

1 Passarino-Veltman decomposition

1.1 Definitions

[1]:

$$A(m) = \frac{1}{i\pi^2} \int d^n q \frac{1}{q^2 + m^2} \quad (1)$$

$$B_0(p, m_1, m_2) = \frac{1}{i\pi^2} \int d^n q \frac{1}{(q^2 + m_1^2)((q+p)^2 + m_2^2)} \quad (2)$$

and apart from their pole term (called Δ - see [1, eq. D.1]), they keep $n = 4$.

[2, 3]:

$$A(m) = \mu^{-\epsilon} \int \frac{d^n q}{(2\pi)^n} \frac{1}{q^2 - m^2} \quad (3)$$

$$B(q_1, m_1, m_2) = \mu^{-\epsilon} \int \frac{d^n q}{(2\pi)^n} \frac{1}{(q^2 - m_1^2)((q+q_1)^2 - m_2^2)} \quad (4)$$

and $n = 4 + \epsilon$. ([2] writes “The notations for the one-, two-, three-, and four-point functions have been taken over from Ref. [1].” - obviously they do not.)

HEPMath[4] and FeynCalc[5, 6] refer to LoopTools[7, 8]. [8, eq. (1.1)] and [9, eq. (2.6)], QCDLoop[10]:

$$T_{\mu_1 \dots \mu_P}^N = \frac{\mu^{4-D}}{i\pi^{D/2} r_\Gamma} \int d^D q \frac{q_{\mu_1} \dots q_{\mu_P}}{[q^2 - m_1^2] [(q+k_1)^2 - m_2^2] \dots [(q+k_{N-1})^2 - m_N^2]} \quad (5)$$

$$r_\Gamma = \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}, \quad D = 4 - 2\epsilon$$

later in the code they use a different signature (to avoid any vector structure):

$$\begin{aligned} A(m^2), B_0(p^2, m_1^2, m_2^2), C_0(p_1^2, p_2^2, (p_1+p_2)^2, m_1^2, m_2^2, m_3^2) \\ D_0(p_1^2, p_2^2, p_3^2, p_4^2, (p_1+p_2)^2, (p_2+p_3)^2, m_1^2, m_2^2, m_3^2, m_4^2) \end{aligned} \quad (6)$$

[11]:

$$T_{\mu_1 \dots \mu_P}^N(p_1, \dots, p_{N-1}, m_0, \dots, m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \frac{q_{\mu_1} \dots q_{\mu_P}}{L_0 L_1 \dots L_{N-1}} \quad (7)$$

$$L_0 = q^2 - m_0^2 + i\epsilon \quad (8)$$

$$L_i = (q+p_i)^2 - m_i^2 + i\epsilon \quad i = 1, \dots, N-1 \quad (9)$$

I will stick to the integrals of [3] as it is the most natural form, I think, and to the non-vector signature, if possible.

The transformation of the *analytic* results from the notation in [10] is given by

$$[10]: \frac{\mu^{4-n}}{i\pi^{n/2}r_\Gamma} \mathcal{I} \leftrightarrow [3]: \frac{\mu^{4-n}}{(2\pi)^n} \mathcal{I} \quad (10)$$

with \mathcal{I} denoting the *raw* integral. We then need to solve (B=Bojak[3],E=Ellis[10]):

$$\begin{aligned} &\Rightarrow iC_\epsilon \left(\frac{a_2^B}{(n-4)^2} + \frac{a_1^B}{n-4} + a_0^B + O(n-4) \right) \\ &\stackrel{!}{=} \frac{\mu^{4-n}}{(2\pi)^n} \frac{i\pi^{n/2}r_\Gamma}{\mu^{4-n}} \left(\frac{a_2^E}{(n-4)^2} + \frac{a_1^E}{n-4} + a_0^E + O(n-4) \right) \end{aligned} \quad (11)$$

$$\Rightarrow a_2^B = a_2^E \quad (12)$$

$$a_1^B = a_1^E - \frac{1}{2}a_2^E \ln(m^2/\mu^2) \quad (13)$$

$$a_0^B = a_0^E - \frac{a_2^E}{8}\zeta(2) + \frac{a_2^E}{8}\ln^2(m^2/\mu^2) - \frac{a_1^E}{2}\ln(m^2/\mu^2) \quad (14)$$

To compare *numeric* results from **LoopTools** or **QCDLoop** one need to solve

$$\begin{aligned} &\Rightarrow \left(\frac{b_2^B}{(n-4)^2} + \frac{b_1^B}{n-4} + b_0^B + O(n-4) \right) \\ &\stackrel{!}{=} \frac{\mu^{4-n}}{(2\pi)^n} \frac{i\pi^{n/2}r_\Gamma}{\mu^{4-n}} \left(\frac{b_2^E}{(n-4)^2} + \frac{b_1^E}{n-4} + b_0^E + O(n-4) \right) \end{aligned} \quad (15)$$

$$\Rightarrow b_2^B = \frac{i}{16\pi^2} b_2^E \quad (16)$$

$$b_1^B = \frac{i}{16\pi^2} \left(b_1^E + \frac{b_2^E}{2}(\gamma_E - \ln(4\pi)) \right) \quad (17)$$

$$b_0^B = \frac{i}{16\pi^2} \left(b_0^E + \frac{b_1^E}{2}(\gamma_E - \ln(4\pi)) + \frac{b_2^E}{8} \left((\gamma_E - \ln(4\pi))^2 - \zeta(2) \right) \right) \quad (18)$$

$$(19)$$

1.2 Decomposition Labeling

[1, 3]:

$$B_\mu(p, m_1, m_2) = p_\mu B_1(p, m_1, m_2) \quad (20)$$

$$B_{\mu\nu} = p_\mu p_\nu B_{21} + g_{\mu\nu} B_{22} \quad (21)$$

$$C_\mu(p_1, p_2, m_1, m_2, m_3) = p_{1,\mu} C_{11} + p_{2,\mu} C_{12} \quad (22)$$

$$C_{\mu\nu} = p_{1,\mu} p_{1,\nu} C_{21} + p_{2,\mu} p_{2,\nu} C_{22} + (p_{1,\mu} p_{2,\nu} + p_{1,\nu} p_{2,\mu}) C_{23} + g_{\mu\nu} C_{24} \quad (23)$$

The arguments of the functions are always inherited.

HEPMath, FeynCalc, LoopTools, [9]:

$$B_\mu(p, m_1, m_2) = p_\mu B_1(p, m_1, m_2) \quad (24)$$

$$B_{\mu\nu} = g_{\mu\nu} B_{00} + p_\mu p_\nu B_{11} \quad (25)$$

$$C_\mu(p_1, p_2, m_1, m_2, m_3) = p_{1,\mu} C_1 + p_{2,\mu} C_2 = \sum_{j=1}^2 p_{j,\mu} C_j \quad (26)$$

$$C_{\mu\nu} = p_{1,\mu} p_{1,\nu} C_{11} + p_{2,\mu} p_{2,\nu} C_{22} + (p_{1,\mu} p_{2,\nu} + p_{1,\nu} p_{2,\mu}) C_{12} + g_{\mu\nu} C_{00} \quad (27)$$

$$= g_{\mu\nu} C_{00} + \sum_{j,k=1}^2 p_{j,\mu} p_{k,\nu} C_{jk} \quad (28)$$

The arguments of the functions are always inherited.

I will stick to HEPMath as it is the more generic and extensible form, I think.

1.3 Decomposition momenta

The tensor coefficients have to be expanded by momenta and there are also different conventions: HEPMath and LoopTools(LT) use the *internal* momenta k_i and [1, 3, 9](E) use the *external* momenta p_i . They are related by

$$k_1 = p_1, \quad k_2 = k_1 + p_2 = p_1 + p_2, \quad k_i = k_{i-1} + p_i, \forall i > 1 \quad (29)$$

So the decomposition is different, e.g. C_μ :

$$C_\mu^{LT} = k_{1\mu} C_1^{LT} + k_{2\mu} C_2^{LT} \quad (30)$$

$$= p_{1\mu} (C_1^{LT} - C_2^{LT}) + p_{2\mu} C_2^{LT} \quad (31)$$

$$C_\mu^E = p_{1\mu} C_1^E + p_{2\mu} C_2^E \quad (32)$$

Scalar PaVe-Coefficients do not need any transformation, because they do not depend on any momenta. B-Coefficients also do not need any transformation, because $k_1 = p_1$. The transformation for all other coefficients is given by:

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix}^E = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}^{LT} \quad \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix}^E = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix}^{LT} \quad (33)$$

$$\begin{pmatrix} C_{00} \\ C_{11} \\ C_{12} \\ C_{22} \end{pmatrix}^E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_{00} \\ C_{11} \\ C_{12} \\ C_{22} \end{pmatrix}^{LT} \quad (34)$$

$$\begin{pmatrix} D_{00} \\ D_{11} \\ D_{12} \\ D_{13} \\ D_{22} \\ D_{23} \\ D_{33} \end{pmatrix}^E = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D_{00} \\ D_{11} \\ D_{12} \\ D_{13} \\ D_{22} \\ D_{23} \\ D_{33} \end{pmatrix}^{LT} \quad (35)$$

$$\begin{pmatrix} D_{001} \\ D_{002} \\ D_{003} \\ D_{111} \\ D_{112} \\ D_{113} \\ D_{122} \\ D_{123} \\ D_{133} \\ D_{222} \\ D_{223} \\ D_{333} \end{pmatrix}^E = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 & 0 & 3 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -2 & 2 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D_{001} \\ D_{002} \\ D_{003} \\ D_{111} \\ D_{112} \\ D_{113} \\ D_{122} \\ D_{123} \\ D_{133} \\ D_{222} \\ D_{223} \\ D_{333} \end{pmatrix}^{LT} \quad (36)$$

1.4 B Decomposition

define

$$f_1 = m_1^2 - m_0^2 - p_1^2 \quad (37)$$

then one finds easily

$$B_1(p_1^2, m_0^2, m_1^2) = \frac{1}{2p_1^2} \left(f_1 B_0(p_1^2, m_0^2, m_1^2) + A_0(m_0^2) - A_0(m_1^2) \right) \quad (38)$$

$$B_{00}(p_1^2, m_0^2, m_1^2) = \frac{1}{2(n-1)} \left(2m_0^2 B_0(p_1^2, m_0^2, m_1^2) + A_0(m_1^2) - f_1 B_1(p_1^2, m_0^2, m_1^2) \right) \quad (39)$$

$$B_{11}(p_1^2, m_0^2, m_1^2) = \frac{1}{2p_1^2} \left(f_1 B_0(p_1^2, m_0^2, m_1^2) + A_0(m_1^2) - 2B_{00}(p_1^2, m_0^2, m_1^2) \right) \quad (40)$$

in accordance with [3, 9].

Concering B_1 [1] and `LoopTools` use the following identity

$$A_0(m_0^2) - A_0(m_1^2) = (m_0^2 - m_1^2) B_0(0, m_0^2, m_1^2) \quad (41)$$

that might help away with

In case m_1 and/or m_2 are very large the expression on the right-hand side of eq. (38) suffers very strong cancellations: the total is very much smaller than the individual terms. For this reason we have not used these algebraic relations, except to rewrite self-energy diagrams as much as possible in a form most suitable for numerical evaluation. ([1, below eq. D.6])

To compare the other results to [1] and `LoopTools` one has to use the *strict* $n \rightarrow 4$ limit and the following identities[12]:

$$(n-4)B_{00}(p_1^2, m_0^2, m_1^2) = \frac{1}{6}(p_1^2 - 3m_0^2 - 3m_1^2) \quad (42)$$

$$(n-4)B_{11}(p_1^2, m_0^2, m_1^2) = -\frac{2}{3} \quad (43)$$

2 Scalar Integrals

We focus on:

$$\gamma^*(q) + g(k_1) \rightarrow Q(p_1) + \bar{Q}(p_2) \quad (44)$$

$$k_1^2 = 0 \quad p_1^2 = p_2^2 = m^2 \quad (p_1 + p_2)^2 = s \quad (p_2 - q)^2 = t \quad (p_1 - q)^2 = u \quad (45)$$

define some shortcuts

$$0 \leq \rho = \frac{4m^2}{s} \leq 1 \quad 0 \leq \beta = \sqrt{1 - \rho} \leq 1 \quad 0 \leq \chi = \frac{1 - \beta}{1 + \beta} \leq 1 \quad (46)$$

$$\rho_q = \frac{4m^2}{q^2} \leq 0 \quad 1 \leq \beta_q = \sqrt{1 - \rho_q} \quad 0 \leq \chi_q = -\frac{1 - \beta_q}{1 + \beta_q} \leq 1 \quad (47)$$

2.1 One-Point Function A_0

[11]:

$$A_0(m^2) = -\frac{i}{16\pi^2} m^2 \left(\frac{m^2}{4\pi\mu^2} \right)^{(n-4)/2} \Gamma(1 - n/2) \quad (48)$$

$$= \frac{im^2}{16\pi^2} \left(\Delta - \log(m^2/\mu^2) + 1 \right) + O(n-4) \quad (49)$$

$$= iC_\epsilon m^2 \left(-\frac{2}{\epsilon} + 1 \right) + O(n-4) \quad (50)$$

$$\Delta = \frac{2}{4-n} - \gamma_E + \log(4\pi) \quad (51)$$

$$C_\epsilon = \frac{1}{16\pi^2} \exp \left(\left(\gamma_E - \log(4\pi) + \log \left(m^2/\mu^2 \right) \right) \frac{\epsilon}{2} \right) \quad (52)$$

this is *up to order* $O(n-4)$ in accordance with [3][13], but NOT beyond - see also [3, eq. (A.12)]. So we can treat C_ϵ and Δ as equal. We also find with [3]

$$A_0(0) = 0 \quad (53)$$

2.2 Two-Point Function B_0

In [11, eq. (4.23)] is a generic function given and we end up with

$$B_0(s, m^2, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 + \beta \log(\chi) \right) \quad (54)$$

$$B_0(q^2, m^2, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 + \beta_q \log(\chi_q) \right) \quad (55)$$

$$B_0(0, m^2, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} \right) \quad (56)$$

$$B_0(m^2, 0, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 \right) \quad (57)$$

$$B_0(t, 0, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 - \frac{t-m^2}{t} \ln \left(-\frac{t-m^2}{m^2} \right) \right) \quad (58)$$

focussing on imaginary part *only*; this in accordance with [3][13]. We also find with [3]

$$B_0(0, 0, 0) = 0 \quad (59)$$

2.3 Three-Point Function C_0

Again, in [11, eq. (4.26)] is a generic function given.

First, we compute $C_0(s, q^2, 0, m^2, m^2, m^2)$ and by taking the limit $k_1^2 \rightarrow 0$ (or equivalently $s_4 \rightarrow 0$) we end up with:

$$C_0(s, q^2, 0, m^2, m^2, m^2) = \frac{i}{16\pi^2} \cdot \frac{1}{s - q^2} \left(\text{Li}_2 \left(\frac{2}{1 + \beta_q} \right) + \text{Li}_2 \left(\frac{2}{1 - \beta_q} \right) - \text{Li}_2 \left(\frac{2}{1 + \beta} \right) - \text{Li}_2 \left(\frac{2}{1 - \beta} \right) \right) \quad (60)$$

with [14] we find:

$$\text{Li}_2 \left(\frac{2}{1 + b} \right) + \text{Li}_2 \left(\frac{2}{1 - b} \right) = 3\zeta(2) + \frac{1}{2} \ln^2 \left(\frac{1 - b}{1 + b} \right) - \ln \left(\frac{1 - b}{1 + b} \right) \ln \left(-\frac{1 - b}{1 + b} \right) \quad (61)$$

and if we focus on real part *only*, we find:

$$\text{Li}_2 \left(\frac{2}{1 + \beta} \right) + \text{Li}_2 \left(\frac{2}{1 - \beta} \right) = 3\zeta(2) - \frac{1}{2} \ln^2(\chi) \quad (62)$$

$$\text{Li}_2 \left(\frac{2}{1 + \beta_q} \right) + \text{Li}_2 \left(\frac{2}{1 - \beta_q} \right) = -\frac{1}{2} \ln^2(\chi_q) \quad (63)$$

Additionally, we find

$$\lim_{q^2 \rightarrow 0} \left[\text{Li}_2 \left(\frac{2}{1 + \beta_q} \right) + \text{Li}_2 \left(\frac{2}{1 - \beta_q} \right) \right] = 0 \quad (64)$$

So we get:

$$C_0(s, q^2, 0, m^2, m^2, m^2) = iC_\epsilon \frac{1}{s - q^2} \left(\frac{1}{2} \ln^2(\chi) - \frac{1}{2} \ln^2(\chi_q) - 3\zeta(2) \right) \quad (65)$$

$$C_0(s, 0, 0, m^2, m^2, m^2) = iC_\epsilon \frac{1}{s} \left(\frac{1}{2} \ln^2(\chi) - 3\zeta(2) \right) \quad (66)$$

in accordance with [3][13][15]. These results can also be obtained by the methods described in [3, chap. 3].

Next, we compute $C_0(m^2, 0, t, 0, m^2, m^2)$ again by taking the limit $k_1^2 \rightarrow 0$ we end up with:

$$C_0(m^2, 0, t, 0, m^2, m^2) = \frac{i}{16\pi^2} \cdot \frac{1}{t - m^2} \left(2\text{Li}_2(2) + \text{Li}_2(m^2/t) - \frac{\pi^2}{6} - \text{Li}_2((t + m^2)/m^2) - \text{Li}_2((m^2 + t)/t) \right) \quad (67)$$

Using [14] and focussing on real part, we find

$$\text{Li}_2(2) = \frac{\pi^2}{4} - i\pi \ln(2) \quad (68)$$

$$2\text{Li}_2(2) + \text{Li}_2(1/z) - \frac{\pi^2}{6} - \text{Li}_2(1 + z) - \text{Li}_2(1 + 1/z) = \frac{\pi^2}{6} - \text{Li}_2(z) \quad (69)$$

So we get:

$$C_0(m^2, 0, t, 0, m^2, m^2) = iC_\epsilon \frac{1}{t - m^2} \left(\zeta(2) - \text{Li}_2(t/m^2) \right) \quad (70)$$

in accordance with [3][13].

To compute $C_0(m^2, s, m^2, 0, m^2, m^2)$ we use [3] and find

$$C_0(m^2, s, m^2, 0, m^2, m^2) = \frac{iC_\epsilon}{s\beta} \left(-\frac{2}{\epsilon} \ln(\chi) - \frac{\pi^2}{2} + \frac{1}{2} \ln^2(\chi) - \ln(\chi) \ln(1 - \chi) - \text{Li}_2(1/(1 - \chi)) + \text{Li}_2(\chi/(\chi - 1)) \right) \quad (71)$$

Using [14] and focussing on real part, we find

$$-\text{Li}_2(1/(1 - \chi)) + \text{Li}_2(\chi/(\chi - 1)) = -2\text{Li}_2(\chi) - \ln(\chi) \ln(1 - \chi) - \frac{\pi^2}{6} \quad (72)$$

So we get:

$$C_0(m^2, s, m^2, 0, m^2, m^2) = iC_\epsilon \frac{1}{s\beta} \left(-\frac{2}{\epsilon} \ln(\chi) - 2 \ln(\chi) \ln(1 - \chi) - 2 \text{Li}_2(\chi) + \frac{1}{2} \ln^2(\chi) - 4\zeta(2) \right) \quad (73)$$

in accordance with [3][13].

To compute $C_0(t, m^2, q^2, 0, m^2, m^2)$ we use [16] and find immediatly:

$$\begin{aligned} C_0(t, m^2, q^2, 0, m^2, m^2) = \frac{iC_\epsilon}{\alpha} & \left[-\zeta(2) + 2 \text{Li}_2 \left(\frac{t_1 + \alpha}{t_1} \right) + \text{Li}_2 \left(\frac{q^2 - t - m^2 + \alpha}{q^2 - t - m^2 - \alpha} \right) \right. \\ & + \text{Li}_2 \left(\frac{t_1 - q^2 \beta_q^2 + \alpha}{t_1 - q^2 \beta_q^2 - \beta_q \alpha} \right) - \text{Li}_2 \left(\frac{t_1 - q^2 \beta_q^2 - \alpha}{t_1 - q^2 \beta_q^2 + \beta_q \alpha} \right) \\ & + \text{Li}_2 \left(\frac{t_1 - q^2 \beta_q^2 + \alpha}{t_1 + q^2 \beta_q^2 - \beta_q \alpha} \right) - \text{Li}_2 \left(\frac{t_1 - q^2 \beta_q^2 - \alpha}{t_1 + q^2 \beta_q^2 + \beta_q \alpha} \right) \\ & - \text{Li}_2 \left(\frac{t_1(q^2 - t - m^2 - \alpha) - 2m^2 \alpha}{t_1(q^2 - t - m^2 + \alpha)} \right) \\ & \left. - \text{Li}_2 \left(\frac{t_1(q^2 - t - m^2 - \alpha) - 2m^2 \alpha}{t_1(q^2 - t - m^2 - \alpha)} \right) \right] \quad (74) \end{aligned}$$

with $\alpha = \kappa(t, q^2, m^2)$ and the Källén function (as defined in [16, eq. (4.27)])

$$\kappa(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 2(xy + xz + yx)} \quad (75)$$

This is in accordance with [15, eq. (A.8)](Note the typo there!).

Additionally, we find

$$\lim_{q^2 \rightarrow 0} C_0(t, m^2, q^2, 0, m^2, m^2) = C_0(t, m^2, 0, 0, m^2, m^2) = C_0(m^2, 0, t, 0, m^2, m^2) \quad (76)$$

To compute $C_0(0, m^2, t, 0, 0, m^2)$ we use [3] and find

$$C_0(0, m^2, t, 0, 0, m^2) = \frac{iC_\epsilon}{t_1} \left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln(-t_1/m^2) + \ln^2(-t_1/m^2) + \text{Li}_2(t/m^2) + \frac{\zeta(2)}{4} \right) \quad (77)$$

2.4 Four-Point Function D_0

To compute $D_0(m^2, 0, q^2, m^2, t, s, 0, m^2, m^2, m^2)$ we follow Ingos way[3] of computing his $D_0(p_1, -k_1, -k_2, 0, m, m, m) = D_0(m^2, 0, 0, m^2, t, s, 0, m^2, m^2, m^2)$ and find

$$\tilde{t} = -\frac{t_1}{m^2} \quad (78)$$

$$K = \frac{x}{\rho\rho_q} [4x(-1+y)yz\rho + yz\rho\rho_q\tilde{t} + x(-4(-1+y)y(-1+z) + \rho - \tilde{t}yz\rho)\rho_q] \quad (79)$$

$$I_{xy} = \frac{2x^{\epsilon/2}\rho\rho_q^{2-\epsilon/2} [\tilde{t}y\rho_q + x(\rho_q + y(4(y-1) - \tilde{t}\rho_q))]}{(-2+\epsilon) [4x(-1+y)\rho + \tilde{t}\rho\rho_q - x(4(y-1) + \tilde{t}\rho)\rho_q]} \quad (80)$$

$$II_{xy} = -\frac{2x^{-1+\epsilon}\rho^{2-\epsilon/2}\rho_q [4(-1+y)y + \rho]^{-1+\epsilon/2}}{(-2+\epsilon) [4x(-1+y)\rho + \tilde{t}\rho\rho_q - x(4(y-1) + \tilde{t}\rho)\rho_q]} \quad (81)$$

“The integration of I_{xy} does not diverge and one easily gets upon setting $\epsilon \rightarrow 0$ ”

$$I = \frac{m^4}{st_1\beta} \left[\ln^2(\chi) + 4\text{Li}_2(-\chi) + \frac{\pi^2}{3} + 2\ln(\chi_q) \ln\left(\frac{\beta_q + \beta}{\beta_q - \beta}\right) - 2\ln(\chi) \ln(1 - q^2/s) \right. \\ \left. + 2\text{Li}_2\left(\frac{\beta_q - 1}{\beta_q - \beta}\right) - 2\text{Li}_2\left(\frac{\beta_q + 1}{\beta_q - \beta}\right) + 2\text{Li}_2\left(\frac{\beta_q + 1}{\beta_q + \beta}\right) - 2\text{Li}_2\left(\frac{\beta_q - 1}{\beta_q + \beta}\right) \right] \quad (82)$$

$$= \frac{m^4}{st_1\beta} \left[\ln^2(\chi) + 4\text{Li}_2(-\chi) + \frac{\pi^2}{3} + \ln\left(\frac{\beta_q^2 - \beta^2}{(\beta_q - 1)^2}\right) \ln\left(\frac{\beta_q - \beta}{\beta_q + \beta}\right) - 2\ln(\chi) \ln(1 - q^2/s) \right. \\ \left. + 2\text{Li}_2\left(\frac{\beta_q - 1}{\beta_q - \beta}\right) + 2\text{Li}_2\left(\frac{\beta_q - \beta}{\beta_q + 1}\right) - 2\text{Li}_2\left(\frac{\beta_q + \beta}{\beta_q + 1}\right) - 2\text{Li}_2\left(\frac{\beta_q - 1}{\beta_q + \beta}\right) \right] \quad (83)$$

with

$$\lim_{q^2 \rightarrow 0} \left[\ln\left(\frac{\beta_q^2 - \beta^2}{(\beta_q - 1)^2}\right) \ln\left(\frac{\beta_q - \beta}{\beta_q + \beta}\right) - 2\ln(\chi) \ln(1 - q^2/s) \right. \\ \left. + 2\text{Li}_2\left(\frac{\beta_q - 1}{\beta_q - \beta}\right) + 2\text{Li}_2\left(\frac{\beta_q - \beta}{\beta_q + 1}\right) - 2\text{Li}_2\left(\frac{\beta_q + \beta}{\beta_q + 1}\right) - 2\text{Li}_2\left(\frac{\beta_q - 1}{\beta_q + \beta}\right) \right] = 0 \quad (84)$$

“Integrating II_{xy} over x gives”

$$II_y = -\frac{2}{\tilde{t}(-2+\epsilon)\epsilon} \left(\frac{\rho - 4y(1-y)}{\rho} \right)^{-1+\epsilon/2} {}_2F_1\left(1, \epsilon; 1+\epsilon; 1 - \frac{4(1-y)(\rho_q - \rho)}{\tilde{t}\rho\rho_q}\right) \quad (85)$$

“The integration over y does not give an additional pole, so we can expand to $O(1)$ using ([3, eq. B.5]) and then integrate to obtain”

$$II = -\frac{m^4}{\beta st_1} \left(\frac{2\ln(\chi)}{\epsilon} + \ln(\chi) \left(1 + 2\ln(\beta\tilde{t}) + \ln(\chi) - 2\ln(1 - q^2/s) \right) + \text{Li}_2(\chi^2) + \frac{5\pi^2}{6} \right) \quad (86)$$

with

$$\lim_{q^2 \rightarrow 0} \ln(1 - q^2/s) = 0 \quad (87)$$

“The final result is then”

$$\begin{aligned} D_0(m^2, 0, q^2, m^2, t, s, 0, m^2, m^2, m^2) \\ = \frac{iC_\epsilon}{\beta st_1} \left[-\frac{2\ln(\chi)}{\epsilon} - 2\ln(\chi) \ln(\beta\tilde{t}) + 2\text{Li}_2(-\chi) - 2\text{Li}_2(\chi) - 3\zeta(2) \right. \\ \left. + \ln\left(\frac{\beta_q^2 - \beta^2}{(\beta_q - 1)^2}\right) \ln\left(\frac{\beta_q - \beta}{\beta_q + \beta}\right) - 2\ln(\chi) \ln(1 - q^2/s) \right. \\ \left. + 2\text{Li}_2\left(\frac{\beta_q - 1}{\beta_q - \beta}\right) + 2\text{Li}_2\left(\frac{\beta_q - \beta}{\beta_q + 1}\right) - 2\text{Li}_2\left(\frac{\beta_q + \beta}{\beta_q + 1}\right) - 2\text{Li}_2\left(\frac{\beta_q - 1}{\beta_q + \beta}\right) \right] \quad (88) \end{aligned}$$

This is NOT in accordance with [15, eq. (A.3)] - but I suspect a bunch of typos there.

We get the match to [3] and [13] by using

$$\begin{aligned} \lim_{q^2 \rightarrow 0} \left[\ln\left(\frac{\beta_q^2 - \beta^2}{(\beta_q - 1)^2}\right) \ln\left(\frac{\beta_q - \beta}{\beta_q + \beta}\right) - 2\ln(\chi) \ln(1 - q^2/s) \right. \\ \left. + 2\text{Li}_2\left(\frac{\beta_q - 1}{\beta_q - \beta}\right) + 2\text{Li}_2\left(\frac{\beta_q - \beta}{\beta_q + 1}\right) - 2\text{Li}_2\left(\frac{\beta_q + \beta}{\beta_q + 1}\right) - 2\text{Li}_2\left(\frac{\beta_q - 1}{\beta_q + \beta}\right) \right] = 0 \quad (89) \end{aligned}$$

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Nevertheless this is probably wrong - because we find with [10, Box 16]:

$$\begin{aligned} D_0(m^2, 0, q^2, m^2, t, s, 0, m^2, m^2, m^2) \\ = \frac{iC_\epsilon}{\beta st_1} \left[-\frac{2\ln(\chi)}{\epsilon} - 2\ln(\chi) \ln(\tilde{t}) + \text{Li}_2(1 - \chi^2) - 4\zeta(2) + \ln^2(\chi_q) + 2\text{Li}_2(-\chi\chi_q) \right. \\ \left. + 2\text{Li}_2(-\chi/\chi_q) + 2\ln(\chi\chi_q) \ln(1 + \chi\chi_q) + 2\ln(\chi/\chi_q) \ln(1 + \chi/\chi_q) \right] \quad (90) \end{aligned}$$

We get the match to [3] and [13] by using

$$\begin{aligned} \lim_{q^2 \rightarrow 0} \left[\text{Li}_2(1 - \chi^2) - 4\zeta(2) + \ln^2(\chi_q) + 2\text{Li}_2(-\chi\chi_q) + 2\text{Li}_2(-\chi/\chi_q) \right. \\ \left. + 2\ln(\chi\chi_q) \ln(1 + \chi\chi_q) + 2\ln(\chi/\chi_q) \ln(1 + \chi/\chi_q) \right] \\ = -2\ln(\chi) \ln(\beta) - 3\zeta(2) + 2\text{Li}_2(2, -\chi) - 2\text{Li}_2(2, \chi) \quad (91) \end{aligned}$$

To compute $D_0(0, m^2, q^2, m^2, t, u, 0, 0, m^2, m^2)$ I neither succeeded with [3] nor [16], but one can use [10, Box 11] (transformation see sec. 1.1) So we get

$$\begin{aligned} D_0(0, m^2, q^2, m^2, t, u, 0, 0, m^2, m^2) = \frac{iC_\epsilon}{t_1 u_1} \left(\frac{4}{\epsilon^2} + \frac{2}{\epsilon} \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) \right) \right. \\ \left. + 2\ln(-t_1/m^2) \ln(-u_1/m^2) - \frac{7}{2}\zeta(2) - \ln^2(\chi_q) \right) \quad (92) \end{aligned}$$

This is in accordance with [15, eq. (A.4)] using [14] (as above):

$$2 \operatorname{Li}_2 \left(\frac{q^2(1+\beta_q)}{2m^2} \right) + 2 \operatorname{Li}_2 \left(\frac{q^2(1-\beta_q)}{2m^2} \right) = 2 \operatorname{Li}_2 \left(\frac{2}{1-\beta_q} \right) + 2 \operatorname{Li}_2 \left(\frac{2}{1+\beta_q} \right) \quad (93)$$

$$= -\ln^2(\chi_q) \quad (94)$$

(The question, why they use a complicated dilogarithm remains open ...)

We get the match to [3] and [13] by using

$$\lim_{q^2 \rightarrow 0} \ln^2(\chi_q) = 0 \quad (95)$$

to find

$$\begin{aligned} D_0(0, m^2, 0, m^2, t, u, 0, 0, m^2, m^2) &= \frac{iC_\epsilon}{t_1 u_1} \left(\frac{4}{\epsilon^2} + \frac{2}{\epsilon} \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) \right) \right. \\ &\quad \left. + 2 \ln(-t_1/m^2) \ln(-u_1/m^2) - \frac{7}{2} \zeta(2) \right) \end{aligned} \quad (96)$$

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List of Corrections

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