1 Feynman Rules

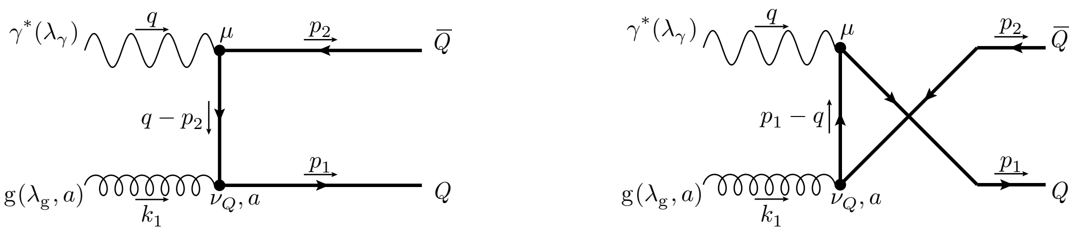
following [1]

FiXme Error: TODO

To perform the calculation of Dirac traces in n dimensions use HEPMath[2] or TRACER[3].

2 Leading Order: $O(\alpha\alpha_s)$

diagramatic:



(a) $i\,\mathcal{M}^{(LO)}_{1,\mu}$ (b) $i\,\mathcal{M}^{(LO)}_{2,\mu}$

Figure 1: LO contributions

formula:

$$i\,\mathcal{M}^{(LO)}_{1,\mu} = \bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon^{(\lambda_\mathrm{g})}_{\nu_Q}(k_1) \tag{1}$$

$$i\,\mathcal{M}^{(LO)}_{2,\mu} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{t_1}(igT_a\gamma^{\nu_Q})v(p_2)\varepsilon^{(\lambda_\mathrm{g})}_{\nu_Q}(k_1) \tag{2}$$

color space:

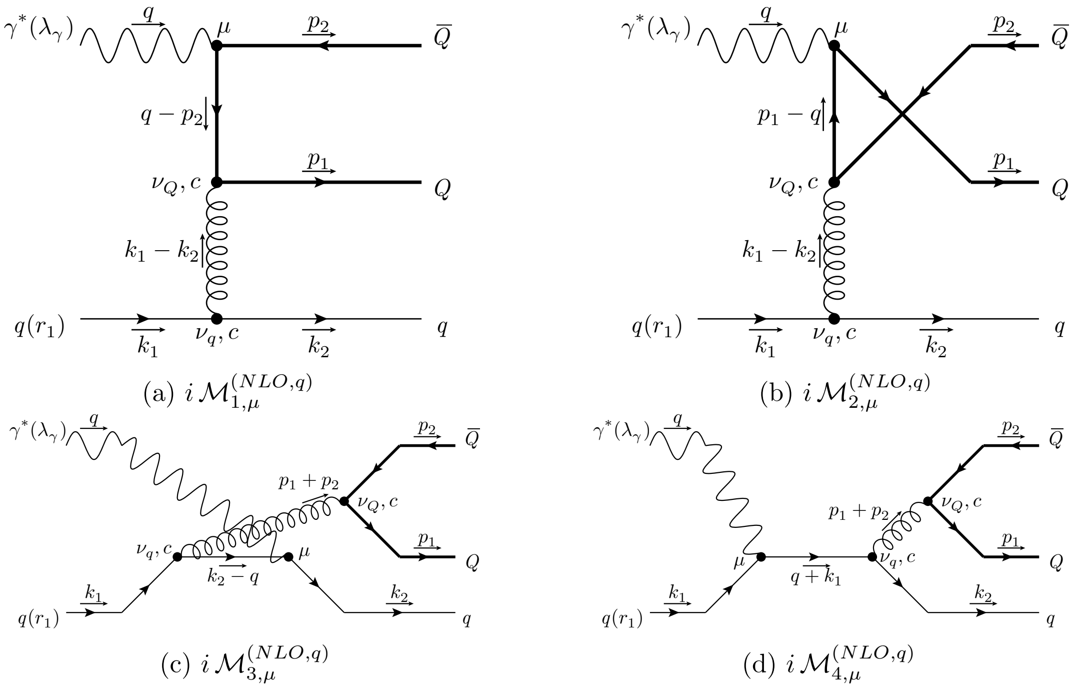
$$\left|\mathcal{M}^{(LO)}_{1,\mu} + \mathcal{M}^{(LO)}_{2,\mu}\right|^2 \sim \mathrm{tr}(T_aT_a) = N_cC_F \tag{3}$$

3 Next-to-leading Order: $O(\alpha\alpha_S^2)$

3.1 Light Quark Contributions

$$\gamma^*(q) + q(k_1) \to \overline{Q}(p_2) + Q(p_1) + q(k_2) \tag{4}$$

diagramatic:



(a) $i\,\mathcal{M}_{1,\mu}^{(NLO,q)}$ (b) $i\,\mathcal{M}_{2,\mu}^{(NLO,q)}$

(c) $i\,\mathcal{M}_{3,\mu}^{(NLO,q)}$ (d) $i\,\mathcal{M}_{4,\mu}^{(NLO,q)}$

Figure 2: NLO contributions by light quarks

formula:

$$
\begin{aligned}
i\,\mathcal{M}_{1,\mu}^{(NLO,q)} &= \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q} - \not{p}_2 + m)}{u_1}(-iee_H\gamma_\mu)v_Q(p_2)\cdot \\
&\qquad \frac{-ig_{\nu_Q,\nu_q}}{t'} \cdot \bar{u}_q(k_2)(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) &(5)\\
i\,\mathcal{M}_{2,\mu}^{(NLO,q)} &= \bar{u}_Q(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{u_7}(igT_c\gamma^{\nu_Q})v_Q(p_2)\cdot \\
&\qquad \frac{-ig_{\nu_Q,\nu_q}}{t'} \cdot \bar{u}_q(k_2)(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) &(6)\\
i\,\mathcal{M}_{3,\mu}^{(NLO,q)} &= \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})v_Q(p_2)\cdot\frac{-ig_{\nu_Q,\nu_q}}{s_5}\cdot \\
&\qquad \bar{u}_q(k_2)(-iee_L\gamma_\mu)\frac{i(\not{k}_2 - \not{q})}{u'}(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) &(7)\\
i\,\mathcal{M}_{4,\mu}^{(NLO,q)} &= \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})v_Q(p_2)\cdot\frac{-ig_{\nu_Q,\nu_q}}{s_5}\cdot \\
&\qquad \bar{u}_q(k_2)(igT_c\gamma^{\nu_q})\frac{i(\not{k}_1 + \not{q})}{s}(-iee_L\gamma_\mu)u_q^{(r_1)}(k_1) &(8)
\end{aligned}
$$

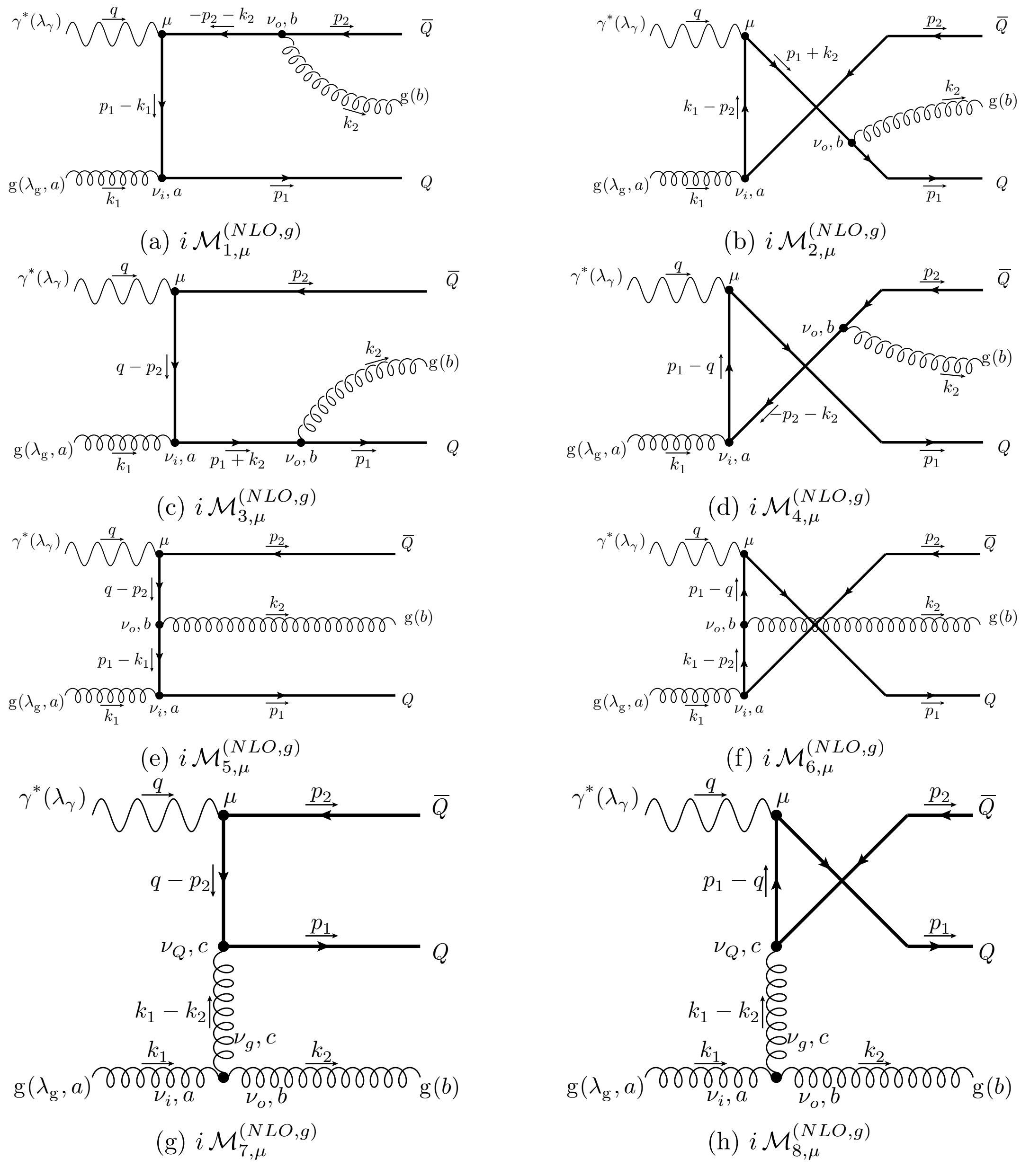
color space:

$$
\left|\mathcal{M}_{1,\mu}^{(NLO,q)} + \mathcal{M}_{2,\mu}^{(NLO,q)} + \mathcal{M}_{3,\mu}^{(NLO,q)} + \mathcal{M}_{4,\mu}^{(NLO,q)}\right|^2 \sim \mathrm{tr}(T_cT_d)\,\mathrm{tr}(T_cT_d) = \frac{1}{2}N_cC_F \quad (9)
$$

3.2 Gluon Bremsstrahlung

$$\gamma^*(q) + \mathrm{g}(k_1) \rightarrow \overline{Q}(p_2) + Q(p_1) + \mathrm{g}(k_2) \tag{10}$$

diagramatic:



(a) $i\,\mathcal{M}_{1,\mu}^{(NLO,g)}$ (b) $i\,\mathcal{M}_{2,\mu}^{(NLO,g)}$

(c) $i\,\mathcal{M}_{3,\mu}^{(NLO,g)}$ (d) $i\,\mathcal{M}_{4,\mu}^{(NLO,g)}$

(e) $i\,\mathcal{M}_{5,\mu}^{(NLO,g)}$ (f) $i\,\mathcal{M}_{6,\mu}^{(NLO,g)}$

(g) $i\,\mathcal{M}_{7,\mu}^{(NLO,g)}$ (h) $i\,\mathcal{M}_{8,\mu}^{(NLO,g)}$

Figure 3: NLO contributions by gluon bremsstrahlung

formula:

$$
\begin{aligned}
i\,\mathcal{M}_{1,\mu}^{(NLO,g)} &= \bar{u}(p_1)(igT_a\gamma^{\nu_i})\frac{i(\not{p}_1-\not{k}_1+m)}{u_6}(-iee_H\gamma_\mu)\cdot \\
&\qquad \frac{i(-\not{p}_2-\not{k}_2+m)}{s_3}(igT_b\gamma^{\nu_o})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \qquad (11)\\
i\,\mathcal{M}_{2,\mu}^{(NLO,g)} &= \bar{u}(p_1)(igT_b\gamma^{\nu_o})\frac{i(\not{p}_1+\not{k}_2+m)}{s_4}(-iee_H\gamma_\mu)\cdot \\
&\qquad \frac{i(\not{k}_1-\not{p}_2+m)}{t_1}(igT_a\gamma^{\nu_i})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \qquad (12)\\
i\,\mathcal{M}_{3,\mu}^{(NLO,g)} &= \bar{u}(p_1)(igT_b\gamma^{\nu_o})\frac{i(\not{p}_1+\not{k}_2+m)}{s_4}(igT_a\gamma^{\nu_i})\cdot \\
&\qquad \frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \qquad (13)\\
i\,\mathcal{M}_{4,\mu}^{(NLO,g)} &= \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_a\gamma^{\nu_i})\cdot \\
&\qquad \frac{i(-\not{p}_2-\not{k}_2+m)}{s_3}(igT_b\gamma^{\nu_o})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \qquad (14)\\
i\,\mathcal{M}_{5,\mu}^{(NLO,g)} &= \bar{u}(p_1)(igT_a\gamma^{\nu_i})\frac{i(\not{p}_1-\not{k}_1+m)}{u_6}(igT_b\gamma^{\nu_o})\cdot \\
&\qquad \frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \qquad (15)\\
i\,\mathcal{M}_{6,\mu}^{(NLO,g)} &= \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_b\gamma^{\nu_o})\cdot \\
&\qquad \frac{i(\not{k}_1-\not{p}_2+m)}{t_1}(igT_a\gamma^{\nu_i})v(p_2)\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2) \qquad (16)\\
i\,\mathcal{M}_{7,\mu}^{(NLO,g)} &= \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_g}}{t'}\cdot\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2)\cdot \\
&\qquad \Big(gf^{acb}\left(g^{\nu_o,\nu_i}(k_1+k_2)^{\nu_g}+g^{\nu_i,\nu_g}(k_2-2k_1)^{\nu_o}+g^{\nu_g,\nu_o}(k_1-2k_2)^{\nu_i}\right)\Big) \qquad (17)\\
i\,\mathcal{M}_{8,\mu}^{(NLO,g)} &= \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_g}}{t'}\cdot\varepsilon_{\nu_i}^{(\lambda_g)}(k_1)\varepsilon_{\nu_o}(k_2)\cdot \\
&\qquad \Big(gf^{acb}\left(g^{\nu_o,\nu_i}(k_1+k_2)^{\nu_g}+g^{\nu_i,\nu_g}(k_2-2k_1)^{\nu_o}+g^{\nu_g,\nu_o}(k_1-2k_2)^{\nu_i}\right)\Big) \qquad (18)
\end{aligned}
$$

color space:

$$
\begin{aligned}
&\sum_{j=1}^{6}\left|\mathcal{M}_{j,\mu}^{(NLO,g)}\right|^2+\mathcal{M}_{1,\mu}^{(NLO,g)}\left(\mathcal{M}_{4,\mu'}^{(NLO,g)}+\mathcal{M}_{5,\mu'}^{(NLO,g)}\right)^*+\mathcal{M}_{3,\mu}^{(NLO,g)}\left(\mathcal{M}_{6,\mu'}^{(NLO,g)}\right)^*+\\
&\mathcal{M}_{2,\mu}^{(NLO,g)}\left(\mathcal{M}_{3,\mu'}^{(NLO,g)}+\mathcal{M}_{6,\mu'}^{(NLO,g)}\right)^*+\mathcal{M}_{4,\mu}^{(NLO,g)}\left(\mathcal{M}_{5,\mu'}^{(NLO,g)}\right)^*\\
&\sim \mathrm{tr}(T_aT_aT_bT_b)=N_CC_F^2 \qquad (19)\\
&\mathcal{M}_{1,\mu}^{(NLO,g)}\left(\mathcal{M}_{2,\mu'}^{(NLO,g)}+\mathcal{M}_{3,\mu'}^{(NLO,g)}+\mathcal{M}_{6,\mu'}^{(NLO,g)}\right)^*+\\
&\left(\mathcal{M}_{2,\mu}^{(NLO,g)}+\mathcal{M}_{3,\mu}^{(NLO,g)}\right)\left(\mathcal{M}_{4,\mu'}^{(NLO,g)}+\mathcal{M}_{5,\mu'}^{(NLO,g)}\right)^*+\\
&\left(\mathcal{M}_{4,\mu}^{(NLO,g)}+\mathcal{M}_{5,\mu}^{(NLO,g)}\right)\left(\mathcal{M}_{6,\mu'}^{(NLO,g)}\right)^*\\
&\sim \mathrm{tr}(T_aT_bT_aT_b)=N_CC_F\left(C_F-\frac{C_A}{2}\right) \qquad (20)\\
&\left(\mathcal{M}_{2,\mu}^{(NLO,g)}+\mathcal{M}_{3,\mu}^{(NLO,g)}+\mathcal{M}_{6,\mu}^{(NLO,g)}\right)\left(\mathcal{M}_{7,\mu'}^{(NLO,g)}+\mathcal{M}_{8,\mu'}^{(NLO,g)}\right)^*\\
&\sim -if_{bda}\,\mathrm{tr}(T_aT_bT_d)=\frac{1}{2}N_CC_FC_A \qquad (21)\\
&\left(\mathcal{M}_{1,\mu}^{(NLO,g)}+\mathcal{M}_{4,\mu}^{(NLO,g)}+\mathcal{M}_{5,\mu}^{(NLO,g)}\right)\left(\mathcal{M}_{7,\mu'}^{(NLO,g)}+\mathcal{M}_{8,\mu'}^{(NLO,g)}\right)^*\\
&\sim -if_{bda}\,\mathrm{tr}(T_bT_aT_d)=if_{bda}\,\mathrm{tr}(T_aT_bT_d)=-\frac{1}{2}N_CC_FC_A \qquad (22)\\
&\left|\mathcal{M}_{7,\mu}^{(NLO,g)}+\mathcal{M}_{8,\mu}^{(NLO,g)}\right|^2\\
&\sim f_{acb}f_{adb}\,\mathrm{tr}(T_cT_d)=N_CC_FC_A \qquad (23)
\end{aligned}
$$

To get the polarisation sums right, one has to subtract the contributions of the Faddeev-Popov ghosts[4, 5]:

diagramatic:

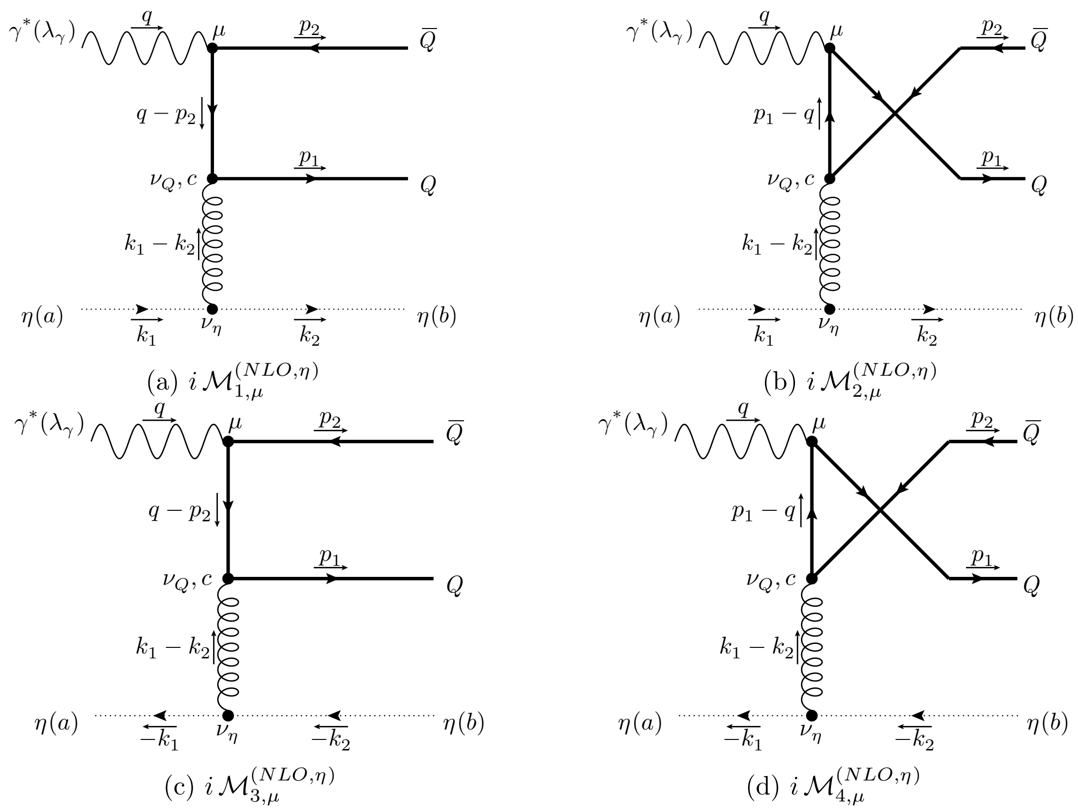


Figure 4: NLO contributions by ghosts

formula:

$$i\,\mathcal{M}_{1,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_\eta}}{t'}\cdot(gf^{acb}k_2^{\nu_\eta}) \tag{24}$$

$$i\,\mathcal{M}_{2,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_\eta}}{t'}\cdot(gf^{acb}k_2^{\nu_\eta}) \tag{25}$$

$$i\,\mathcal{M}_{3,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_\eta}}{t'}\cdot(gf^{cab}(-k_1)^{\nu_\eta}) \tag{26}$$

$$i\,\mathcal{M}_{4,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2)\cdot\frac{-ig_{\nu_Q,\nu_\eta}}{t'}\cdot(gf^{cab}(-k_1)^{\nu_\eta}) \tag{27}$$

color space:

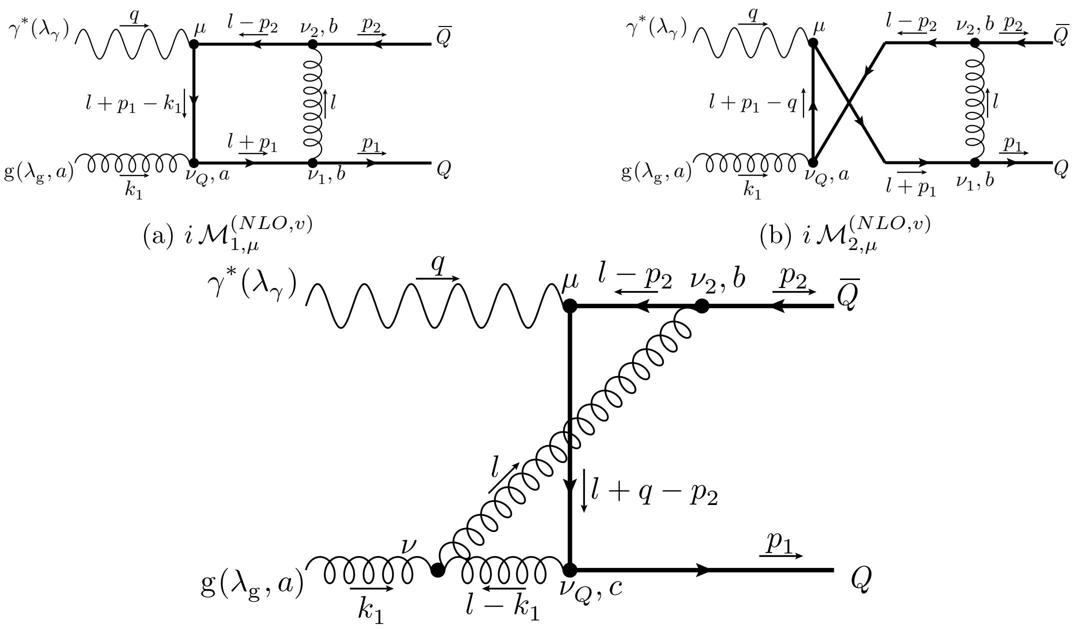
$$\left|\mathcal{M}_{1,\mu}^{(NLO,\eta)} + \mathcal{M}_{2,\mu}^{(NLO,\eta)}\right|^2 \sim f_{acb} f_{adb}\, \mathrm{tr}(T_c T_d) = N_C C_F C_A \tag{28}$$

$$\left|\mathcal{M}_{3,\mu}^{(NLO,\eta)} + \mathcal{M}_{4,\mu}^{(NLO,\eta)}\right|^2 \sim f_{cab} f_{dab}\, \mathrm{tr}(T_c T_d) = f_{acb} f_{adb}\, \mathrm{tr}(T_c T_d) = N_C C_F C_A \tag{29}$$

3.3 Virtual Contributions

$$\gamma^*(q) + \mathrm{g}(k_1) \to \overline{Q}(p_2) + Q(p_1) \tag{30}$$

3.3.1 Loops



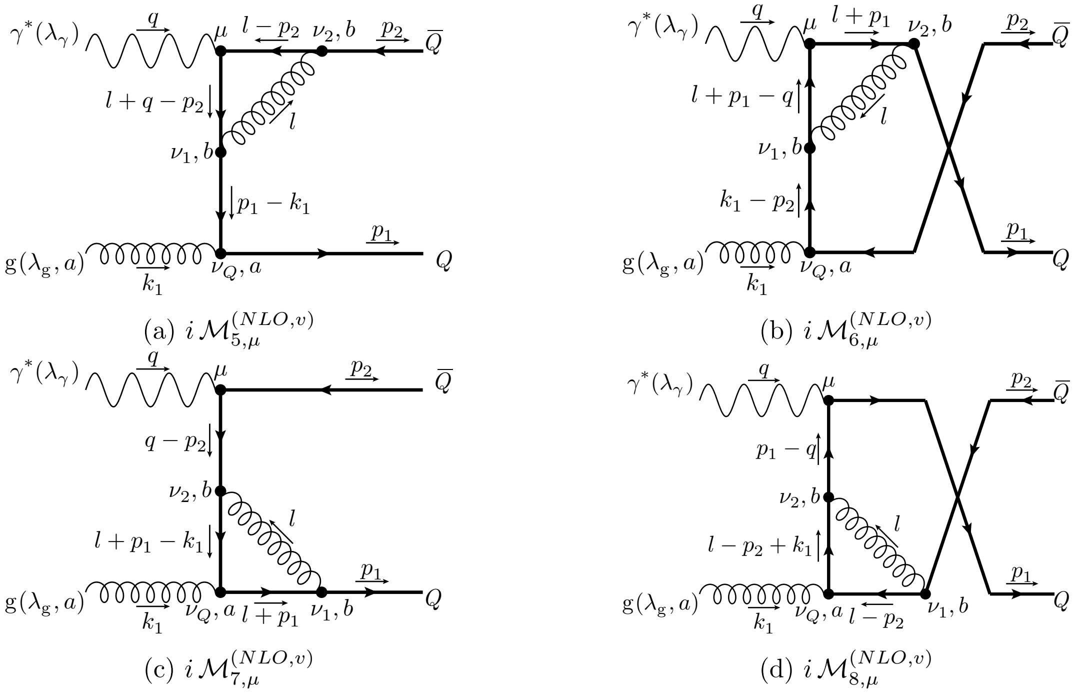
(a) $i\,\mathcal{M}_{1,\mu}^{(NLO,v)}$ (b) $i\,\mathcal{M}_{2,\mu}^{(NLO,v)}$ (c) $i\,\mathcal{M}_{3,\mu}^{(NLO,v)}$

Figure 5: NLO contributions by one loop

$$i\,\mathcal{M}_{1,\mu}^{(NLO,v)} = \mu_D^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(igT_b\gamma^{\nu_1})\frac{i(\not{l}+\not{p}_1+m)}{(l+p_1)^2-m^2}(igT_a\gamma^{\nu_Q})\frac{i(\not{l}+\not{p}_1-\not{k}_1+m)}{(l+p_1-k_1)^2-m^2}\cdot$$
$$(-iee_H\gamma_\mu)\frac{i(\not{l}-\not{p}_2+m)}{(l-p_2)^2-m^2}(igT_b\gamma^{\nu_2})\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon_{\nu_Q}^{(\lambda_{\mathrm{g}})}(k_1) \tag{31}$$

$$i\,\mathcal{M}_{2,\mu}^{(NLO,v)} = \mu_D^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(igT_b\gamma^{\nu_1})\frac{i(\not{l}+\not{p}_1+m)}{(l+p_1)^2-m^2}(igT_a\gamma^{\nu_Q})\frac{i(\not{l}+\not{p}_1-\not{q}+m)}{(l+p_1-q)^2-m^2}\cdot$$
$$(-iee_H\gamma_\mu)\frac{i(\not{l}-\not{p}_2+m)}{(l-p_2)^2-m^2}(igT_b\gamma^{\nu_2})\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon_{\nu_Q}^{(\lambda_{\mathrm{g}})}(k_1) \tag{32}$$

$$i\,\mathcal{M}_{3,\mu}^{(NLO,v)} = \mu_D^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{l}+\not{q}_1-\not{p}_2+m)}{(l+q-p_2)^2-m^2}(-iee_H\gamma_\mu)\frac{i(\not{l}-\not{p}_2+m)}{(l-p_2)^2-m^2}\cdot$$
$$(igT_b\gamma^{\nu_2})\frac{(-i)^2}{l^2(l-k_1)^2}v(p_2)\varepsilon^{\nu,(\lambda_{\mathrm{g}})}(k_1)\cdot$$
$$\Big(gf_{abc}\Big(g_{\nu_2\nu_Q}(k_1-2l)_\nu+g_{\nu_Q\nu}(l-2k_1)_{\nu_2}+g_{\nu\nu_2}(k_1+l)_{\nu_Q}\Big)\Big) \tag{33}$$



(a) $i\,\mathcal{M}_{5,\mu}^{(NLO,v)}$ (b) $i\,\mathcal{M}_{6,\mu}^{(NLO,v)}$ (c) $i\,\mathcal{M}_{7,\mu}^{(NLO,v)}$ (d) $i\,\mathcal{M}_{8,\mu}^{(NLO,v)}$

Figure 6: NLO contributions by one loop (cont'ed)

$$i\,\mathcal{M}_{5,\mu}^{(NLO,v)} = \mu_D^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not{p}_1-\not{k}_1+m)}{u_1}(igT_b\gamma^{\nu_1})\frac{i(\not{l}+\not{q}-\not{p}_2+m)}{(l+q-p_2)^2-m^2}\cdot$$
$$(-iee_H\gamma_\mu)\frac{i(\not{l}-\not{p}_2+m)}{(l-p_2)^2-m^2}(igT_b\gamma^{\nu_2})\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon_{\nu_Q}^{(\lambda_\mathrm{g})}(k_1) \tag{34}$$

$$i\,\mathcal{M}_{6,\mu}^{(NLO,v)} = \mu_D^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(igT_b\gamma^{\nu_2})\frac{i(\not{l}+\not{p}_1+m)}{(l+p_1)^2-m^2}(-iee_H\gamma_\mu)\frac{i(\not{l}+\not{p}_1-\not{q}+m)}{(l+p_1-q)^2-m^2}\cdot$$
$$(igT_b\gamma^{\nu_1})\frac{i(\not{k}_1-\not{p}_2+m)}{t_1}(igT_a\gamma^{\nu_Q})\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon_{\nu_Q}^{(\lambda_\mathrm{g})}(k_1) \tag{35}$$

$$i\,\mathcal{M}_{7,\mu}^{(NLO,v)} = \mu_D^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(igT_b\gamma^{\nu_1})\frac{i(\not{l}+\not{p}_1+m)}{(l+p_1)^2-m^2}(igT_a\gamma^{\nu_Q})\frac{i(\not{l}+\not{p}_1-\not{k}_1+m)}{(l+p_1-k_1)^2-m^2}\cdot$$
$$(igT_b\gamma^{\nu_2})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon_{\nu_Q}^{(\lambda_\mathrm{g})}(k_1) \tag{36}$$

$$i\,\mathcal{M}_{8,\mu}^{(NLO,v)} = \mu_D^{4-n}\int\frac{d^n l}{(2\pi)^n}\,\bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{t_1}(igT_b\gamma^{\nu_2})\frac{i(\not{l}-\not{p}_2+\not{k}_1+m)}{(l-p_2+k_1)^2-m^2}\cdot$$
$$(igT_a\gamma^{\nu_Q})\frac{i(\not{l}-\not{p}_2+m)}{(l-p_2)^2-m^2}(igT_b\gamma^{\nu_1})\frac{-ig_{\nu_1,\nu_2}}{l^2}v(p_2)\varepsilon_{\nu_Q}^{(\lambda_\mathrm{g})}(k_1) \tag{37}$$

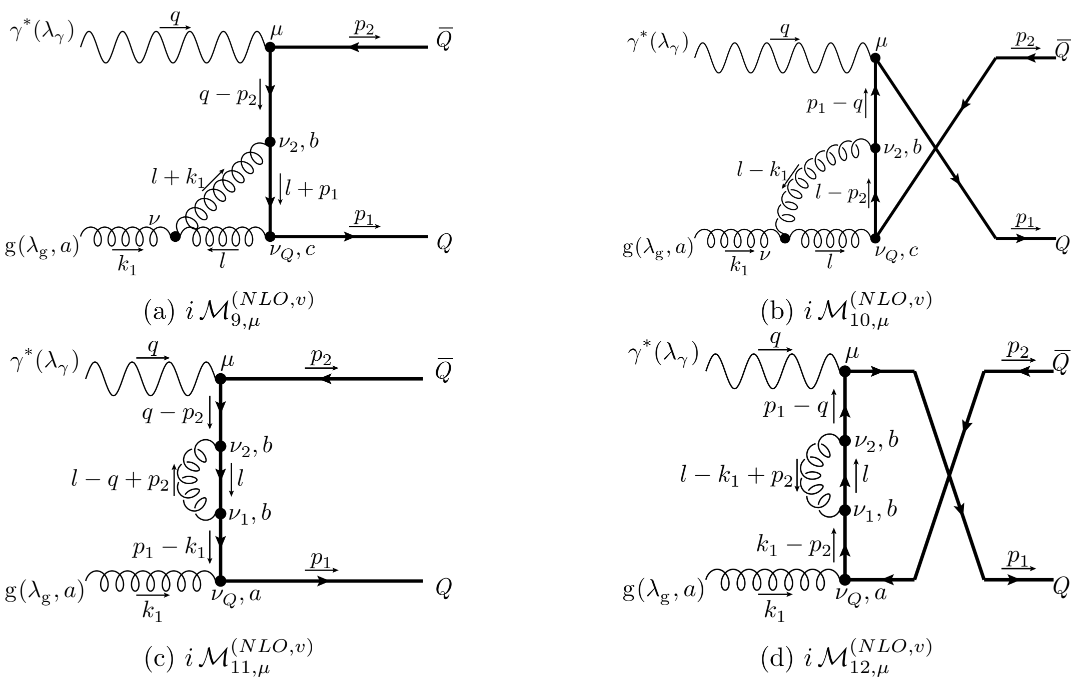


Figure 7: NLO contributions by one loop (cont'ed)

$$
\begin{aligned}
i\,\mathcal{M}^{(NLO,v)}_{9,\mu} = \mu_D^{4-n}\int\frac{d^n l}{(2\pi)^n}\,&\bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{l}+\not{p}_1+m)}{(l+p_1)^2-m^2}(igT_b\gamma^{\nu_2})\frac{i(\not{q}-\not{p}_2+m)}{u_1}\cdot\\
&(-iee_H\gamma_\mu)\frac{(-i)^2}{l^2(l+k_1)^2}v(p_2)\varepsilon^{\nu,(\lambda_\mathrm{g})}(k_1)\cdot\\
&\Big(gf_{abc}\Big(g_{\nu\nu_2}(2k_1+l)_{\nu_Q}+g_{\nu_2\nu_Q}(-2l-k_1)_\nu+g_{\nu_Q\nu}(l-k_1)_{\nu_2}\Big)\Big)
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
i\,\mathcal{M}^{(NLO,v)}_{10,\mu} = \mu_D^{4-n}\int\frac{d^n l}{(2\pi)^n}\,&\bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{t_1}(igT_b\gamma^{\nu_2})\frac{i(\not{l}-\not{p}_2+m)}{(l-p_2)^2-m^2}\cdot\\
&(igT_c\gamma^{\nu_Q})\frac{(-i)^2}{l^2(l-k_1)^2}v(p_2)\varepsilon^{\nu,(\lambda_\mathrm{g})}(k_1)\cdot\\
&\Big(gf_{abc}\Big(g_{\nu\nu_2}(2k_1-l)_{\nu_Q}+g_{\nu_2\nu_Q}(2l-k_1)_\nu+g_{\nu_Q\nu}(-l-k_1)_{\nu_2}\Big)\Big)
\end{aligned}
\tag{39}
$$

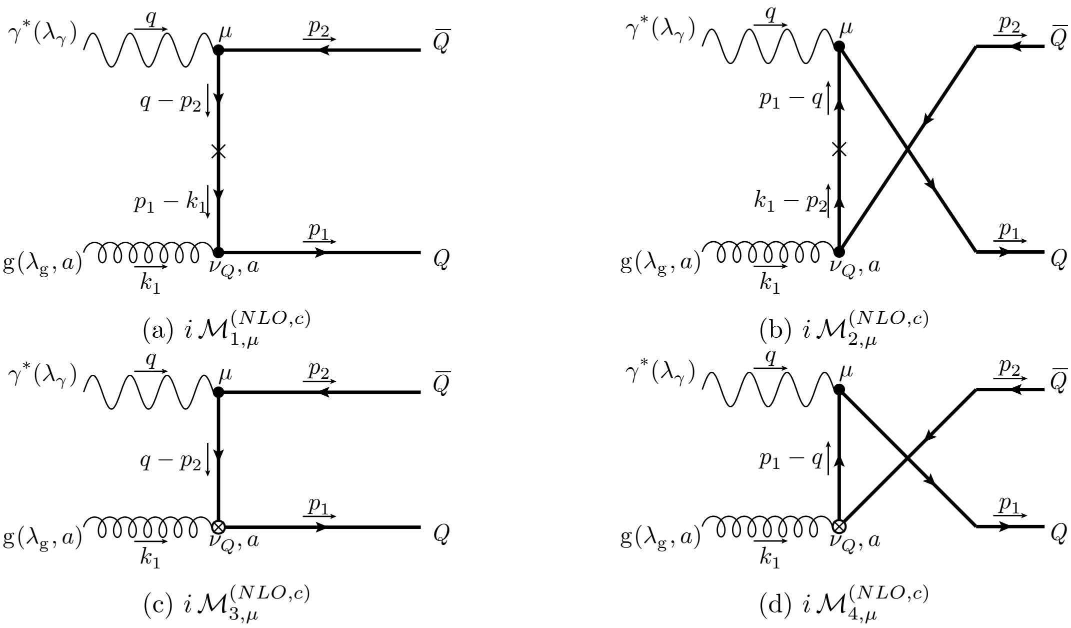
$$
\begin{aligned}
i\,\mathcal{M}^{(NLO,v)}_{11,\mu} = \mu_D^{4-n}\int\frac{d^n l}{(2\pi)^n}\,&\bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not{p}_1-\not{k}_1+m)}{u_1}(igT_b\gamma^{\nu_1})\frac{i(\not{l}+m)}{l^2-m^2}\cdot\\
&(igT_b\gamma^{\nu_2})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)\frac{-ig_{\nu_1,\nu_2}}{(l-q+p_2)^2}v(p_2)\varepsilon^{(\lambda_\mathrm{g})}_{\nu_Q}(k_1)
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
i\,\mathcal{M}^{(NLO,v)}_{12,\mu} = \mu_D^{4-n}\int\frac{d^n l}{(2\pi)^n}\,&\bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{t_1}(igT_b\gamma^{\nu_2})\frac{i(\not{l}+m)}{l^2-m^2}\cdot\\
&(igT_b\gamma^{\nu_1})\frac{i(\not{k}_1-\not{p}_2+m)}{t_1}(igT_a\gamma^{\nu_Q})\frac{-ig_{\nu_1,\nu_2}}{(l-k_1+p_2)^2}v(p_2)\varepsilon^{(\lambda_\mathrm{g})}_{\nu_Q}(k_1)
\end{aligned}
\tag{41}
$$

Color space:

$$
\begin{aligned}
&\left(\mathcal{M}_{1,\mu}^{(NLO,v)}+\mathcal{M}_{2,\mu}^{(NLO,v)}+\mathcal{M}_{7,\mu}^{(NLO,v)}+\mathcal{M}_{8,\mu}^{(NLO,v)}\right)\\
&\quad\cdot\left(\mathcal{M}_{1,\mu'}^{(LO)}+\mathcal{M}_{2,\mu'}^{(LO)}\right)^{*}\sim -i\,\mathrm{tr}(T_aT_bT_aT_b)=-iN_CC_F\left(C_F-\frac{C_A}{2}\right) \qquad (42)\\
&\left(\mathcal{M}_{3,\mu}^{(NLO,v)}+\mathcal{M}_{9,\mu}^{(NLO,v)}+\mathcal{M}_{10,\mu}^{(NLO,v)}\right)\\
&\quad\cdot\left(\mathcal{M}_{1,\mu'}^{(LO)}+\mathcal{M}_{2,\mu'}^{(LO)}\right)^{*}\sim f_{abc}\,\mathrm{tr}(T_cT_bT_a)=-\frac{i}{2}N_CC_FC_A \qquad (43)\\
&\left(\mathcal{M}_{5,\mu}^{(NLO,v)}+\mathcal{M}_{6,\mu}^{(NLO,v)}+\mathcal{M}_{11,\mu}^{(NLO,v)}+\mathcal{M}_{12,\mu}^{(NLO,v)}\right)\\
&\quad\cdot\left(\mathcal{M}_{1,\mu'}^{(LO)}+\mathcal{M}_{2,\mu'}^{(LO)}\right)^{*}\sim -i\,\mathrm{tr}(T_aT_aT_bT_b)=-iN_CC_F^2 \qquad (44)
\end{aligned}
$$

3.3.2 Counter terms



(a) $i\,\mathcal{M}_{1,\mu}^{(NLO,c)}$ (b) $i\,\mathcal{M}_{2,\mu}^{(NLO,c)}$

(c) $i\,\mathcal{M}_{3,\mu}^{(NLO,c)}$ (d) $i\,\mathcal{M}_{4,\mu}^{(NLO,c)}$

Figure 8: NLO contributions by counter terms

$$-i\,\mathcal{M}^{(NLO,c)}_{1,\mu} = \bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not{p}_1-\not{k}_1+m)}{u_1}\Big(i((Z_2-1)(\not{q}-\not{p}_2-m)-(Z_m-1)m)\Big)$$
$$\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon^{(\lambda_g)}_{\nu_Q}(k_1) \tag{45}$$
$$-i\,\mathcal{M}^{(NLO,c)}_{2,\mu} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{t_1}\Big(i((Z_2-1)(\not{p}_1-\not{q}-m)-(Z_m-1)m)\Big)$$
$$\frac{i(\not{k}_1-\not{p}_2+m)}{t_1}(igT_a\gamma^{\nu_Q})v(p_2)\varepsilon^{(\lambda_g)}_{\nu_Q}(k_1) \tag{46}$$
$$-i\,\mathcal{M}^{(NLO,c)}_{3,\mu} = \bar{u}(p_1)(-i(Z_{1f}-1)gT_a\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon^{(\lambda_g)}_{\nu_Q}(k_1) \tag{47}$$
$$-i\,\mathcal{M}^{(NLO,c)}_{4,\mu} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{t_1}(-i(Z_{1f}-1)gT_a\gamma^{\nu_Q})v(p_2)\varepsilon^{(\lambda_g)}_{\nu_Q}(k_1) \tag{48}$$

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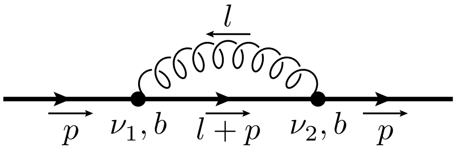
$$\left(\mathcal{M}^{(NLO,c)}_{1,\mu}+\mathcal{M}^{(NLO,c)}_{2,\mu}\right)\left(\mathcal{M}^{(LO)}_{1,\mu'}+\mathcal{M}^{(LO)}_{2,\mu'}\right)^* \sim \mathrm{tr}(T_aT_a)(Z_2-1) = N_C C_F^2 \tag{49}$$
$$\left(\mathcal{M}^{(NLO,c)}_{3,\mu}+\mathcal{M}^{(NLO,c)}_{4,\mu}\right)\left(\mathcal{M}^{(LO)}_{1,\mu'}+\mathcal{M}^{(LO)}_{2,\mu'}\right)^* \sim \mathrm{tr}(T_aT_a)(Z_{1f}-1) = N_C C_F(C_F+C_A) \tag{50}$$

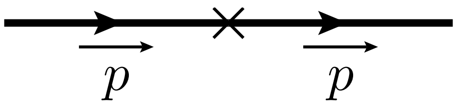
3.3.3 Quark Self Energy

To compute self energies, we follow [6]. It is

$$\{\gamma_\mu,\gamma_\nu\} = 2g_{\mu\nu} \tag{51}$$
$$\gamma_\mu\gamma^\mu = g^\mu_\mu = n \tag{52}$$
$$\gamma_\mu\gamma_\nu\gamma^\mu = (2-n)\gamma_\nu \tag{53}$$



(a) $-i\Sigma(p)$



(b) $-i\Sigma_C(p)$

Figure 9: NLO contributions by quark self energy

$$
\begin{aligned}
-i\Sigma(p) &= \mu_D^{4-n}\int\frac{d^n l}{(2\pi)^n}(igT_b\gamma_{\nu_1})\frac{i(\not{l}+\not{p}+m)}{(l+p)^2-m^2}(igT_b\gamma_{\nu_2})\frac{-ig^{\nu_1,\nu_2}}{l^2} && (54)\\
&= -\mu_D^{4-n}g^2C_F\int\frac{d^n l}{(2\pi)^n}\frac{n\cdot m+(2-n)\not{p}+(2-n)\not{l}}{l^2((l+p)^2-m^2)} && (55)\\
&= -g^2C_F\left(\left(n\cdot m+(2-n)\not{p}\right)B_0(p^2,0,m^2)+(2-n)\not{p}B_1(p^2,0,m^2)\right) && (56)\\
&= -g^2C_F\left(B_0(p^2,0,m^2)\left(n\cdot m+(2-n)\not{p}\frac{p^2+m^2}{2p^2}\right)-(2-n)\not{p}\frac{1}{2p^2}A_0(m^2)\right) && (57)
\end{aligned}
$$

Using [6] we find

$$
\begin{aligned}
C_\epsilon &= \frac{1}{16\pi^2}\exp\left((\gamma_E-\log(4\pi))\frac{\epsilon}{2}\right)\left(m^2/\mu_D^2\right)^{\epsilon/2} && (58)\\
A_0(m^2) &= iC_\epsilon\left(-\frac{2}{\epsilon}+1\right) && (59)\\
B_0(p^2,0,m^2) &= iC_\epsilon\left(-\frac{2}{\epsilon}+2+\frac{m^2-p^2}{p^2}\ln\left(\frac{m^2-p^2}{m^2}\right)\right) && (60)
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow -i\Sigma(p) &= -ig^2C_FC_\epsilon\left[\frac{2\not{p}-8m}{\epsilon}+2m\left(3-2\left(1-\frac{m^2}{p^2}\right)\ln\left(1-\frac{p^2}{m^2}\right)\right)\right.\\
&\qquad\left.-\not{p}\left(1+\frac{m^2}{p^2}\right)\left(1-\left(1-\frac{m^2}{p^2}\right)\ln\left(1-\frac{p^2}{m^2}\right)\right)\right] && (61)\\
&\stackrel{!}{=} -i(Am+B(\not{p}-m)) && (62)
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow A &= \frac{1}{m}\left.\Sigma(p)\right|_{\not{p}=m} && (63)\\
&= -g^2C_FC_\epsilon\left(\frac{6}{\epsilon}-5+\frac{m^2}{p^2}+\left(3-4\frac{m^2}{p^2}+\frac{m^4}{p^4}\right)\ln\left(1-\frac{p^2}{m^2}\right)\right) && (64)\\
\Rightarrow B &= \frac{1}{m}\left.\frac{d\Sigma(p)}{d\not{p}}\right|_{\not{p}=m} && (65)\\
&= \frac{g^2}{16\pi^2}C_F\left(\frac{2}{\hat{\epsilon}_m}-1-\frac{m^2}{p^2}+\left(1-\frac{m^4}{p^4}\right)\ln\left(1-\frac{p^2}{m^2}\right)\right) && (66)
\end{aligned}
$$

Counterterm:

$$
\begin{aligned}
-i\Sigma_C(p) &= i((Z_2-1)\not{p}-(Z_2Z_m-1)m) && (67)\\
&= i((Z_2-1)(\not{p}-m)-(Z_m-1)m)+O(\alpha_S^2) && (68)
\end{aligned}
$$

Use on-shell renormalization:

$$0 \stackrel{!}{=} (-i\Sigma(p) - i\Sigma_C(p))\big|_{\not{p}=m} \tag{69}$$

$$= i\left(((Z_m - 1) + A)m + (B - (Z_2 - 1))(\not{p} - m)\right)\Big|_{\not{p}=m} \tag{70}$$

$$\Rightarrow (Z_m - 1) = -A\big|_{\not{p}=m} \tag{71}$$

$$= \frac{g^2}{16\pi^2} C_F \left(\frac{6}{\hat{\epsilon}_m} - 4\right) \tag{72}$$

such that $m \to Z_m m$.

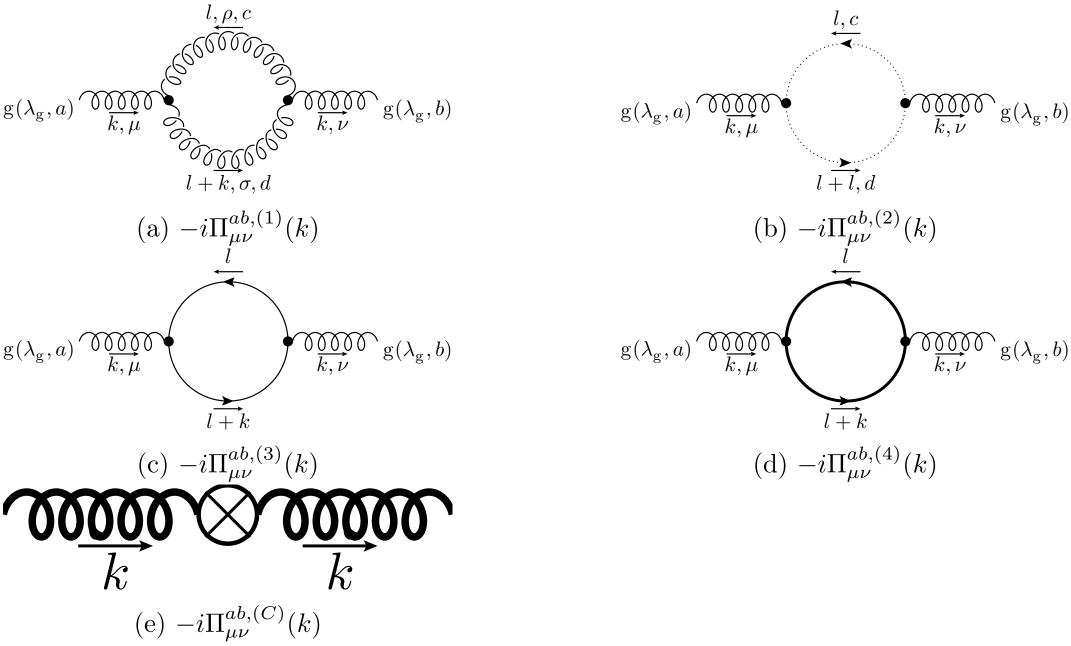
We also choose

$$Z_2 - 1 = \frac{g^2}{16\pi^2} C_F \frac{2}{\hat{\epsilon}_m} \tag{73}$$

$$Z_{1f} = Z_2 + \frac{g^2}{16\pi^2} C_A \frac{2}{\hat{\epsilon}} \tag{74}$$

3.3.4 Gluon Self Energy (Off shell)

Again, we follow [6].



(a) $-i\Pi^{ab,(1)}_{\mu\nu}(k)$ (b) $-i\Pi^{ab,(2)}_{\mu\nu}(k)$

(c) $-i\Pi^{ab,(3)}_{\mu\nu}(k)$ (d) $-i\Pi^{ab,(4)}_{\mu\nu}(k)$

(e) $-i\Pi^{ab,(C)}_{\mu\nu}(k)$

Figure 10: NLO contributions by quark self energy

$$
\begin{aligned}
-i\Pi^{ab,(1)}_{\mu\nu}(k) = \frac{1}{2!}\mu_R^{4-n}\int \frac{d^n l}{(2\pi)^n} g f_{cda}\left(g_{\rho\sigma}(2l+k)_\mu + g_{\sigma\mu}(-l-2k)_\rho + g_{\mu\rho}(k-l)_\sigma\right) \\
\cdot\, g f_{cbd}\left(g^{\rho}{}_{\nu}(k-l)^\sigma + g^{\sigma}{}_{\nu}(-l-2k)^\rho + g^{\sigma\rho}(2l+k)_\nu\right) \cdot \frac{(-i)^2}{l^2(l+k)^2} \qquad (75)
\end{aligned}
$$

$$
-i\Pi^{ab,(2)}_{\mu\nu}(k) = -\mu_R^{4-n}\int \frac{d^n l}{(2\pi)^n} g^2 f_{adc} f_{bcd} l_\mu (l+k)_\nu \frac{i^2}{l^2(l+k)^2} \qquad (76)
$$

$$
-i\Pi^{ab,(3)}_{\mu\nu}(k) = -\mu_R^{4-n}\int \frac{d^n l}{(2\pi)^n} \frac{\mathrm{tr}\big((ig\gamma_\mu)(i\not{l})\,(ig\gamma_\nu)\,(i(\not{l}+\not{k}))\big)\,\mathrm{tr}(T_a T_b)}{l^2(l+k)^2} \qquad (77)
$$

$$
-i\Pi^{ab,(4)}_{\mu\nu}(k) = -\mu_R^{4-n}\int \frac{d^n l}{(2\pi)^n} \frac{\mathrm{tr}\big((ig\gamma_\mu)(i(\not{l}+m))\,(ig\gamma_\nu)\,(i(\not{l}+\not{k}+m))\big)\,\mathrm{tr}(T_a T_b)}{(l^2-m^2)((l+k)^2-m^2)} \qquad (78)
$$

$$
-i\Pi^{ab,(C)}_{\mu\nu}(k) = i(Z_3-1)\delta^{ab}(k_\mu k_\nu - k^2 g_{\mu\nu}) \qquad (79)
$$

Color space:

$$
f_{acd} f_{bdc} = -\delta_{ab} C_A = f_{dca} f_{dbc} \qquad (80)
$$

$$
\mathrm{tr}(T_a T_b) = \frac{1}{2}\delta_{ab} \qquad (81)
$$

By Slavnov-Taylor we know:

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$$\Pi^{ab}_{\mu\nu}(k) = \delta^{ab}(k_\mu k_\nu - k^2 g_{\mu\nu})\Pi(k^2) \tag{82}$$

$$\Rightarrow \Pi(k^2) = -\frac{\delta_{ab}}{N_c^2-1}\frac{1}{(3+\epsilon)k^2}g^{\mu\nu}\Pi^{ab}_{\mu\nu}(k) \tag{83}$$

Gluon + Ghost loop:

$$\Pi^{(1+2)}(k^2) = -ig^2 C_A \frac{1}{(3+\epsilon)k^2}\mu_R^{-\epsilon}\int\frac{d^n l}{(2\pi)^n}\frac{(8+3\epsilon)k\cdot l + (9+3\epsilon)k^2 + (8+3\epsilon)l^2}{l^2(l+k)^2} \tag{84}$$

$$= -ig^2 C_A \frac{10+3\epsilon}{2(3+\epsilon)}\mu_R^{-\epsilon}\int\frac{d^n l}{(2\pi)^n}\frac{1}{l^2(l+k)^2} \tag{85}$$

with

$$B_0(k^2,0,0) = \mu_R^{-\epsilon}\int\frac{d^n l}{(2\pi)^n}\frac{1}{l^2(l+k)^2} \tag{86}$$

$$= \frac{i}{16\pi^2}\left(-\frac{2}{\hat{\epsilon}} - \ln(-k^2/\mu_R^2) + 2\right) \tag{87}$$

We find

$$\Rightarrow \Pi^{(1+2)}(k^2) = g^2 C_A\left(-\frac{10}{3\hat{\epsilon}} - \frac{5}{3}\ln(-k^2/\mu_R^2) + \frac{31}{9}\right) \tag{88}$$

$$= -C_A\frac{g^2}{16\pi^2}\frac{5}{3}\left(\frac{2}{\hat{\epsilon}} + \ln(-k^2/\mu_R^2) - \frac{31}{15}\right) \tag{89}$$

Heavy quark loop:

$$\Pi^{(4)}(k^2) = ig^2\frac{1}{2(3+\epsilon)k^2}\mu_R^{-\epsilon}\int\frac{d^n l}{(2\pi)^n}\frac{4((2+\epsilon)(m^2 - k\cdot l - l^2) + 2m^2)}{(l^2-m^2)((l+k)^2-m^2)} \tag{90}$$

$$= ig^2\frac{2}{(3+\epsilon)k^2}\Big(2m^2 B_0(k^2,m^2,m^2) - (2+\epsilon)A_0(m^2)$$
$$-(2+\epsilon)k^2 B_1(k^2,m^2,m^2)\Big) \tag{91}$$

with

$$A_0(m^2) = \frac{im^2}{16\pi^2}\left(-\frac{2}{\hat{\epsilon}_m} + 1\right) \tag{92}$$

$$B_1(k^2,m^2,m^2) = -\frac{1}{2}B_0(k^2,m^2,m^2) \tag{93}$$

$$B_0(k^2,m^2,m^2) = \frac{i}{16\pi^2}\left(-\frac{2}{\hat{\epsilon}_m} + 2 + \beta_k\ln(\chi_k)\right) \tag{94}$$

and $\beta_k = \sqrt{1-4m^2/k^2}$, $\chi_k = (\beta_k - 1)/(1+\beta_k)$. We then find

$$\Rightarrow \Pi^{(4)}(k^2) = \frac{2g^2}{16\pi^2}\left(\frac{2}{3\hat{\epsilon}_m} - \frac{5}{9} - \frac{4m^2}{3k^2} - \frac{1}{3}\left(1+2\frac{m^2}{k^2}\right)\beta_k \ln(\chi_k)\right) \tag{95}$$

$$= \frac{g^2}{16\pi^2}\frac{2}{3}\left(\frac{2}{\hat{\epsilon}_m} - \frac{5}{3} - 4\frac{m^2}{k^2} - \left(1+2\frac{m^2}{k^2}\right)\beta_k \ln(\chi_k)\right) \tag{96}$$

Light quark loop:

$$\Pi^{(3)}(k^2) = -ig^2\frac{2(2+\epsilon)}{(3+\epsilon)}B_1(k^2,0,0) \tag{97}$$

$$= ig^2\frac{(2+\epsilon)}{(3+\epsilon)}B_0(k^2,0,0) \tag{98}$$

$$= \frac{g^2}{16\pi^2}\frac{2}{3}\left(\frac{2}{\hat{\epsilon}} + \ln(-k^2/\mu_R^2) - \frac{5}{3}\right) \tag{99}$$

We choose the counterterm to be:

$$-\Pi^{(C)}(k^2) = Z_3 - 1 = \frac{g^2}{16\pi^2}\left(-\frac{5}{3}C_A\frac{2}{\hat{\epsilon}} + n_{lf}\frac{2}{3}\frac{2}{\hat{\epsilon}} + \frac{2}{3}\frac{2}{\hat{\epsilon}_m}\right) \tag{100}$$

$$= \frac{g^2}{16\pi^2}\left((2C_A - \beta_0^{lf})\frac{2}{\hat{\epsilon}} + \frac{2}{3}\frac{2}{\hat{\epsilon}_m}\right) \tag{101}$$

$$= \frac{g^2}{16\pi^2}\left((2C_A - \beta_0^{f})\frac{2}{\hat{\epsilon}} + \frac{2}{3}\ln(m^2/\mu_R^2)\right) \tag{102}$$

with $\beta_0^{lf} = (11C_A - 2n_{lf})/3$, $\beta_0^f = (11C_A - 2n_f)/3$ and $n_f = n_{lf} + 1$.

We also find

$$Z_g - 1 = \frac{g^2}{16\pi^2}\frac{1}{2}\left(\beta_0^f\frac{2}{\hat{\epsilon}} - \frac{2}{3}\ln(m^2/\mu_R^2)\right) \tag{103}$$

such that $g \to Z_g g$.

3.3.5 Gluon Self Energy (On shell)

We have[7, (A.18)-(A.20)]:

$$I_0 = \int \frac{d^n p}{(2\pi)^n} \frac{1}{(p^2 + 2kp + M^2 + i\eta)^\alpha} \tag{104}$$

$$= i\frac{(-\pi)^{n/2}}{(2\pi)^n} \frac{\Gamma(\alpha - n/2)}{\Gamma(\alpha)} \frac{1}{(M^2 - k^2 + i\eta)^{\alpha - n/2}} \tag{105}$$

$$I_\mu = \int \frac{d^n p}{(2\pi)^n} \frac{p_\mu}{(p^2 + 2kp + M^2 + i\eta)^\alpha} \tag{106}$$

$$= -k_\mu I_0 \tag{107}$$

$$I_{\mu\nu} = \int \frac{d^n p}{(2\pi)^n} \frac{p_\mu p_\nu}{(p^2 + 2kp + M^2 + i\eta)^\alpha} \tag{108}$$

$$= I_0 \left(k_\mu k_\nu + \frac{1}{2} g_{\mu\nu} (M^2 - k^2) \frac{1}{\alpha - n/2 - 1} \right) \tag{109}$$

Heavy Quark loop:

$$-i\Pi^{ab,(4)}_{\mu\nu}(k) = -\mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{\mathrm{tr}\big((ig\gamma_\mu)(i(\not{l} + m))\,(ig\gamma_\nu)\,(i(\not{l} + \not{k} + m))\big)\,\mathrm{tr}(T_a T_b)}{(l^2 - m^2)((l+k)^2 - m^2)} \tag{110}$$

$$= -\frac{\delta^{ab}}{2} g^2 \mu_R^{4-n} \int_0^1 dx \int \frac{d^n l}{(2\pi)^n} 4 \frac{2 l_\mu l_\nu + (l_\mu k_\nu + l_\nu k_\mu) + (m^2 - k \cdot l - l^2) g_{\mu\nu}}{(l^2 + 2xk \cdot l - m^2 + k^2 x)^2} \tag{111}$$

$$= -2\delta^{ab} g^2 \mu_R^{4-n} \int_0^1 dx \Big(2I_{\mu\nu} + \big(k_\nu I_\nu + k_\nu I_\mu\big) + g_{\mu\nu} \left(m^2 I_0 - k^\rho I_\rho - g_\rho^{\ \sigma} I_{\rho\sigma} \right) \Big) \tag{112}$$

$$= -2\delta^{ab} g^2 \mu_R^{4-n} \int_0^1 dx\, I_0 \Big[k_\mu k_\nu (2x^2 - 2x) + g_{\mu\nu} \left((k^2 x(1-x) - m^2) \frac{1 - n/2}{1 - n/2} + m^2 + k^2 x(1-x) \right) \Big] \tag{113}$$

$$= 4\delta^{ab} g^2 (k_\mu k_\nu - g_{\mu\nu} k^2) \mu_R^{4-n} \int_0^1 dx\, x(1-x) I_0 \tag{114}$$

where

$$\mu_R^{4-n} I_0 = i \frac{(-\pi)^{n/2}}{(2\pi)^n} \frac{\Gamma(2 - n/2)}{\Gamma(2)} \left(\frac{\mu_R^2}{k^2 x(1-x) - m^2} \right)^{2 - n/2} \tag{115}$$

$$= -\frac{i}{16\pi^2} \left(\frac{2}{\hat{\epsilon}} - \ln((k^2 x(1-x) - m^2)/\mu_R^2) \right) + O(\epsilon) \tag{116}$$

so for $k^2 = 0$ (on shell) we end up with

$$- i\Pi^{ab,(4)}_{\mu\nu}(k) = -i\delta^{ab}\frac{g^2}{16\pi^2}(k_\mu k_\nu - g_{\mu\nu}k^2)\frac{4}{3\hat{\epsilon}_m} \tag{117}$$

Light Quark loop:

$$-i\Pi^{ab,(3)}_{\mu\nu}(k) = -\mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\frac{\mathrm{tr}\big((ig\gamma_\mu)(i\not{l})\,(ig\gamma_\nu)\,(i(\not{l}+\not{k}))\big)\,\mathrm{tr}(T_a T_b)}{l^2(l+k)^2} \tag{118}$$

$$= 4\delta^{ab}g^2(k_\mu k_\nu - g_{\mu\nu}k^2)\mu_R^{4-n}\int_0^1 dx\, x(1-x)I_0 \tag{119}$$

where

$$\mu_R^{4-n}I_0 = \mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\frac{1}{(l^2+2xk\cdot l+k^2x)^2} \tag{120}$$

$$= \mu_R^{4-n}\int\frac{d^n l}{(2\pi)^n}\frac{1}{((l+xk)^2+k^2x(1-x))^2} \tag{121}$$

so for $k^2 = 0$ (on shell) we get $I_0 = 0$ and end up with

$$- i\Pi^{ab,(3)}_{\mu\nu}(k) = 0 \tag{122}$$

Gluon + Ghost loop vanish by the same arguments as the light quark loop:

$$- i\Pi^{ab,(1+2)}_{\mu\nu}(k) = 0 \tag{123}$$

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List of Corrections

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