1 Feynman Rules

following [1]

To perform the calculation of Dirac traces in n dimensions use HEPMath[2] or TRACER[3].

2 Leading Order: $O(\alpha\alpha_s)$

diagramatic:

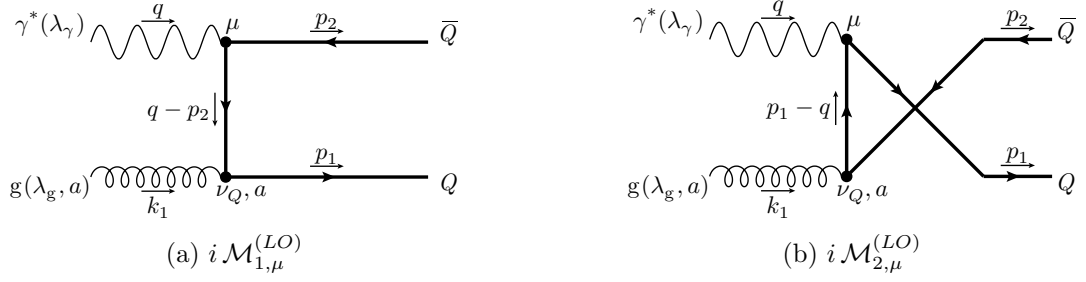(a) $i\,\mathcal{M}_{1,\mu}^{(LO)}$ (b) $i\,\mathcal{M}_{2,\mu}^{(LO)}$

Figure 1: LO contributions

formula:

$$i\,\mathcal{M}_{1,\mu}^{(LO)} = \bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\slashed{q}-\slashed{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_Q}^{(\lambda_{\mathrm{g}})}(k_1) \tag{1}$$

$$i\,\mathcal{M}_{2,\mu}^{(LO)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\slashed{p}_1-\slashed{q}+m)}{t_1}(igT_a\gamma^{\nu_Q})v(p_2)\varepsilon_{\nu_Q}^{(\lambda_{\mathrm{g}})}(k_1) \tag{2}$$

color space:

$$\left|\mathcal{M}_{1,\mu}^{(LO)}+\mathcal{M}_{2,\mu}^{(LO)}\right|^2 \sim \mathrm{tr}(T_aT_a) = N_cC_F \tag{3}$$

3 Next-to-leading Order: $O(\alpha\alpha_S^2)$

3.1 Light Quark Contributions

$$\gamma^*(q) + q(k_1) \to \overline{Q}(p_2) + Q(p_1) + q(k_2) \tag{4}$$

diagramatic:

1

(a) $i\,\mathcal{M}_{1,\mu}^{(NLO,q)}$

(b) $i\,\mathcal{M}_{2,\mu}^{(NLO,q)}$

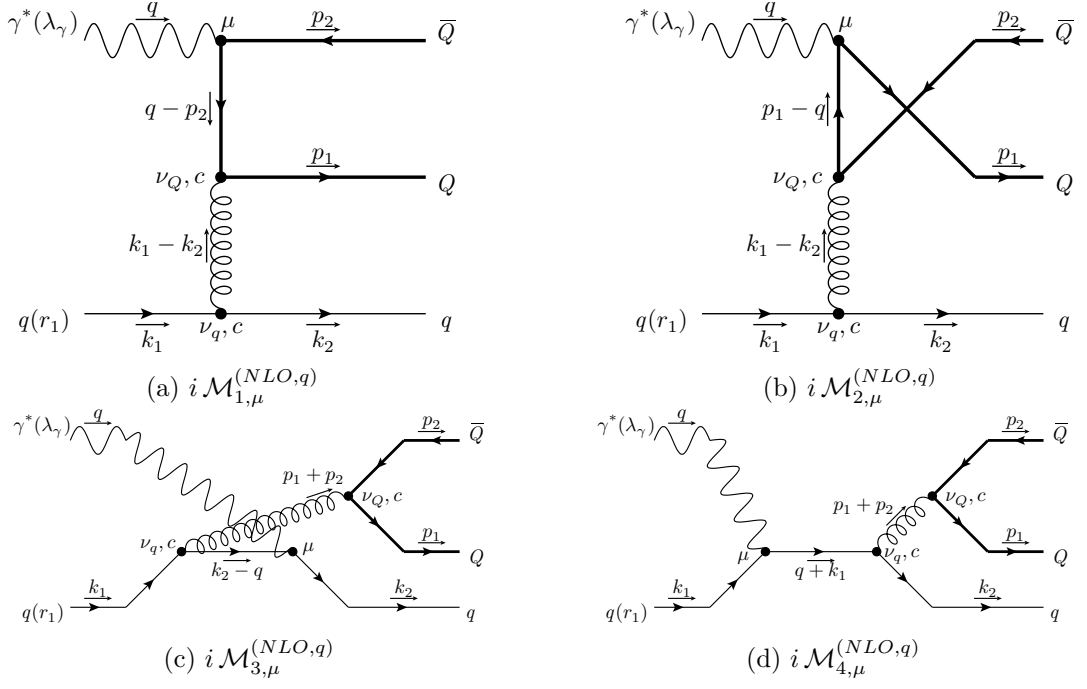(c) $i\,\mathcal{M}_{3,\mu}^{(NLO,q)}$

(d) $i\,\mathcal{M}_{4,\mu}^{(NLO,q)}$

Figure 2: NLO contributions by light quarks

formula:

$$i\,\mathcal{M}_{1,\mu}^{(NLO,q)} = \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\slashed{q}-\slashed{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v_Q(p_2)\cdot$$
$$\frac{-ig_{\nu_Q,\nu_q}}{t'}\cdot\bar{u}_q(k_2)(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) \tag{5}$$

$$i\,\mathcal{M}_{2,\mu}^{(NLO,q)} = \bar{u}_Q(p_1)(-iee_H\gamma_\mu)\frac{i(\slashed{p}_1-\slashed{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v_Q(p_2)\cdot$$
$$\frac{-ig_{\nu_Q,\nu_q}}{t'}\cdot\bar{u}_q(k_2)(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) \tag{6}$$

$$i\,\mathcal{M}_{3,\mu}^{(NLO,q)} = \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})v_Q(p_2)\cdot\frac{-ig_{\nu_Q,\nu_q}}{s_5}\cdot$$
$$\bar{u}_q(k_2)(-iee_L\gamma_\mu)\frac{i(\slashed{k}_2-\slashed{q})}{u'}(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) \tag{7}$$

$$i\,\mathcal{M}_{4,\mu}^{(NLO,q)} = \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})v_Q(p_2)\cdot\frac{-ig_{\nu_Q,\nu_q}}{s_5}\cdot$$
$$\bar{u}_q(k_2)(igT_c\gamma^{\nu_q})\frac{i(\slashed{k}_1+\slashed{q})}{s}(-iee_L\gamma_\mu)u_q^{(r_1)}(k_1) \tag{8}$$

color space:

$$\left|\mathcal{M}_{1,\mu}^{(NLO,q)}+\mathcal{M}_{2,\mu}^{(NLO,q)}+\mathcal{M}_{3,\mu}^{(NLO,q)}+\mathcal{M}_{4,\mu}^{(NLO,q)}\right|^2 \sim \mathrm{tr}(T_cT_d)\,\mathrm{tr}(T_cT_d) = \frac{1}{2}N_cC_F \tag{9}$$

2

3.2 Gluon Bremsstrahlung

$$\gamma^*(q) + g(k_1) \rightarrow \overline{Q}(p_2) + Q(p_1) + g(k_2) \tag{10}$$

diagramatic:

(a) $i\,\mathcal{M}_{1,\mu}^{(NLO,g)}$

(b) $i\,\mathcal{M}_{2,\mu}^{(NLO,g)}$

(c) $i\,\mathcal{M}_{3,\mu}^{(NLO,g)}$

(d) $i\,\mathcal{M}_{4,\mu}^{(NLO,g)}$

(e) $i\,\mathcal{M}_{5,\mu}^{(NLO,g)}$

(f) $i\,\mathcal{M}_{6,\mu}^{(NLO,g)}$

(g) $i\,\mathcal{M}_{7,\mu}^{(NLO,g)}$

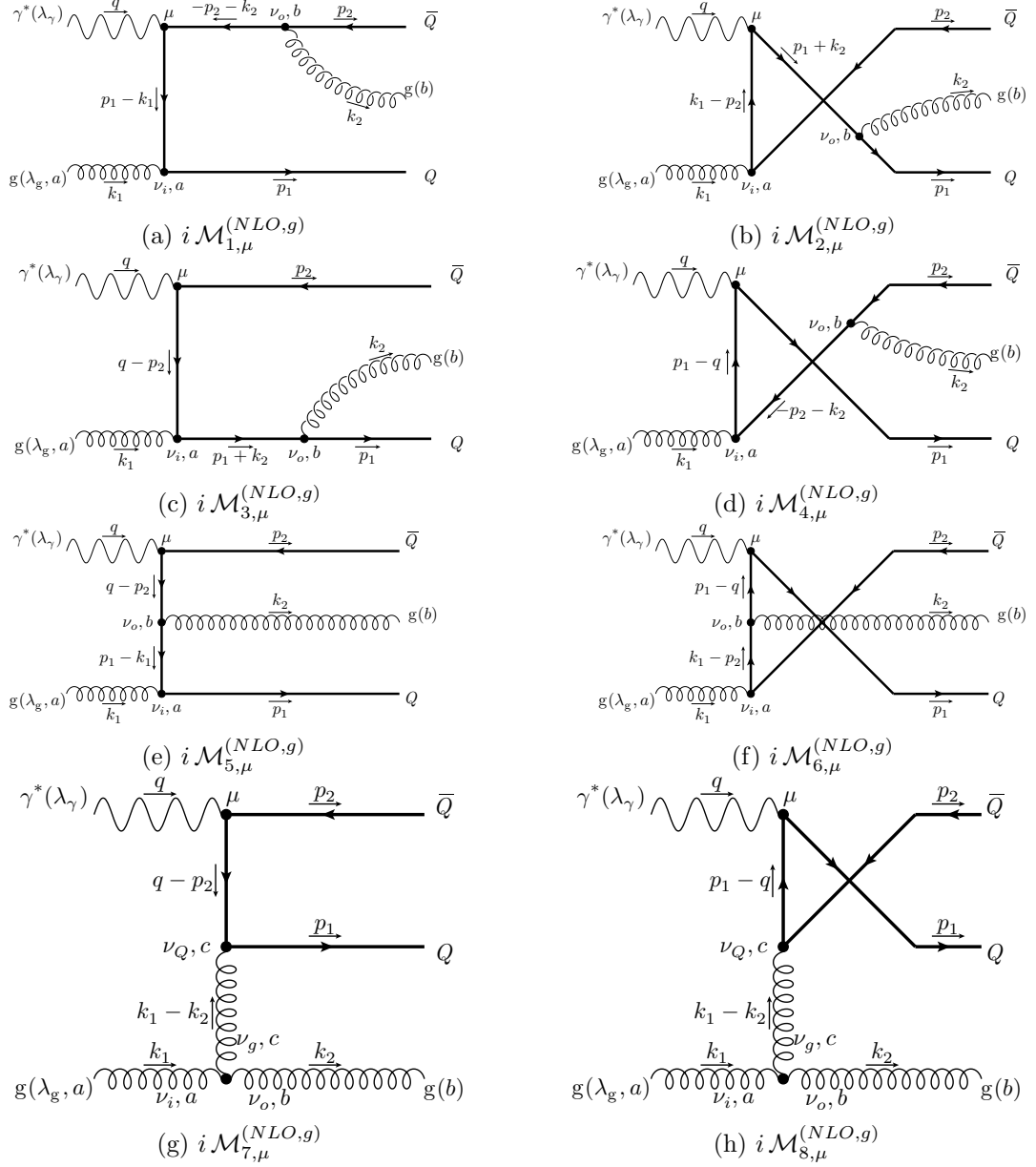(h) $i\,\mathcal{M}_{8,\mu}^{(NLO,g)}$

Figure 3: NLO contributions by gluon bremsstrahlung

3

formula:

$$i\,\mathcal{M}_{1,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_a\gamma^{\nu_i})\frac{i(\not{p}_1 - \not{k}_1 + m)}{u_6}(-iee_H\gamma_\mu)\cdot$$

$$\frac{i(-\not{p}_2 - \not{k}_2 + m)}{s_3}(igT_b\gamma^{\nu_o})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \tag{11}$$

$$i\,\mathcal{M}_{2,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_b\gamma^{\nu_o})\frac{i(\not{p}_1 + \not{k}_2 + m)}{s_4}(-iee_H\gamma_\mu)\cdot$$

$$\frac{i(\not{k}_1 - \not{p}_2 + m)}{t_1}(igT_a\gamma^{\nu_i})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \tag{12}$$

$$i\,\mathcal{M}_{3,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_b\gamma^{\nu_o})\frac{i(\not{p}_1 + \not{k}_2 + m)}{s_4}(igT_a\gamma^{\nu_i})\cdot$$

$$\frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \tag{13}$$

$$i\,\mathcal{M}_{4,\mu}^{(NLO,g)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{u_7}(igT_a\gamma^{\nu_i})\cdot$$

$$\frac{i(-\not{p}_2 - \not{k}_2 + m)}{s_3}(igT_b\gamma^{\nu_o})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \tag{14}$$

$$i\,\mathcal{M}_{5,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_a\gamma^{\nu_i})\frac{i(\not{p}_1 - \not{k}_1 + m)}{u_6}(igT_b\gamma^{\nu_o})\cdot$$

$$\frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \tag{15}$$

$$i\,\mathcal{M}_{6,\mu}^{(NLO,g)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{u_7}(igT_b\gamma^{\nu_o})\cdot$$

$$\frac{i(\not{k}_1 - \not{p}_2 + m)}{t_1}(igT_a\gamma^{\nu_i})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \tag{16}$$

$$i\,\mathcal{M}_{7,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_g}}{t'}\cdot\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2)\cdot$$

$$\left(gf^{acb}\left(g^{\nu_o,\nu_i}(k_1+k_2)^{\nu_g} + g^{\nu_i,\nu_g}(k_2-2k_1)^{\nu_o} + g^{\nu_g,\nu_o}(k_1-2k_2)^{\nu_i}\right)\right) \tag{17}$$

$$i\,\mathcal{M}_{8,\mu}^{(NLO,g)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_g}}{t'}\cdot\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2)\cdot$$

$$\left(gf^{acb}\left(g^{\nu_o,\nu_i}(k_1+k_2)^{\nu_g} + g^{\nu_i,\nu_g}(k_2-2k_1)^{\nu_o} + g^{\nu_g,\nu_o}(k_1-2k_2)^{\nu_i}\right)\right) \tag{18}$$

color space:

$$\sum_{j=1}^{6} \left| \mathcal{M}_{j,\mu}^{(NLO,g)} \right|^2 + \mathcal{M}_{1,\mu}^{(NLO,g)} \left(\mathcal{M}_{4,\mu'}^{(NLO,g)} + \mathcal{M}_{5,\mu'}^{(NLO,g)} \right)^* + \mathcal{M}_{3,\mu}^{(NLO,g)} \left(\mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^* +$$

$$\mathcal{M}_{2,\mu}^{(NLO,g)} \left(\mathcal{M}_{3,\mu'}^{(NLO,g)} + \mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^* + \mathcal{M}_{4,\mu}^{(NLO,g)} \left(\mathcal{M}_{5,\mu'}^{(NLO,g)} \right)^*$$

$$\sim \mathrm{tr}(T_a T_a T_b T_b) = N_C C_F^2 \tag{19}$$

$$\mathcal{M}_{1,\mu}^{(NLO,g)} \left(\mathcal{M}_{2,\mu'}^{(NLO,g)} + \mathcal{M}_{3,\mu'}^{(NLO,g)} + \mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^* +$$

$$\left(\mathcal{M}_{2,\mu}^{(NLO,g)} + \mathcal{M}_{3,\mu}^{(NLO,g)} \right) \left(\mathcal{M}_{4,\mu'}^{(NLO,g)} + \mathcal{M}_{5,\mu'}^{(NLO,g)} \right)^* +$$

$$\left(\mathcal{M}_{4,\mu}^{(NLO,g)} + \mathcal{M}_{5,\mu}^{(NLO,g)} \right) \left(\mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^*$$

$$\sim \mathrm{tr}(T_a T_b T_a T_b) = N_C C_F \left(C_F - \frac{C_A}{2} \right) \tag{20}$$

$$\left(\mathcal{M}_{2,\mu}^{(NLO,g)} + \mathcal{M}_{3,\mu}^{(NLO,g)} + \mathcal{M}_{6,\mu}^{(NLO,g)} \right) \left(\mathcal{M}_{7,\mu'}^{(NLO,g)} + \mathcal{M}_{8,\mu'}^{(NLO,g)} \right)^*$$

$$\sim -i f_{bda} \, \mathrm{tr}(T_a T_b T_d) = \frac{1}{2} N_C C_F C_A \tag{21}$$

$$\left(\mathcal{M}_{1,\mu}^{(NLO,g)} + \mathcal{M}_{4,\mu}^{(NLO,g)} + \mathcal{M}_{5,\mu}^{(NLO,g)} \right) \left(\mathcal{M}_{7,\mu'}^{(NLO,g)} + \mathcal{M}_{8,\mu'}^{(NLO,g)} \right)^*$$

$$\sim -i f_{bda} \, \mathrm{tr}(T_b T_a T_d) = i f_{bda} \, \mathrm{tr}(T_a T_b T_d) = -\frac{1}{2} N_C C_F C_A \tag{22}$$

$$\left| \mathcal{M}_{7,\mu}^{(NLO,g)} + \mathcal{M}_{8,\mu}^{(NLO,g)} \right|^2$$

$$\sim f_{acb} f_{adb} \, \mathrm{tr}(T_c T_d) = N_C C_F C_A \tag{23}$$

To get the polarisation sums right, one has to subtract the contributions of the Faddeev-Popov ghosts[4, 5]:

diagramatic:

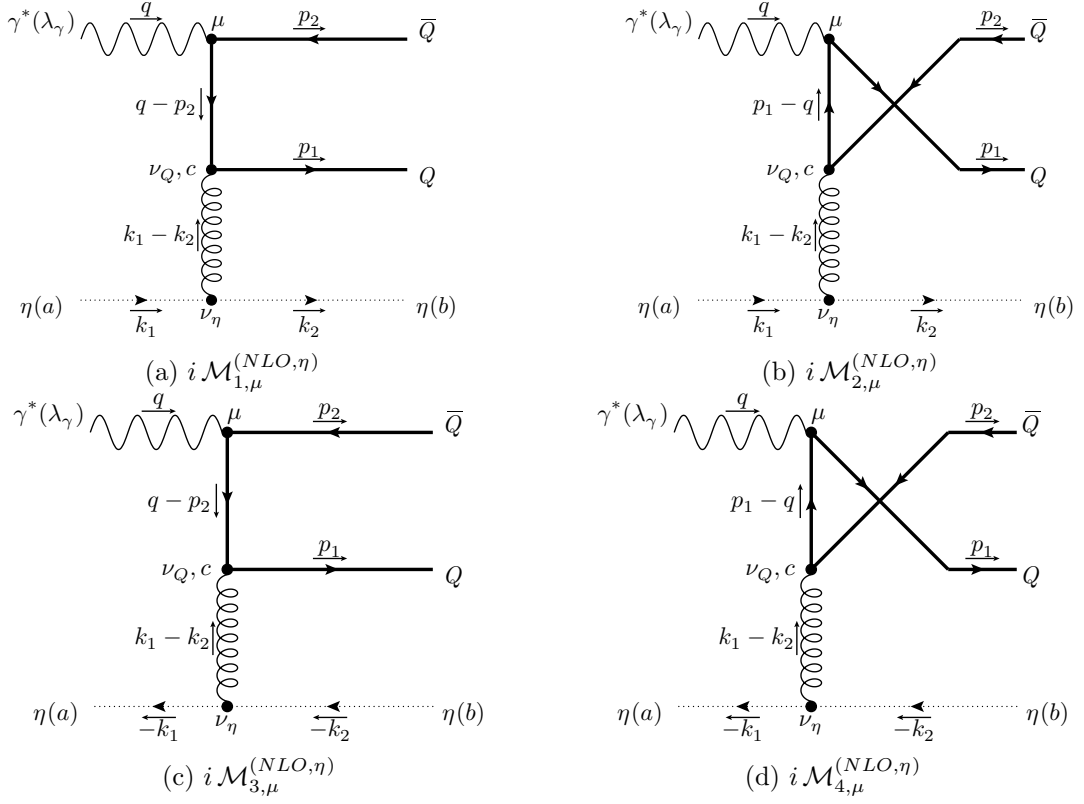(a) $i\,\mathcal{M}^{(NLO,\eta)}_{1,\mu}$

(b) $i\,\mathcal{M}^{(NLO,\eta)}_{2,\mu}$

(c) $i\,\mathcal{M}^{(NLO,\eta)}_{3,\mu}$

(d) $i\,\mathcal{M}^{(NLO,\eta)}_{4,\mu}$

Figure 4: NLO contributions by ghosts

formula:

$$i\,\mathcal{M}^{(NLO,\eta)}_{1,\mu} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\slashed{q}-\slashed{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_\eta}}{t'} \cdot (gf^{acb}k_2^{\nu_\eta}) \quad (24)$$

$$i\,\mathcal{M}^{(NLO,\eta)}_{2,\mu} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\slashed{p}_1-\slashed{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_\eta}}{t'} \cdot (gf^{acb}k_2^{\nu_\eta}) \quad (25)$$

$$i\,\mathcal{M}^{(NLO,\eta)}_{3,\mu} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\slashed{q}-\slashed{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_\eta}}{t'} \cdot (gf^{cab}(-k_1)^{\nu_\eta})$$
$$(26)$$

$$i\,\mathcal{M}^{(NLO,\eta)}_{4,\mu} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\slashed{p}_1-\slashed{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_\eta}}{t'} \cdot (gf^{cab}(-k_1)^{\nu_\eta})$$
$$(27)$$

color space:

$$\left|\mathcal{M}_{1,\mu}^{(NLO,\eta)} + \mathcal{M}_{2,\mu}^{(NLO,\eta)}\right|^2 \sim f_{acb}f_{adb}\,\mathrm{tr}(T_cT_d) = N_CC_FC_A \tag{28}$$

$$\left|\mathcal{M}_{3,\mu}^{(NLO,\eta)} + \mathcal{M}_{4,\mu}^{(NLO,\eta)}\right|^2 \sim f_{cab}f_{dab}\,\mathrm{tr}(T_cT_d) = f_{acb}f_{adb}\,\mathrm{tr}(T_cT_d) = N_CC_FC_A \tag{29}$$

3.3 Virtual Contributions

$$\gamma^*(q) + \mathrm{g}(k_1) \rightarrow \overline{Q}(p_2) + Q(p_1) \tag{30}$$
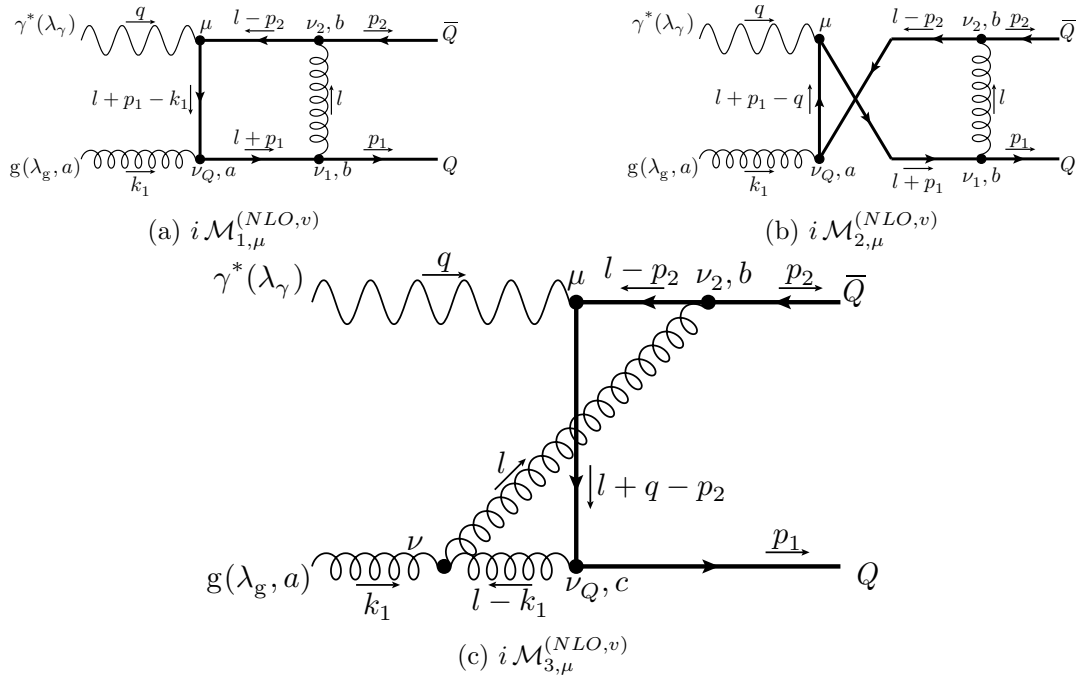
3.3.1 Loops

Figure 5: NLO contributions by one loop

$$i\,\mathcal{M}_{1,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n}\, \bar{u}(p_1)(igT_b\gamma^{\nu_1}) \frac{i(\slashed{l}+\slashed{p}_1+m)}{(l+p_1)^2-m^2}(igT_a\gamma^{\nu_Q}) \frac{i(\slashed{l}+\slashed{p}_1-\slashed{k}_1+m)}{(l+p_1-k_1)^2-m^2}\cdot$$

$$(-iee_H\gamma_\mu)\frac{i(\slashed{l}-\slashed{p}_2+m)}{(l-p_2)^2-m^2}(igT_b\gamma^{\nu_2})\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \qquad (31)$$

$$i\,\mathcal{M}_{2,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n}\, \bar{u}(p_1)(igT_b\gamma^{\nu_1}) \frac{i(\slashed{l}+\slashed{p}_1+m)}{(l+p_1)^2-m^2}(igT_a\gamma^{\nu_Q}) \frac{i(\slashed{l}+\slashed{p}_1-\slashed{q}+m)}{(l+p_1-q)^2-m^2}\cdot$$

$$(-iee_H\gamma_\mu)\frac{i(\slashed{l}-\slashed{p}_2+m)}{(l-p_2)^2-m^2}(igT_b\gamma^{\nu_2})\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \qquad (32)$$

$$i\,\mathcal{M}_{3,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n}\, \bar{u}(p_1)(igT_c\gamma^{\nu_Q}) \frac{i(\slashed{l}+\slashed{q}_1-\slashed{p}_2+m)}{(l+q-p_2)^2-m^2}(-iee_H\gamma_\mu)\frac{i(\slashed{l}-\slashed{p}_2+m)}{(l-p_2)^2-m^2}\cdot$$

$$(igT_b\gamma^{\nu_2})\frac{(-i)^2}{l^2(l-k_1)^2}v(p_2)\varepsilon^{\nu,(\lambda_g)}(k_1)\cdot$$

$$\left(gf_{abc}\left(g_{\nu_2\nu_Q}(k_1-2l)_\nu + g_{\nu_Q\nu}(l-2k_1)_{\nu_2} + g_{\nu\nu_2}(k_1+l)_{\nu_Q}\right)\right) \qquad (33)$$

(a) $i \mathcal{M}_{5,\mu}^{(NLO,v)}$

(b) $i \mathcal{M}_{6,\mu}^{(NLO,v)}$

(c) $i \mathcal{M}_{7,\mu}^{(NLO,v)}$
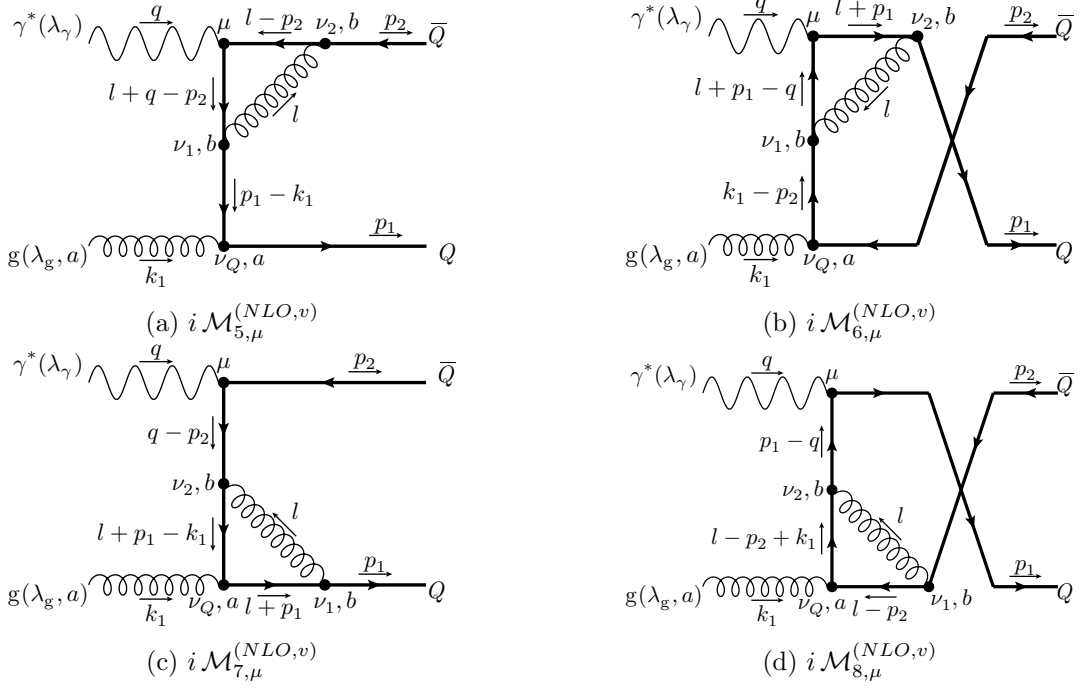
(d) $i \mathcal{M}_{8,\mu}^{(NLO,v)}$

Figure 6: NLO contributions by one loop (cont'ed)

$$
i \mathcal{M}_{5,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \, \bar{u}(p_1)(ig T_a \gamma^{\nu_Q}) \frac{i(\slashed{p}_1 - \slashed{k}_1 + m)}{u_1} (ig T_b \gamma^{\nu_1}) \frac{i(\slashed{l} + \slashed{q} - \slashed{p}_2 + m)}{(l+q-p_2)^2 - m^2} \cdot
$$
$$
(-iee_H \gamma_\mu) \frac{i(\slashed{l} - \slashed{p}_2 + m)}{(l-p_2)^2 - m^2} (ig T_b \gamma^{\nu_2}) \frac{-ig_{\nu_1,\nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \tag{34}
$$

$$
i \mathcal{M}_{6,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \, \bar{u}(p_1)(ig T_b \gamma^{\nu_2}) \frac{i(\slashed{l} + \slashed{p}_1 + m)}{(l+p_1)^2 - m^2} (-iee_H \gamma_\mu) \frac{i(\slashed{l} + \slashed{p}_1 - \slashed{q} + m)}{(l+p_1-q)^2 - m^2} \cdot
$$
$$
(ig T_b \gamma^{\nu_1}) \frac{i(\slashed{k}_1 - \slashed{p}_2 + m)}{t_1} (ig T_a \gamma^{\nu_Q}) \frac{-ig_{\nu_1,\nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \tag{35}
$$

$$
i \mathcal{M}_{7,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \, \bar{u}(p_1)(ig T_b \gamma^{\nu_1}) \frac{i(\slashed{l} + \slashed{p}_1 + m)}{(l+p_1)^2 - m^2} (ig T_a \gamma^{\nu_Q}) \frac{i(\slashed{l} + \slashed{p}_1 - \slashed{k}_1 + m)}{(l+p_1-k_1)^2 - m^2} \cdot
$$
$$
(ig T_b \gamma^{\nu_2}) \frac{i(\slashed{q} - \slashed{p}_2 + m)}{u_1} (-iee_H \gamma_\mu) \frac{-ig_{\nu_1,\nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \tag{36}
$$

$$
i \mathcal{M}_{8,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \, \bar{u}(p_1)(-iee_H \gamma_\mu) \frac{i(\slashed{p}_1 - \slashed{q} + m)}{t_1} (ig T_b \gamma^{\nu_2}) \frac{i(\slashed{l} - \slashed{p}_2 + \slashed{k}_1 + m)}{(l-p_2+k_1)^2 - m^2} \cdot
$$
$$
(ig T_a \gamma^{\nu_Q}) \frac{i(\slashed{l} - \slashed{p}_2 + m)}{(l-p_2)^2 - m^2} (ig T_b \gamma^{\nu_1}) \frac{-ig_{\nu_1,\nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \tag{37}
$$

(a) $i\,\mathcal{M}_{9,\mu}^{(NLO,v)}$

(b) $i\,\mathcal{M}_{10,\mu}^{(NLO,v)}$

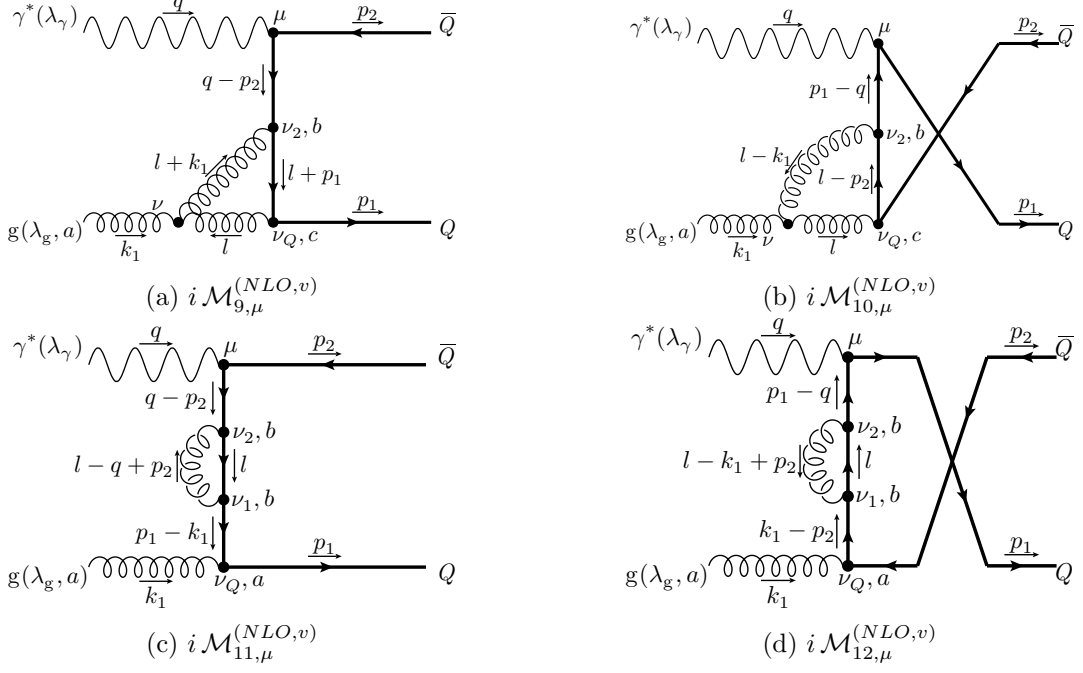(c) $i\,\mathcal{M}_{11,\mu}^{(NLO,v)}$

(d) $i\,\mathcal{M}_{12,\mu}^{(NLO,v)}$

Figure 7: NLO contributions by one loop (cont'ed)

$$
i\,\mathcal{M}_{9,\mu}^{(NLO,v)} = \mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\slashed{l}+\slashed{p}_1+m)}{(l+p_1)^2-m^2}(igT_b\gamma^{\nu_2})\frac{i(\slashed{q}-\slashed{p}_2+m)}{u_1}\cdot
$$

$$
(-iee_H\gamma_\mu)\frac{(-i)^2}{l^2(l+k_1)^2}v(p_2)\varepsilon^{\nu,(\lambda_g)}(k_1)\cdot
$$

$$
\Big(gf_{abc}\Big(g_{\nu\nu_2}(2k_1+l)_{\nu_Q}+g_{\nu_2\nu_Q}(-2l-k_1)_\nu+g_{\nu_Q\nu}(l-k_1)_{\nu_2}\Big)\Big) \tag{38}
$$

$$
i\,\mathcal{M}_{10,\mu}^{(NLO,v)} = \mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\slashed{p}_1-\slashed{q}+m)}{t_1}(igT_b\gamma^{\nu_2})\frac{i(\slashed{l}-\slashed{p}_2+m)}{(l-p_2)^2-m^2}\cdot
$$

$$
(igT_c\gamma^{\nu_Q})\frac{(-i)^2}{l^2(l-k_1)^2}v(p_2)\varepsilon^{\nu,(\lambda_g)}(k_1)\cdot
$$

$$
\Big(gf_{abc}\Big(g_{\nu\nu_2}(2k_1-l)_{\nu_Q}+g_{\nu_2\nu_Q}(2l-k_1)_\nu+g_{\nu_Q\nu}(-l-k_1)_{\nu_2}\Big)\Big) \tag{39}
$$

$$
i\,\mathcal{M}_{11,\mu}^{(NLO,v)} = \mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\slashed{p}_1-\slashed{k}_1+m)}{u_1}(igT_b\gamma^{\nu_1})\frac{i(\slashed{l}+m)}{l^2-m^2}\cdot
$$

$$
(igT_b\gamma^{\nu_2})\frac{i(\slashed{q}-\slashed{p}_2+m)}{u_1}(-iee_H\gamma_\mu)\frac{-ig_{\nu_1,\nu_2}}{(l-q+p_2)^2}v(p_2)\varepsilon^{(\lambda_g)}_{\nu_Q}(k_1) \tag{40}
$$

$$
i\,\mathcal{M}_{12,\mu}^{(NLO,v)} = \mu^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\slashed{p}_1-\slashed{q}+m)}{t_1}(igT_b\gamma^{\nu_2})\frac{i(\slashed{l}+m)}{l^2-m^2}\cdot
$$

$$
(igT_b\gamma^{\nu_1})\frac{i(\slashed{k}_1-\slashed{p}_2+m)}{t_1}(igT_a\gamma^{\nu_Q})\frac{-ig_{\nu_1,\nu_2}}{(l-k_1+p_2)^2}v(p_2)\varepsilon^{(\lambda_g)}_{\nu_Q}(k_1) \tag{41}
$$

Color space:

$$\left(\mathcal{M}_{1,\mu}^{(NLO,v)} + \mathcal{M}_{2,\mu}^{(NLO,v)} + \mathcal{M}_{7,\mu}^{(NLO,v)} + \mathcal{M}_{8,\mu}^{(NLO,v)}\right)$$
$$\cdot \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)}\right)^* \sim -i\operatorname{tr}(T_a T_b T_a T_b) = -iN_C C_F\left(C_F - \frac{C_A}{2}\right) \qquad (42)$$

$$\left(\mathcal{M}_{3,\mu}^{(NLO,v)} + \mathcal{M}_{9,\mu}^{(NLO,v)} + \mathcal{M}_{10,\mu}^{(NLO,v)}\right)$$
$$\cdot \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)}\right)^* \sim f_{abc}\operatorname{tr}(T_c T_b T_a) = -\frac{i}{2}N_C C_F C_A \qquad (43)$$

$$\left(\mathcal{M}_{5,\mu}^{(NLO,v)} + \mathcal{M}_{6,\mu}^{(NLO,v)} + \mathcal{M}_{11,\mu}^{(NLO,v)} + \mathcal{M}_{12,\mu}^{(NLO,v)}\right)$$
$$\cdot \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)}\right)^* \sim -i\operatorname{tr}(T_a T_a T_b T_b) = -iN_C C_F^2 \qquad (44)$$

3.3.2 Counter terms

(a) $i\,\mathcal{M}_{1,\mu}^{(NLO,c)}$

(b) $i\,\mathcal{M}_{2,\mu}^{(NLO,c)}$

(c) $i\,\mathcal{M}_{3,\mu}^{(NLO,c)}$

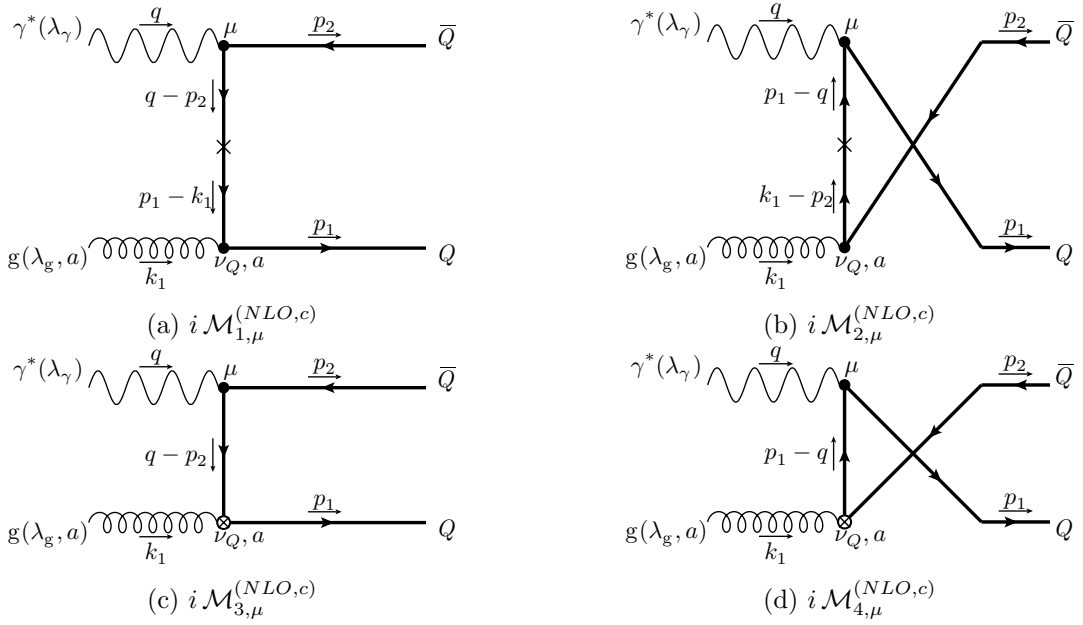(d) $i\,\mathcal{M}_{4,\mu}^{(NLO,c)}$

Figure 8: NLO contributions by counter terms

$$-i\,\mathcal{M}_{1,\mu}^{(NLO,c)} = \bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not{p}_1 - \not{k}_1 + m)}{u_1}\Big(i((Z_2-1)(\not{q}-\not{p}_2-m)-(Z_m-1)m)\Big)$$

$$\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \tag{45}$$

$$-i\,\mathcal{M}_{2,\mu}^{(NLO,c)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{t_1}\Big(i((Z_2-1)(\not{p}_1-\not{q}-m)-(Z_m-1)m)\Big)$$

$$\frac{i(\not{k}_1-\not{p}_2+m)}{t_1}(igT_a\gamma^{\nu_Q})v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \tag{46}$$

$$-i\,\mathcal{M}_{3,\mu}^{(NLO,c)} = \bar{u}(p_1)(-i(Z_{1f}-1)gT_a\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \tag{47}$$

$$-i\,\mathcal{M}_{4,\mu}^{(NLO,c)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{t_1}(-i(Z_{1f}-1)gT_a\gamma^{\nu_Q})v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \tag{48}$$

Color space:

$$\left(\mathcal{M}_{1,\mu}^{(NLO,c)}+\mathcal{M}_{2,\mu}^{(NLO,c)}\right)\left(\mathcal{M}_{1,\mu'}^{(LO)}+\mathcal{M}_{2,\mu'}^{(LO)}\right)^* \sim \text{tr}(T_aT_a)(Z_2-1) = C_F^2 \tag{49}$$

$$\left(\mathcal{M}_{3,\mu}^{(NLO,c)}+\mathcal{M}_{4,\mu}^{(NLO,c)}\right)\left(\mathcal{M}_{1,\mu'}^{(LO)}+\mathcal{M}_{2,\mu'}^{(LO)}\right)^* \sim \text{tr}(T_aT_a)(Z_{1f}-1) = C_F(C_F+C_A) \tag{50}$$

3.3.3 Quark Self Energy

To compute self energies, we follow [6]. It is

$$\{\gamma_\mu,\gamma_\nu\} = 2g_{\mu\nu} \tag{51}$$

$$\gamma_\mu\gamma^\mu = g_\mu^\mu = n \tag{52}$$

$$\gamma_\mu\gamma_\nu\gamma^\mu = (2-n)\gamma_\nu \tag{53}$$
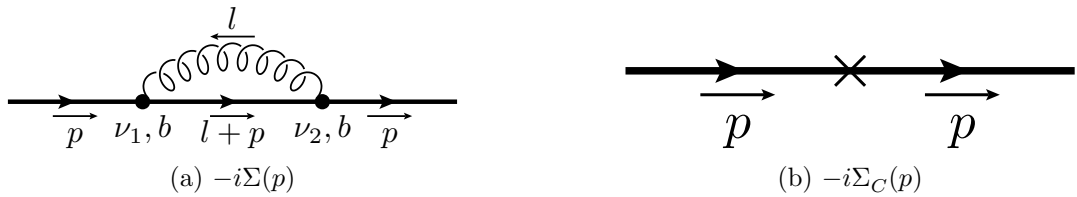

(a) $-i\Sigma(p)$ (b) $-i\Sigma_C(p)$

Figure 9: NLO contributions by quark self energy

$$-i\Sigma(p) = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} (igT_b\gamma_{\nu_1}) \frac{i(\slashed{l} + \slashed{p} + m)}{(l+p)^2 - m^2} (igT_b\gamma_{\nu_2}) \frac{-ig^{\nu_1,\nu_2}}{l^2} \tag{54}$$

$$= -\mu_R^{4-n} g^2 C_F \int \frac{d^n l}{(2\pi)^n} \frac{2m + (2-n)\slashed{p} + (2-n)\slashed{l}}{l^2((l+p)^2 - m^2)} \tag{55}$$

$$= -g^2 C_F \left(\left(2m + (2-n)\slashed{p}\right) B_0(p^2, 0, m^2) + (2-n) B_1(p^2, 0, m^2) \right) \tag{56}$$

$$= -g^2 C_F \left(B_0(p^2, 0, m^2) \left(n \cdot m + (2-n)\slashed{p}\frac{p^2 + m^2}{2p^2} \right) - (2-n)\slashed{p}\frac{1}{2p^2} A_0(m^2) \right) \tag{57}$$

Using [6] we find

$$C_\epsilon = \frac{1}{16\pi^2} \exp\left((\gamma_E - \log(4\pi)) \frac{\epsilon}{2} \right) \left(m^2/\mu^2 \right)^{\epsilon/2} \tag{58}$$

$$A_0(m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 1 \right) \tag{59}$$

$$B_0(p^2, 0, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 + \frac{m^2 - p^2}{p^2} \ln\left(\frac{m^2 - p^2}{m^2} \right) \right) \tag{60}$$

$$\Rightarrow -i\Sigma(p) = -ig^2 C_F C_\epsilon \left[\frac{2\slashed{p} - 8m}{\epsilon} + 2m \left(3 - 2\left(1 - \frac{m^2}{p^2}\right) \ln\left(1 - \frac{p^2}{m^2}\right) \right) \right.$$
$$\left. -\slashed{p} \left(1 + \frac{m^2}{p^2}\right) \left(1 - \left(1 - \frac{m^2}{p^2}\right) \ln\left(1 - \frac{p^2}{m^2}\right) \right) \right] \tag{61}$$

$$\overset{!}{=} -i(Am + B(\slashed{p} - m)) \tag{62}$$

$$\Rightarrow A = \frac{1}{m} \left. \Sigma(p) \right|_{\slashed{p}=m} \tag{63}$$

$$= -g^2 C_F C_\epsilon \left(\frac{6}{\epsilon} - 5 + \frac{m^2}{p^2} + \left(3 - 4\frac{m^2}{p^2} + \frac{m^4}{p^4} \right) \ln\left(1 - \frac{p^2}{m^2}\right) \right) \tag{64}$$

$$\Rightarrow B = \frac{1}{m} \left. \frac{d\Sigma(p)}{d\slashed{p}} \right|_{\slashed{p}=m} \tag{65}$$

$$= \frac{g^2}{16\pi^2} C_F \left(\frac{2}{\hat{\epsilon}_m} - 1 - \frac{m^2}{p^2} + \left(1 - \frac{m^4}{p^4} \right) \ln\left(1 - \frac{p^2}{m^2}\right) \right) \tag{66}$$

Counterterm:

$$-i\Sigma_C(p) = i((Z_2 - 1)\slashed{p} - (Z_2 Z_m - 1)m) \tag{67}$$

$$= i((Z_2 - 1)(\slashed{p} - m) - (Z_m - 1)m) + O(\alpha_S^2) \tag{68}$$

Use on-shell renormalization:

$$0 \overset{!}{=} \left(-i\Sigma(p) - i\Sigma_C(p) \right)\big|_{\slashed{p}=m} \tag{69}$$

$$= i \left(((Z_m - 1) + A)m + (B - (Z_2 - 1))(\slashed{p} - m) \right)\big|_{\slashed{p}=m} \tag{70}$$

$$\Rightarrow (Z_m - 1) = -A\big|_{\slashed{p}=m} \tag{71}$$

$$= \frac{g^2}{16\pi^2} C_F \left(\frac{6}{\hat{\epsilon}_m} - 4 \right) \tag{72}$$

such that $m \to Z_m m$.

We also choose

$$Z_2 - 1 = \frac{g^2}{16\pi^2} C_F \frac{2}{\hat{\epsilon}_m} \tag{73}$$

$$Z_{1f} = Z_2 + \frac{g^2}{16\pi^2} C_A \frac{2}{\hat{\epsilon}} \tag{74}$$

3.3.4 Gluon Self Energy (Off shell)

Again, we follow [6].

(a) $-i\Pi_{\mu\nu}^{ab,(1)}(k)$

(b) $-i\Pi_{\mu\nu}^{ab,(2)}(k)$

(c) $-i\Pi_{\mu\nu}^{ab,(3)}(k)$

(d) $-i\Pi_{\mu\nu}^{ab,(4)}(k)$
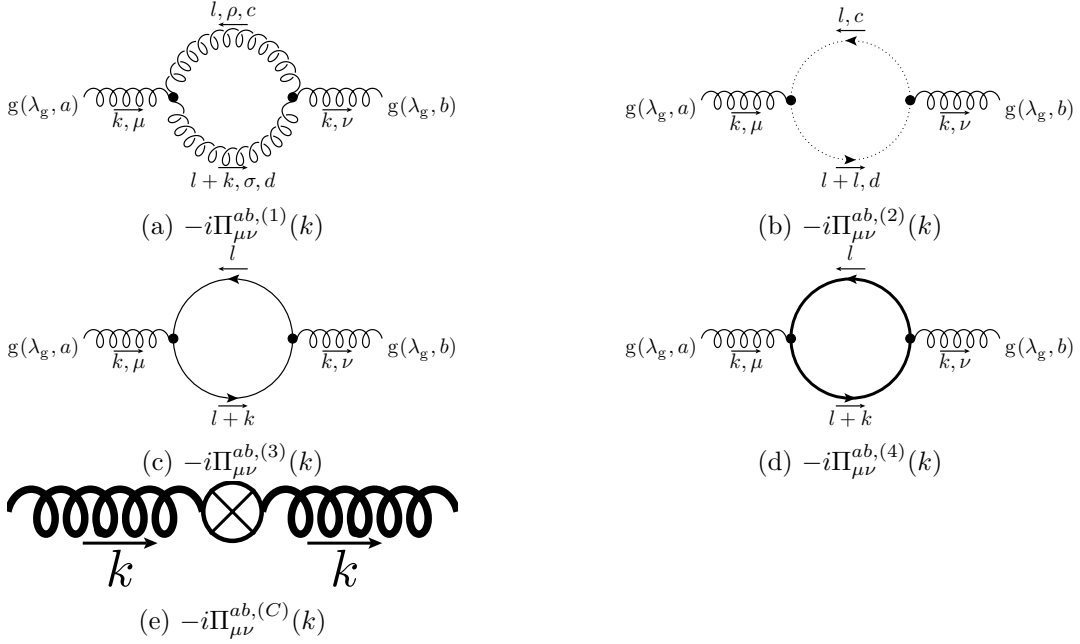
(e) $-i\Pi_{\mu\nu}^{ab,(C)}(k)$

Figure 10: NLO contributions by quark self energy

$$-i\Pi_{\mu\nu}^{ab,(1)}(k) = \frac{1}{2!}\mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n} g f_{cda}\left(g_{\rho\sigma}(2l+k)_\mu + g_{\sigma\mu}(-l-2k)_\rho + g_{\mu\rho}(k-l)_\sigma\right)$$
$$\cdot g f_{cbd}\left(g^\rho{}_\nu(k-l)^\sigma + g^\sigma{}_\nu(-l-2k)^\rho + g^{\sigma\rho}(2l+k)_\nu\right)\cdot\frac{(-i)^2}{l^2(l+k)^2} \quad (75)$$

$$-i\Pi_{\mu\nu}^{ab,(2)}(k) = -\mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n} g^2 f_{adc}f_{bcd} l_\mu(l+k)_\nu \frac{i^2}{l^2(l+k)^2} \quad (76)$$

$$-i\Pi_{\mu\nu}^{ab,(3)}(k) = -\mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\frac{\mathrm{tr}\left((ig\gamma_\mu)(i\slashed{l})(ig\gamma_\nu)(i(\slashed{l}+\slashed{k}))\right)\mathrm{tr}(T_a T_b)}{l^2(l+k)^2} \quad (77)$$

$$-i\Pi_{\mu\nu}^{ab,(4)}(k) = -\mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\frac{\mathrm{tr}\left((ig\gamma_\mu)(i(\slashed{l}+m))(ig\gamma_\nu)(i(\slashed{l}+\slashed{k}+m))\right)\mathrm{tr}(T_a T_b)}{(l^2-m^2)((l+k)^2-m^2)} \quad (78)$$

$$-i\Pi_{\mu\nu}^{ab,(C)}(k) = i(Z_3-1)\delta^{ab}(k_\mu k_\nu - k^2 g_{\mu\nu}) \quad (79)$$

Color space:

$$f_{acd}f_{bdc} = -\delta_{ab}C_A = f_{dca}f_{dbc} \quad (80)$$

$$\mathrm{tr}(T_a T_b) = \frac{1}{2}\delta_{ab} \quad (81)$$

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By Slavnov-Taylor we know:

$$\Pi_{\mu\nu}^{ab}(k) = \delta^{ab}(k_\mu k_\nu - k^2 g_{\mu\nu})\Pi(k^2) \tag{82}$$

$$\Rightarrow \Pi(k^2) = -\frac{\delta_{ab}}{N_c^2 - 1}\frac{1}{(3+\epsilon)k^2}g^{\mu\nu}\Pi_{\mu\nu}^{ab}(k) \tag{83}$$

Gluon + Ghost loop:

$$\Pi^{(1+2)}(k^2) = -ig^2 C_A \frac{1}{(3+\epsilon)k^2}\mu_R^{-\epsilon}\int \frac{d^n l}{(2\pi)^n}\frac{(8+3\epsilon)k\cdot l + (9+3\epsilon)k^2 + (8+3\epsilon)l^2}{l^2(l+k)^2} \tag{84}$$

$$= -ig^2 C_A \frac{10+3\epsilon}{2(3+\epsilon)}\mu_R^{-\epsilon}\int \frac{d^n l}{(2\pi)^n}\frac{1}{l^2(l+k)^2} \tag{85}$$

with

$$B_0(k^2,0,0) = \mu_R^{-\epsilon}\int \frac{d^n l}{(2\pi)^n}\frac{1}{l^2(l+k)^2} \tag{86}$$

$$= \frac{i}{16\pi^2}\left(-\frac{2}{\hat{\epsilon}} - \ln(-k^2/\mu_R^2) + 2\right) \tag{87}$$

We find

$$\Rightarrow \Pi^{(1+2)}(k^2) = g^2 C_A\left(-\frac{10}{3\hat{\epsilon}} - \frac{5}{3}\ln(-k^2/\mu_R^2) + \frac{31}{9}\right) \tag{88}$$

$$= -C_A\frac{g^2}{16\pi^2}\frac{5}{3}\left(\frac{2}{\hat{\epsilon}} + \ln(-k^2/\mu_R^2) - \frac{31}{15}\right) \tag{89}$$

Heavy quark loop:

$$\Pi^{(4)}(k^2) = ig^2\frac{1}{2(3+\epsilon)k^2}\mu_R^{-\epsilon}\int \frac{d^n l}{(2\pi)^n}\frac{4((2+\epsilon)(m^2 - k\cdot l - l^2) + 2m^2)}{(l^2-m^2)((l+k)^2-m^2)} \tag{90}$$

$$= ig^2\frac{2}{(3+\epsilon)k^2}\left(2m^2 B_0(k^2,m^2,m^2) - (2+\epsilon)A_0(m^2)\right.$$

$$\left. - (2+\epsilon)k^2 B_1(k^2,m^2,m^2)\right) \tag{91}$$

with

$$A_0(m^2) = \frac{im^2}{16\pi^2}\left(-\frac{2}{\hat{\epsilon}_m} + 1\right) \tag{92}$$

$$B_1(k^2,m^2,m^2) = -\frac{1}{2}B_0(k^2,m^2,m^2) \tag{93}$$

$$B_0(k^2,m^2,m^2) = \frac{i}{16\pi^2}\left(-\frac{2}{\hat{\epsilon}_m} + 2 + \beta_k\ln(\chi_k)\right) \tag{94}$$

and $\beta_k = \sqrt{1 - 4m^2/k^2}$, $\chi_k = (\beta_k - 1)/(1 + \beta_k)$. We then find

$$\Rightarrow \Pi^{(4)}(k^2) = \frac{2g^2}{16\pi^2}\left(\frac{2}{3\hat{\epsilon}_m} - \frac{5}{9} - \frac{4m^2}{3k^2} - \frac{1}{3}\left(1 + 2\frac{m^2}{k^2}\right)\beta_k \ln(\chi_k)\right) \tag{95}$$

$$= \frac{g^2}{16\pi^2}\frac{2}{3}\left(\frac{2}{\hat{\epsilon}_m} - \frac{5}{3} - 4\frac{m^2}{k^2} - \left(1 + 2\frac{m^2}{k^2}\right)\beta_k \ln(\chi_k)\right) \tag{96}$$

Light quark loop:

$$\Pi^{(3)}(k^2) = -ig^2\frac{2(2 + \epsilon)}{(3 + \epsilon)}B_1(k^2, 0, 0) \tag{97}$$

$$= ig^2\frac{(2 + \epsilon)}{(3 + \epsilon)}B_0(k^2, 0, 0) \tag{98}$$

$$= \frac{g^2}{16\pi^2}\frac{2}{3}\left(\frac{2}{\hat{\epsilon}} + \ln(-k^2/\mu_R^2) - \frac{5}{3}\right) \tag{99}$$

We choose the counterterm to be:

$$-\Pi^{(C)}(k^2) = Z_3 - 1 = \frac{g^2}{16\pi^2}\left(-\frac{5}{3}C_A\frac{2}{\hat{\epsilon}} + n_{lf}\frac{2}{3}\frac{2}{\hat{\epsilon}} + \frac{2}{3}\frac{2}{\hat{\epsilon}_m}\right) \tag{100}$$

$$= \frac{g^2}{16\pi^2}\left((2C_A - \beta_0^{lf})\frac{2}{\hat{\epsilon}} + \frac{2}{3}\frac{2}{\hat{\epsilon}_m}\right) \tag{101}$$

$$= \frac{g^2}{16\pi^2}\left((2C_A - \beta_0^f)\frac{2}{\hat{\epsilon}} + \frac{2}{3}\ln(m^2/\mu_R^2)\right) \tag{102}$$

with $\beta_0^{lf} = (11C_A - 2n_{lf})/3$, $\beta_0^f = (11C_A - 2n_f)/3$ and $n_f = n_{lf} + 1$.

We also find

$$Z_g - 1 = \frac{g^2}{16\pi^2}\frac{1}{2}\left(\beta_0^f\frac{2}{\hat{\epsilon}} - \frac{2}{3}\ln(m^2/\mu_R^2)\right) \tag{103}$$

such that $g \rightarrow Z_g g$.

3.3.5 Gluon Self Energy (On shell)

We have[7, (A.18)-(A.20)]:

$$I_0 = \int \frac{d^n p}{(2\pi)^n} \frac{1}{(p^2 + 2kp + M^2 + i\eta)^\alpha} \tag{104}$$

$$= i \frac{(-\pi)^{n/2}}{(2\pi)^n} \frac{\Gamma(\alpha - n/2)}{\Gamma(\alpha)} \frac{1}{(M^2 - k^2 + i\eta)^{\alpha - n/2}} \tag{105}$$

$$I_\mu = \int \frac{d^n p}{(2\pi)^n} \frac{p_\mu}{(p^2 + 2kp + M^2 + i\eta)^\alpha} \tag{106}$$

$$= -k_\mu I_0 \tag{107}$$

$$I_{\mu\nu} = \int \frac{d^n p}{(2\pi)^n} \frac{p_\mu p_\nu}{(p^2 + 2kp + M^2 + i\eta)^\alpha} \tag{108}$$

$$= I_0 \left(k_\mu k_\nu + \frac{1}{2} g_{\mu\nu} (M^2 - k^2) \frac{1}{\alpha - n/2 - 1} \right) \tag{109}$$

Heavy Quark loop:

$$-i\Pi_{\mu\nu}^{ab,(4)}(k) = -\mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{\mathrm{tr}\big((ig\gamma_\mu)(i(\slashed{l} + m))(ig\gamma_\nu)(i(\slashed{l} + \slashed{k} + m))\big) \mathrm{tr}(T_a T_b)}{(l^2 - m^2)((l+k)^2 - m^2)} \tag{110}$$

$$= -\frac{\delta^{ab}}{2} g^2 \mu_R^{4-n} \int\limits_0^1 dx \int \frac{d^n l}{(2\pi)^n} 4 \frac{2l_\mu l_\nu + (l_\mu k_\nu + l_\nu k_\mu) + (m^2 - k\cdot l - l^2) g_{\mu\nu}}{(l^2 + 2xk\cdot l - m^2 + k^2 x)^2} \tag{111}$$

$$= -2\delta^{ab} g^2 \mu_R^{4-n} \int\limits_0^1 dx \left(2I_{\mu\nu} + (k_\nu I_\nu + k_\nu I_\mu) + g_{\mu\nu} \left(m^2 I_0 - k^\rho I_\rho - g_\rho^\sigma I_{\rho\sigma} \right) \right) \tag{112}$$

$$= -2\delta^{ab} g^2 \mu_R^{4-n} \int\limits_0^1 dx\, I_0 \Big[k_\mu k_\nu (2x^2 - 2x) + $$

$$g_{\mu\nu} \left((k^2 x(1-x) - m^2) \frac{1 - n/2}{1 - n/2} + m^2 + k^2 x(1-x) \right) \Big] \tag{113}$$

$$= 4\delta^{ab} g^2 (k_\mu k_\nu - g_{\mu\nu} k^2) \mu_R^{4-n} \int\limits_0^1 dx\, x(1-x) I_0 \tag{114}$$

where

$$\mu_R^{4-n} I_0 = i \frac{(-\pi)^{n/2}}{(2\pi)^n} \frac{\Gamma(2 - n/2)}{\Gamma(2)} \left(\frac{\mu_R^2}{k^2 x(1-x) - m^2} \right)^{2 - n/2} \tag{115}$$

$$= -\frac{i}{16\pi^2} \left(\frac{2}{\hat\epsilon} - \ln((k^2 x(1-x) - m^2)/\mu_R^2) \right) + O(\epsilon) \tag{116}$$

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so for $k^2 = 0$ (on shell) we end up with

$$- i\Pi_{\mu\nu}^{ab,(4)}(k) = -i\delta^{ab}\frac{g^2}{16\pi^2}(k_\mu k_\nu - g_{\mu\nu}k^2)\frac{4}{3\hat{\epsilon}} \tag{117}$$

Light Quark loop:

$$-i\Pi_{\mu\nu}^{ab,(3)}(k) = -\mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\frac{\text{tr}\big((ig\gamma_\mu)(i\slashed{l})(ig\gamma_\nu)(i(\slashed{l}+\slashed{k}))\big)\,\text{tr}(T_a T_b)}{l^2(l+k)^2} \tag{118}$$

$$= 4\delta^{ab}g^2(k_\mu k_\nu - g_{\mu\nu}k^2)\mu_R^{4-n}\int_0^1 dx\,x(1-x)I_0 \tag{119}$$

where

$$\mu_R^{4-n}I_0 = \mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\frac{1}{(l^2 + 2xk\cdot l + k^2 x)^2} \tag{120}$$

$$= \mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\frac{1}{((l+xk)^2 + k^2 x(1-x))^2} \tag{121}$$

so for $k^2 = 0$ (on shell) we get $I_0 = 0$ and end up with

$$- i\Pi_{\mu\nu}^{ab,(3)}(k) = 0 \tag{122}$$

Gluon + Ghost loop vanish by the same arguments as the light quark loop:

$$- i\Pi_{\mu\nu}^{ab,(1+2)}(k) = 0 \tag{123}$$

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List of Corrections