Next-to-Leading Order QCD Corrections to Inclusive Heavy-Flavor Production in Polarized Deep-Inelastic Scattering

Felix Hekhorn and Marco Stratmann

Institute for Theoretical Physics, University of Tübingen

March, 2018

Outline

Introduction - Heavy Quarks (HQ)

- Heavy Quarks (HQ): c$(m_c = 1.5\,\mathrm{GeV})$, b$(m_b = 4.75\,\mathrm{GeV})$, t$(m_t = 175\,\mathrm{GeV})$
- EIC will reach region with HQ relevant to structure functions
- compare unpolarized case HERA@DESY: at small $x \sim 30\%$ charm contributions [Laenen,Riemersma,Smith,van Neerven]

Introduction - Heavy Quarks (HQ)

- Heavy Quarks (HQ): $\mathrm{c}(m_\mathrm{c} = 1.5\,\mathrm{GeV})$, $\mathrm{b}(m_\mathrm{b} = 4.75\,\mathrm{GeV})$, $\mathrm{t}(m_\mathrm{t} = 175\,\mathrm{GeV})$
- EIC will reach region with HQ relevant to structure functions
- compare unpolarized case HERA@DESY: at small $x \sim 30\%$ charm contributions [Laenen,Riemersma,Smith,van Neerven]
- measure Δg as dominated by photon-gluon fusion (PGF)
- first full NLO computation of polarized process [Blümlein,Bojak,Stratmann]

Introduction - Heavy Quarks (HQ)

- Heavy Quarks (HQ): $c(m_c = 1.5\,\text{GeV})$, $b(m_b = 4.75\,\text{GeV})$, $t(m_t = 175\,\text{GeV})$
- EIC will reach region with HQ relevant to structure functions
- compare unpolarized case HERA@DESY: at small $x \sim 30\%$ charm contributions [Laenen,Riemersma,Smith,van Neerven]
- measure Δg as dominated by photon-gluon fusion (PGF)
- first full NLO computation of polarized process [Blümlein,Bojak,Stratmann]

- need improved charm tagging
- fully inclusive cross section is complicated to reconstruct
- no hadronization here

Introduction - Heavy Quarks (HQ)

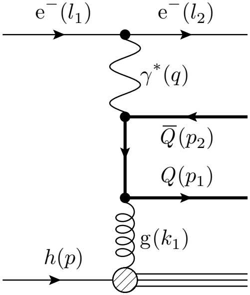
- scale of hard process is in a pertubative regime
  $m > \Lambda_{QCD}$
- finite mass m provides total cross sections

Introduction - Heavy Quarks (HQ)

- scale of hard process is in a pertubative regime $m > \Lambda_{QCD}$
- finite mass m provides total cross sections
- full m^2 dependence makes computations complicated: phase space + matrix elements
- 2-scale problem: $\ln\left(\frac{s-4m^2}{4m^2}\right)$ and/or $\ln(Q^2/m^2)$
- keep analytic expressions

Introduction - DIS Setup

$$e^-(l_1) + h(p) \to e^-(l_2) + \bar{Q}(p_2) + X[Q]$$

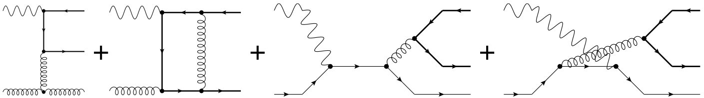


- $S_h = (p + l_1)^2 = x\,y\,Q^2,\ x,\ y,$
  $Q^2 = -q^2 = -(l_1 - l_2)^2 \ll M_Z^2$
- unpolarized cross section: [PDG]
  $$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2}{xyQ^2}\left(Y_+ F_2(x,Q^2) - y^2 F_L(x,Q^2)\right)$$
  $$2x F_1(x,Q^2) = F_2(x,Q^2) - F_L(x,Q^2)$$
- polarized cross section: [PDG]
  $$\frac{d^2\Delta\sigma}{dxdy} = \frac{4\pi\alpha^2}{xyQ^2} Y_- \cdot 2x g_1(x,Q^2)$$
- with $Y_\pm = 1 \pm (1-y)^2$

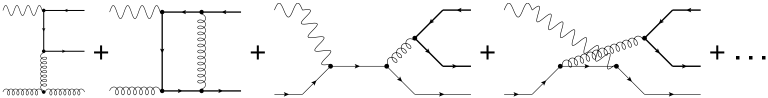
Computation Review

- use factorisation theorem: PDF and $s = \xi S_h$
- PGF: $g(k_1) + \gamma^*(q) \to \bar{Q}(p_2) + Q(p_1)$
- three massive particles: $m^2 > 0, q^2 = -Q^2 < 0$

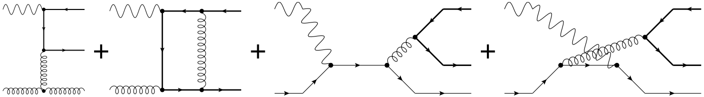
Computation Review

- use factorisation theorem: PDF and $s = \xi S_h$
- PGF: $\mathrm{g}(k_1) + \gamma^*(q) \to \bar{Q}(p_2) + Q(p_1)$
- three massive particles: $m^2 > 0, q^2 = -Q^2 < 0$
- compute 2-to-3-phase space: e.g. $dPS_3 \sim dt_1 du_1 d\Omega_n d\hat{\mathcal{I}}$ [Laenen, Bojak]
- diagrams:  $+ \ldots$

Computation Review

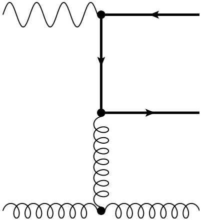
- use factorisation theorem: PDF and $s = \xi S_h$
- PGF: $\mathrm{g}(k_1) + \gamma^*(q) \to \bar{Q}(p_2) + Q(p_1)$
- three massive particles: $m^2 > 0, q^2 = -Q^2 < 0$
- compute 2-to-3-phase space: e.g. $dPS_3 \sim dt_1 du_1 d\Omega_n d\hat{\mathcal{I}}$ [Laenen, Bojak]
- diagrams: 
- $\Rightarrow g_1 \sim e_u^2 \cdot \Delta u \otimes d_{P,q}^{(1)}$
- $d_{P,q}^{(1)}(\chi, \chi') = c_1(\chi, \chi') \ln(\chi) + c_2(\chi, \chi') \operatorname{Li}_2\left(\frac{1+\chi'}{1+\chi}\right) + \ldots$ ✓ [Blümlein,Falcioni,De Freitas]
- $\frac{m^2}{s} = \frac{\chi}{(1+\chi)^2}$ and $\frac{m^2}{s+Q^2} = \frac{m^2}{s'} = \frac{\chi'}{(1+\chi')^2}$ and $\frac{m^2}{Q^2} = \frac{\chi_q}{(1-\chi_q)^2}$

Computation Review

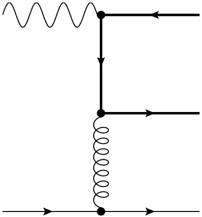
- use factorisation theorem: PDF and $s = \xi S_h$
- PGF: $g(k_1) + \gamma^*(q) \to \bar{Q}(p_2) + Q(p_1)$
- three massive particles: $m^2 > 0, q^2 = -Q^2 < 0$
- compute 2-to-3-phase space: e.g. $dPS_3 \sim dt_1\, du_1\, d\Omega_n d\hat{\mathcal{I}}$ [Laenen, Bojak]
- diagrams:  $+ \dots$
- $\Rightarrow g_1 \sim e_u^2 \cdot \Delta u \otimes d_{P,q}^{(1)}$
- $d_{P,q}^{(1)}(\chi,\chi') = c_1(\chi,\chi')\ln(\chi) + c_2(\chi,\chi')\,\mathrm{Li}_2\left(\frac{1+\chi'}{1+\chi}\right) + \dots$ ✓ [Blümlein,Falcioni,De Freitas]
- $\frac{m^2}{s} = \frac{\chi}{(1+\chi)^2}$ and $\frac{m^2}{s+Q^2} = \frac{m^2}{s'} = \frac{\chi'}{(1+\chi')^2}$ and $\frac{m^2}{Q^2} = \frac{\chi_q}{(1-\chi_q)^2}$
- γ_5 and $\varepsilon_{\mu\nu\rho\sigma}$ in n-dimension? $\to$ HVBM scheme ['t Hooft,Veltman,Breitenlohner,Maison]

Computation Review - Collinear Poles

collinear poles appear in, e.g.,



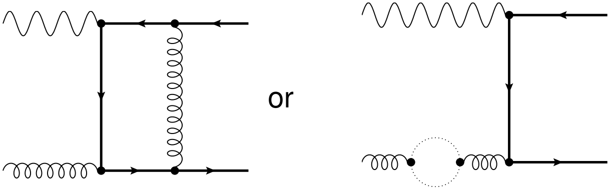
or



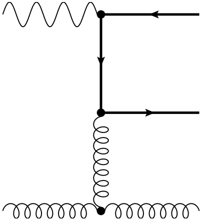
- remove by mass factorization $\rightarrow \overline{\text{MS}}$
- $\Rightarrow g_1 \sim e_H^2 \cdot \Delta g \otimes \ln(\mu_F^2/m^2)\bar{c}_{P,g}^{F,(1)}$
- $\bar{c}_{P,g}^{F,(1)}(\chi,\chi_q) = c_1(\chi,\chi_q)\ln(\chi) + c_2(\chi,\chi_q)\,\mathrm{Li}_2\left(\frac{1-\chi_q}{1+\chi}\right) + \ldots$ (✓for $Q^2 \gg m^2$ [Buza,Matiounine,Smith,van Neerven])

Computation Review - UV and IR Poles

virtual diagrams are, e.g.,



soft poles appear in the limit of a soft gluon, e.g.,



soft + virtual + renormalization ($\overline{\text{MS}}_m$) + factorization is finite! [Laenen,Bojak]

Computation Review - Analytic Expressions

$$D_0(m^2, 0, q^2, m^2, t, s, 0, m^2, m^2, m^2) = \frac{iC_\epsilon}{\beta s t_1} \times \Bigg[-\frac{2}{\epsilon}\ln(\chi) - 2\ln(\chi)\ln\left(\frac{-t_1}{m^2}\right)$$
$$+ \mathrm{Li}_2(1-\chi^2) - 4\zeta(2) + \ln^2(\chi_q) + 2\,\mathrm{Li}_2(-\chi\chi_q) + 2\,\mathrm{Li}_2\left(\frac{-\chi}{\chi_q}\right)$$
$$+ 2\ln(\chi\chi_q)\ln(1+\chi\chi_q) + 2\ln\left(\frac{\chi}{\chi_q}\right)\ln\left(1+\frac{\chi}{\chi_q}\right)\Bigg]$$

$$\int \frac{d\Omega_n}{t' {u_7}^2} = -\frac{2\pi(m^2+s_4)(s'+t_1)}{s_4 {t_1}^2 {u_1}^2}\Bigg[-2 + \frac{t_1 u_1(-q^2 s_4 + (2m^2+s_4)(s'+u_1))}{(s'+t_1)\left(q^2 s_4 t_1 + m^2(s'+u_1)^2\right)}$$
$$+ \frac{2}{\epsilon} + \ln\left(\frac{{t_1}^2 {u_1}^2 (m^2+s_4)}{(s'+t_1)^2\left(m^2(s'+u_1)^2 + q^2 t_1 s_4\right)}\right)\Bigg]$$

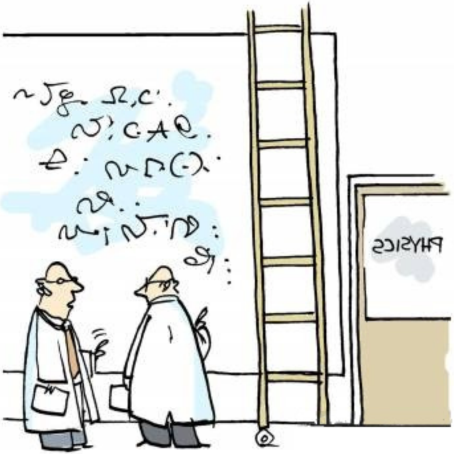
$$D_0(m^2, 0, q^2, m^2, t, s, 0, m^2, m^2, m^2) = \frac{iC}{\beta s}$$

$$+ \mathrm{Li}_2(1 - \chi^2) - 4\zeta(2) + \ln^2(\chi_q) + 2\,\mathrm{Li}_2(-$$

$$+ 2\ln(\chi\chi_q)\ln(1 + \chi\chi_q) + 2\ln\left(\frac{\chi}{\chi_q}\right)\ln\left(\right.$$

$$\int \frac{d\Omega_n}{t' {u_7}^2} = -\frac{2\pi(m^2 + s_4)(s' + t_1)}{s_4 {t_1}^2 {u_1}^2}\left[-2 + \right.$$

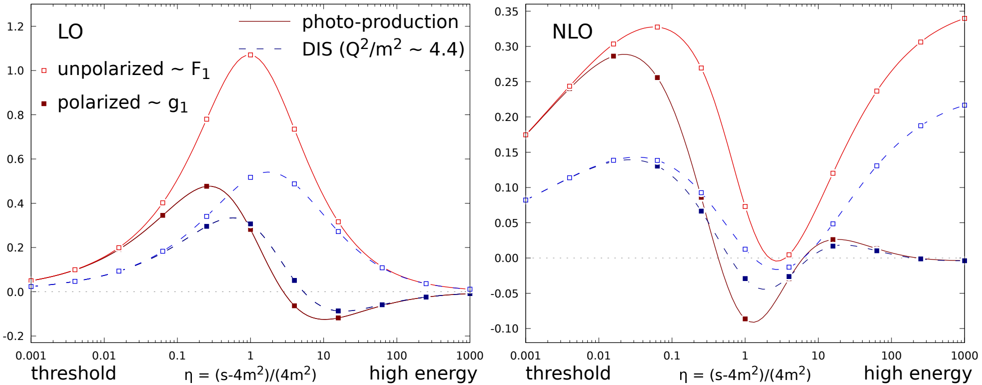
$$+ \frac{2}{\epsilon} + \ln\left(\frac{{t_1}^2 {u_1}^2 (m^2 + s}{(s' + t_1)^2 \left(m^2(s' + u_1)\right.}\right.$$



OOO, I'VE THOUGHT OF A NEW ONE! TWO SQUIGGLES AND A BACKWARDS 6!

Partonic Results - Gluon Channel

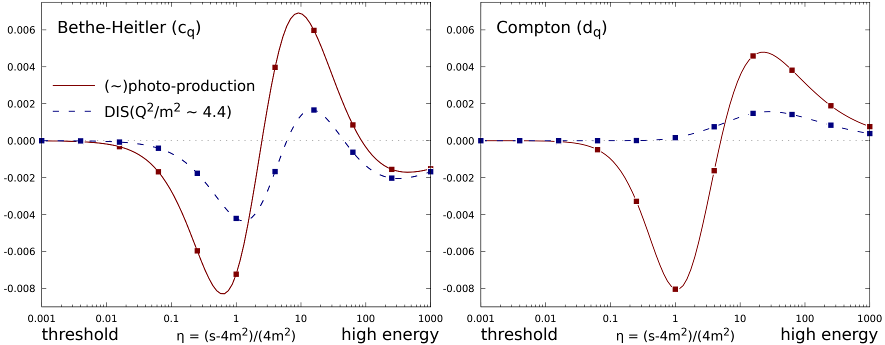
$$g_1 \sim \alpha_s \cdot \Delta g \otimes \left(c^{(0)}_{P,g} + 4\pi\alpha_s \left[c^{(1)}_{P,g} + \ln\left(\frac{\mu^2}{m^2}\right) \bar{c}^{(1)}_{P,g} \right] \right)$$



- polarized $\sim$ unpolarized near threshold, but not at high energy

Partonic Results - Light Quark Channel

$$g_1 \sim \alpha_s^2 \sum_q (\Delta q + \Delta\bar{q}) \otimes \left(e_H^2 \left[c_{P,q}^{(1)} + \ln\left(\frac{\mu_F^2}{m^2}\right) \bar{c}_{P,q}^{(1)} \right] + e_q^2 d_{P,q}^{(1)} \right)$$

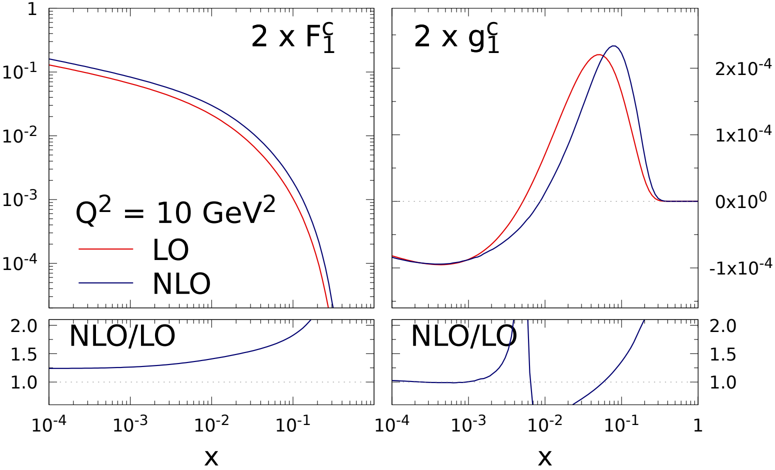


- no interference term $\sim e_H e_q$
- Compton subprocess contains $\ln(Q^2/m^2)$

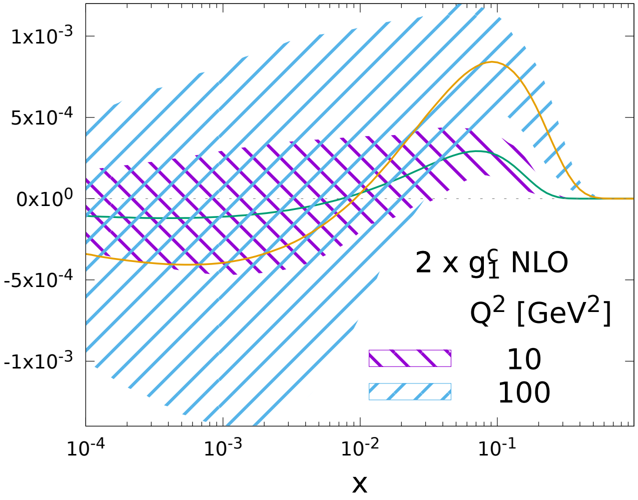
Hadronic Results - Unpolarized vs. Polarized

unpolarized $\sim$ MSTW2008 $\leftrightarrow$ polarized $\sim$ DSSV2014

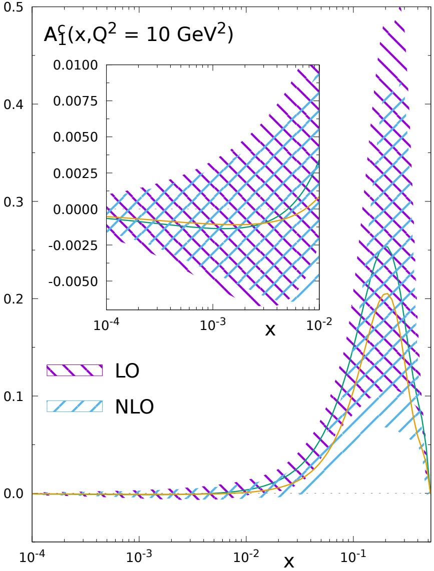
[Martin,Stirling,Thorne,Watt] $\leftrightarrow$ [de Florian,Sassot,Stratmann,Vogelsang]



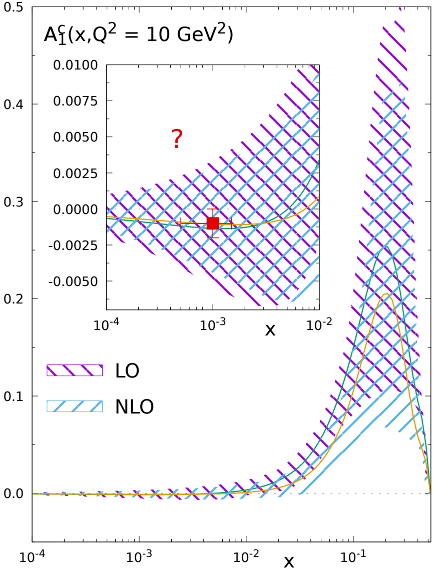
Hadronic Results - PDF Uncertainties DSSV (I)



- quark error band is small
- sign unconstrained
- gluon dominates (?)
- default gluon is small

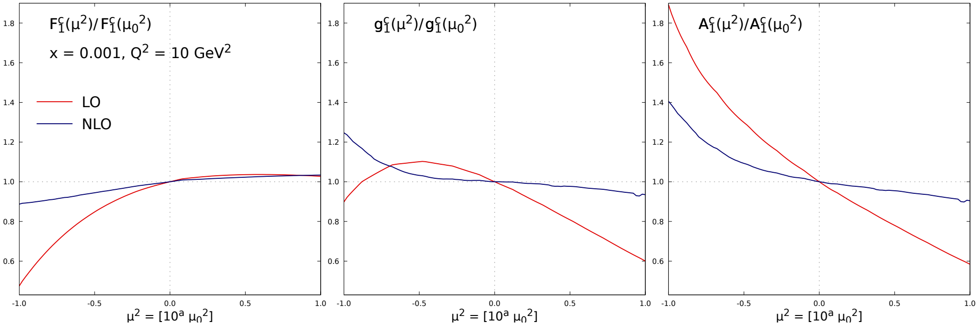


- $A_1^c(x,Q^2) = \frac{g_1^c(x,Q^2)}{F_1^c(x,Q^2)}$
- error band are only due to DSSV uncertainties (no correlations!)

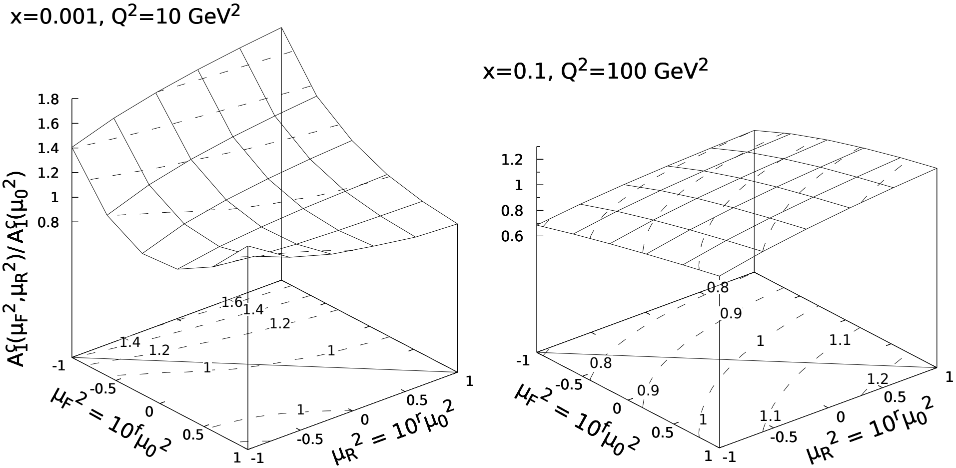


- $A_1^c(x,Q^2) = \frac{g_1^c(x,Q^2)}{F_1^c(x,Q^2)}$
- error band are only due to DSSV uncertainties (no correlations!)
- sign unconstrained
- need measurement of $\mathcal{O}(10^{-3})$
- NLO $\lesssim\approx$ LO

Hadronic Results - Scale Uncertainties (I)



$$\mu_F^2 = \mu_R^2 = 10^a \mu_0^2 \text{ with } \mu_0^2 = 4m^2 + Q^2$$


x=0.001, Q2=10 GeV2
1.8
1.6
1.4
1.2
1
0.8
-1
-0.5
0
0.5
1
μF2 = 10f μ02
μR2 = 10r μ02
x=0.1, Q2=100 GeV2
1.2
1
0.8
0.6

$$\mu_0^2 = 4m^2 + Q^2$$

Outlook

- inclusive distributions: $\frac{dg_1}{dp_{T,\bar{Q}}}, \frac{dg_1}{dy_{\bar{Q}}}, \ldots$ [Laenen,Riemersma,Smith,van Neerven]
- correlated distributions: $\frac{dg_1}{dM^2_{Q\bar{Q}}}, \frac{dg_1}{d\phi_{Q\bar{Q}}}$, TMD, $\ldots$ [Harris,Smith]

Outlook

- inclusive distributions: $\frac{dg_1}{dp_{T,\bar{Q}}}, \frac{dg_1}{dy_{\bar{Q}}}, \ldots$ [Laenen,Riemersma,Smith,van Neerven]
- correlated distributions: $\frac{dg_1}{dM^2_{Q\bar{Q}}}, \frac{dg_1}{d\phi_{Q\bar{Q}}}$, TMD, $\ldots$ [Harris,Smith]
- full neutral current (NC) contributions: $F_3^{Z\gamma}, g_4^{Z\gamma}, g_5^{Z\gamma}$ and F_2^Z, F_L^Z, g_1^Z
- distributions of full NC structure functions: $\frac{dg_1^{NC}}{dp_{T,\bar{Q}}}, \frac{dg_1^{NC}}{dM^2_{Q\bar{Q}}}, \ldots$

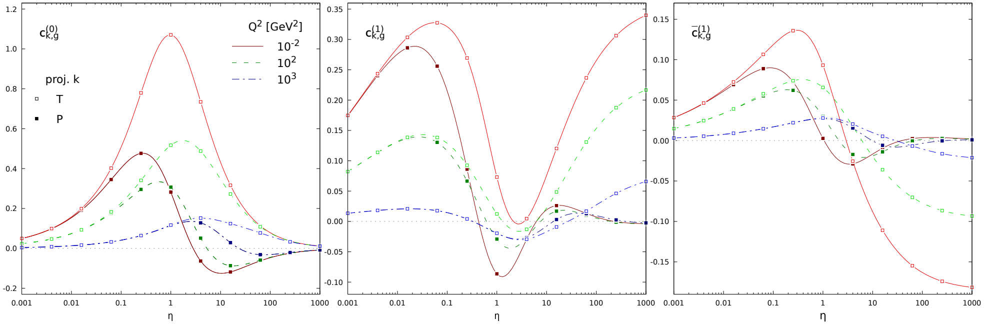
Outlook

- inclusive distributions: $\frac{dg_1}{dp_{T,\bar{Q}}}, \frac{dg_1}{dy_{\bar{Q}}}, \ldots$ [Laenen,Riemersma,Smith,van Neerven]
- correlated distributions: $\frac{dg_1}{dM^2_{Q\bar{Q}}}, \frac{dg_1}{d\phi_{Q\bar{Q}}}$, TMD, $\ldots$ [Harris,Smith]
- full neutral current (NC) contributions: $F_3^{Z\gamma}, g_4^{Z\gamma}, g_5^{Z\gamma}$ and F_2^Z, F_L^Z, g_1^Z
- distributions of full NC structure functions: $\frac{dg_1^{NC}}{dp_{T,\bar{Q}}}, \frac{dg_1^{NC}}{dM^2_{Q\bar{Q}}}, \ldots$

Thank you for your attention!

Backup: Partonic Results - Gluon Channel

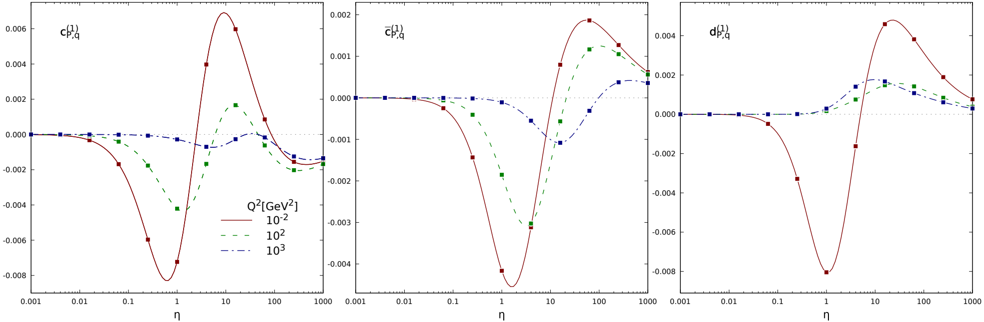
$$g_1 \sim \alpha_s \cdot \Delta g \otimes \left(c^{(0)}_{P,g} + 4\pi\alpha_s \left[c^{(1)}_{P,g} + \ln\left(\frac{\mu^2}{m^2}\right) \bar{c}^{(1)}_{P,g} \right] \right)$$



$$\eta = \frac{s - 4m^2}{4m^2}, \quad m = m_b = 4.75\,\mathrm{GeV}$$

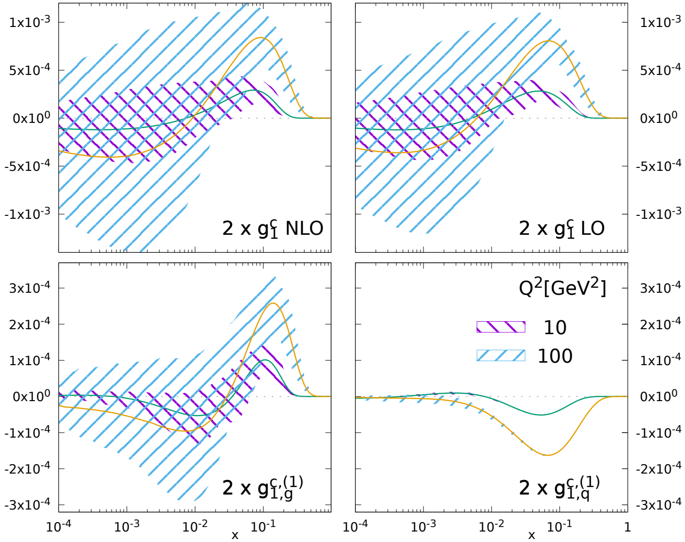
Backup: Partonic Results - Light Quark Channel

$$g_1 \sim \alpha_s^2 \sum_q (\Delta q + \Delta\bar{q}) \otimes \left(e_H^2 \left[c_{P,q}^{(1)} + \ln\left(\frac{\mu^2}{m^2}\right) \bar{c}_{P,q}^{(1)} \right] + e_q^2 d_{P,q}^{(1)} \right)$$



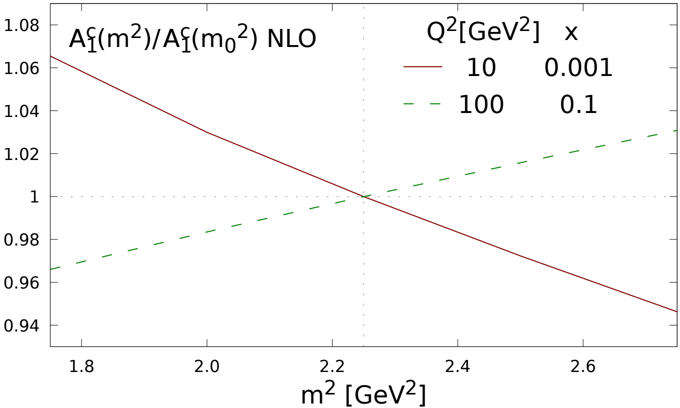
$$\eta = \frac{s - 4m^2}{4m^2}, \quad m = m_b = 4.75\,\text{GeV}$$

Backup: Hadronic Results - PDF Uncertainties DSSV



- gluon dominates
- sign uncontrained
- quark band small
- default gluon is small

Backup: Hadronic Results - Mass Variation



$$m_0 = 1.5\,\mathrm{GeV}$$