1 Introduction

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1.1 Motivation

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1.2 Notation

We study the neutral-current (NC) deep inelastic scattering (DIS) reaction

$$\ell(k) + \mathrm{N}(P) \to \ell'(k') + \overline{Q}(p_2) + X[Q] \tag{1}$$

and define the usual set of kinematic variables

$$q = k - k', \qquad Q^2 = -q^2, \qquad x = \frac{Q^2}{2q\cdot P}, \qquad y = \frac{q\cdot P}{k\cdot P} \tag{2}$$

Assuming $k^2 = m_\ell^2 = 0$, we can write the hadronic tensor (adopting the naming convention of [1]):

$$\begin{aligned} W_{\mu\mu'} = &\left(-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2}\right) F_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_{\mu'}}{P\cdot q} F_2(x,Q^2) - i\varepsilon_{\mu\mu'\alpha\beta}\frac{q^\alpha P^\beta}{2P\cdot q}F_3(x,Q^2) \\ &+ i\varepsilon_{\mu\mu'\alpha\beta}\frac{q^\alpha S^\beta}{P\cdot q}g_1(x,Q^2) + \frac{S\cdot q}{P\cdot q}\left[\frac{\hat{P}_\mu \hat{P}_{\mu'}}{P\cdot q} g_4(x,Q^2) + \left(-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2}\right) g_5(x,Q^2)\right] \end{aligned} \tag{3}$$

with $\hat{P}_\mu = P_\mu - \frac{P\cdot q}{q^2} q_\mu$ and S denoting the spin vector of the nucleon. We further introduce the more convenient structure functions

$$F_L(x,Q^2) = F_2(x,Q^2) - 2xF_1(x,Q^2), \qquad g_L(x,Q^2) = g_4(x,Q^2) - 2xg_5(x,Q^2)\,. \tag{4}$$

As we are interested in the full neutral current contributions, we allow for both, the virtual photon γ^* and the virtual Z^*-boson exchange. The structure functions can then be decomposed by

$$H = H^\gamma - \left(g_V^\ell \pm \lambda g_A^\ell\right)\eta_{\gamma\mathrm{Z}} H^{\gamma\mathrm{Z}} + \left((g_V^\ell)^2 + (g_A^\ell)^2 \pm 2\lambda g_V^\ell g_A^\ell\right)\eta_\mathrm{Z} H^\mathrm{Z},\ H \in \{F_2, F_L, 2xg_1\} \tag{5}$$

$$H = -\left(g_A^\ell \pm \lambda g_V^\ell\right)\eta_{\gamma\mathrm{Z}} H^{\gamma\mathrm{Z}} + \left(2g_V^\ell g_A^\ell \pm \lambda\left((g_V^\ell)^2 + (g_A^\ell)^2\right)\right)\eta_\mathrm{Z} H^\mathrm{Z},\ H \in \{xF_3, g_4, g_L\} \tag{6}$$

where λ defines the helicity of the incoming lepton and

$$\eta_{\gamma Z} = \left(\frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\right)\left(\frac{Q^2}{Q^2+M_Z^2}\right) \qquad \eta_Z = \eta_{\gamma Z}^2 \tag{7}$$

We use the framework of the collinear factorization, so the leading order partonic reaction is given by

$$b(q) + \mathrm{g}(k_1) \to \overline{Q}(p_2) + Q(p_1), \quad b \in \{\gamma^*, Z^*\} \tag{8}$$

where $k_1 = \xi P$ and the relevent kinematic variables are

$$\begin{aligned} z &= \frac{Q^2}{2q\cdot k_1} = \frac{x}{\xi}, & s &= (k_1+q)^2, & s' &= s - q^2 \\ t_1 &= (k_1 - p_2) - m^2, & u_1 &= (q-p_2)^2 - m^2, & u_1' &= u_1 - q^2\,. \end{aligned} \tag{9}$$

The hadronic tensor 3 can then be mapped onto a partonic level by

$$\begin{aligned} \frac{1}{2z}\hat{w}_{\mu,\mu'} = &(-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2})\hat{F}_1(z,Q^2) + \frac{\hat{k}_{1,\mu}\hat{k}_{1,\mu'}}{k_1\cdot q}\hat{F}_2(z,Q^2) - i\varepsilon_{\mu\mu'\alpha\beta}\frac{q^\alpha k_1^\beta}{2k_1\cdot q}\hat{F}_3(z,Q^2) \\ &+ \frac{q_\mu q_{\mu'}}{q^2}\hat{F}_4(z,Q^2) + \frac{q_\mu k_{1,\mu'} + q_{\mu'}k_{1,\mu}}{2k_1\cdot q}\hat{F}_5(z,Q^2) \\ &+ i\varepsilon_{\mu\mu'\alpha\beta}\frac{q^\alpha S^\beta}{k_1\cdot q}\hat{g}_1(z,Q^2) + \frac{S\cdot q}{k_1\cdot q}\left[\frac{\hat{k}_{1,\mu}\hat{k}_{1,\mu'}}{k_1\cdot q}\hat{g}_4(z,Q^2) + (-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2})\hat{g}_5(z,Q^2)\right] \\ &+ \frac{S\cdot q}{k_1\cdot q}\left[\frac{q_\mu q_{\mu'}}{q^2}\hat{g}_6(z,Q^2) + \frac{q_\mu k_{1,\mu'} + q_{\mu'}k_{1,\mu}}{2k_1\cdot q}\hat{g}_7(z,Q^2)\right] \end{aligned} \tag{10}$$

with $\hat{k}_{1,\mu} = k_{1,\mu} - \frac{k_1\cdot q}{q^2}q_\mu$ and

$$\hat{F}_L(z,Q^2) = \hat{F}_2(z,Q^2) - 2z\hat{F}_1(z,Q^2), \qquad \hat{g}_L(z,Q^2) = \hat{g}_4(z,Q^2) - 2z\hat{g}_5(z,Q^2)\,. \tag{11}$$

Note that we have to include $\hat{F}_4, \hat{F}_5, \hat{g}_6$ and $\hat{g}_7$ into Eq. 10 as we are interested in the full neutral current case, that is, we allow for Z-bosons to be exchanged and due to this we can no longer rely on the Ward-identity, because the contraction of the axial current $q_\mu J_5^\mu$ does not vanish for massive particles:

$$q_\mu J_5^\mu = q_\mu\bar{\psi}\gamma^\mu\gamma^5\psi = 2m\,\bar{\psi}i\gamma^5\psi\,. \tag{12}$$

It is convenient to rescale the structure functions and so we focus in the following on the six structure functions

$$\hat{F}_2, \hat{F}_L, z\hat{F}_3, 2z\hat{g}_1, \hat{g}_4 \text{ and } \hat{g}_L \tag{13}$$

or their hadronic counter parts.

This way we can define the projections onto the structure functions by

$$\hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{F}_2} = \frac{-g^{\mu\mu'}}{n-2} - \frac{n-1}{n-2} \cdot \frac{4z^2 k_1^{\mu} k_1^{\mu'}}{q^2} - \frac{q^{\mu} q^{\mu'}}{q^2} - \frac{n-2}{n-1} \cdot \frac{2z(q^{\mu} k_1^{\mu'} + q^{\mu'} k_1^{\mu})}{q^2} \tag{14}$$

$$\hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{F}_L} = -\frac{4z^2 k_1^{\mu} k_1^{\mu'}}{q^2} - \frac{q^{\mu} q^{\mu'}}{q^2} - \frac{2z(q^{\mu} k_1^{\mu'} + q^{\mu'} k_1^{\mu})}{q^2} \tag{15}$$

$$\hat{\mathcal{P}}^{b,\mu\mu'}_{z\hat{F}_3} = -\frac{iz\varepsilon^{\mu\mu'\alpha\beta} k_{1,\alpha} q_{\beta}}{q^2} \tag{16}$$

and, due to the symmetry in the Lorentz structure,

$$\hat{\mathcal{P}}^{b,\mu\mu'}_{2z\hat{g}_1} = \hat{\mathcal{P}}^{b,\mu\mu'}_{z\hat{F}_3} \qquad \hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{g}_4} = -\hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{F}_2} \qquad \hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{g}_L} = -\hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{F}_L} . \tag{17}$$

This way we have

$$\hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{h}} \hat{w}_{\mu,\mu'} = \hat{h} \quad \text{for } \hat{h} \in \{\hat{F}_2, \hat{F}_L, z\hat{F}_3, 2z\hat{g}_1, \hat{g}_4, \hat{g}_L\} \tag{18}$$

For the unpolarized structure functions $\hat{F}_2, \hat{F}_L$ and $z\hat{F}_3$ the helicity of the parton, either gluon or (anti-)quark, has to be averaged, whereas for the polarized $2z\hat{g}_1, \hat{g}_4$ and $\hat{g}_L$ we have to consider the helicity difference. For the gluons this is achieved by

$$\hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{F} = -g^{\nu\nu'} \qquad \hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{g} = 2i\varepsilon^{\nu\nu'\alpha\beta} \frac{k_{1,\alpha} q_{\beta}}{2k_1 \cdot q} \tag{19}$$

and by choosing just $-g^{\nu\nu'}$, we decided to include incoming external ghost to cancel all unphysical gluon polarization. All initial-state (anti-)quarks are taken as massless partons, so the relevant projection operators onto definitive helicity states are given by

$$\begin{aligned} \hat{\mathcal{P}}^{q,aa'}_{F} &= (\not{k}_1)_{aa'}, & \hat{\mathcal{P}}^{q,aa'}_{g} &= -(\gamma_5 \not{k}_1)_{aa'}, \\ \hat{\mathcal{P}}^{\bar{q},bb'}_{F} &= (\not{k}_1)_{bb'}, & \hat{\mathcal{P}}^{\bar{q},bb'}_{g} &= (\gamma_5 \not{k}_1)_{bb'} \end{aligned} \tag{20}$$

where a and a' (b and b') refer to the Dirac-index of the initial (anti-)quark spinor in the relevant matrix elements given below.

In order to compute the exchange of scattered vector boson $b = \{\gamma, \mathrm{Z}\}$ with a common notation, we write the coupling of b to the hadronic process by

$$\Gamma^{\mu}_{b,j} = g^{V}_{b,j} \Gamma^{\mu}_{V} + g^{A}_{b,j} \Gamma^{\mu}_{A} = g^{V}_{b,j} \gamma^{\mu} + g^{A}_{b,j} \gamma^{\mu} \gamma^{5}, \quad j \in \{q, Q\}. \tag{21}$$

Due to symmetry reasons the parity-violating structure functions $z\hat{F}_3, g_4, g_L$ can only recieve contributions from $\Gamma^{\mu}_{V} \Gamma^{\mu'}_{A}$, where μ refer to the Lorentz index of the boson in matrix amplitude and the μ' to the index in the complex conjungate. Likewise the parity conserving structure functions can only recieve contributions from either $\Gamma^{\mu}_{V} \Gamma^{\mu'}_{V}$ or $\Gamma^{\mu}_{A} \Gamma^{\mu'}_{A}$. To introduce a compact notation we write

$$\vec{\kappa} = (\kappa_1, \kappa_2) \quad \kappa_1 \in \{\mathrm{VV}, \mathrm{VA}, \mathrm{AA}\}, \, \kappa_2 \in \{\hat{F}_2, \hat{F}_L, z\hat{F}_3, 2z\hat{g}_1, \hat{g}_4, \hat{g}_L\} . \tag{22}$$

We define an additional set of partonic variables, that will simplify many partonic expressions:

$$0 \leq \rho = \frac{4m^2}{s} \leq 1 \qquad 0 \leq \beta = \sqrt{1-\rho} \leq 1 \qquad 0 \leq \chi = \frac{1-\beta}{1+\beta} \leq 1 \tag{23}$$

$$0 \leq \rho' = \frac{4m^2}{s'} \leq 1 \qquad 0 \leq \beta' = \sqrt{1-\rho'} \leq 1 \qquad 0 \leq \chi' = \frac{1-\beta'}{1+\beta'} \leq 1 \tag{24}$$

$$\rho_q = \frac{4m^2}{q^2} \leq 0 \qquad 1 \leq \beta_q = \sqrt{1-\rho_q} \qquad 0 \leq \chi_q = \frac{\beta_q - 1}{\beta_q + 1} \leq 1 \tag{25}$$

which obey the further inequalities

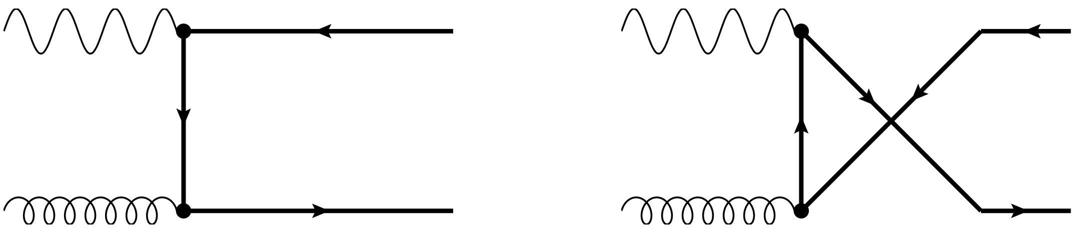
$$\rho' < \rho, \qquad \rho' < \frac{\rho_q}{\rho_q - 1}, \qquad \beta < \beta', \qquad \beta' < \frac{1}{\beta_q}, \qquad \chi' < \chi, \qquad \chi' < \chi_q\,. \tag{26}$$

2 Leading Order Calculations

In leading order we have to consider photon-gluon-fusion (PGF), that is

$$b(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2), \quad b \in \{\gamma^*, Z^*\} \tag{27}$$

with two contributing diagrams depicted in figure 1.



(a) $i\varepsilon_b^\mu(q)\varepsilon_\mathrm{g}^\nu(k_1)\,\mathcal{M}_{\mu\nu}^{(0),1}$ (b) $i\varepsilon_b^\mu(q)\varepsilon_\mathrm{g}^\nu(k_1)\,\mathcal{M}_{\mu\nu}^{(0),2}$

Figure 1: leading order Feynman diagrams
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The result can then be written as

$$M_{\vec{\kappa}}^{(0)} = \hat{\mathcal{P}}_{\vec{\kappa}}^{b,\mu\mu'} \hat{\mathcal{P}}_{\kappa_2}^{\mathrm{g},\nu\nu'} \sum_{j,j'=1}^{2} \mathcal{M}_{\mu\nu}^{(0),j} \left(\mathcal{M}_{\mu'\nu'}^{(0),j'}\right)^* = 8g^2\mu_D^{-\epsilon}e^2e_H^2 N_C C_F B_{\vec{\kappa},\mathrm{QED}} \tag{28}$$

where g and e are the strong and electromagnetic coupling constants respectively, μ_D is an arbitray mass parameter introduced to keep the couplings dimensionless and e_H is the magnitude of the heavy quark in units of e. Further $N_C = 3$ corresponds to the gauge group $SU(N_C)$ and the color factor $C_F = (N_C^2 - 1)/(2N_C)$ refers to the second Casimir constant of the fundamental representation for the quarks. We then find:

$$B_{\mathrm{VV},F_2,\mathrm{QED}} = \left[-1 - \frac{6q^2}{s'} - \frac{6q^4}{s'^2} + \frac{q^2(6m^2+s)+2m^2s+s'^2/2}{t_1u_1} - \frac{(2m^2+q^2)m^2s'^2}{(t_1u_1)^2}\right] + \frac{\epsilon}{2}\left[-1+\frac{s^2-q^2s'}{t_1u_1} - \frac{m^2q^2s'^2}{t_1^2u_1^2}\right] + \epsilon^2\frac{s'^2}{8t_1u_1} \tag{29}$$

$$B_{\mathrm{VV},F_L,\mathrm{QED}} = -\frac{4q^2}{s'}\left(\frac{s}{s'} - \frac{m^2s'}{t_1u_1}\right) \tag{30}$$

$$B_{\mathrm{VV},2xg_1,\mathrm{QED}} = \left\{1 + \frac{2q^2}{s'} - \frac{s'(2(2m^2+q^2)+s')}{2t_1u_1} + \frac{m^2s'^3}{(t_1u_1)^2} + \epsilon\left(-\frac{1}{2} + \frac{s'^2}{4t_1u_1}\right)\right\}(1+\epsilon) \tag{31}$$

$$\begin{aligned}B_{\mathrm{AA},F_2,\mathrm{QED}} = &\frac{m^2s'^2(1+\epsilon)(2+\epsilon)(12m^2(-1+\epsilon)+q^2(-6+(-3+\epsilon)\epsilon))}{12(t_1u_1)^2} - \\ &\frac{(1+\epsilon)\left(8s'^3\epsilon + 12q^6(2+\epsilon)+12q^4s'(2+\epsilon)+q^2s'^2(4+\epsilon(20-(-3+\epsilon)\epsilon))\right)}{4q^2s'^2} - \\ &\frac{(1+\epsilon)}{48q^2(t_1u_1)}\left(q^2(2+\epsilon)(-6+(-3+\epsilon)\epsilon)\left(4q^4+4q^2s'+s'^2(2+\epsilon)\right)\right. \\ &\left. +48m^2\left(-s'^2(-2+\epsilon)+q^4(-4+\epsilon)(2+\epsilon)+q^2s'\left(-2+\epsilon+\epsilon^2\right)\right)\right)\end{aligned} \tag{32}$$

$$\begin{aligned}B_{\mathrm{AA},F_L,\mathrm{QED}} = &-\frac{m^2s'^2(1+\epsilon)(2+\epsilon)(12m^2+q^2\epsilon)}{6(t_1u_1)^2} - \\ &\frac{(1+\epsilon)\left(4s'^3\epsilon+4q^6(2+\epsilon)+4q^4s'(2+\epsilon)+q^2s'^2\epsilon(6+\epsilon)\right)}{2q^2s'^2} + \\ &\frac{(1+\epsilon)}{24q^2(t_1u_1)}\left(24m^2\left(s'^2(-2+\epsilon)+4q^4(2+\epsilon)+2q^2s'(2+\epsilon)\right)+\right. \\ &\left. q^2\epsilon(2+\epsilon)\left(4q^4+4q^2s'+s'^2(2+\epsilon)\right)\right)\end{aligned} \tag{33}$$

$$B_{\mathrm{AA},2xg_1,\mathrm{QED}} = \frac{(1+\epsilon)(2-\epsilon)}{2}B_{\mathrm{VV},2xg_1,\mathrm{QED}} \tag{34}$$

$$B_{\mathrm{VA},xF_3,\mathrm{QED}} = -(1+\epsilon)(2+\epsilon)(t_1^2-u_1^2)\left\{-\frac{m^2q^2}{2(t_1u_1)^2} + \frac{4q^2(q^2+s')+s'^2(2+\epsilon)}{8s'^2t_1u_1}\right\} \tag{35}$$

$$B_{\mathrm{VA},g_4,\mathrm{QED}} = (1+\epsilon)(t_1-u_1)\left\{-\frac{m^2s'^2}{(t_1u_1)^2} + \frac{4q^2+s'(2-\epsilon)}{4t_1u_1}\right\} \tag{36}$$

$$B_{\mathrm{VA},g_L,\mathrm{QED}} = 0 \tag{37}$$

We will decompose the Born cross section further by their dependence on ϵ

$$B_{\vec{\kappa},\mathrm{QED}} = B^{(0)}_{\vec{\kappa},\mathrm{QED}} + \epsilon B^{(1)}_{\vec{\kappa},\mathrm{QED}} + \epsilon^2 B^{(2)}_{\vec{\kappa},\mathrm{QED}} \tag{38}$$

and do find $B^{(0)}_{\mathrm{VV},2xg_1,\mathrm{QED}} = B^{(0)}_{\mathrm{AA},2xg_1,\mathrm{QED}}$, but $B^{(1)}_{\mathrm{VV},2xg_1,\mathrm{QED}} \neq B^{(1)}_{\mathrm{AA},2xg_1,\mathrm{QED}}$.

The required n-dimensional 2-to-2-particle phase space dPS_2 is given by

$$\mathrm{dPS}_2 = \frac{2\pi S_\epsilon}{s'\Gamma(1+\epsilon/2)} \left(\frac{(t_1 u_1' - s'm^2)s' - q^2 t_1^2}{s'^2}\right)^{\epsilon/2} \delta(s'+t_1+u_1)\, dt_1 du_1\,. \tag{39}$$

The spin and color averaged partonic cross section is then given by

$$d\sigma^{(0)}_{\vec{\kappa},\mathrm{g}} = \frac{1}{2s'}\frac{1}{2} E_{\kappa_2}(\epsilon) K_{\mathrm{g}\gamma} M^{(0)}_{\vec{\kappa}}\, \mathrm{dPS}_2 \tag{40}$$

where

$$E_F(\epsilon) = \frac{1}{1+\epsilon/2}\,, \qquad\qquad E_g(\epsilon) = 1 \tag{41}$$

accounts for additional degrees of freedom in n dimensions for initial-state bosons.

3 Next-To-Leading Order Calculations

3.1 One Loop Virtual Contributions

At NLO we have to consider the one-loop virtual corrections to the PGF process. In this order two different color structures arise: the abelian QED part and the non-Abelian OK part, so we decompose our result by this structure. The needed matrix elements can then be written analog to [2]

$$\begin{aligned} M^{(1),V}_{\vec{\kappa}} &= \hat{\mathcal{P}}^{b,\mu\mu'}_{\vec{\kappa}} \hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{\kappa_2} \sum_{j,j'} 2\mathrm{Re}\left[\mathcal{M}^{(1),V}_{j,\mu\nu}\left(\mathcal{M}^{(0)}_{j',\mu'\nu'}\right)^*\right] \\ &= 8g^4\mu_D^{-\epsilon} e^2 e_H^2 N_C C_F C_\epsilon \left(C_A V_{\vec{\kappa},\mathrm{OK}} + 2C_F V_{\vec{\kappa},\mathrm{QED}}\right) \end{aligned} \tag{42}$$

where $C_\epsilon = \exp(\epsilon/2(\gamma_E - \ln(4\pi)))/(16\pi^2)$, γ_E is the Euler-Mascheroni constant and $C_A = N_C$ is the second Casimir constant of the adjoint representation for the gluon (that introduces the non-abelian OK part).

The details of the calculation for the $V_{\vec{\kappa},\mathrm{OK}}, V_{\vec{\kappa},\mathrm{QED}}$ are outlined in our previous paper [2] along with a correction for an integral in [3]. As we adopted the Larin-scheme here to deal with the subtleties of γ_5 instead of the HVBM-scheme there, the expressions in the intermediate steps do not match. Nevertheless the arising poles can be cast in the very same form, i.e.,

$$V_{\vec{\kappa},\mathrm{OK}} = -2B_{\vec{\kappa},\mathrm{QED}}\left(\frac{4}{\epsilon^2} + \frac{2}{\epsilon}\left[\ln(-t_1/m^2) + \ln(-u_1/m^2) + \frac{s-2m^2}{s\beta}\ln(\chi)\right]\right) + O(\epsilon^0) \tag{43}$$

$$V_{\vec{\kappa},\mathrm{QED}} = -2B_{\vec{\kappa},\mathrm{QED}}\left(1 - \frac{s-2m^2}{s\beta}\ln(\chi)\right)\frac{2}{\epsilon} + O(\epsilon^0) \tag{44}$$

Note that these results completely factorize and the $B_{\vec{\kappa},\mathrm{QED}}$ carry the only dependence on the projection $\vec{\kappa}$. The above results do not include self-energies on external legs. We renormalize the HQs *on-shell* and m thus refers to the pole mass of the HQ. For the renormalization of the strong coupling we use the same $\overline{\mathrm{MS}}_m$ scheme as defined in [2] and so the full (remaining) renormalization of all ultra-violates poles can be achieved by

$$\frac{d^2\sigma^{(1),V}_{\vec{\kappa},\mathrm{g}}}{dt_1du_1} = \left.\frac{d^2\sigma^{(1),V}_{\vec{\kappa},\mathrm{g}}}{dt_1du_1}\right|_{\mathrm{bare}} + 4\pi\alpha_s(\mu_R^2)C_\epsilon\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}$$
$$\left[\left(\frac{2}{\epsilon}+\ln(\mu_R^2/m^2)\right)\beta_0^f + \frac{2}{3}\ln(\mu_R^2/m^2)\right]\frac{d^2\sigma^{(0)}_{\vec{\kappa},\mathrm{g}}}{dt_1du_1} \tag{45}$$

with μ_R the renormalization scale, $\beta_0^f = (11C_A - 2n_f)/3$ the first coefficient of the beta function and n_f the number of *total* flavours (i.e. $n_{lf} = n_f - 1$ active (light) flavours and one heavy flavour). The double poles in $V_{\vec{\kappa},\mathrm{OK}}$ originate from diagrams where soft and collinear singularities can coincide. In what follows, we will often drop the scale in the strong coupling, i.e., α_s has to be understood as $\alpha_s(\mu_R)$.

The bare double differential partonic one-loop virtual PGF cross section is given by

$$\left. d\sigma^{(1),V}_{\vec{\kappa},\mathrm{g}}\right|_{\mathrm{bare}} = \frac{1}{2s'}\frac{1}{2}E_{\kappa_2}(\epsilon)K_{\mathrm{g}\gamma}M^{(1),V}_{\vec{\kappa}}\,\mathrm{dPS}_2 \tag{46}$$

3.2 Single Gluon Radiation

In addition to the virtual corrections we have to consider the radiation of an additional gluon at next-to-leading order, i. e., the $2 \to 3$ process

$$b(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) + \mathrm{g}(k_2), \quad b \in \{\gamma^*, Z^*\} \tag{47}$$

As before, we split the results according to their color structure into QED and OK parts and matrix elements can then be written as

$$M^{(1),\mathrm{g}}_{\vec{\kappa}} = \hat{\mathcal{P}}^{b,\mu\mu'}_{\vec{\kappa}}\hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{\kappa_2}\sum_{j,j'}\mathcal{M}^{(1),\mathrm{g}}_{j,\mu\nu}\left(\mathcal{M}^{(1),\mathrm{g}}_{j',\mu'\nu'}\right)^* \tag{48}$$
$$= 8g^4\mu_D^{-2\epsilon}e^2e_H^2N_CC_F\left(C_AR_{\vec{\kappa},\mathrm{OK}} + 2C_FR_{\vec{\kappa},\mathrm{QED}}\right)\,. \tag{49}$$

The required n-dimensional phase space dPS_3 was computed in [2] and is given by

$$\mathrm{dPS}_3 = \frac{S_\epsilon^2}{\Gamma(1+\epsilon)s'}\frac{s_4^{1+\epsilon}}{(s_4+m^2)^{1+\epsilon/2}}\left(\frac{(t_1u_1' - s'm^2)s' - q^2t_1^2}{s'^2}\right)^{\epsilon/2}dt_1du_1d\Omega_n \tag{50}$$

with $d\Omega_n = \sin^{n-3}(\theta_1)d\theta_1\sin^{n-4}(\theta_2)d\theta_2$. Note that we do not have to consider the hat momenta space, as we use the Larin-scheme here.

Most of needed the phase space integrals have already been calculated for [2], but the new projections here do bring up some more integrals that are obtained straight forward using the list in [4]. The collinear poles can then be given in a compact form by

$$\frac{s_4}{2\pi(s_4+m^2)}\int d\Omega_n\, C_A R_{\vec{\kappa},OK} = -\frac{2}{u_1} B_{\vec{\kappa},\mathrm{QED}} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} P^{(0),H}_{\kappa_2,\mathrm{gg}}(x_1)\frac{2}{\epsilon} + O(\epsilon^0) \quad (51)$$

with $x_1 = -u_1/(s'+t_1)$ and the hard part of the LO Altarelli-Parisi splitting functions $P^{(0),H}_{\kappa_2,\mathrm{gg}}$ [5]:

$$P^{(0),H}_{F_2,\mathrm{gg}}(x) = P^{(0),H}_{F_L,\mathrm{gg}}(x) = P^{(0),H}_{xF_3,\mathrm{gg}}(x) = C_A\left(\frac{2}{1-x} + \frac{2}{x} - 4 + 2x - 2x^2\right) \quad (52)$$

$$P^{(0),H}_{2xg_1,\mathrm{gg}}(x) = P^{(0),H}_{g_4,\mathrm{gg}}(x) = P^{(0),H}_{g_L,\mathrm{gg}}(x) = C_A\left(\frac{2}{1-x} - 4x + 2\right) \quad (53)$$

The hard abelian QED part $R_{\vec{\kappa},\mathrm{QED}}$ does not contain any collinear poles.

The spin and color averaged partonic cross section for the real corrections (R) to the PGF process is then given by

$$d\sigma^{(1),R}_{\vec{\kappa},\mathrm{g}} = \frac{1}{2s'}\frac{1}{2} E_{\kappa_2}(\epsilon) K_{\mathrm{g}\gamma} M^{(1),\mathrm{g}}_{\vec{\kappa}}\, \mathrm{dPS}_3\ . \quad (54)$$

Following the phase space slicing methods introduced in [2], we split the calculation of the real contributions into a hard part $s_4 > \Delta$ and a soft part $s_4 < \Delta$. The soft limit of the matrix elements is obtained by taking the limit $k_2 \to 0$ and we find

$$\lim_{k_2\to 0}\left(C_A R_{\vec{\kappa},\mathrm{OK}} + 2C_F R_{\vec{\kappa},\mathrm{QED}}\right) = \left(C_A S_{\vec{\kappa},\mathrm{OK}} + 2C_F S_{\vec{\kappa},\mathrm{QED}}\right) + O(1/s_4, 1/s_3, 1/t') \quad (55)$$

where

$$S_{\vec{\kappa},\mathrm{OK}} = 2\left(\frac{t_1}{t' s_3} + \frac{u_1}{t' s_4} - \frac{s-2m^2}{s_3 s_4}\right) B_{\vec{\kappa},\mathrm{QED}} \quad (56)$$

$$S_{\vec{\kappa},\mathrm{QED}} = 2\left(\frac{s-2m^2}{s_3 s_4} - \frac{m^2}{s_3^2} - \frac{m^2}{s_4^2}\right) B_{\vec{\kappa},\mathrm{QED}}\ . \quad (57)$$

Note that the einkonal factors multiplying the Born functions $B_{\vec{\kappa},\mathrm{QED}}$ neither depend on the photon's virtuality q^2 nor on the projection $\vec{\kappa}$.

3.3 Light Quark Processes

In next-to-leading order a new production mechanism enters that is induced by a light quark, so we have to consider the process

$$b(q) + q(k_1) \to Q(p_1) + \overline{Q}(p_2) + q(k_2), \quad b \in \{\gamma^*, Z^*\} \quad (58)$$

and the obtained matrix elements can then be written as

$$M_{\vec{\kappa}}^{(1),q} = \hat{\mathcal{P}}_{\vec{\kappa}}^{b,\mu\mu'} \hat{\mathcal{P}}_{\vec{\kappa}}^{q,aa'} \sum_{j,j'=1}^{4} \mathcal{M}_{j,\mu a}^{(1),q} \left(\mathcal{M}_{j',\mu' a'}^{(1),q} \right)^* \tag{59}$$

$$= 8 g^4 \mu_D^{-2\epsilon} e^2 N_C C_F \left(e_H^2 A_{\vec{\kappa},1} + e_L^2 A_{\vec{\kappa},2} + e_L e_H A_{\vec{\kappa},3} \right) \tag{60}$$

where e_L denotes the charge of the light quark q in units of e. We will refer to the $A_{\vec{\kappa},1}$ as Bethe-Heitler subprocess and to the $A_{\vec{\kappa},2}$ as Compton subprocess.

The needed 2-to-3-particle phase space has already been calculated in section 3.2, so we can immediately quote the collinear poles here

$$\frac{s_4}{4\pi(s_4+m^2)} \int d\Omega_n \, C_F A_{\vec{\kappa},1} = -\frac{1}{u_1} B_{\vec{\kappa},\text{QED}} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} P^{(0)}_{\kappa_2,\text{g}\,q}(x_1) \frac{2}{\epsilon} + O(\epsilon^0) \tag{61}$$

with $x_1 = -u_1/(s'+t_1)$ and the LO Altarelli-Parisi splitting functions $P^{(0)}_{\kappa_2,\text{g}\,q}$[5]:

$$P^{(0)}_{F_2,\text{g}\,q}(x) = P^{(0)}_{F_L,\text{g}\,q}(x) = P^{(0)}_{xF_3,\text{g}\,q}(x) = C_F \left(\frac{1}{x} + \frac{(1-x)^2}{x} \right) \tag{62}$$

$$P^{(0)}_{2xg_1,\text{g}\,q}(x) = P^{(0)}_{g_4,\text{g}\,q}(x) = P^{(0)}_{g_L,\text{g}\,q}(x) = C_F \left(2 - x\right) \tag{63}$$

For the current case of DIS ($q^2 \neq 0$) the Coulomb subprocess $A_{\vec{\kappa},2}$ does not contain collinear poles. Due to Furry's theorem we find $\int dt_1 du_1 \int d\Omega_4 \, A_{\vec{\kappa},3} = 0$.

The spin and color averaged partonic cross section is then given by

$$d\sigma^{(1)}_{\vec{\kappa},q} = \frac{1}{2s'} \frac{1}{2} K_{q\gamma} M_{\vec{\kappa}}^{(1),q} \, \text{dPS}_3 \tag{64}$$

4 Mass Factorization

All collinear poles that arise in the NLO corrections to PGF subprocess $\sigma^{(1)}_{\vec{\kappa},\text{g}}$ or the Bethe-Heitler subprocess $\sigma^{(1)}_{\vec{\kappa},q}$ can be removed by a standard mass factorization

$$s'^2 \frac{d^2\sigma^{(1),fin}_{\vec{\kappa},j}(s',t_1,u_1,\mu_F)}{dt_1 du_1} = \lim_{\epsilon\to 0} \left[s'^2 \frac{d^2\sigma^{(1)}_{\vec{\kappa},j}(s',t_1,u_1,\epsilon)}{dt_1 du_1} \right.$$
$$\left. - \int_0^1 \frac{dx_1}{x_1} \Gamma^{(1)}_{\kappa_2,\text{g}j}(x_1,\mu_F^2,\mu_D,\epsilon)(x_1 s')^2 \frac{d^2\sigma^{(0)}_{\vec{\kappa},\text{g}}(x_1 s', x_1 t_1, u_1, \epsilon)}{d(x_1 t_1) du_1} \right] \tag{65}$$

for $j = \{\text{g}, q\}$ and where $\Gamma^{(1)}_{\kappa_2,ij}$ is the first order correction to the transition functions $\Gamma_{\kappa_2,ij}$ for *incoming* particle j and *outgoing* particle i in projection κ_2:

$$\Gamma^{(1)}_{\kappa_2,ij}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_s}{2\pi} \left(P^{(0)}_{\kappa_2,ij}(x) \frac{2}{\epsilon} + f^{(1)}_{\kappa_2,ij}(x,\mu_F^2,\mu_D^2) \right) \tag{66}$$

In the adopted $\overline{\mathrm{MS}}$-scheme the $f^{(1)}_{\kappa_2,ij}$ take their usual form and we find

$$\Gamma^{(1),\overline{\mathrm{MS}}}_{\kappa_2,ij}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_s}{2\pi}P^{(0)}_{\kappa_2,ij}(x)\left(\frac{2}{\epsilon}+\gamma_E-\ln(4\pi)+\ln(\mu_F^2/m^2)-\ln(\mu_D^2/m^2)\right) \quad (67)$$

$$= 8\pi\alpha_s P^{(0)}_{\kappa_2,ij}(x)C_\epsilon\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}\left(\frac{2}{\epsilon}+\ln(\mu_F^2/m^2)\right) \quad (68)$$

For the PGF subprocess we need the LO gluon-to-gluon splitting functions $P^{(0)}_{\kappa_2,\mathrm{gg}}(x)$ given by[5]

$$P^{(0)}_{\kappa_2,\mathrm{gg}}(x) = \Theta(1-\delta-x)P^{(0),H}_{\kappa_2,\mathrm{gg}}(x)+\delta(1-x)\left(2C_A\ln(\delta)+\frac{\beta_0}{2}\right) \quad (69)$$

where we introduced another infrared cut-off δ to seperate soft collinear ($x \geq 1-\delta$) and hard collinear ($x < 1-\delta$) gluons. By simple kinematics we find the relation $\delta = \Delta/(s'+t_1)$ that allows us to divide the collinear part again into a hard and a soft part, rendering each into a finite result. The functions $P^{(0),H}_{\kappa_2,\mathrm{gg}}(x)$ have been already quoted in Eq. 52 and 53 and the needed splitting functions $P^{(0)}_{\kappa_2,\mathrm{g}q}$ for the Bethe-Heitler subprocess have been given in Eqs. (62) and (63), to denote the collinear poles there.

The final, finite partonic cross sections for the PGF subprocess is split, as discussed, into a hard contribution given by

$$\begin{aligned} s'^2\frac{d^2\sigma^{(1),H,fin}_{\vec{\kappa},\mathrm{g}}}{dt_1du_1} = \alpha\alpha_S e_H^2 K_{\mathrm{g}\gamma}N_C C_F\Bigg[&-\frac{2}{u_1}P^H_{\kappa_2,\mathrm{gg}}(x_1) \\ &\left\{B^{(0)}_{\vec{\kappa},\mathrm{QED}}\begin{pmatrix} s'\to x_1 s' \\ t_1\to x_1 t_1\end{pmatrix}\left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)}\right)-\ln(\mu_F^2/m^2)\right)\right. \\ &\left.-2B^{(1)}_{\vec{\kappa},\mathrm{QED}}\begin{pmatrix} s'\to x_1 s' \\ t_1\to x_1 t_1\end{pmatrix}\right\} \\ &+C_A\frac{s_4}{2\pi(s_4+m^2)}\left(\int d\Omega_n\, R_{\vec{\kappa},\mathrm{OK}}\right)^{finite} \\ &+2C_F\frac{s_4}{2\pi(s_4+m^2)}\int d\Omega_4\, R_{\vec{\kappa},\mathrm{QED}}\Bigg] \end{aligned} \quad (70)$$

and the soft plus virtual contributions given by

$$\begin{aligned} s'^2\frac{d^2\sigma^{(1),S+V,fin}_{\vec{\kappa},\mathrm{g}}}{dt_1du_1} = 4\alpha\alpha_S e_H^2 K_{\mathrm{g}\gamma}N_C C_F B^{(0)}_{\vec{\kappa},\mathrm{QED}}\delta(s'+t_1+u_1)\Bigg[&C_A\ln^2(\Delta/m^2) \\ &+\ln(\Delta/m^2)\left(\left(\ln(-t_1/m^2)-\ln(-u_1/m^2)-\ln(\mu_F^2/m^2)\right)C_A\right. \\ &\left.-\frac{2m^2-s}{s\beta}\ln(\chi)(C_A-2C_F)-2C_F\right) \\ &+\frac{\beta_0^{lf}}{4}\left(\ln(\mu_R^2/m^2)-\ln(\mu_F^2/m^2)\right)+f_{\vec{\kappa}}(s',u_1,t_1,q^2)\Bigg] \end{aligned} \quad (71)$$

where the functions $f_{\vec{\kappa}}$ in Eq. 71 contain logarithms and dilogarithms with different, complicated arguments, but they do not depend on Δ, μ_F^2, μ_R^2 nor n_f and $\beta_0^{lf} = (11C_A - 2n_{lf})/3$.

The corresponding finite, reduced partonic cross section for the light quark initiated processes reads

$$
\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),fin}_{\vec{\kappa},q}}{dt_1 du_1} = \alpha\alpha_S K_{q\gamma} N_C \Bigg[& -\frac{1}{u_1} e_H^2 P_{\kappa_2,\mathrm{g}q}(x_1) \\
& \left\{ B^{(0)}_{\vec{\kappa},\mathrm{QED}} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)} \right) - \ln(\mu_F^2/m^2) - 2\partial_\epsilon E_{\vec{\kappa}}(\epsilon=0) \right) \right. \\
& \left. -2 B^{(1)}_{\vec{\kappa},\mathrm{QED}} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \right\} \\
& + C_F \frac{s_4}{4\pi(s_4+m^2)} \left(\int d\Omega_n \, e_H^2 A_{\vec{\kappa},1} \right)^{finite} \\
& + C_F \frac{s_4}{4\pi(s_4+m^2)} \int d\Omega_4 \, e_L^2 A_{\vec{\kappa},2} + C_F \frac{s_4}{4\pi(s_4+m^2)} \int d\Omega_4 \, e_H e_L A_{\vec{\kappa},3} \Bigg] .
\end{aligned} \tag{72}
$$

Note that the hat contributions for $g_1(x,Q^2)$ to the NLO PGF and the Bethe-Heitler process as computed in [2] this way can be directly related to $B^{(1)}_{\mathrm{VV},2xg_1,\mathrm{QED}}$, as in this calculation this variable is not longer vanishing. This way we have shown that the Larin-scheme and the HVBM-scheme are equivalent for this process up to NLO.

5 Partonic Results

We decompose the partonic cross section by their order in QCD pertubation theory and their charge structure and write

$$
\begin{aligned}
\sigma_{\vec{\kappa}}(m^2,q^2,s) = \frac{\alpha\alpha_s}{m^2} \Big\{ & e_H^2 c^{(0)}_{\vec{\kappa},\mathrm{g}} + 4\pi\alpha_s \left[e_H^2 \left(c^{(1)}_{\vec{\kappa},\mathrm{g}} + \bar{c}^{(1),F}_{\vec{\kappa},\mathrm{g}} \ln(\mu_F^2/m^2) + \bar{c}^{(1),R}_{\vec{\kappa},\mathrm{g}} \ln(\mu_R^2/m^2) \right) \right] \\
& + 4\pi\alpha_s \left[e_H^2 c^{(1)}_{\vec{\kappa},q} + e_L^2 d^{(1)}_{\vec{\kappa},q} \right] \Big\}
\end{aligned} \tag{73}
$$

The LO coefficient functions $c^{(0)}_{\vec{\kappa},\mathrm{g}}$ can be computed from Eq. 40 by analytical means. The parity violating $B_{\mathrm{VA},\kappa_2,\mathrm{QED}}$ are anti-symmetric with respect to the t_1-integration so they do not contribute to the full inclusive cross sections and we find

$$
c^{(0)}_{\mathrm{VA},\kappa_2,\mathrm{g}} = 0 \quad \kappa_2 \in \{xF_3, g_4, g_L\} . \tag{74}
$$

This also holds for the NLO factorization scaling functions $\bar{c}^{(1),F}_{\mathrm{VA},\kappa_2,\mathrm{g}}$ and $\bar{c}^{(1),F}_{\mathrm{VA},\kappa_2,q}$ due to the Eqs. 70-72. Their simplified structure actually allows us to give analytical results for these coefficient functions, given in the Appendices A.3 and A.5 respectively.

From Eq. 71 we can also find immediately the renormalization scaling functions

$$\bar{c}^{(1),R}_{\vec{\kappa},\mathrm{g}} = \frac{\beta_0^{lf}}{16\pi^2} c^{(0)}_{\vec{\kappa},\mathrm{g}} \tag{75}$$

and for convenience we also define $\bar{c}^{(1)}_{\vec{\kappa},\mathrm{g}} = \bar{c}^{(1),F}_{\vec{\kappa},\mathrm{g}} + \bar{c}^{(1),R}_{\vec{\kappa},\mathrm{g}}$.

The Coulomb coefficient function $d^{(1)}_{\vec{\kappa},q}$ also have a simplified analytic structure, so we can give analytic results for these as well (see Appendix A.6). We find at this order

$$d^{(1)}_{\mathrm{VA},g_4,q} = d^{(1)}_{\mathrm{VA},g_L,q} = 0 = d^{(1)}_{\mathrm{AA},2xg_1,q} \tag{76}$$

and

$$d^{(1)}_{\mathrm{VV},F_2,q} = d^{(1)}_{\mathrm{AA},F_2,q}\,, \qquad d^{(1)}_{\mathrm{VV},F_L,q} = d^{(1)}_{\mathrm{AA},F_L,q}\,, \qquad d^{(1)}_{\mathrm{VV},2xg_1,q} = d^{(1)}_{\mathrm{VA},xF_3,q}\,. \tag{77}$$

So here the coefficient functions for the parity conserving unpolarized F_2 and F_L do show the V-A structure of the massless case, i.e. $d^{(1)}_{\mathrm{VV},F_2,q} - d^{(1)}_{\mathrm{AA},F_2,q} = 0$, as the exchanged boson couples to the massless quark q. This does not hold for the polarized $2xg_1$, instead the matrix element to the axial-axial couling does vanish, i.e. $A_{\mathrm{AA},2xg_1,2} = 0$.

6 Hadronic Results

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7 Summary

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A Partonic Results

A.1 $c^{(0)}_{\vec{\kappa},\mathrm{g}}$

In leading order, we find

$$c^{(0)}_{\mathrm{VV},F_2,\mathrm{g}} = -\frac{\pi\rho'^3}{4\rho^2{\rho_q}^2}\Big[2\beta\left(\rho^2+{\rho_q}^2+\rho\rho_q(6+\rho_q)\right) + \left(2{\rho_q}^2+2\rho{\rho_q}^2+\rho^2(2-(-4+\rho_q)\rho_q)\right)\ln(\chi)\Big] \tag{78}$$

$$c^{(0)}_{\mathrm{VV},F_L,\mathrm{g}} = -\frac{\pi\rho'^3}{\rho\rho_q}\left[2\beta+\rho\ln(\chi)\right] \tag{79}$$

$$c^{(0)}_{\mathrm{VV},2xg_1,\mathrm{g}} = \frac{\pi\rho'^2}{2\rho\rho_q}\left[\beta(\rho+3\rho_q)+(\rho+\rho_q)\ln(\chi)\right] \tag{80}$$

$$c^{(0)}_{\mathrm{AA},F_2,\mathrm{g}} = \frac{\pi\rho'^3}{4\rho^2{\rho_q}^2}\Big[2\beta\left(\rho^2+{\rho_q}^2+\rho\rho_q(6+\rho_q)\right) - \left(-6\rho{\rho_q}^2+2(-1+\rho_q){\rho_q}^2+\rho^2(-2+(-2+\rho_q)\rho_q)\right)\ln(\chi)\Big] \tag{81}$$

$$c^{(0)}_{\mathrm{AA},F_L,\mathrm{g}} = -\frac{\pi\rho'^3}{2\rho^2\rho_q}\Big[2\beta\rho(2+\rho_q)-\left(\rho^2(-1+\rho_q)-4\rho\rho_q+{\rho_q}^2\right)\ln(\chi)\Big] \tag{82}$$

$$c^{(0)}_{\mathrm{AA},2xg_1,\mathrm{g}} = c^{(0)}_{\mathrm{VV},2xg_1,\mathrm{g}} \tag{83}$$

$$c^{(0)}_{\mathrm{VA},xF_3,\mathrm{g}} = c^{(0)}_{\mathrm{VA},g_4,\mathrm{g}} = c^{(0)}_{\mathrm{VA},g_L,\mathrm{g}} = 0 \tag{84}$$

Near threshold we find

$$c^{(0),\mathrm{thr}}_{\mathrm{VV},F_2,\mathrm{g}} = \frac{\pi\beta\rho_q}{2(\rho_q-1)} \tag{85}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{VV},F_L,\mathrm{g}} = \frac{4\pi\beta^3\rho_q^2}{3(1-\rho_q)^3} \tag{86}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{VV},2xg_1,\mathrm{g}} = c^{(0),\mathrm{thr}}_{\mathrm{AA},2xg_1,\mathrm{g}} = c^{(0),\mathrm{thr}}_{\mathrm{VV},F_2,\mathrm{g}} \tag{87}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{AA},F_2,\mathrm{g}} = \frac{\pi\beta\rho_q^2}{1-\rho_q} \tag{88}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{AA},F_2,\mathrm{g}} = \frac{\pi\beta(1-2\rho_q)\rho_q}{2(\rho_q-1)} \tag{89}$$

A.2 $c^{(1)}_{\vec{\kappa},\mathrm{g}}$

Near threshold, we find

$$c^{(1),\mathrm{thr}}_{\vec{k},\mathrm{g}} = c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{1}{\pi^2} \Bigg[C_A \left(a^{(1,2)}_{\vec{k},\mathrm{g}} \ln^2(\beta) + a^{(1,1)}_{\vec{k},\mathrm{g}} \ln(\beta) - \frac{\pi^2}{16\beta} + a^{(1,0)}_{\vec{k},\mathrm{g},\mathrm{OK}} \right) \\ + 2C_F \left(\frac{\pi^2}{16\beta} + a^{(1,0)}_{\vec{k},\mathrm{g},\mathrm{QED}} \right) \Bigg] , \tag{90}$$

with

$$a^{(1,2)}_{\vec{k},\mathrm{g}} = 1 \tag{91}$$

$$a^{(1,1)}_{\mathrm{VV},F_2,\mathrm{g}} = -\frac{5}{2} + 3\ln(2) \tag{92}$$

$$a^{(1,1)}_{\mathrm{VV},F_L,\mathrm{g}} = a^{(1,1)}_{\mathrm{VV},F_2,\mathrm{g}} - \frac{2}{3} \tag{93}$$

$$a^{(1,1)}_{\mathrm{VV},2xg_1,\mathrm{g}} = a^{(1,1)}_{\mathrm{AA},F_2,\mathrm{g}} = a^{(1,1)}_{\mathrm{AA},F_L,\mathrm{g}} = a^{(1,1)}_{\mathrm{AA},2xg_1,\mathrm{g}} = a^{(1,1)}_{\mathrm{VV},F_2,\mathrm{g}} \tag{94}$$

A.3 $\bar{c}^{(1)}_{\vec{\kappa},\mathrm{g}}$

Near threshold, we find

$$\bar{c}^{(1),\mathrm{thr}}_{\vec{k},\mathrm{g}} = c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{1}{\pi^2} C_A \left(\bar{a}^{(1,1)}_{\vec{k},\mathrm{g}} \ln(\beta) + \bar{a}^{(1,0)}_{\vec{k},\mathrm{g}} \right) , \tag{95}$$

with

$$\bar{a}^{(1,1)}_{\vec{k},\mathrm{g}} = -\frac{1}{2} \tag{96}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_2,\mathrm{g}} = -\frac{1}{4} \ln\left(\frac{16\chi_q}{(1+\chi_q)^2} \right) + \frac{1}{2} \tag{97}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_L,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,\mathrm{g}} + \frac{1}{6} \tag{98}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},2xg_1,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{AA},F_2,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{AA},F_L,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{AA},2xg_1,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,\mathrm{g}} \tag{99}$$

A.4 $c^{(1)}_{\vec{\kappa},\mathrm{q}}$

Near threshold, we find

$$c^{(1),\mathrm{thr}}_{\vec{k},q} = c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{\beta^2 \rho_q}{\pi^2(\rho_q - 1)} \frac{K_{q\gamma}}{6K_{\mathrm{g}\gamma}} \left[a^{(1,1)}_{\vec{k},q} \ln(\beta) + a^{(1,0)}_{\vec{k},q} \right] , \tag{100}$$

with

$$a^{(1,1)}_{\mathrm{VV},F_2,q} = 1 \tag{101}$$
$$a^{(1,1)}_{\mathrm{VV},F_L,q} = a^{(1,1)}_{\mathrm{VV},F_2,q} - \frac{2}{3} \tag{102}$$
$$a^{(1,1)}_{\mathrm{VV},2xg_1,q} = a^{(1,1)}_{\mathrm{AA},F_2,q} = a^{(1,1)}_{\mathrm{AA},F_L,q} = a^{(1,1)}_{\mathrm{AA},2xg_1,q} = a^{(1,1)}_{\mathrm{VV},F_2,q} \tag{103}$$
$$a^{(1,0)}_{\mathrm{VV},F_2,q} = -\frac{13}{12} + \frac{3}{2}\ln(2) \tag{104}$$
$$a^{(1,0)}_{\mathrm{VV},F_L,q} = -\frac{77}{100} + \frac{9}{10}\ln(2) \tag{105}$$
$$a^{(1,0)}_{\mathrm{VV},2xg_1,q} = a^{(1,0)}_{\mathrm{VV},F_2,q} - \frac{1}{4} \tag{106}$$
$$a^{(1,0)}_{\mathrm{AA},F_2,q} = a^{(1,0)}_{\mathrm{AA},F_L,q} = a^{(1,0)}_{\mathrm{AA},2xg_1,q} = a^{(1,0)}_{\mathrm{VV},F_2,q} \tag{107}$$

A.5 $\bar{c}^{(1),F}_{\vec{\kappa},\mathbf{q}}$

Near threshold, we find

$$\bar{c}^{(1),F,\mathrm{thr}}_{\vec{k},q} = -c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{\beta^2\rho_q}{\pi^2(\rho_q-1)} \frac{K_{q\gamma}}{24K_{\mathrm{g}\gamma}} \cdot \bar{a}^{(1,0)}_{\vec{k},q} \tag{108}$$

with

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_2,q} = 1 \tag{109}$$
$$\bar{a}^{(1,0)}_{\mathrm{VV},F_L,q} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,q} - \frac{2}{3} \tag{110}$$
$$\bar{a}^{(1,0)}_{\mathrm{VV},2xg_1,q} = \bar{a}^{(1,0)}_{\mathrm{AA},F_2,q} = \bar{a}^{(1,0)}_{\mathrm{AA},F_L,q} = \bar{a}^{(1,0)}_{\mathrm{AA},2xg_1,q} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,q} \tag{111}$$

A.6 $d^{(1)}_{\vec{\kappa},\mathbf{q}}$

For $\chi' \to 0$ we find:

$$\begin{aligned} d^{(1)}_{\mathrm{VV},F_2,q} = &-\frac{\rho}{9\pi}\left(\mathrm{Li}_2(-\chi) - \frac{1}{4}\ln^2(\chi) + \frac{\pi^2}{12} + \ln(\chi)\ln(1+\chi)\right) \\ &+ \beta\frac{\rho(718+5\rho)}{2592\pi} + \frac{\rho(232+9\rho^2)}{1728\pi}\ln(\chi) + \mathcal{O}(\chi') \end{aligned} \tag{112}$$
$$d^{(1)}_{\mathrm{VV},F_L,q} = \left(\beta\frac{-38+23\rho}{54\pi} + \frac{-8+3\rho^2}{36\pi}\ln(\chi)\right)\chi' + \mathcal{O}(\chi'^2) \tag{113}$$
$$d^{(1)}_{\mathrm{VV},2xg_1,q} = d^{(1)}_{\mathrm{AA},F_2,q} = d^{(1)}_{\mathrm{VA},xF_3,q} \tag{114}$$
$$d^{(1)}_{\mathrm{AA},F_L,q} = d^{(1)}_{\mathrm{VV},F_L,q} \tag{115}$$

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List of Corrections