated), 4.24 applies for atoms of equal arity.

[692]Note a very similar discussion in [151, p. 206] wrt HiLog. Moreover, the problem of equal categories with distinct extensions separates between the use of the π- and ν-semantics introduced in [600] for ontological purposes. ν-semantics, like HiLog's, by construction has only models which comply with the single extension property. In contrast, π-semantics allows for models where this is not the case [600, p. 629]. Both works are mentioned anew at the end of this sect. 7.2.2.2.

[693]The only predicate $\mathcal{P}$ (not to be confused with our sign for the powerset operator) has no fixed arity, but is anadic, cf. [619, p. 87].

first-order predicate logic [619, p. 87; 126; 209] with a single predicate "$\mathcal{P}$". Initially, [619, esp. sect. II.3] argues that predication should be understood as a name for the speech act of characterizing entities by means of using a predicate for them [ibid., p. 33]. After further analysis, Neuhaus concludes that a sign for predication falls into the category of 'copula', i.e., has a status comparable to that of parentheses in an atomic formula $Q(x)$ [ibid., p. 38]. In the language itself that copula is introduced as the only predicate symbol $\mathcal{P}$ in order to avoid non-standard extensions to FO syntax [619, p. 83].

Neuhaus adopts a truth-value semantics according to Hugues Leblanc [512, 513][694] with the motivation of providing a logical language that avoids ontological presuppositions [619, p. 104]. Truth-value semantics leads to a substitutional approach to quantification, which requires some justification [ibid., p. 105–112], ending in the author's remark that a truth-value semantics is well-suited for logical matters, but "It is completely inexpedient for the task of examining the relationship of language and world. That makes it inappropriate for ontologists, but does not bother a logician." [619, p. 112, own transl.].

In addition to the truth-value semantics, [619, sect. III.10] defines a referential semantics for NPL, which Neuhaus discusses and questions critically.[695] In its definition of predication semantics, this referential semantics parallels features of CL semantics, in particular the case of predication is similar, cf. (7.1) above. Using a different syntax with $\mathcal{P}$ and a nominalized / reified predicate F^d, $\mathcal{P}(k_1, \ldots, k_d, F^d)$ is true in an interpreting structure, which has two functions Ext and I, iff $(I(k_1), \ldots, I(k_d)) \in Ext(I(F^d))$, i.e., an atomic formula is satisfied if the tuple of the referent interpretations of the arguments is a member of the set that is assigned to the referent interpretation of the predicate. The fourth chapter is devoted to the analysis of known paradoxes (Russell, Grelling, and the Liar paradox) and their non-occurrence in NPL, as a logic that allows for quantifying over (nominalized) properties.

RELATIONSHIP WITH ONTOLOGICAL SEMANTICS

The quotation on "inappropriateness for ontologists" may not sound promising for the enterprise of formalizing ontologies at first glance, and there are various differences in the basic positions and starting points between NPL and ontological semantics as proposed herein. With a closer look and with hindsight[696], and especially from an application-oriented point of view (as concerns each logical language and what these offer to language users), some of our results of ch. 3 and 4 are closely related to and in harmony with a number of quintessences drawn in [619]. Moreover, the development of NPL shares motivations from sect. 1.2, e.g., of a "free" / less restricted use of predicates (e.g. in argument positions, but also other aspects) and in avoiding ontological presuppositions in the logical language.

Neuhaus likewise realizes that most predicates can be "paraphrased away" [619, e.g. p. 210] in terms of nominalized predicates. Essentially and in AI-terminology, this means reifying all predicates, which basically corresponds to the approach of first introducing logical individual constants for everything that is to be denoted, as in the method $\mathcal{OS}$ proposed in sect. 5.1.

The basic views on predication are very different, in that ontological semantics assumes that predication[697] can be ontologically analyzed – in possibly different ways – and can be defined, except for fundamental predicates. In $\mathcal{CR}$ terminology, these definitions establish a (specific) relationship for the components of an atomic formula, commonly one between the predicate referent and the referents of its argument. Accordingly and usually, the definitions should involve reflection constants, which can be understood as nominalized / reified predicates.

From a more formal point of view, due to the proximity to CL, the above remarks on CL wrt the linkage of referent interpretation and the satisfaction of atomic formulas should transfer correspondingly. Furthermore, it remains future work to study in detail whether NPL may be seen as an instance of ontological semantics, resulting from a predication system that declares $\mathcal{P}$ as a fundamental predicate (for which no axioms are assumed, apart from tautologies of classical logic) and that otherwise uses only constants in the language.

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[694][514] appears in a more recent edition of the handbook which contains [513].

[695]A.o., it is stated that one cannot learn anything about the nature of properties (in the sense of a "formal property theory" [619, p. 145, own transl.]) from the interpretation structures and the respective sets used therein, s.t. also the referential version of NPL is regarded to be compatible with realist, conceptualist, or nominalist formal property theories [619, p. 145–146].

[696]Despite its publication year of 2004, we discovered NPL only in a late stage of developing ontological semantics.

[697]and more generally, syntactic objects composed of other syntactic objects

Overall and despite differing basic assumptions, we see various observations in the foundation and development of NPL that can be shared against the background of ontological semantics, and several characteristics that apply to NPL as well as to ontological semantics. Without having introduced and discussed the mentioned paradoxes herein, let us mention that their treatment in [619, ch. IV] appears largely acceptable and adoptable from the point of view of ontological semantics, such that this chapter in [ibid.] should form a suitable starting point for future analyses of ontological semantics wrt those issues.

7.2.3 Pointers into Weakly Related Fields

We see plenty of further publications that are, and thus should be explicitly, related to one or another aspect that occurs in the previous chapters. Albeit not pursuing any more detailed investigations in the framework of the present document, this claim is to be substantiated by indicating the following selection of connections. Actually, we begin with weakly related approaches that carry the same name 'ontological semantics'. Comments and pointers are arranged in a way oriented at smooth transitions.

- The extensive account of 'ontological semantics' by Sergei Nirenburg and Viktor Raskin [627, 628] is largely a work on / in text meaning and computational linguistics, originating from machine translation. For this reason we see the approach as only weakly related, although it is true otherwise that [628] touches several related aspects, e.g., on meaning in [628, ch. 3] and on ontology in [628, ch. 5]. Notably, there is also a component of axiomatic ontology [628, sect. 7.1.6] that is based on/integrates the Mikrokosmos ontology, cf. [289, sect. 2.3.4], which is represented in a frame-based format.
- [523] speaks of ontological semantics in the context of conceptual modeling and the use of ontologies therefore, similar to and building on earlier work of Yair Wand, Ron Weber and Jörg Evermann, cf. e.g. [231, 862]. In particular, the aim is "[...] to assign real-world semantics to a core set of UML constructs by proposing a set of principles for mapping these constructs to the formal ontology of Mario Bunge, [...]" [523, p. 179]. This can be compared to ontological usage schemes and the latter might offer a formalized account for the same purpose.
- Similarly, OUS appear associable with Domain-Specific Modeling Languages, cf. [190], e.g. for their ontological (re-)interpretation, itself allowing for bridging / merging of domains-specific languages (DSLs) [190, 578]. [186] concerns reference domain ontologies to define the real-world semantics of DSLs.
- In general, formal semantic approaches to integration and interoperability, e.g. [226, 228, 472, 733, 734], are related to this thesis (and have been mentioned above). This extends to the context of databases, e.g., 'instance-based integration' in [847, esp. sect. 3] relates to ontological usage schemes. [103]
- The Uni-Level Description (ULD) [103] is a meta data model in a flat, FO representation that is claimed to provide uniform representation and access to data model, schema, and data. We see connections to our considerations on the meta-ontological architecture in sect. 2.3.3 as well as to aspects of ontological semantics.
- Further investigations may profit from earlier publications dealing with reification. For instance, [39] is a 1997 publication on reification in DLs, using a role for membership with an adapted language semantics, such that it remains first-order.
- Object Role Modeling (ORM) [@89] [368] is closely related with relations and roles in $\mathcal{CR}$. [815] discusses formalizations of ORM.
- Ontological semantics shares much motivation and some features with situation theory / semantics [56, 201, 202]. E.g., [202, p. 606–607] considers a notion of unrestricted, yet situated quantification. Overall, reading about situation semantics has certainly influenced the development of ontological semantics. An actual comparison with hindsight is of interest, but is not accomplished herein.
- Intensional logics, see e.g. [246, ch. 2], are very occasionally considered herein, although close connections must be expected. E.g., and remembering CL and NPL, note that George Bealer's intensional logic, the Theory of Properties, Relations, Propositions (PRPs) relies on uniform predication: "[...] treats the copula in natural language as a distinguished (2-place) logical predicate that expresses the

fundamental logical relation of predication." [65, p. 6] (in clear contrast to NPL).

Among our reasons not to adopt an intensional logic immediately is the "remaining" problem of extensionality in Montague semantics (for the latter, see [656]). However, further accounts in response to that remain to be investigated; cf. "It was mentioned in Section 3.5 that one criticism of Montague's semantics was that intensions analyzed as functions from possible worlds to extensions are not intensional enough: logically equivalent expressions of any category are then counted as semantically identical. Of the many responses to this problem, one that has a wide range of potential consequences is to replace the background metatheory, substituting a property theory for the normally presupposed set theory. (The principal feature which distinguishes all property theories from all set theories is the rejection of the axiom of extensionality.)" [504, sect. 4.2].

- More broadly, we are well aware that numerous positions that are discussed or merely touched in ch. 4 are well-known in philosophy, and find an established context there, while the latter is (almost certainly) misrepresented herein. However, this thesis has been written in applied ontology from a theoretical computer science background, such that only glimpses to individual pieces from philosophy could be added where this appeared adequate to us. To name a few examples of further interest: [833] on "Topics on General and Formal Ontology" has many interconnections with the chapters herein. [161] tackling (a.o.) denotation and reference relates to our discussions, esp. in ch. 4 on ontological semantics.

  Edward N. Zalta offers a Theory of Abstract Objects [890, 891] [@122], cf. also [892]. Of particular interest for us is its distinction between two types of predication, namely and very roughly speaking, one of *exemplifying* a property (for concrete objects), another of *encoding* a property (for abstract objects, incl. fictional ones, a.o.). The existing weak connection to ontological semantics is to consider different types / kinds of predication, where ontological semantics allows for more kinds, which are not predetermined. However, the two kinds in Abstract Objects may relate to envisioned versions of intensional equivalence.

- [77, 249, 250] may be beneficial regarding theories of concepts, e.g., for extensions of $\mathcal{CR}$ or for approaching notion(s) of intensional / conceptual equivalence.

7.3 Conclusions

7.3.1 Reconsideration of Objectives

RECAPITULATION

We recapitulate the objectives established at the beginning of this thesis in sect. 1.3.2 and 1.3.3. Note that the presentation follows a dependency-based order, whereas the numbering is due to the sequence as originally conceived. As summarized at the end of sect. 1.3.3, the following objectives are pursued herein:

- an in-depth analysis (Obj. 1) of accounts and open problems of
  - ontology representation, primarily by FOL theories
  - semantic translations on the basis of ontologies
- a theoretical framework that
  - addresses the problem of formally representing ontologies themselves (Obj. 3)
  - serves as a fundament for the question of how ontologies can be used for semantics / intension-preserving translations (Obj. 2)
  - provides a solution that can reuse current reasoners to a large extent (Obj. 4)
- contributions to ontologies (Obj. 6), incl. an ontology that makes the theoretical framework applicable, e.g. to GFO (Obj. 5)
- contributions to methodological / engineering aspects and applications of ontologies (Obj. 7)

OBJECTIVES ARE ACCOMPLISHED, SOME AS FIRST STEPS

At a glance and remembering a fairly detailed "outline with hindsight" in the summary sect. 7.1 above, all objectives are tackled and pursued in the preceding sections. We defend the position to have them basically achieved, perhaps with limitations wrt Obj. 2. Clearly, there are various "places" / aspects where further steps can be taken (and are desirable). In slightly more detail, Obj. 1 is addressed in ch. 2, in its sect. 2.2, in particular. The major outcomes there are the discussion of the encoding problem wrt (usual) formal semantics of languages as well as the observation that equivalence modulo a(n ontological) theory is an inappropriate form of ontology-based / intensional / conceptual equivalence. Ontological semantics as presented in ch. 4 together with the formalization method $\mathcal{OS}$ of sect. 5.1 is oriented at Obj. 2–4. Adopting the perspective of analysis explication, we argue that it solves the encoding problem (Obj. 3) on the basis of a referential semantic approach. This is visible by its ontological neutrality which applies, depending on the definition, at least to the class of $\mathcal{OS}$ theories, see sect. 4.7. Although we believe that ontological semantics is therefore also promising wrt the overarching, long-term target of proposals for understanding intensional equivalence – in favor of Obj. 2 – a "proof" / verifying support for that, ideally in the form of such proposed notions, is still missing – which is a qualification / limitation of convincingly arguing for Obj. 2 – and remains future work. At least, see the next sect. 7.4 with some indications. Obj. 4 is dealt with in sect. 4.6 and widely achieved by Th. 4.44 of transferring inconsistency and entailment / reasoning results from classical to ontological semantics limited to set-compatible ontological models, indicating further generalization (without a constructive solution) in Obs. 4.46.

The proposal of the ontology of categories and relations $\mathcal{CR}$, presented as a conceptualization in sect. 2.4 and formalized in sect. 6.1, accounts for the achievement of Obj. 5, at least, for an initial contribution with this intent. The link to the applicability of ontological semantics, via method $\mathcal{OS}$ and ontological usage schemes, is established by relying on $\mathcal{CR}$ in several such schemes in ch. 5 and ch. 6. Regarding ontologies in general, we mention a number of further contributions in sect. 6.2, of which only the topic of time is elaborated in detail in the framework of the present work itself, see sect. 6.2.2 and sect. 6.3, in particular. Eventually, results gained from pursuing the seventh objective are primarily described in ch. 5, starting with method $\mathcal{OS}$ and continuing via ontological usage schemes to applications in the biomedical domain in sect. 5.4.

7.3.2 Concluding Remarks

FOCUS ON ONTOLOGICAL SEMANTICS

Without aiming at diminishing the value that we see in components such as $\mathcal{CR}$, the ontologies of time $\mathcal{BT}^{\mathcal{C}}$ and $\mathcal{BT}^{\mathcal{R}}$, contributions to ontologies and representation in the biomedical domain as well as the early analyses provided herein, the concluding remarks of this thesis – except for indications of future work and next steps that appear below – are devoted to ontological semantics[698] and its understanding in a broader context. Indeed, we certainly see a potential for doubts about the approach of ontological semantics, possibly regarding its definition, and maybe (more?) its utility in the light of returning / sticking to FOL with its classical semantics, in some sense. Hence it is in order to comment on a number of issues and questions in these regards, which we have asked ourselves repeatedly in the course of writing this thesis.

IS ONTOLOGICAL SEMANTICS ONTOLOGICAL OR ONTOLOGICALLY NEUTRAL?

The youngest among those questions concerns the terminologically somewhat contradictory relation between 'ontological' semantics which is shown to be 'ontologically neutral'. If the latter is the case, why is it reasonable to call the semantics 'ontological', rather than 'ontologically neutral'? We see the answer(s) to relate to distinct levels at which ontological semantics is considered herein. On the one hand, ontological semantics is designed for not making any ontological assumptions – at a general level, concerning ontological structures, which do not presuppose any kind of entities, and the definition of satisfaction of logical connectives and quantifiers. This also applies to the generic definition of predication systems, but this is a parametrized notion. Thus, on the other hand, particular instances of ontological semantics presuppose the set of predication definitions resulting from the predication system as tautologies[699] together with the

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[698] incl. method $\mathcal{OS}$ and ontological usage schemes
[699] denoted by Π in sect. 4.4.4

meta-level assumptions that reflect as further tautologies into the object level,[700] both forming a "built-in" theory / ontology. The latter justifies the name 'ontological semantics' in addition to the consideration of ontological structures instead of mathematical ones. Therefore and clearly, specific instances of ontological semantics are not (necessarily) neutral in the sense of being free of ontological assumptions. In terms of the definitions of ontological neutrality in sect. 3.4, they can be neutral in the sense that they do not change ontological assumptions (about referent entities) between the meta level and the object level. As already stated and in contrast to set-theoretic semantics, ontological semantics does not introduce new kinds of (referent) entities[701] that are inaccessible from within a theory. Altogether and with hindsight, one may have considered renaming ontological *neutrality* into ontological *faithfulness*. Then, the term 'ontological neutrality' may be reserved for an ontological semantics without any meta-level assumptions / additional tautologies[702] compared to the classical ones. Note that this corresponds to the approach of a naïve definition of predication semantics discussed in sect. 4.4.1, together with no meta-level assumptions on interrelations among predicates / predications.

RELATION OF OBJECT AND META LEVELS WRT ONTOLOGICAL SEMANTICS

Ontological neutrality, in the sense of accepting the same theory of entities at the object and the meta level, gives rise to another puzzle: Why is it actually possible for a semantics that does not presuppose any entities beyond those that the theories refer to stated in them? One may expect a problem of self-reference here, in line with widely discussed paradoxes such as Russell's, as well as one may see a problematic connection to the motivation of Tarski in actually introducing the distinction between object and meta level. The main point of support for the property[703] of ontological neutrality is that the object- to meta-level translations considered in this connection are only concerned with entities that act as referents at the side of the meta-level. That means, for example, syntactic entities and meta-level relations between syntactic entities and referent entities such as denotation, or other meta-level relations like satisfiability or entailment exist at the meta-level *in addition* to the referent entities. Insofar it is not the case that object and meta level are equated in totality – this concerns only those entities that can be referred to by the symbols of the object language.

If the argument of having only those entities available as referents that an object-level theory postulates, this appears reasonable to us in particular for ontological theories, as they are intended to declare which entities there are (and how they are interrelated). According to our knowledge, an at least partially analogous approach appears to be taken in dealing with set-theories stated in FOL, where sets are assumed at the meta level, either adopting the same, weakened, or just different axioms for set membership.

METHODOLOGICAL APPLICABILITY EVEN IN CASE OF DOUBTS

We can imagine a fair amount of skepticism wrt ontological semantics and the justification that we attempt to provide herein. Yet this does not hamper adopting the method $\mathcal{OS}$ for FOL derived from it, if only plainly on pragmatic grounds, e.g., in terms of (then possibly only seeming) expressiveness.[704] From a practical point of view of formalizing ontologies in FOL, one may surely adopt the policy of representing everything in terms of logical individual constants, adopt some predicates as primitives and define all others, and from

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[700]Where possible, F denotes an axiomatization of these, e.g., in sect. 4.4.4.

[701]beyond those associated with a signature, from an analysis explication point of view

[702]i.e., with $F = \emptyset$ or $F \cup \Pi = \emptyset$.

[703]'properties' would be more precise, given distinct definitional variants

[704]Compared to HOL, FOL appears "equally expressive" in the following sense, where we follow the thesis of Ebbinghaus, Flum and Thomas' on the relationship between first-order logic and the development of mathematics [213, p. 106–107], [214, p. 115–116; p. 149]. They state that, according to all experience, all mathematical deductions can in principle be carried out in a first-order proof system, if a foundational axiomatization of set theory is presupposed and all mathematical notions are reduced to set-theoretic expressions. The most relevant piece is the following, where L^S is a first-order language for an adopted set-theory, axiomatized by the formal theory Φ_0:

> More generally: Experience shows that all mathematical propositions can be formalized in L^S (or in variants of it), and that mathematically provable propositions have formalizations which are derivable from Φ_0. Thus it is in principle possible to imitate all mathematical reasoning in L^S using the rules of sequent calculus. In this sense, first-order logic is sufficient for mathematics.
> [213, p. 106], in the German equivalent [214, p. 115].

Wolfgang Rautenberg agrees with this view [691, p. 90], yet also suggesting not too overestimate it because it is not of any practical relevance for (doing) mathematics.

then on establish an axiomatization. Indeed, we claim that efforts of a similar[705] kind exist, remembering the case of SUMO [@119], for example, where reification and axioms of meta-ontological kind within the theory appear to be implemented, cf. sect. 6.1. Of course, adopting the method in connection with FOL under any non-referential semantics is likewise possible, up to considering FOL only for reasoning, based on a proof theory. Nevertheless, personally, we prefer and advocate having a clearly defined semantics available for ontology representation.

In our opinion, the mere method leaves a number of questions open. What is the relationship between logical individual constants and predicates that may represent the same notion / entity (by intention of the language user)? What is the relationship to the, then classical, formal semantics of FOL? On the ground of just basic logical foundations, which define, e.g., FOL and DL syntax and their set-theoretic semantics, however, we saw no straightforward answers to these questions, as it was not obvious to us what constitutes an adequate use of logic and where one would run into problems, as stated, in the case of representing categories (or "even" sets) by means of logical individual constants. Insofar, the present work is our attempt of studying corresponding questions/issues and effects. Surely, this is a contribution of theoretical character, and within this realm, primarily one of an explanatory kind.

Based on method $\mathcal{OS}$ and theories adhering to it, ontological usage schemes allow for a more convenient use of FOL, e.g., by sticking to the assumption that quantification ranges only over the *ontological* individuals of a domain of reality / knowledge, not at the same time over its categories and relations. An ontological usage scheme provides some ontological grounding for domain-specific notions even with abstract core ontologies, e.g. by stating that a category, denoted by a predicate symbol, is a category of individuals. The main utility in our own case should concern the further development of GFO, as the case of the ontology of time in sect. 6.3 illustrates.

DOES THIS AFFECT PRACTICAL ONTOLOGY DEVELOPMENT?

Approaching the very end, there is the question for practical implications. Is there any need for ontological semantics, if we can adopt $\mathcal{OS}$ theories (as foundation for ontological usage schemes) basically[706] by returning to FOL under classical semantics, without differing reasoning results? First of all, according to our impression it is not uncommon that basic logical training leads to ontologies that are represented intuitively / unconsciously along the lines that, as we argue, can now be theoretically supported by ontological semantics and / or ontological usage schemes based on $\mathcal{OS}$ theories. For instance, many ontologies in FOL and DL can likely be appropriately interpreted by a parametric ontological usage scheme that classifies unary predicates as categories, n-ary predicates as relations, and logical individual constants as ontological individuals. Indeed, if accepted, this is a positive observation, in that the presented theoretical account covers a large number of cases. Independently of falling under a generic, parametric scheme or not, we defend the view that it is useful if an ontology engineer is aware of this meta-ontological analysis and of the difference, up to complete independence, from the abstract syntax *and* the formal semantic categories of the language. Moreover, though without any comprehensive validation herein, we expect that ontological semantics and its surrounding theory can prove beneficial beyond illustrating cases that we have considered in previous chapters. In our own case, the theory allows for inspiration of further solutions, cf. the outlined ideas on intensional equivalence in the next section.

QUINTESSENCE

For us and after all, ontological semantics (as developed herein, in connection with FOL) yields further justification for relying on the axiomatic method for ontology development,[707] in a way that includes the semantics of languages that we have not seen before. Regarding formalization, we argue that a language with an ontological semantics accounts for the conceptualization underlying that ontological semantics, without any necessary consideration of additional entities / notions, including formal ones. Hence, for us it resolves the problem of providing the "right" / an adequate formalization, given a certain formal semantics

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[705]without, e.g., the notion of predication definitions

[706]As stated in sect. 5.1, OS theories can be considered under both semantics, with comparable effects wrt ontological neutrality.

[707]To what extent this might be a "rediscovery" of an earlier function of the axiomatic method requires further studies of the development of early ideas in mathematical logic. Clearly, the axiomatic method precedes model-theoretic semantics, in that the former has been subject to discussion, cf. David Hilbert's [401] of 1918, prior to the early work of Alfred Tarski on metamathematics in the 1920s and 1930s, cf. [806].

and the goal of encoding a conceptualization therein, by allowing for actually stating the conceptualization in a formalism without its own / additional assumptions, and possibly constraints. That means one can view the axiomatic method, especially in connection with the formalization method $\mathcal{OS}$ for FOL, as an approach that leads to theories that can establish "their own" semantics – as is required for representing ontologies.

7.4 Beginnings of Future Work

COMMENTS ON SELECTION

After concluding the theory elaborated herein, it remains to spend at least a few words on future work and next steps that we can foresee. We shall focus on few selected items in the present section, although previous chapters contain a number of "open ends", i.e., tasks that are worthwhile to pursue further, already noted there. However, listing all of them here, briefly and without their context, is hardly likely to be beneficial to the reader, whereas we refrain from presenting them in an understandable, but correspondingly more detailed manner. At least an equal treatment of all those items would lead to an inappropriate length of the section. Note that sect. 6.3.13 comprises notes on future work wrt the ontologies of time presented in sect. 6.3. In the present section, remarks on open issues in the theory as established are followed by initial comments on proceeding toward the open, long-term goal of intensional / conceptual equivalence.

7.4.1 Extending Results and Relationships

EXTENDING RELATIONSHIPS

Remembering sect. 7.2 on related work above, we see numerous ways of more in-depth comparison with other approaches, typically on particular aspects, as well as for the substantiation of aspects herein from fields such as philosophy, cognitive sciences, semantics and linguistics. Indeed, especially the pointers to related work in sect. 7.2.3 form a list of candidates wrt which our investigations should be further interlinked.

ELABORATION ON ONTOLOGICAL NEUTRALITY

Besides "merely" establishing relations to other works and fields, previous chapters leave room for extending some of the results presented. Indeed, we are aware that the present work takes only small steps in the analysis of several of the notions established. A first example is given by the definitions of ontological neutrality. As already noted in sect. 3.4 and due to not yet being embedded into a more comprehensive theory, the current definitions of ontological neutrality may require future revision.[708] This does not only refer to the notion of equivalence or entailment employed in those definitions, cf. also sect. 4.7, but the next section reveals a potential connection with notions of intensional equivalence, whose further elaboration may retroact on the understanding of ontological neutrality. Furthermore and in addition to the present definitions, several aspects could be singled out and be defined separately, e.g., focusing on the maintenance of existential claims or of ontological commitments in general.

ONTOLOGICAL SEMANTICS: CONNECTIONS TO CLASSICAL CASE AND METALOGICAL ANALYSIS

Regarding ontological semantics, but ignoring that various aspects of our considerations during its introduction can be further rooted / anchored in accounts and literature of other areas, a next step in generalizing the results on ontological semantics itself is to establish a proposition on consistency transfer from set-theoretic semantics "complementary" to the inconsistency transfer shown. Consistency transfer may have the form that a classically consistent theory that satisfies all tautologies of a corresponding ontological semantics is ontologically consistent, as well. Similarly, Obs. 4.46 should be "implemented" or, better, be accompanied by a theorem that generalizes Th. 4.44. In both cases of consistency transfer and the (constructive) generalization of inconsistency transfer, there are open questions as to whether the respective results can be achieved[709] in combination with assuming an alternative set-theory, e.g., with a universal set, at the meta level or with "standard" ZFC set theory.

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[708]Which is not to say that we were aware of actual problems of the current definitions. We do defend the current versions as part of this thesis.

[709]at all, and if so, if more easily

Another issue concerns the metalogical features of ontological semantics, in general or wrt specific instances. Interlinked with the outcomes of the previous issues, properties of FOL under classical semantics may be transferable to / derivable for ontological semantics rather straightforwardly.

Regarding ontological usage schemes, it appears worthwhile to study the relationship between ontological usage schemes and the classification of meta properties that has led to OntoClean, cf. the discussion by Nicola Guarino in [326, sect. 2] up to further elaboration by Giancarlo Guizzardi [351], see also sect. 7.2.1.

ONTOLOGY OF CATEGORIES AND RELATIONS

Thinking of $\mathcal{CR}$, the present formalizations cover a very abstract level as required in this thesis. Moreover, the need for further elaboration esp. of the first-order formalization is already discussed in sect. 6.1.2, including the meta-theoretic analysis. For example, based on first steps in that direction, we expect a consistency proof to be challenging, in particular if the reflection axioms (6.11)–(6.13) for the basic $\mathcal{CR}$ signature of instantiation, role-playing, and role-having are included in the theory. Wrt the latter, it is of further interest to examine the relationship between theories with and without those axioms, studying in which theories they exhibit "significant" effects. A natural extension beyond the current formalized theories of $\mathcal{CR}$ in the context of the General Formal Ontology (GFO) [@46] is to augment these on the basis of taking the overall conceptualization of categories in GFO into account.

7.4.2 Notions of Intensional Equivalence

EQUIVALENCE UNDER ONTOLOGICAL SEMANTICS IS NO INTENSIONAL EQUIVALENCE

Despite an early original inspiration of this thesis by the problem of an insufficient account of intensional / conceptual equivalence, the development of an ontologically neutral semantic foundation has not left sufficient resources thus far in order to maintain this ambition and substantially tackle the problem of intensional / conceptual equivalence in itself. Clearly and albeit we find that a language that does not require encoding is an important prerequisite, looking at the arguments in the analysis of the available account of ontology-based equivalence, we do not think that the problem of intensional equivalence is resolved by pleading ontological semantics, incl. not by adopting a modification of Def. 2.3, employing consequence and equivalence under ontological semantics. In particular, the problem of the equivalence of all formulas that follow from a theory applies under ontological semantics, as well.

ONTOLOGICAL ANALYSIS IS REQUIRED AND EQUAL SIGNATURE AS A FIRST APPROXIMATION

In order to approach this topic, we are first interested in and working on an ontological analysis of the notion of intension. In this regard, we observe first, based on the development of this thesis, that the consideration of categories – as "intensional entities" – in the domain of discourse (i.e., their reification / nominalization) does not a priori yield a solution / an approach that elucidates structural relationships among intensions or categories. Moreover, we observe that intensional equivalence is of interest for two kinds of entities: for categories themselves as well as for formulas.

Wrt to the latter, a first approximation of the intensional equivalence *of formulas* may be attempted as follows. Starting from the definition of ontology-based equivalence in Def. 2.3, but avoiding the assumption of any ontological theory[710] we reach (classical) logical equivalence. From the perspective of analysis explication and assuming intended interpretations of predications (in the sense of predication system) and of logical individual constants, one may further constrain that to the idea that two formulas are intensionally equivalent :iff they are logically equivalent *and* if they "speak about the same", reflected by the requirement of having equal signature. This yields a first consideration for formulas, namely ϕ and ψ are intensionally equivalent :iff $\phi \equiv \psi$ and $Sig(\phi) = Sig(\psi)$. While this is just one working hypothesis which still has issues, e.g., in the case of tautologies, we observe – with hindsight – that the basic idea is actually hidden in (at least) Def. 3.20, p. 121 of ontological neutrality, which adopts an "equal language" requirement.

EQUIVALENCE MODULO SPECIAL THEORIES AND ROUTE FOR INTENSION OF CATEGORIES

A related occurrence / case, there of already declaring two statements to be conceptually equivalent, is given in Ex. 5.12, p. 199, in connection with GFO-Bio. This case is different, in that conceptual equivalence is defined by equivalence modulo a theory – possibly surprisingly, given our earlier criticism. However,

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[710]as already considered in sect. 2.2.2.2 and rejected as a solution as such

that theory does not cover all of GFO-Bio. Much more its basic idea is very close to that of expecting equal signature, but now allowing for (mainly)[711] the application of predication definitions. Note that this approach becomes possible by the background of ontological semantics.

Analogously, one may investigate admitting analysis definitions. This leads also a step further towards intensions of categories, where we currently investigate their connection to (primarily) analysis definitions and generalizations from that starting point. This is based on the idea that categorial intensions become visible through the axioms postulated within an ontological theory, possibly relating well to the theory theory of concepts [610, ch. 3, p. 60–64][712]. In these regards, the $\mathcal{CR}$ taxonomy in sect. 6.1.1 with its asserted definitions of OWL classes and the resulting equivalences based on the classification by the OWL reasoner appears very instructive, cf. the discussion in that section.

Nevertheless, even if one might see some routes for working towards intensional equivalence to start based on these rough notes, we expect the provision of a coherent, well-analyzed initial proposal to require much further work – of expectedly very interesting kind.

> *"Now, endings normally happen at the end.*
> *But as we all know, endings are just beginnings."*
>
> from the lyrics of the album
> *Amarok* by Mike Oldfield, 1990

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[711] For required auxiliary formulas, see FN 594, p. 199.

[712] Although the term 'knowledge approach' is preferred by Gregory L. Murphy over 'theory theory' [610, p. 61]

Appendix

Appendix A

Additional Preliminaries

This appendix chapter comprises a number of specific formal definitions and axiomatic systems, in addition to the basic formal preliminaries gathered in sect. 1.5.

A.1 Logical Notions

A.1.1 Relativation

Relativation refers to the restriction of formulas to a unary predicate, say P, (or a formula with a single free variable). The *P-relativized* $\phi^{|P}$ of ϕ results from ϕ by replacing all subformulas of the form $\exists x . \alpha$ by $\exists x . P(x) \wedge \alpha$, and $\forall x . \alpha$ by $\forall x . P(x) \rightarrow \alpha$ (recursively in both cases), cf. [693, sect. 6.6, p. 258], [214, sect. VIII.2.D-E, p. 130, 134–135]. The term 'relativized' appears only in side remarks in [222, p. 170, 297].

The next definition is adapted from [693, sect. 6.6, p. 258].

A.1 Definition (relativation of formulas and theories to a predicate symbol P)
Let L a FO language, $x \in Var(L)$ a variable, $\phi, \psi \in L$ formulas, and $T \subseteq L$ a set of formulas. In addition, let P a unary predicate, which may or may not be a member of $Sig(L)$.

A precise definition of $\phi^{|P}$ runs by induction: $\phi^{|P} := \phi$ if ϕ is an atomic formula, $(\neg \phi)^{|P} := \neg \phi^{|P}$, $(\phi \wedge \psi)^{|P} := \phi^{|P} \wedge \psi^{|P}$, and $(\exists x . \phi)^{|P} := \exists x . P(x) \wedge \phi^{|P}$. For sets of formulas, $T^{|P} := \{\phi^{|P} \mid \phi \in T\}$. ❑

The next definition is translated and adapted from Def. 5.4 and its subsequent text in [214, p. 45–46].

A.2 Definition (substructures, Σ-closed set in a structure, X generated substructure)
Let $\mathcal{A}$ and $\mathcal{B}$ Σ structures. Then $\mathcal{A}$ is called *substructure* of $\mathcal{B}$, :iff

- $A \subset B$
- and these closure conditions are satisfied
  - for n-ary predicate symbol $R \in \Sigma$: $R^{\mathcal{A}} := R^{\mathcal{B}} \cap A^n$
    (i.e., for all $a_1, \ldots, a_n \in A$: $(a_1, \ldots, a_n) \in R^{\mathcal{A}}$ iff $(a_1, \ldots, a_n) \in R^{\mathcal{B}}$)
  - for n-ary function symbol $f \in \Sigma$: $f^{\mathcal{A}}$ is the restriction of $f^{\mathcal{B}}$ to A^n
  - for a (logical individual) constant symbol $c \in \Sigma$: $c^{\mathcal{A}} = c^{\mathcal{B}}$

If $\mathcal{A}$ is a substructure of $\mathcal{B}$, then the universe A of $\mathcal{A}$ is Σ-closed / closed under Σ (in $\mathcal{B}$), i.e., $A \neq \emptyset$, for n-ary $f \in \Sigma$ and $a_1, \ldots, a_n \in A$ follows $f^{\mathcal{B}}(a_1, \ldots, a_n) \in \mathcal{A}$, and for $c \in \Sigma$, $c^{\mathcal{B}} \in A$.

Conversely, every Σ-closed set of the universe of $\mathcal{B}$ forms the universe of a single substructure of $\mathcal{B}$, the X *generated substructure* of $\mathcal{B}$, which is denoted by $[X]^{\mathcal{B}}$; because the above conditions on substructures determine a unique substructure with universe X. ❑

The relativation lemma is translated and adapted from [214, Relativierungslemma 2.3, p. 135]. Cf. also Prop. A.7 in sect. A.1.2.2.

A.3 Proposition (relativation lemma)
Let $\mathcal{A}$ a $\Sigma \cup \{P\}$ structure with universe A, where $P \notin \Sigma$ and unary. Let the set $P^{\mathcal{A}} \subset A$ Σ closed in $\mathcal{A}$. Then for all $\psi \in Lg(\Sigma)$: $\quad [P^A]^{\mathcal{A}} \models \psi \quad$ iff $\quad \mathcal{A} \models \psi^{|P}$.

  Intuitively: The relativation of $\psi^{|P}$ means in $\mathcal{A}$ the same as ψ in $[P^A]^{\mathcal{A}}$.   ❑

A.1.2   Theory Interpretations

A.1.2.1   General form (adapted from [733, p. 134])

Marco Schorlemmer and Yannis Kalfoglou present a generic definition of theory interpretation in [733, p. 134], therein actually in a formal framework based on the theory of institutions [283].[713] However, just for a generic definition, we can abstract away from that background.

A.4 Definition ((general) theory interpretation)
Given logical theories T and T', a function
$\alpha : Lg(T) \to Lg(T')$ is a *theory interpretation* of T into T' iff T' and α preserve T theorems/consequences, i.e., for all $\phi \in Lg(T)$, $T \models \phi$ implies $T' \models \alpha(\phi)$.
α is a *faithful* theory interpretation of T into T' iff $T \models \phi$ iff $T' \models \alpha(\phi)$.   ❑

A.1.2.2   First-order form (following [693, p. 258f.])

Besides "interpretability from Rabin, called *model interpretability*" [693, p. 258] (which we do not repeat here), Wolfgang Rautenberg introduces "interpretability from Tarski (also called *relative interpretability*)" [ibid.], which he defines by means of sets of explicit definitions. Note that Rautenberg's original and our adapted / definitions and proposition assume that theories are consistent. Adaptations primarily concern our notation herein, occasionally they connect to concepts or notation in [214]. $\cdot^{|P}$ stands for the P-relativized, cf. sect. A.1.1. The summarizing phrases are added.

  The next definition is quoted from [693, p. 258] and solely adapted in notation.

A.5 Definition (relative interpretability)
$T_0 \subseteq L_0$ is called *interpretable* in $T_1 \subseteq L_1$ (where for simplicity we assume that T_0 has finite signature) if there is a list Δ of explicit definitions legitimate in T_1 of the symbols of T_0 not occurring in T_1 and of a new unary predicate symbol P such that $T_0^{|P} \subseteq T_1 \cup \Delta$, the *definitorial extension of T_1 by Δ*.   ❑

  This definition is compiled from text in [693, p. 258f.], modifying notation.

A.6 Definition (closure axioms)
Let P the relativation predicate in the context of a relative interpretation from $T_0 \subseteq L_0 = Lg(\Sigma_0)$ to $T_1 \subseteq L_1^{\Delta}$. The set of the so-called *closure axioms* wrt P, denoted by CA_P, is the set
$$\{\exists x . \, P(x), \;\; P(c), \;\; \forall x_1, \ldots x_n . \, \bigwedge_{i=1}^{n} P(x_i) \to P(f(x_1, \ldots, x_n)) \mid c, f \in Sig(L_0)\}$$
For any given L_1^{Δ} structure $\mathcal{B}$ s.t. $\mathcal{B} \models \Delta$, $\mathcal{B}_{\Delta} := [P^{\mathcal{B}}]^{\mathcal{B}|\Sigma_0}$ denotes the substructure generated from $P^{\mathcal{B}}$ of the $Sig(L_0)$ reduct of $\mathcal{B}$.   ❑

  Minor modifications apply to the proposition, incl. adding the preconditions on P and L_0. Cf. also Prop. A.3 (the relativation lemma in [214, p. 135]) in sect. A.1.1.

A.7 Proposition (transfer of satisfaction based on an interpretation (relativation lemma))
Let P the relativation predicate in the context of a relative interpretation with source language L_0, and let $\mathcal{B} \models CA_P$. Then $\quad \mathcal{B}_{\Delta} \models \alpha \quad$ iff $\quad \mathcal{B} \models \alpha^P$, for all sentences $\alpha \in L_0$.

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[713]Cf. further recent work such as [499, 596, 597, 686] on an at least very similar basis, and directed at formalizing ontologies. The notion of institutions itself is founded on concepts from category theory, cf. [3, 52, 544].

A.1.2.3 First-order form (following [221])

A form of theory interpretation developed, much earlier and originally, for first-order logic is comprehensively presented in [221, sect. 2.7, p. 154–163]. It is summarized and presented more compactly in [733, p. 134] and [156, p. 541], and is reformulated below for the purposes and in the style of this thesis. Note that we separate terminologically between (a) signature interpretation translations, (b) formula interpretation translations, and (c) first-order theory interpretations, whereas Enderton uses the term 'interpretation' in all three cases (distinguishable by the arguments of the respective functions). The reformulation is further similar to [214, ch. VIII, esp. sect. E], which is kept under the heading "syntactic interpretations" by Ebbinghaus et al..

The next definition assumes as basic logical constants only $\neg, \rightarrow, \forall$ and uses all others as abbreviations. $\dot{=}$ expresses equality of two terms as an object level atomic formula.

A.8 Definition (signature and formula interpretation translations)
Let Σ a signature, $L = Lg(\Sigma)$, and let T a theory in a language $Lg(T)$ (possibly distinct or equal to L). A translation function $\pi : \Sigma \cup \{\forall\} \rightarrow Lg(T)$ is a *signature interpretation translation* of Σ into $Lg(T)$ iff

- π assigns to $\forall$ a formula $\pi_\forall \in Lg(T)$ in which at most one variable x occurs free and $T \models \exists x . \pi_\forall$.
- π assigns to each n-ary predicate symbol P a formula $\pi_P \in Lg(T)$ in which at most n variables occur free.
- π assigns to each n-ary function symbol f a formula $\pi_f \in Lg(T)$ in which at most $n+1$ variables $x_1, \ldots, x_{n+1}$ occur free and
$$T \models \forall x_1 \ldots x_n \left(\pi_\forall(x_1) \wedge \ldots \wedge \pi_\forall(x_n) \quad \rightarrow \right.$$
$$\left. \exists x_{n+1} \left(\pi_\forall(x_{n+1}) \wedge \forall y \left(\pi_f(x_1, \ldots, x_n, y) \leftrightarrow y \dot{=} x_{n+1} \right) \right) \right).$$

Given a signature translation π of Σ to $Lg(T)$, a *formula interpretation translation* of L into $Lg(T)$ is a function $\alpha : L \rightarrow Lg(T)$ defined as follows.

- Each atomic formula $\phi_P(t_1, \ldots, t_k)$ is recursively (regarding its terms) translated to ϕ_P^α, by replacing the non-variable terms with new variables and constructing intermediate formulas in parallel. The recursion starts from $k+1$ and works towards index 1, for $k+1 \geq i \geq 1$ with a formula sequence ϕ_P^i with $\phi_P^{k+1} := \phi_P(t_1, \ldots, t_k)$ and a formula sequence[714] $\phi_{P,i}^\alpha$ with $\phi_{P,k+1}^\alpha := \phi_P(t_1, \ldots, t_k)$. The recursion completes with applying π_P to $\phi_{P,1}^\alpha$. During the recursion the subsequent conditions apply.

  - For every variable term $t_i = x$, $\phi_P^i = \phi_P^{i+1}$ and $\phi_{P,i}^\alpha = \phi_{P,i+1}^\alpha$.
  - For every non-variable term t_i with function symbol f and $\pi_f(x_1, \ldots, x_{n+1})$, let y be a new variable. Then $\phi_P^i = \phi_P^{i+1}[t_i/y]$. Let $\sigma = \forall y(\pi_f(x_1, \ldots, x_n, y) \rightarrow \phi_P^i)$. Then $\phi_{P,i}^\alpha = \phi_{P,i+1}^\alpha [\phi_P^{i+1}/\sigma]$

- For non-atomic formulas, $(\neg\phi)^\alpha := \neg\phi^\alpha$, $(\phi \rightarrow \psi)^\alpha := \phi^\alpha \rightarrow \psi^\alpha$, $(\forall x\phi)^\alpha := \forall x(\pi_\forall(x) \rightarrow \phi^\alpha)$. □

Enderton provides an example at [221, p. 161].

The interpretation translation of an atomic formula $P(z_1, \ldots, z_n)$ where all z_i are variables simplifies to $\alpha\left(P(z_1, \ldots, z_n)\right) = \pi_P[x_i/z_i]_{1 \leq i \leq n}$ (where the x_i capture the free variables in π_P), cf. "α^π" for zero function symbols in an atomic formula "α" at [221, p. 160].

This can be extended to the case that $P(t_1, \ldots, t_n)$ comprises as terms t_i only variables or (functional) constants: "if φ is an atomic formula with $n$ary relation symbol r, $\pi_\varphi = \pi_r$ applied to the same set of variables and constants;" [733, p. 134]. To see this, we first note that the (functional) constants of L must be available in $Lg(T)$, as well. Then, denoting constants by $c.$ and variables by $z.$, let

$$\{t_1, \ldots, t_n\} = \begin{cases} \{c_1, \ldots, c_k\} \cup \{z_{k+1}, \ldots, z_n\} & \text{if } k < n \\ \{c_1, \ldots, c_n\} & \text{otherwise} \end{cases},$$

assume that $\pi_{c_i} = x \dot{=} c_i$ for $1 \leq i \leq k$ and use $\bar{t} = (t_1, \ldots, t_n)$ as short form of the argument sequence. Then applying the recursive definition from above (and joining and reordering the arising sequence of

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[714]In those formulas, non-variable symbols of L and $Lg(T)$ are mixed, i.e., their language is $L \cup Lg(T)$.

premises $\forall y_k(\pi_{c_k}[x/y_k] \rightarrow \forall y_{k-1}(\pi_{c_{k-1}}[x/y_{k-1}] \rightarrow \ldots \forall y_1(\pi_{c_1}[x/y_1] \rightarrow \pi_P(\bar{t}[c_i/y_i]_{1\leq i\leq k}))\ldots)))$ yields

$$\alpha(P(\bar{t})) = \forall y_1 \ldots y_k \cdot (\bigwedge_{1\leq i\leq k} \pi_{c_i}[x/y_i]) \rightarrow \pi_P(\bar{t}[c_i/y_i]_{1\leq i\leq k})$$

$$= \forall y_1 \ldots y_k \cdot (\bigwedge_{1\leq i\leq k} y_i \doteq c_i) \rightarrow \pi_P(\bar{t}[c_i/y_i]_{1\leq i\leq k})$$

Since the latter formula is logically equivalent to $\pi_P(\bar{t}[c_i/y_i]_{1\leq i\leq n})[y_i/c_i]_{1\leq i\leq n}$ and the substitutions neutralize each other, $\alpha(P(\bar{t})) = \pi_P[x_i/t_i]_{1\leq i\leq n}$ is correct (again, with free variables x_i in π_P). Notably, this line of reasoning can be easily generalized to allow for a renaming of the (functional) constants between L and $Lg(T)$, instead of being restricted to the very same constants. With a bijection ρ on constant symbols and with interpretation definitions $\pi_{c_i} = x \doteq \rho(c_i)$, the corresponding translation of an atomic formula involving constants (and possibly variables) becomes $\alpha(P(\bar{t})) = \pi_P[x_i/t_i][c_i/\rho(c_i)]$.

Eventually, a first-order theory interpretation (as defined in [221, p. 162]) is a (first-order) formula interpretation translation that preserves theorems / consequences of the interpreted theory, just as in the generic definition A.4.

A.9 Definition (first-order theory interpretation (Enderton))
Let T and T' two first-order theories over possibly distinct signatures. A formula interpretation translation $\alpha : Lg(T) \rightarrow Lg(T')$ is a *first-order theory interpretation* of T into T' iff $\llcorner$ for all $\phi \in Lg(T)$, $T \models \phi$ implies $T' \models \alpha(\phi)$ $\lrcorner$. α is *faithful* iff $\llcorner T \models \phi$ iff $T' \models \alpha(\phi)$ $\lrcorner$. $\Box$

A.2   Axiomatic Systems of Set and Number Theory

A.2.1   Zermelo-Fraenkel Set Theory (With Axiom of Choice)

There are various, provably equivalent presentations of "the" axioms of Zermelo-Fraenkel set theory with the Axiom of Choice (ZFC). The subsequent compilation of corresponding axioms and axiom systems relevant herein (ZF^0, ZF, ZFC) is due to the presentation in [212] (with minor modifications in style), cf. esp. the listing at [212, p. 241–242]. Where possible, the axiom names follow [200, p. 44–45]. Otherwise and for additional remarks, the English Wikipedia entry "Zermelo-Fraenkel set theory" was used [@138]. Finally, note that the axioms are presented for "pure" set theory, i.e., disregarding urelements, see also sect. [3.1, p. 103].

Signature $\Sigma_{ZFC}^{basic} := (\in)$, where $\in$ is a binary predicate symbol with the membership relation as its intended meaning.

$\Sigma_{ZFC}^{def} := (\subseteq, \cap, \cup, \emptyset)$ with a binary predicate symbol $\subseteq$, unary functions $\cap$ and $\cup$ and a (functional, 0-ary) constant $\emptyset$. The obvious intended meanings are the subset relation, the operations of intersection and union, and the empty set.

Defined Symbols

$$x \subseteq y := \forall z \,.\, z \in x \rightarrow z \in y$$
$$x = y \cap z := \forall u \,.\, u \in x \leftrightarrow u \in y \wedge u \in z$$
$$x = y \cup z := \forall u \,.\, u \in x \leftrightarrow u \in y \vee u \in z$$
$$x = \emptyset := \neg \exists y \,.\, y \in x$$

Axioms

Ex	Axiom of Existence[715]
	$\exists x \,.\, x = x$
Ext	Axiom of Extensionality
	$\forall xy \,.\, \forall z(z \in x \leftrightarrow z \in y) \rightarrow x = y$
Sep	Axiom Schema of Restricted Comprehension / of Separation / of Specification
	For every formula $\phi(z, \bar{w})$ the axiom
	$\forall \bar{w} \forall x \exists y \forall z(z \in y \leftrightarrow z \in x \wedge \phi(z, \bar{w}))$
$\cup$-**Ax**	Axiom of Small Union
	$\forall xy \exists w \forall z \,.\, z \in x \vee z \in y \rightarrow z \in w$
$\bigcup$-**Ax**	Axiom of Union
	$\forall u \exists w \forall xz \,.\, x \in u \wedge z \in x \rightarrow z \in w$
Pow	Power Set Axiom
	$\forall x \exists y \forall z \,.\, z \subseteq x \rightarrow z \in y$
Inf	Axiom of Infinity
	$\exists x \,.\, \emptyset \in x \wedge \forall z(z \in x \rightarrow z \cup \{z\} \in x)$
Rep	Axiom of Replacement / Collection
	For every formula $\phi(x, y, \bar{z})$ the axiom
	$\forall \bar{z} \,.\, (\forall x \exists! y \phi(x, y, \bar{z})) \rightarrow \forall u \exists v \forall xy(x \in u \wedge \phi(x, y, \bar{z}) \rightarrow y \in v)$
Found	Axiom of Foundation / Regularity
	$\forall x \,.\, x \neq \emptyset \rightarrow \exists y(y \in x \wedge x \cap y = \emptyset)$
AC	Axiom of Choice

$$\forall x \,.\, \forall uv(u \in x \wedge v \in x \rightarrow u \neq \emptyset \wedge (u = v \vee u \cap v = \emptyset))) \rightarrow$$
$$\exists y \forall w(w \in x \rightarrow \exists! z(z \in y \cap w))$$

Axiom systems of Zermelo-Fraenkel

$\mathbf{Z}^0$ comprises Ex, Ext, Sep, $\cup$-Ax, $\bigcup$-Ax, Pow, Inf
$\mathbf{Z}$ is Z^0 extended with Found
$\mathbf{ZF}^0$ is Z^0 extended with Rep
$\mathbf{ZF}$ is ZF^0 extended with Found
$\mathbf{ZFC}$ is ZF extended with AC

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[715]The Null Set Axiom [200, p. 44] requires $\exists x \,.\, x = \emptyset$.

A.2.2 Peano Arithmetic PA

The presentation below is based on [693, sect. 3.3, p. 105–106], [214, sect. III.7.3], and [212, sect. V.1, p. 65–66]. [@93] is more closely oriented at Peano's original work, cf. [840] for a translated version. [221, ch. 3] more generally discusses 'number theory' in various forms / versions (up to additionally including exponentiation) wrt undecidability.

Signature $\Sigma_{PA}^{basic} := (S, +, \cdot, 0)$, where S is a unary function symbol, $+$ and $\cdot$ are binary function symbols, and 0 is a (functional, 0-ary) constant. The obvious intended meanings are the successor function, addition and multiplication of natural numbers, and the individual number zero, respectively.

$\Sigma_{PA}^{def} := (\le)$, where $\le$ is a binary predicate symbol, intended to stand for the smaller-than(-or-equal) ordering relation between natural numbers.

Defined Symbols   While the axioms below in a sense define addition and multiplication, they are kept among the basic symbols, e.g., in line with [693].

$$x \le y := \exists z \,.\, x + z = y$$

Axioms

P1   No number is a successor of zero.
$\forall x \,.\, S(x) \neq 0$

P2   The successor function is injective.
$\forall xy \,.\, S(x) = S(y) \to x = y$

P3   Recursive definition of addition: base case, 0 is neutral wrt addition.
$\forall x \,.\, x + 0 = x$

P4   Recursive definition of addition: recursion.
$\forall xy \,.\, x + S(y) = S(x + y)$

P5   Recursive definition of multiplication: base case, 0 is absorbing wrt multiplication.
$\forall x \,.\, x \cdot 0 = x$

P6   Recursive definition of multiplication: recursion.
$\forall xy \,.\, x \cdot S(y) = (x \cdot y) + x$

PIS   first-order induction schema:
For every formula $\phi(x, \bar{y})$:
$\forall \bar{y} \,.\, \phi[x/0] \wedge \forall x(\phi \to \phi[x/S(x)]) \to \forall x \phi$

PIA   second-order induction axiom:
$\forall P \,.\, P(0) \wedge \forall x(P(x) \to P(S(x))) \to \forall x P(x)$

Axiom systems of Peano Arithmetic   For any of the following, the 'first-order variant' includes PIS for *PI*, the 'second-order variant' PIA instead.

PA^{S} covers P1–P2 and *PI*
PA   comprises P1–P6 and *PI*

Appendix B

Axioms of the $\mathcal{CR}$ Taxonomy in OWL

An axiomatization of a (mainly taxonomic) $\mathcal{CR}$ fragment in OWL and thus description logic (DL) is presented in this chapter. Its major aspects are described in the corresponding sect. 6.1.1, starting on p. 209 in the main part of this thesis. Accordingly, we issue only a few more technical or specific comments here.

The list of axioms is specified in the more compact DL notation[716] and has been generated originally from the file `cr-dl.owl` [@21]. On that basis we have edited the axioms manually for reasons of ordering and layout, as well as in order to "cure" effects resulting from an implementation issue[717] and from the export into the LATEX format, such as repetitions of disjointness axioms in all concerned classes or properties. Below a disjointness axiom appears only at the class or property whose name is lexicographically ordered prior to other arguments in the axiom.

Note that Entity is equivalent to $\top$ in DL / Thing in OWL. Moreover, we manually insert the general concept inclusion axioms in `cr-dl.owl` together with the category (a named class in all cases) that appears in the superclass role of the subsumption. 'RL' at the end of a class name abbreviates 'role'. It is introduced in order to distinguish between RoleRL (a role category) and RoleRole (a player category).

The next two sections plainly present all asserted axioms, those on OWL classes being followed by those on OWL object properties. There is no use of OWL individuals nor datatypes. The resulting inferred hierarchy of classes is presented in Fig. 6.1 on p. 210.

B.1 Asserted OWL Class Axioms

BasePlayerAbleCategory $\sqsubseteq$ Category

BasePlayerCategory $\sqsubseteq$ Category

BaseRoleCategory $\equiv$ $=1$ hasBaseRole$^-$ Relation
BaseRoleCategory $\sqsubseteq$ RoleCategory

Categorizable $\sqsubseteq$ Entity
$\exists$ instanceOf Entity $\sqsubseteq$ Categorizable

Category $\sqsubseteq$ Entity
Category $\sqsubseteq$ Instantiable
Category $\sqsubseteq$ NonRelating
Category $\sqsubseteq$ $\neg$ Individual
DisjointUnion Instantiated NonInstantiated

CategoryCategory $\sqsubseteq$ Category
CategoryCategory $\sqsubseteq$ $\neg$ EmptyCategory

CategoryCategory $\sqsubseteq$ $\neg$ IndividualCategory
CategoryCategory $\sqsubseteq$ $\neg$ MixCategory
CategoryCategory $\sqsubseteq$ $\forall$ instantiatedBy Category
Instantiated $\sqcap$ ($\forall$ instantiatedBy Category) $\sqsubseteq$
                          CategoryCategory

Context $\equiv$ ≥ 2 hasRole Role
Context $\sqsubseteq$ Player
Context $\sqsubseteq$ Relator

ContextRL $\sqsubseteq$ Role
ContextRL $\sqsubseteq$ $\neg$ InstanceRL
ContextRL $\sqsubseteq$ $\neg$ InstantiatedRL
ContextRL $\sqsubseteq$ $\neg$ PlayedRL
ContextRL $\sqsubseteq$ $\neg$ PlayerRL
ContextRL $\sqsubseteq$ $\neg$ RoleRL

---

[716]Some "relics" from OWL are the DisjointUnion axiom, see Category, and characterizations of object properties like TransitiveProperty, see the OWL object property elsA.

[717]In `cr-dl.owl`, Individual is named Individual (ending in capital 'i' instead of capital 'l').

EmptyCategory ⊑ NonInstantiated

Entity ≡ Category ⊔ Individual
Entity ≡ NonRelating ⊔ Relator ⊔ Role
Entity ⊑ Categorizable
Entity ⊑ Instance
Entity ⊑ Player
Entity ⊑ PlayerAble

Individual ⊑ ¬ Instantiable
Individual ⊑ ¬ Category

IndividualCategory ⊑ Category
IndividualCategory ⊑ ¬ EmptyCategory
IndividualCategory ⊑ ¬ MixCategory
IndividualCategory ⊑ ∀ instantiatedBy Individual
Instantiated ⊓ (∀ instantiatedBy Individual) ⊑
                                        IndividualCategory

Instance ≡ ∃ instanceOf Entity
Instance ⊑ Categorizable
Instance ⊑ Player

InstanceRL ⊑ Role
InstanceRL ⊑ ¬ InstantiatedRL
InstanceRL ⊑ ¬ PlayedRL
InstanceRL ⊑ ¬ PlayerRL
InstanceRL ⊑ ¬ RoleRL

Instantiable ⊑ Category
∃ instantiatedBy Entity ⊑ Instantiable

Instantiated ⊑ Instantiable
Instantiated ⊑ Player
Instantiated ⊑ ¬ NonInstantiated
Instantiated ≡ ∃ instantiatedBy Entity

InstantiatedRL ⊑ Role
InstantiatedRL ⊑ ¬ PlayedRL
InstantiatedRL ⊑ ¬ PlayerRL
InstantiatedRL ⊑ ¬ RoleRL

MixCategory ⊑ Category
(∃ instantiatedBy Category) ⊓
   (∃ instantiatedBy Individual)   ⊑   MixCategory

NonInstantiated ≡
      Instantiable ⊓ ¬(∃ instantiatedBy Entity)
NonInstantiated ⊑ ¬ Instantiated

NonRelating ≡ Entity ⊓ ¬Relating
NonRelating ⊑ NonRole
NonRelating ⊑ NonRelator
NonRelating ⊑ ¬ Relating

NonRelator ≡ NonRelating ⊔ Role
NonRelator ≡ Entity ⊓ ¬Relator
NonRelator ⊑ ¬ Relator

NonRole ≡ NonRelating ⊔ Relator
NonRole ≡ Entity ⊓ ¬Role
NonRole ⊑ ¬ Role

Playable ⊑ Role
∃ playedBy Entity ⊑ Playable

Played ≡ =1 playedBy Player
Played ⊑ Player
Played ⊑ Role

PlayedRL ⊑ Role
PlayedRL ⊑ ¬ PlayerRL
PlayedRL ⊑ ¬ RoleRL

Player ≡ ∃ plays Role
Player ⊑ Entity

PlayerAble ⊑ Entity
∃ plays Entity ⊑ PlayerAble

PlayerRL ⊑ Role
PlayerRL ⊑ ¬ RoleRL

Relating ≡ Relator ⊔ Role
Relating ⊑ Individual
Relating ⊑ ¬ NonRelating

Relation ≡ ∃ hasBaseRole BaseRoleCategory
Relation ⊑ RelatorCategory

Relator ⊑ Context
Relator ⊑ Relating
Relator ⊑ RoleHaveAble
Relator ⊑ ¬ NonRelator
Relator ⊑ ¬ Role

RelatorCategory ⊑ IndividualCategory
RelatorCategory ⊑ ¬ RoleCategory
RelatorCategory ⊑ ∀ instantiatedBy Relator
Instantiated ⊓ (∀ instantiatedBy Relator) ⊑
                                        RelatorCategory

Role ⊑ Playable
Role ⊑ Played
Role ⊑ Relating
Role ⊑ RoleBeAble
Role ⊑ RoleRole
Role ⊑ ¬ Relator
Role ⊑ ¬ NonRole

RoleBeAble ⊑ Role
∃ roleOf Entity ⊑ RoleBeAble

RoleCategory ⊑ IndividualCategory
RoleCategory ⊑ ¬ RelatorCategory
RoleCategory ⊑ ∀ instantiatedBy Role

RoleHaveAble ⊑ Relator
∃ hasRole Entity ⊑ RoleHaveAble

RoleRL ⊑ Role

RoleRole ≡ =1 roleOf Context
RoleRole ⊑ Played
RoleRole ⊑ Role

B.2   Asserted OWL Object Property Axioms

NB on abbreviations: eIsA stands for 'extensional is-a'.

baseEIsA ⊑ eIsA
∃ baseEIsA Entity ⊑ BasePlayerAbleCategory
⊤ ⊑ ∀ baseEIsA Category

∃ **basePlayedBy** Entity ⊑ BaseRoleCategory
⊤ ⊑ ∀ basePlayedBy BasePlayerCategory
⊤ ⊑ ≤ 1 basePlayedBy Entity
⊤ ⊑ ≤ 1 basePlayedBy⁻ Entity
DisjointObjectProperties basePlayedBy hasBaseRole
DisjointObjectProperties basePlayedBy plays
DisjointObjectProperties basePlayedBy roleOf

basePlayerAble ⊑ eIsA
∃ basePlayerAble Entity ⊑ BasePlayerCategory
⊤ ⊑ ∀ basePlayerAble BasePlayerAbleCategory
⊤ ⊑ ≤1 basePlayerAble Entity
⊤ ⊑ ≤1 basePlayerAble⁻ Entity

∃ **eIsA** Entity ⊑ Category
⊤ ⊑ ∀ eIsA Category
TransitiveProperty eIsA

∃ **hasBaseRole** Entity ⊑ Relation
⊤ ⊑ ∀ hasBaseRole BaseRoleCategory
⊤ ⊑ ≤ 1 hasBaseRole⁻ Entity
DisjointObjectProperties hasBaseRole instanceOf
DisjointObjectProperties hasBaseRole plays
DisjointObjectProperties hasBaseRole roleOf
AsymmetricProperty hasBaseRole
IrreflexiveObjectProperty hasBaseRole

hasRole ≡ roleOf⁻
∃ hasRole Entity ⊑ Relator
⊤ ⊑ ∀ hasRole Role

∃ **instanceOf** Entity ⊑ Entity
⊤ ⊑ ∀ instanceOf Category
DisjointObjectProperties instanceOf plays
DisjointObjectProperties instanceOf roleOf

instantiatedBy ≡ instanceOf⁻
∃ instantiatedBy Entity ⊑ Category
⊤ ⊑ ∀ instantiatedBy Entity

playedBy ≡ plays⁻
∃ playedBy Entity ⊑ Role
⊤ ⊑ ∀ playedBy Entity

∃ **plays** Entity ⊑ Entity
⊤ ⊑ ∀ plays Role
⊤ ⊑ ≤ 1 plays⁻ Entity
DisjointObjectProperties plays roleOf
AsymmetricProperty plays
IrreflexiveObjectProperty plays

∃ **roleOf** Entity ⊑ Role
⊤ ⊑ ∀ roleOf Relator
⊤ ⊑ ≤ 1 roleOf Entity
AsymmetricProperty roleOf
IrreflexiveObjectProperty roleOf

Appendix C

Lists of Figures and Tables

C.1 List of Figures

C.2   List of Tables

Appendix D

Abbreviations, Acronyms and Names

D.1 Abbreviations

Citations appear in double quotation marks (" "), text in brackets within a quotation indicate modifications in the quoted text (e.g., capitalization changes), [. . .] stands for pieces left out in the quotation.

a.o.	among others
App.	Appendix *or* Appendices
cf.	confer
ch.	chapter *or* chapters
Cond.	Condition *or* Conditions
Conj.	Conjecture *or* Conjectures
Def.	Definition *or* Definitions
ed.	edition
e.g.	for example [718]
esp.	especially
etc	et cetera
Ex.	Example *or* Examples
f.	and the following page
ff.	and the following pages
Fig.	Figure
FN	footnote
ibid.	ibidem = the same place
i.e.	id est = that means
iff	if and only if
incl.	including
NB	nota bene
Obj.	Objective
Obs.	Observation
p.	page *or* pages
par.	paragraph *or* paragraphs
prim.	primarily
Prop.	Proposition *or* Propositions
resp.	respective *or* respectively
sect.	section *or* sections
s.t.	such that

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[718]Note on (the non-uniformity wrt punctuation of) our use of 'e.g.' that a subsequent comma suggests a short pause / stop (in vocalization), which is not assumed where no comma is set.

Table	Table (*only for completeness of this record*)
transl.	translation *or* translated
vs	versus
wlog	without loss of generalization
wrt	with respect to

D.2 Acronyms and Names

Many acronyms and names can be associated with a respective web reference. To avoid duplication, the subsequent list of acronyms and names comprises only entries that are not already captured among web references in sect. E.2.

ABox	assertional part of a DL theory
ACO	abstract core ontology
AI	artificial intelligence
API	application programming interface
ASP	answer set programming
CO	core ontology
CWA	closed world assumption
DAG	directed acyclic graph
DLP	description logic programs [308]
DSL	domain-specific language
DSML	domain-specific modeling language
DSO	domain-specific ontology
DTD	document type definition [112, sect. 2.8]
EU	European Union
FCA	Formal Concept Analysis [261]
F-Logic	*a logical framework with object-oriented features, developed in the context of logic programming* [474, 476]
FO	first order
FOL	first-order logic
GALEN	*name of a medical terminology*
GF	guarded fragment
GOL	General Ontological Language, preceding project name of GFO [@46]
HiLog	*a logical framework with metaprogramming features, developed in the context of logic programming* [151]
HO	higher order
HOL	higher order logic(s)
HTML	Hypertext Markup Language
JTC	Joint Technical Committee
KB	knowledge base
KBS	knowledge based system(s)
KR	knowledge representation
KR&R	knowlege representation and reasoning
LP	logic programming
NBG	von Neumann-Bernays-Gödel (set theory)
OO	object-oriented / object-orientation
OOP	object-oriented programming
OUS	ontological usage scheme
OWA	open world assumption
OWL	Web Ontology Language [@91] [572, 661]
OWL 2	*second version of OWL, new standard* [@91] [174, 855]

PL	propositional logic
pOUS	parametrized OUS
pUML	precise UML [229]
RFP	Request for Proposal
SC	Subcommittee
SNOMED	Systematized Nomenclature of Medicine
SOL	second-order logic
SQL	Structured Query Language
SW	Semantic Web
SWRL	Semantic Web Rule Language
TBox	terminological part of a DL theory
TLO	top-level ontology
TM	topic map(s) [448, 654]
UK	United Kingdom
URI	Uniform Resource Identifier
URL	Uniform Resource Locator
U.S.	United States of America
ZF	Zermelo-Fraenkel (set theory)
ZFC	ZF set theory extended with the Axiom of Choice

Appendix E

References

E.1 Literature References

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E.2   Web References / List of URLs

Note: For comprehensible associations and easier access, web references are labeled with descriptions and, where available, with appropriate acronyms first. The latter are used for sorting the web references, using the descriptive phrases if no acronym exists. We omit the specification of access dates, but all entries were checked for accessibility (unless otherwise stated) in the period of October to December 2014. Where versioning is available, the link to the version referred to is provided in addition, including the creation date of that version. In some cases we specify time spans that indicate the primary periods of access while working intensely wrt the respective sources.

[@1] AAAI, Association for the Advancement of Artificial Intelligence, website
`http://www.aaai.org`

[@2] AAAI 1999 Panel "Ontologies: Expert Systems all over again?"
Organizer: Christopher A. Welty
Panelists (acc. to panel website): Michael Grüninger, Fritz Lehmann, Deborah McGuiness, Michael Uschold
Panelists (acc. to guide): David Fogel, Mark Fox, Nicola Guarino, Doug Lenat, Deborah McGuiness, Michael Uschold
`http://www.cs.vassar.edu/~weltyc/aaai-99/` (panel website)
`http://www.cs.vassar.edu/~weltyc/aaai-99/AAAI-99-panel.PPT` (panel slide set)
`http://www.aaai.org/Conferences/AAAI/1999/aaai99program.pdf` (program and exhibit guide)

[@3] Abstract Syntax of Programming Languages, content slide by John McCarthy, cited in [381, sect. 1]
`http://www-formal.stanford.edu/jmc/towards/node12.html`

[@4] AI, Artificial Intelligence, journal homepage
`http://www.journals.elsevier.com/artificial-intelligence`

[@5] Alfred Horn, Wikipedia entry
`http://en.wikipedia.org/wiki/Alfred_Horn` version: `http://en.wikipedia.org/w/index.php?title=Alfred_Horn&oldid=621844476` (18 Aug 2014)

[@6] AO, Applied Ontology, journal homepage
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[@7] Arabidopsis thaliana, Wikipedia entry
`http://en.wikipedia.org/wiki/Arabidopsis_thaliana`

[@8] Arabidopsis thaliana in GFO-Bio [@47]
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[@9] Arabidopsis thaliana in NCBI Taxon [@69]
`http://purl.bioontology.org/ontology/NCBITAXON/3702`

[@10] ASCII, American Standard Code for Information Interchange [22], Wikipedia entry
`http://en.wikipedia.org/wiki/ASCII`
version: `http://en.wikipedia.org/w/index.php?title=ASCII&oldid=620713247` (11 Aug 2014)

[@11] ATP, automated theorem proving, Wikipedia entry
`http://en.wikipedia.org/wiki/Automated_theorem_proving`   (version link below: 9 Oct 2014)
`http://en.wikipedia.org/w/index.php?title=Automated_theorem_proving&oldid=628958083`

[@12] BFO, Basic Formal Ontology, a TLO, project website
http://www.ifomis.org/bfo/

[@13] Bibliography: Set Theory with a Universal Set, bibliography page
http://math.boisestate.edu/~holmes/holmes/setbiblio.html

[@14] Biomedical Ontologies, a thematic series of the Journal of Biomedical Semantics, homepage
http://www.jbiomedsem.com/series/BIOONT

[@15] BioPortal, web portal and repository for ontologies, website
http://bioportal.bioontology.org/

[@16] BioTop, "A top-domain ontology for the life sciences", project website
http://www2.imbi.uni-freiburg.de/ontology/biotop/

[@17] CC BY 2.0, Creative Commons license Attribution 2.0
http://creativecommons.org/licenses/by/2.0/
http://creativecommons.org/licenses/by/2.0/legalcode

[@18] CL, Common Logic [452], an ISO [@61] standard
http://www.iso-commonlogic.org/, an alias to http://philebus.tamu.edu/cl/ (project website)
formerly, but no longer used alias: http://www.commonlogic.org
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[@19] CLIB, Component Library, a TLO developed for RKF [@107]
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Appendix F

Work and Author Information

Selbständigkeitserklärung (Declaration of Authorship)

Hiermit erkläre ich, die vorliegende Dissertation selbständig und ohne unzulässige fremde Hilfe angefertigt zu haben. Ich habe keine anderen als die angeführten Quellen und Hilfsmittel benutzt und sämtliche Textstellen, die wörtlich oder sinngemäß aus veröffentlichten oder unveröffentlichten Schriften entnommen wurden, und alle Angaben, die auf mündlichen Auskünften beruhen, als solche kenntlich gemacht. Ebenfalls sind alle von anderen Personen bereitgestellten Materialien oder erbrachten Dienstleistungen als solche gekennzeichnet.

(Ort, Datum)

(Unterschrift)

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Number of Tables	11

Scientific Record

F.1.1   Education and Affiliations

<div align="right">unless otherwise stated, the country is Germany</div>

1996	General Degree for University Entrance ('Abitur'), Gymnasium Markranstädt
1997 – 2003	Study of Computer Science ('Diplom'), University of Leipzig
2003 – 2006	Graduate Program "Knowledge Representation", University of Leipzig
2006 – 2007	Institute of Medical Informatics, Statistics and Epidemiology (IMISE), University of Leipzig
2007 – 2015	Department of Computer Science, University of Leipzig

Extended Visits

2000 – 2001 *(4 months)*	Middlesex University, London, UK
2001 *(4 months)*	German Research Center for Artificial Intelligence (DFKI), Saarbrücken
2012 *(10 weeks)*	SRI International, Menlo Park, California, USA

F.1.2   List of Publications

Each sublist is sorted first in reverse chronological order, within years lexicographically by the lastnames of the authors (or editors).

F.1.2.1   Journal Articles or Book Chapters

1. **Baumann R**, **Loebe F**, **Herre H**, **2014**. *Axiomatic theories of the ontology of time in GFO*. Applied Ontology 9(3-4):171–215. Reprinted with permission from IOS Press as sect. 6.3.

2. **Loebe F**, **Stumpf F**, **Hoehndorf R**, **Herre H**, **2012**. *Towards Improving Phenotype Representation in OWL*. Journal of Biomedical Semantics 3(Suppl 2):S5.1–17.

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7. **Hoehndorf R**, **Bacher J**, **Backhaus M**, **Gregorio SEJ**, **Loebe F**, **Prüfer K**, **Uciteli A**, **Visagie J**, **Herre H**, **Kelso J**, **2009**. *BOWiki: An ontology-based wiki for annotation of data and integration of knowledge in biology*. BMC Bioinformatics 10(Suppl 5):S5.1–S5.8.

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F.1.2.2   In Conference or Workshop Proceedings

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F.1.2.4 Further Publications

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37. **Loebe F**, **2006**. *A Comparison of Roles in the General Formal Ontology and the HL7 Reference Information Model.* In: Löffler M, Winter A, (editors), Klinische Forschung Vernetzen: Proceedings of the 51. Annual Conference of the Deutsche Gesellschaft für Medizinische Informatik, Biometrie und Epidemiologie (GMDS), Leipzig, Germany, Sep 10-14. University of Leipzig, pp. 131–132. Also published as open-access by: Düsseldorf (Germany): German Medical Science.

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