1 Introduction

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1.1 Motivation

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1.2 Notation

We study the neutral-current (NC) deep inelastic scattering (DIS) reaction

$$\ell(k) + \mathrm{N}(P) \to \ell'(k') + \overline{Q}(p_2) + X[Q] \tag{1}$$

and define the usual set of kinematic variables

$$q = k - k', \qquad Q^2 = -q^2, \qquad x = \frac{Q^2}{2q \cdot P}, \qquad y = \frac{q \cdot P}{k \cdot P} \tag{2}$$

Assuming $k^2 = m_\ell^2 = 0$, we can write the hadronic tensor (adopting the naming convention of [1]):

$$\begin{aligned} W_{\mu\mu'} = &\left(-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2}\right) F_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_{\mu'}}{P \cdot q} F_2(x,Q^2) - i\varepsilon_{\mu\mu'\alpha\beta}\frac{q^\alpha P^\beta}{2P \cdot q} F_3(x,Q^2) \\ &+ i\varepsilon_{\mu\mu'\alpha\beta}\frac{q^\alpha S^\beta}{P \cdot q} g_1(x,Q^2) + \frac{S \cdot q}{P \cdot q}\left[\frac{\hat{P}_\mu \hat{P}_{\mu'}}{P \cdot q} g_4(x,Q^2) + \left(-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2}\right) g_5(x,Q^2)\right] \end{aligned} \tag{3}$$

with $\hat{P}_\mu = P_\mu - \frac{P \cdot q}{q^2} q_\mu$ and S denoting the spin vector of the nucleon. We further introduce the more convenient structure functions

$$F_L(x,Q^2) = F_2(x,Q^2) - 2xF_1(x,Q^2), \qquad g_L(x,Q^2) = g_4(x,Q^2) - 2xg_5(x,Q^2)\,. \tag{4}$$

As we are interested in the full neutral current contributions, we allow for both, the virtual photon γ^* and the virtual Z^*-boson exchange. The structure functions can then be decomposed by

$$H = H^\gamma - \left(g_V^\ell \pm \lambda g_A^\ell\right)\eta_{\gamma\mathrm{Z}} H^{\gamma\mathrm{Z}} + \left((g_V^\ell)^2 + (g_A^\ell)^2 \pm 2\lambda g_V^\ell g_A^\ell\right)\eta_{\mathrm{Z}} H^{\mathrm{Z}},\ H \in \{F_2, F_L, 2xg_1\} \tag{5}$$

$$H = -\left(g_A^\ell \pm \lambda g_V^\ell\right)\eta_{\gamma\mathrm{Z}} H^{\gamma\mathrm{Z}} + \left(2g_V^\ell g_A^\ell \pm \lambda\left((g_V^\ell)^2 + (g_A^\ell)^2\right)\right)\eta_{\mathrm{Z}} H^{\mathrm{Z}},\ H \in \{xF_3, g_4, g_L\} \tag{6}$$

where λ defines the helicity of the incoming lepton and

$$\eta_{\gamma Z} = \left(\frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\right)\left(\frac{Q^2}{Q^2+M_Z^2}\right) \qquad\qquad \eta_Z = \eta_{\gamma Z}^2 \tag{7}$$

We use the framework of the collinear factorization, so the leading order partonic reaction is given by

$$b(q) + \mathrm{g}(k_1) \to \overline{Q}(p_2) + Q(p_1), \quad b \in \{\gamma^*, Z^*\} \tag{8}$$

where $k_1 = \xi P$ and the relevent kinematic variables are

$$\begin{aligned} z &= \frac{Q^2}{2q\cdot k_1} = \frac{x}{\xi}, & s &= (k_1+q)^2, & s' &= s - q^2 \\ t_1 &= (k_1 - p_2) - m^2, & u_1 &= (q-p_2)^2 - m^2, & u_1' &= u_1 - q^2\,. \end{aligned} \tag{9}$$

The hadronic tensor 3 can then be mapped onto a partonic level by

$$\begin{aligned} \frac{1}{2z}\hat{w}_{\mu,\mu'} = &(-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2})\hat{F}_1(z,Q^2) + \frac{\hat{k}_{1,\mu}\hat{k}_{1,\mu'}}{k_1\cdot q}\hat{F}_2(z,Q^2) - i\varepsilon_{\mu\mu'\alpha\beta}\frac{q^\alpha k_1^\beta}{2k_1\cdot q}\hat{F}_3(z,Q^2) \\ &+ \frac{q_\mu q_{\mu'}}{q^2}\hat{F}_4(z,Q^2) + \frac{q_\mu k_{1,\mu'} + q_{\mu'}k_{1,\mu}}{2k_1\cdot q}\hat{F}_5(z,Q^2) \\ &+ i\varepsilon_{\mu\mu'\alpha\beta}\frac{q^\alpha S^\beta}{k_1\cdot q}\hat{g}_1(z,Q^2) + \frac{S\cdot q}{k_1\cdot q}\left[\frac{\hat{k}_{1,\mu}\hat{k}_{1,\mu'}}{k_1\cdot q}\hat{g}_4(z,Q^2) + (-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2})\hat{g}_5(z,Q^2)\right] \\ &+ \frac{S\cdot q}{k_1\cdot q}\left[\frac{q_\mu q_{\mu'}}{q^2}\hat{g}_6(z,Q^2) + \frac{q_\mu k_{1,\mu'} + q_{\mu'}k_{1,\mu}}{2k_1\cdot q}\hat{g}_7(z,Q^2)\right] \end{aligned} \tag{10}$$

with $\hat{k}_{1,\mu} = k_{1,\mu} - \frac{k_1\cdot q}{q^2}q_\mu$ and

$$\hat{F}_L(z,Q^2) = \hat{F}_2(z,Q^2) - 2z\hat{F}_1(z,Q^2), \qquad \hat{g}_L(z,Q^2) = \hat{g}_4(z,Q^2) - 2z\hat{g}_5(z,Q^2)\,. \tag{11}$$

Note that we have to include $\hat{F}_4, \hat{F}_5, \hat{g}_6$ and $\hat{g}_7$ into Eq. 10 as we are interested in the full neutral current case, that is, we allow for Z-bosons to be exchanged and due to this we can no longer rely on the Ward-identity, because the contraction of the axial current $q_\mu J_5^\mu$ does not vanish for massive particles:

$$q_\mu J_5^\mu = q_\mu \bar{\psi}\gamma^\mu\gamma^5\psi = 2m\,\bar{\psi}i\gamma^5\psi\,. \tag{12}$$

It is convenient to rescale the structure functions and so we focus in the following on the six structure functions

$$\hat{F}_2, \hat{F}_L, z\hat{F}_3, 2z\hat{g}_1, \hat{g}_4 \text{ and } \hat{g}_L \tag{13}$$

or their hadronic counter parts.

This way we can define the projections onto the structure functions by

$$\hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{F}_2} = \frac{-g^{\mu\mu'}}{n-2} - \frac{n-1}{n-2}\cdot\frac{4z^2 k_1^\mu k_1^{\mu'}}{q^2} - \frac{q^\mu q^{\mu'}}{q^2} - \frac{n-2}{n-1}\cdot\frac{2z(q^\mu k_1^{\mu'} + q^{\mu'} k_1^\mu)}{q^2} \tag{14}$$

$$\hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{F}_L} = -\frac{4z^2 k_1^\mu k_1^{\mu'}}{q^2} - \frac{q^\mu q^{\mu'}}{q^2} - \frac{2z(q^\mu k_1^{\mu'} + q^{\mu'} k_1^\mu)}{q^2} \tag{15}$$

$$\hat{\mathcal{P}}^{b,\mu\mu'}_{z\hat{F}_3} = -\frac{iz\varepsilon^{\mu\mu'\alpha\beta} k_{1,\alpha} q_\beta}{q^2} \tag{16}$$

and, due to the symmetry in the Lorentz structure,

$$\hat{\mathcal{P}}^{b,\mu\mu'}_{2z\hat{g}_1} = \hat{\mathcal{P}}^{b,\mu\mu'}_{z\hat{F}_3} \qquad \hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{g}_4} = -\hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{F}_2} \qquad \hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{g}_L} = -\hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{F}_L}\,. \tag{17}$$

This way we have

$$\hat{\mathcal{P}}^{b,\mu\mu'}_{\hat{h}} \hat{w}_{\mu,\mu'} = \hat{h} \quad \text{for } \hat{h} \in \{\hat{F}_2, \hat{F}_L, z\hat{F}_3, 2z\hat{g}_1, \hat{g}_4, \hat{g}_L\} \tag{18}$$

For the unpolarized structure functions $\hat{F}_2, \hat{F}_L$ and $z\hat{F}_3$ the helicity of the parton, either gluon or (anti-)quark, has to be averaged, whereas for the polarized $2z\hat{g}_1, \hat{g}_4$ and $\hat{g}_L$ we have to consider the helicity difference. For the gluons this is achieved by

$$\hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{F} = -g^{\nu\nu'} \qquad \hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{g} = 2i\varepsilon^{\nu\nu'\alpha\beta}\frac{k_{1,\alpha} q_\beta}{2k_1\cdot q} \tag{19}$$

and by choosing just $-g^{\nu\nu'}$, we decided to include incoming external ghost to cancel all unphysical gluon polarization. All initial-state (anti-)quarks are taken as massless partons, so the relevant projection operators onto definitive helicity states are given by

$$\begin{aligned}\hat{\mathcal{P}}^{q,aa'}_{F} &= (\not{k}_1)_{aa'}, & \hat{\mathcal{P}}^{q,aa'}_{g} &= -(\gamma_5\not{k}_1)_{aa'},\\ \hat{\mathcal{P}}^{\bar{q},bb'}_{F} &= (\not{k}_1)_{bb'}, & \hat{\mathcal{P}}^{\bar{q},bb'}_{g} &= (\gamma_5\not{k}_1)_{bb'}\end{aligned} \tag{20}$$

where a and a' (b and b') refer to the Dirac-index of the initial (anti-)quark spinor in the relevant matrix elements given below.

In order to compute the exchange of scattered vector boson $b = \{\gamma, \mathrm{Z}\}$ with a common notation, we write the coupling of b to the hadronic process by

$$\Gamma^\mu_{b,j} = g^V_{b,j}\Gamma^\mu_V + g^A_{b,j}\Gamma^\mu_A = g^V_{b,j}\gamma^\mu + g^A_{b,j}\gamma^\mu\gamma^5, \quad j \in \{q, Q\}. \tag{21}$$

Due to symmetry reasons the parity-violating structure functions $z\hat{F}_3, g_4, g_L$ can only recieve contributions from $\Gamma^\mu_V\Gamma^{\mu'}_A$, where μ refer to the Lorentz index of the boson in matrix amplitude and the μ' to the index in the complex conjungate. Likewise the parity conserving structure functions can only recieve contributions from either $\Gamma^\mu_V\Gamma^{\mu'}_V$ or $\Gamma^\mu_A\Gamma^{\mu'}_A$. To introduce a compact notation we write

$$\vec{\kappa} = (\kappa_1, \kappa_2) \quad \kappa_1 \in \{\mathrm{VV}, \mathrm{VA}, \mathrm{AA}\},\, \kappa_2 \in \{\hat{F}_2, \hat{F}_L, z\hat{F}_3, 2z\hat{g}_1, \hat{g}_4, \hat{g}_L\}\,. \tag{22}$$

We define an additional set of partonic variables, that will simplify many partonic expressions:

$$0 \le \rho = \frac{4m^2}{s} \le 1 \qquad 0 \le \beta = \sqrt{1-\rho} \le 1 \qquad 0 \le \chi = \frac{1-\beta}{1+\beta} \le 1 \tag{23}$$

$$0 \le \rho' = \frac{4m^2}{s'} \le 1 \qquad 0 \le \beta' = \sqrt{1-\rho'} \le 1 \qquad 0 \le \chi' = \frac{1-\beta'}{1+\beta'} \le 1 \tag{24}$$

$$\rho_q = \frac{4m^2}{q^2} \le 0 \qquad 1 \le \beta_q = \sqrt{1-\rho_q} \qquad 0 \le \chi_q = \frac{\beta_q-1}{\beta_q+1} \le 1 \tag{25}$$

which obey the further inequalities

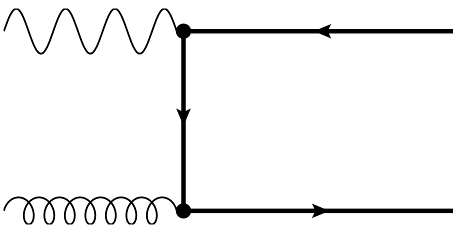
$$\rho' < \rho, \qquad \rho' < \frac{\rho_q}{\rho_q - 1}, \qquad \beta < \beta', \qquad \beta' < \frac{1}{\beta_q}, \qquad \chi' < \chi, \qquad \chi' < \chi_q\,. \tag{26}$$

2 Leading Order Calculations

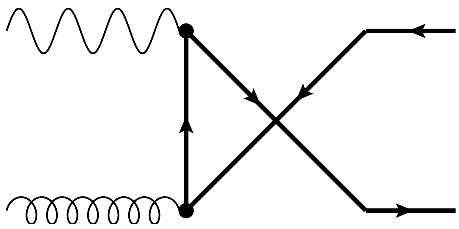
In leading order we have to consider photon-gluon-fusion (PGF), that is

$$b(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2), \quad b \in \{\gamma^*, Z^*\} \tag{27}$$

with two contributing diagrams depicted in figure 1.



(a) $i\varepsilon_b^\mu(q)\varepsilon_\mathrm{g}^\nu(k_1)\,\mathcal{M}^{(0),1}_{\mu\nu}$



(b) $i\varepsilon_b^\mu(q)\varepsilon_\mathrm{g}^\nu(k_1)\,\mathcal{M}^{(0),2}_{\mu\nu}$

Figure 1: leading order Feynman diagrams
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The result can then be written as

$$M^{(0)}_{\vec\kappa} = \hat{\mathcal{P}}^{b,\mu\mu'}_{\vec\kappa}\hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{\kappa_2}\sum_{j,j'=1}^{2}\mathcal{M}^{(0),j}_{\mu\nu}\left(\mathcal{M}^{(0),j'}_{\mu'\nu'}\right)^* = 8g^2\mu_D^{-\epsilon}e^2e_H^2N_CC_FB_{\vec\kappa,\mathrm{QED}} \tag{28}$$

where g and e are the strong and electromagnetic coupling constants respectively, μ_D is an arbitray mass parameter introduced to keep the couplings dimensionless and e_H is the magnitude of the heavy quark in units of e. Further $N_C = 3$ corresponds to the gauge group $SU(N_C)$ and the color factor $C_F = (N_C^2-1)/(2N_C)$ refers to the second Casimir constant of the fundamental representation for the quarks. We then find:

$$B_{\mathrm{VV},F_2,\mathrm{QED}} = \left[-1 - \frac{6q^2}{s'} - \frac{6q^4}{s'^2} + \frac{q^2(6m^2+s)+2m^2s+s'^2/2}{t_1u_1} - \frac{(2m^2+q^2)m^2s'^2}{(t_1u_1)^2}\right] + \frac{\epsilon}{2}\left[-1 + \frac{s^2-q^2s'}{t_1u_1} - \frac{m^2q^2s'^2}{t_1^2u_1^2}\right] + \epsilon^2\frac{s'^2}{8t_1u_1} \tag{29}$$

$$B_{\mathrm{VV},F_L,\mathrm{QED}} = -\frac{4q^2}{s'}\left(\frac{s}{s'} - \frac{m^2s'}{t_1u_1}\right) \tag{30}$$

$$B_{\mathrm{VV},2xg_1,\mathrm{QED}} = \left\{1 + \frac{2q^2}{s'} - \frac{s'(2(2m^2+q^2)+s')}{2t_1u_1} + \frac{m^2s'^3}{(t_1u_1)^2} + \epsilon\left(-\frac{1}{2} + \frac{s'^2}{4t_1u_1}\right)\right\}(1+\epsilon) \tag{31}$$

$$\begin{aligned} B_{\mathrm{AA},F_2,\mathrm{QED}} = {} & \frac{m^2s'^2(1+\epsilon)(2+\epsilon)(12m^2(-1+\epsilon)+q^2(-6+(-3+\epsilon)\epsilon))}{12(t_1u_1)^2} - \\ & \frac{(1+\epsilon)\left(8s'^3\epsilon + 12q^6(2+\epsilon) + 12q^4s'(2+\epsilon) + q^2s'^2(4+\epsilon(20-(-3+\epsilon)\epsilon))\right)}{4q^2s'^2} - \\ & \frac{(1+\epsilon)}{48q^2(t_1u_1)}\left(q^2(2+\epsilon)(-6+(-3+\epsilon)\epsilon)\left(4q^4+4q^2s'+s'^2(2+\epsilon)\right)\right. \\ & \left. +48m^2\left(-s'^2(-2+\epsilon)+q^4(-4+\epsilon)(2+\epsilon)+q^2s'\left(-2+\epsilon+\epsilon^2\right)\right)\right) \end{aligned} \tag{32}$$

$$\begin{aligned} B_{\mathrm{AA},F_L,\mathrm{QED}} = {} & -\frac{m^2s'^2(1+\epsilon)(2+\epsilon)(12m^2+q^2\epsilon)}{6(t_1u_1)^2} - \\ & \frac{(1+\epsilon)\left(4s'^3\epsilon + 4q^6(2+\epsilon) + 4q^4s'(2+\epsilon) + q^2s'^2\epsilon(6+\epsilon)\right)}{2q^2s'^2} + \\ & \frac{(1+\epsilon)}{24q^2(t_1u_1)}\left(24m^2\left(s'^2(-2+\epsilon)+4q^4(2+\epsilon)+2q^2s'(2+\epsilon)\right)+\right. \\ & \left. q^2\epsilon(2+\epsilon)\left(4q^4+4q^2s'+s'^2(2+\epsilon)\right)\right) \end{aligned} \tag{33}$$

$$B_{\mathrm{AA},2xg_1,\mathrm{QED}} = \frac{(1+\epsilon)(2-\epsilon)}{2}B_{\mathrm{VV},2xg_1,\mathrm{QED}} \tag{34}$$

$$B_{\mathrm{VA},xF_3,\mathrm{QED}} = -(1+\epsilon)(2+\epsilon)(t_1^2-u_1^2)\left\{-\frac{m^2q^2}{2(t_1u_1)^2} + \frac{4q^2(q^2+s')+s'^2(2+\epsilon)}{8s'^2t_1u_1}\right\} \tag{35}$$

$$B_{\mathrm{VA},g_4,\mathrm{QED}} = (1+\epsilon)(t_1-u_1)\left\{-\frac{m^2s'^2}{(t_1u_1)^2} + \frac{4q^2+s'(2-\epsilon)}{4t_1u_1}\right\} \tag{36}$$

$$B_{\mathrm{VA},g_L,\mathrm{QED}} = 0 \tag{37}$$

We will decompose the Born cross section further by their dependence on ϵ

$$B_{\vec{\kappa},\mathrm{QED}} = B^{(0)}_{\vec{\kappa},\mathrm{QED}} + \epsilon B^{(1)}_{\vec{\kappa},\mathrm{QED}} + \epsilon^2 B^{(2)}_{\vec{\kappa},\mathrm{QED}} \tag{38}$$

and do find $B^{(0)}_{\mathrm{VV},2xg_1,\mathrm{QED}} = B^{(0)}_{\mathrm{AA},2xg_1,\mathrm{QED}}$, but $B^{(1)}_{\mathrm{VV},2xg_1,\mathrm{QED}} \neq B^{(1)}_{\mathrm{AA},2xg_1,\mathrm{QED}}$.

The required n-dimensional 2-to-2-particle phase space dPS_2 is given by

$$\mathrm{dPS}_2 = \frac{2\pi S_\epsilon}{s'\Gamma(1+\epsilon/2)} \left(\frac{(t_1u_1' - s'm^2)s' - q^2t_1^2}{s'^2}\right)^{\epsilon/2} \delta(s' + t_1 + u_1)\, dt_1 du_1\,. \tag{39}$$

The spin and color averaged partonic cross section is then given by

$$d\sigma^{(0)}_{\vec{\kappa},\mathrm{g}} = \frac{1}{2s'}\frac{1}{2}E_{\kappa_2}(\epsilon)K_{\mathrm{g}\gamma}M^{(0)}_{\vec{\kappa}}\,\mathrm{dPS}_2 \tag{40}$$

where

$$E_F(\epsilon) = \frac{1}{1+\epsilon/2}\,, \qquad\qquad E_g(\epsilon) = 1 \tag{41}$$

accounts for additional degrees of freedom in n dimensions for initial-state bosons.

3 Next-To-Leading Order Calculations

3.1 One Loop Virtual Contributions

$$\begin{aligned} M^{(1),V}_{\vec{\kappa}} &= \hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\vec{\kappa}}\hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{\kappa_2}\sum_{j,j'} 2\mathrm{Re}\left[\mathcal{M}^{(1),V}_{j,\mu\nu}\left(\mathcal{M}^{(0)}_{j',\mu'\nu'}\right)^*\right] \\ &= 8g^4\mu_D^{-\epsilon}e^2e_H^2N_CC_FC_\epsilon\left(C_AV_{\vec{\kappa},\mathrm{OK}} + 2C_FV_{\vec{\kappa},\mathrm{QED}}\right) \end{aligned} \tag{42}$$

where $C_\epsilon = \exp(\epsilon/2(\gamma_E - \ln(4\pi)))/(16\pi^2)$ and C_A is the second Casimir constant of the adjoint representation for the gluon (that introduces a non-abelian part).

The full expressions for the $V_{\vec{\kappa},\mathrm{OK}}, V_{\vec{\kappa},\mathrm{QED}}$ are quite complicated and are too long to be presented here, nevertheless the arising poles are quite compact:

$$V_{\vec{\kappa},\mathrm{OK}} = -2B_{\vec{\kappa},\mathrm{QED}}\left(\frac{4}{\epsilon^2} + \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) + \frac{s-2m^2}{s\beta}\ln(\chi)\right)\frac{2}{\epsilon}\right) + O(\epsilon^0) \tag{43}$$

$$V_{\vec{\kappa},\mathrm{QED}} = -2B_{\vec{\kappa},\mathrm{QED}}\left(1 - \frac{s-2m^2}{s\beta}\ln(\chi)\right)\frac{2}{\epsilon} + O(\epsilon^0) \tag{44}$$

The above results already include the mass renormalization that we have performed *on-shell*, so all ultra-violet poles have been removed. For the renormalization of the strong coupling we use the $\overline{\mathrm{MS}}_m$ scheme defined in [2] and so the full (remaining) renormalization can be achieved by

$$\frac{d^2\sigma^{(1),V}_{\vec{\kappa},\mathrm{g}}}{dt_1 du_1} = \left.\frac{d^2\sigma^{(1),V}_{\vec{\kappa},\mathrm{g}}}{dt_1 du_1}\right|_{\mathrm{bare}} + 4\pi\alpha_s(\mu_R^2)C_\epsilon \left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2} \left[\left(\frac{2}{\epsilon} + \ln(\mu_R^2/m^2)\right)\beta_0^f + \frac{2}{3}\ln(\mu_R^2/m^2)\right] \frac{d^2\sigma^{(0)}_{\vec{\kappa},\mathrm{g}}}{dt_1 du_1} \tag{45}$$

with μ_R the renormalization scale introduced by the RGE, $\beta_0^f = (11C_A - 2n_f)/3$ the first coefficient of the beta function and n_f the number of *total* flavours (i.e. $n_{lf} = n_f - 1$ active (light) flavours and one heavy flavour). The double poles occuring in $V_{\vec{\kappa},\mathrm{OK}}$ are introduced by the diagrams **FiXme Error: do** when the soft and collinear singularities coincide. FiXme Error!

The partonic cross section is given by

$$d\sigma^{(1),V}_{\vec{\kappa},\mathrm{g}} = \frac{1}{2s'}\frac{1}{2}E_{\kappa_2}(\epsilon)M^{(1),V}_{\vec{\kappa}}\,\mathrm{dPS}_2 \tag{46}$$

3.2 Single Gluon Radiation

In addition to the virtual corrections we have to consider the radiation of an additional gluon at next-to-leading order, i. e., the $2 \to 3$ process

$$b(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) + \mathrm{g}(k_2), \quad b \in \{\gamma^*, Z^*\} \tag{47}$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as FiXme Error!

$$\begin{aligned} M^{(1),\mathrm{g}}_{\vec{\kappa}} &= \hat{\mathcal{P}}^{b,\mu\mu'}_{\vec{\kappa}}\hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{\kappa_2}\sum_{j,j'}\mathcal{M}^{(1),\mathrm{g}}_{j,\mu\nu}\left(\mathcal{M}^{(1),\mathrm{g}}_{j',\mu'\nu'}\right)^* &(48)\\ &= 8g^4\mu_D^{-2\epsilon}e^2e_H^2N_CC_F\left(C_AR_{\vec{\kappa},\mathrm{OK}} + 2C_FR_{\vec{\kappa},\mathrm{QED}}\right) &(49)\end{aligned}$$

The required n-dimensional phase space dPS_3 is given by

$$\mathrm{dPS}_3 = \frac{S_\epsilon^2}{\Gamma(1+\epsilon)s'}\frac{s_4^{1+\epsilon}}{(s_4+m^2)^{1+\epsilon/2}}\left(\frac{(t_1u_1' - s'm^2)s' - q^2t_1^2}{s'^2}\right)^{\epsilon/2} dt_1du_1d\Omega_n \tag{50}$$

with $d\Omega_n = \sin^{n-3}(\theta_1)d\theta_1\sin^{n-4}(\theta_2)d\theta_2$.

Again when integrating the phase space angles the expressions become quite lengthy, but the collinear poles can be given in a compact form by

$$\frac{s_4}{2\pi(s_4+m^2)}\int d\Omega_n\, C_AR_{\vec{\kappa},OK} = -\frac{2}{u_1}B_{\vec{\kappa},\mathrm{QED}}\begin{pmatrix} s' \to x_1s' \\ t_1 \to x_1t_1\end{pmatrix}P^{(0),H}_{\kappa_2,\mathrm{gg}}(x_1)\frac{2}{\epsilon} + O(\epsilon^0) \tag{51}$$

with $x_1 = -u_1/(s' + t_1)$ and the hard part of the LO Altarelli-Parisi splitting functions $P^{(0),H}_{\kappa_2,\mathrm{gg}}$ [3, 4]:

$$P^{(0),H}_{F_2,\mathrm{gg}}(x) = P^{(0),H}_{F_L,\mathrm{gg}}(x) = P^{(0),H}_{xF_3,\mathrm{gg}}(x) = C_A \left(\frac{2}{1-x} + \frac{2}{x} - 4 + 2x - 2x^2 \right) \tag{52}$$

$$P^{(0),H}_{2xg_1,\mathrm{gg}}(x) = P^{(0),H}_{g_4,\mathrm{gg}}(x) = P^{(0),H}_{g_L,\mathrm{gg}}(x) = C_A \left(\frac{2}{1-x} - 4x + 2 \right) \tag{53}$$

The hard abelian QED part $R_{\vec{\kappa},\mathrm{QED}}$ does not contain collinear poles.

The spin and color averaged partonic cross section for the real corrections to the PGF process is then given by

$$d\sigma^{(1),R}_{\vec{\kappa},\mathrm{g}} = \frac{1}{2s'} \frac{1}{2} E_{\kappa_2}(\epsilon) K_{\mathrm{g}\gamma} M^{(1),\mathrm{g}}_{\vec{\kappa}} \, \mathrm{dPS}_3 \tag{54}$$

From these expressions we can obtain the soft gluon limit $k_2 \to 0$ and seperate their calculations:

$$\lim_{k_2 \to 0} \left(C_A R_{\vec{\kappa},\mathrm{OK}} + 2C_F R_{\vec{\kappa},\mathrm{QED}} \right) = \left(C_A S_{\vec{\kappa},\mathrm{OK}} + 2C_F S_{\vec{\kappa},\mathrm{QED}} \right) + O(1/s_4, 1/s_3, 1/t') \tag{55}$$

where

$$S_{\vec{\kappa},\mathrm{OK}} = 2 \left(\frac{t_1}{t' s_3} + \frac{u_1}{t' s_4} - \frac{s - 2m^2}{s_3 s_4} \right) B_{\vec{\kappa},\mathrm{QED}} \tag{56}$$

$$S_{\vec{\kappa},\mathrm{QED}} = 2 \left(\frac{s - 2m^2}{s_3 s_4} - \frac{m^2}{s_3^2} - \frac{m^2}{s_4^2} \right) B_{\vec{\kappa},\mathrm{QED}} \tag{57}$$

Note that the einkonal factors multiplying the Born functions $B_{\vec{\kappa},\mathrm{QED}}$ neither depend on the photon's virtuality q^2 nor on the projection $\vec{\kappa}$.

3.3 Light Quark Processes

In next-to-leading order a new production mechanism enters that is induced by a light quark, so we have to consider the process

$$b(q) + q(k_1) \to Q(p_1) + \overline{Q}(p_2) + q(k_2), \quad b \in \{\gamma^*, Z^*\} \tag{58}$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as FiXme Error!

$$M^{(1),q}_{\vec{\kappa}} = \hat{\mathcal{P}}^{b,\mu\mu'}_{\vec{\kappa}} \hat{\mathcal{P}}^{q,aa'}_{\vec{\kappa}} \sum_{j,j'=1}^{4} \mathcal{M}^{(1),q}_{j,\mu a} \left(\mathcal{M}^{(1),q}_{j',\mu' a'} \right)^* \tag{59}$$

$$= 8g^4 \mu_D^{-2\epsilon} e^2 N_C C_F \left(e_H^2 A_{\vec{\kappa},1} + e_L^2 A_{\vec{\kappa},2} + e_L e_H A_{\vec{\kappa},3} \right) \tag{60}$$

where e_L denotes the charge of the light quark q in units of e.

The needed 2-to-3-particle phase space has already been calculated in section 3.2, so we can immediately quote the collinear poles here

$$\frac{s_4}{4\pi(s_4+m^2)}\int d\Omega_n\, C_F A_{\vec{\kappa},1} = -\frac{1}{u_1}B_{\vec{\kappa},\mathrm{QED}}\begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1\end{pmatrix} P^{(0)}_{\kappa_2,\mathrm{g}\,q}(x_1)\frac{2}{\epsilon} + O(\epsilon^0) \tag{61}$$

with $x_1 = -u_1/(s'+t_1)$ and the LO Altarelli-Parisi splitting functions $P^{(0)}_{\kappa_2,\mathrm{g}\,q}$[3, 4]:

$$P^{(0)}_{F_2,\mathrm{g}\,q}(x) = P^{(0)}_{F_L,\mathrm{g}\,q}(x) = P^{(0)}_{xF_3,\mathrm{g}\,q}(x) = C_F\left(\frac{1}{x}+\frac{(1-x)^2}{x}\right) \tag{62}$$

$$P^{(0)}_{2xg_1,\mathrm{g}\,q}(x) = P^{(0)}_{g_4,\mathrm{g}\,q}(x) = P^{(0)}_{g_L,\mathrm{g}\,q}(x) = C_F\,(2-x) \tag{63}$$

For the current case of $q^2 \neq 0$ $A_{\vec{\kappa},2}$ does not contain collinear poles. Due to Furry's theorem we find $\int dt_1 du_1 \int d\Omega_4\, A_{\vec{\kappa},3} = 0$.

The spin and color averaged partonic cross section is then given by

$$d\sigma^{(1)}_{\vec{\kappa},q} = \frac{1}{2s'}\frac{1}{2}K_{q\gamma}M^{(1),q}_{\vec{\kappa}}\,\mathrm{dPS}_3 \tag{64}$$

4 Mass Factorization

All collinear poles that arise in the NLO corrections to PGF subprocess $\sigma^{(1)}_{\vec{\kappa},\mathrm{g}}$ or the NLO corrections to the Bethe-Heitler subprocess $\sigma^{(1)}_{\vec{\kappa},q}$ can be removed by standard mass factorization in the following way:

$$\begin{aligned} s'^2\frac{d^2\sigma^{(1),fin}_{\vec{\kappa},j}(s',t_1,u_1,\mu_F)}{dt_1 du_1} &= \lim_{\epsilon\to 0}\Bigg[s'^2\frac{d^2\sigma^{(1)}_{\vec{\kappa},j}(s',t_1,u_1,\epsilon)}{dt_1du_1} \\ &- \int_0^1 \frac{dx_1}{x_1}\Gamma^{(1)}_{\kappa_2,\mathrm{g}j}(x_1,\mu_F^2,\mu_D,\epsilon)(x_1 s')^2\frac{d^2\sigma^{(0)}_{\vec{\kappa},\mathrm{g}}(x_1s',x_1t_1,u_1,\epsilon)}{d(x_1t_1)du_1}\Bigg] \end{aligned} \tag{65}$$

where $\Gamma^{(1)}_{\kappa_2,ij}$ is the first order correction to the transition functions $\Gamma_{\kappa_2,ij}$ for *incoming* particle j and *outgoing* particle i in projection κ_2:

$$\Gamma^{(1)}_{\kappa_2,ij}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_s}{2\pi}\left(P^{(0)}_{\kappa_2,ij}(x)\frac{2}{\epsilon} + f^{(1)}_{\kappa_2,ij}(x,\mu_F^2,\mu_D^2)\right) \tag{66}$$

In the adopted $\overline{\mathrm{MS}}$-scheme the $f^{(1)}_{\kappa_2,ij}$ take their usual form and we find

$$\begin{aligned}\Gamma^{(1),\overline{\mathrm{MS}}}_{\kappa_2,ij}(x,\mu_F^2,\mu_D,\epsilon) &= \frac{\alpha_s}{2\pi}P^{(0)}_{\kappa_2,ij}(x)\left(\frac{2}{\epsilon}+\gamma_E - \ln(4\pi) + \ln(\mu_F^2/m^2) - \ln(\mu_D^2/m^2)\right) &(67)\\ &= 8\pi\alpha_s P^{(0)}_{\kappa_2,ij}(x) C_\epsilon\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}\left(\frac{2}{\epsilon}+\ln(\mu_F^2/m^2)\right) &(68)\end{aligned}$$

For the PGF subprocess we need the LO gluon-to-gluon splitting functions $P_{\kappa_2,\mathrm{gg}}(x)$ given by[3]

$$P^{(0)}_{\kappa_2,\mathrm{gg}}(x) = \Theta(1-\delta-x)P^{(0),H}_{\kappa_2,\mathrm{gg}}(x) + \delta(1-x)\left(2C_A\ln(\delta)+\frac{\beta_0}{2}\right) \tag{69}$$

where we introduced another infrared cut-off δ to seperate soft collinear ($x \geq 1-\delta$) and hard collinear ($x < 1-\delta$) gluons. By simple kinematics we find the relation $\delta = \Delta/(s'+t_1)$ that allows us to divide the collinear part again into a hard and a soft part, rendering each into a finite result. The functions $P^{(0),H}_{\kappa_2,\mathrm{gg}}(x)$ have been already quoted in Eq. 52 and 53 and the needed splitting functions $P^{(0)}_{\kappa_2,\mathrm{g}q}$ for the Bethe-Heitler subprocess have been given in Eqs. (62) and (63), to denote the collinear poles there.

The final, finite partonic cross sections for the PGF subprocess can be split into the hard contributions given by

$$\begin{aligned}
s'^2\frac{d^2\sigma^{(1),H,fin}_{\vec\kappa,\mathrm{g}}}{dt_1du_1} = {}& \alpha\alpha_S e_H^2 K_{\mathrm{g}\gamma}N_C C_F\Bigg[-\frac{2}{u_1}P^H_{\kappa_2,\mathrm{gg}}(x_1) \\
& \Bigg\{B^{(0)}_{\vec\kappa,\mathrm{QED}}\begin{pmatrix} s'\to x_1 s' \\ t_1\to x_1t_1\end{pmatrix}\left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)}\right)-\ln(\mu_F^2/m^2)\right) \\
& -2B^{(1)}_{\vec\kappa,\mathrm{QED}}\begin{pmatrix} s'\to x_1 s' \\ t_1\to x_1t_1\end{pmatrix}\Bigg\} \\
& + C_A\frac{s_4}{2\pi(s_4+m^2)}\left(\int d\Omega_n\, R_{\vec\kappa,\mathrm{OK}}\right)^{finite} \\
& +2C_F\frac{s_4}{2\pi(s_4+m^2)}\int d\Omega_4\, R_{\vec\kappa,\mathrm{QED}}\Bigg]
\end{aligned} \tag{70}$$

and the soft plus virtual contributions given by

$$\begin{aligned}
s'^2\frac{d^2\sigma^{(1),S+V,fin}_{\vec\kappa,\mathrm{g}}}{dt_1du_1} = {}& 4\alpha\alpha_S e_H^2 K_{\mathrm{g}\gamma}N_C C_F B^{(0)}_{\vec\kappa,\mathrm{QED}}\delta(s'+t_1+u_1)\Big[C_A\ln^2(\Delta/m^2) \\
& +\ln(\Delta/m^2)\Bigg(\Big(\ln(-t_1/m^2)-\ln(-u_1/m^2)-\ln(\mu_F^2/m^2)\Big)C_A \\
& -\frac{2m^2-s}{s\beta}\ln(\chi)(C_A-2C_F)-2C_F\Bigg) \\
& +\frac{\beta_0^{lf}}{4}\Big(\ln(\mu_R^2/m^2)-\ln(\mu_F^2/m^2)\Big)+f_{\vec\kappa}(s',u_1,t_1,q^2)\Big]
\end{aligned} \tag{71}$$

where the functions $f_{\vec\kappa}$ in Eq. 71 contain logarithms and dilogarithms with different, complicated arguments, but they do not depend on Δ, μ_F^2, μ_R^2 nor n_f and $\beta_0^{lf} = (11C_A - 2n_{lf})/3$.

The corresponding finite, reduced partonic cross section for the light quark initiated

Bethe-Heitler and Compton processes reads

$$s'^2 \frac{d^2\sigma^{(1),fin}_{\vec{\kappa},q}}{dt_1 du_1} = \alpha\alpha_S K_{q\gamma} N_C \Bigg[-\frac{1}{u_1} e_H^2 P_{\kappa_2,\mathrm{g}q}(x_1)$$
$$\left\{ B^{(0)}_{\vec{\kappa},\mathrm{QED}} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)} \right) - \ln(\mu_F^2/m^2) - 2\partial_\epsilon E_{\vec{\kappa}}(\epsilon=0) \right) \right.$$
$$\left. -2B^{(1)}_{\vec{\kappa},\mathrm{QED}} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \right\}$$
$$+ C_F \frac{s_4}{4\pi(s_4+m^2)} \left(\int d\Omega_n \, e_H^2 A_{\vec{\kappa},1} \right)^{finite}$$
$$+ C_F \frac{s_4}{4\pi(s_4+m^2)} \int d\Omega_4 \, e_L^2 A_{\vec{\kappa},2} + C_F \frac{s_4}{4\pi(s_4+m^2)} \int d\Omega_4 \, e_H e_L A_{\vec{\kappa},3} \Bigg] \tag{72}$$

5 Partonic Results

FiXme Error!

FiXme Error: write partonic

We decompose the partonic structure functions by their charge structure and write

$$\sigma_{\vec{\kappa}}(m^2, q^2, s) = \frac{\alpha\alpha_s}{m^2} \left\{ e_H^2 c^{(0)}_{\vec{\kappa},\mathrm{g}} + 4\pi\alpha_s \left[e_H^2 \left(c^{(1)}_{\vec{\kappa},\mathrm{g}} + \bar{c}^{(1),F}_{\vec{\kappa},\mathrm{g}} \ln(\mu_F^2/m^2) + \bar{c}^{(1),R}_{\vec{\kappa},\mathrm{g}} \ln(\mu_R^2/m^2) \right) \right] \right.$$
$$\left. +4\pi\alpha_s \left[e_H^2 c^{(1)}_{\vec{\kappa},q} + e_L^2 d^{(1)}_{\vec{\kappa},q} \right] \right\} \tag{73}$$

From Eq. 71 we find immediately

$$\bar{c}^{(1),R}_{\vec{\kappa},\mathrm{g}} = \frac{\beta_0^{lf}}{16\pi^2} c^{(0)}_{\vec{\kappa},\mathrm{g}} \tag{74}$$

and for convenience we then also define $\bar{c}^{(1)}_{\vec{\kappa},\mathrm{g}} = \bar{c}^{(1),F}_{\vec{\kappa},\mathrm{g}} + \bar{c}^{(1),R}_{\vec{\kappa},\mathrm{g}}$.

5.1 $c^{(0)}_{\vec{\kappa},\mathrm{g}}$

In leading order, we find

$$c^{(0)}_{\mathrm{VV},F_2,\mathrm{g}} = -\frac{\pi {\rho'}^3}{4\rho^2 {\rho_q}^2}\Big[2\beta\left(\rho^2+{\rho_q}^2+\rho\rho_q(6+\rho_q)\right) + \left(2{\rho_q}^2+2\rho{\rho_q}^2+\rho^2(2-(-4+\rho_q)\rho_q)\right)\ln(\chi)\Big] \tag{75}$$

$$c^{(0)}_{\mathrm{VV},F_L,\mathrm{g}} = -\frac{\pi {\rho'}^3}{\rho\rho_q}\left[2\beta+\rho\ln(\chi)\right] \tag{76}$$

$$c^{(0)}_{\mathrm{VV},2xg_1,\mathrm{g}} = \frac{\pi {\rho'}^2}{2\rho\rho_q}\left[\beta(\rho+3\rho_q)+(\rho+\rho_q)\ln(\chi)\right] \tag{77}$$

$$c^{(0)}_{\mathrm{AA},F_2,\mathrm{g}} = \frac{\pi {\rho'}^3}{4\rho^2 {\rho_q}^2}\Big[2\beta\left(\rho^2+{\rho_q}^2+\rho\rho_q(6+\rho_q)\right) - \left(-6\rho{\rho_q}^2+2(-1+\rho_q){\rho_q}^2+\rho^2(-2+(-2+\rho_q)\rho_q)\right)\ln(\chi)\Big] \tag{78}$$

$$c^{(0)}_{\mathrm{AA},F_L,\mathrm{g}} = -\frac{\pi {\rho'}^3}{2\rho^2\rho_q}\Big[2\beta\rho(2+\rho_q)-\left(\rho^2(-1+\rho_q)-4\rho\rho_q+{\rho_q}^2\right)\ln(\chi)\Big] \tag{79}$$

$$c^{(0)}_{\mathrm{AA},2xg_1,\mathrm{g}} = c^{(0)}_{\mathrm{VV},2xg_1,\mathrm{g}} \tag{80}$$

$$c^{(0)}_{\mathrm{VA},xF_3,\mathrm{g}} = c^{(0)}_{\mathrm{VA},g_4,\mathrm{g}} = c^{(0)}_{\mathrm{VA},g_L,\mathrm{g}} = 0 \tag{81}$$

Near threshold we find

$$c^{(0),\mathrm{thr}}_{\mathrm{VV},F_2,\mathrm{g}} = \frac{\pi\beta\rho_q}{2(\rho_q-1)} \tag{82}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{VV},F_L,\mathrm{g}} = \frac{4\pi\beta^3\rho_q^2}{3(1-\rho_q)^3} \tag{83}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{VV},2xg_1,\mathrm{g}} = c^{(0),\mathrm{thr}}_{\mathrm{AA},2xg_1,\mathrm{g}} = c^{(0),\mathrm{thr}}_{\mathrm{VV},F_2,\mathrm{g}} \tag{84}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{AA},F_2,\mathrm{g}} = \frac{\pi\beta\rho_q^2}{1-\rho_q} \tag{85}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{AA},F_2,\mathrm{g}} = \frac{\pi\beta(1-2\rho_q)\rho_q}{2(\rho_q-1)} \tag{86}$$

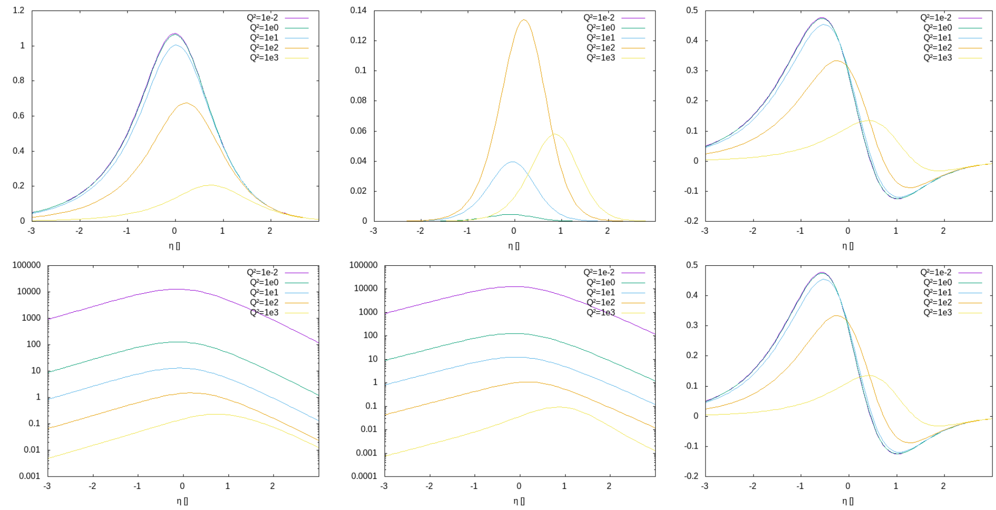

Figure 2: leading order scaling functions $c^{(0)}_{k,\mathrm{g}}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2)-1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.2 $c^{(1)}_{\vec{\kappa},\mathbf{g}}$

Near threshold, we find

$$c^{(1),\mathrm{thr}}_{\vec{k},\mathrm{g}} = c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{1}{\pi^2}\left[C_A\left(a^{(1,2)}_{\vec{k},\mathrm{g}}\ln^2(\beta) + a^{(1,1)}_{\vec{k},\mathrm{g}}\ln(\beta) - \frac{\pi^2}{16\beta} + a^{(1,0)}_{\vec{k},\mathrm{g,OK}}\right)\right.$$
$$\left. + 2C_F\left(\frac{\pi^2}{16\beta} + a^{(1,0)}_{\vec{k},\mathrm{g,QED}}\right)\right], \tag{87}$$

with

$$a^{(1,2)}_{\vec{k},\mathrm{g}} = 1 \tag{88}$$
$$a^{(1,1)}_{\mathrm{VV},F_2,\mathrm{g}} = -\frac{5}{2} + 3\ln(2) \tag{89}$$
$$a^{(1,1)}_{\mathrm{VV},F_L,\mathrm{g}} = a^{(1,1)}_{\mathrm{VV},F_2,\mathrm{g}} - \frac{2}{3} \tag{90}$$
$$a^{(1,1)}_{\mathrm{VV},2xg_1,\mathrm{g}} = a^{(1,1)}_{\mathrm{AA},F_2,\mathrm{g}} = a^{(1,1)}_{\mathrm{AA},F_L,\mathrm{g}} = a^{(1,1)}_{\mathrm{AA},2xg_1,\mathrm{g}} = a^{(1,1)}_{\mathrm{VV},F_2,\mathrm{g}} \tag{91}$$

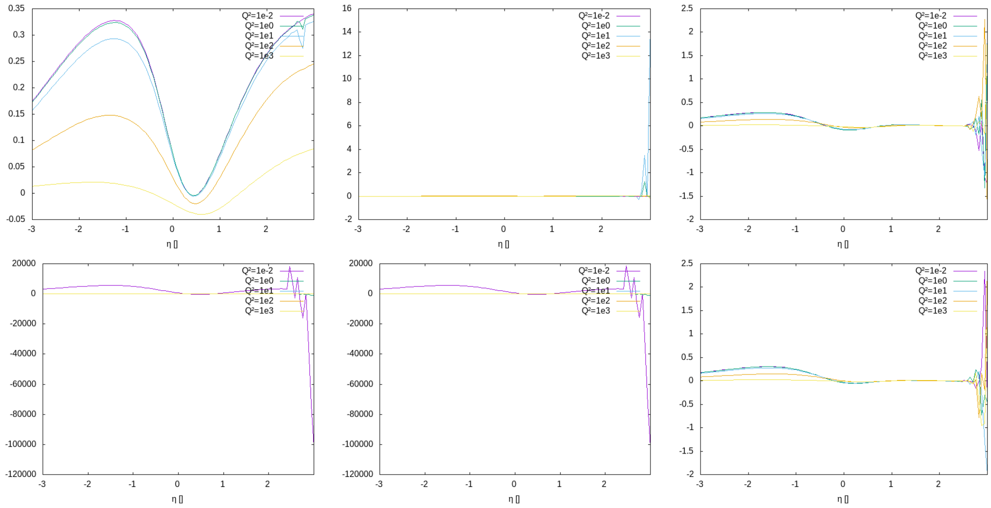

Figure 3: next-to-leading order scaling functions $c^{(1)}_{k,\mathrm{g}}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.3 $\bar{c}^{(1)}_{\vec{\kappa},\mathbf{g}}$

For the scaling functions we find at this order:

$$\bar{c}^{(1)}_{\mathrm{VA},xF_3,\mathrm{g}} = \bar{c}^{(1)}_{\mathrm{VA},g_4,\mathrm{g}} = \bar{c}^{(1)}_{\mathrm{VA},g_L,\mathrm{g}} = 0 \tag{92}$$

and

$$\bar{c}^{(1)}_{\mathrm{VV},2xg_1,\mathrm{g}} = \bar{c}^{(1)}_{\mathrm{AA},2xg_1,\mathrm{g}} \tag{93}$$

and furthermore near threshold, we find

$$\bar{c}^{(1),\mathrm{thr}}_{\vec{k},\mathrm{g}} = c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{1}{\pi^2} C_A \left(\bar{a}^{(1,1)}_{\vec{k},\mathrm{g}} \ln(\beta) + \bar{a}^{(1,0)}_{\vec{k},\mathrm{g}} \right) \,, \tag{94}$$

with

$$\bar{a}^{(1,1)}_{\vec{k},\mathrm{g}} = -\frac{1}{2} \tag{95}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_2,\mathrm{g}} = -\frac{1}{4} \ln\left(\frac{16\chi_q}{(1+\chi_q)^2}\right) + \frac{1}{2} \tag{96}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_L,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,\mathrm{g}} + \frac{1}{6} \tag{97}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},2xg_1,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{AA},F_2,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{AA},F_L,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{AA},2xg_1,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,\mathrm{g}} \tag{98}$$

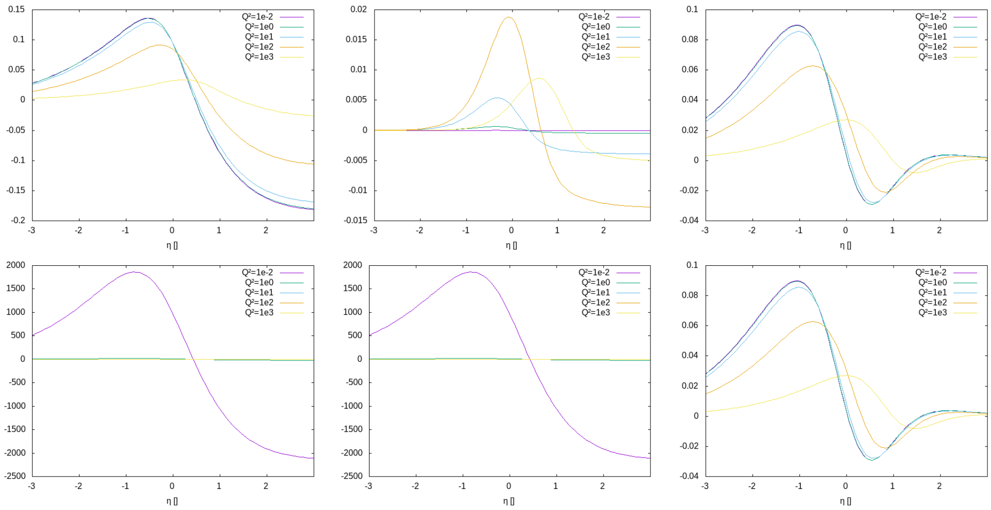

Figure 4: next-to-leading order scaling functions $\bar{c}^{(1)}_{k,\mathrm{g}}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.4 $c^{(1)}_{\vec{\kappa},\mathbf{q}}$

Near threshold, we find

$$c_{\vec{k},q}^{(1),\mathrm{thr}} = c_{\vec{k},\mathrm{g}}^{(0),\mathrm{thr}} \frac{\beta^2 \rho_q}{\pi^2(\rho_q - 1)} \frac{K_{q\gamma}}{6K_{\mathrm{g}\gamma}} \left[a_{\vec{k},q}^{(1,1)} \ln(\beta) + a_{\vec{k},q}^{(1,0)} \right], \tag{99}$$

with

$$a_{\mathrm{VV},F_2,q}^{(1,1)} = 1 \tag{100}$$

$$a_{\mathrm{VV},F_L,q}^{(1,1)} = a_{\mathrm{VV},F_2,q}^{(1,1)} - \frac{2}{3} \tag{101}$$

$$a_{\mathrm{VV},2xg_1,q}^{(1,1)} = a_{\mathrm{AA},F_2,q}^{(1,1)} = a_{\mathrm{AA},F_L,q}^{(1,1)} = a_{\mathrm{AA},2xg_1,q}^{(1,1)} = a_{\mathrm{VV},F_2,q}^{(1,1)} \tag{102}$$

$$a_{\mathrm{VV},F_2,q}^{(1,0)} = -\frac{13}{12} + \frac{3}{2}\ln(2) \tag{103}$$

$$a_{\mathrm{VV},F_L,q}^{(1,0)} = -\frac{77}{100} + \frac{9}{10}\ln(2) \tag{104}$$

$$a_{\mathrm{VV},2xg_1,q}^{(1,0)} = a_{\mathrm{VV},F_2,q}^{(1,0)} - \frac{1}{4} \tag{105}$$

$$a_{\mathrm{AA},F_2,q}^{(1,0)} = a_{\mathrm{AA},F_L,q}^{(1,0)} = a_{\mathrm{AA},2xg_1,q}^{(1,0)} = a_{\mathrm{VV},F_2,q}^{(1,0)} \tag{106}$$

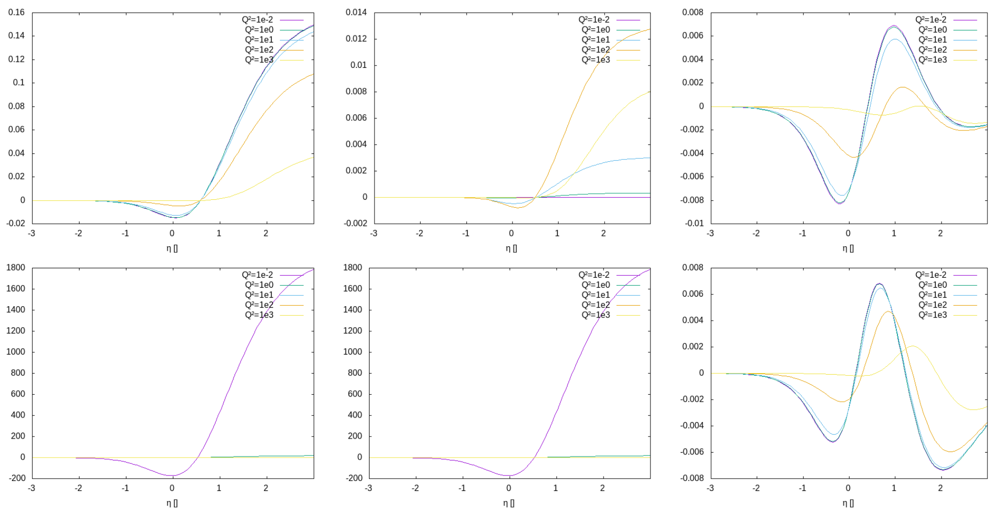

Figure 5: next-to-leading order scaling functions $c_{k,q}^{(1)}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.5 $\bar{c}^{(1),F}_{\vec{\kappa},\mathbf{q}}$

For the scaling functions we find at this order:

$$\bar{c}^{(1),F}_{\mathrm{VA},xF_3,q} = \bar{c}^{(1),F}_{\mathrm{VA},g_4,q} = \bar{c}^{(1),F}_{\mathrm{VA},g_L,q} = 0 \tag{107}$$

and

$$\bar{c}^{(1),F}_{\mathrm{VV},2xg_1,q} = \bar{c}^{(1),F}_{\mathrm{AA},2xg_1,q} \tag{108}$$

and furthermore near threshold, we find

$$\bar{c}^{(1),F,\mathrm{thr}}_{\vec{k},q} = -c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{\beta^2\rho_q}{\pi^2(\rho_q-1)} \frac{K_{q\gamma}}{24K_{\mathrm{g}\gamma}} \cdot \bar{a}^{(1,0)}_{\vec{k},q} \tag{109}$$

with

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_2,q} = 1 \tag{110}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_L,q} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,q} - \frac{2}{3} \tag{111}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},2xg_1,q} = \bar{a}^{(1,0)}_{\mathrm{AA},F_2,q} = \bar{a}^{(1,0)}_{\mathrm{AA},F_L,q} = \bar{a}^{(1,0)}_{\mathrm{AA},2xg_1,q} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,q} \tag{112}$$

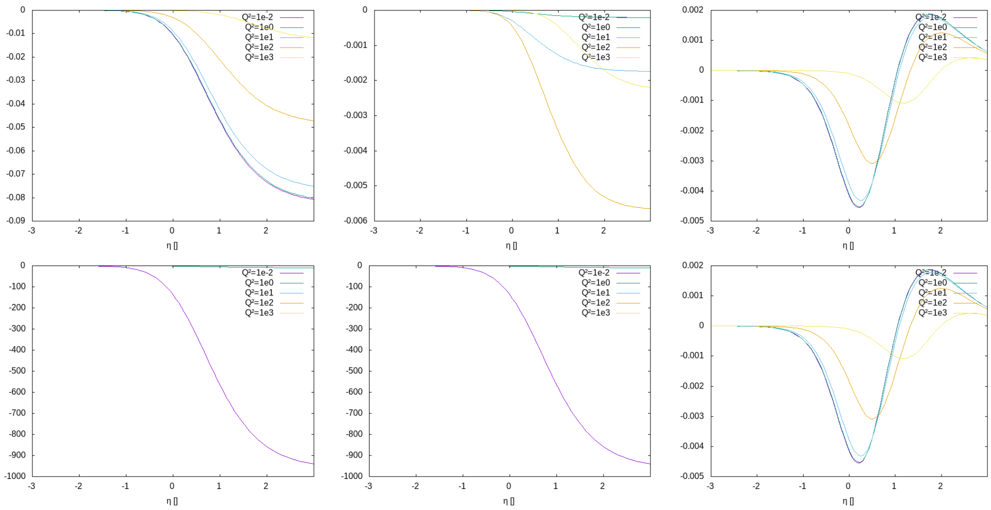

Figure 6: next-to-leading order scaling functions $\bar{c}^{(1),F}_{k,q}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2)-1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.6 $d^{(1)}_{\vec{\kappa},\mathbf{q}}$

For the scaling functions we find at this order:

$$d^{(1)}_{\mathrm{VA},g_4,q} = d^{(1)}_{\mathrm{VA},g_L,q} = 0 = d^{(1)}_{\mathrm{AA},2xg_1,q} \tag{113}$$

and

$$d^{(1)}_{\mathrm{VV},F_2,q} = d^{(1)}_{\mathrm{AA},F_2,q} \qquad d^{(1)}_{\mathrm{VV},F_L,q} = d^{(1)}_{\mathrm{AA},F_L,q} \qquad d^{(1)}_{\mathrm{VA},xF_3,q} = d^{(1)}_{\mathrm{VV},2xg_1,q} \tag{114}$$

For $\chi' \to 0$ we find:

$$\begin{aligned} d^{(1)}_{\mathrm{VV},F_2,q} &= -\frac{\rho}{9\pi}\left(\mathrm{Li}_2(-\chi) - \frac{1}{4}\ln^2(\chi) + \frac{\pi^2}{12} + \ln(\chi)\ln(1+\chi)\right) \\ &\qquad + \beta\frac{\rho(718+5\rho)}{2592\pi} + \frac{\rho(232+9\rho^2)}{1728\pi}\ln(\chi) + \mathcal{O}(\chi') \end{aligned} \tag{115}$$

$$d^{(1)}_{\mathrm{VV},F_L,q} = \left(\beta\frac{-38+23\rho}{54\pi} + \frac{-8+3\rho^2}{36\pi}\ln(\chi)\right)\chi' + \mathcal{O}({\chi'}^2) \tag{116}$$

$$d^{(1)}_{\mathrm{VV},2xg_1,q} = d^{(1)}_{\mathrm{AA},F_2,q} = d^{(1)}_{\mathrm{VA},xF_3,q} \tag{117}$$

$$d^{(1)}_{\mathrm{AA},F_L,q} = d^{(1)}_{\mathrm{VV},F_L,q} \tag{118}$$

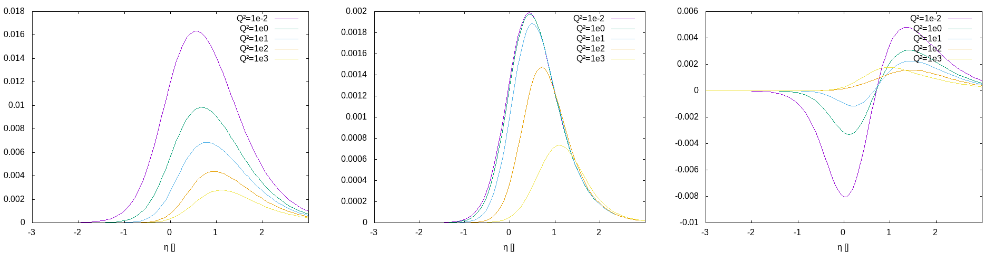

Figure 7: next-to-leading order scaling functions $\bar{c}^{(1),F}_{k,q}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2)-1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

6 Hadronic Results

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FiXme Error: write hadronic

7 Summary

FiXme Error!

FiXme Error: write summary

A References

[1] **Particle Data Group** Collaboration, C. Patrignani et al., "Review of Particle Physics," Chin. Phys. **C40** no. 10, (2016) 100001.

[2] I. Bojak, NLO QCD corrections to the polarized photoproduction and hadroproduction of heavy quarks. PhD thesis, Dortmund U., 2000. `arXiv:hep-ph/0005120 [hep-ph]`.

[3] G. Altarelli and G. Parisi, "Asymptotic Freedom in Parton Language," Nucl. Phys. **B126** (1977) 298–318.

[4] W. Vogelsang, "A Rederivation of the spin dependent next-to-leading order splitting functions," Phys. Rev. **D54** (1996) 2023–2029, `arXiv:hep-ph/9512218 [hep-ph]`.

List of Corrections