1 Passarino-Veltman decomposition

1.1 Definitions

[1]:

$$A(m) = \frac{1}{i\pi^2}\int d^nq \frac{1}{q^2+m^2} \tag{1}$$

$$B_0(p,m_1,m_2) = \frac{1}{i\pi^2}\int d^nq \frac{1}{(q^2+m_1^2)((q+p)^2+m_2^2)} \tag{2}$$

and apart from their pole term (called Δ - see [1, eq. D.1]), they keep $n=4$.

[2, 3]:

$$A(m) = \mu^{-\epsilon}\int \frac{d^nq}{(2\pi)^n}\frac{1}{q^2-m^2} \tag{3}$$

$$B(q_1,m_1,m_2) = \mu^{-\epsilon}\int \frac{d^nq}{(2\pi)^n}\frac{1}{(q^2-m_1^2)((q+q_1)^2-m_2^2)} \tag{4}$$

and $n=4+\epsilon$. ([2] writes "The notations for the one-, two-, three-, and four-point functions have been taken over from Ref. [1]." - obviously they do not.)

`HEPMath`[4] and `FeynCalc`[5, 6] refer to `LoopTools`[7, 8]. [8, eq. (1.1)] and [9, eq. (2.6)]:

$$T^N_{\mu_1\ldots\mu_P} = \frac{\mu^{4-D}}{i\pi^{D/2}\, r_\Gamma}\int d^Dq \frac{q_{\mu_1}\cdots q_{\mu_P}}{\left[q^2-m_1^2\right]\left[(q+k_1)^2-m_2^2\right]\cdots\left[(q+k_{N-1})^2-m_N^2\right]} \tag{5}$$

$$r_\Gamma = \frac{\Gamma^2(1-\varepsilon)\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}, \quad D = 4-2\varepsilon$$

later in the code they use a different signature (to avoid any vector structure):

$$\begin{aligned} &A(m^2), B_0(p^2,m_1^2,m_2^2), C_0(p_1^2,p_2^2,(p_1+p_2)^2,m_1^2,m_2^2,m_3^2) \\ &D_0(p_1^2,p_2^2,p_3^2,p_4^2,(p_1+p_2)^2,(p_2+p_3)^2,m_1^2,m_2^2,m_3^2,m_4^2) \end{aligned} \tag{6}$$

[10]:

$$T^N_{\mu_1\ldots\mu_P}(p_1,\ldots,p_{N-1},m_0,\ldots,m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq \frac{q_{\mu_1}\cdots q_{\mu_P}}{L_0L_1\cdots L_{N-1}} \tag{7}$$

$$L_0 = q^2-m_0^2+i\varepsilon \tag{8}$$

$$L_i = (q+p_i)^2-m_i^2+i\varepsilon\, i=1,\ldots,N-1 \tag{9}$$

I will stick to the integrals of [3] as it is the most natural form, I think, and to the non-vector signature, if possible.

1.2 Decomposition Labeling

[1, 3]:

$$B_\mu(p,m_1,m_2) = p_\mu B_1(p,m_1,m_2) \tag{10}$$
$$B_{\mu\nu} = p_\mu p_\nu B_{21} + g_{\mu\nu} B_{22} \tag{11}$$
$$C_\mu(p_1,p_2,m_1,m_2,m_3) = p_{1,\mu} C_{11} + p_{2,\mu} C_{12} \tag{12}$$
$$C_{\mu\nu} = p_{1,\mu} p_{1,\nu} C_{21} + p_{2,\mu} p_{2,\nu} C_{22} + (p_{1,\mu} p_{2,\nu} + p_{1,\nu} p_{2,\mu}) C_{23} + g_{\mu\nu} C_{24} \tag{13}$$

The arguments of the functions are always inherited.

`HEPMath`, `FeynCalc`, `LoopTools`, [9]:

$$B_\mu(p,m_1,m_2) = p_\mu B_1(p,m_1,m_2) \tag{14}$$
$$B_{\mu\nu} = g_{\mu\nu} B_{00} + p_\mu p_\nu B_{11} \tag{15}$$
$$C_\mu(p_1,p_2,m_1,m_2,m_3) = p_{1,\mu} C_1 + p_{2,\mu} C_2 = \sum_{j=1}^{2} p_{j,\mu} C_j \tag{16}$$
$$C_{\mu\nu} = p_{1,\mu} p_{1,\nu} C_{11} + p_{2,\mu} p_{2,\nu} C_{22} + (p_{1,\mu} p_{2,\nu} + p_{1,\nu} p_{2,\mu}) C_{12} + g_{\mu\nu} C_{00} \tag{17}$$
$$= g_{\mu\nu} C_{00} + \sum_{j,k=1}^{2} p_{j,\mu} p_{k,\nu} C_{jk} \tag{18}$$

The arguments of the functions are always inherited.

I will stick to `HEPMath` as it is the more generic and extensible form, I think.

1.3 B Decomposition

define

$$f_1 = m_1^2 - m_0^2 - p^2 \tag{19}$$

then one finds easily

$$B_1(p^2,m_0^2,m_1^2) = \frac{1}{2p^2}\left(f_1 B_0(p^2,m_0^2,m_1^2) + A_0(m_0^2) - A_0(m_1^2)\right) \tag{20}$$
$$B_{00}(p^2,m_0^2,m_1^2) = \frac{1}{2(n-1)}\left(2m_0^2 B_0(p^2,m_0^2,m_1^2) + A_0(m_1^2) - f_1 B_1(p^2,m_0^2,m_1^2)\right) \tag{21}$$
$$B_{11}(p^2,m_0^2,m_1^2) = \frac{1}{2p^2}\left(f_1 B_0(p^2,m_0^2,m_1^2) + A_0(m_1^2) - 2B_{00}(p^2,m_0^2,m_1^2)\right) \tag{22}$$

in accordance with [3, 9].

Concering B_1 [1] and `LoopTools` use the following identity

$$A_0(m_0^2) - A_0(m_1^2) = (m_0^2 - m_1^2)B_0(0, m_0^2, m_1^2) \tag{23}$$

that might help away with

> In case m_1 and/or m_2 are very large the expression on the right-hand side of eq. (20) suffers very strong cancellations: the total is very much smaller than the individual terms. For this reason we have not used these algebraic relations, except to rewrite self-energy diagrams as much as possible in a form most suitable for numerical evaluation. ([1, below eq. D.6])

To compare the other results to [1] and `LoopTools` one has to use the *strict* $n \to 4$ limit and the following identities[10]:

$$(n-4)B_{00}(p^2, m_0^2, m_1^2) = \frac{1}{6}(p^2 - 3m_0^2 - 3m_1^2) \tag{24}$$

$$(n-4)B_{11}(p^2, m_0^2, m_1^2) = -\frac{2}{3} \tag{25}$$

2 Scalar Integrals

We focus on:

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) \tag{26}$$

$$k_1^2 = 0 \quad p_1^2 = p_2^2 = m^2 \quad (p_1 + p_2)^2 = s \quad (p_2 - q)^2 = t \quad (p_1 - q)^2 = u \tag{27}$$

define some shortcuts

$$0 \leq \rho = \frac{4m^2}{s} \leq 1 \qquad 0 \leq \beta = \sqrt{1-\rho} \leq 1 \qquad 0 \leq \chi = \frac{1-\beta}{1+\beta} \leq 1 \tag{28}$$

$$\rho_q = \frac{4m^2}{q^2} \leq 0 \qquad 1 \leq \beta_q = \sqrt{1-\rho_q} \qquad 0 \leq \chi_q = -\frac{1-\beta_q}{1+\beta_q} \leq 1 \tag{29}$$

2.1 One-Point Function A_0

[10]:

$$A_0(m^2) = -\frac{i}{16\pi^2} m^2 \left(\frac{m^2}{4\pi\mu^2}\right)^{(n-4)/2} \Gamma(1-n/2) \tag{30}$$

$$= \frac{im^2}{16\pi^2}\left(\Delta - \log(m^2/\mu^2) + 1\right) + O(n-4) \tag{31}$$

$$= iC_\epsilon m^2 \left(-\frac{2}{\epsilon} + 1\right) + O(n-4) \tag{32}$$

$$\Delta = \frac{2}{4-n} - \gamma_E + \log(4\pi) \tag{33}$$

$$C_\epsilon = \frac{1}{16\pi^2}\exp\left(\left(\gamma_E - \log(4\pi) + \log\left(m^2/\mu^2\right)\right)\frac{\epsilon}{2}\right) \tag{34}$$

this is *up to order* $O(n-4)$ in accordance with [3][11], but NOT beyond - see also [3, eq. (A.12)]. So we can treat C_ϵ and Δ as equal.

2.2 Two-Point Function B_0

In [10, eq. (4.23)] is a generic function given and we end up with

$$B_0(s, m^2, m^2) = iC_\epsilon\left(-\frac{2}{\epsilon} + 2 + \beta\log(\chi)\right) \tag{35}$$

$$B_0(q^2, m^2, m^2) = iC_\epsilon\left(-\frac{2}{\epsilon} + 2 + \beta_q\log(\chi_q)\right) \tag{36}$$

$$B_0(0, m^2, m^2) = iC_\epsilon\left(-\frac{2}{\epsilon}\right) \tag{37}$$

$$B_0(m^2, 0, m^2) = iC_\epsilon\left(-\frac{2}{\epsilon} + 2\right) \tag{38}$$

$$B_0(t, 0, m^2) = iC_\epsilon\left(-\frac{2}{\epsilon} + 2 - \frac{t-m^2}{t}\ln\left(-\frac{t-m^2}{m^2}\right)\right) \tag{39}$$

focussing on imaginary part *only*; this in accordance with [3][11].

2.3 Three-Point Function C_0

Again, in [10, eq. (4.26)] is a generic function given.

First, we compute $C_0(s, q^2, 0, m^2, m^2, m^2)$ and by taking the limit $k_1^2 \to 0$ (or equivalenty

$s_4 \to 0$) we end up with:

$$C_0(s, q^2, 0, m^2, m^2, m^2) = \frac{i}{16\pi^2} \cdot \frac{1}{s - q^2} \left(\mathrm{Li}_2\left(\frac{2}{1+\beta_q}\right) + \mathrm{Li}_2\left(\frac{2}{1-\beta_q}\right) \right.$$
$$\left. - \mathrm{Li}_2\left(\frac{2}{1+\beta}\right) - \mathrm{Li}_2\left(\frac{2}{1-\beta}\right)\right) \tag{40}$$

with [12] we find:

$$\mathrm{Li}_2\left(\frac{2}{1+b}\right) + \mathrm{Li}_2\left(\frac{2}{1-b}\right) = 3\zeta(2) + \frac{1}{2}\ln^2\left(\frac{1-b}{1+b}\right) - \ln\left(\frac{1-b}{1+b}\right)\ln\left(-\frac{1-b}{1+b}\right) \tag{41}$$

and if we focus on real part *only*, we find:

$$\mathrm{Li}_2\left(\frac{2}{1+\beta}\right) + \mathrm{Li}_2\left(\frac{2}{1-\beta}\right) = 3\zeta(2) - \frac{1}{2}\ln^2(\chi) \tag{42}$$

$$\mathrm{Li}_2\left(\frac{2}{1+\beta_q}\right) + \mathrm{Li}_2\left(\frac{2}{1-\beta_q}\right) = -\frac{1}{2}\ln^2(\chi_q) \tag{43}$$

Moreover it is

$$\lim_{q^2 \to 0}\left[\mathrm{Li}_2\left(\frac{2}{1+\beta_q}\right) + \mathrm{Li}_2\left(\frac{2}{1-\beta_q}\right)\right] = 0 \tag{44}$$

So we get:

$$C_0(s, q^2, 0, m^2, m^2, m^2) = iC_\epsilon \frac{1}{s - q^2}\left(\frac{1}{2}\ln^2(\chi) - \frac{1}{2}\ln^2(\chi_q) - 3\zeta(2)\right) \tag{45}$$

$$C_0(s, 0, 0, m^2, m^2, m^2) = iC_\epsilon \frac{1}{s}\left(\frac{1}{2}\ln^2(\chi) - 3\zeta(2)\right) \tag{46}$$

in accordance with [3][11][13]. These results can also be obtained by the methods described in [3, chap. 3].

Next, we compute $C_0(m^2, 0, t, 0, m^2, m^2)$ again by taking the limit $k_1^2 \to 0$ we end up with:

$$C_0(m^2, 0, t, 0, m^2, m^2) = \frac{i}{16\pi^2} \cdot \frac{1}{t - m^2}\left(2\,\mathrm{Li}_2(2) + \mathrm{Li}_2(m^2/t) - \frac{\pi^2}{6}\right.$$
$$\left. - \mathrm{Li}_2((t + m^2)/m^2) - \mathrm{Li}_2((m^2 + t)/t)\right) \tag{47}$$

Using [12] and focussing on real part, we find

$$\mathrm{Li}_2(2) = \frac{\pi^2}{4} - i\pi\ln(2) \tag{48}$$

$$2\,\mathrm{Li}_2(2) + \mathrm{Li}_2(1/z) - \frac{\pi^2}{6} - \mathrm{Li}_2(1+z) - \mathrm{Li}_2(1+1/z) = \frac{\pi^2}{6} - \mathrm{Li}_2(z) \tag{49}$$

So we get:

$$C_0(m^2, 0, t, 0, m^2, m^2) = iC_\epsilon \frac{1}{t - m^2}\left(\zeta(2) - \mathrm{Li}_2(t/m^2)\right) \tag{50}$$

in accordance with [3][11].

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List of Corrections