1 Introduction

This work is mainly based on the paper "Complete $O(\alpha_S)$ corrections to heavy-flavour structure functions in electroproduction" by Laenen et. al.[1] - that is, it recalculates all properties and formulas. It extends then the application to the equivalent *polarized* processes. The treating of the polarized processes can for example be found in [2] and we will use many ideas and technices from there. **FiXme Error: more** FiXme Error!

1.1 Motivation

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1.2 Notation

To collect all soft and collinear poles we have to calculate in $n = 4 + \epsilon$ dimension. Unfortunaly the extension for *polarized* processes is nontrivial, because the occuring Levi-Civita tensors $\varepsilon_{\mu\nu\rho\sigma}$ and γ_5. A common choice to deal with these objects is the HVBM prescription[3] that keeps those two objects four dimensional at the price for splitting the full n-dimensional space into a $(n-4)$-dimensional space, called "hat-space", and a four-dimensional space (that is actually never used).

In leading order (LO) we have to consider the following processes

$$\gamma^*(q;\sigma_q) + \mathrm{g}(k_1;\sigma_{k_1}) \to Q(p_1) + \overline{Q}(p_2) \tag{1}$$

The corresponding parton structure tensor $W^{(0)}_{\mu\mu'}$ can then be written as **FiXme Error: avoid all order expr?** FiXme Error!

$$\begin{aligned}
&W^{(0)}_{\mu\mu'}(k_1,q;s,t_1,u_1,q^2;\sigma_{k_1}\sigma_q) \\
&= \frac{1}{2}E_\epsilon K_{\gamma\mathrm{g}}\int\frac{d^{n-1}p_1}{2E_1(2\pi)^{n-1}}\int\frac{d^{n-1}p_2}{2E_2(2\pi)^{n-1}}\delta(p_1^2-m^2)\delta(p_2^2-m^2) \\
&\quad (2\pi)^n\delta^{(n)}(k_1+q-p_1-p_2)\,\mathcal{M}^{(0)}_{\mu}(\sigma_{k_1},\sigma_q)\,\mathcal{M}^{(0)}_{\mu'}(\sigma_{k_1},\sigma_q)
\end{aligned} \tag{2}$$

where the initial 1/2 is the initial state spin average, $K_{\gamma\mathrm{g}}$ is the color average,

$$E_\epsilon := \begin{cases} 1/(1+\epsilon/2) & \text{unpolarized} \\ 1 & \text{polarized} \end{cases} \tag{3}$$

accounts for initial freedom in n dimensions for bosons and we defined the following Mandelstam variables:

$$s = (q+k_1)^2, \quad t_1 = t - m^2 = (k_1-p_2)^2 - m^2, \quad u_1 = u - m^2 = (q-p_2)^2 - m^2 \tag{4}$$

$$s' = s - q^2, \quad u_1' = u_1 - q^2 \tag{5}$$

FiXme Error: move to LO? The Lorentz indices μ and μ' refer to the virtual photon that is exchanged with the scattering lepton. FiXme Error!

By using Lorentz covariance, hermiticity, parity invariance and current conservation the parton structure tensor can be decomposed into several parts:

$$\begin{aligned} W_{\mu\mu'}(k_1, q; s, t_1, u_1, q^2; \sigma_{k_1}, \sigma_q) = &\left(-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2}\right) \frac{d^2\sigma_T(s, t_1, u_1, q^2)}{dt_1 du_1} \\ &+ \left(k_{1,\mu} - \frac{k_1 \cdot q}{q^2} q_\mu\right) \left(k_{1,\mu'} - \frac{k_1 \cdot q}{q^2} q_{\mu'}\right) \left(\frac{-4q^2}{s'^2}\right) \\ &\cdot \left(\frac{d^2\sigma_T(s, t_1, u_1, q^2)}{dt_1 du_1} + \frac{d^2\sigma_L(s, t_1, u_1, q^2)}{dt_1 du_1}\right) \end{aligned} \tag{6}$$

FiXme Error: extend We can then define appropiate projection operators[1, 4]: FiXme Error!

$$\mathcal{P}_{G,\mu\mu'} = -g_{\mu\mu'} \qquad b_G(\epsilon) = \frac{1}{2(1+\epsilon/2)} \tag{7}$$

$$\mathcal{P}_{L,\mu\mu'} = -\frac{4q^2}{s'^2} k_{1,\mu} k_{1,\mu'} \qquad b_L(\epsilon) = 1 \tag{8}$$

$$\mathcal{P}_{P,\mu\mu'} = i\varepsilon_{\mu\mu'\rho\rho'} \frac{q^\rho k_1^{\rho'}}{s'} \qquad b_P(\epsilon) = 1 \tag{9}$$

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$$\frac{d^2\sigma_k(s, t_1, u_1, q^2)}{dt_1 tu_1} = b_k(\epsilon) \mathcal{P}_{k,\mu\mu'} W^{\mu\mu'} \tag{10}$$

with $k \in \{G, L, P\}$ denoting (here and mostly ever after) the projection type. The transverse partonic cross section $d\sigma_T$ can be reconstructed from the above definitions by using

$$\frac{d^2\sigma_T}{dt_1 du_1} = \frac{d^2\sigma_G}{dt_1 du_1} + b_G(\epsilon) \frac{d^2\sigma_L}{dt_1 du_1} \tag{11}$$

The final state spins are always summed over, but the initial spins have to be treated seperately: for unpolarized projections $k \in G, L$ they are also summed over, but for polarized $k = P$ they are combined as follows

$$\hat{\sum}_{G,\sigma} f(\sigma_{k_1}, \sigma_q) = \hat{\sum}_{L,\sigma} f(\sigma_{k_1}, \sigma_q) = \sum_{\sigma_{k_1}, \sigma_q \in \{+,-\}} f(\sigma_{k_1}, \sigma_q) \tag{12}$$

$$\hat{\sum}_{P,\sigma} f(\sigma_{k_1}, \sigma_q) = f(+,+) + f(-,-) - f(+,-) - f(-,+) \tag{13}$$

which keeps spin asymmetries well behaving.

When computing total cross section we define a set of partonic variables:

$$0 \leq \rho = \frac{4m^2}{s} \leq 1 \qquad 0 \leq \beta = \sqrt{1-\rho} \leq 1 \qquad 0 \leq \chi = \frac{1-\beta}{1+\beta} \leq 1 \tag{14}$$

$$\rho_q = \frac{4m^2}{q^2} \leq 0 \qquad 1 \leq \beta_q = \sqrt{1-\rho_q} \qquad 0 \leq \chi_q = -\frac{1-\beta_q}{1+\beta_q} \leq 1 \tag{15}$$

When computing Feynman diagrams a computer algebra system (CAS) is almost obligatory: common choices are `FORM`[5] or `Mathematica`[6] - for the later the most common choice is `TRACER`[7], but we have chosen `HEPMath`[8]. We used the Feynman rules given by [9].**FiXme Error: explain ghosts?** FiXme Error!

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2 Leading Order Calculations

In leading order we have to consider photon-gluon-fusion (PGF), that is

$$\gamma^*(q;\sigma_q) + \mathrm{g}(k_1;\sigma_{k_1}) \to Q(p_1) + \overline{Q}(p_2) \tag{16}$$

with two contributing diagrams depicted in figure **FiXme Error: todo**. The result can then be written as FiXme Error!

$$\hat{\sum}_{k,\sigma} \mathcal{P}_k^{\mu\mu'} \sum_{j=1}^{2} \mathcal{M}_{j,\mu}^{(0)}(\sigma_{k_1},\sigma_q)\, \mathcal{M}_{j,\mu'}^{(0)*}(\sigma_{k_1},\sigma_q) = 8g^2e^2e_H^2 N_C C_F B_{k,QED} \tag{17}$$

where g and e are the strong and electromagnetic coupling constants respectively and e_H is the magnitude of the heavy quark in units e. Further N_C corresponds to the gauge group $SU(N_C)$ and the color factor $C_F = (N_C^2-1)/(2N_C)$ refers to the second Casimir constant of the fundamental representation for the quarks. We then find:

$$\begin{aligned} B_{G,QED} &= \frac{t_1}{u_1} + \frac{u_1}{t_1} + \frac{4m^2s'}{t_1u_1}\left(1 - \frac{m^2s'}{t_1u_1}\right) + \frac{2s'q^2}{t_1u_1} + \frac{2q^4}{t_1u_1} + \frac{2m^2q^2}{t_1u_1}\left(2 - \frac{s'^2}{t_1u_1}\right) \\ &\quad + \epsilon\left\{-1 + \frac{s'^2}{t_1u_1} + \frac{s'q^2}{t_1u_1} - \frac{q^4}{t_1u_1} - \frac{m^2q^2s'^2}{t_1^2u_1^2}\right\} + \epsilon^2\frac{s'^2}{4t_1u_1} \end{aligned} \tag{18}$$

$$B_{L,QED} = -\frac{4q^2}{s'}\left(\frac{s}{s'} - \frac{m^2s'}{t_1u_1}\right) \tag{19}$$

$$B_{P,QED} = \frac{1}{2}\left(\frac{t_1}{u_1} + \frac{u_1}{t_1}\right)\left(\frac{2m^2s'}{t_1u_1} - 1 - \frac{2q^2}{s'}\right) \tag{20}$$

By using eq. (2) we can derive the n-dimensional $2 \to 2$ phase space

$$dPS_2 = \int\frac{d^np_1}{(2\pi)^{n-1}}\int\frac{d^np_2}{(2\pi)^{n-1}}\Theta(E_1)\delta(p_1^2-m^2)\Theta(E_2)\delta(p_2^2-m^2)(2\pi)^n\delta^{(n)}(k_1+q-p_1-p_2) \tag{21}$$

that can be solved by using the center-of-mass system (CMS) of the incoming particles[2]

$$q = \left(\frac{s+q^2}{2\sqrt{s}}, 0, 0, -\frac{s-q^2}{2\sqrt{s}}, \hat{0} \right) \qquad k_1 = \frac{s-q^2}{2\sqrt{s}} \left(1, 0, 0, 1, \hat{0}\right) \tag{22}$$

such that $q + k_1 = (\sqrt{s}, \vec{0})$ and $k_1^2 = 0$. For the outgoing particles it follows

$$p_1 = \frac{\sqrt{s}}{2} \left(1, 0, \beta \sin\theta, \beta \cos\theta, \hat{0}\right) \qquad p_2 = \frac{\sqrt{s}}{2} \left(1, 0, -\beta \sin\theta, -\beta \cos\theta, \hat{0}\right) \tag{23}$$

such that $p_1 + p_2 = (\sqrt{s}, \vec{0})$ and $p_1^2 = p_2^2 = m^2$. Finally we have to use the n-sphere

$$d^n x = \Omega_n x^{n-1} dx = \frac{2\pi^{n/2}}{\Gamma(n/2)} x^{n-1} dx = \frac{\pi^{n/2}}{\Gamma(n/2)} (x^2)^{(n-2)/2} dx^2 \tag{24}$$

and arrive at the well known result[1]

$$dPS_2 = \frac{\delta(s' + t_1 + u_1)}{2s'\Gamma((n-2)/2)(4\pi)^{(n-2)/2}} \left(\frac{(t_1 u_1' - s' m^2)s' - q^2 t_1^2}{s'^2} \right)^{(n-4)/2} dt_1 du_1 \tag{25}$$

$$= \frac{\delta(s' + t_1 + u_1) S_\epsilon}{2s'\Gamma(1+\epsilon/2)} \left(\frac{(t_1 u_1' - s' m^2)s' - q^2 t_1^2}{s'^2} \right)^{\epsilon/2} dt_1 du_1 \tag{26}$$

with $S_\epsilon = (4\pi)^{(-2-\epsilon/2)}$.

The final LO cross section can then be written as

$$\frac{d^2\sigma_k^{(0)}(s', t_1, u_1, q^2)}{dt_1 du_1} = \alpha\alpha_s E_\epsilon b_k(\epsilon) \delta(s' + t_1 + u_1) B_{k,QED} \tag{27}$$

where we used $e^2 = 4\pi\alpha$ and $g^2 = 4\pi\alpha_s$. **FiXme Error: do** FiXme Error!

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3 Next-To-Leading Order Calculations

Next-to-leading order contributions can be split into three parts: one loop virtual contributions, one gluon radiation and light quark processes. **FiXme Error: more?** FiXme Error!

3.1 One Loop Virtual Contributions

Virtual contributions have the same initial and final state as the Born process, but have a looping particle. All contributing Feynman diagrams are depicted in figure **FiXme** FiXme Error!

Error: do. The result can then be written as

$$\hat{\sum}_{k,\sigma}\mathcal{P}_k^{\mu\mu'}\sum_j\left[\mathcal{M}_{j,\mu}^{(1),v}\left(\mathcal{M}_{1,\mu'}^{(0)}+\mathcal{M}_{2,\mu'}^{(0)}\right)^*+c.c.\right]=8g^4e^2e_H^2N_CC_F\left(C_AV_{k,OK}+2C_FV_{k,QED}\right) \tag{28}$$

where C_A is the second Casimir constant of the adjoint representation for the gluon.

For the computation of the loops the Passarino-Veltman-decomposition[10] is used as far as possible. The decomposition is based on Lorentz invariance and a good explanation is for example given in [2]. The needed scalar integrals are given in [11] and [1], but there is also one wrong integral: we find with [12, Box 16]:

$$\begin{aligned}&D_0(m^2,0,q^2,m^2,t,s,0,m^2,m^2,m^2)\\&=\frac{iC_\epsilon}{\beta st_1}\left[-\frac{2\ln(\chi)}{\epsilon}-2\ln(\chi)\ln(\tilde{t})+\mathrm{Li}_2(1-\chi^2)-4\zeta(2)+\ln^2(\chi_q)+2\,\mathrm{Li}_2(-\chi\chi_q)\right.\\&\qquad\left.+2\,\mathrm{Li}_2(-\chi/\chi_q)+2\ln(\chi\chi_q)\ln(1+\chi\chi_q)+2\ln(\chi/\chi_q)\ln(1+\chi/\chi_q)\right]\end{aligned} \tag{29}$$

where we used the argument ordering of `LoopTools`[13, 14] (and also checked it against `LoopTools`).

3.2 2-to-3-Phasespace

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3.3 Single Gluon Radiation

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3.4 Light Quark Processes

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4 Mass Factorization

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5 Partonic Results

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6 Hadronic Results

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7 Summary

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A References

[1] E. Laenen, S. Riemersma, J. Smith, and W. van Neerven, "Complete $O(\alpha_S)$ corrections to heavy-flavour structure functions in electroproduction," Nuclear Physics B **392** no. 1, (1993) 162 – 228. http://www.sciencedirect.com/science/article/pii/055032139390201Y.

[2] I. Bojak, NLO QCD corrections to the polarized photoproduction and hadroproduction of heavy quarks. PhD thesis, Dortmund U., 2000. arXiv:hep-ph/0005120 [hep-ph].

[3] P. Breitenlohner and D. Maison, "Dimensional renormalization and the action principle," Comm. Math. Phys. **52** no. 1, (1977) 11–38. http://projecteuclid.org/euclid.cmp/1103900439.

[4] W. Vogelsang, Tests and signatures of spin dependent parton distributions in leading and next-to-leading order o PhD thesis, Dortmund U., 1993. http://alice.cern.ch/format/showfull?sysnb=0171841.

[5] J. A. M. Vermaseren, "New features of FORM," arXiv:math-ph/0010025 [math-ph].

[6] S. Wolfram, "Mathematica." Wolfram research, 1997. Ver. 3 or higher.

[7] M. Jamin and M. E. Lautenbacher, "TRACER version 1.1: A mathematica package for γ-algebra in arbitrary dimensions," Computer Physics Communications **74** no. 2, (1993) 265 – 288. http://www.sciencedirect.com/science/article/pii/001046559390097V.

[8] M. Wiebusch, "HEPMath 1.4: A Mathematica Package for Semi-Automatic Computations in High Energy Physics," Computer Physics Communications **195** (Oct., 2015) 172–190. http://arxiv.org/abs/1412.6102. arXiv: 1412.6102.

[9] E. Leader and E. Predazzi, An introduction to Gauge theories and modern particle physics. Univ. Pr., Cambridge.

[10] G. Passarino and M. J. G. Veltman, "One Loop Corrections for e+ e- Annihilation Into mu+ mu- in the Weinberg Model," Nucl. Phys. **B160** (1979) 151.

[11] Beenakker, W. and Kuijf, H. and van Neerven, W. L. and Smith, J., "Qcd corrections to heavy-quark production in $p\overline{p}$ collisions," Phys. Rev. D **40** (Jul, 1989) 54–82. http://link.aps.org/doi/10.1103/PhysRevD.40.54.

[12] R. K. Ellis and G. Zanderighi, "Scalar one-loop integrals for QCD," JHEP **02** (2008) 002, arXiv:0712.1851 [hep-ph].

[13] T. Hahn and M. Perez-Victoria, "Automatized one loop calculations in four-dimensions and D-dimensions," Comput. Phys. Commun. **118** (1999) 153–165, arXiv:hep-ph/9807565 [hep-ph].

[14] T. Hahn, "LoopTools 2.12 User's Guide." http://www.feynarts.de/looptools/, 2014.

List of Corrections