

## 1 Feynman Rules

following [1]

To perform the calculation of Dirac traces in  $n$  dimensions use HEPMath[2] or TRACER[3].

FiXme  
Error:  
TODO

## 2 Leading Order: $O(\alpha\alpha_s)$

diagramatic:

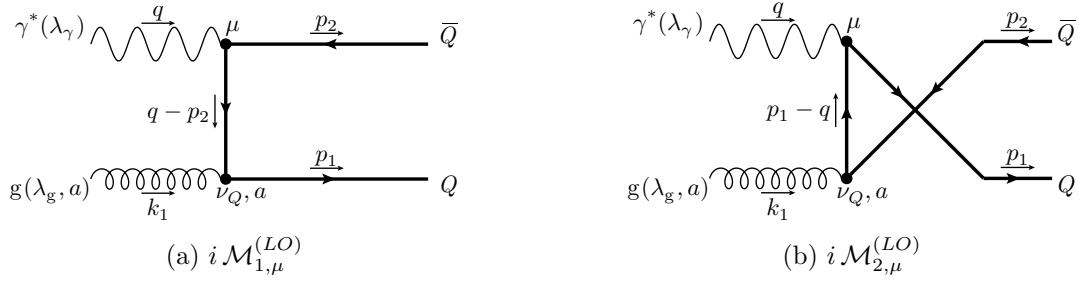


Figure 1: LO contributions

formula:

$$i\mathcal{M}_{1,\mu}^{(LO)} = \bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (1)$$

$$i\mathcal{M}_{2,\mu}^{(LO)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{t_1}(igT_a\gamma^{\nu_Q})v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (2)$$

color space:

$$\left|\mathcal{M}_{1,\mu}^{(LO)} + \mathcal{M}_{2,\mu}^{(LO)}\right|^2 \sim \text{tr}(T_a T_a) = N_c C_F \quad (3)$$

## 3 Next-to-leading Order: $O(\alpha\alpha_s^2)$

### 3.1 Light Quark Contributions

$$\gamma^*(q) + q(k_1) \rightarrow \bar{Q}(p_2) + Q(p_1) + q(k_2) \quad (4)$$

diagramatic:

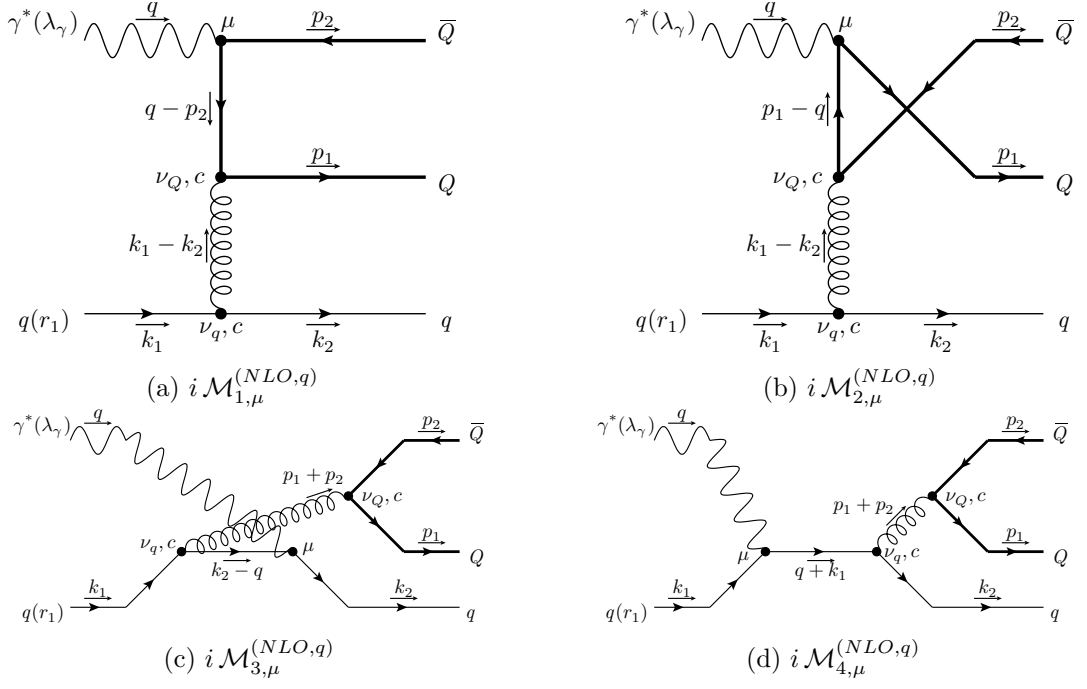


Figure 2: NLO contributions by light quarks

formula:

$$i\mathcal{M}_{1,\mu}^{(NLO,q)} = \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v_Q(p_2) \cdot \frac{-ig_{\nu_Q,\nu_q}}{t'} \cdot \bar{u}_q(k_2)(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) \quad (5)$$

$$i\mathcal{M}_{2,\mu}^{(NLO,q)} = \bar{u}_Q(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{u_7}(igT_c\gamma^{\nu_Q})v_Q(p_2) \cdot \frac{-ig_{\nu_Q,\nu_q}}{t'} \cdot \bar{u}_q(k_2)(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) \quad (6)$$

$$i\mathcal{M}_{3,\mu}^{(NLO,q)} = \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})v_Q(p_2) \cdot \frac{-ig_{\nu_Q,\nu_q}}{s_5} \cdot \bar{u}_q(k_2)(-iee_L\gamma_\mu)\frac{i(\not{k}_2 - \not{q})}{u'}(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) \quad (7)$$

$$i\mathcal{M}_{4,\mu}^{(NLO,q)} = \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})v_Q(p_2) \cdot \frac{-ig_{\nu_Q,\nu_q}}{s_5} \cdot \bar{u}_q(k_2)(igT_c\gamma^{\nu_q})\frac{i(\not{k}_1 + \not{q})}{s}(-iee_L\gamma_\mu)u_q^{(r_1)}(k_1) \quad (8)$$

color space:

$$\left| \mathcal{M}_{1,\mu}^{(NLO,q)} + \mathcal{M}_{2,\mu}^{(NLO,q)} + \mathcal{M}_{3,\mu}^{(NLO,q)} + \mathcal{M}_{4,\mu}^{(NLO,q)} \right|^2 \sim \text{tr}(T_c T_d) \text{tr}(T_c T_d) = \frac{1}{2} N_c C_F \quad (9)$$

### 3.2 Gluon Bremsstrahlung

$$\gamma^*(q) + g(k_1) \rightarrow \bar{Q}(p_2) + Q(p_1) + g(k_2) \quad (10)$$

diagramatic:

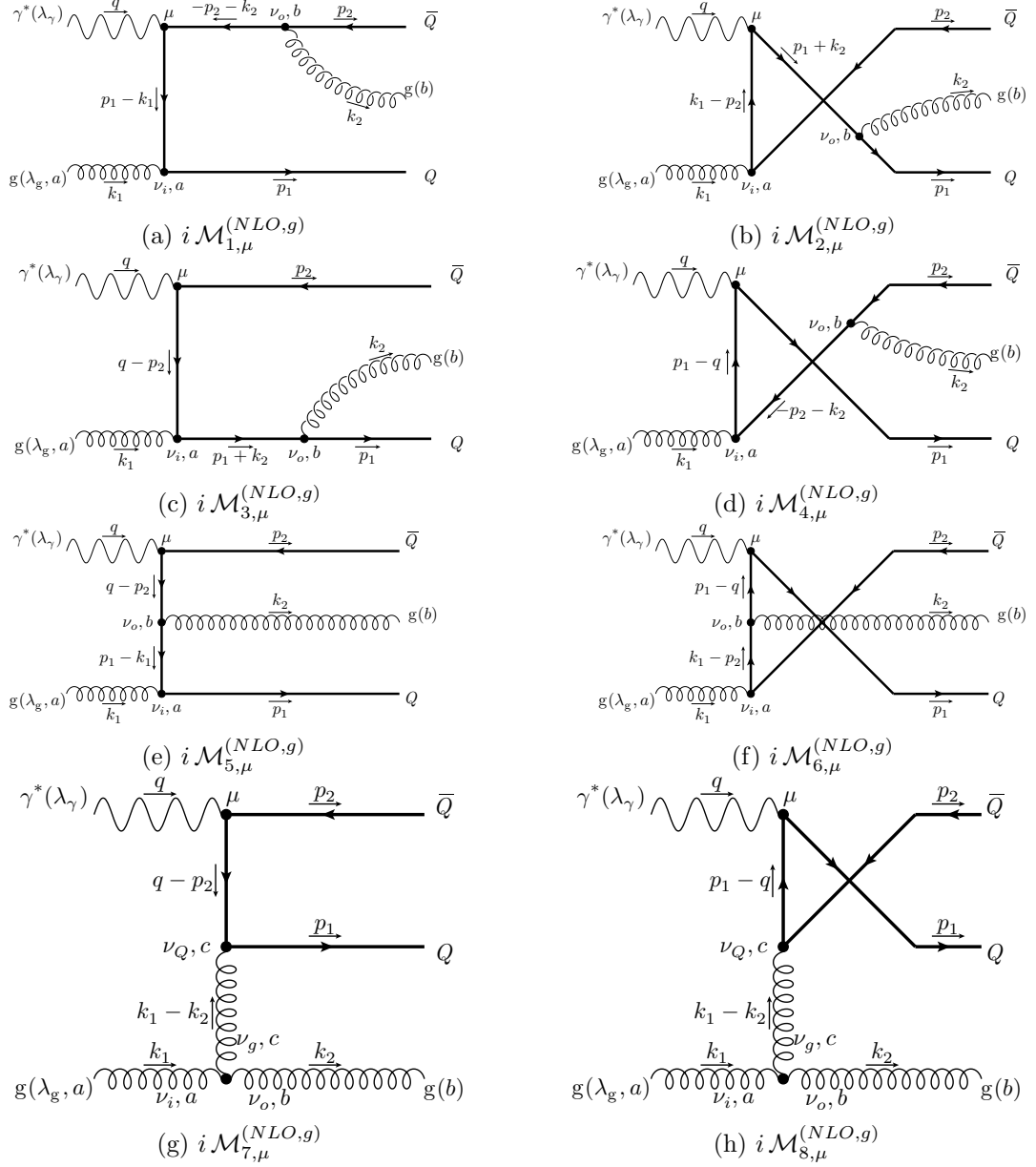


Figure 3: NLO contributions by gluon bremsstrahlung

formula:

$$i\mathcal{M}_{1,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_a\gamma^{\nu_i})\frac{i(\not{p}_1 - \not{k}_1 + m)}{u_6}(-iee_H\gamma_\mu) \cdot \frac{i(-\not{p}_2 - \not{k}_2 + m)}{s_3}(igT_b\gamma^{\nu_o})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \quad (11)$$

$$i\mathcal{M}_{2,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_b\gamma^{\nu_o})\frac{i(\not{p}_1 + \not{k}_2 + m)}{s_4}(-iee_H\gamma_\mu) \cdot \frac{i(\not{k}_1 - \not{p}_2 + m)}{t_1}(igT_a\gamma^{\nu_i})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \quad (12)$$

$$i\mathcal{M}_{3,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_b\gamma^{\nu_o})\frac{i(\not{p}_1 + \not{k}_2 + m)}{s_4}(igT_a\gamma^{\nu_i}) \cdot \frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \quad (13)$$

$$i\mathcal{M}_{4,\mu}^{(NLO,g)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{u_7}(igT_a\gamma^{\nu_i}) \cdot \frac{i(-\not{p}_2 - \not{k}_2 + m)}{s_3}(igT_b\gamma^{\nu_o})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \quad (14)$$

$$i\mathcal{M}_{5,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_a\gamma^{\nu_i})\frac{i(\not{p}_1 - \not{k}_1 + m)}{u_6}(igT_b\gamma^{\nu_o}) \cdot \frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \quad (15)$$

$$i\mathcal{M}_{6,\mu}^{(NLO,g)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{u_7}(igT_b\gamma^{\nu_o}) \cdot \frac{i(\not{k}_1 - \not{p}_2 + m)}{t_1}(igT_a\gamma^{\nu_i})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \quad (16)$$

$$i\mathcal{M}_{7,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_g}}{t'} \cdot \varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \cdot \left(gf^{acb}(g^{\nu_o,\nu_i}(k_1+k_2)^{\nu_g} + g^{\nu_i,\nu_g}(k_2-2k_1)^{\nu_o} + g^{\nu_g,\nu_o}(k_1-2k_2)^{\nu_i})\right) \quad (17)$$

$$i\mathcal{M}_{8,\mu}^{(NLO,g)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_g}}{t'} \cdot \varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \cdot \left(gf^{acb}(g^{\nu_o,\nu_i}(k_1+k_2)^{\nu_g} + g^{\nu_i,\nu_g}(k_2-2k_1)^{\nu_o} + g^{\nu_g,\nu_o}(k_1-2k_2)^{\nu_i})\right) \quad (18)$$

color space:

$$\begin{aligned}
& \sum_{j=1}^6 \left| \mathcal{M}_{j,\mu}^{(NLO,g)} \right|^2 + \mathcal{M}_{1,\mu}^{(NLO,g)} \left( \mathcal{M}_{4,\mu'}^{(NLO,g)} + \mathcal{M}_{5,\mu'}^{(NLO,g)} \right)^* + \mathcal{M}_{3,\mu}^{(NLO,g)} \left( \mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^* + \\
& \mathcal{M}_{2,\mu}^{(NLO,g)} \left( \mathcal{M}_{3,\mu'}^{(NLO,g)} + \mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^* + \mathcal{M}_{4,\mu}^{(NLO,g)} \left( \mathcal{M}_{5,\mu'}^{(NLO,g)} \right)^* \\
& \sim \text{tr}(T_a T_a T_b T_b) = N_C C_F^2
\end{aligned} \tag{19}$$

$$\begin{aligned}
& \mathcal{M}_{1,\mu}^{(NLO,g)} \left( \mathcal{M}_{2,\mu'}^{(NLO,g)} + \mathcal{M}_{3,\mu'}^{(NLO,g)} + \mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^* + \\
& \left( \mathcal{M}_{2,\mu}^{(NLO,g)} + \mathcal{M}_{3,\mu}^{(NLO,g)} \right) \left( \mathcal{M}_{4,\mu'}^{(NLO,g)} + \mathcal{M}_{5,\mu'}^{(NLO,g)} \right)^* + \\
& \left( \mathcal{M}_{4,\mu}^{(NLO,g)} + \mathcal{M}_{5,\mu}^{(NLO,g)} \right) \left( \mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^* \\
& \sim \text{tr}(T_a T_b T_a T_b) = N_C C_F \left( C_F - \frac{C_A}{2} \right)
\end{aligned} \tag{20}$$

$$\begin{aligned}
& \left( \mathcal{M}_{2,\mu}^{(NLO,g)} + \mathcal{M}_{3,\mu}^{(NLO,g)} + \mathcal{M}_{6,\mu}^{(NLO,g)} \right) \left( \mathcal{M}_{7,\mu'}^{(NLO,g)} + \mathcal{M}_{8,\mu'}^{(NLO,g)} \right)^* \\
& \sim -i f_{bda} \text{tr}(T_a T_b T_d) = \frac{1}{2} N_C C_F C_A
\end{aligned} \tag{21}$$

$$\begin{aligned}
& \left( \mathcal{M}_{1,\mu}^{(NLO,g)} + \mathcal{M}_{4,\mu}^{(NLO,g)} + \mathcal{M}_{5,\mu}^{(NLO,g)} \right) \left( \mathcal{M}_{7,\mu'}^{(NLO,g)} + \mathcal{M}_{8,\mu'}^{(NLO,g)} \right)^* \\
& \sim -i f_{bda} \text{tr}(T_b T_a T_d) = i f_{bda} \text{tr}(T_a T_b T_d) = -\frac{1}{2} N_C C_F C_A
\end{aligned} \tag{22}$$

$$\begin{aligned}
& \left| \mathcal{M}_{7,\mu}^{(NLO,g)} + \mathcal{M}_{8,\mu}^{(NLO,g)} \right|^2 \\
& \sim f_{acb} f_{adb} \text{tr}(T_c T_d) = N_C C_F C_A
\end{aligned} \tag{23}$$

To get the polarisation sums right, one has to subtract the contributions of the Faddeev-Popov ghosts[4, 5]:

diagrammatic:

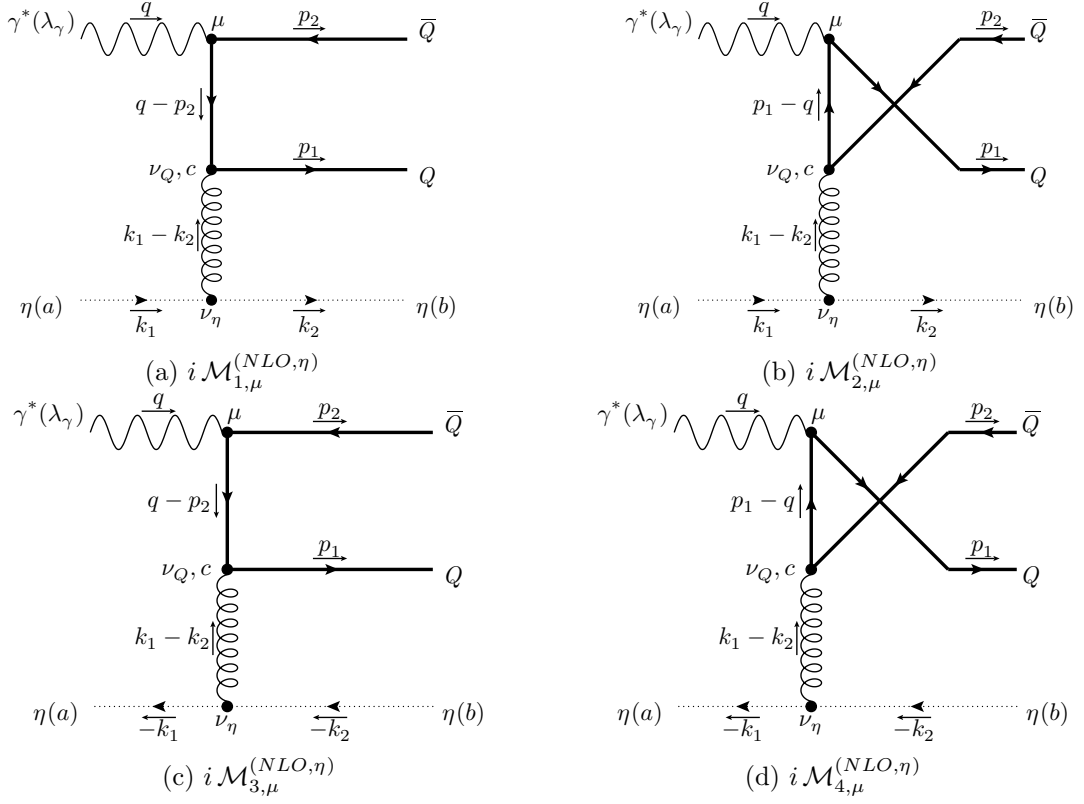


Figure 4: NLO contributions by ghosts

formula:

$$i \mathcal{M}_{1,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_\eta}}{t'} \cdot (gf^{acb}k_2^{\nu_\eta}) \quad (24)$$

$$i \mathcal{M}_{2,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_\eta}}{t'} \cdot (gf^{acb}k_2^{\nu_\eta}) \quad (25)$$

$$i \mathcal{M}_{3,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_\eta}}{t'} \cdot (gf^{cab}(-k_1)^{\nu_\eta}) \quad (26)$$

$$i \mathcal{M}_{4,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_\eta}}{t'} \cdot (gf^{cab}(-k_1)^{\nu_\eta}) \quad (27)$$

color space:

$$\left| \mathcal{M}_{1,\mu}^{(NLO,\eta)} + \mathcal{M}_{2,\mu}^{(NLO,\eta)} \right|^2 \sim f_{acb} f_{adb} \text{tr}(T_c T_d) = N_C C_F C_A \quad (28)$$

$$\left| \mathcal{M}_{3,\mu}^{(NLO,\eta)} + \mathcal{M}_{4,\mu}^{(NLO,\eta)} \right|^2 \sim f_{cab} f_{dab} \text{tr}(T_c T_d) = f_{acb} f_{adb} \text{tr}(T_c T_d) = N_C C_F C_A \quad (29)$$

### 3.3 Virtual Contributions

$$\gamma^*(q) + g(k_1) \rightarrow \bar{Q}(p_2) + Q(p_1) \quad (30)$$

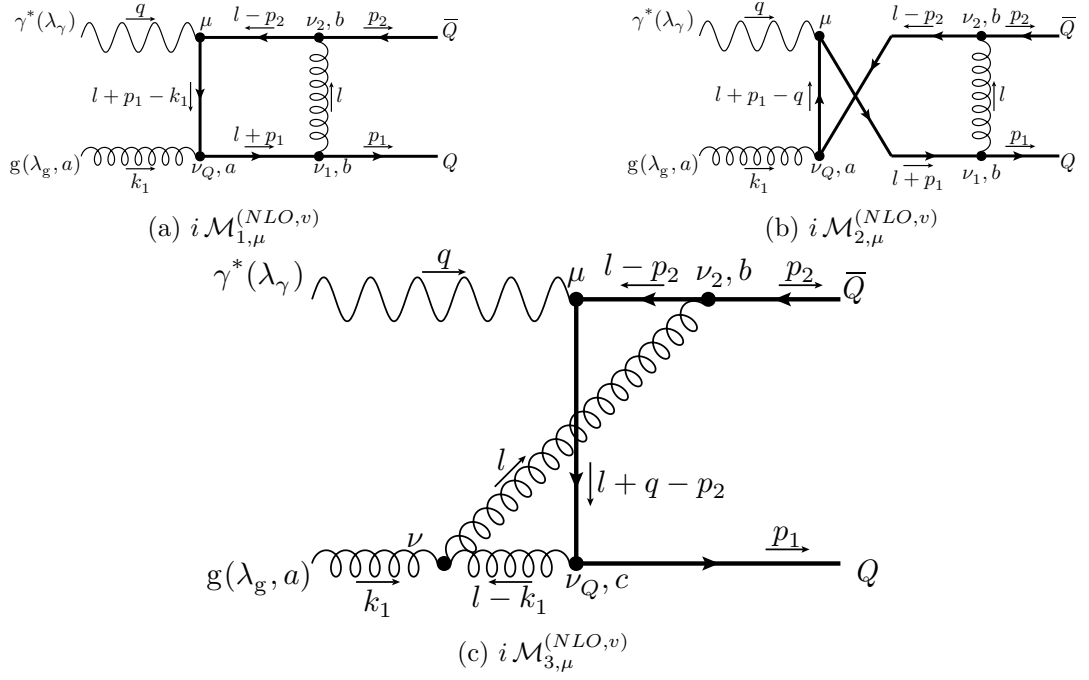


Figure 5: NLO contributions by one loop

$$i\mathcal{M}_{1,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_b \gamma^{\nu_1}) \frac{i(\not{l} + \not{p}_1 + m)}{(l+p_1)^2 - m^2} (igT_a \gamma^{\nu_Q}) \frac{i(\not{l} + \not{p}_1 - \not{k}_1 + m)}{(l+p_1-k_1)^2 - m^2} \cdot$$

$$(-iee_H \gamma_\mu) \frac{i(\not{l} - \not{p}_2 + m)}{(l-p_2)^2 - m^2} (igT_b \gamma^{\nu_2}) \frac{-ig_{\nu_1, \nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (31)$$

$$i\mathcal{M}_{2,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_b \gamma^{\nu_1}) \frac{i(\not{l} + \not{p}_1 + m)}{(l+p_1)^2 - m^2} (igT_a \gamma^{\nu_Q}) \frac{i(\not{l} + \not{p}_1 - \not{q} + m)}{(l+p_1-q)^2 - m^2} \cdot$$

$$(-iee_H \gamma_\mu) \frac{i(\not{l} - \not{p}_2 + m)}{(l-p_2)^2 - m^2} (igT_b \gamma^{\nu_2}) \frac{-ig_{\nu_1, \nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (32)$$

$$i\mathcal{M}_{3,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_c \gamma^{\nu_Q}) \frac{i(\not{l} + \not{q}_1 - \not{p}_2 + m)}{(l+q-p_2)^2 - m^2} (-iee_H \gamma_\mu) \frac{i(\not{l} - \not{p}_2 + m)}{(l-p_2)^2 - m^2} \cdot$$

$$(igT_b \gamma^{\nu_2}) \frac{(-i)^2}{l^2(l-k_1)^2} v(p_2) \varepsilon^{\nu, (\lambda_g)}(k_1) \cdot$$

$$\left( gf_{abc} \left( g_{\nu_2 \nu_Q} (k_1 - 2l)_\nu + g_{\nu_Q \nu} (l - 2k_1)_{\nu_2} + g_{\nu \nu_2} (k_1 + l)_{\nu_Q} \right) \right) \quad (33)$$



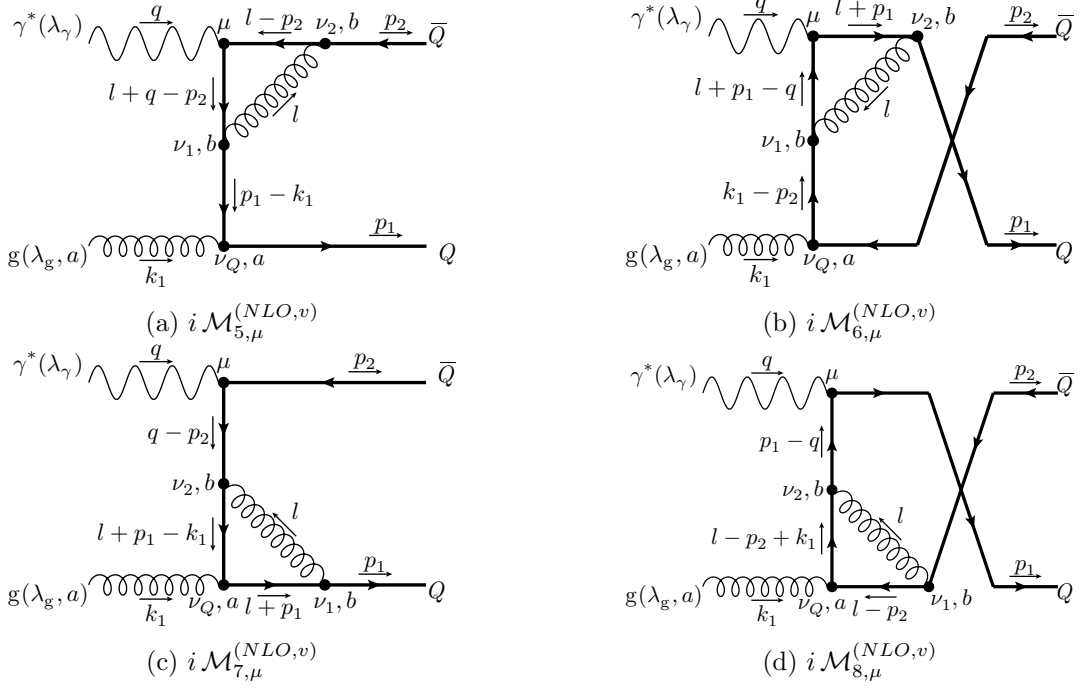


Figure 6: NLO contributions by one loop (cont'd)

$$i\mathcal{M}_{5,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_a \gamma^{\nu_Q}) \frac{i(\not{p}_1 - \not{k}_1 + m)}{u_1} (igT_b \gamma^{\nu_1}) \frac{i(\not{l} + \not{q} - \not{p}_2 + m)}{(l+q-p_2)^2 - m^2} \cdot$$

$$(-iee_H \gamma_\mu) \frac{i(\not{l} - \not{p}_2 + m)}{(l-p_2)^2 - m^2} (igT_b \gamma^{\nu_2}) \frac{-ig_{\nu_1, \nu_2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)}{l^2} \quad (34)$$

$$i\mathcal{M}_{6,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_b \gamma^{\nu_2}) \frac{i(\not{l} + \not{p}_1 + m)}{(l+p_1)^2 - m^2} (-iee_H \gamma_\mu) \frac{i(\not{l} + \not{p}_1 - \not{q} + m)}{(l+p_1-q)^2 - m^2} \cdot$$

$$(igT_b \gamma^{\nu_1}) \frac{i(\not{k}_1 - \not{p}_2 + m)}{t_1} (igT_a \gamma^{\nu_Q}) \frac{-ig_{\nu_1, \nu_2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)}{l^2} \quad (35)$$

$$i\mathcal{M}_{7,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_b \gamma^{\nu_1}) \frac{i(\not{l} + \not{p}_1 + m)}{(l+p_1)^2 - m^2} (igT_a \gamma^{\nu_Q}) \frac{i(\not{l} + \not{p}_1 - \not{k}_1 + m)}{(l+p_1-k_1)^2 - m^2} \cdot$$

$$(igT_b \gamma^{\nu_2}) \frac{i(\not{q} - \not{p}_2 + m)}{u_1} (-iee_H \gamma_\mu) \frac{-ig_{\nu_1, \nu_2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)}{l^2} \quad (36)$$

$$i\mathcal{M}_{8,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (-iee_H \gamma_\mu) \frac{i(\not{p}_1 - \not{q} + m)}{t_1} (igT_b \gamma^{\nu_2}) \frac{i(\not{l} - \not{p}_2 + \not{k}_1 + m)}{(l-p_2+k_1)^2 - m^2} \cdot$$

$$(igT_a \gamma^{\nu_Q}) \frac{i(\not{l} - \not{p}_2 + m)}{(l-p_2)^2 - m^2} (igT_b \gamma^{\nu_1}) \frac{-ig_{\nu_1, \nu_2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)}{l^2} \quad (37)$$

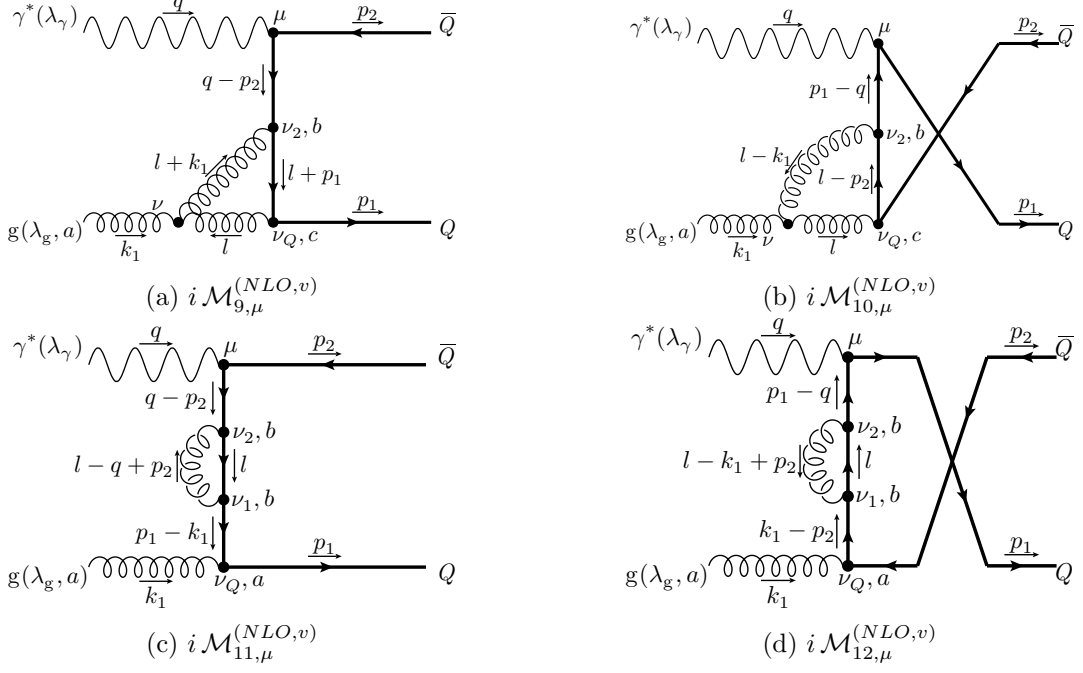


Figure 7: NLO contributions by one loop (cont'd)

$$\begin{aligned}
 i\mathcal{M}_{9,\mu}^{(NLO,v)} &= \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_c \gamma^{\nu_Q}) \frac{i(\not{l} + \not{p}_1 + m)}{(l + p_1)^2 - m^2} (igT_b \gamma^{\nu_2}) \frac{i(\not{q} - \not{p}_2 + m)}{u_1} \\
 &\quad (-iee_H \gamma_\mu) \frac{(-i)^2}{l^2 (l + k_1)^2} v(p_2) \varepsilon^{\nu, (\lambda_g)}(k_1) \cdot \\
 &\quad \left( gf_{abc} \left( g_{\nu\nu_2} (2k_1 + l)_{\nu_Q} + g_{\nu_2\nu_Q} (-2l - k_1)_\nu + g_{\nu_Q\nu} (l - k_1)_{\nu_2} \right) \right) \quad (38)
 \end{aligned}$$

$$\begin{aligned}
 i\mathcal{M}_{10,\mu}^{(NLO,v)} &= \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (-iee_H \gamma_\mu) \frac{i(\not{p}_1 - \not{q} + m)}{t_1} (igT_b \gamma^{\nu_2}) \frac{i(\not{l} - \not{p}_2 + m)}{(l - p_2)^2 - m^2} \\
 &\quad (igT_c \gamma^{\nu_Q}) \frac{(-i)^2}{l^2 (l - k_1)^2} v(p_2) \varepsilon^{\nu, (\lambda_g)}(k_1) \cdot \\
 &\quad \left( gf_{abc} \left( g_{\nu\nu_2} (2k_1 - l)_{\nu_Q} + g_{\nu_2\nu_Q} (2l - k_1)_\nu + g_{\nu_Q\nu} (-l - k_1)_{\nu_2} \right) \right) \quad (39)
 \end{aligned}$$

$$\begin{aligned}
 i\mathcal{M}_{11,\mu}^{(NLO,v)} &= \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_a \gamma^{\nu_Q}) \frac{i(\not{p}_1 - \not{k}_1 + m)}{t_1} (igT_b \gamma^{\nu_1}) \frac{i(\not{l} + m)}{l^2 - m^2} \\
 &\quad (igT_b \gamma^{\nu_2}) \frac{i(\not{q} - \not{p}_2 + m)}{t_1} (-iee_H \gamma_\mu) \frac{-ig_{\nu_1\nu_2}}{(l - q + p_2)^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (40)
 \end{aligned}$$

$$\begin{aligned}
 i\mathcal{M}_{12,\mu}^{(NLO,v)} &= \mu^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (-iee_H \gamma_\mu) \frac{i(\not{p}_1 - \not{q} + m)}{u_1} (igT_b \gamma^{\nu_2}) \frac{i(\not{l} + m)}{l^2 - m^2} \\
 &\quad (igT_b \gamma^{\nu_1}) \frac{i(\not{k}_1 - \not{p}_2 + m)}{u_1} (igT_a \gamma^{\nu_Q}) \frac{-ig_{\nu_1\nu_2}}{(l - k_1 + p_2)^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (41)
 \end{aligned}$$

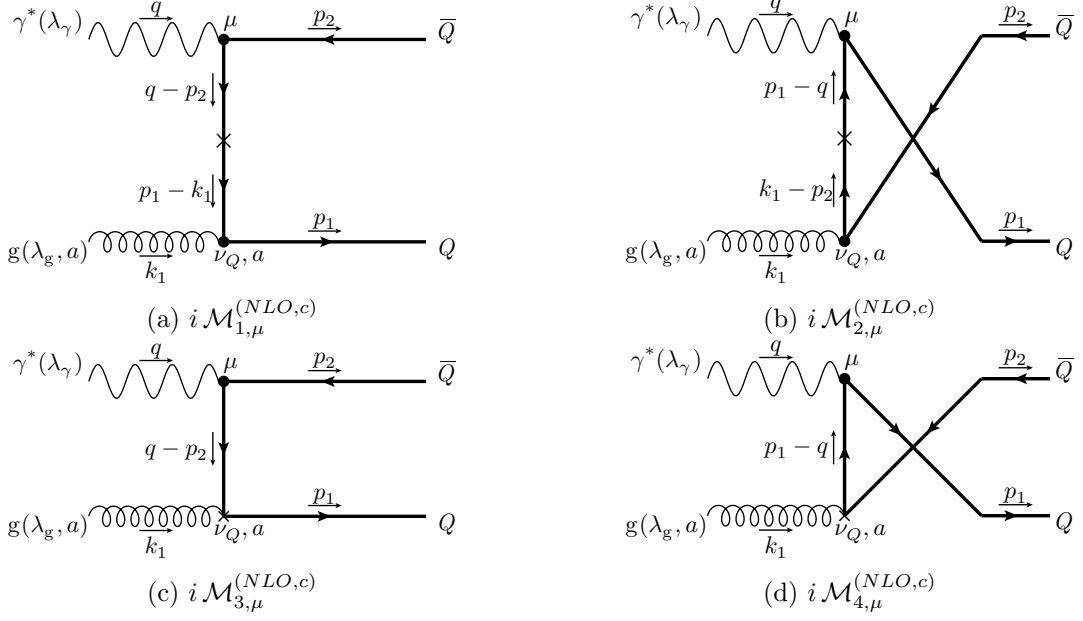


Figure 8: NLO contributions by counter terms

$$-i \mathcal{M}_{1,\mu}^{(NLO,c)} = \bar{u}(p_1) (ig T_a \gamma^{\nu_Q}) \frac{i(\not{p}_1 - \not{k}_1 + m)}{t_1} \left( i((Z_2 - 1)(\not{q} - \not{p}_2 - m) - (Z_m - 1)m) \right) \frac{i(\not{q} - \not{p}_2 + m)}{t_1} (-ie e_H \gamma_\mu) v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (42)$$

$$-i \mathcal{M}_{2,\mu}^{(NLO,c)} = \bar{u}(p_1) (-ie e_H \gamma_\mu) \frac{i(\not{p}_1 - \not{q} + m)}{u_1} \left( i((Z_2 - 1)(\not{p}_1 - \not{q} - m) - (Z_m - 1)m) \right) \frac{i(\not{k}_1 - \not{p}_2 + m)}{u_1} (ig T_a \gamma^{\nu_Q}) v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (43)$$

$$-i \mathcal{M}_{3,\mu}^{(NLO,c)} = \bar{u}(p_1) (-i(Z_{1f} - 1)g T_a \gamma^{\nu_Q}) \frac{i(\not{q} - \not{p}_2 + m)}{u_1} (-ie e_H \gamma_\mu) v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (44)$$

$$-i \mathcal{M}_{4,\mu}^{(NLO,c)} = \bar{u}(p_1) (-ie e_H \gamma_\mu) \frac{i(\not{p}_1 - \not{q} + m)}{t_1} (-i(Z_{1f} - 1)g T_a \gamma^{\nu_Q}) v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (45)$$

Color space:

$$\left( \mathcal{M}_{1,\mu}^{(NLO,v)} + \mathcal{M}_{2,\mu}^{(NLO,v)} \right) \left( \mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)} \right)^* \sim -i \text{tr}(T_a T_b T_a T_b) = -i N_C C_F \left( C_F - \frac{C_A}{2} \right) \quad (46)$$

$$\left( \mathcal{M}_{3,\mu}^{(NLO,v)} \right) \left( \mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)} \right)^* \sim f_{abc} \text{tr}(T_c T_b T_a) = -\frac{i}{2} N_C C_F C_A \quad (47)$$

$$\left( \mathcal{M}_{5,\mu}^{(NLO,v)} + \mathcal{M}_{6,\mu}^{(NLO,v)} \right) \left( \mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)} \right)^* \sim -i \text{tr}(T_a T_a T_b T_b) = -i N_C C_F^2 \quad (48)$$

$$(49)$$

To compute self energies, we follow [6]. It is

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \quad (50)$$

$$\gamma_\mu \gamma^\mu = g_\mu^\mu = n \quad (51)$$

$$\gamma_\mu \gamma_\nu \gamma^\mu = (2-n)\gamma_\nu \quad (52)$$

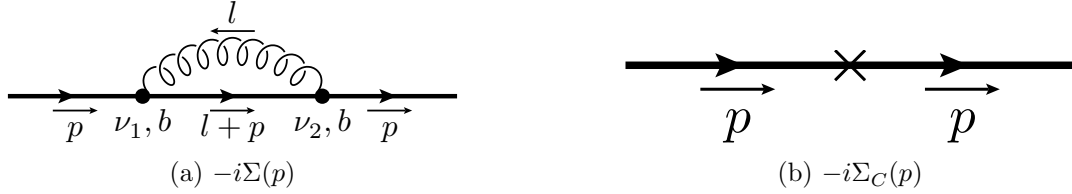


Figure 9: NLO contributions by quark self energy

$$-i\Sigma(p) = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} (igT_b \gamma_{\nu_1}) \frac{i(\not{l} + \not{p} + m)}{(l+p)^2 - m^2} (igT_b \gamma_{\nu_2}) \frac{-ig^{\nu_1, \nu_2}}{l^2} \quad (53)$$

$$= -\mu_R^{4-n} g^2 C_F \int \frac{d^n l}{(2\pi)^n} \frac{2m + (2-n)\not{p} + (2-n)\not{l}}{l^2 ((l+p)^2 - m^2)} \quad (54)$$

$$= -g^2 C_F \left( (2m + (2-n)\not{p}) B_0(p^2, 0, m^2) + (2-n) B_1(p^2, 0, m^2) \right) \quad (55)$$

$$= -g^2 C_F \left( B_0(p^2, 0, m^2) \left( n \cdot m + (2-n)\not{p} \frac{p^2 + m^2}{2p^2} \right) - (2-n)\not{p} \frac{1}{2p^2} A_0(m^2) \right) \quad (56)$$

Using [6] we find

$$C_\epsilon = \frac{1}{16\pi^2} \exp \left( (\gamma_E - \log(4\pi)) \frac{\epsilon}{2} \right) \left( m^2/\mu^2 \right)^{\epsilon/2} \quad (57)$$

$$A_0(m^2) = iC_\epsilon \left( -\frac{2}{\epsilon} + 1 \right) \quad (58)$$

$$B_0(p^2, 0, m^2) = iC_\epsilon \left( -\frac{2}{\epsilon} + 2 + \frac{m^2 - p^2}{p^2} \ln \left( \frac{m^2 - p^2}{m^2} \right) \right) \quad (59)$$

$$\Rightarrow -i\Sigma(p) = -ig^2 C_F C_\epsilon \left[ \frac{2\not{p} - 8m}{\epsilon} + 2m \left( 3 - 2 \left( 1 - \frac{m^2}{p^2} \right) \ln \left( 1 - \frac{p^2}{m^2} \right) \right) - \not{p} \left( 1 + \frac{m^2}{p^2} \right) \left( 1 - \left( 1 - \frac{m^2}{p^2} \right) \ln \left( 1 - \frac{p^2}{m^2} \right) \right) \right] \quad (60)$$

$$\stackrel{!}{=} -i(Am + B(\not{p} - m)) \quad (61)$$

$$\Rightarrow A = \frac{1}{m} \Sigma(p)|_{\not{p}=m} \quad (62)$$

$$= -g^2 C_F C_\epsilon \left( \frac{6}{\epsilon} - 5 + \frac{m^2}{p^2} + \left( 3 - 4 \frac{m^2}{p^2} + \frac{m^4}{p^4} \right) \ln \left( 1 - \frac{p^2}{m^2} \right) \right) \quad (63)$$

$$\Rightarrow B = \frac{1}{m} \frac{d\Sigma(p)}{d\not{p}} \Big|_{\not{p}=m} \quad (64)$$

$$= g^2 C_F C_\epsilon \left( \frac{2}{\epsilon} - 1 - \frac{m^2}{p^2} + \left( 1 - \frac{m^4}{p^4} \right) \ln \left( 1 - \frac{p^2}{m^2} \right) \right) \quad (65)$$

Counterterm:

$$-i\Sigma_C(p) = i((Z_2 - 1)\not{p} - (Z_2 Z_m - 1)m) \quad (66)$$

$$= i((Z_2 - 1)(\not{p} - m) - (Z_m - 1)m) + O(\alpha_S^2) \quad (67)$$

Use on-shell renormalisation:

$$0 \stackrel{!}{=} (-i\Sigma(p) - i\Sigma_C(p))|_{\not{p}=m} \quad (68)$$

$$= i((Z_m - 1) + A)m + (B - (Z_2 - 1))(\not{p} - m)|_{\not{p}=m} \quad (69)$$

$$\Rightarrow (Z_m - 1) = -A|_{\not{p}=m} \quad (70)$$

$$= g^2 C_F C_\epsilon \left( \frac{6}{\epsilon} - 4 \right) \quad (71)$$

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## List of Corrections

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