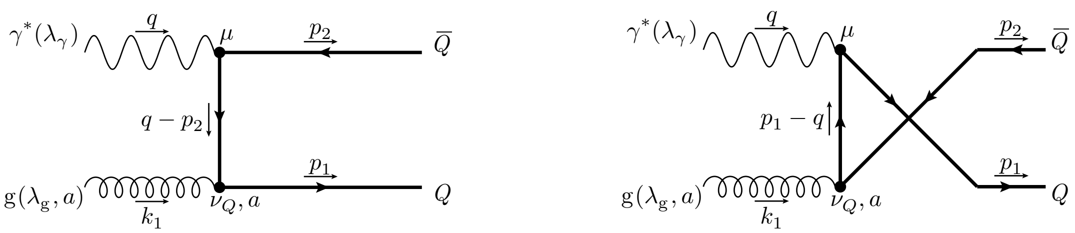
1 Feynman Rules

following [1] FiXme Error: TODO

To perform the calculation of Dirac traces in n dimensions use HEPMath[2] or TRACER[3].

2 Leading Order: $O(\alpha\alpha_s)$

diagramatic:



(a) $i\,\mathcal{M}^{(LO)}_{1,\mu}$ (b) $i\,\mathcal{M}^{(LO)}_{2,\mu}$

Figure 1: LO contributions

formula:

$$i\,\mathcal{M}^{(LO)}_{1,\mu} = \bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon^{(\lambda_\mathrm{g})}_{\nu_Q}(k_1) \tag{1}$$

$$i\,\mathcal{M}^{(LO)}_{2,\mu} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{t_1}(igT_a\gamma^{\nu_Q})v(p_2)\varepsilon^{(\lambda_\mathrm{g})}_{\nu_Q}(k_1) \tag{2}$$

color space:

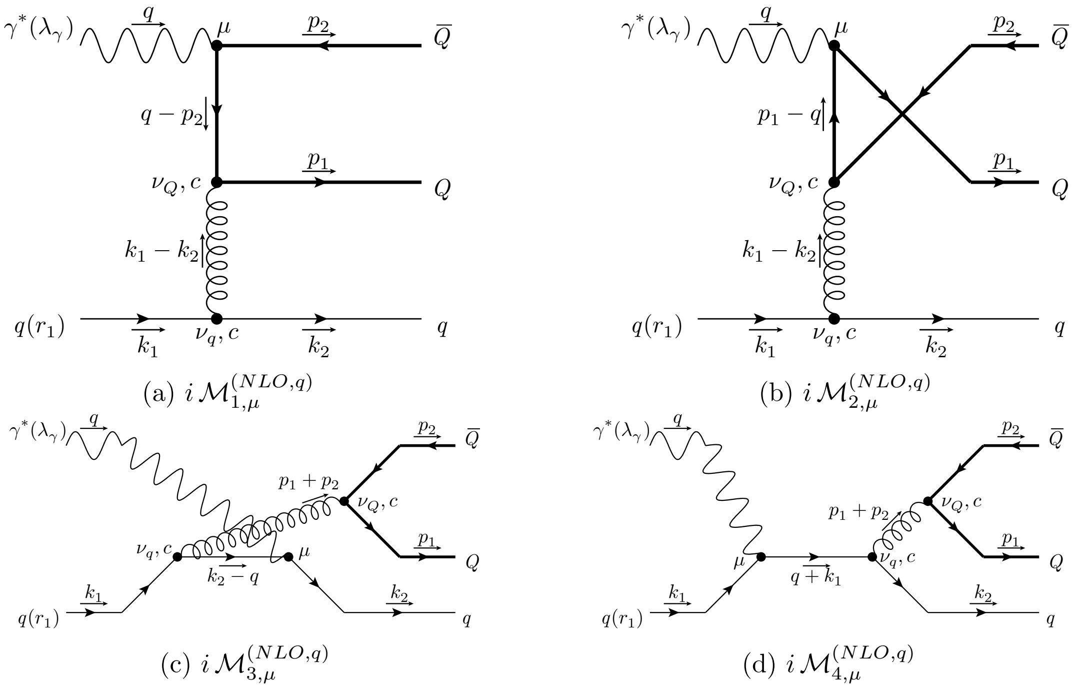
$$\left|\mathcal{M}^{(LO)}_{1,\mu} + \mathcal{M}^{(LO)}_{2,\mu}\right|^2 \sim \mathrm{tr}(T_aT_a) = N_cC_F \tag{3}$$

3 Next-to-leading Order: $O(\alpha\alpha_S^2)$

3.1 Light Quark Contributions

$$\gamma^*(q) + q(k_1) \to \overline{Q}(p_2) + Q(p_1) + q(k_2) \tag{4}$$

diagramatic:



(a) $i\,\mathcal{M}_{1,\mu}^{(NLO,q)}$ (b) $i\,\mathcal{M}_{2,\mu}^{(NLO,q)}$ (c) $i\,\mathcal{M}_{3,\mu}^{(NLO,q)}$ (d) $i\,\mathcal{M}_{4,\mu}^{(NLO,q)}$

Figure 2: NLO contributions by light quarks

formula:

$$
\begin{aligned}
i\,\mathcal{M}_{1,\mu}^{(NLO,q)} &= \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{t_1}(-iee_H\gamma_\mu)v_Q(p_2)\cdot \\
&\qquad \frac{-ig_{\nu_Q,\nu_q}}{t'}\cdot\bar{u}_q(k_2)(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) && (5)\\
i\,\mathcal{M}_{2,\mu}^{(NLO,q)} &= \bar{u}_Q(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v_Q(p_2)\cdot \\
&\qquad \frac{-ig_{\nu_Q,\nu_q}}{t'}\cdot\bar{u}_q(k_2)(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) && (6)\\
i\,\mathcal{M}_{3,\mu}^{(NLO,q)} &= \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})v_Q(p_2)\cdot\frac{-ig_{\nu_Q,\nu_q}}{s_5}\cdot \\
&\qquad \bar{u}_q(k_2)(-iee_L\gamma_\mu)\frac{i(\not{k}_2-\not{q})}{u'}(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) && (7)\\
i\,\mathcal{M}_{4,\mu}^{(NLO,q)} &= \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})v_Q(p_2)\cdot\frac{-ig_{\nu_Q,\nu_q}}{s_5}\cdot \\
&\qquad \bar{u}_q(k_2)(igT_c\gamma^{\nu_q})\frac{i(\not{k}_1+\not{q})}{s}(-iee_L\gamma_\mu)u_q^{(r_1)}(k_1) && (8)
\end{aligned}
$$

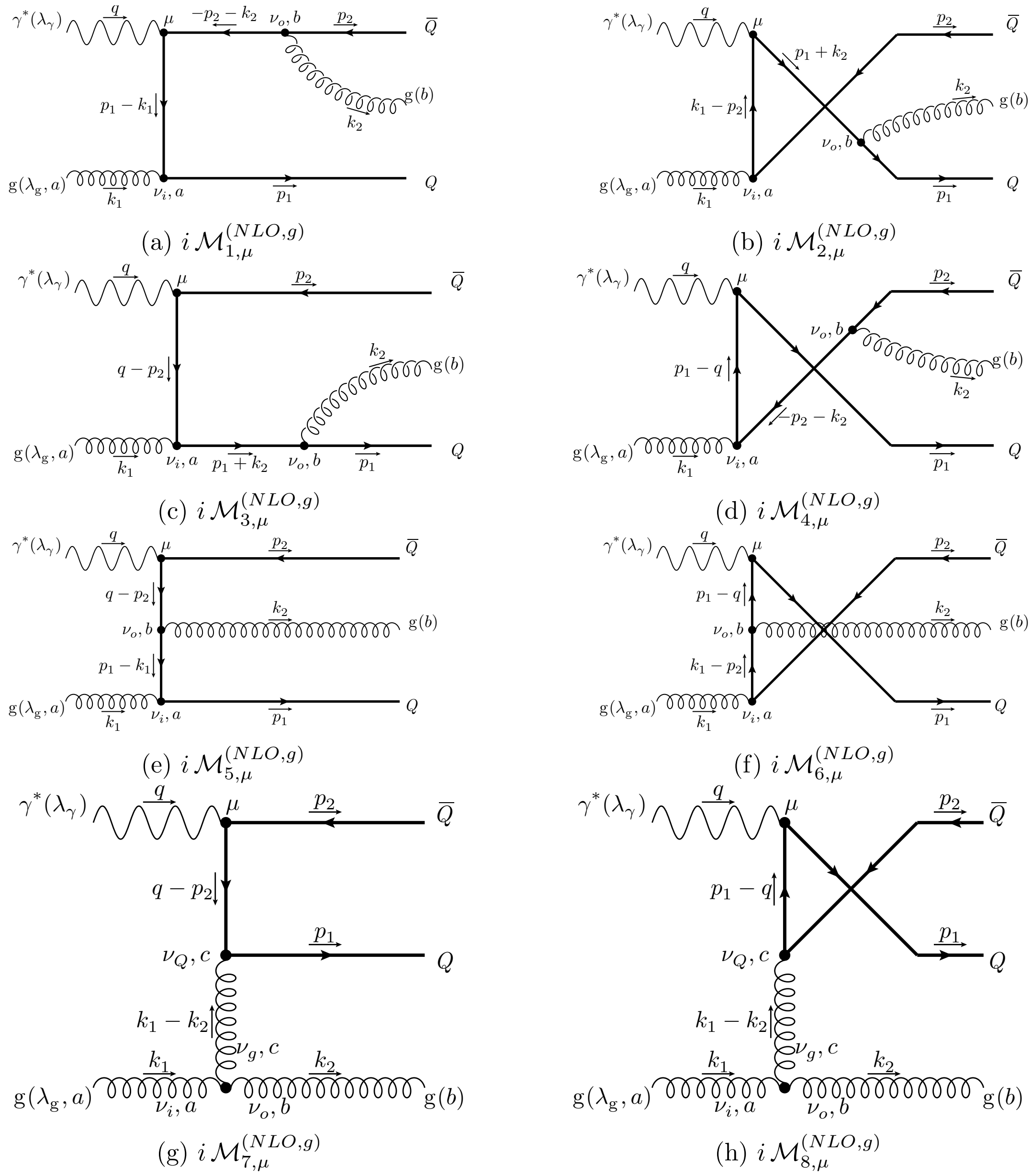
color space:

$$\left|\mathcal{M}_{1,\mu}^{(NLO,q)}+\mathcal{M}_{2,\mu}^{(NLO,q)}+\mathcal{M}_{3,\mu}^{(NLO,q)}+\mathcal{M}_{4,\mu}^{(NLO,q)}\right|^2\sim\mathrm{tr}(T_cT_d)\,\mathrm{tr}(T_cT_d)=\frac{1}{2}N_cC_F \quad (9)$$

3.2 Gluon Bremsstrahlung

$$\gamma^*(q) + \mathrm{g}(k_1) \to \overline{Q}(p_2) + Q(p_1) + \mathrm{g}(k_2) \tag{10}$$

diagramatic:



(a) $i\,\mathcal{M}^{(NLO,g)}_{1,\mu}$ (b) $i\,\mathcal{M}^{(NLO,g)}_{2,\mu}$

(c) $i\,\mathcal{M}^{(NLO,g)}_{3,\mu}$ (d) $i\,\mathcal{M}^{(NLO,g)}_{4,\mu}$

(e) $i\,\mathcal{M}^{(NLO,g)}_{5,\mu}$ (f) $i\,\mathcal{M}^{(NLO,g)}_{6,\mu}$

(g) $i\,\mathcal{M}^{(NLO,g)}_{7,\mu}$ (h) $i\,\mathcal{M}^{(NLO,g)}_{8,\mu}$

Figure 3: NLO contributions by gluon bremsstrahlung

formula:

$$i\,\mathcal{M}_{1,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_a\gamma^{\nu_i})\frac{i(\not{p}_1-\not{k}_1+m)}{u_6}(-iee_H\gamma_\mu)\cdot \frac{i(-\not{p}_2-\not{k}_2+m)}{s_3}(igT_b\gamma^{\nu_o})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \tag{11}$$

$$i\,\mathcal{M}_{2,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_b\gamma^{\nu_o})\frac{i(\not{p}_1+\not{k}_2+m)}{s_4}(-iee_H\gamma_\mu)\cdot \frac{i(\not{k}_1-\not{p}_2+m)}{t_1}(igT_a\gamma^{\nu_i})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \tag{12}$$

$$i\,\mathcal{M}_{3,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_b\gamma^{\nu_o})\frac{i(\not{p}_1+\not{k}_2+m)}{s_4}(igT_a\gamma^{\nu_i})\cdot \frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \tag{13}$$

$$i\,\mathcal{M}_{4,\mu}^{(NLO,g)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_a\gamma^{\nu_i})\cdot \frac{i(-\not{p}_2-\not{k}_2+m)}{s_3}(igT_b\gamma^{\nu_o})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \tag{14}$$

$$i\,\mathcal{M}_{5,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_a\gamma^{\nu_i})\frac{i(\not{p}_1-\not{k}_1+m)}{u_6}(igT_b\gamma^{\nu_o})\cdot \frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \tag{15}$$

$$i\,\mathcal{M}_{6,\mu}^{(NLO,g)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_b\gamma^{\nu_o})\cdot \frac{i(\not{k}_1-\not{p}_2+m)}{t_1}(igT_a\gamma^{\nu_i})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \tag{16}$$

$$i\,\mathcal{M}_{7,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_g}}{t'}\cdot\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2)\cdot \Big(gf^{acb}\left(g^{\nu_o,\nu_i}(k_1+k_2)^{\nu_g}+g^{\nu_i,\nu_g}(k_2-2k_1)^{\nu_o}+g^{\nu_g,\nu_o}(k_1-2k_2)^{\nu_i}\right)\Big) \tag{17}$$

$$i\,\mathcal{M}_{8,\mu}^{(NLO,g)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_g}}{t'}\cdot\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2)\cdot \Big(gf^{acb}\left(g^{\nu_o,\nu_i}(k_1+k_2)^{\nu_g}+g^{\nu_i,\nu_g}(k_2-2k_1)^{\nu_o}+g^{\nu_g,\nu_o}(k_1-2k_2)^{\nu_i}\right)\Big) \tag{18}$$

color space:

$$
\begin{aligned}
&\sum_{j=1}^{6}\left|\mathcal{M}_{j,\mu}^{(NLO,g)}\right|^2+\mathcal{M}_{1,\mu}^{(NLO,g)}\left(\mathcal{M}_{4,\mu'}^{(NLO,g)}+\mathcal{M}_{5,\mu'}^{(NLO,g)}\right)^*+\mathcal{M}_{3,\mu}^{(NLO,g)}\left(\mathcal{M}_{6,\mu'}^{(NLO,g)}\right)^*+\\
&\mathcal{M}_{2,\mu}^{(NLO,g)}\left(\mathcal{M}_{3,\mu'}^{(NLO,g)}+\mathcal{M}_{6,\mu'}^{(NLO,g)}\right)^*+\mathcal{M}_{4,\mu}^{(NLO,g)}\left(\mathcal{M}_{5,\mu'}^{(NLO,g)}\right)^*\\
&\sim \mathrm{tr}(T_aT_aT_bT_b)=N_CC_F^2 \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{M}_{1,\mu}^{(NLO,g)}\left(\mathcal{M}_{2,\mu'}^{(NLO,g)}+\mathcal{M}_{3,\mu'}^{(NLO,g)}+\mathcal{M}_{6,\mu'}^{(NLO,g)}\right)^*+\\
&\left(\mathcal{M}_{2,\mu}^{(NLO,g)}+\mathcal{M}_{3,\mu}^{(NLO,g)}\right)\left(\mathcal{M}_{4,\mu'}^{(NLO,g)}+\mathcal{M}_{5,\mu'}^{(NLO,g)}\right)^*+\\
&\left(\mathcal{M}_{4,\mu}^{(NLO,g)}+\mathcal{M}_{5,\mu}^{(NLO,g)}\right)\left(\mathcal{M}_{6,\mu'}^{(NLO,g)}\right)^*\\
&\sim \mathrm{tr}(T_aT_bT_aT_b)=N_CC_F\left(C_F-\frac{C_A}{2}\right) \qquad (20)
\end{aligned}
$$

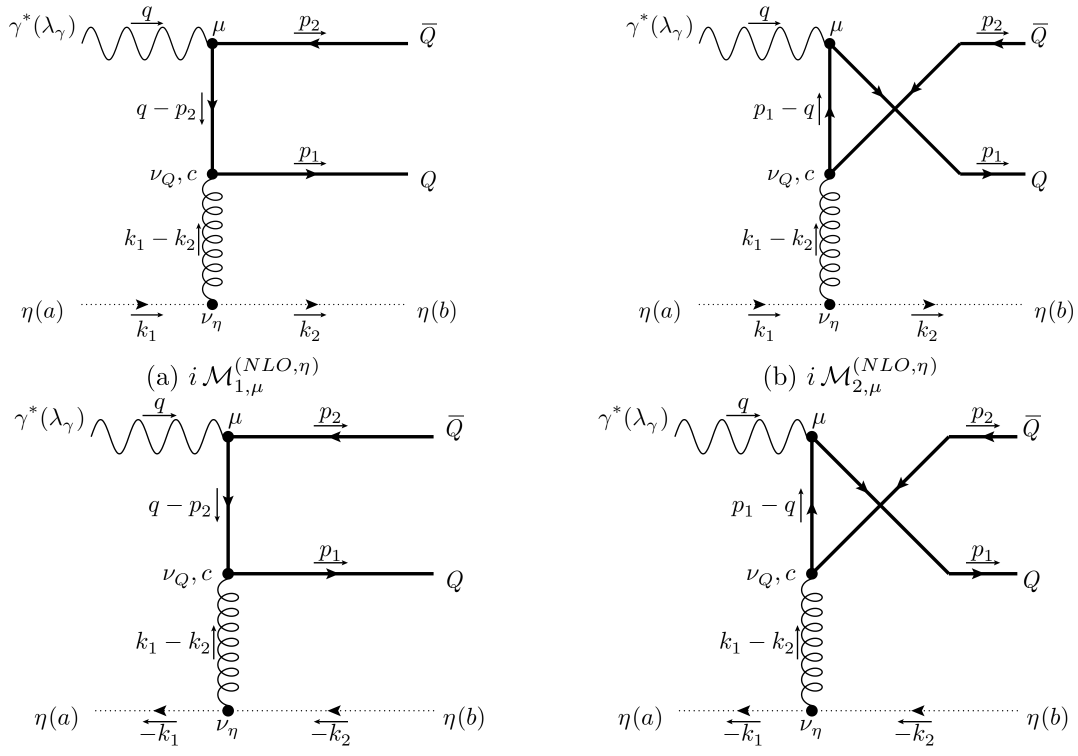
$$
\begin{aligned}
&\left(\mathcal{M}_{2,\mu}^{(NLO,g)}+\mathcal{M}_{3,\mu}^{(NLO,g)}+\mathcal{M}_{6,\mu}^{(NLO,g)}\right)\left(\mathcal{M}_{7,\mu'}^{(NLO,g)}+\mathcal{M}_{8,\mu'}^{(NLO,g)}\right)^*\\
&\sim -if_{bda}\,\mathrm{tr}(T_aT_bT_d)=\frac{1}{2}N_CC_FC_A \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
&\left(\mathcal{M}_{1,\mu}^{(NLO,g)}+\mathcal{M}_{4,\mu}^{(NLO,g)}+\mathcal{M}_{5,\mu}^{(NLO,g)}\right)\left(\mathcal{M}_{7,\mu'}^{(NLO,g)}+\mathcal{M}_{8,\mu'}^{(NLO,g)}\right)^*\\
&\sim -if_{bda}\,\mathrm{tr}(T_bT_aT_d)=if_{bda}\,\mathrm{tr}(T_aT_bT_d)=-\frac{1}{2}N_CC_FC_A \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
&\left|\mathcal{M}_{7,\mu}^{(NLO,g)}+\mathcal{M}_{8,\mu}^{(NLO,g)}\right|^2\\
&\sim f_{acb}f_{adb}\,\mathrm{tr}(T_cT_d)=N_CC_FC_A \qquad (23)
\end{aligned}
$$

To get the polarisation sums right, one has to subtract the contributions of the Faddeev-Popov ghosts[4, 5]:

diagramatic:



(a) $i\,\mathcal{M}_{1,\mu}^{(NLO,\eta)}$ (b) $i\,\mathcal{M}_{2,\mu}^{(NLO,\eta)}$ (c) $i\,\mathcal{M}_{3,\mu}^{(NLO,\eta)}$ (d) $i\,\mathcal{M}_{4,\mu}^{(NLO,\eta)}$

Figure 4: NLO contributions by ghosts

formula:

$$i\,\mathcal{M}_{1,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_\eta}}{t'}\cdot(gf^{acb}k_2^{\nu_\eta}) \quad (24)$$

$$i\,\mathcal{M}_{2,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_\eta}}{t'}\cdot(gf^{acb}k_2^{\nu_\eta}) \quad (25)$$

$$i\,\mathcal{M}_{3,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_\eta}}{t'}\cdot(gf^{cab}(-k_1)^{\nu_\eta}) \quad (26)$$

$$i\,\mathcal{M}_{4,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_\eta}}{t'}\cdot(gf^{cab}(-k_1)^{\nu_\eta}) \quad (27)$$

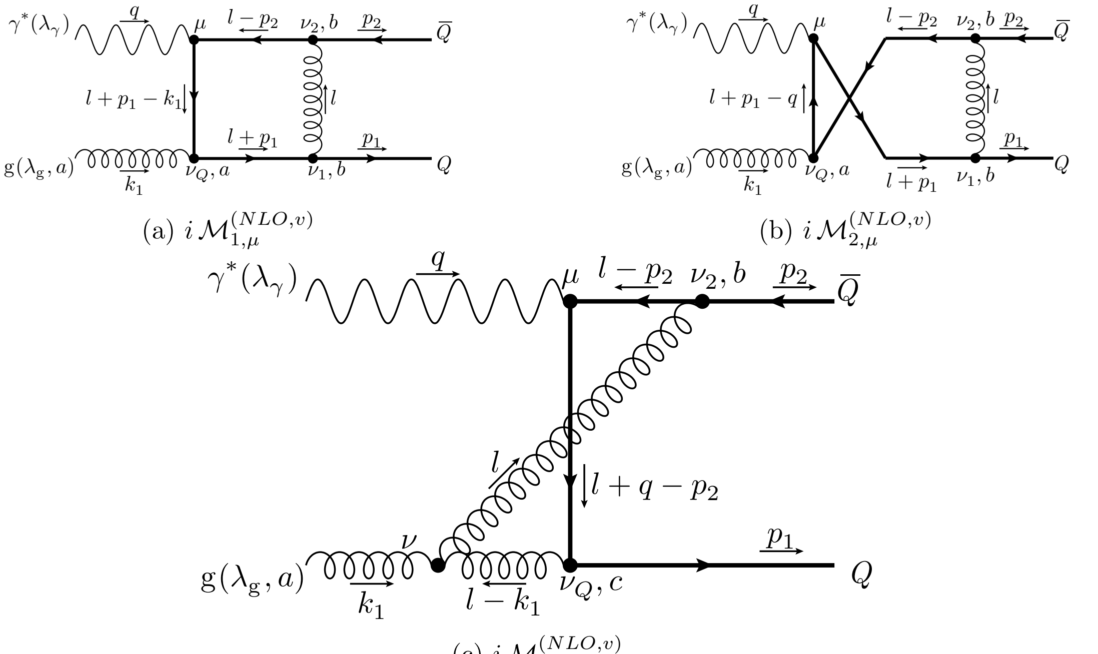
color space:

$$\left|\mathcal{M}_{1,\mu}^{(NLO,\eta)} + \mathcal{M}_{2,\mu}^{(NLO,\eta)}\right|^2 \sim f_{acb} f_{adb}\, \mathrm{tr}(T_c T_d) = N_C C_F C_A \tag{28}$$

$$\left|\mathcal{M}_{3,\mu}^{(NLO,\eta)} + \mathcal{M}_{4,\mu}^{(NLO,\eta)}\right|^2 \sim f_{cab} f_{dab}\, \mathrm{tr}(T_c T_d) = f_{acb} f_{adb}\, \mathrm{tr}(T_c T_d) = N_C C_F C_A \tag{29}$$

3.3 Virtual Contributions

$$\gamma^*(q) + \mathrm{g}(k_1) \to \overline{Q}(p_2) + Q(p_1) \tag{30}$$



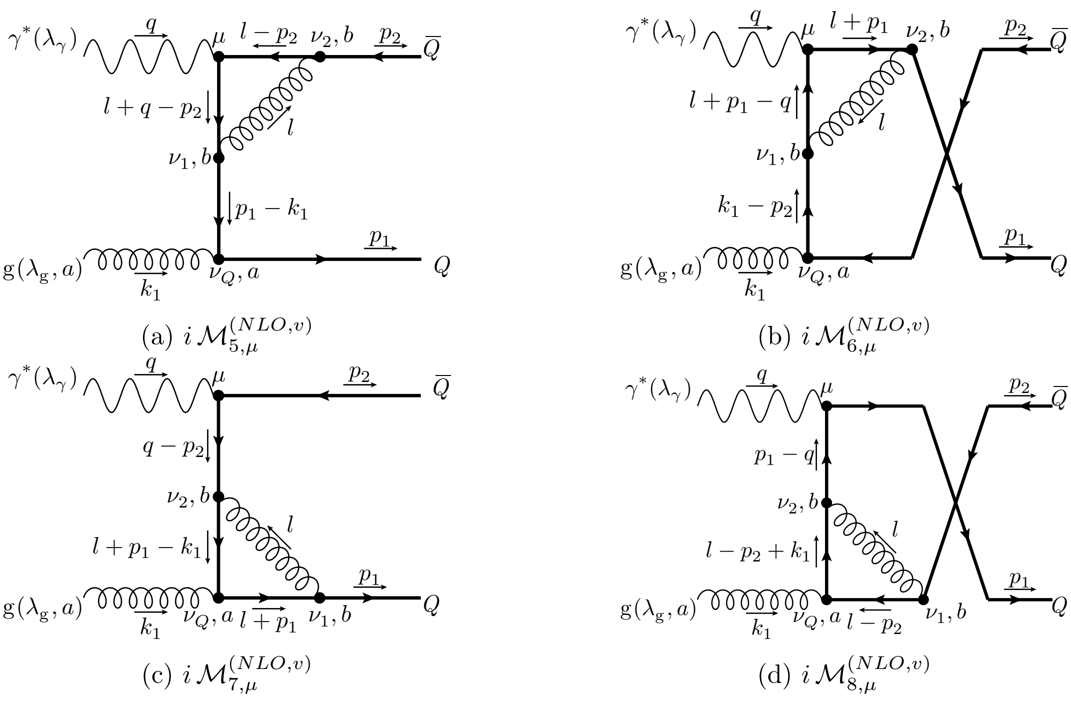
(a) $i\,\mathcal{M}_{1,\mu}^{(NLO,v)}$ (b) $i\,\mathcal{M}_{2,\mu}^{(NLO,v)}$ (c) $i\,\mathcal{M}_{3,\mu}^{(NLO,v)}$

Figure 5: NLO contributions by one loop

$$
\begin{aligned}
i\,\mathcal{M}_{1,\mu}^{(NLO,v)} = \mu_R^{4-n}\!\int\! \frac{d^n l}{(2\pi)^n}\, \bar{u}(p_1)(igT_b\gamma^{\nu_1})\frac{i(\not{l}+\not{p}_1+m)}{(l+p_1)^2-m^2}(igT_a\gamma^{\nu_Q})\frac{i(\not{l}+\not{p}_1-\not{k}_1+m)}{(l+p_1-k_1)^2-m^2}\cdot \\
(-iee_H\gamma_\mu)\frac{i(\not{l}-\not{p}_2+m)}{(l-p_2)^2-m^2}(igT_b\gamma^{\nu_2})\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon_{\nu_Q}^{(\lambda_{\rm g})}(k_1)
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
i\,\mathcal{M}_{2,\mu}^{(NLO,v)} = \mu_R^{4-n}\!\int\! \frac{d^n l}{(2\pi)^n}\, \bar{u}(p_1)(igT_b\gamma^{\nu_1})\frac{i(\not{l}+\not{p}_1+m)}{(l+p_1)^2-m^2}(igT_a\gamma^{\nu_Q})\frac{i(\not{l}+\not{p}_1-\not{q}+m)}{(l+p_1-q)^2-m^2}\cdot \\
(-iee_H\gamma_\mu)\frac{i(\not{l}-\not{p}_2+m)}{(l-p_2)^2-m^2}(igT_b\gamma^{\nu_2})\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon_{\nu_Q}^{(\lambda_{\rm g})}(k_1)
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
i\,\mathcal{M}_{3,\mu}^{(NLO,v)} = \mu_R^{4-n}\!\int\! \frac{d^n l}{(2\pi)^n}\, \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{l}+\not{q}_1-\not{p}_2+m)}{(l+q-p_2)^2-m^2}(-iee_H\gamma_\mu)\frac{i(\not{l}-\not{p}_2+m)}{(l-p_2)^2-m^2}\cdot \\
(igT_b\gamma^{\nu_2})\frac{(-i)^2}{l^2(l-k_1)^2}v(p_2)\varepsilon^{\nu,(\lambda_{\rm g})}(k_1)\cdot \\
\Big(gf_{abc}\Big(g_{\nu_2\nu_Q}(k_1-2l)_\nu + g_{\nu_Q\nu}(l-2k_1)_{\nu_2} + g_{\nu\nu_2}(k_1+l)_{\nu_Q}\Big)\Big)
\end{aligned}
\tag{33}
$$



(a) $i\,\mathcal{M}_{5,\mu}^{(NLO,v)}$
(b) $i\,\mathcal{M}_{6,\mu}^{(NLO,v)}$
(c) $i\,\mathcal{M}_{7,\mu}^{(NLO,v)}$
(d) $i\,\mathcal{M}_{8,\mu}^{(NLO,v)}$

Figure 6: NLO contributions by one loop (cont'ed)

$$i\,\mathcal{M}^{(NLO,v)}_{5,\mu} = \mu_R^{4-n}\int\frac{d^nl}{(2\pi)^n}\,\bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not{p}_1-\not{k}_1+m)}{t_1}(igT_b\gamma^{\nu_1})\frac{i(\not{l}+\not{q}-\not{p}_2+m)}{(l+q-p_2)^2-m^2}\cdot$$

$$(-iee_H\gamma_\mu)\frac{i(\not{l}-\not{p}_2+m)}{(l-p_2)^2-m^2}(igT_b\gamma^{\nu_2})\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon^{(\lambda_{\rm g})}_{\nu_Q}(k_1) \tag{34}$$

$$i\,\mathcal{M}^{(NLO,v)}_{6,\mu} = \mu_R^{4-n}\int\frac{d^nl}{(2\pi)^n}\,\bar{u}(p_1)(igT_b\gamma^{\nu_2})\frac{i(\not{l}+\not{p}_1+m)}{(l+p_1)^2-m^2}(-iee_H\gamma_\mu)\frac{i(\not{l}+\not{p}_1-\not{q}+m)}{(l+p_1-q)^2-m^2}\cdot$$

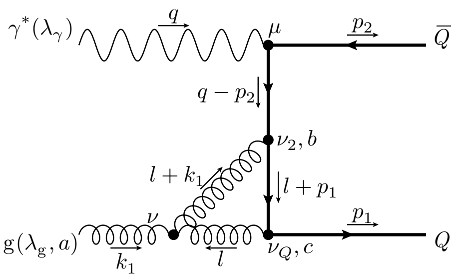
$$(igT_b\gamma^{\nu_1})\frac{i(\not{k}_1-\not{p}_2+m)}{u_1}(igT_a\gamma^{\nu_Q})\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon^{(\lambda_{\rm g})}_{\nu_Q}(k_1) \tag{35}$$

$$i\,\mathcal{M}^{(NLO,v)}_{7,\mu} = \mu_R^{4-n}\int\frac{d^nl}{(2\pi)^n}\,\bar{u}(p_1)(igT_b\gamma^{\nu_1})\frac{i(\not{l}+\not{p}_1+m)}{(l+p_1)^2-m^2}(igT_a\gamma^{\nu_Q})\frac{i(\not{l}+\not{p}_1-\not{k}_1+m)}{(l+p_1-k_1)^2-m^2}\cdot$$

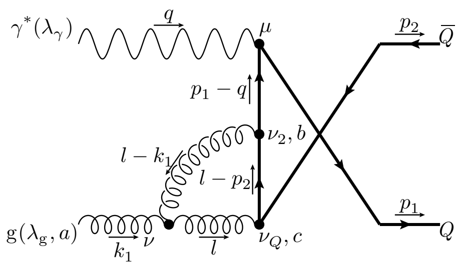
$$(igT_b\gamma^{\nu_2})\frac{i(\not{q}-\not{p}_2+m)}{t_1}(-iee_H\gamma_\mu)\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon^{(\lambda_{\rm g})}_{\nu_Q}(k_1) \tag{36}$$

$$i\,\mathcal{M}^{(NLO,v)}_{8,\mu} = \mu_R^{4-n}\int\frac{d^nl}{(2\pi)^n}\,\bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_1}(igT_b\gamma^{\nu_2})\frac{i(\not{l}-\not{p}_2+\not{k}_1+m)}{(l-p_2+k_1)^2-m^2}\cdot$$

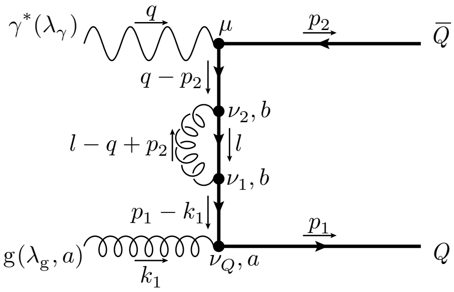
$$(igT_a\gamma^{\nu_Q})\frac{i(\not{l}-\not{p}_2+m)}{(l-p_2)^2-m^2}(igT_b\gamma^{\nu_1})\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon^{(\lambda_{\rm g})}_{\nu_Q}(k_1) \tag{37}$$



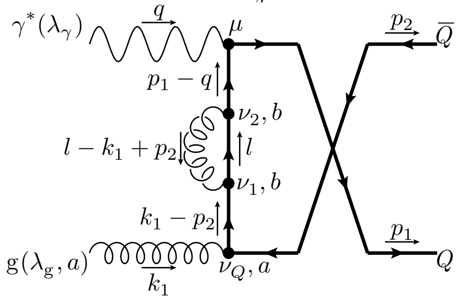
(a) $i\,\mathcal{M}^{(NLO,v)}_{9,\mu}$



(b) $i\,\mathcal{M}^{(NLO,v)}_{10,\mu}$



(c) $i\,\mathcal{M}^{(NLO,v)}_{11,\mu}$



(d) $i\,\mathcal{M}^{(NLO,v)}_{12,\mu}$

Figure 7: NLO contributions by one loop (cont'ed)

$$
\begin{aligned}
i\,\mathcal{M}^{(NLO,v)}_{9,\mu} &= \mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{l}+\not{p}_1+m)}{(l+p_1)^2-m^2}(igT_b\gamma^{\nu_2})\frac{i(\not{q}-\not{p}_2+m)}{t_1}\cdot\\
&\quad(-iee_H\gamma_\mu)\frac{(-i)^2}{l^2(l+k_1)^2}v(p_2)\varepsilon^{\nu,(\lambda_{\rm g})}(k_1)\cdot\\
&\quad\Big(gf_{abc}\Big(g_{\nu\nu_2}(2k_1+l)_{\nu_Q}+g_{\nu_2\nu_Q}(-2l-k_1)_\nu+g_{\nu_Q\nu}(l-k_1)_{\nu_2}\Big)\Big) \qquad (38)\\
i\,\mathcal{M}^{(NLO,v)}_{10,\mu} &= \mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_1}(igT_b\gamma^{\nu_2})\frac{i(\not{l}-\not{p}_2+m)}{(l-p_2)^2-m^2}\cdot\\
&\quad(igT_c\gamma^{\nu_Q})\frac{(-i)^2}{l^2(l-k_1)^2}v(p_2)\varepsilon^{\nu,(\lambda_{\rm g})}(k_1)\cdot\\
&\quad\Big(gf_{abc}\Big(g_{\nu\nu_2}(2k_1-l)_{\nu_Q}+g_{\nu_2\nu_Q}(2l-k_1)_\nu+g_{\nu_Q\nu}(-l-k_1)_{\nu_2}\Big)\Big) \qquad (39)\\
i\,\mathcal{M}^{(NLO,v)}_{11,\mu} &= \mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not{p}_1-\not{k}_1+m)}{t_1}(igT_b\gamma^{\nu_1})\frac{i(\not{l}+m)}{l^2-m^2}\cdot\\
&\quad(igT_b\gamma^{\nu_2})\frac{i(\not{q}-\not{p}_2+m)}{t_1}(-iee_H\gamma_\mu)\frac{-ig_{\nu_1,\nu_2}}{(l-q+p_2)^2}v(p_2)\varepsilon^{(\lambda_{\rm g})}_{\nu_Q}(k_1) \qquad (40)\\
i\,\mathcal{M}^{(NLO,v)}_{12,\mu} &= \mu^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_1}(igT_b\gamma^{\nu_2})\frac{i(\not{l}+m)}{l^2-m^2}\cdot\\
&\quad(igT_b\gamma^{\nu_1})\frac{i(\not{k}_1-\not{p}_2+m)}{u_1}(igT_a\gamma^{\nu_Q})\frac{-ig_{\nu_1,\nu_2}}{(l-k_1+p_2)^2}v(p_2)\varepsilon^{(\lambda_{\rm g})}_{\nu_Q}(k_1) \qquad (41)
\end{aligned}
$$

color space:

$$\left(\mathcal{M}^{(NLO,v)}_{1,\mu}+\mathcal{M}^{(NLO,v)}_{2,\mu}\right)\left(\mathcal{M}^{(LO)}_{1,\mu'}+\mathcal{M}^{(LO)}_{2,\mu'}\right)^*\sim -i\,\mathrm{tr}(T_aT_bT_aT_b)=-iN_CC_F\left(C_F-\frac{C_A}{2}\right) \qquad (42)$$

$$\left(\mathcal{M}^{(NLO,v)}_{3,\mu}\right)\left(\mathcal{M}^{(LO)}_{1,\mu'}+\mathcal{M}^{(LO)}_{2,\mu'}\right)^*\sim f_{abc}\,\mathrm{tr}(T_cT_bT_a)=-\frac{i}{2}N_CC_FC_A \qquad (43)$$

$$\left(\mathcal{M}^{(NLO,v)}_{5,\mu}+\mathcal{M}^{(NLO,v)}_{6,\mu}\right)\left(\mathcal{M}^{(LO)}_{1,\mu'}+\mathcal{M}^{(LO)}_{2,\mu'}\right)^*\sim -i\,\mathrm{tr}(T_aT_aT_bT_b)=-iN_CC_F^2 \qquad (44)$$

$$(45)$$

to compute self energies, we follow [6]. It is

$$\{\gamma_\mu,\gamma_\nu\}=2g_{\mu\nu} \qquad (46)$$

$$\gamma_\mu\gamma^\mu=g^\mu_\mu=n \qquad (47)$$

$$\gamma_\mu\gamma_\nu\gamma^\mu=(2-n)\gamma_\nu \qquad (48)$$

(a) $-i\Sigma(p)$ (b) $-i\Sigma_C(p)$

Figure 8: NLO contributions by quark self energy

$$
\begin{aligned}
-i\Sigma(p) &= \mu_R^{4-n}\int\frac{d^nl}{(2\pi)^n}(igT_b\gamma_{\nu_1})\frac{i(\not{l}+\not{p}+m)}{(l+p)^2-m^2}(igT_b\gamma_{\nu_2})\frac{-ig^{\nu_1,\nu_2}}{l^2} && (49)\\
&= -\mu_R^{4-n}g^2C_F\int\frac{d^nl}{(2\pi)^n}\frac{2m+(2-n)\not{p}+(2-n)\not{l}}{l^2((l+p)^2-m^2)} && (50)\\
&= -g^2C_F\left(\left(2m+(2-n)\not{p}\right)B_0(p^2,0,m^2)+(2-n)B_1(p^2,0,m^2)\right) && (51)\\
&= -g^2C_F\left(B_0(p^2,0,m^2)\left(n\cdot m+(2-n)\not{p}\frac{p^2+m^2}{2p^2}\right)-(2-n)\not{p}\frac{1}{2p^2}A_0(m^2)\right) && (52)
\end{aligned}
$$

Using [6] we find

$$
\begin{aligned}
C_\epsilon &= \frac{1}{16\pi^2}\exp\left(\left(\gamma_E-\log(4\pi)\right)\frac{\epsilon}{2}\right)\left(m^2/\mu^2\right)^{\epsilon/2} && (53)\\
A_0(m^2) &= iC_\epsilon\left(-\frac{2}{\epsilon}+1\right) && (54)\\
B_0(p^2,0,m^2) &= iC_\epsilon\left(-\frac{2}{\epsilon}+2+\frac{m^2-p^2}{p^2}\ln\left(\frac{m^2-p^2}{m^2}\right)\right) && (55)
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow -i\Sigma(p) &= -ig^2C_FC_\epsilon\left[\frac{2\not{p}-8m}{\epsilon}+2m\left(3-2\left(1-\frac{m^2}{p^2}\right)\ln\left(1-\frac{p^2}{m^2}\right)\right)\right.\\
&\qquad\left.-\not{p}\left(1+\frac{m^2}{p^2}\right)\left(1-\left(1-\frac{m^2}{p^2}\right)\ln\left(1-\frac{p^2}{m^2}\right)\right)\right] && (56)\\
&\overset{!}{=} -i(Am+B(\not{p}-m)) && (57)
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow A &= \frac{1}{m}\left.\Sigma(p)\right|_{\not{p}=m} && (58)\\
&= -g^2C_FC_\epsilon\left(\frac{6}{\epsilon}-5+\frac{m^2}{p^2}+\left(3-4\frac{m^2}{p^2}+\frac{m^4}{p^4}\right)\ln\left(1-\frac{p^2}{m^2}\right)\right) && (59)\\
\Rightarrow B &= \frac{1}{m}\left.\frac{d\Sigma(p)}{d\not{p}}\right|_{\not{p}=m} && (60)\\
&= g^2C_FC_\epsilon\left(\frac{2}{\epsilon}-1-\frac{m^2}{p^2}+\left(1-\frac{m^4}{p^4}\right)\ln\left(1-\frac{p^2}{m^2}\right)\right) && (61)
\end{aligned}
$$

Counterterm:

$$-i\Sigma_C(p) = i((Z_2-1)\not{p} - (Z_2Z_m-1)m) \tag{62}$$

$$= i((Z_2-1)(\not{p}-m) - (Z_m-1)m) + O(\alpha_S^2) \tag{63}$$

Use on-shell renormalisation:

$$0 \stackrel{!}{=} (-i\Sigma(p) - i\Sigma_C(p))|_{\not{p}=m} \tag{64}$$

$$= i(((Z_m-1)+A)m + (B-(Z_2-1))(\not{p}-m)) \tag{65}$$

$$\Rightarrow (Z_m-1) = -A|_{\not{p}=m} \tag{66}$$

$$= g^2 C_F C_\epsilon \left(\frac{6}{\epsilon} - 4\right) \tag{67}$$

A References

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List of Corrections