1 Introduction

This work is mainly based on the paper "Heavy quark correlations in deep inelastic electroproduction" by Harris et. al.[1] - that is, it recalculates all properties and formulas. It extends then the application to the equivalent *polarized* processes. The treating of the polarized processes can for example be found in [2] and we will use many ideas and technices from there. **FiXme Error: more** FiXme Error!

1.1 Motivation

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FiXme Error: why do we do this

1.2 Notation

We calculate in $n = 4 + \epsilon$ dimension to regularize all soft, collinear and ultraviolet poles. Unfortunaly this extension for *polarized* processes is nontrivial, due to the occurance of the Levi-Civita tensors $\varepsilon_{\mu\nu\rho\sigma}$ and γ_5. A common choice to deal with these objects is the HVBM prescription[3] that keeps those two objects four dimensional at the price of splitting the full n-dimensional space into a $(n-4)$-dimensional space, called "hat-space", and a four-dimensional space (that is actually never used).

In leading order (LO) we have to consider the following process:

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) \tag{1}$$

The corresponding parton structure tensor $W^{(0)}_{\mu\mu'}$ can then be written as **FiXme Error: avoid all order expr? FiXme Error: remove?** FiXme Error! FiXme Error!

$$\begin{aligned} &W^{(0)}_{\mu\mu'}(k_1, q; s, t_1, u_1, q^2; \sigma_q) \\ &= \frac{1}{2} E_\epsilon K_{\mathrm{g}\gamma} \frac{1}{2s'} \int \frac{d^{n-1}p_1}{2E_1(2\pi)^{n-1}} \int \frac{d^{n-1}p_2}{2E_2(2\pi)^{n-1}} \delta(p_1^2 - m^2)\delta(p_2^2 - m^2) \\ &\quad (2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2)\, \mathcal{M}^{(0)}_\mu \mathcal{M}^{(0)}_{\mu'} \end{aligned} \tag{2}$$

where the initial 1/2 is the initial state spin average, $K_{\mathrm{g}\gamma}$ is the color average,

$$E_\epsilon := \begin{cases} 1/(1+\epsilon/2) & \text{unpolarized} \\ 1 & \text{polarized} \end{cases} \tag{3}$$

accounts for additional degrees of freedom in n dimensions for initial bosons. The Lorentz indices μ and μ' refer to the virtual photon that is exchanged with the scattering lepton. We have chosen to detect the heavy *antiquark* $\overline{Q}(p_2)$ and so we define the following Mandelstam variables:

$$s = (q + k_1)^2, \quad t_1 = t - m^2 = (k_1 - p_2)^2 - m^2, \quad u_1 = u - m^2 = (q - p_2)^2 - m^2 \tag{4}$$

For convenience we also define $s' = s - q^2$ and $u_1' = u_1 - q^2$. If the heavy *quark* $\overline{Q}(p_1)$ is detected, p_2 in eq. (4) has to be replaced by p_1 which effectively interchanges $t_1 \leftrightarrow u_1$. **FiXme Error: move to LO?** FiXme Error!

By using Lorentz covariance, hermiticity, parity invariance and current conservation the parton structure tensor can be decomposed into several parts:

$$\begin{aligned} W_{\mu\mu'}(k_1, q; s, t_1, u_1, q^2; \sigma_q) = &\left(-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2}\right) \frac{d^2\sigma_T(s, t_1, u_1, q^2)}{dt_1 du_1} \\ &+ \left(k_{1,\mu} - \frac{k_1 \cdot q}{q^2} q_\mu\right)\left(k_{1,\mu'} - \frac{k_1 \cdot q}{q^2} q_{\mu'}\right)\left(\frac{-4q^2}{s'^2}\right) \\ &\cdot \left(\frac{d^2\sigma_T(s, t_1, u_1, q^2)}{dt_1 du_1} + \frac{d^2\sigma_L(s, t_1, u_1, q^2)}{dt_1 du_1}\right) \end{aligned} \tag{5}$$

FiXme Error: extend We can then define appropiate projection operators[4, 5] FiXme Error!

$$\hat{\mathcal{P}}^{\gamma}_{G,\mu\mu'} = -g_{\mu\mu'} \qquad b_G(\epsilon) = \frac{1}{2(1+\epsilon/2)} \tag{6}$$

$$\hat{\mathcal{P}}^{\gamma}_{L,\mu\mu'} = -\frac{4q^2}{s'^2} k_{1,\mu} k_{1,\mu'} \qquad b_L(\epsilon) = 1 \tag{7}$$

$$\hat{\mathcal{P}}^{\gamma}_{P,\mu\mu'} = i\varepsilon_{\mu\mu'\rho\rho'} \frac{q^\rho k_1^{\rho'}}{s'} \qquad b_P(\epsilon) = 1 \tag{8}$$

FiXme Error: justify avoidance of $\boldsymbol{\Delta}$? and we then find FiXme Error!

$$\frac{d^2\sigma_k(s, t_1, u_1, q^2)}{dt_1 t u_1} = b_k(\epsilon) \hat{\mathcal{P}}^{\gamma}_{k,\mu\mu'} W^{\mu\mu'} \tag{9}$$

FiXme Error: how to insert 2nd projector? with $k \in \{G, L, P\}$ denoting (here and mostly ever after) the projection type. The transverse partonic cross section $d\sigma_T$ can be reconstructed from the above definitions by using FiXme Error!

$$d\sigma_T = d\sigma_G + b_G(\epsilon) d\sigma_L \tag{10}$$

We also define accordingly

$$E_G(\epsilon) = E_L(\epsilon) = \frac{1}{1+\epsilon/2} \qquad E_P(\epsilon) = 1 \tag{11}$$

The final state spins are always summed over, but the initial spins have to be treated seperately: for unpolarized projections $k \in \{G, L\}$ they are also summed over, but for the polarized projection $k = P$ they are projected on their asymmetric part:

$$\hat{\mathcal{P}}^{\mathrm{g}}_{G,\nu\nu'} = \hat{\mathcal{P}}^{\mathrm{g}}_{L,\nu\nu'} = -g_{\nu\nu'} \qquad \hat{\mathcal{P}}^{\mathrm{g}}_{P,\nu\nu'} = 2i\epsilon_{\nu\nu'\rho\rho'} \frac{k_1^\rho q^\rho}{s'} \tag{12}$$

where ν and ν' refer to the initial gluon. By writing $\hat{\mathcal{P}}^{\mathrm{g}}_{G,\nu\nu'}$ in eq. (12) we decided to introduce Fadeev-Popov ghosts[2] as we got a single diagram in next-to-leading order with a triple-gluon vertex **FiXme Error: explain ghosts?**. As we can consider all quarks in the initial state as massless, we further find FiXme Error!

$$\hat{\mathcal{P}}^q_{G,aa'} = \hat{\mathcal{P}}^q_{L,aa'} = (\not{k}_1)_{aa'} \qquad \hat{\mathcal{P}}^q_{P,aa'} = -(\gamma_5\not{k}_1)_{aa'} \tag{13}$$

$$\hat{\mathcal{P}}^{\bar{q}}_{G,bb'} = \hat{\mathcal{P}}^{\bar{q}}_{L,bb'} = (\not{k}_1)_{bb'} \qquad \hat{\mathcal{P}}^{\bar{q}}_{P,bb'} = (\gamma_5\not{k}_1)_{bb'} \tag{14}$$

where a and a' refer to the Dirac-index of the intial quark spinor in next-to-leading order - analogous for b and the antiquark.

We further define a set of partonic variables:

$$0 \leq \rho = \frac{4m^2}{s} \leq 1 \qquad 0 \leq \beta = \sqrt{1-\rho} \leq 1 \qquad 0 \leq \chi = \frac{1-\beta}{1+\beta} \leq 1 \tag{15}$$

$$\rho_q = \frac{4m^2}{q^2} \leq 0 \qquad 1 \leq \beta_q = \sqrt{1-\rho_q} \qquad 0 \leq \chi_q = \frac{\beta_q - 1}{\beta_q + 1} \leq 1 \tag{16}$$

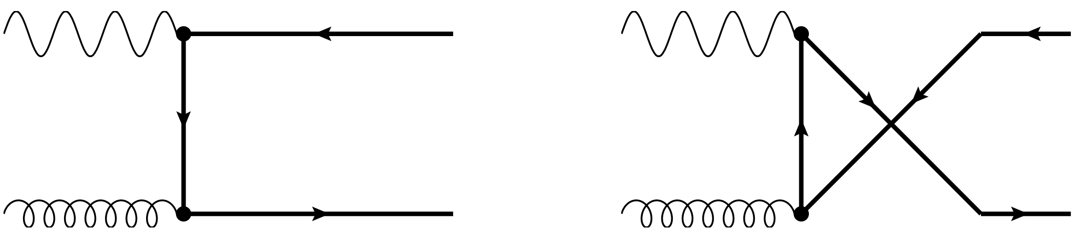
When computing Feynman diagrams a computer algebra system (CAS) is almost obligatory: common choices are `FORM`[6] or `Mathematica`[7] - for the later the most common choice is `TRACER`[8], but we have chosen `HEPMath`[9]. We used the Feynman rules given by [10] and [2].

2 Leading Order Calculations

In leading order we have to consider photon-gluon-fusion (PGF), that is

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) \tag{17}$$

with two contributing diagrams depicted in figure 1.



(a) $i\varepsilon^\mu_\gamma(q)\varepsilon^\nu_{\mathrm{g}}(k_1)\,\mathcal{M}^{(0),1}_{\mu\nu}$ (b) $i\varepsilon^\mu_\gamma(q)\varepsilon^\nu_{\mathrm{g}}(k_1)\,\mathcal{M}^{(0),2}_{\mu\nu}$

Figure 1: leading order Feynman diagrams
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The result can then be written as

$$M^{(0)}_k = \hat{\mathcal{P}}^{\gamma,\mu\mu'}_k \hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_k \sum_{j,j'=1}^{2} \mathcal{M}^{(0),j}_{\mu\nu} \left(\mathcal{M}^{(0),j'}_{\mu'\nu'}\right)^* = 8g^2\mu_D^{-\epsilon} e^2 e_H^2 N_C C_F B_{k,QED} \tag{18}$$

where g and e are the strong and electromagnetic coupling constants respectively, μ_D is an arbitray mass parameter introduced to keep the couplings dimensionless and e_H is the magnitude of the heavy quark in units of e. Further N_C corresponds to the gauge group $SU(N_C)$ and the color factor $C_F = (N_C^2 - 1)/(2N_C)$ refers to the second Casimir constant of the fundamental representation for the quarks. We then find:

$$\begin{aligned} B_{G,QED} &= \frac{t_1}{u_1} + \frac{u_1}{t_1} + \frac{4m^2 s'}{t_1 u_1}\left(1 - \frac{m^2 s'}{t_1 u_1}\right) + \frac{2s' q^2}{t_1 u_1} + \frac{2q^4}{t_1 u_1} + \frac{2m^2 q^2}{t_1 u_1}\left(2 - \frac{s'^2}{t_1 u_1}\right) \\ &\quad + \epsilon\left\{-1 + \frac{s'^2}{t_1 u_1} + \frac{s' q^2}{t_1 u_1} - \frac{q^4}{t_1 u_1} - \frac{m^2 q^2 s'^2}{t_1^2 u_1^2}\right\} + \epsilon^2 \frac{s'^2}{4 t_1 u_1} \qquad (19) \\ B_{L,QED} &= -\frac{4q^2}{s'}\left(\frac{s}{s'} - \frac{m^2 s'}{t_1 u_1}\right) \qquad (20) \\ B_{P,QED} &= \frac{1}{2}\left(\frac{t_1}{u_1} + \frac{u_1}{t_1}\right)\left(\frac{2m^2 s'}{t_1 u_1} - 1 - \frac{2q^2}{s'}\right) \qquad (21) \\ B_{k,QED} &= B_{k,QED}^{(0)} + \epsilon B_{k,QED}^{(1)} + \epsilon^2 B_{k,QED}^{(2)} \qquad (22) \end{aligned}$$

By using eq. (2) we can derive the n-dimensional $2 \to 2$ phase space

$$dPS_2 = \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_2}{(2\pi)^{n-1}} \Theta(p_{1,0})\delta(p_1^2 - m^2)\Theta(p_{2,0})\delta(p_2^2 - m^2)(2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2) \qquad (23)$$

that can be solved by using the center-of-mass system (CMS) of the incoming particles[2]

$$q = \left(\frac{s + q^2}{2\sqrt{s}}, 0, 0, -\frac{s - q^2}{2\sqrt{s}}, \hat{0}\right) \qquad k_1 = \frac{s - q^2}{2\sqrt{s}}\left(1, 0, 0, 1, \hat{0}\right) \qquad (24)$$

such that $q + k_1 = (\sqrt{s}, \vec{0})$ and $k_1^2 = 0$. For the outgoing particles it follows

$$p_1 = \frac{\sqrt{s}}{2}\left(1, 0, -\beta \sin\theta_1, -\beta \cos\theta_1, \hat{0}\right) \qquad p_2 = \frac{\sqrt{s}}{2}\left(1, 0, \beta \sin\theta_1, \beta \cos\theta_1, \hat{0}\right) \qquad (25)$$

such that $p_1 + p_2 = (\sqrt{s}, \vec{0})$ and $p_1^2 = p_2^2 = m^2$ and

$$t_1 = -\frac{s'}{2}(1 - \beta\cos(\theta_1)), \quad u_1 = -\frac{s'}{2}(1 + \beta\cos(\theta_1)) \qquad (26)$$

We then arrive at the well known result[1]:

$$dPS_2 = \frac{\beta \sin(\theta_1)}{16\pi\Gamma(1 + \epsilon/2)}\left(\frac{s\beta^2 \sin^2(\theta_1)}{16\pi}\right)^{\epsilon/2} d\theta_1 \qquad (27)$$

The cross sections are then given by:

$$d\sigma_k^{(0)} = \frac{1}{2s}\frac{1}{2} b_k(\epsilon) E_k(\epsilon) K_{\gamma g} M_k^{(0)} dPS_2 \qquad (28)$$

The procedure is completely analogous to the inclusive case **FiXme Error: cite** and the results agree to there. FiXme Error!

3 Next-To-Leading Order Calculations

Next-to-leading order contributions can be split into three parts: one loop virtual contributions, one gluon radiation and the light quark processes. **FiXme Error: more?** FiXme Error!

3.1 One Loop Virtual Contributions

Virtual contributions have the same initial and final state as the Born process, but have a looping particle. All contributing Feynman diagrams are depicted in figure **FiXme Error: do**. The result can then be written as FiXme Error!

$$\begin{aligned} M_k^{(1),V} &= \hat{\mathcal{P}}_k^{\gamma,\mu\mu'}\hat{\mathcal{P}}_k^{g}\sum_j \left[\mathcal{M}_{j,\mu}^{(1),V}\left(\mathcal{M}_{1,\mu'}^{(0)}+\mathcal{M}_{2,\mu'}^{(0)}\right)^* + c.c.\right] \\ &= 8g^4\mu_D^{-\epsilon}e^2e_H^2N_CC_FC_\epsilon\left(C_AV_{k,OK}+2C_FV_{k,QED}\right) \end{aligned} \tag{29}$$

where $C_\epsilon = \exp(\epsilon/2(\gamma_E - \ln(4\pi)))/(16\pi^2)$ and $C_A = N_C$ is the second Casimir constant of the adjoint representation for the gluon (that introduces a non-abelian part).

For the computation of the loops the Passarino-Veltman-decomposition[11] in $n = 4+\epsilon$ dimension is used as far as possible. The decomposition is based on Lorentz invariance and a good explanation is for example given in [2]. The needed scalar integrals are given in [12] and [4], but there is also one wrong integral: we find with [13, Box 16]:

$$\begin{aligned} &D_0(m^2,0,q^2,m^2,t,s,0,m^2,m^2,m^2) \\ &= \frac{iC_\epsilon}{\beta s t_1}\Big[-\frac{2\ln(\chi)}{\epsilon} - 2\ln(\chi)\ln(-t_1/m^2) + \mathrm{Li}_2(1-\chi^2) - 4\zeta(2) + \ln^2(\chi_q) + 2\,\mathrm{Li}_2(-\chi\chi_q) \\ &\qquad + 2\,\mathrm{Li}_2(-\chi/\chi_q) + 2\ln(\chi\chi_q)\ln(1+\chi\chi_q) + 2\ln(\chi/\chi_q)\ln(1+\chi/\chi_q)\Big] \end{aligned} \tag{30}$$

where we used the argument ordering of `LoopTools`[14, 15] (and also checked it against `LoopTools`).

As the short example above shows, the full expressions for the $V_{k,OK}, V_{k,QED}$ are quite complicated and too long to be presented here, nevertheless the arising poles are quite compact:

$$V_{k,OK} = -2B_{k,QED}\left(\frac{4}{\epsilon^2} + \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) - \frac{2m^2-s}{s}\ln(\chi)\right)\frac{2}{\epsilon}\right) + O(\epsilon^0) \tag{31}$$

$$V_{k,QED} = -2B_{k,QED}\left(1 - \frac{2m^2-s}{s}\ln(\chi)\right)\frac{2}{\epsilon} + O(\epsilon^0) \tag{32}$$

The above results already include the mass renormalization that we have performed *on-shell*, so all ultra-violet poles have been removed. For the renormalization of the strong coupling we use the $\overline{\mathrm{MS}}_m$ scheme defined in [2] and so the full (remaining) renormalization can be achieved by

$$\frac{d^2\sigma_k^{(1),V,ren.}}{dt_1du_1} = \frac{d^2\sigma_k^{(1),V}}{dt_1du_1} + \frac{\alpha_s(\mu_R^2)}{4\pi}\left[\left(\frac{2}{\epsilon}+\gamma_E-\ln(4\pi)+\ln(\mu_R^2/m^2)-\ln(\mu_D^2/m^2)\right)\beta_0^f\right.$$
$$\left.+\frac{2}{3}\ln(\mu_R^2/m^2)\right]\frac{d^2\sigma_k^{(0)}}{dt_1du_1} \quad (33)$$
$$= \frac{d^2\sigma_k^{(1),V}}{dt_1du_1} + 4\pi\alpha_s(\mu_R^2)C_\epsilon\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}\left[\left(\frac{2}{\epsilon}+\ln(\mu_R^2/m^2)\right)\beta_0^f\right.$$
$$\left.+\frac{2}{3}\ln(\mu_R^2/m^2)\right]\frac{d^2\sigma_k^{(0)}}{dt_1du_1} \quad (34)$$

with μ_R the renormalization scale introduced by the renormalization group equation (RGE), $\beta_0^f = (11C_A - 2n_f)/3$ the first coefficient of the beta function and n_f the number of *total* flavours (i.e. $n_{lf} = n_f - 1$ active (light) flavours and one heavy flavour). The double poles occuring in $V_{k,OK}$ are introduced by the diagrams **FiXme Error: do** when the soft and collinear singularities coincide. FiXme Error!

The results agree in the photo-production limit ($q^2 \to 0$) with [16] **FiXme Error: Matrix elements available upon request**. FiXme Error!

The procedure is completely analogous to the inclusive case **FiXme Error: cite**. FiXme Error!

3.2 2-to-3 particle phase space

In next-to-leading order we have to consider processes which involve an additional particle in the final state. The matrix elements will then depend on ten kinematical invariants:

$$s = (q+k_1)^2 \qquad t_1 = (k_1-p_2)^2 - m^2 \qquad u_1 = (q-p_2)^2 - m^2 \quad (35)$$
$$s_3 = (k_2+p_2)^2 - m^2 \qquad s_4 = (k_2+p_1)^2 - m^2 \qquad s_5 = (p_1+p_2)^2 = -u_5 \quad (36)$$
$$t' = (k_1-k_2)^2 \quad (37)$$
$$u' = (q-k_2)^2 \qquad u_6 = (k_1-p_1)^2 - m^2 \qquad u_7 = (q-p_1)^2 - m^2 \quad (38)$$

from which only five are independent as can be seen from momentum conservation $k_1+q = p_1+p_2+k_2$ and s, t_1, u_1 match to their leading order definition.

The $2 \to 3$ n-dimensional phase space is given by

$$dPS_3 = \int \frac{d^np_1}{(2\pi)^{n-1}}\frac{d^np_2}{(2\pi)^{n-1}}\frac{d^nk_2}{(2\pi)^{n-1}}(2\pi)^n\delta^{(n)}(k_1+q-p_1-p_2-k_2)$$
$$\Theta(p_{1,0})\delta(p_1^2-m^2)\Theta(p_{2,0})\delta(p_2^2-m^2)\Theta(k_{2,0})\delta(k_2^2) \quad (39)$$

This can be solved by writing eq. (39) as product of a $2 \to 2$ decay and a subsequent $1 \to 2$ decay[12]. We choose the following decomposition[1]:

$$q = (q`0, 0, |\vec{q}|) \tag{40}$$
$$k_1 = k_0(1, 0, \sin\psi, \cos\psi) \tag{41}$$
$$p_1 = \frac{\sqrt{s_5}}{2}(1, \beta_5 \sin\theta_2 \sin\theta_1, \beta_5 \sin\theta_2 \cos\theta_1, \beta_5 \cos\theta_1) \tag{42}$$
$$p_2 = \frac{\sqrt{s_5}}{2}(1, -\beta_5 \sin\theta_2 \sin\theta_1, -\beta_5 \sin\theta_2 \cos\theta_1, -\beta_5 \cos\theta_1) \tag{43}$$
$$k_2 = (k_2^0, 0, k_1 \sin\psi, |\vec{q}| + k_1^0 \cos\psi) \tag{44}$$

where

$$q_0 = \frac{s+u'}{2\sqrt{s_5}}, \qquad |\vec{q}| = \frac{1}{2\sqrt{s_5}}\sqrt{(s+u')^2 - 4s_5q^2}, \tag{45}$$
$$k_1^0 = \frac{s_5 - u'}{2\sqrt{s_5}}, \qquad \cos\psi = \frac{2k_1^0q^0 - s'}{2k_1^0\,|\vec{q}|}, \qquad \beta_5 = \sqrt{1 - 4m^2/s_5}, \tag{46}$$
$$k_2^0 = \frac{s - s_5}{2\sqrt{s_5}} \tag{47}$$

We further introduce $\rho^* = \frac{4m^2 - q^2}{s - q^2} \leq x = \frac{s_5 - q^2}{s - q^2} \leq 1$ and $-1 \leq y \leq 1$ where y is the cosine of the angle between $\vec{q}$ and $\vec{k}_2$ in the system with $\vec{q} + \vec{k}_2 = 0$. We then find[1]:

$$dPS_3 = \frac{T_\epsilon}{2\pi}\left(\frac{s'^2}{s}\right)^{1+\epsilon/2}(1-x)^{1+\epsilon}(1-y^2)^{\epsilon/2}dPS_2^{(5)}dy\sin^\epsilon(\theta_1)d\theta_1 d\theta_2 \tag{48}$$

with $0 \leq \theta_1 \leq \pi, 0 \leq \theta_2 \leq \pi, \rho^* \leq x \leq 1, -1 \leq y \leq 1$ and

$$S_\epsilon = (4\pi)^{-2-\epsilon/2} \tag{49}$$
$$T_\epsilon = \frac{\Gamma(1+\epsilon/2)}{\Gamma(1+\epsilon)}S_\epsilon = \frac{1}{16\pi^2}\left(1 + \frac{\epsilon}{2}(\gamma_E - \ln(4\pi)) + O(\epsilon^2)\right) \tag{50}$$
$$dPS_2^{(5)} = \frac{\beta_5 \sin(\theta_1)}{16\pi\Gamma(1+\epsilon/2)}\left(\frac{s_5\beta_5^2\sin^2(\theta_1)}{16\pi}\right)^{\epsilon/2} d\theta_1 dx = dPS_2(s \to s_5)dx \tag{51}$$

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3.3 Single Gluon Radiation

In next-to-leading order we have to consider the following process:

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) + \mathrm{g}(k_2) \tag{52}$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can FiXme Error!

be written as

$$M_k^{(1),\mathrm{g}} = \hat{\mathcal{P}}_k^{\gamma,\mu\mu'} \hat{\mathcal{P}}_k^{\mathrm{g}} \sum_{j,j'} \mathcal{M}_{j,\mu}^{(1),g} \mathcal{M}_{j',\mu'}^{(1),g^*} = 8g^4 \mu_D^{-2\epsilon} e^2 e_H^2 N_C C_F \left(C_A R_{k,OK} + 2C_F R_{k,QED}\right) \tag{53}$$

For the gluonic part we shift the occuring soft ($x \to 1$) and collinear ($y \to -1$) poles from the matrix elements to the phase space by dividing by $t' \propto (1+y)(1-x)$ and $u' - q^2 s_5/s \propto (1-x)$:

$$dPS'_{3,g} = \frac{dPS_3}{t'(u' - q^2 s_5/s)} = dPS_3 \cdot \left(\frac{2s}{s'^2}\right)^2 \frac{(1-x)^2}{(1-y)(1+y)} \tag{54}$$

$$= \frac{2T_\epsilon}{\pi} \left(\frac{s'^2}{s}\right)^{-1+\epsilon/2} (1-x)^{-1+\epsilon} (1-y^2)^{-1+\epsilon/2} dPS_2^{(5)} dy \sin^\epsilon(\theta_2) d\theta_2 \tag{55}$$

The soft and collinear factors $(1-x)^{-1+\epsilon}$ and $(1-y^2)^{-1+\epsilon/2}$ can be replaced by generalized plus distributions[1]

$$(1-x)^{-1+\epsilon} \sim \left(\frac{1}{1-x}\right)_{\tilde{\rho}} + \epsilon \left(\frac{\ln(1-x)}{1-x}\right)_{\tilde{\rho}} + \delta(1-x)\left(\frac{1}{\epsilon} + 2\ln\tilde{\beta} + 2\epsilon \ln^2(\tilde{\beta})\right) + O(\epsilon^2) \tag{56}$$

$$(1-y^2)^{-1+\epsilon} \sim \frac{1}{2}\left(\left(\frac{1}{1+y}\right)_\omega + \left(\frac{1}{1-y}\right)_\omega\right) + (\delta(1+y) + \delta(1-y))\left(\frac{1}{2\epsilon} + \frac{1}{2}\ln(2\omega)\right) + O(\epsilon) \tag{57}$$

$$(1+y)^{-1+\epsilon} \sim \left(\frac{1}{1+y}\right)_\omega + \delta(1+y)\left(\frac{1}{\epsilon} + \ln\omega\right) + O(\epsilon) \tag{58}$$

inside integration over smooth functions with $\tilde{\beta} = \sqrt{1-\tilde{\rho}}$. The distributions are defined by

$$\int_{\tilde{\rho}}^{1} dx\, f(x) \left(\frac{1}{1-x}\right)_{\tilde{\rho}} = \int_{\tilde{\rho}}^{1} dx\, \frac{f(x) - f(1)}{1-x} \tag{59}$$

$$\int_{\tilde{\rho}}^{1} dx\, f(x) \left(\frac{\ln(1-x)}{1-x}\right)_{\tilde{\rho}} = \int_{\tilde{\rho}}^{1} dx\, \frac{f(x) - f(1)}{1-x} \ln(1-x) \tag{60}$$

$$\int_{-1}^{-1+\omega} dy\, f(y) \left(\frac{1}{1+y}\right)_\omega = \int_{-1}^{-1+\omega} dy\, \frac{f(y) - f(-1)}{1+y} \tag{61}$$

$$\int_{1-\omega}^{1} dy\, f(y) \left(\frac{1}{1-y}\right)_\omega = \int_{1-\omega}^{1} dy\, \frac{f(y) - f(1)}{1-y} \tag{62}$$

with $\rho^* \leq \tilde{\rho} < 1$ and $0 < \omega \leq 2$. If the integration does not include a singularity the distribution sign can be dropped. From an analytical point of view the results may not depend on the specific choice of the regularisation parameters $\tilde{\rho}$ and ω but for any numerical purpose they may influence the rate of convergence or stability. For numerical computations we must also cut the poles out of the integrations

$$\int_{\rho^*}^{1} dx \to \int_{\rho^*}^{1-\delta_x} dx \qquad\qquad \int_{-1}^{1} dy \to \int_{-1+\delta_y}^{1} dy \tag{63}$$

If not stated otherwise we use as a default setup:

$$\tilde{\rho} = \rho^* + \tilde{x}(1-\rho^*) \,\text{with}\, \tilde{x} = 0.8 \qquad \omega = 1.0 \tag{64}$$

$$\delta_x = 1\times 10^{-6} \qquad \delta_y = 7\times 10^{-6} \tag{65}$$

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From the above expression we can obtain the soft limit $k_2 \to 0$ and seperate their calculations:

$$\lim_{k_2\to 0}\left(C_A R_{k,OK} + 2C_F R_{k,QED}\right) = \left(C_A S_{k,OK} + 2C_F S_{k,QED}\right) + O(1/s_4, 1/s_3, 1/t') \tag{66}$$

$$S_{k,OK} = 2\left(\frac{t_1}{t's_3} + \frac{u_1}{t's_4} - \frac{s-2m^2}{s_3 s_4}\right) B_{k,QED} \tag{67}$$

$$S_{k,QED} = 2\left(\frac{s-2m^2}{s_3 s_4} - \frac{m^2}{s_3^2} - \frac{m^2}{s_4^2}\right) B_{k,QED} \tag{68}$$

Note that the einkonal factors multiplying the Born functions $B_{k,QED}$ neither depend on q^2 nor on the projection k. But the integrated expressions do depend on the regularization scheme, i.e. will here depend on $\tilde{\rho}$ rather then on a phasespace slicing parameter Δ as for inclusive calculations[4] **FiXme Error: add my cite**. We find for the integrated expressions FiXme Error!

$$d\sigma_{k,g}^{(1),S} = \frac{1}{2s'}\frac{1}{2}E_k(\epsilon)b_k(\epsilon)M_k^{(1),S}dPS_2 \tag{69}$$

$$M_k^{(1),S} = 8g^4\mu_D^{-\epsilon}e^2e_H^2N_CC_FC_\epsilon\left(C_A\tilde{S}_{k,OK} + 2C_F\tilde{S}_{k,QED}\right) \tag{70}$$

where the expressions for $\tilde{S}_k$ can be found in [1].

The results agree in the photo-production limit ($q^2 \to 0$) with [16].

3.4 Light Quark Processes

In next-to-leading order a new production mechanism enters that is induced by a light quark, so we have to consider the process

$$\gamma^*(q) + q(k_1) \to Q(p_1) + \overline{Q}(p_2) + q(k_2) \tag{71}$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as

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$$\hat{\mathcal{P}}_k^{\gamma,\mu\mu'}\hat{\mathcal{P}}_k^{q}\sum_{j,j'}\mathcal{M}_{j,\mu}^{(1),q}\mathcal{M}_{j',\mu'}^{(1),q*} = 8g^4\mu_D^{-2\epsilon}e^2 N_C C_F\left(e_H^2 A_{k,1} + e_L^2 A_{k,2} + e_L e_H A_{k,3}\right) \quad (72)$$

where e_L denotes the charge of the light quark q in units of e.

The results agree in the photo-production limit ($q^2 \to 0$) with [16].

4 Mass Factorization

All collinear poles in the gluonic subprocess can be removed by mass factorization in the following way: **FiXme Error: todo**

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The light quark process can be regularized in a complete analogous way: **FiXme Error: todo**

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The final finite cross sections are then **FiXme Error: todo**

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5 Partonic Results

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6 Hadronic Results

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7 Summary

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List of Corrections