

1 Feynman Rules

following [1]

To perform the calculation of Dirac traces in n dimensions use HEPMath[2] or TRACER[3].

FiXme
Error:
TODO

2 Leading Order: $O(\alpha\alpha_s)$

diagramatic:

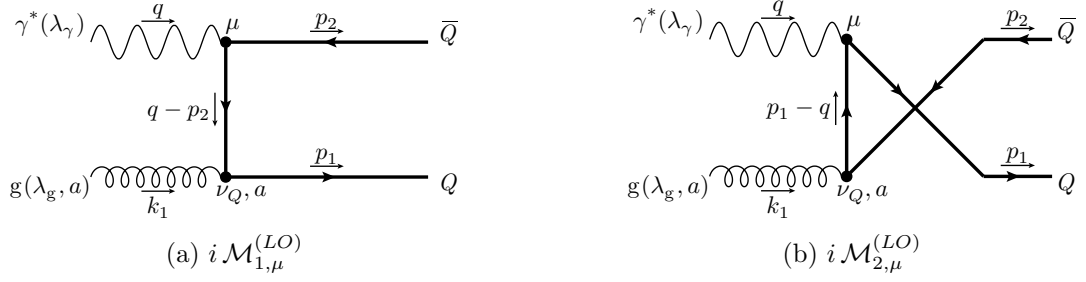


Figure 1: LO contributions

formula:

$$i\mathcal{M}_{1,\mu}^{(LO)} = \bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (1)$$

$$i\mathcal{M}_{2,\mu}^{(LO)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{t_1}(igT_a\gamma^{\nu_Q})v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (2)$$

color space:

$$\left|\mathcal{M}_{1,\mu}^{(LO)} + \mathcal{M}_{2,\mu}^{(LO)}\right|^2 \sim \text{tr}(T_a T_a) = N_c C_F \quad (3)$$

3 Next-to-leading Order: $O(\alpha\alpha_s^2)$

3.1 Light Quark Contributions

$$\gamma^*(q) + q(k_1) \rightarrow \bar{Q}(p_2) + Q(p_1) + q(k_2) \quad (4)$$

diagramatic:

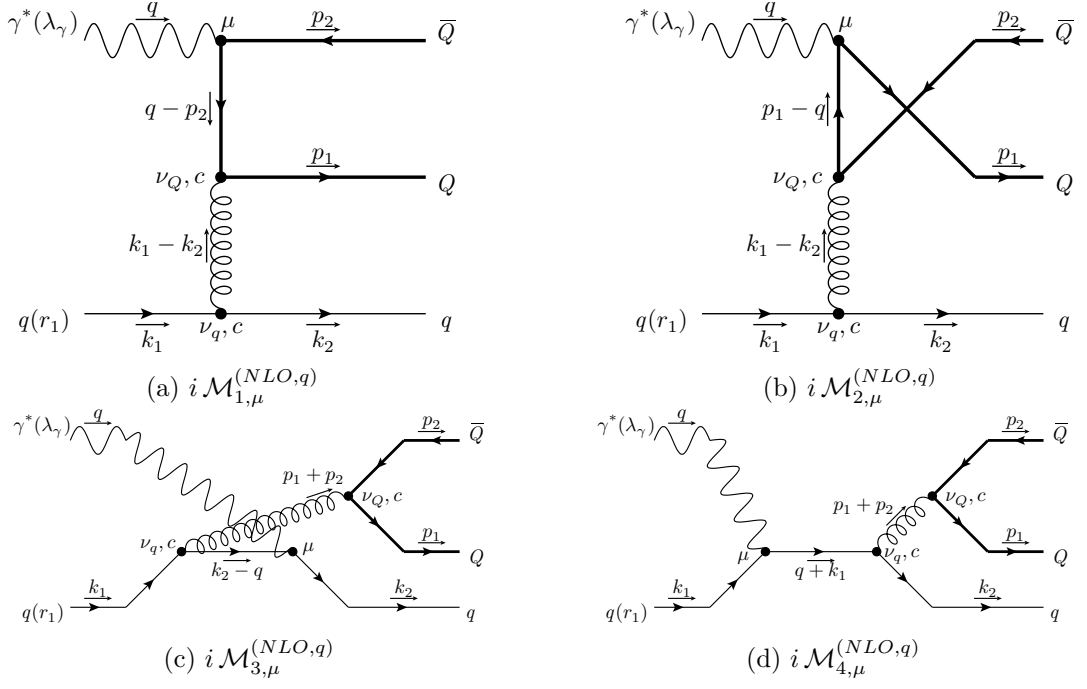


Figure 2: NLO contributions by light quarks

formula:

$$i \mathcal{M}_{1,\mu}^{(NLO,q)} = \bar{u}_Q(p_1) (ig T_c \gamma^{\nu_Q}) \frac{i(\not{q} - \not{p}_2 + m)}{u_1} (-ie e_H \gamma_\mu) v_Q(p_2) \cdot \frac{-ig_{\nu_Q, \nu_q}}{t'} \cdot \bar{u}_q(k_2) (ig T_c \gamma^{\nu_q}) u_q^{(r_1)}(k_1) \quad (5)$$

$$i \mathcal{M}_{2,\mu}^{(NLO,q)} = \bar{u}_Q(p_1) (-ie e_H \gamma_\mu) \frac{i(\not{p}_1 - \not{q} + m)}{u_7} (ig T_c \gamma^{\nu_Q}) v_Q(p_2) \cdot \frac{-ig_{\nu_Q, \nu_q}}{t'} \cdot \bar{u}_q(k_2) (ig T_c \gamma^{\nu_q}) u_q^{(r_1)}(k_1) \quad (6)$$

$$i \mathcal{M}_{3,\mu}^{(NLO,q)} = \bar{u}_Q(p_1) (ig T_c \gamma^{\nu_Q}) v_Q(p_2) \cdot \frac{-ig_{\nu_Q, \nu_q}}{s_5} \cdot \bar{u}_q(k_2) (-ie e_L \gamma_\mu) \frac{i(\not{k}_2 - \not{q})}{u'} (ig T_c \gamma^{\nu_q}) u_q^{(r_1)}(k_1) \quad (7)$$

$$i \mathcal{M}_{4,\mu}^{(NLO,q)} = \bar{u}_Q(p_1) (ig T_c \gamma^{\nu_Q}) v_Q(p_2) \cdot \frac{-ig_{\nu_Q, \nu_q}}{s_5} \cdot \bar{u}_q(k_2) (ig T_c \gamma^{\nu_q}) \frac{i(\not{k}_1 + \not{q})}{s} (-ie e_L \gamma_\mu) u_q^{(r_1)}(k_1) \quad (8)$$

color space:

$$\left| \mathcal{M}_{1,\mu}^{(NLO,q)} + \mathcal{M}_{2,\mu}^{(NLO,q)} + \mathcal{M}_{3,\mu}^{(NLO,q)} + \mathcal{M}_{4,\mu}^{(NLO,q)} \right|^2 \sim \text{tr}(T_c T_d) \text{tr}(T_c T_d) = \frac{1}{2} N_c C_F \quad (9)$$

3.2 Gluon Bremsstrahlung

$$\gamma^*(q) + g(k_1) \rightarrow \bar{Q}(p_2) + Q(p_1) + g(k_2) \quad (10)$$

diagramatic:

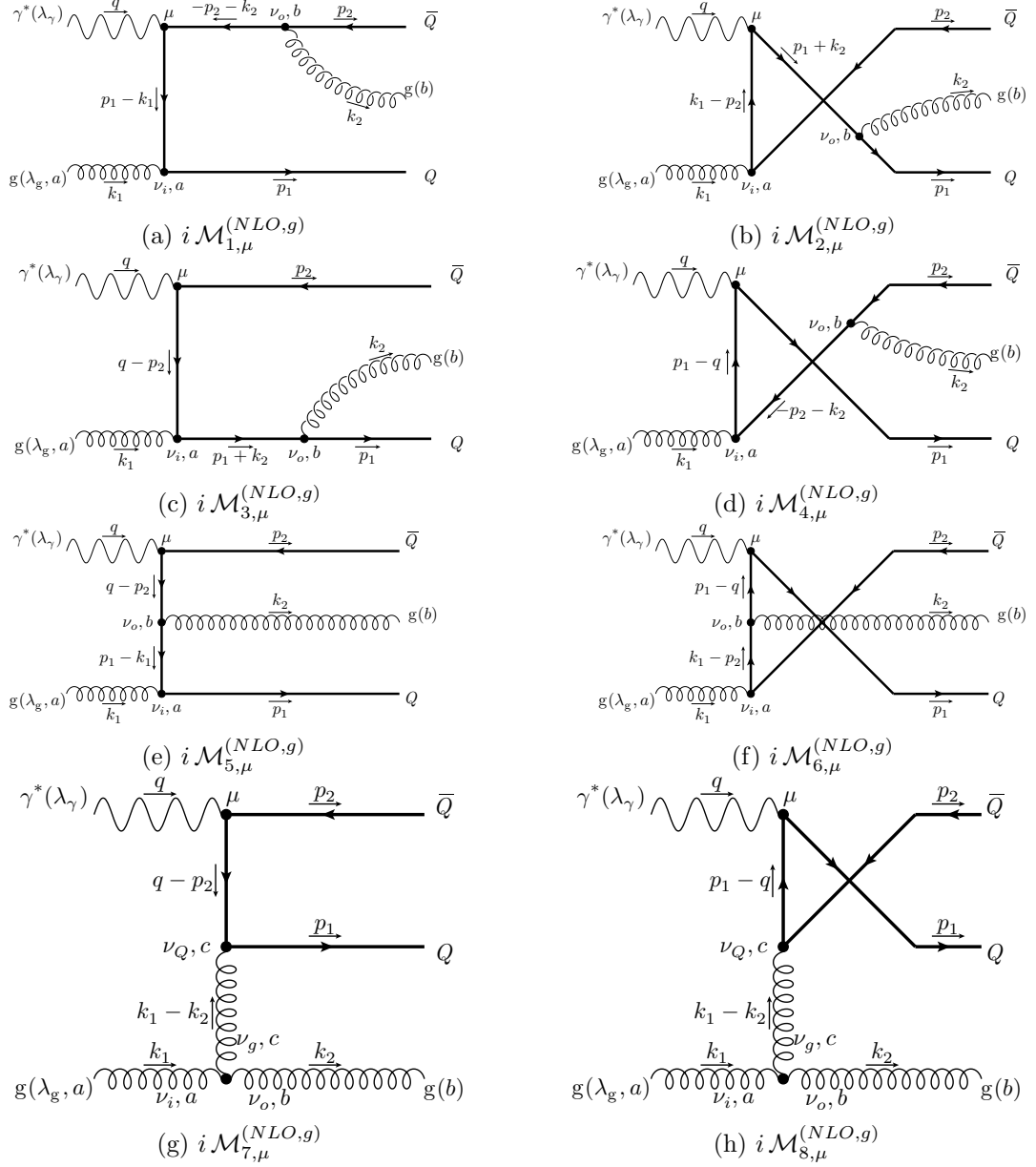


Figure 3: NLO contributions by gluon bremsstrahlung

formula:

$$i\mathcal{M}_{1,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_a\gamma^{\nu_i})\frac{i(\not{p}_1 - \not{k}_1 + m)}{u_6}(-iee_H\gamma_\mu) \cdot \frac{i(-\not{p}_2 - \not{k}_2 + m)}{s_3}(igT_b\gamma^{\nu_o})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \quad (11)$$

$$i\mathcal{M}_{2,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_b\gamma^{\nu_o})\frac{i(\not{p}_1 + \not{k}_2 + m)}{s_4}(-iee_H\gamma_\mu) \cdot \frac{i(\not{k}_1 - \not{p}_2 + m)}{t_1}(igT_a\gamma^{\nu_i})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \quad (12)$$

$$i\mathcal{M}_{3,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_b\gamma^{\nu_o})\frac{i(\not{p}_1 + \not{k}_2 + m)}{s_4}(igT_a\gamma^{\nu_i}) \cdot \frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \quad (13)$$

$$i\mathcal{M}_{4,\mu}^{(NLO,g)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{u_7}(igT_a\gamma^{\nu_i}) \cdot \frac{i(-\not{p}_2 - \not{k}_2 + m)}{s_3}(igT_b\gamma^{\nu_o})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \quad (14)$$

$$i\mathcal{M}_{5,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_a\gamma^{\nu_i})\frac{i(\not{p}_1 - \not{k}_1 + m)}{u_6}(igT_b\gamma^{\nu_o}) \cdot \frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \quad (15)$$

$$i\mathcal{M}_{6,\mu}^{(NLO,g)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{u_7}(igT_b\gamma^{\nu_o}) \cdot \frac{i(\not{k}_1 - \not{p}_2 + m)}{t_1}(igT_a\gamma^{\nu_i})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \quad (16)$$

$$i\mathcal{M}_{7,\mu}^{(NLO,g)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_g}}{t'} \cdot \varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \cdot \left(gf^{acb}(g^{\nu_o,\nu_i}(k_1+k_2)^{\nu_g} + g^{\nu_i,\nu_g}(k_2-2k_1)^{\nu_o} + g^{\nu_g,\nu_o}(k_1-2k_2)^{\nu_i})\right) \quad (17)$$

$$i\mathcal{M}_{8,\mu}^{(NLO,g)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_g}}{t'} \cdot \varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \cdot \left(gf^{acb}(g^{\nu_o,\nu_i}(k_1+k_2)^{\nu_g} + g^{\nu_i,\nu_g}(k_2-2k_1)^{\nu_o} + g^{\nu_g,\nu_o}(k_1-2k_2)^{\nu_i})\right) \quad (18)$$

color space:

$$\begin{aligned}
& \sum_{j=1}^6 \left| \mathcal{M}_{j,\mu}^{(NLO,g)} \right|^2 + \mathcal{M}_{1,\mu}^{(NLO,g)} \left(\mathcal{M}_{4,\mu'}^{(NLO,g)} + \mathcal{M}_{5,\mu'}^{(NLO,g)} \right)^* + \mathcal{M}_{3,\mu}^{(NLO,g)} \left(\mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^* + \\
& \mathcal{M}_{2,\mu}^{(NLO,g)} \left(\mathcal{M}_{3,\mu'}^{(NLO,g)} + \mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^* + \mathcal{M}_{4,\mu}^{(NLO,g)} \left(\mathcal{M}_{5,\mu'}^{(NLO,g)} \right)^* \\
& \sim \text{tr}(T_a T_a T_b T_b) = N_C C_F^2
\end{aligned} \tag{19}$$

$$\begin{aligned}
& \mathcal{M}_{1,\mu}^{(NLO,g)} \left(\mathcal{M}_{2,\mu'}^{(NLO,g)} + \mathcal{M}_{3,\mu'}^{(NLO,g)} + \mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^* + \\
& \left(\mathcal{M}_{2,\mu}^{(NLO,g)} + \mathcal{M}_{3,\mu}^{(NLO,g)} \right) \left(\mathcal{M}_{4,\mu'}^{(NLO,g)} + \mathcal{M}_{5,\mu'}^{(NLO,g)} \right)^* + \\
& \left(\mathcal{M}_{4,\mu}^{(NLO,g)} + \mathcal{M}_{5,\mu}^{(NLO,g)} \right) \left(\mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^* \\
& \sim \text{tr}(T_a T_b T_a T_b) = N_C C_F \left(C_F - \frac{C_A}{2} \right)
\end{aligned} \tag{20}$$

$$\begin{aligned}
& \left(\mathcal{M}_{2,\mu}^{(NLO,g)} + \mathcal{M}_{3,\mu}^{(NLO,g)} + \mathcal{M}_{6,\mu}^{(NLO,g)} \right) \left(\mathcal{M}_{7,\mu'}^{(NLO,g)} + \mathcal{M}_{8,\mu'}^{(NLO,g)} \right)^* \\
& \sim -i f_{bda} \text{tr}(T_a T_b T_d) = \frac{1}{2} N_C C_F C_A
\end{aligned} \tag{21}$$

$$\begin{aligned}
& \left(\mathcal{M}_{1,\mu}^{(NLO,g)} + \mathcal{M}_{4,\mu}^{(NLO,g)} + \mathcal{M}_{5,\mu}^{(NLO,g)} \right) \left(\mathcal{M}_{7,\mu'}^{(NLO,g)} + \mathcal{M}_{8,\mu'}^{(NLO,g)} \right)^* \\
& \sim -i f_{bda} \text{tr}(T_b T_a T_d) = i f_{bda} \text{tr}(T_a T_b T_d) = -\frac{1}{2} N_C C_F C_A
\end{aligned} \tag{22}$$

$$\begin{aligned}
& \left| \mathcal{M}_{7,\mu}^{(NLO,g)} + \mathcal{M}_{8,\mu}^{(NLO,g)} \right|^2 \\
& \sim f_{acb} f_{adb} \text{tr}(T_c T_d) = N_C C_F C_A
\end{aligned} \tag{23}$$

To get the polarisation sums right, one has to subtract the contributions of the Faddeev-Popov ghosts[4, 5]:

diagrammatic:

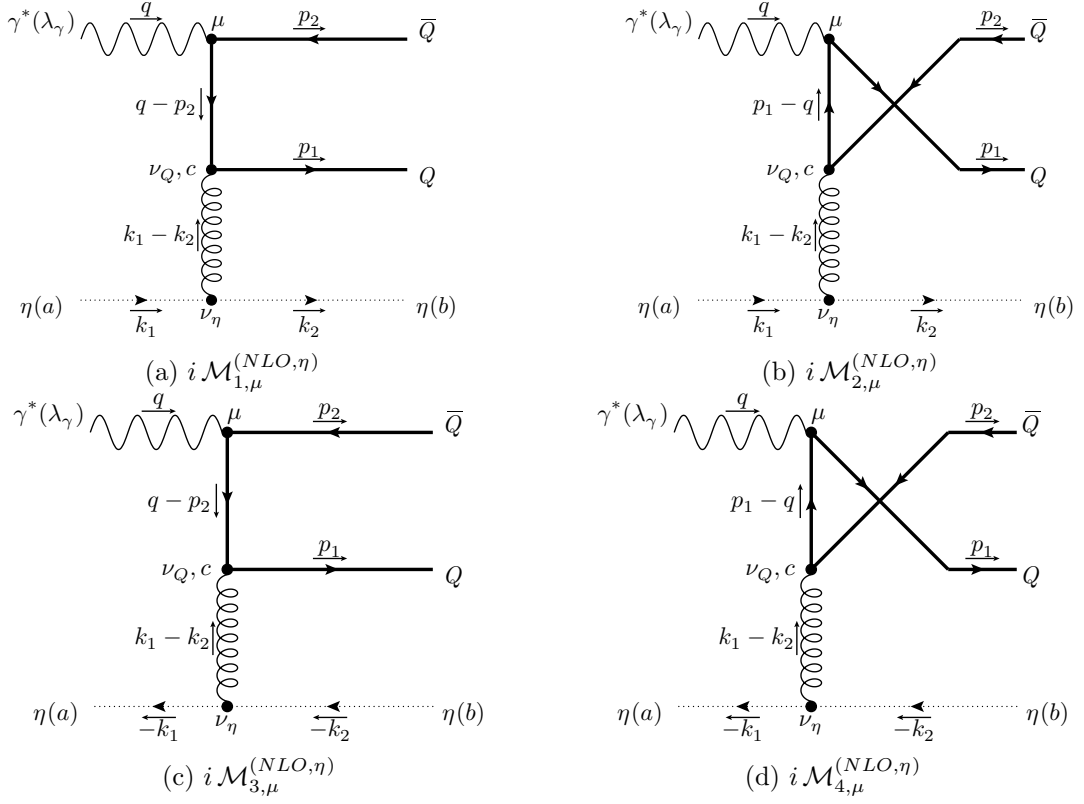


Figure 4: NLO contributions by ghosts

formula:

$$i \mathcal{M}_{1,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_\eta}}{t'} \cdot (gf^{acb}k_2^{\nu_\eta}) \quad (24)$$

$$i \mathcal{M}_{2,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_\eta}}{t'} \cdot (gf^{acb}k_2^{\nu_\eta}) \quad (25)$$

$$i \mathcal{M}_{3,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_\eta}}{t'} \cdot (gf^{cab}(-k_1)^{\nu_\eta}) \quad (26)$$

$$i \mathcal{M}_{4,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2) \cdot \frac{-ig_{\nu_Q,\nu_\eta}}{t'} \cdot (gf^{cab}(-k_1)^{\nu_\eta}) \quad (27)$$

color space:

$$\left| \mathcal{M}_{1,\mu}^{(NLO,\eta)} + \mathcal{M}_{2,\mu}^{(NLO,\eta)} \right|^2 \sim f_{acb} f_{adb} \text{tr}(T_c T_d) = N_C C_F C_A \quad (28)$$

$$\left| \mathcal{M}_{3,\mu}^{(NLO,\eta)} + \mathcal{M}_{4,\mu}^{(NLO,\eta)} \right|^2 \sim f_{cab} f_{dab} \text{tr}(T_c T_d) = f_{acb} f_{adb} \text{tr}(T_c T_d) = N_C C_F C_A \quad (29)$$

3.3 Virtual Contributions

$$\gamma^*(q) + g(k_1) \rightarrow \bar{Q}(p_2) + Q(p_1) \quad (30)$$

3.3.1 Loops

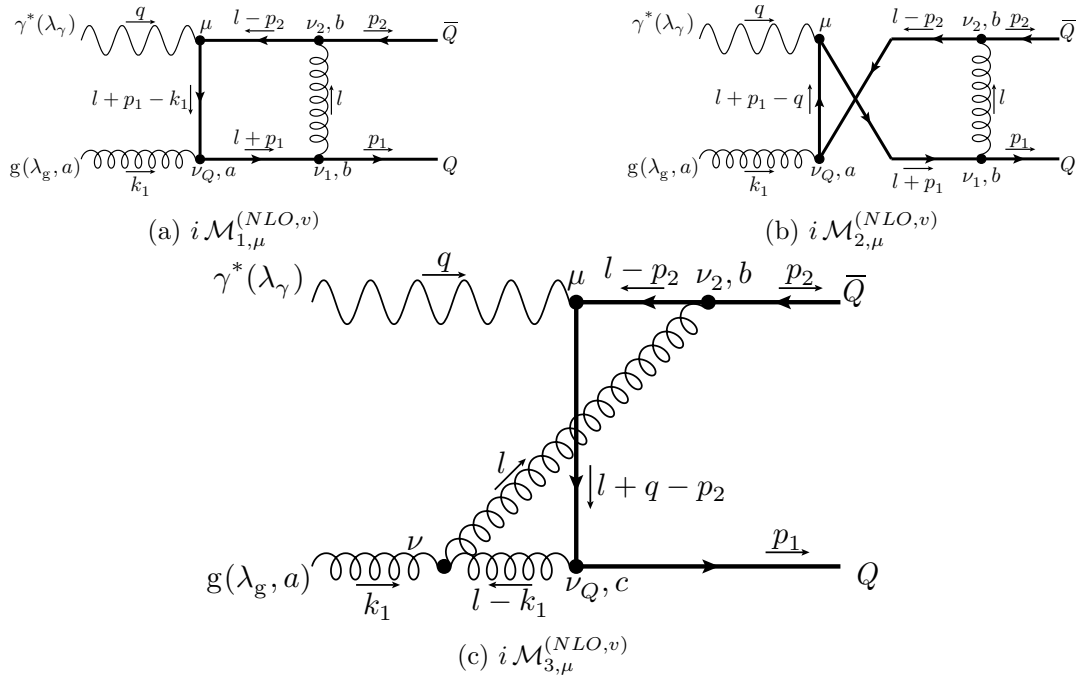


Figure 5: NLO contributions by one loop

$$i\mathcal{M}_{1,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_b \gamma^{\nu_1}) \frac{i(\not{l} + \not{p}_1 + m)}{(l+p_1)^2 - m^2} (igT_a \gamma^{\nu_Q}) \frac{i(\not{l} + \not{p}_1 - \not{k}_1 + m)}{(l+p_1-k_1)^2 - m^2} \cdot$$

$$(-iee_H \gamma_\mu) \frac{i(\not{l} - \not{p}_2 + m)}{(l-p_2)^2 - m^2} (igT_b \gamma^{\nu_2}) \frac{-ig_{\nu_1, \nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (31)$$

$$i\mathcal{M}_{2,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_b \gamma^{\nu_1}) \frac{i(\not{l} + \not{p}_1 + m)}{(l+p_1)^2 - m^2} (igT_a \gamma^{\nu_Q}) \frac{i(\not{l} + \not{p}_1 - \not{q} + m)}{(l+p_1-q)^2 - m^2} \cdot$$

$$(-iee_H \gamma_\mu) \frac{i(\not{l} - \not{p}_2 + m)}{(l-p_2)^2 - m^2} (igT_b \gamma^{\nu_2}) \frac{-ig_{\nu_1, \nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (32)$$

$$i\mathcal{M}_{3,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_c \gamma^{\nu_Q}) \frac{i(\not{l} + \not{q}_1 - \not{p}_2 + m)}{(l+q-p_2)^2 - m^2} (-iee_H \gamma_\mu) \frac{i(\not{l} - \not{p}_2 + m)}{(l-p_2)^2 - m^2} \cdot$$

$$(igT_b \gamma^{\nu_2}) \frac{(-i)^2}{l^2(l-k_1)^2} v(p_2) \varepsilon^{\nu, (\lambda_g)}(k_1) \cdot$$

$$\left(gf_{abc} \left(g_{\nu_2 \nu_Q} (k_1 - 2l)_\nu + g_{\nu_Q \nu} (l - 2k_1)_{\nu_2} + g_{\nu \nu_2} (k_1 + l)_{\nu_Q} \right) \right) \quad (33)$$

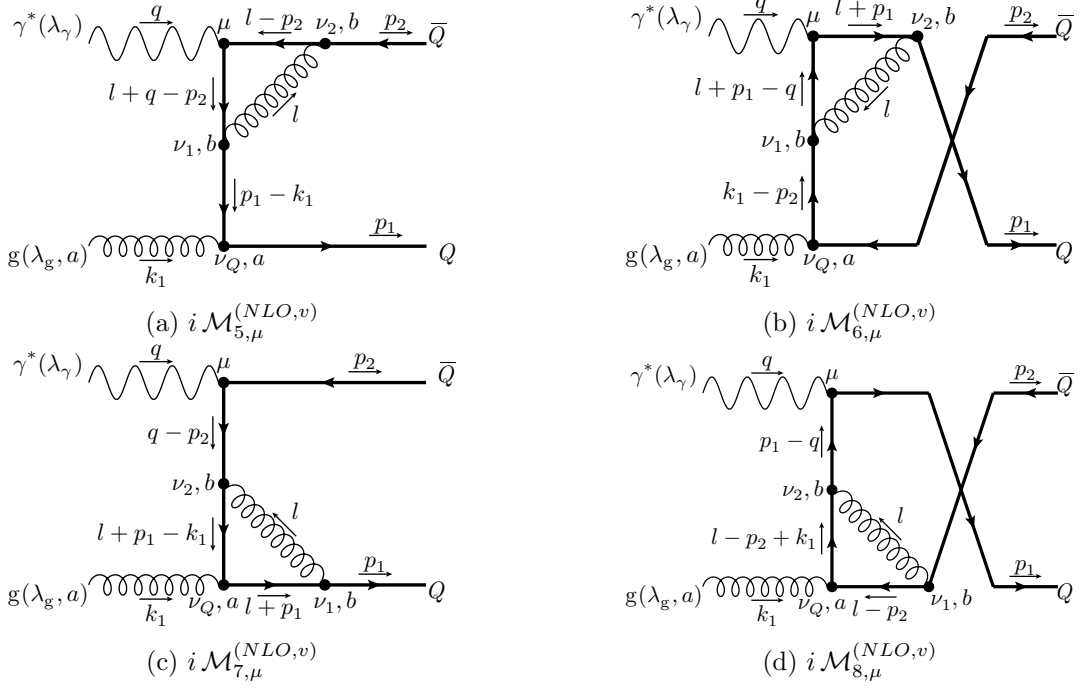


Figure 6: NLO contributions by one loop (cont'd)

$$i\mathcal{M}_{5,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_a \gamma^{\nu_Q}) \frac{i(\not{p}_1 - \not{k}_1 + m)}{u_1} (igT_b \gamma^{\nu_1}) \frac{i(\not{l} + \not{q} - \not{p}_2 + m)}{(l+q-p_2)^2 - m^2} \cdot$$

$$(-iee_H \gamma_\mu) \frac{i(\not{l} - \not{p}_2 + m)}{(l-p_2)^2 - m^2} (igT_b \gamma^{\nu_2}) \frac{-ig_{\nu_1, \nu_2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)}{l^2} \quad (34)$$

$$i\mathcal{M}_{6,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_b \gamma^{\nu_2}) \frac{i(\not{l} + \not{p}_1 + m)}{(l+p_1)^2 - m^2} (-iee_H \gamma_\mu) \frac{i(\not{l} + \not{p}_1 - \not{q} + m)}{(l+p_1-q)^2 - m^2} \cdot$$

$$(igT_b \gamma^{\nu_1}) \frac{i(\not{k}_1 - \not{p}_2 + m)}{t_1} (igT_a \gamma^{\nu_Q}) \frac{-ig_{\nu_1, \nu_2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)}{l^2} \quad (35)$$

$$i\mathcal{M}_{7,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_b \gamma^{\nu_1}) \frac{i(\not{l} + \not{p}_1 + m)}{(l+p_1)^2 - m^2} (igT_a \gamma^{\nu_Q}) \frac{i(\not{l} + \not{p}_1 - \not{k}_1 + m)}{(l+p_1-k_1)^2 - m^2} \cdot$$

$$(igT_b \gamma^{\nu_2}) \frac{i(\not{q} - \not{p}_2 + m)}{u_1} (-iee_H \gamma_\mu) \frac{-ig_{\nu_1, \nu_2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)}{l^2} \quad (36)$$

$$i\mathcal{M}_{8,\mu}^{(NLO,v)} = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (-iee_H \gamma_\mu) \frac{i(\not{p}_1 - \not{q} + m)}{t_1} (igT_b \gamma^{\nu_2}) \frac{i(\not{l} - \not{p}_2 + \not{k}_1 + m)}{(l-p_2+k_1)^2 - m^2} \cdot$$

$$(igT_a \gamma^{\nu_Q}) \frac{i(\not{l} - \not{p}_2 + m)}{(l-p_2)^2 - m^2} (igT_b \gamma^{\nu_1}) \frac{-ig_{\nu_1, \nu_2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)}{l^2} \quad (37)$$

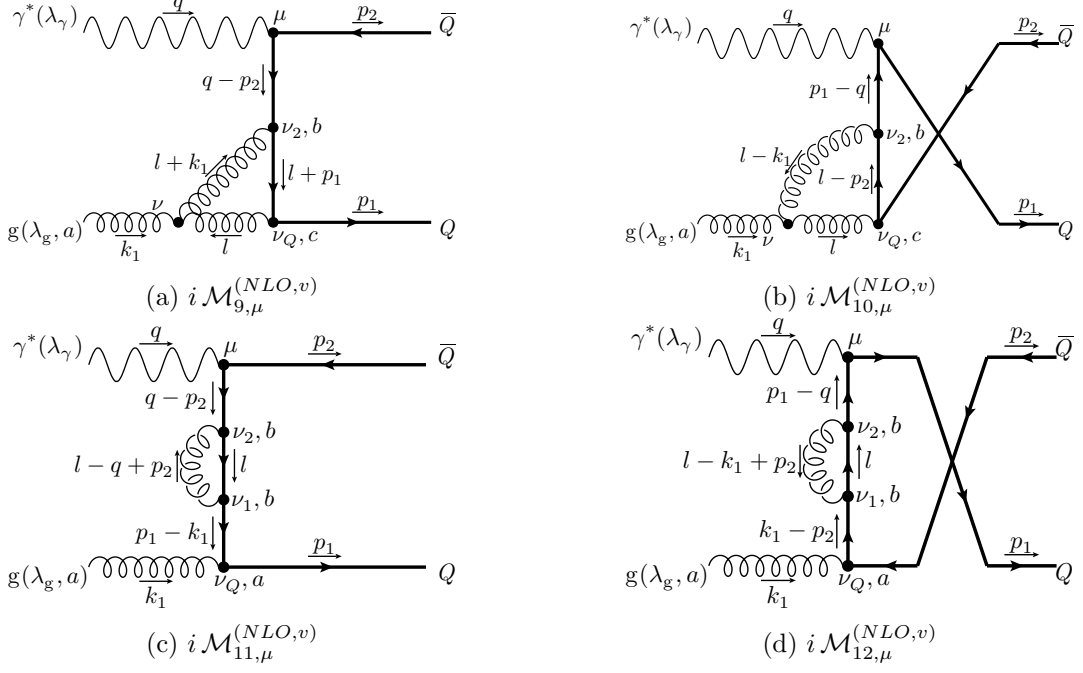


Figure 7: NLO contributions by one loop (cont'd)

$$\begin{aligned}
i\mathcal{M}_{9,\mu}^{(NLO,v)} &= \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_c \gamma^{\nu_Q}) \frac{i(\not{l} + \not{p}_1 + m)}{(l + p_1)^2 - m^2} (igT_b \gamma^{\nu_2}) \frac{i(\not{q} - \not{p}_2 + m)}{u_1} \\
&\quad (-iee_H \gamma_\mu) \frac{(-i)^2}{l^2 (l + k_1)^2} v(p_2) \varepsilon^{\nu, (\lambda_g)}(k_1) \cdot \\
&\quad \left(gf_{abc} \left(g_{\nu\nu_2} (2k_1 + l)_{\nu_Q} + g_{\nu_2\nu_Q} (-2l - k_1)_\nu + g_{\nu_Q\nu} (l - k_1)_{\nu_2} \right) \right) \quad (38)
\end{aligned}$$

$$\begin{aligned}
i\mathcal{M}_{10,\mu}^{(NLO,v)} &= \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (-iee_H \gamma_\mu) \frac{i(\not{p}_1 - \not{q} + m)}{t_1} (igT_b \gamma^{\nu_2}) \frac{i(\not{l} - \not{p}_2 + m)}{(l - p_2)^2 - m^2} \\
&\quad (igT_c \gamma^{\nu_Q}) \frac{(-i)^2}{l^2 (l - k_1)^2} v(p_2) \varepsilon^{\nu, (\lambda_g)}(k_1) \cdot \\
&\quad \left(gf_{abc} \left(g_{\nu\nu_2} (2k_1 - l)_{\nu_Q} + g_{\nu_2\nu_Q} (2l - k_1)_\nu + g_{\nu_Q\nu} (-l - k_1)_{\nu_2} \right) \right) \quad (39)
\end{aligned}$$

$$\begin{aligned}
i\mathcal{M}_{11,\mu}^{(NLO,v)} &= \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (igT_a \gamma^{\nu_Q}) \frac{i(\not{p}_1 - \not{k}_1 + m)}{u_1} (igT_b \gamma^{\nu_1}) \frac{i(\not{l} + m)}{l^2 - m^2} \\
&\quad (igT_b \gamma^{\nu_2}) \frac{i(\not{q} - \not{p}_2 + m)}{u_1} (-iee_H \gamma_\mu) \frac{-ig_{\nu_1\nu_2}}{(l - q + p_2)^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (40)
\end{aligned}$$

$$\begin{aligned}
i\mathcal{M}_{12,\mu}^{(NLO,v)} &= \mu^{4-n} \int \frac{d^n l}{(2\pi)^n} \bar{u}(p_1) (-iee_H \gamma_\mu) \frac{i(\not{p}_1 - \not{q} + m)}{t_1} (igT_b \gamma^{\nu_2}) \frac{i(\not{l} + m)}{l^2 - m^2} \\
&\quad (igT_b \gamma^{\nu_1}) \frac{i(\not{k}_1 - \not{p}_2 + m)}{t_1} (igT_a \gamma^{\nu_Q}) \frac{-ig_{\nu_1\nu_2}}{(l - k_1 + p_2)^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (41)
\end{aligned}$$

Color space:

$$\begin{aligned} & \left(\mathcal{M}_{1,\mu}^{(NLO,v)} + \mathcal{M}_{2,\mu}^{(NLO,v)} + \mathcal{M}_{7,\mu}^{(NLO,v)} + \mathcal{M}_{8,\mu}^{(NLO,v)} \right) \\ & \cdot \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)} \right)^* \sim -i \operatorname{tr}(T_a T_b T_a T_b) = -i N_C C_F \left(C_F - \frac{C_A}{2} \right) \end{aligned} \quad (42)$$

$$\begin{aligned} & \left(\mathcal{M}_{3,\mu}^{(NLO,v)} + \mathcal{M}_{9,\mu}^{(NLO,v)} + \mathcal{M}_{10,\mu}^{(NLO,v)} \right) \\ & \cdot \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)} \right)^* \sim f_{abc} \operatorname{tr}(T_c T_b T_a) = -\frac{i}{2} N_C C_F C_A \end{aligned} \quad (43)$$

$$\begin{aligned} & \left(\mathcal{M}_{5,\mu}^{(NLO,v)} + \mathcal{M}_{6,\mu}^{(NLO,v)} + \mathcal{M}_{11,\mu}^{(NLO,v)} + \mathcal{M}_{12,\mu}^{(NLO,v)} \right) \\ & \cdot \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)} \right)^* \sim -i \operatorname{tr}(T_a T_a T_b T_b) = -i N_C C_F^2 \end{aligned} \quad (44)$$

3.3.2 Counter terms

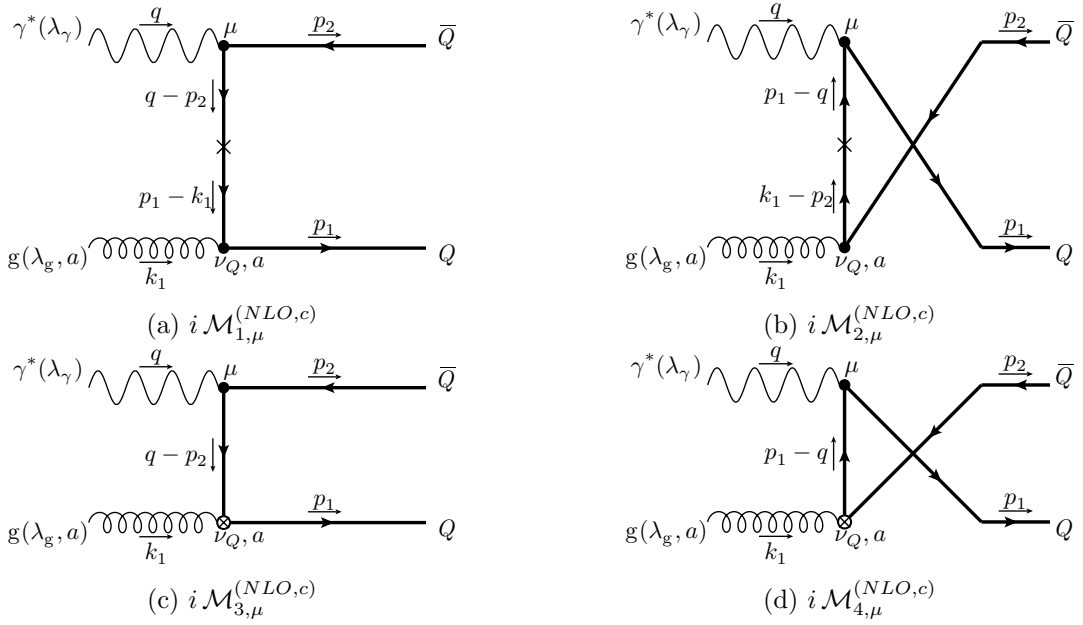


Figure 8: NLO contributions by counter terms

$$-i\mathcal{M}_{1,\mu}^{(NLO,c)} = \bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not{p}_1 - \not{k}_1 + m)}{u_1} \left(i((Z_2 - 1)(\not{q} - \not{p}_2 - m) - (Z_m - 1)m) \right) \frac{i(\not{q} - \not{p}_2 + m)}{u_1} (-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (45)$$

$$-i\mathcal{M}_{2,\mu}^{(NLO,c)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{t_1} \left(i((Z_2 - 1)(\not{p}_1 - \not{q} - m) - (Z_m - 1)m) \right) \frac{i(\not{k}_1 - \not{p}_2 + m)}{t_1} (igT_a\gamma^{\nu_Q})v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (46)$$

$$-i\mathcal{M}_{3,\mu}^{(NLO,c)} = \bar{u}(p_1)(-i(Z_{1f} - 1)gT_a\gamma^{\nu_Q})\frac{i(\not{q} - \not{p}_2 + m)}{u_1} (-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (47)$$

$$-i\mathcal{M}_{4,\mu}^{(NLO,c)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{t_1} (-i(Z_{1f} - 1)gT_a\gamma^{\nu_Q})v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \quad (48)$$

Color space:

$$\left(\mathcal{M}_{1,\mu}^{(NLO,c)} + \mathcal{M}_{2,\mu}^{(NLO,c)} \right) \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)} \right)^* \sim \text{tr}(T_a T_a)(Z_2 - 1) = C_F^2 \quad (49)$$

$$\left(\mathcal{M}_{3,\mu}^{(NLO,c)} + \mathcal{M}_{4,\mu}^{(NLO,c)} \right) \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)} \right)^* \sim \text{tr}(T_a T_a)(Z_{1f} - 1) = C_F(C_F + C_A) \quad (50)$$

3.3.3 Quark Self Energy

To compute self energies, we follow [6]. It is

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \quad (51)$$

$$\gamma_\mu \gamma^\mu = g_\mu^\mu = n \quad (52)$$

$$\gamma_\mu \gamma_\nu \gamma^\mu = (2 - n)\gamma_\nu \quad (53)$$

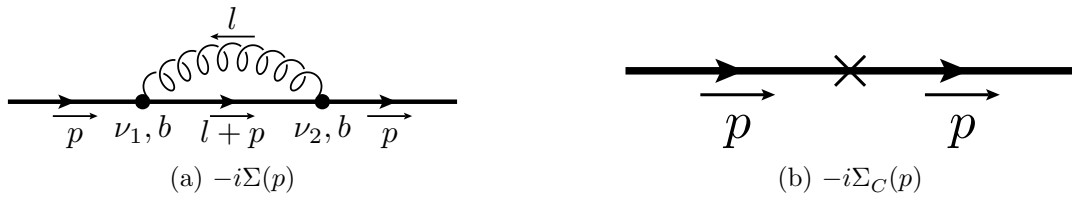


Figure 9: NLO contributions by quark self energy

$$-i\Sigma(p) = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} (igT_b \gamma_{\nu_1}) \frac{i(\not{l} + \not{p} + m)}{(l+p)^2 - m^2} (igT_b \gamma_{\nu_2}) \frac{-ig^{\nu_1, \nu_2}}{l^2} \quad (54)$$

$$= -\mu_R^{4-n} g^2 C_F \int \frac{d^n l}{(2\pi)^n} \frac{2m + (2-n)\not{p} + (2-n)\not{l}}{l^2((l+p)^2 - m^2)} \quad (55)$$

$$= -g^2 C_F \left((2m + (2-n)\not{p}) B_0(p^2, 0, m^2) + (2-n)B_1(p^2, 0, m^2) \right) \quad (56)$$

$$= -g^2 C_F \left(B_0(p^2, 0, m^2) \left(n \cdot m + (2-n)\not{p} \frac{p^2 + m^2}{2p^2} \right) - (2-n)\not{p} \frac{1}{2p^2} A_0(m^2) \right) \quad (57)$$

Using [6] we find

$$C_\epsilon = \frac{1}{16\pi^2} \exp \left((\gamma_E - \log(4\pi)) \frac{\epsilon}{2} \right) \left(m^2/\mu^2 \right)^{\epsilon/2} \quad (58)$$

$$A_0(m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 1 \right) \quad (59)$$

$$B_0(p^2, 0, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 + \frac{m^2 - p^2}{p^2} \ln \left(\frac{m^2 - p^2}{m^2} \right) \right) \quad (60)$$

$$\Rightarrow -i\Sigma(p) = -ig^2 C_F C_\epsilon \left[\frac{2\not{p} - 8m}{\epsilon} + 2m \left(3 - 2 \left(1 - \frac{m^2}{p^2} \right) \ln \left(1 - \frac{p^2}{m^2} \right) \right) \right. \\ \left. - \not{p} \left(1 + \frac{m^2}{p^2} \right) \left(1 - \left(1 - \frac{m^2}{p^2} \right) \ln \left(1 - \frac{p^2}{m^2} \right) \right) \right] \quad (61)$$

$$\stackrel{!}{=} -i(Am + B(\not{p} - m)) \quad (62)$$

$$\Rightarrow A = \frac{1}{m} \Sigma(p)|_{\not{p}=m} \quad (63)$$

$$= -g^2 C_F C_\epsilon \left(\frac{6}{\epsilon} - 5 + \frac{m^2}{p^2} + \left(3 - 4\frac{m^2}{p^2} + \frac{m^4}{p^4} \right) \ln \left(1 - \frac{p^2}{m^2} \right) \right) \quad (64)$$

$$\Rightarrow B = \frac{1}{m} \frac{d\Sigma(p)}{d\not{p}} \Big|_{\not{p}=m} \quad (65)$$

$$= \frac{g^2}{16\pi^2} C_F \left(\frac{2}{\epsilon_m} - 1 - \frac{m^2}{p^2} + \left(1 - \frac{m^4}{p^4} \right) \ln \left(1 - \frac{p^2}{m^2} \right) \right) \quad (66)$$

Counterterm:

$$-i\Sigma_C(p) = i((Z_2 - 1)\not{p} - (Z_2 Z_m - 1)m) \quad (67)$$

$$= i((Z_2 - 1)(\not{p} - m) - (Z_m - 1)m) + O(\alpha_S^2) \quad (68)$$

Use on-shell renormalisation:

$$0 \stackrel{!}{=} (-i\Sigma(p) - i\Sigma_C(p))|_{\not{p}=m} \quad (69)$$

$$= i \left(((Z_m - 1) + A)m + (B - (Z_2 - 1))(\not{p} - m) \right)|_{\not{p}=m} \quad (70)$$

$$\Rightarrow (Z_m - 1) = -A|_{\not{p}=m} \quad (71)$$

$$= \frac{g^2}{16\pi^2} C_F \left(\frac{6}{\hat{\epsilon}_m} - 4 \right) \quad (72)$$

We also choose

$$Z_2 - 1 = \frac{g^2}{16\pi^2} C_F \frac{2}{\hat{\epsilon}_m} \quad (73)$$

$$Z_{1f} = Z_2 + \frac{g^2}{16\pi^2} C_A \frac{2}{\hat{\epsilon}} \quad (74)$$

3.3.4 Gluon Self Energy (Off shell)

Again, we follow [6].

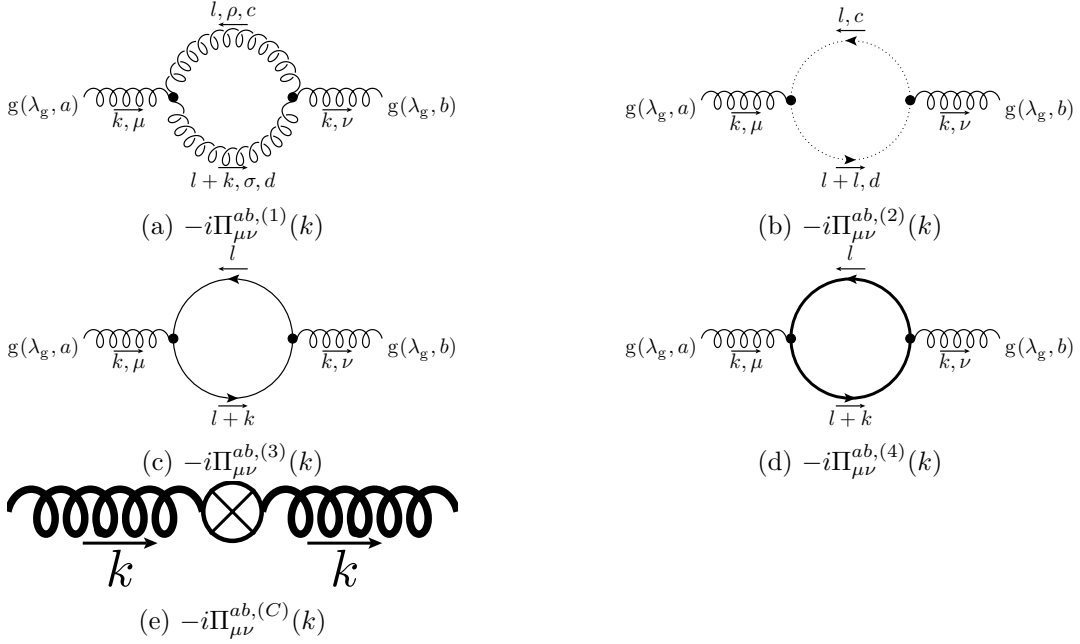


Figure 10: NLO contributions by quark self energy

$$-i\Pi_{\mu\nu}^{ab,(1)}(k) = \frac{1}{2!}\mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}gf_{cda}\left(g_{\rho\sigma}(2l+k)_\mu+g_{\sigma\mu}(-l-2k)_\rho+g_{\mu\rho}(k-l)_\sigma\right) \\ \cdot gf_{cbd}\left(g^\rho_\nu(k-l)^\sigma+g^\sigma_\nu(-l-2k)^\rho+g^{\sigma\rho}(2l+k)_\nu\right)\cdot\frac{(-i)^2}{l^2(l+k)^2} \quad (75)$$

$$-i\Pi_{\mu\nu}^{ab,(2)}(k) = -\mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}g^2f_{adc}f_{bcd}l_\mu(l+k)_\nu\frac{i^2}{l^2(l+k)^2} \quad (76)$$

$$-i\Pi_{\mu\nu}^{ab,(3)}(k) = -\mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\frac{\text{tr}\left((ig\gamma_\mu)(i\cancel{l})(ig\gamma_\nu)(i(\cancel{l}+\cancel{k}))\right)\text{tr}(T_aT_b)}{l^2(l+k)^2} \quad (77)$$

$$-i\Pi_{\mu\nu}^{ab,(4)}(k) = -\mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\frac{\text{tr}\left((ig\gamma_\mu)(i(\cancel{l}+m))(ig\gamma_\nu)(i(\cancel{l}+\cancel{k}+m))\right)\text{tr}(T_aT_b)}{(l^2-m^2)((l+k)^2-m^2)} \quad (78)$$

$$-i\Pi_{\mu\nu}^{ab,(C)}(k) = i(Z_3-1)\delta^{ab}(k_\mu k_\nu - k^2 g_{\mu\nu}) \quad (79)$$

Color space:

$$f_{acd}f_{bdc} = -\delta_{ab}C_A = f_{dca}f_{dbc} \quad (80)$$

$$\text{tr}(T_aT_b) = \frac{1}{2}\delta_{ab} \quad (81)$$

By Slavnov-Taylor we know:

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$$\Pi_{\mu\nu}^{ab}(k) = \delta^{ab}(k_\mu k_\nu - k^2 g_{\mu\nu})\Pi(k^2) \quad (82)$$

$$\Rightarrow \Pi(k^2) = -\frac{\delta_{ab}}{N_c^2 - 1} \frac{1}{(3 + \epsilon)k^2} g^{\mu\nu} \Pi_{\mu\nu}^{ab}(k) \quad (83)$$

Gluon + Ghost loop:

$$\Pi^{(1+2)}(k^2) = -ig^2 C_A \frac{1}{(3 + \epsilon)k^2} \mu_R^{-\epsilon} \int \frac{d^n l}{(2\pi)^n} \frac{(8 + 3\epsilon)k \cdot l + (9 + 3\epsilon)k^2 + (8 + 3\epsilon)l^2}{l^2(l + k)^2} \quad (84)$$

$$= -ig^2 C_A \frac{10 + 3\epsilon}{2(3 + \epsilon)} \mu_R^{-\epsilon} \int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2(l + k)^2} \quad (85)$$

with

$$B_0(k^2, 0, 0) = \mu_R^{-\epsilon} \int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2(l + k)^2} \quad (86)$$

$$= \frac{i}{16\pi^2} \left(-\frac{2}{\hat{\epsilon}} - \ln(-k^2/\mu_R^2) + 2 \right) \quad (87)$$

We find

$$\Rightarrow \Pi^{(1+2)}(k^2) = g^2 C_A \left(-\frac{10}{3\hat{\epsilon}} - \frac{5}{3} \ln(-k^2/\mu_R^2) + \frac{31}{9} \right) \quad (88)$$

$$= -C_A \frac{g^2}{16\pi^2} \frac{5}{3} \left(\frac{2}{\hat{\epsilon}} + \ln(-k^2/\mu_R^2) - \frac{31}{15} \right) \quad (89)$$

Heavy quark loop:

$$\Pi^{(4)}(k^2) = ig^2 \frac{1}{2(3 + \epsilon)k^2} \mu_R^{-\epsilon} \int \frac{d^n l}{(2\pi)^n} \frac{4((2 + \epsilon)(m^2 - k \cdot l - l^2) + 2m^2)}{(l^2 - m^2)((l + k)^2 - m^2)} \quad (90)$$

$$= ig^2 \frac{2}{(3 + \epsilon)k^2} \left(2m^2 B_0(k^2, m^2, m^2) - (2 + \epsilon) A_0(m^2) - (2 + \epsilon) k^2 B_1(k^2, m^2, m^2) \right) \quad (91)$$

with

$$A_0(m^2) = \frac{im^2}{16\pi^2} \left(-\frac{2}{\hat{\epsilon}_m} + 1 \right) \quad (92)$$

$$B_1(k^2, m^2, m^2) = -\frac{1}{2} B_0(k^2, m^2, m^2) \quad (93)$$

$$B_0(k^2, m^2, m^2) = \frac{i}{16\pi^2} \left(-\frac{2}{\hat{\epsilon}_m} + 2 + \beta_k \ln(\chi_k) \right) \quad (94)$$

and $\beta_k = \sqrt{1 - 4m^2/k^2}$, $\chi_k = (\beta_k - 1)/(1 + \beta_k)$. We then find

$$\Rightarrow \Pi^{(4)}(k^2) = \frac{2g^2}{16\pi^2} \left(\frac{2}{3\hat{\epsilon}_m} - \frac{5}{9} - \frac{4m^2}{3k^2} - \frac{1}{3} \left(1 + 2\frac{m^2}{k^2} \right) \beta_k \ln(\chi_k) \right) \quad (95)$$

$$= \frac{g^2}{16\pi^2} \frac{2}{3} \left(\frac{2}{\hat{\epsilon}_m} - \frac{5}{3} - 4\frac{m^2}{k^2} - \left(1 + 2\frac{m^2}{k^2} \right) \beta_k \ln(\chi_k) \right) \quad (96)$$

Light quark loop:

$$\Pi^{(3)}(k^2) = -ig^2 \frac{2(2+\epsilon)}{(3+\epsilon)} B_1(k^2, 0, 0) \quad (97)$$

$$= ig^2 \frac{2(2+\epsilon)}{(3+\epsilon)} B_0(k^2, 0, 0) \quad (98)$$

$$= \frac{g^2}{16\pi^2} \frac{2}{3} \left(\frac{2}{\hat{\epsilon}} + \ln(-k^2/\mu_R^2) - \frac{5}{3} \right) \quad (99)$$

We choose the counterterm to be:

$$-\Pi^{(C)}(k^2) = Z_3 - 1 = \frac{g^2}{16\pi^2} \left(-\frac{5}{3} C_A \frac{2}{\hat{\epsilon}} + n_{lf} \frac{2}{3} \frac{2}{\hat{\epsilon}} + \frac{2}{3} \frac{2}{\hat{\epsilon}_m} \right) \quad (100)$$

$$= \frac{g^2}{16\pi^2} \left((2C_A - \beta_0) \frac{2}{\hat{\epsilon}} + \frac{2}{3} \frac{2}{\hat{\epsilon}_m} \right) \quad (101)$$

$$= \frac{g^2}{16\pi^2} \left((2C_A - \beta_0^f) \frac{2}{\hat{\epsilon}} + \frac{2}{3} \ln(m^2/\mu^2) \right) \quad (102)$$

with $\beta_0 = (11C_A - 2n_{lf})/3$ and $\beta_0^f = (11C_A - 2n_f)/3$.

3.3.5 Gluon Self Energy (On shell)

We have[7, (A.18)-(A.20)]:

$$I_0 = \int \frac{d^n p}{(2\pi)^n} \frac{1}{(p^2 + 2kp + M^2 + i\eta)^\alpha} \quad (103)$$

$$= i \frac{(-\pi)^{n/2}}{(2\pi)^n} \frac{\Gamma(\alpha - n/2)}{\Gamma(\alpha)} \frac{1}{(M^2 - k^2 + i\eta)^{\alpha - n/2}} \quad (104)$$

$$I_\mu = \int \frac{d^n p}{(2\pi)^n} \frac{p_\mu}{(p^2 + 2kp + M^2 + i\eta)^\alpha} \quad (105)$$

$$= -k_\mu I_0 \quad (106)$$

$$I_{\mu\nu} = \int \frac{d^n p}{(2\pi)^n} \frac{p_\mu p_\nu}{(p^2 + 2kp + M^2 + i\eta)^\alpha} \quad (107)$$

$$= I_0 \left(k_\mu k_\nu + \frac{1}{2} g_{\mu\nu} (M^2 - k^2) \frac{1}{\alpha - n/2 - 1} \right) \quad (108)$$

Heavy Quark loop:

$$-i\Pi_{\mu\nu}^{ab,(4)}(k) = -\mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{\text{tr}((ig\gamma_\mu)(i(\not{l} + m))(ig\gamma_\nu)(i(\not{l} + \not{k} + m))) \text{tr}(T_a T_b)}{(l^2 - m^2)((l+k)^2 - m^2)} \quad (109)$$

$$= -\frac{\delta^{ab}}{2} g^2 \mu_R^{4-n} \int_0^1 dx \int \frac{d^n l}{(2\pi)^n} 4 \frac{2l_\mu l_\nu + (l_\mu k_\nu + l_\nu k_\mu) + (m^2 - k \cdot l - l^2) g_{\mu\nu}}{(l^2 + 2xk \cdot l - m^2 + k^2 x)^2} \quad (110)$$

$$= -2\delta^{ab} g^2 \mu_R^{4-n} \int_0^1 dx \left(2I_{\mu\nu} + (k_\nu I_\mu + k_\mu I_\nu) + g_{\mu\nu} \left(m^2 I_0 - k^\rho I_\rho - g_\rho^\sigma I_{\rho\sigma} \right) \right) \quad (111)$$

$$= -2\delta^{ab} g^2 \mu_R^{4-n} \int_0^1 dx I_0 \left[k_\mu k_\nu (2x^2 - 2x) + g_{\mu\nu} \left((k^2 x(1-x) - m^2) \frac{1-n/2}{1-n/2} + m^2 + k^2 x(1-x) \right) \right] \quad (112)$$

$$= 4\delta^{ab} g^2 (k_\mu k_\nu - g_{\mu\nu} k^2) \mu_R^{4-n} \int_0^1 dx x(1-x) I_0 \quad (113)$$

where

$$\mu_R^{4-n} I_0 = i \frac{(-\pi)^{n/2}}{(2\pi)^n} \frac{\Gamma(2-n/2)}{\Gamma(2)} \left(\frac{\mu_R^2}{k^2 x(1-x) - m^2} \right)^{2-n/2} \quad (114)$$

$$= -\frac{i}{16\pi^2} \left(\frac{2}{\epsilon} - \gamma_E - \ln(4\pi) - \ln((k^2 x(1-x) - m^2)/\mu_R^2) \right) + O(\epsilon) \quad (115)$$

so for $k^2 = 0$ (on shell) we end up with

$$-i\Pi_{\mu\nu}^{ab,(4)}(k) = -i\delta^{ab} g^2 (k_\mu k_\nu - g_{\mu\nu} k^2) C_\epsilon \frac{4}{3\epsilon} \quad (116)$$

Light Quark loop:

$$-i\Pi_{\mu\nu}^{ab,(3)}(k) = -\mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{\text{tr}((ig\gamma_\mu)(i\not{l})(ig\gamma_\nu)(i(\not{l} + \not{k}))) \text{tr}(T_a T_b)}{l^2(l+k)^2} \quad (117)$$

$$= 4\delta^{ab} g^2 (k_\mu k_\nu - g_{\mu\nu} k^2) \mu_R^{4-n} \int_0^1 dx x(1-x) I_0 \quad (118)$$

where

$$\mu_R^{4-n} I_0 = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{1}{(l^2 + 2xk \cdot l + k^2 x)^2} \quad (119)$$

$$= \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{1}{((l+xk)^2 + k^2 x(1-x))^2} \quad (120)$$

so for $k^2 = 0$ (on shell) we get $I_0 = 0$ and end up with

$$-i\Pi_{\mu\nu}^{ab,(3)}(k) = 0 \quad (121)$$

Gluon + Ghost loop vanish by the same arguments as the light quark loop:

$$-i\Pi_{\mu\nu}^{ab,(1+2)}(k) = 0 \quad (122)$$

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List of Corrections

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