

1 2 to 2 phase space

process:

$$\gamma^*(q) + g(k_1) \rightarrow Q(p_1) + \bar{Q}(p_2) \quad (1)$$

use c.m.s. of incoming particles:

$$q = \left(\frac{s + q^2}{2\sqrt{s}}, 0, 0, \dots, -\frac{s - q^2}{2\sqrt{s}} \right) \quad (2)$$

$$k_1 = \frac{s - q^2}{2\sqrt{s}} (1, 0, 0, \dots, 1) \quad (3)$$

such that

$$q + k_1 = (\sqrt{s}, \vec{0}) \quad k_1^2 = 0 \quad (4)$$

for the outgoing particles it follows

$$p_1 = \frac{\sqrt{s}}{2} (1, 0, \beta \sin \theta, \dots, \beta \cos \theta) \quad (5)$$

$$p_2 = \frac{\sqrt{s}}{2} (1, 0, -\beta \sin \theta, \dots, -\beta \cos \theta) \quad (6)$$

with $\beta = \sqrt{1 - 4m^2/s}$ such that

$$p_1 + p_2 = (\sqrt{s}, \vec{0}) \quad p_1^2 = p_2^2 = m^2 \quad (7)$$

use n-sphere:

$$d^D x = \Omega_D x^{D-1} dx = \frac{2\pi^{D/2}}{\Gamma(D/2)} x^{D-1} dx = \frac{\pi^{D/2}}{\Gamma(D/2)} (x^2)^{(D-2)/2} dx^2 \quad (8)$$

compute phase space:

$$PS_2 = \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_2}{(2\pi)^{n-1}} (2\pi)^n \delta^{(n)}(q + k_1 - p_1 - p_2) \delta(p_1^2 - m^2) \delta(p_2^2 - m^2) \quad (9)$$

$$= \frac{1}{(2\pi)^{n-2}} \int d^n p_1 \delta((q + k_1 - p_1)^2 - m^2) \delta(p_1^2 - m^2) \quad (10)$$

$$= \frac{1}{(2\pi)^{n-2}} \int dp_{1,0} dp_{1,\parallel} d^2 p_{1,\perp} d^{n-4} \hat{p}_1 \delta(s - 2p_{1,0}\sqrt{s}) \delta(p_{1,0}^2 - p_{1,\parallel}^2 - p_{1,\perp}^2 - \hat{p}_1^2 - m^2) \quad (11)$$

$$= \frac{\pi}{(2\pi)^{n-2} 2\sqrt{s}} \int dp_{1,\parallel} dp_{1,\perp}^2 d^{n-4} \hat{p}_1 \delta(s/4 - p_{1,\parallel}^2 - p_{1,\perp}^2 - \hat{p}_1^2 - m^2) \quad (12)$$

$$= \frac{\pi}{(2\pi)^{n-2} 2\sqrt{s}} \int dp_{1,\parallel} d\hat{p}_1^2 \frac{\pi^{(n-4)/2}}{\Gamma((n-4)/2)} (\hat{p}_1^2)^{(n-6)/2} \quad (13)$$

$$= \frac{1}{2\sqrt{s} \Gamma((n-4)/2) (4\pi)^{(n-2)/2}} \int dp_{1,\parallel} d\hat{p}_1^2 (\hat{p}_1^2)^{(n-6)/2} \quad (14)$$

Integration borders are

$$p_{1,\parallel} \in \frac{\sqrt{s}}{2} \beta \cdot [-1, 1] \quad \hat{p}_1^2 \in \left(\frac{s\beta^2}{4} - p_{1,\parallel}^2 \right) \cdot [0, 1] \quad (15)$$

if cross section does not depend on hat-space:

$$\int d\hat{p}_1^2 (\hat{p}_1^2)^{(n-6)/2} = \frac{2}{n-4} \left(\frac{s\beta^2}{4} - p_{1,\parallel}^2 \right)^{(n-4)/2} \quad (16)$$

$$\Rightarrow PS_2 = \frac{1}{2\sqrt{s}\Gamma((n-2)/2)(4\pi)^{(n-2)/2}} \int dp_{1,\parallel} \left(\frac{s\beta^2}{4} - p_{1,\parallel}^2 \right)^{(n-4)/2} \quad (17)$$

rewrite $p_{1,\parallel}$ to $\cos\theta$:

$$p_{1,\parallel} = \frac{\sqrt{s}}{2}\beta \cos\theta \Rightarrow dp_{1,\parallel} = \frac{\sqrt{s}}{2}\beta d\cos\theta, \quad \cos\theta \in [-1, 1], \quad \hat{p}_1^2 \in \frac{s\beta^2}{4} (1 - \cos^2\theta) \cdot [0, 1] \quad (18)$$

rewrite $\cos\theta$ to $t_1 = (k_1 - p_2)^2 - m^2$:

$$\cos\theta = t_1 \Rightarrow d\cos\theta = dt_1, \quad t_1 \in [0, 1], \quad \hat{p}_1^2 \in [0, 1] \quad (19)$$

2 2 to 3 phase space

process:

$$\gamma^*(q) + q(k_1) \rightarrow Q(p_1) + \bar{Q}(p_2) + q(k_2) \quad (20)$$

2.1 kinematic constraints

definitions of kinematic variables:

$$s = (q + k_1)^2 \Rightarrow 2qk_1 = s - q^2 \quad (21)$$

$$s_3 = (k_2 + p_2)^2 - m^2 \Rightarrow 2k_2p_2 = s_3 \quad (22)$$

$$s_4 = (k_2 + p_1)^2 - m^2 \Rightarrow 2k_2p_1 = s_4 \quad (23)$$

$$s_5 = (p_1 + p_2)^2 = -u_5 \Rightarrow 2p_1p_2 = s_5 - 2m^2 \quad (24)$$

$$t_1 = (k_1 - p_2)^2 - m^2 = t - m^2 \Rightarrow 2k_1p_2 = -t_1 \quad (25)$$

$$t' = (k_1 - k_2)^2 \Rightarrow 2k_1k_2 = -t' \quad (26)$$

$$u_1 = (q - p_2)^2 - m^2 = u - m^2 \Rightarrow 2qp_2 = -u_1 + q^2 \quad (27)$$

$$u_6 = (k_1 - p_1)^2 - m^2 \Rightarrow 2k_1p_1 = -u_6 \quad (28)$$

$$u_7 = (q - p_1)^2 - m^2 \Rightarrow 2qp_1 = -u_7 + q^2 \quad (29)$$

$$u' = (q - k_2)^2 \Rightarrow 2qk_2 = -u' + q^2 \quad (30)$$

impose momentum conservation:

$$q + k_1 = p_1 + p_2 + k_2 \quad (31)$$

contract with 2 times momentum:

$$2q^2 + s - q^2 = -u_7 + q^2 \quad -u_1 + q^2 \quad -u' + q^2 \quad \Leftrightarrow \quad 0 = s + u_1 + u_7 + u' - 2q^2 \quad (32)$$

$$s - q^2 + 0 = -u_6 \quad -t_1 \quad -t' \quad \Leftrightarrow \quad 0 = s + t_1 + t' + u_6 - q^2 \quad (33)$$

$$-u_7 + q^2 \quad -u_6 = 2m^2 \quad +s_5 - 2m^2 \quad +s_4 \quad \Leftrightarrow \quad 0 = s_4 + s_5 + u_6 + u_7 - q^2 \quad (34)$$

$$-u_1 + q^2 \quad -t_1 = s_5 - 2m^2 \quad +2m^2 \quad +s_3 \quad \Leftrightarrow \quad 0 = s_3 + s_5 + t_1 + u_1 - q^2 \quad (35)$$

$$-u' + q^2 \quad -t' = s_4 \quad +s_3 \quad +0 \quad \Leftrightarrow \quad 0 = s_3 + s_4 + t' + u' - q^2 \quad (36)$$

$$\frac{1}{2} ((32) + (33) + (35) - (34) - (36)) = 0 = s - q^2 + t_1 + u_1 - s_4 \quad (37)$$

2.2 computation

use c.m.s. of recoiling heavy and light quark ($Q(p_1)$ and $q(k_2)$):

$$k_2 = (\omega_2, 0, \dots, 0, \omega_2 \sin \theta \sin \phi, \omega_2 \sin \theta \cos \phi, \omega_2 \cos \theta) \quad (38)$$

$$p_1 = (E_1, 0, \dots, 0, -\omega_2 \sin \theta \sin \phi, -\omega_2 \sin \theta \cos \phi, -\omega_2 \cos \theta) \quad (39)$$

$$k_1 = (\omega_1, 0, \dots, 0, 0, 0, \omega_1) \quad (40)$$

$$q = (q_0, 0, \dots, 0, |\vec{p}_2| \sin \psi, |\vec{p}_2| \cos \psi - \omega_1) \quad (41)$$

$$p_2 = (E_2, 0, \dots, 0, |\vec{p}_2| \sin \psi, |\vec{p}_2| \cos \psi) \quad (42)$$

light quark masss already taken into account: $k_1^2 = 0 = k_2^2$

constraints:

$$q_0 + \omega_1 = E_1 + E_2 + \omega_2 \quad (43)$$

$$m^2 = p_1^2 = E_1^2 - \omega_1^2 \quad (44)$$

$$m^2 = p_2^2 = E_2^2 - |\vec{p}_2|^2 \quad (45)$$

$$q^2 = q_0^2 - |\vec{p}_2|^2 + 2|\vec{p}_2|\omega_1 \cos \psi - \omega_1^2 \quad (46)$$

$$s = (q + k_1)^2 = (q_0 - \omega_1)^2 - |\vec{p}_2|^2 \quad (47)$$

$$t = (k_1 - p_2)^2 = (\omega_1 - E_2)^2 - |\vec{p}_2|^2 + 2|\vec{p}_2|\omega_1 \cos \psi - \omega_1^2 \quad (48)$$

$$u = (q - p_2)^2 = (q_0 - E_2)^2 - \omega_1^2 \quad (49)$$

$$(47) - (46) + (48) - (45) + (49) = s - q^2 + t - m^2 + u \quad (50)$$

$$= s_4 + m^2 = (E_1 + \omega)^2 \quad (51)$$

$$(48) + (49) - (46) = t + u - q^2 = -2(E_1 + \omega_2)E_2 \quad (52)$$

$$\Rightarrow E_2 = -\frac{t + u - q^2}{2\sqrt{s_4 + m^2}} = \frac{s_4 + 2m^2}{2\sqrt{s_4 + m^2}} \quad (53)$$

$$\Rightarrow |\vec{p}_2| = \sqrt{E_2^2 - m^2} = \frac{s_4}{2\sqrt{s_4 + m^2}} \quad (54)$$

$$(47) + (49) - (45) = s + u - m^2 = 2q_0(E_1 + \omega_2) \quad (55)$$

$$\Rightarrow q_0 = \frac{s + u - m^2}{2\sqrt{s_4 + m^2}} \quad (56)$$

$$(48) - (46) = t - q^2 = (\omega_1 - E_2)^2 - q_0^2 \quad (57)$$

$$\Rightarrow \omega_1 = \frac{s_4 + m^2 - u}{2\sqrt{s_4 + m^2}} \quad (58)$$