1 Introduction

This work is mainly based on the paper “Heavy quark correlations in deep inelastic electroproduction” by Harris et. al.[1] - that is, it recalculates all properties and formulas. It extends then the application to the equivalent *polarized* processes. The treating of the polarized processes can for example be found in [2] and we will use many ideas and technices from there. **FiXme Error: more** FiXme Error!

1.1 Motivation

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FiXme Error: why do we do this

1.2 Notation

We calculate in $n = 4 + \epsilon$ dimension to regularize all soft, collinear and ultraviolet poles. Unfortunaly this extension for *polarized* processes is nontrivial, due to the occurance of the Levi-Civita tensors $\varepsilon_{\mu\nu\rho\sigma}$ and γ_5. We avoid these problems here by shifting the poles (apart from ultraviolet) to the phasespace resulting in generalized plus distributions and thus resulting in matrix elements in just $n = 4$ dimensions.

In leading order (LO) we have to consider the following process:

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) \tag{1}$$

The corresponding parton structure tensor $W^{(0)}_{\mu\mu'}$ can then be written as **FiXme Error: avoid all order expr? FiXme Error: remove?** FiXme Error! FiXme Error!

$$\begin{aligned}
&W^{(0)}_{\mu\mu'}(k_1, q; s, t_1, u_1, q^2; \sigma_q) \\
&= \frac{1}{2} E_\epsilon K_{\mathrm{g}\gamma} \frac{1}{2s'} \int \frac{d^{n-1}p_1}{2E_1(2\pi)^{n-1}} \int \frac{d^{n-1}p_2}{2E_2(2\pi)^{n-1}} \delta(p_1^2 - m^2)\delta(p_2^2 - m^2) \\
&\quad (2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2)\, \mathcal{M}^{(0)}_{\mu}\, \mathcal{M}^{(0)}_{\mu'}
\end{aligned} \tag{2}$$

where the initial 1/2 is the initial state spin average, $K_{\mathrm{g}\gamma}$ is the color average,

$$E_\epsilon := \begin{cases} 1/(1+\epsilon/2) & \text{unpolarized} \\ 1 & \text{polarized} \end{cases} \tag{3}$$

accounts for additional degrees of freedom in n dimensions for initial bosons. The Lorentz indices μ and μ' refer to the virtual photon that is exchanged with the scattering lepton. We have chosen to detect the heavy *antiquark* $\overline{Q}(p_2)$ and so we define the following Mandelstam variables:

$$s = (q + k_1)^2, \quad t_1 = t - m^2 = (k_1 - p_2)^2 - m^2, \quad u_1 = u - m^2 = (q - p_2)^2 - m^2 \tag{4}$$

For convenience we also define $s' = s - q^2$ and $u_1' = u_1 - q^2$. If the heavy *quark* $\overline{Q}(p_1)$ is detected, p_2 in eq. (4) has to be replaced by p_1 which effectively interchanges $t_1 \leftrightarrow u_1$. **FiXme Error: move to LO?** FiXme Error!

By using Lorentz covariance, hermiticity, parity invariance and current conservation the parton structure tensor can be decomposed into several parts:

$$\begin{aligned} W_{\mu\mu'}(k_1, q; s, t_1, u_1, q^2; \sigma_q) = &\left(-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2}\right) \frac{d^2\sigma_T(s, t_1, u_1, q^2)}{dt_1 du_1} \\ &+ \left(k_{1,\mu} - \frac{k_1 \cdot q}{q^2} q_\mu\right)\left(k_{1,\mu'} - \frac{k_1 \cdot q}{q^2} q_{\mu'}\right)\left(\frac{-4q^2}{s'^2}\right) \\ &\cdot \left(\frac{d^2\sigma_T(s, t_1, u_1, q^2)}{dt_1 du_1} + \frac{d^2\sigma_L(s, t_1, u_1, q^2)}{dt_1 du_1}\right) \end{aligned} \tag{5}$$

FiXme Error: extend We can then define appropiate projection operators[3, 4] FiXme Error!

$$\hat{\mathcal{P}}^\gamma_{G,\mu\mu'} = -g_{\mu\mu'} \qquad b_G(\epsilon) = \frac{1}{2(1+\epsilon/2)} \tag{6}$$

$$\hat{\mathcal{P}}^\gamma_{L,\mu\mu'} = -\frac{4q^2}{s'^2} k_{1,\mu} k_{1,\mu'} \qquad b_L(\epsilon) = 1 \tag{7}$$

$$\hat{\mathcal{P}}^\gamma_{P,\mu\mu'} = i\varepsilon_{\mu\mu'\rho\rho'} \frac{q^\rho k_1^{\rho'}}{s'} \qquad b_P(\epsilon) = 1 \tag{8}$$

FiXme Error: justify avoidance of $\boldsymbol{\Delta}$ (such as $\boldsymbol{\Delta\hat{\mathcal{P}}}$)? and we then find FiXme Error!

$$\frac{d^2\sigma_k(s, t_1, u_1, q^2)}{dt_1 t u_1} = b_k(\epsilon) \hat{\mathcal{P}}^\gamma_{k,\mu\mu'} W^{\mu\mu'} \tag{9}$$

FiXme Error: how to insert 2nd projector? with $k \in \{G, L, P\}$ denoting (here and mostly ever after) the projection type. The transverse partonic cross section $d\sigma_T$ can be reconstructed from the above definitions by using FiXme Error!

$$d\sigma_T = d\sigma_G + b_G(\epsilon) d\sigma_L \tag{10}$$

We also define accordingly

$$E_G(\epsilon) = E_L(\epsilon) = \frac{1}{1+\epsilon/2} \qquad E_P(\epsilon) = 1 \tag{11}$$

The final state spins are always summed over, but the initial spins have to be treated seperately: for unpolarized projections $k \in \{G, L\}$ they are also summed over, but for the polarized projection $k = P$ they are projected on their asymmetric part:

$$\hat{\mathcal{P}}^{\mathrm{g}}_{G,\nu\nu'} = \hat{\mathcal{P}}^{\mathrm{g}}_{L,\nu\nu'} = -g_{\nu\nu'} \qquad \hat{\mathcal{P}}^{\mathrm{g}}_{P,\nu\nu'} = 2i\epsilon_{\nu\nu'\rho\rho'} \frac{k_1^\rho q^\rho}{s'} \tag{12}$$

where ν and ν' refer to the initial gluon. By writing $\hat{\mathcal{P}}^{\mathrm{g}}_{G,\nu\nu'}$ in eq. (12) we decided to introduce Fadeev-Popov ghosts[2] as we got a single diagram in next-to-leading order with a triple-gluon vertex **FiXme Error: explain ghosts?**. As we can consider all quarks in the initial state as massless, we further find FiXme Error!

$$\hat{\mathcal{P}}^{q}_{G,aa'} = \hat{\mathcal{P}}^{q}_{L,aa'} = (\not{k}_1)_{aa'} \qquad \hat{\mathcal{P}}^{q}_{P,aa'} = -(\gamma_5\not{k}_1)_{aa'} \tag{13}$$

$$\hat{\mathcal{P}}^{\bar{q}}_{G,bb'} = \hat{\mathcal{P}}^{\bar{q}}_{L,bb'} = (\not{k}_1)_{bb'} \qquad \hat{\mathcal{P}}^{\bar{q}}_{P,bb'} = (\gamma_5\not{k}_1)_{bb'} \tag{14}$$

where a and a' refer to the Dirac-index of the intial quark spinor in next-to-leading order - analogous for b and the antiquark.

We further define a set of partonic variables:

$$0 \leq \rho = \frac{4m^2}{s} \leq 1 \qquad 0 \leq \beta = \sqrt{1-\rho} \leq 1 \qquad 0 \leq \chi = \frac{1-\beta}{1+\beta} \leq 1 \tag{15}$$

$$\rho_q = \frac{4m^2}{q^2} \leq 0 \qquad 1 \leq \beta_q = \sqrt{1-\rho_q} \qquad 0 \leq \chi_q = \frac{\beta_q - 1}{\beta_q + 1} \leq 1 \tag{16}$$

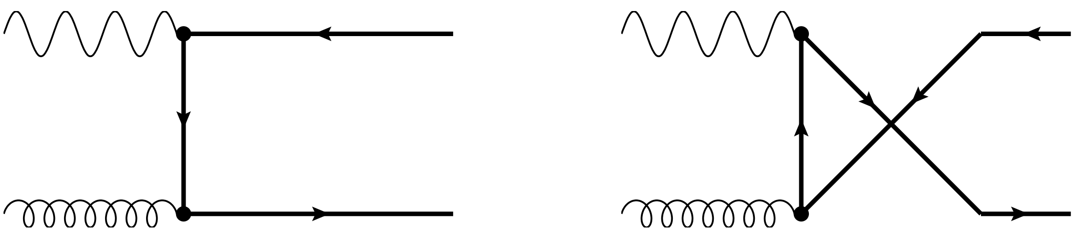
When computing Feynman diagrams a computer algebra system (CAS) is almost obligatory: common choices are `FORM`[5] or `Mathematica`[6] - for the later the most common choice is `TRACER`[7], but we have chosen `HEPMath`[8]. We used the Feynman rules given by [9] and [2].

2 Leading Order Calculations

In leading order we have to consider photon-gluon-fusion (PGF), that is

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) \tag{17}$$

with two contributing diagrams depicted in figure 1.



(a) $i\varepsilon^{\mu}_{\gamma}(q)\varepsilon^{\nu}_{\mathrm{g}}(k_1)\,\mathcal{M}^{(0),1}_{\mu\nu}$ (b) $i\varepsilon^{\mu}_{\gamma}(q)\varepsilon^{\nu}_{\mathrm{g}}(k_1)\,\mathcal{M}^{(0),2}_{\mu\nu}$

Figure 1: leading order Feynman diagrams
FiXme Error: shift to appendix?

The result can then be written as

$$M^{(0)}_k = \hat{\mathcal{P}}^{\gamma,\mu\mu'}_k \hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_k \sum_{j,j'=1}^{2} \mathcal{M}^{(0),j}_{\mu\nu} \left(\mathcal{M}^{(0),j'}_{\mu'\nu'}\right)^* = 8g^2\mu_D^{-\epsilon}e^2e_H^2 N_C C_F B_{k,QED} \tag{18}$$

where g and e are the strong and electromagnetic coupling constants respectively, μ_D is an arbitray mass parameter introduced to keep the couplings dimensionless and e_H is the magnitude of the heavy quark in units of e. Further N_C corresponds to the gauge group $SU(N_C)$ and the color factor $C_F = (N_C^2 - 1)/(2N_C)$ refers to the second Casimir constant of the fundamental representation for the quarks. We then find:

$$
\begin{aligned}
B_{G,QED} &= \frac{t_1}{u_1} + \frac{u_1}{t_1} + \frac{4m^2 s'}{t_1 u_1}\left(1 - \frac{m^2 s'}{t_1 u_1}\right) + \frac{2s' q^2}{t_1 u_1} + \frac{2q^4}{t_1 u_1} + \frac{2m^2 q^2}{t_1 u_1}\left(2 - \frac{s'^2}{t_1 u_1}\right) \\
&\quad + \epsilon \left\{ -1 + \frac{s'^2}{t_1 u_1} + \frac{s' q^2}{t_1 u_1} - \frac{q^4}{t_1 u_1} - \frac{m^2 q^2 s'^2}{t_1^2 u_1^2} \right\} + \epsilon^2 \frac{s'^2}{4 t_1 u_1} \qquad (19)
\end{aligned}
$$

$$
B_{L,QED} = -\frac{4q^2}{s'}\left(\frac{s}{s'} - \frac{m^2 s'}{t_1 u_1}\right) \qquad (20)
$$

$$
B_{P,QED} = \frac{1}{2}\left(\frac{t_1}{u_1} + \frac{u_1}{t_1}\right)\left(\frac{2m^2 s'}{t_1 u_1} - 1 - \frac{2q^2}{s'}\right) \qquad (21)
$$

$$
B_{k,QED} = B_{k,QED}^{(0)} + \epsilon B_{k,QED}^{(1)} + \epsilon^2 B_{k,QED}^{(2)} \qquad (22)
$$

By using eq. (2) we can derive the n-dimensional $2 \to 2$ phase space

$$
dPS_2 = \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_2}{(2\pi)^{n-1}} \Theta(p_{1,0}) \delta(p_1^2 - m^2) \Theta(p_{2,0}) \delta(p_2^2 - m^2) (2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2) \qquad (23)
$$

that can be solved by using the center-of-mass system (CMS) of the incoming particles[2]

$$
q = \left(\frac{s + q^2}{2\sqrt{s}}, 0, 0, -\frac{s - q^2}{2\sqrt{s}}, \hat{0}\right) \qquad k_1 = \frac{s - q^2}{2\sqrt{s}}\left(1, 0, 0, 1, \hat{0}\right) \qquad (24)
$$

such that $q + k_1 = (\sqrt{s}, \vec{0})$ and $k_1^2 = 0$. For the outgoing particles it follows

$$
p_1 = \frac{\sqrt{s}}{2}\left(1, 0, -\beta \sin\theta_1, -\beta\cos\theta_1, \hat{0}\right) \qquad p_2 = \frac{\sqrt{s}}{2}\left(1, 0, \beta\sin\theta_1, \beta\cos\theta_1, \hat{0}\right) \qquad (25)
$$

such that $p_1 + p_2 = (\sqrt{s}, \vec{0})$ and $p_1^2 = p_2^2 = m^2$ and

$$
t_1 = -\frac{s'}{2}(1 - \beta\cos(\theta_1)), \quad u_1 = -\frac{s'}{2}(1 + \beta\cos(\theta_1)) \qquad (26)
$$

We then arrive at the well known result[1]:

$$
dPS_2 = \frac{\beta \sin(\theta_1)}{16\pi\Gamma(1 + \epsilon/2)}\left(\frac{s\beta^2 \sin^2(\theta_1)}{16\pi}\right)^{\epsilon/2} d\theta_1 \qquad (27)
$$

The cross sections are then given by:

$$
d\sigma_k^{(0)} = \frac{1}{2s}\frac{K_{\gamma g} E_k(\epsilon)}{2} b_k(\epsilon) M_k^{(0)} dPS_2 \qquad (28)
$$

The procedure is completely analogous to the inclusive case **FiXme Error: cite** and the results agree to there. FiXme Error!

3 Next-To-Leading Order Calculations

Next-to-leading order contributions can be split into three parts: one loop virtual contributions, one gluon radiation and the light quark processes. **FiXme Error: more?** FiXme Error!

3.1 One Loop Virtual Contributions

Virtual contributions have the same initial and final state as the Born process, but have a looping particle. All contributing Feynman diagrams are depicted in figure **FiXme Error: do**. The result can then be written as FiXme Error!

$$\begin{aligned} M_k^{(1),V} &= \hat{\mathcal{P}}_k^{\gamma,\mu\mu'}\hat{\mathcal{P}}_k^{g,\nu\nu'}\sum_j\left[\mathcal{M}_{j,\mu\nu}^{(1),V}\left(\mathcal{M}_{1,\mu'\nu'}^{(0)}+\mathcal{M}_{2,\mu'\nu'}^{(0)}\right)^*+c.c.\right] \\ &= 8g^4\mu_D^{-\epsilon}e^2e_H^2N_CC_FC_\epsilon\left(C_AV_{k,OK}+2C_FV_{k,QED}\right) \end{aligned} \tag{29}$$

where $C_\epsilon = \exp(\epsilon/2(\gamma_E - \ln(4\pi)))/(16\pi^2)$ and $C_A = N_C$ is the second Casimir constant of the adjoint representation for the gluon (that introduces a non-abelian part).

For the computation of the loops the Passarino-Veltman-decomposition[10] in $n = 4 + \epsilon$ dimension is used as far as possible. The decomposition is based on Lorentz invariance and a good explanation is for example given in [2]. The needed scalar integrals are given in [11] and [3], but there is also one wrong integral: we find with [12, Box 16]:

$$\begin{aligned} &D_0(m^2,0,q^2,m^2,t,s,0,m^2,m^2,m^2) \\ &= \frac{iC_\epsilon}{\beta st_1}\left[-\frac{2\ln(\chi)}{\epsilon}-2\ln(\chi)\ln(-t_1/m^2)+\mathrm{Li}_2(1-\chi^2)-4\zeta(2)+\ln^2(\chi_q)+2\,\mathrm{Li}_2(-\chi\chi_q)\right. \\ &\qquad\left.+2\,\mathrm{Li}_2(-\chi/\chi_q)+2\ln(\chi\chi_q)\ln(1+\chi\chi_q)+2\ln(\chi/\chi_q)\ln(1+\chi/\chi_q)\right] \end{aligned} \tag{30}$$

where we used the argument ordering of `LoopTools`[13, 14] (and also checked it against `LoopTools`).

As the short example above shows, the full expressions for the $V_{k,OK}, V_{k,QED}$ are quite complicated and too long to be presented here, nevertheless the arising poles are quite compact:

$$V_{k,OK} = -2B_{k,QED}\left(\frac{4}{\epsilon^2}+\left(\ln(-t_1/m^2)+\ln(-u_1/m^2)-\frac{2m^2-s}{s\beta}\ln(\chi)\right)\frac{2}{\epsilon}\right)+O(\epsilon^0) \tag{31}$$

$$V_{k,QED} = -2B_{k,QED}\left(1+\frac{2m^2-s}{s\beta}\ln(\chi)\right)\frac{2}{\epsilon}+O(\epsilon^0) \tag{32}$$

The above results already include the mass renormalization that we have performed *on-shell*, so all ultra-violet poles have been removed. For the renormalization of the strong coupling we use the $\overline{\mathrm{MS}}_m$ scheme defined in [2] and so the full (remaining) renormalization can be achieved by

$$
\begin{aligned}
d\sigma_k^{(1),V,ren.} &= d\sigma_k^{(1),V} + \frac{\alpha_s(\mu_R^2)}{4\pi}\left[\left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln(\mu_R^2/m^2) - \ln(\mu_D^2/m^2)\right)\beta_0^f \right.\\
&\qquad\qquad \left. + \frac{2}{3}\ln(\mu_R^2/m^2)\right] d\sigma_k^{(0)} \qquad (33)\\
&= d\sigma_k^{(1),V} + 4\pi\alpha_s(\mu_R^2)C_\epsilon\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}\left[\left(\frac{2}{\epsilon} + \ln(\mu_R^2/m^2)\right)\beta_0^f \right.\\
&\qquad\qquad \left. + \frac{2}{3}\ln(\mu_R^2/m^2)\right] d\sigma_k^{(0)} \qquad (34)
\end{aligned}
$$

with μ_R the renormalization scale introduced by the renormalization group equation (RGE), $\beta_0^f = (11C_A - 2n_f)/3$ the first coefficient of the beta function and n_f the number of *total* flavours (i.e. $n_{lf} = n_f - 1$ active (light) flavours and one heavy flavour). The double poles occuring in $V_{k,OK}$ are introduced by the diagrams **FiXme Error: do** when the soft and collinear singularities coincide. FiXme Error!

The results agree in the photo-production limit ($q^2 \to 0$) with [15] **FiXme Error: Matrix elements available upon request**. FiXme Error!

The procedure is completely analogous to the inclusive case **FiXme Error: cite**. FiXme Error!

3.2 2-to-3 particle phase space

In next-to-leading order we have to consider processes which involve an additional particle in the final state. The matrix elements will then depend on ten kinematical invariants:

$$
\begin{aligned}
&s = (q + k_1)^2 && t_1 = (k_1 - p_2)^2 - m^2 && u_1 = (q - p_2)^2 - m^2 && (35)\\
&s_3 = (k_2 + p_2)^2 - m^2 && s_4 = (k_2 + p_1)^2 - m^2 && s_5 = (p_1 + p_2)^2 = -u_5 && (36)\\
&t' = (k_1 - k_2)^2 && && && (37)\\
&u' = (q - k_2)^2 && u_6 = (k_1 - p_1)^2 - m^2 && u_7 = (q - p_1)^2 - m^2 && (38)
\end{aligned}
$$

from which only five are independent as can be seen from momentum conservation $k_1 + q = p_1 + p_2 + k_2$ and s, t_1, u_1 match to their leading order definition.

The $2 \to 3$ n-dimensional phase space is given by

$$
\begin{aligned}
dPS_3 = \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_2}{(2\pi)^{n-1}} \frac{d^n k_2}{(2\pi)^{n-1}} (2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2 - k_2)&\\
\Theta(p_{1,0})\delta(p_1^2 - m^2)\Theta(p_{2,0})\delta(p_2^2 - m^2)\Theta(k_{2,0})\delta(k_2^2)& \qquad (39)
\end{aligned}
$$

This can be solved by writing eq. (39) as product of a $2 \to 2$ decay and a subsequent $1 \to 2$ decay[11]. We choose the following decomposition[1]:

$$
\begin{aligned}
q &= (q\text{'}0, 0, |\vec{q}|) && (40)\\
k_1 &= k_0(1, 0, \sin\psi, \cos\psi) && (41)\\
p_1 &= \frac{\sqrt{s_5}}{2}(1, \beta_5 \sin\theta_2 \sin\theta_1, \beta_5 \sin\theta_2 \cos\theta_1, \beta_5 \cos\theta_1) && (42)\\
p_2 &= \frac{\sqrt{s_5}}{2}(1, -\beta_5 \sin\theta_2 \sin\theta_1, -\beta_5 \sin\theta_2 \cos\theta_1, -\beta_5 \cos\theta_1) && (43)\\
k_2 &= (k_2^0, 0, k_1 \sin\psi, |\vec{q}| + k_1^0 \cos\psi) && (44)
\end{aligned}
$$

where

$$
\begin{aligned}
q_0 &= \frac{s+u'}{2\sqrt{s_5}}, \qquad |\vec{q}| = \frac{1}{2\sqrt{s_5}}\sqrt{(s+u')^2 - 4s_5q^2}, && (45)\\
k_1^0 &= \frac{s_5-u'}{2\sqrt{s_5}}, \qquad \cos\psi = \frac{2k_1^0q^0 - s'}{2k_1^0\,|\vec{q}|}, \qquad \beta_5 = \sqrt{1-4m^2/s_5}, && (46)\\
k_2^0 &= \frac{s-s_5}{2\sqrt{s_5}} && (47)
\end{aligned}
$$

We further introduce $\rho^* = \frac{4m^2-q^2}{s-q^2} \leq x = \frac{s_5-q^2}{s-q^2} \leq 1$ and $-1 \leq y \leq 1$ where y is the cosine of the angle between $\vec{q}$ and $\vec{k}_2$ in the system with $\vec{q} + \vec{k}_2 = 0$. We then find[1]:

$$
dPS_3 = \frac{T_\epsilon}{2\pi}\left(\frac{s'^2}{s}\right)^{1+\epsilon/2}(1-x)^{1+\epsilon}(1-y^2)^{\epsilon/2} dPS_2^{(5)} dy \sin^\epsilon(\theta_1) d\theta_1 d\theta_2 \qquad (48)
$$

with $0 \leq \theta_1 \leq \pi, 0 \leq \theta_2 \leq \pi, \rho^* \leq x \leq 1, -1 \leq y \leq 1$ and

$$
\begin{aligned}
S_\epsilon &= (4\pi)^{-2-\epsilon/2} && (49)\\
T_\epsilon &= \frac{\Gamma(1+\epsilon/2)}{\Gamma(1+\epsilon)} S_\epsilon = \frac{1}{16\pi^2}\left(1 + \frac{\epsilon}{2}(\gamma_E - \ln(4\pi)) + O(\epsilon^2)\right) && (50)\\
dPS_2^{(5)} &= \frac{\beta_5 \sin(\theta_1)}{16\pi\Gamma(1+\epsilon/2)}\left(\frac{s_5\beta_5^2 \sin^2(\theta_1)}{16\pi}\right)^{\epsilon/2} d\theta_1 dx = dPS_2(s \to s_5) dx && (51)
\end{aligned}
$$

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3.3 Single Gluon Radiation

In next-to-leading order we have to consider the following process:

$$
\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) + \mathrm{g}(k_2) \qquad (52)
$$

where all contributing diagrams are depicted in figure 2.

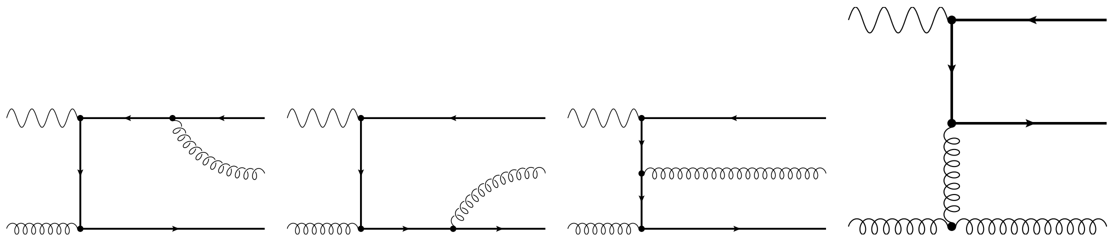

Figure 2: next-to-leading order Feynman diagrams for the single gluon radiation $i\varepsilon_\gamma^\mu(q)\varepsilon_{\mathrm{g}}^\nu(k_1)\,\mathcal{M}^{(1),g}_{j,\mu\nu}$. Four additional graphs are obtained by crossing the final heavy quark pair. With our choice of $\hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{G/L}$ the diagram 2d has to be regularized with the ghost contributions.

FiXme Error: shift to appendix?

The result can than be written as

$$M_k^{(1),\mathrm{g}} = \hat{\mathcal{P}}_k^{\gamma,\mu\mu'}\hat{\mathcal{P}}_k^{\mathrm{g},\nu\nu'}\sum_{j,j'}\mathcal{M}^{(1),g}_{j,\mu\nu}\,\mathcal{M}^{(1),g}_{j',\mu'\nu'}{}^{*} \tag{53}$$

$$= 8g^4\mu_D^{-2\epsilon}e^2e_H^2N_CC_F\left(C_AR_{k,OK}+2C_FR_{k,QED}\right) \tag{54}$$

The partonic cross section is then given by:

$$d\sigma^{(1)}_{k,\mathrm{g}} = \frac{1}{2s'}\frac{K_{\gamma\mathrm{g}}E_k(\epsilon)}{2}b_k(\epsilon)M_k^{(1),\mathrm{g}}dPS_3 \tag{55}$$

For the gluonic part we shift the occuring soft ($x\to 1$) and collinear ($y\to -1$) poles from the matrix elements to the phase space by dividing by $t'\propto(1+y)(1-x)$ and $u'-q^2s_5/s\propto(1-x)$:

$$dPS'_{3,\mathrm{g}} = \frac{dPS_3}{t'(u'-q^2s_5/s)} = dPS_3\cdot\left(\frac{2s}{s'^2}\right)^2\frac{1}{(1-x)^2(1-y)(1+y)} \tag{56}$$

$$= \frac{2T_\epsilon}{\pi}\left(\frac{s'^2}{s}\right)^{-1+\epsilon/2}(1-x)^{-1+\epsilon}(1-y^2)^{-1+\epsilon/2}dPS_2^{(5)}dy\sin^\epsilon(\theta_2)d\theta_2 \tag{57}$$

$$M_k^{(1),\mathrm{g}'} = t'(u'-q^2s_5/s)M_k^{(1),\mathrm{g}} \tag{58}$$

$$\Rightarrow d\sigma^{(1)}_{k,\mathrm{g}} = \frac{1}{2s'}\frac{K_{\mathrm{g}\gamma}E_k(\epsilon)}{2}b_k(\epsilon)M_k^{(1),\mathrm{g}'}dPS'_{3,\mathrm{g}} \tag{59}$$

The soft and collinear factors $(1-x)^{-1+\epsilon}$ and $(1-y^2)^{-1+\epsilon/2}$ can be replaced by generalized plus distributions[1]

$$(1-x)^{-1+\epsilon} \sim \left(\frac{1}{1-x}\right)_{\tilde{\rho}} + \epsilon \left(\frac{\ln(1-x)}{1-x}\right)_{\tilde{\rho}} + \delta(1-x)\left(\frac{1}{\epsilon} + 2\ln\tilde{\beta} + 2\epsilon\ln^2(\tilde{\beta})\right) + O(\epsilon^2) \tag{60}$$

$$\begin{aligned}(1-y^2)^{-1+\epsilon} \sim \frac{1}{2}\left(\left(\frac{1}{1+y}\right)_{\omega} + \left(\frac{1}{1-y}\right)_{\omega}\right)& \\ + (\delta(1+y)+\delta(1-y))\left(\frac{1}{2\epsilon} + \frac{1}{2}\ln(2\omega)\right) + O(\epsilon)&\end{aligned} \tag{61}$$

$$(1+y)^{-1+\epsilon} \sim \left(\frac{1}{1+y}\right)_{\omega} + \delta(1+y)\left(\frac{1}{\epsilon} + \ln\omega\right) + O(\epsilon) \tag{62}$$

inside integration over smooth functions with $\tilde{\beta} = \sqrt{1-\tilde{\rho}}$. The distributions are defined by

$$\int_{\tilde{\rho}}^{1} dx\, f(x)\left(\frac{1}{1-x}\right)_{\tilde{\rho}} = \int_{\tilde{\rho}}^{1} dx\, \frac{f(x)-f(1)}{1-x} \tag{63}$$

$$\int_{\tilde{\rho}}^{1} dx\, f(x)\left(\frac{\ln(1-x)}{1-x}\right)_{\tilde{\rho}} = \int_{\tilde{\rho}}^{1} dx\, \frac{f(x)-f(1)}{1-x}\ln(1-x) \tag{64}$$

$$\int_{-1}^{-1+\omega} dy\, f(y)\left(\frac{1}{1+y}\right)_{\omega} = \int_{-1}^{-1+\omega} dy\, \frac{f(y)-f(-1)}{1+y} \tag{65}$$

$$\int_{1-\omega}^{1} dy\, f(y)\left(\frac{1}{1-y}\right)_{\omega} = \int_{1-\omega}^{1} dy\, \frac{f(y)-f(1)}{1-y} \tag{66}$$

with $\rho^* \leq \tilde{\rho} < 1$ and $0 < \omega \leq 2$. If the integration does not include a singularity the distribution sign can be dropped. From an analytical point of view the results may not depend on the specific choice of the regularisation parameters $\tilde{\rho}$ and ω but for any numerical purpose they may influence the rate of convergence or stability. For numerical computations we must also cut the poles out of the integrations

$$\int_{\rho^*}^{1} dx \to \int_{\rho^*}^{1-\delta_x} dx \qquad\qquad \int_{-1}^{1} dy \to \int_{-1+\delta_y}^{1} dy \tag{67}$$

If not stated otherwise we use as a default setup **FiXme Error: justify numbers?**: FiXme Error!

$$\tilde{\rho} = \rho^* + \tilde{x}(1-\rho^*)\text{ with }\tilde{x} = 0.8 \qquad\qquad \omega = 1.0 \tag{68}$$

$$\delta_x = 1\times 10^{-6} \qquad\qquad \delta_y = 7\times 10^{-6} \tag{69}$$

FiXme Error: shift (parts) to appendix? FiXme Error!

With the given distribution we can split the gluonic NLO part into three pieces[1]:

$$d\sigma^{(1)}_{k,\mathrm{g}} = d\sigma^{(1),s}_{k,\mathrm{g}} + d\sigma^{(1),c-}_{k,\mathrm{g}} + d\sigma^{(1),f}_{k,\mathrm{g}} \tag{70}$$

corresponding to the soft ($d\sigma^{(1),s}_{k,\mathrm{g}} \sim \delta(1-x)$), the collinear ($d\sigma^{(1),c-}_{k,\mathrm{g}} \sim \delta(1+y)$) and the finite parts ($d\sigma^{(1),f}_{k,\mathrm{g}} \sim \left(\frac{1}{1-x}\right)_{\tilde\rho}\left(\frac{1}{1+y}\right)_{\omega}$). Note that for $q^2 < 0$ there is only a single collinear contribution ($y \to -1$).

The soft matrix elements can be obtained from the above expressions by taking the soft limit $k_2 \to 0$:

$$\lim_{k_2\to 0}\left(C_A R_{k,OK} + 2C_F R_{k,QED}\right) = \left(C_A S_{k,OK} + 2C_F S_{k,QED}\right) + O(1/s_4, 1/s_3, 1/t') \quad (71)$$

with

$$S_{k,OK} = 2\left(\frac{t_1}{t' s_3} + \frac{u_1}{t' s_4} - \frac{s-2m^2}{s_3 s_4}\right) B_{k,QED} \quad (72)$$

$$S_{k,QED} = 2\left(\frac{s-2m^2}{s_3 s_4} - \frac{m^2}{s_3^2} - \frac{m^2}{s_4^2}\right) B_{k,QED} \quad (73)$$

Note that the einkonal factors multiplying the Born functions $B_{k,QED}$ neither depend on q^2 nor on the projection k. But the integrated expressions do depend on the regularization scheme, i.e. will here depend on $\tilde\rho$ rather then on a phasespace slicing parameter Δ as for inclusive calculations[3] **FiXme Error: add my cite**. We find for the integrated expressions FiXme Error!

$$d\sigma^{(1),s}_{k,\mathrm{g}} = \frac{1}{2s'}\frac{K_{\mathrm{g}\gamma}E_k(\epsilon)}{2} b_k(\epsilon) M_k^{(1),S} dPS_2 \quad (74)$$

$$M_k^{(1),S} = 8g^4\mu_D^{-\epsilon}e^2e_H^2 N_C C_F C_\epsilon\left(C_A\tilde S_{OK} + 2C_F\tilde S_{QED}\right) B_{k,QED} \quad (75)$$

where the full expressions for $\tilde S$ can be found in [1] and the poles are given by

$$\tilde S_{OK} = 2\left(\frac{4}{\epsilon^2} + \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) - \frac{2m^2-s}{s\beta}\ln(\chi) + 4\ln(\tilde\beta)\right)\frac{2}{\epsilon}\right) + O(\epsilon^0) \quad (76)$$

$$\tilde S_{QED} = -2\cdot\left(1 - \frac{2m^2-s}{s\beta}\ln(\chi)\right)\frac{2}{\epsilon} + O(\epsilon^0) \quad (77)$$

The results agree in the photo-production limit ($q^2 \to 0$) with [15].

3.4 Light Quark Processes

In next-to-leading order a new production mechanism enters that is induced by a light quark, so we have to consider the process

$$\gamma^*(q) + q(k_1) \to Q(p_1) + \overline{Q}(p_2) + q(k_2) \quad (78)$$

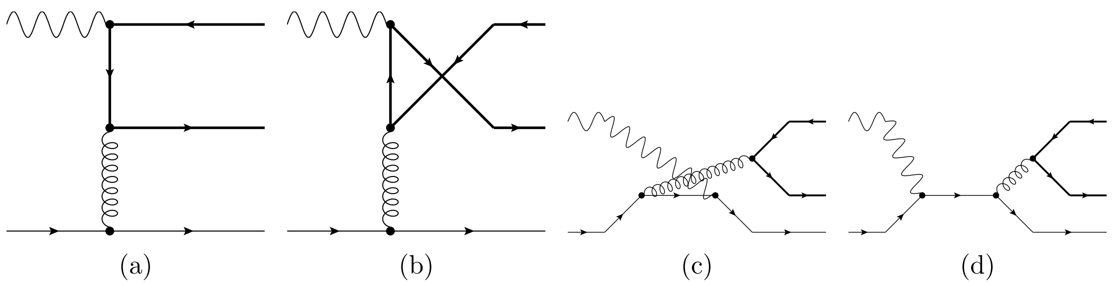

Figure 3: next-to-leading order Feynman diagrams for the light quark processes $i\varepsilon_\gamma^\mu(q)u_q^a(k_1)\,\mathcal{M}_{j,\mu a}^{(1),q}$

FiXme Error: shift to appendix?

where all contributing diagrams are depicted in figure 3.

The result can then be written as

$$M_k^{(1),q} = \hat{\mathcal{P}}_k^{\gamma,\mu\mu'}\hat{\mathcal{P}}_k^{q,aa'}\sum_{j,j'=1}^{4}\mathcal{M}_{j,\mu a}^{(1),q}\left(\mathcal{M}_{j',\mu' a'}^{(1),q}\right)^* \tag{79}$$

$$= 8g^4\mu_D^{-2\epsilon}e^2N_CC_F\left(e_H^2A_{k,1}+e_L^2A_{k,2}+e_Le_HA_{k,3}\right) \tag{80}$$

where e_L denotes the charge of the light quark q in units of e.

For the light quark part we shift the occuring collinear $(y\to-1)$ poles from the matrix elements to the phase space by dividing by $t'\propto(1+y)(1-x)$:

$$dPS'_{3,q} = \frac{dPS_3}{t'} = dPS_3\cdot\left(\frac{2s}{s'^2}\right)\frac{1}{(1-x)(1+y)} \tag{81}$$

$$= \frac{T_\epsilon}{\pi}\left(\frac{s'^2}{s}\right)^{\epsilon/2}(1-x)^\epsilon(1-y)^{\epsilon/2}(1+y)^{-1+\epsilon/2}dPS_2^{(5)}dy\sin^\epsilon(\theta_2)d\theta_2 \tag{82}$$

$$M_k^{(1),q'} = t'M_k^{(1),q} \tag{83}$$

$$\Rightarrow d\sigma_{k,q}^{(1)} = \frac{1}{2s'}\frac{K_{q\gamma}}{2}b_k(\epsilon)M_k^{(1),q'}dPS'_{3,q} \tag{84}$$

We can now use again the above defined distributions to replace the divergent factor $(1+y)^{-1+\epsilon/2}$ to split the light quark NLO part into two pieces[1]

$$d\sigma_{k,q}^{(1)} = d\sigma_{k,q}^{(1),c-}+d\sigma_{k,q}^{(1),f} \tag{85}$$

corresponding to the collinear part ($d\sigma_{k,q}^{(1),c-}\sim\delta(1+y)$) and the finite part ($d\sigma_{k,q}^{(1),f}\sim\left(\frac{1}{1+y}\right)_\omega$). Note that for $q^2<0$ there is only a single collinear contribution $(y\to-1)$ in $A_{k,1}$, i.e. $A_{k,2}$ does not contain any poles.

The results agree in the photo-production limit $(q^2\to0)$ with [15].

4 Mass Factorization

Due to the factorization theorem [16] all remaining poles in the gluonic subprocess can be removed by mass factorization in the following way:

$$d\sigma_{k,\mathrm{g}}^{(1),fin}(\mu_F) = \lim_{\epsilon\to 0}\left[d\sigma_{k,\mathrm{g}}^{(1)}(\mu_D,\epsilon) - \int_0^1 dx\,\Gamma_{k,\mathrm{gg}}^{(1)}(x,\mu_F^2,\mu_D,\epsilon)d\sigma_{k,\mathrm{g}}^{(0)}(xk_1,\epsilon)\right] \tag{86}$$

$$\Gamma_{k,ij}^{(1)}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_S}{2\pi}\left(P_{k,ij}(x)\frac{2}{\epsilon} + f_{k,ij}(x,\mu_F^2,\mu_D^2)\right) \tag{87}$$

where $\Gamma_{k,ij}^{(1)}$ is the first order correction to the transition functions $\Gamma_{k,ij}$ for *incoming* particle j and *outgoing* particle i in projection k. In the $\overline{\mathrm{MS}}$-scheme the $f_{k,ij}$ take their usual form and we find

$$\Gamma_{k,ij}^{(1),\overline{\mathrm{MS}}}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_s}{2\pi}P_{k,ij}(x)\left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln(\mu_F^2/m^2) - \ln(\mu_D^2/m^2)\right) \tag{88}$$

$$= 8\pi\alpha_s P_{k,ij}(x)C_\epsilon\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}\left(\frac{2}{\epsilon} + \ln(\mu_F^2/m^2)\right) \tag{89}$$

The $P_{k,ij}(x)$ are the Altarelli-Parisi splitting functions for which we find[17, 18, 19]

$$P_{k,\mathrm{gg}}(x) = \left(P_{k,\mathrm{gg}}^{H,(0)}(x) + \epsilon P_{k,\mathrm{gg}}^{H,(1)}(x)\right) + \delta(1-x)\left(P_{k,\mathrm{gg}}^{S,(0)} + \epsilon P_{k,\mathrm{gg}}^{S,(1)}\right) + O(\epsilon^2) \tag{90}$$

$$P_{G,\mathrm{gg}}(x) = P_{L,\mathrm{gg}}(x) \tag{91}$$

$$= 2C_A\left(\frac{x}{(1-x)_{\tilde\rho}} + \frac{1-x}{x} + x(1-x)\right) + \delta(1-x)\left(\frac{b_0^{lf}}{2} + 4C_A\ln(\tilde\beta)\right) \tag{92}$$

$$P_{P,\mathrm{gg}}(x) = 2C_A\left(\frac{1}{(1-x)_{\tilde\rho}} - 2x + 1 - \epsilon(1-x)\right) + \delta(1-x)\left(\frac{b_0^{lf}}{2} + 4C_A\ln(\tilde\beta) - \epsilon\frac{N_C}{6}\right) \tag{93}$$

with $b_0^{lf} = \frac{11}{6}C_A - \frac{2}{3}T_F n_{lf}$ the first coefficient of the QCD beta function and $T_F = 1/2$ the normalization of the generators in color space.

The poles introduced by eq. (89) multiplied together with the hard part $P_{k,\mathrm{gg}}^{H}(x)$ remove all remaining collinear poles. The non vanishing $P_{P,\mathrm{gg}}^{H,(1)}$ represents the hat contributions that arise when the n-dimensional spacetime is treated by the HVBM scheme[20]. This prescription is avoided in our approach by the introduction of the generalised plus distribution. The poles multiplied with the soft part $P_{k,\mathrm{gg}}^{S}$ remove all remaining soft poles. We have

$$\lim_{x\to 1}(1-x)P_{k,\mathrm{gg}}^{H,(0)} = 2C_A \tag{94}$$

The light quark process can be regularized in a complete analogous way:

$$d\sigma_{k,q}^{(1),fin}(\mu_F) = \lim_{\epsilon\to 0}\left[d\sigma_{k,q}^{(1)}(\mu_D,\epsilon) - \int_0^1 dx\,\Gamma_{k,\mathrm{g}q}^{(1)}(x,\mu_F^2,\mu_D,\epsilon)d\sigma_{k,\mathrm{g}}^{(0)}(xk_1,\epsilon)\right] \tag{95}$$

For the Altarelli-Parisi splitting functions $P_{k,\mathrm{g}q}$ we find[17, 18, 19]:

$$P_{k,\mathrm{g}q}(x) = P^{(0)}_{k,\mathrm{g}q}(x) + \epsilon P^{(1)}_{k,\mathrm{g}q}(x) + O(\epsilon^2) \tag{96}$$

$$P_{G,\mathrm{g}q}(x) = P_{L,\mathrm{g}q}(x) = 2C_F \frac{1+(1-x)^2}{x} + \epsilon \frac{C_F}{2} x \tag{97}$$

$$P_{P,\mathrm{g}q}(x) = C_F(2-x) - \epsilon C_F(1-z) \tag{98}$$

The poles introduced by eq. (89) multiplied together with $P_{k,\mathrm{g}q}(x)$ remove all remaining collinear poles. Note that $K_{q\gamma} = 1/(N_C) = 2C_F K_{\mathrm{g}\gamma}$.

The final finite cross sections are then for the gluonic part

$$\begin{aligned} d\sigma^{(1),s+v}_{k,\mathrm{g}} &= \alpha_S^2 \alpha_{em} \frac{1}{2s'} \frac{K_{\mathrm{g}\gamma} E_k(0)}{2} b_k(0) N_C C_F B_{k,QED} \cdot 2^7 \Big[4C_A \ln^2(\tilde{\beta}) \\ &\quad + \ln(\tilde{\beta}) \Big(2 \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) - \ln(\mu_F^2/m^2) \right) C_A \\ &\quad -2C_F + \frac{s-2m^2}{s\beta} \ln(\chi)(C_A - 2C_F) \Big) \\ &\quad + \frac{\beta_0^{lf}}{4} \left(\ln(\mu_R^2/m^2) - \ln(\mu_F^2/m^2) \right) + f_k(s, q^2, \theta_1) \Big] dPS_2 \end{aligned} \tag{99}$$

$$\begin{aligned} d\sigma^{(1),c-}_{k,\mathrm{g}} &= 16 \cdot \frac{1}{2xs'} \frac{K_{\mathrm{g}\gamma} E_k(0)}{2} b_k(0) \cdot \alpha_S^2 e^2 e_H^2 N_C C_F B_{k,QED}(xk_1) dPS_2^{(5)} \\ &\quad \cdot \Bigg[(1-x) P^{H,(0)}_{k,\mathrm{gg}}(x) \left(\frac{1}{(1-x)_{\tilde{\rho}}} \left(\ln(s'/\mu_F^2) + \ln(s'/s) + \ln(\omega/2) \right) \right. \\ &\quad \left. +2 \left(\frac{\ln(1-x)}{1-x} \right)_{\tilde{\rho}} \right) + 2P^{H,(1)}_{k,\mathrm{gg}}(x) \Bigg] \end{aligned} \tag{100}$$

$$\begin{aligned} d\sigma^{(1),f}_{k,\mathrm{g}} &= 2 \left(\frac{1}{4\pi} \right)^4 \frac{1}{2s'} \frac{K_{\mathrm{g}\gamma} E_k(0)}{2} b_k(0) \frac{s\beta_5}{(s')^2} \left(\frac{1}{1-x} \right)_{\tilde{\rho}} \left(\frac{1}{1+y} \right)_{\omega} \frac{1}{1-y} \\ &\quad \cdot M_k^{(1),\mathrm{g}'} dx dy \sin(\theta_1) d\theta_1 d\theta_2 \end{aligned} \tag{101}$$

$$\begin{aligned} &= \alpha_S^2 \alpha_{em} e_H^2 \cdot K_{\mathrm{g}\gamma} N_C C_F \frac{s \sin(\theta_1)}{\pi (s')^3} \frac{\beta_5}{1-y} \left(\frac{1}{1-x} \right)_{\tilde{\rho}} \left(\frac{1}{1+y} \right)_{\omega} \\ &\quad \cdot b_k(0) t'(u' - q^2 s_5/s) \left(C_A R_{k,OK} + 2C_F R_{k,QED} \right) dx dy d\theta_1 d\theta_2 \end{aligned} \tag{102}$$

where $d\sigma^{(1),s+v}_{k,\mathrm{g}}$ collects all 2-to-2-phasespace contributions, that is soft contributions $d\sigma^{(1),S}_{k,\mathrm{g}}$, virtual contributions $d\sigma^{(1),V}_{k,\mathrm{g}}$, quark self-energies and factorization contributions and f_k contains lots of logarithms and dilogarithms, but does not depend on $\tilde{\rho}, \mu_F^2, \mu_R^2$ nor n_f nor $\beta_0^{lf} = (11C_A - 2n_{lf})/3$.

For the light quark process we find:

$$d\sigma_{k,q}^{(1),c-} = 8 \cdot \frac{1}{2xs'} \frac{K_{q\gamma}}{2} b_k(0) \cdot \alpha_S^2 e^2 e_H^2 N_C B_{k,QED}(xk_1) dPS_2^{(5)} \\ \cdot \left(P_{k,gq}^{(0)}(x)\left(\ln(s'/\mu_F^2) + \ln(s'/s) + \ln(\omega/2) + 2\ln(1-x)\right) + 2P_{k,gq}^{(1)}(x)\right) \tag{103}$$

$$d\sigma_{k,q}^{(1),f} = -\left(\frac{1}{4\pi}\right)^4 \frac{1}{2s'} \frac{K_{q\gamma}}{2} b_k(0)\beta_5 \left(\frac{1}{1+y}\right)_\omega M_k^{(1),q'} dxdy \sin(\theta_1) d\theta_1 d\theta_2 \tag{104}$$

$$= \alpha_S^2 \alpha_{em} \cdot K_{q\gamma} N_C C_F \left(-\frac{1}{2\pi}\right) \frac{\beta_5 \sin(\theta_1)}{s'} \left(\frac{1}{1+y}\right)_\omega \\ \cdot b_k(0) t' \left(e_H^2 A_{k,1} + e_L^2 A_{k,2} + e_H e_L A_{k,3}\right) dxdyd\theta_1 d\theta_2 \tag{105}$$

5 Partonic Results

FiXme Error!

FiXme Error: do anything here? results agree mostly with inclusive counterpart - discuss numerical issues, such as errors/speed, on 2- (inclusive) vs. 4- (fully differential) dimensional integration? Note there are still subtles differences between the two codes, i.e. cqbarF1 and dq1

6 Hadronic Results

FiXme Error!

FiXme Error: todo: repeat plots from NPB452-109[1]? redo them for $k = P$? define a differential spin asymmetry $(dF_P/d\square)/(dF_T/d\square)$?

7 Summary

FiXme Error!

FiXme Error: do

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List of Corrections