1 Passarino-Veltman decomposition

1.1 Definitions

[1]:

$$A(m) = \frac{1}{i\pi^2}\int d^n q \frac{1}{q^2+m^2} \tag{1}$$

$$B_0(p,m_1,m_2) = \frac{1}{i\pi^2}\int d^n q \frac{1}{(q^2+m_1^2)((q+p)^2+m_2^2)} \tag{2}$$

and apart from their pole term (called Δ - see [1, eq. D.1]), they keep $n=4$.

[2, 3]:

$$A(m) = \mu^{-\epsilon}\int \frac{d^n q}{(2\pi)^n}\frac{1}{q^2-m^2} \tag{3}$$

$$B(q_1,m_1,m_2) = \mu^{-\epsilon}\int \frac{d^n q}{(2\pi)^n}\frac{1}{(q^2-m_1^2)((q+q_1)^2-m_2^2)} \tag{4}$$

and $n = 4+\epsilon$. ([2] writes "The notations for the one-, two-, three-, and four-point functions have been taken over from Ref. [1]." - obviously they do not.)

`HEPMath`[4] and `FeynCalc`[5, 6] refer to `LoopTools`[7, 8]. [8, eq. (1.1)] and [9, eq. (2.6)],`QCDLoop`[10]:

$$T^N_{\mu_1\cdots\mu_P} = \frac{\mu^{4-D}}{i\pi^{D/2}r_\Gamma}\int d^D q\, \frac{q_{\mu_1}\cdots q_{\mu_P}}{\left[q^2-m_1^2\right]\left[(q+k_1)^2-m_2^2\right]\cdots\left[(q+k_{N-1})^2-m_N^2\right]} \tag{5}$$

$$r_\Gamma = \frac{\Gamma^2(1-\varepsilon)\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}, \quad D = 4-2\varepsilon$$

later in the code they use a different signature (to avoid any vector structure):

$$\begin{aligned} &A(m^2), B_0(p^2,m_1^2,m_2^2), C_0(p_1^2,p_2^2,(p_1+p_2)^2,m_1^2,m_2^2,m_3^2)\\ &D_0(p_1^2,p_2^2,p_3^2,p_4^2,(p_1+p_2)^2,(p_2+p_3)^2,m_1^2,m_2^2,m_3^2,m_4^2)\end{aligned} \tag{6}$$

[11]:

$$T^N_{\mu_1\cdots\mu_P}(p_1,\ldots,p_{N-1},m_0,\ldots,m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^D q \frac{q_{\mu_1}\cdots q_{\mu_P}}{L_0 L_1\cdots L_{N-1}} \tag{7}$$

$$L_0 = q^2 - m_0^2 + i\varepsilon \tag{8}$$

$$L_i = (q+p_i)^2 - m_i^2 + i\varepsilon\, i = 1,\ldots,N-1 \tag{9}$$

I will stick to the integrals of [3] as it is the most natural form, I think, and to the non-vector signature, if possible.

The transformation of the *analytic* results from the notation in [10] is given by

$$[10]\text{: } \frac{\mu^{4-n}}{i\pi^{n/2} r_\Gamma}\mathcal{I} \leftrightarrow [3]\text{: } \frac{\mu^{4-n}}{(2\pi)^n}\mathcal{I} \tag{10}$$

with $\mathcal{I}$ denonting the *raw* integral. We then need to solve (B=Bojak[3],E=Ellis[10]):

$$\begin{aligned}\Rightarrow iC_\epsilon &\left(\frac{a_2^B}{(n-4)^2} + \frac{a_1^B}{n-4} + a_0^B + O(n-4)\right) \\ &\stackrel{!}{=} \frac{\mu^{4-n}}{(2\pi)^n}\frac{i\pi^{n/2} r_\Gamma}{\mu^{4-n}}\left(\frac{a_2^E}{(n-4)^2} + \frac{a_1^E}{n-4} + a_0^E + O(n-4)\right)\end{aligned} \tag{11}$$

$$\Rightarrow a_2^B = a_2^E \tag{12}$$

$$a_1^B = a_1^E - \frac{1}{2}a_2^E \ln(m^2/\mu^2) \tag{13}$$

$$a_0^B = a_0^E - \frac{a_2^E}{8}\zeta(2) + \frac{a_2^E}{8}\ln^2(m^2/\mu^2) - \frac{a_1^E}{2}\ln(m^2/\mu^2) \tag{14}$$

To compare *numeric* results form `LoopTools` or `QCDLoop` one need to solve

$$\begin{aligned}\Rightarrow &\left(\frac{b_2^B}{(n-4)^2} + \frac{b_1^B}{n-4} + b_0^B + O(n-4)\right) \\ &\stackrel{!}{=} \frac{\mu^{4-n}}{(2\pi)^n}\frac{i\pi^{n/2} r_\Gamma}{\mu^{4-n}}\left(\frac{b_2^E}{(n-4)^2} + \frac{b_1^E}{n-4} + b_0^E + O(n-4)\right)\end{aligned} \tag{15}$$

$$\Rightarrow b_2^B = \frac{i}{16\pi^2} b_2^E \tag{16}$$

$$b_1^B = \frac{i}{16\pi^2}\left(b_1^E + \frac{b_2^E}{2}(\gamma_E - \ln(4\pi))\right) \tag{17}$$

$$b_0^B = \frac{i}{16\pi^2}\left(b_0^E + \frac{b_1^E}{2}(\gamma_E - \ln(4\pi)) + \frac{b_2^E}{8}\left((\gamma_E - \ln(4\pi))^2 - \zeta(2)\right)\right) \tag{18}$$

$$\tag{19}$$

1.2 Decomposition Labeling

[1, 3]:

$$B_\mu(p, m_1, m_2) = p_\mu B_1(p, m_1, m_2) \tag{20}$$

$$B_{\mu\nu} = p_\mu p_\nu B_{21} + g_{\mu\nu} B_{22} \tag{21}$$

$$C_\mu(p_1, p_2, m_1, m_2, m_3) = p_{1,\mu} C_{11} + p_{2,\mu} C_{12} \tag{22}$$

$$C_{\mu\nu} = p_{1,\mu}p_{1,\nu}C_{21} + p_{2,\mu}p_{2,\nu}C_{22} + (p_{1,\mu}p_{2,\nu} + p_{1,\nu}p_{2,\mu})C_{23} + g_{\mu\nu}C_{24} \tag{23}$$

The arguments of the functions are always inherited.

`HEPMath`, `FeynCalc`, `LoopTools`, [9]:

$$B_\mu(p, m_1, m_2) = p_\mu B_1(p, m_1, m_2) \tag{24}$$

$$B_{\mu\nu} = g_{\mu\nu} B_{00} + p_\mu p_\nu B_{11} \tag{25}$$

$$C_\mu(p_1, p_2, m_1, m_2, m_3) = p_{1,\mu} C_1 + p_{2,\mu} C_2 = \sum_{j=1}^{2} p_{j,\mu} C_j \tag{26}$$

$$C_{\mu\nu} = p_{1,\mu} p_{1,\nu} C_{11} + p_{2,\mu} p_{2,\nu} C_{22} + (p_{1,\mu} p_{2,\nu} + p_{1,\nu} p_{2,\mu}) C_{12} + g_{\mu\nu} C_{00} \tag{27}$$

$$= g_{\mu\nu} C_{00} + \sum_{j,k=1}^{2} p_{j,\mu} p_{k,\nu} C_{jk} \tag{28}$$

The arguments of the functions are always inherited.

I will stick to `HEPMath` as it is the more generic and extensible form, I think.

1.3 B Decomposition

define

$$f_1 = m_1^2 - m_0^2 - p^2 \tag{29}$$

then one finds easily

$$B_1(p^2, m_0^2, m_1^2) = \frac{1}{2p^2} \left(f_1 B_0(p^2, m_0^2, m_1^2) + A_0(m_0^2) - A_0(m_1^2) \right) \tag{30}$$

$$B_{00}(p^2, m_0^2, m_1^2) = \frac{1}{2(n-1)} \left(2m_0^2 B_0(p^2, m_0^2, m_1^2) + A_0(m_1^2) - f_1 B_1(p^2, m_0^2, m_1^2) \right) \tag{31}$$

$$B_{11}(p^2, m_0^2, m_1^2) = \frac{1}{2p^2} \left(f_1 B_0(p^2, m_0^2, m_1^2) + A_0(m_1^2) - 2B_{00}(p^2, m_0^2, m_1^2) \right) \tag{32}$$

in accordance with [3, 9].

Concering B_1 [1] and `LoopTools` use the following identity

$$A_0(m_0^2) - A_0(m_1^2) = (m_0^2 - m_1^2) B_0(0, m_0^2, m_1^2) \tag{33}$$

that might help away with

> In case m_1 and/or m_2 are very large the expression on the right-hand side of eq. (30) suffers very strong cancellations: the total is very much smaller than the individual terms. For this reason we have not used these algebraic relations, except to rewrite self-energy diagrams as much as possible in a form most suitable for numerical evaluation. ([1, below eq. D.6])

To compare the other results to [1] and `LoopTools` one has to use the *strict* $n \to 4$ limit and the following identities[11]:

$$(n-4)B_{00}(p^2, m_0^2, m_1^2) = \frac{1}{6}(p^2 - 3m_0^2 - 3m_1^2) \tag{34}$$

$$(n-4)B_{11}(p^2, m_0^2, m_1^2) = -\frac{2}{3} \tag{35}$$

2 Scalar Integrals

We focus on:

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) \tag{36}$$

$$k_1^2 = 0 \quad p_1^2 = p_2^2 = m^2 \quad (p_1+p_2)^2 = s \quad (p_2-q)^2 = t \quad (p_1-q)^2 = u \tag{37}$$

define some shortcuts

$$0 \le \rho = \frac{4m^2}{s} \le 1 \qquad 0 \le \beta = \sqrt{1-\rho} \le 1 \qquad 0 \le \chi = \frac{1-\beta}{1+\beta} \le 1 \tag{38}$$

$$\rho_q = \frac{4m^2}{q^2} \le 0 \qquad 1 \le \beta_q = \sqrt{1-\rho_q} \qquad 0 \le \chi_q = -\frac{1-\beta_q}{1+\beta_q} \le 1 \tag{39}$$

2.1 One-Point Function A_0

[11]:

$$A_0(m^2) = -\frac{i}{16\pi^2} m^2 \left(\frac{m^2}{4\pi\mu^2}\right)^{(n-4)/2} \Gamma(1-n/2) \tag{40}$$

$$= \frac{im^2}{16\pi^2}\Big(\Delta - \log(m^2/\mu^2) + 1\Big) + O(n-4) \tag{41}$$

$$= iC_\epsilon m^2 \left(-\frac{2}{\epsilon} + 1\right) + O(n-4) \tag{42}$$

$$\Delta = \frac{2}{4-n} - \gamma_E + \log(4\pi) \tag{43}$$

$$C_\epsilon = \frac{1}{16\pi^2} \exp\left(\left(\gamma_E - \log(4\pi) + \log\left(m^2/\mu^2\right)\right)\frac{\epsilon}{2}\right) \tag{44}$$

this is *up to order* $O(n-4)$ in accordance with [3][12], but NOT beyond - see also [3, eq. (A.12)]. So we can treat C_ϵ and Δ as equal.

2.2 Two-Point Function B_0

In [11, eq. (4.23)] is a generic function given and we end up with

$$B_0(s, m^2, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 + \beta \log(\chi) \right) \tag{45}$$

$$B_0(q^2, m^2, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 + \beta_q \log(\chi_q) \right) \tag{46}$$

$$B_0(0, m^2, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} \right) \tag{47}$$

$$B_0(m^2, 0, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 \right) \tag{48}$$

$$B_0(t, 0, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 - \frac{t-m^2}{t} \ln\left(-\frac{t-m^2}{m^2} \right) \right) \tag{49}$$

focussing on imaginary part *only*; this in accordance with [3][12].

2.3 Three-Point Function C_0

Again, in [11, eq. (4.26)] is a generic function given.

First, we compute $C_0(s, q^2, 0, m^2, m^2, m^2)$ and by taking the limit $k_1^2 \to 0$ (or equivalenty $s_4 \to 0$) we end up with:

$$\begin{aligned} C_0(s, q^2, 0, m^2, m^2, m^2) = \frac{i}{16\pi^2} \cdot \frac{1}{s-q^2} \Bigg(&\mathrm{Li}_2\left(\frac{2}{1+\beta_q}\right) + \mathrm{Li}_2\left(\frac{2}{1-\beta_q}\right) \\ &- \mathrm{Li}_2\left(\frac{2}{1+\beta}\right) - \mathrm{Li}_2\left(\frac{2}{1-\beta}\right) \Bigg) \end{aligned} \tag{50}$$

with [13] we find:

$$\mathrm{Li}_2\left(\frac{2}{1+b}\right) + \mathrm{Li}_2\left(\frac{2}{1-b}\right) = 3\zeta(2) + \frac{1}{2}\ln^2\left(\frac{1-b}{1+b}\right) - \ln\left(\frac{1-b}{1+b}\right)\ln\left(-\frac{1-b}{1+b}\right) \tag{51}$$

and if we focus on real part *only*, we find:

$$\mathrm{Li}_2\left(\frac{2}{1+\beta}\right) + \mathrm{Li}_2\left(\frac{2}{1-\beta}\right) = 3\zeta(2) - \frac{1}{2}\ln^2(\chi) \tag{52}$$

$$\mathrm{Li}_2\left(\frac{2}{1+\beta_q}\right) + \mathrm{Li}_2\left(\frac{2}{1-\beta_q}\right) = -\frac{1}{2}\ln^2(\chi_q) \tag{53}$$

Additionally, we find

$$\lim_{q^2\to 0}\left[\mathrm{Li}_2\left(\frac{2}{1+\beta_q}\right) + \mathrm{Li}_2\left(\frac{2}{1-\beta_q}\right)\right] = 0 \tag{54}$$

So we get:

$$C_0(s,q^2,0,m^2,m^2,m^2) = iC_\epsilon \frac{1}{s-q^2}\left(\frac{1}{2}\ln^2(\chi) - \frac{1}{2}\ln^2(\chi_q) - 3\zeta(2)\right) \tag{55}$$

$$C_0(s,0,0,m^2,m^2,m^2) = iC_\epsilon \frac{1}{s}\left(\frac{1}{2}\ln^2(\chi) - 3\zeta(2)\right) \tag{56}$$

in accordance with [3][12][14]. These results can also be obtained by the methods described in [3, chap. 3].

Next, we compute $C_0(m^2,0,t,0,m^2,m^2)$ again by taking the limit $k_1^2 \to 0$ we end up with:

$$\begin{aligned} C_0(m^2,0,t,0,m^2,m^2) = \frac{i}{16\pi^2}\cdot\frac{1}{t-m^2}\Bigg(&2\,\mathrm{Li}_2(2) + \mathrm{Li}_2(m^2/t) - \frac{\pi^2}{6} \\ &- \mathrm{Li}_2((t+m^2)/m^2) - \mathrm{Li}_2((m^2+t)/t)\Bigg) \end{aligned} \tag{57}$$

Using [13] and focussing on real part, we find

$$\mathrm{Li}_2(2) = \frac{\pi^2}{4} - i\pi\ln(2) \tag{58}$$

$$2\,\mathrm{Li}_2(2) + \mathrm{Li}_2(1/z) - \frac{\pi^2}{6} - \mathrm{Li}_2(1+z) - \mathrm{Li}_2(1+1/z) = \frac{\pi^2}{6} - \mathrm{Li}_2(z) \tag{59}$$

So we get:

$$C_0(m^2,0,t,0,m^2,m^2) = iC_\epsilon \frac{1}{t-m^2}\left(\zeta(2) - \mathrm{Li}_2(t/m^2)\right) \tag{60}$$

in accordance with [3][12].

To compute $C_0(m^2,s,m^2,0,m^2,m^2)$ we use [3] and find

$$\begin{aligned} C_0(m^2,s,m^2,0,m^2,m^2) = \frac{iC_\epsilon}{s\beta}\Bigg(&-\frac{2}{\epsilon}\ln(\chi) - \frac{\pi^2}{2} + \frac{1}{2}\ln^2(\chi) - \ln(\chi)\ln(1-\chi) \\ &- \mathrm{Li}_2(1/(1-\chi)) + \mathrm{Li}_2(\chi/(\chi-1))) \end{aligned} \tag{61}$$

Using [13] and focussing on real part, we find

$$-\mathrm{Li}_2(1/(1-\chi)) + \mathrm{Li}_2(\chi/(\chi-1)) = -2\,\mathrm{Li}_2(\chi) - \ln(\chi)\ln(1-\chi) - \frac{\pi^2}{6} \tag{62}$$

So we get:

$$\begin{aligned} C_0(m^2,s,m^2,0,m^2,m^2) = iC_\epsilon \frac{1}{s\beta}\Bigg(&-\frac{2}{\epsilon}\ln(\chi) - 2\ln(\chi)\ln(1-\chi) - 2\,\mathrm{Li}_2(\chi) \\ &+\frac{1}{2}\ln^2(\chi) - 4\zeta(2)\Bigg) \end{aligned} \tag{63}$$

in accordance with [3][12].

To compute $C_0(t,m^2,q^2,0,m^2,m^2)$ we use [11] and find immediatelty:

$$
\begin{aligned}
C_0(t,m^2,q^2,0,m^2,m^2) = \frac{iC_\epsilon}{\alpha}\Bigg[&-\zeta(2) + 2\,\mathrm{Li}_2\left(\frac{t_1+\alpha}{t_1}\right) + \mathrm{Li}_2\left(\frac{q^2-t-m^2+\alpha}{q^2-t-m^2-\alpha}\right) \\
&\mathrm{Li}_2\left(\frac{t_1-q^2\beta_q^2+\alpha}{t_1-q^2\beta_q^2-\beta_q\alpha}\right) - \mathrm{Li}_2\left(\frac{t_1-q^2\beta_q^2-\alpha}{t_1-q^2\beta_q^2+\beta_q\alpha}\right) \\
&\mathrm{Li}_2\left(\frac{t_1-q^2\beta_q^2+\alpha}{t_1+q^2\beta_q^2-\beta_q\alpha}\right) - \mathrm{Li}_2\left(\frac{t_1-q^2\beta_q^2-\alpha}{t_1+q^2\beta_q^2+\beta_q\alpha}\right) \\
&-\mathrm{Li}_2\left(\frac{t_1(q^2-t-m^2-\alpha)-2m^2\alpha}{t_1(q^2-t-m^2+\alpha)}\right) \\
&-\mathrm{Li}_2\left(\frac{t_1(q^2-t-m^2-\alpha)-2m^2\alpha}{t_1(q^2-t-m^2-\alpha)}\right)\Bigg]
\end{aligned} \tag{64}
$$

with $\alpha = \kappa(t,q^2,m^2)$ and the Källén function (as defined in [11, eq. (4.27)])

$$
\kappa(x,y,z) = \sqrt{x^2+y^2+z^2-2(xy+xz+yx)} \tag{65}
$$

This is in accordance with [14, eq. (A.8)](Note the typo there!).

Additionally, we find

$$
\lim_{q^2\to 0} C_0(t,m^2,q^2,0,m^2,m^2) = C_0(t,m^2,0,0,m^2,m^2) = C_0(m^2,0,t,0,m^2,m^2) \tag{66}
$$

2.4 Four-Point Function D_0

To compute $D_0(m^2,0,q^2,m^2,t,s,0,m^2,m^2,m^2)$ we follow Ingos way[3] of computing his $D_0(p_1,-k_1,-k_2,0,m,m,m) = D_0(m^2,0,0,m^2,t,s,0,m^2,m^2,m^2)$ and find

$$
\tilde{t} = -\frac{t_1}{m^2} \tag{67}
$$

$$
K = \frac{x}{\rho\rho_q}\left[4x(-1+y)yz\rho + yz\rho\rho_q\tilde{t} + x(-4(-1+y)y(-1+z)+\rho-\tilde{t}yz\rho)\rho_q\right] \tag{68}
$$

$$
I_{xy} = \frac{2x^{\epsilon/2}\rho\rho_q^{2-\epsilon/2}\left[\tilde{t}y\rho_q + x(\rho_q+y(4(y-1)-\tilde{t}\rho_q))\right]^{-1+\epsilon/2}}{(-2+\epsilon)\left[4x(-1+y)\rho+\tilde{t}\rho\rho_q-x(4(y-1)+\tilde{t}\rho)\rho_q\right]} \tag{69}
$$

$$
II_{xy} = -\frac{2x^{-1+\epsilon}\rho^{2-\epsilon/2}\rho_q\left[4(-1+y)y+\rho\right]^{-1+\epsilon/2}}{(-2+\epsilon)\left[4x(-1+y)\rho+\tilde{t}\rho\rho_q-x(4(y-1)+\tilde{t}\rho)\rho_q\right]} \tag{70}
$$

"The integration of I_{xy} does not diverge and one easily gets upon setting $\epsilon \to 0$"

$$I = \frac{m^4}{st_1\beta}\left[\ln^2(\chi) + 4\,\mathrm{Li}_2(-\chi) + \frac{\pi^2}{3} + 2\ln(\chi_q)\ln\left(\frac{\beta_q+\beta}{\beta_q-\beta}\right) - 2\ln(\chi)\ln(1-q^2/s)\right.$$
$$\left. +2\,\mathrm{Li}_2\left(\frac{\beta_q-1}{\beta_q-\beta}\right) - 2\,\mathrm{Li}_2\left(\frac{\beta_q+1}{\beta_q-\beta}\right) + 2\,\mathrm{Li}_2\left(\frac{\beta_q+1}{\beta_q+\beta}\right) - 2\,\mathrm{Li}_2\left(\frac{\beta_q-1}{\beta_q+\beta}\right)\right] \tag{71}$$

$$= \frac{m^4}{st_1\beta}\left[\ln^2(\chi) + 4\,\mathrm{Li}_2(-\chi) + \frac{\pi^2}{3} + \ln\left(\frac{\beta_q^2-\beta^2}{(\beta_q-1)^2}\right)\ln\left(\frac{\beta_q-\beta}{\beta_q+\beta}\right) - 2\ln(\chi)\ln(1-q^2/s)\right.$$
$$\left. +2\,\mathrm{Li}_2\left(\frac{\beta_q-1}{\beta_q-\beta}\right) + 2\,\mathrm{Li}_2\left(\frac{\beta_q-\beta}{\beta_q+1}\right) - 2\,\mathrm{Li}_2\left(\frac{\beta_q+\beta}{\beta_q+1}\right) - 2\,\mathrm{Li}_2\left(\frac{\beta_q-1}{\beta_q+\beta}\right)\right] \tag{72}$$

with

$$\lim_{q^2\to 0}\left[\ln\left(\frac{\beta_q^2-\beta^2}{(\beta_q-1)^2}\right)\ln\left(\frac{\beta_q-\beta}{\beta_q+\beta}\right) - 2\ln(\chi)\ln(1-q^2/s)\right.$$
$$\left. +2\,\mathrm{Li}_2\left(\frac{\beta_q-1}{\beta_q-\beta}\right) + 2\,\mathrm{Li}_2\left(\frac{\beta_q-\beta}{\beta_q+1}\right) - 2\,\mathrm{Li}_2\left(\frac{\beta_q+\beta}{\beta_q+1}\right) - 2\,\mathrm{Li}_2\left(\frac{\beta_q-1}{\beta_q+\beta}\right)\right] = 0 \tag{73}$$

"Integrating II_{xy} over x gives"

$$II_y = -\frac{2}{\tilde{t}(-2+\epsilon)\epsilon}\left(\frac{\rho-4y(1-y)}{\rho}\right)^{-1+\epsilon/2} {}_2F_1\left(1,\epsilon;1+\epsilon;1-\frac{4(1-y)(\rho_q-\rho)}{\tilde{t}\rho\rho_q}\right) \tag{74}$$

"The integration over y does not give an additional pole, so we can expand to $O(1)$ using ([3, eq. B.5]) and then integrate to obtain"

$$II = -\frac{m^4}{\beta st_1}\left(\frac{2\ln(\chi)}{\epsilon} + \ln(\chi)\left(1+2\ln(\beta\tilde{t}) + \ln(\chi) - 2\ln\left(1-q^2/s\right)\right) + \mathrm{Li}_2(\chi^2) + \frac{5\pi^2}{6}\right) \tag{75}$$

with

$$\lim_{q^2\to 0}\ln(1-q^2/s) = 0 \tag{76}$$

"The final result is then"

$$D_0(m^2,0,q^2,m^2,t,s,0,m^2,m^2,m^2)$$
$$= \frac{iC_\epsilon}{\beta st_1}\left[-\frac{2\ln(\chi)}{\epsilon} - 2\ln(\chi)\ln(\beta\tilde{t}) + 2\,\mathrm{Li}_2(-\chi) - 2\,\mathrm{Li}_2(\chi) - 3\zeta(2)\right.$$
$$+\ln\left(\frac{\beta_q^2-\beta^2}{(\beta_q-1)^2}\right)\ln\left(\frac{\beta_q-\beta}{\beta_q+\beta}\right) - 2\ln(\chi)\ln(1-q^2/s)$$
$$\left. +2\,\mathrm{Li}_2\left(\frac{\beta_q-1}{\beta_q-\beta}\right) + 2\,\mathrm{Li}_2\left(\frac{\beta_q-\beta}{\beta_q+1}\right) - 2\,\mathrm{Li}_2\left(\frac{\beta_q+\beta}{\beta_q+1}\right) - 2\,\mathrm{Li}_2\left(\frac{\beta_q-1}{\beta_q+\beta}\right)\right] \tag{77}$$

This is NOT in accordance with [14, eq. (A.3)] - but I suspect a bunch of typos there.

We get the match to [3] and [12] by using

$$\lim_{q^2\to 0}\Bigg[\ln\left(\frac{\beta_q^2-\beta^2}{(\beta_q-1)^2}\right)\ln\left(\frac{\beta_q-\beta}{\beta_q+\beta}\right)-2\ln(\chi)\ln(1-q^2/s)$$
$$2\,\mathrm{Li}_2\left(\frac{\beta_q-1}{\beta_q-\beta}\right)+2\,\mathrm{Li}_2\left(\frac{\beta_q-\beta}{\beta_q+1}\right)-2\,\mathrm{Li}_2\left(\frac{\beta_q+\beta}{\beta_q+1}\right)-2\,\mathrm{Li}_2\left(\frac{\beta_q-1}{\beta_q+\beta}\right)\Bigg]\quad=0 \quad (78)$$

FiXme Error: fix

Nevertheless this is probably wrong - because we find with [10, Box 16]:

$$\begin{aligned}&D_0(m^2,0,q^2,m^2,t,s,0,m^2,m^2,m^2)\\&=\frac{iC_\epsilon}{\beta s t_1}\Bigg[-\frac{2\ln(\chi)}{\epsilon}-2\ln(\chi)\ln(\tilde{t})+\mathrm{Li}_2(1-\chi^2)-4\zeta(2)+\ln^2(\chi_q)+2\,\mathrm{Li}_2(-\chi\chi_q)\\&\qquad+2\,\mathrm{Li}_2(-\chi/\chi_q)+2\ln(\chi\chi_q)\ln(1+\chi\chi_q)+2\ln(\chi/\chi_q)\ln(1+\chi/\chi_q)\Bigg]\end{aligned}\tag{79}$$

We get the match to [3] and [12] by using

$$\begin{aligned}&\lim_{q^2\to 0}\Big[\mathrm{Li}_2(1-\chi^2)-4\zeta(2)+\ln^2(\chi_q)+2\,\mathrm{Li}_2(-\chi\chi_q)+2\,\mathrm{Li}_2(-\chi/\chi_q)\\&\qquad+2\ln(\chi\chi_q)\ln(1+\chi\chi_q)+2\ln(\chi/\chi_q)\ln(1+\chi/\chi_q)\Big]\\&=-2\ln(\chi)\ln(\beta)-3\zeta(2)+2\,\mathrm{Li}_2(2,-\chi)-2\,\mathrm{Li}_2(2,\chi)\end{aligned}\tag{80}$$

To compute $D_0(0,m^2,q^2,m^2,t,u,0,0,m^2,m^2)$ I neither suceeded with [3] nor [15], but one can use [10, Box 11] (transformation see sec. 1.1) So we get

$$\begin{aligned}D_0(0,m^2,q^2,m^2,t,u,0,0,m^2,m^2)=\frac{iC_\epsilon}{t_1u_1}\Bigg(&\frac{4}{\epsilon^2}+\frac{2}{\epsilon}\Big(\ln(-t_1/m^2)+\ln(-u_1/m^2)\Big)\\&+2\ln(-t_1/m^2)\ln(-u_1/m^2)-\frac{7}{2}\zeta(2)-\ln^2(\chi_q)\Bigg)\end{aligned}\tag{81}$$

This is in accordance with [14, eq. (A.4)] using [13] (as above):

$$2\,\mathrm{Li}_2\left(\frac{q^2(1+\beta_q)}{2m^2}\right)+2\,\mathrm{Li}_2\left(\frac{q^2(1-\beta_q)}{2m^2}\right)=2\,\mathrm{Li}_2\left(\frac{2}{1-\beta_q}\right)+2\,\mathrm{Li}_2\left(\frac{2}{1-\beta_q}\right)\tag{82}$$
$$=-\ln^2(\chi_q)\tag{83}$$

(The question, why they use a complicated dilogarithm remains open ...)

We get the match to [3] and [12] by using

$$\lim_{q^2\to 0}\ln^2(\chi_q)=0\tag{84}$$

to find

$$D_0(0, m^2, 0, m^2, t, u, 0, 0, m^2, m^2) = \frac{iC_\epsilon}{t_1 u_1} \left(\frac{4}{\epsilon^2} + \frac{2}{\epsilon} \Big(\ln(-t_1/m^2) + \ln(-u_1/m^2) \Big) \right.$$
$$\left. + 2 \ln(-t_1/m^2) \ln(-u_1/m^2) - \frac{7}{2}\zeta(2) \right) \qquad (85)$$

A References

[1] G. Passarino and M. J. G. Veltman, "One Loop Corrections for e+ e- Annihilation Into mu+ mu- in the Weinberg Model," Nucl. Phys. **B160** (1979) 151.

[2] Beenakker, W. and Kuijf, H. and van Neerven, W. L. and Smith, J., "Qcd corrections to heavy-quark production in $p\overline{p}$ collisions," Phys. Rev. D **40** (Jul, 1989) 54–82. `http://link.aps.org/doi/10.1103/PhysRevD.40.54`.

[3] I. Bojak, NLO QCD corrections to the polarized photoproduction and hadroproduction of heavy quarks. PhD thesis, Dortmund U., 2000. `arXiv:hep-ph/0005120 [hep-ph]`.

[4] M. Wiebusch, "HEPMath 1.4: A Mathematica Package for Semi-Automatic Computations in High Energy Physics," Computer Physics Communications **195** (Oct., 2015) 172–190. `http://arxiv.org/abs/1412.6102`. arXiv: 1412.6102.

[5] R. Mertig, M. Bohm, and A. Denner, "FEYN CALC: Computer algebraic calculation of Feynman amplitudes," Comput. Phys. Commun. **64** (1991) 345–359.

[6] V. Shtabovenko, R. Mertig, and F. Orellana, "New Developments in FeynCalc 9.0," `arXiv:1601.01167 [hep-ph]`.

[7] T. Hahn and M. Perez-Victoria, "Automatized one loop calculations in four-dimensions and D-dimensions," Comput. Phys. Commun. **118** (1999) 153–165, `arXiv:hep-ph/9807565 [hep-ph]`.

[8] T. Hahn, "LoopTools 2.12 User's Guide." `http://www.feynarts.de/looptools/`, 2014.

[9] R. K. Ellis, Z. Kunszt, K. Melnikov, and G. Zanderighi, "One-loop calculations in quantum field theory: from Feynman diagrams to unitarity cuts," Phys. Rept. **518** (2012) 141–250, `arXiv:1105.4319 [hep-ph]`.

[10] R. K. Ellis and G. Zanderighi, "Scalar one-loop integrals for QCD," JHEP **02** (2008) 002, `arXiv:0712.1851 [hep-ph]`.

[11] A. Denner and S. Dittmaier, "Reduction schemes for one-loop tensor integrals," Nucl. Phys. **B734** (2006) 62–115, `arXiv:hep-ph/0509141 [hep-ph]`.

[12] W. Beenakker, H. Kuijf, W. L. van Neerven, and J. Smith, "QCD Corrections to Heavy Quark Production in p anti-p Collisions," Phys. Rev. **D40** (1989) 54–82.

[13] D. Zagier, Frontiers in Number Theory, Physics, and Geometry II: On Conformal Field Theories, Discrete Groups and Renormalization, ch. The Dilogarithm Function, pp. 3–65. Springer Berlin Heidelberg, Berlin, Heidelberg, 2007. `http://dx.doi.org/10.1007/978-3-540-30308-4_1`.

[14] E. Laenen, S. Riemersma, J. Smith, and W. van Neerven, "Complete $O(\alpha_S)$ corrections to heavy-flavour structure functions in electroproduction," Nuclear Physics B **392** no. 1, (1993) 162 – 228. `http://www.sciencedirect.com/science/article/pii/055032139390201Y`.

[15] A. Denner, U. Nierste, and R. Scharf, "A Compact expression for the scalar one loop four point function," Nucl. Phys. **B367** (1991) 637–656.

List of Corrections