1 2 to 2 phase space

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process:

$$\gamma^*(q) + g(k_1) \to Q(p_1) + \bar{Q}(p_2) \tag{1}$$

kinematics:

$$s = (q + k_1)^2 \qquad s' = s - q^2 \tag{2}$$
$$t = (k_1 - p_1)^2 \qquad t_1 = t - m^2 \tag{3}$$
$$u = (k_1 - p_2)^2 \qquad u_1 = u - m^2 \tag{4}$$

use c.m.s. of incoming particles:

$$q = \left(\frac{s+q^2}{2\sqrt{s}}, 0, 0, \ldots, -\frac{s-q^2}{2\sqrt{s}}\right) \tag{5}$$
$$k_1 = \frac{s-q^2}{2\sqrt{s}}\,(1, 0, 0, \ldots, 1) \tag{6}$$

such that

$$q + k_1 = (\sqrt{s}, \vec{0}) \qquad k_1^2 = 0 \tag{7}$$

for the outgoing particles it follows

$$p_1 = \frac{\sqrt{s}}{2}\,(1, 0, \beta\sin\theta, \ldots, \beta\cos\theta) \tag{8}$$
$$p_2 = \frac{\sqrt{s}}{2}\,(1, 0, -\beta\sin\theta, \ldots, -\beta\cos\theta) \tag{9}$$

with $\beta = \sqrt{1 - 4m^2/s}$ such that

$$p_1 + p_2 = (\sqrt{s}, \vec{0}) \qquad p_1^2 = p_2^2 = m^2 \tag{10}$$

use n-sphere:

$$d^D x = \Omega_D x^{D-1} dx = \frac{2\pi^{D/2}}{\Gamma(D/2)} x^{D-1} dx = \frac{\pi^{D/2}}{\Gamma(D/2)} (x^2)^{(D-2)/2} dx^2 \tag{11}$$

compute phase space:

$$PS_2 = \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_1}{(2\pi)^{n-1}} (2\pi)^n \delta^{(n)}(q + k_1 - p_1 - p_2)\delta(p_1^2 - m^2)\delta(p_2^2 - m^2) \tag{12}$$
$$= \frac{1}{(2\pi)^{n-2}} \int d^n p_1\, \delta((q + k_1 - p_2)^2 - m^2)\delta(p_1^2 - m^2) \tag{13}$$
$$= \frac{1}{(2\pi)^{n-2}} \int dp_{1,0} dp_{1,||} d^2 p_{1,\perp} d^{n-4}\hat{p}_1\, \delta(s - 2p_{1,0}\sqrt{s})\delta(p_{1,0}^2 - p_{1,||}^2 - p_{1,\perp}^2 - \hat{p}_1^2 - m^2) \tag{14}$$
$$= \frac{\pi}{(2\pi)^{n-2} 2\sqrt{s}} \int dp_{1,||} dp_{1,\perp}^2 d^{n-4}\hat{p}_1\, \delta(s/4 - p_{1,||}^2 - p_{1,\perp}^2 - \hat{p}_1^2 - m^2) \tag{15}$$
$$= \frac{\pi}{(2\pi)^{n-2} 2\sqrt{s}} \int dp_{1,||} d\hat{p}_1^2\, \frac{\pi^{(n-4)/2}}{\Gamma((n-4)/2)} (\hat{p}_1^2)^{(n-6)/2} \tag{16}$$
$$= \frac{1}{2\sqrt{s}\Gamma((n-4)/2)(4\pi)^{(n-2)/2}} \int dp_{1,||} d\hat{p}_1^2\, (\hat{p}_1^2)^{(n-6)/2} \tag{17}$$

Integration borders are

$$p_{1,||} \in \frac{\sqrt{s}}{2}\beta \cdot [-1,1] \qquad \hat{p}_1^2 \in \left(\frac{s\beta^2}{4} - p_{1,||}^2\right) \cdot [0,1] \tag{18}$$

if cross section does not depend on hat-space:

$$\int d\hat{p}_1^2\, (\hat{p}_1^2)^{(n-6)/2} = \frac{2}{n-4}\left(\frac{s\beta^2}{4} - p_{1,||}^2\right)^{(n-4)/2} \tag{19}$$

$$\Rightarrow PS_2 = \frac{1}{2\sqrt{s}\Gamma((n-2)/2)(4\pi)^{(n-2)/2}} \int dp_{1,||} \left(\frac{s\beta^2}{4} - p_{1,||}^2\right)^{(n-4)/2} \tag{20}$$

rewrite $p_{1,||}$ to $\cos\theta$:

$$p_{1,||} = \frac{\sqrt{s}}{2}\beta\cos\theta \Rightarrow \quad dp_{1,||} = \frac{\sqrt{s}}{2}\beta\, d\cos\theta, \quad \cos\theta \in [-1,1],\, \hat{p}_1^2 \in \frac{s\beta^2}{4}\left(1-\cos^2\theta\right)\cdot[0,1] \tag{21}$$

rewrite $\cos\theta$ to $t_1 = (k_1 - p_2)^2 - m^2$:

$$\cos\theta = \frac{2t_1/s' + 1}{\beta} \Rightarrow \quad d\cos\theta = \frac{2}{\beta s'}dt_1, \quad t_1 \in \frac{s'}{2}[-\beta-1, \beta-1],\, \hat{p}_1^2 \in (-m^2 - \frac{st_1}{s'^2}(s'+t_1))\cdot[0,1] \tag{22}$$

$$p_{1,||} = \sqrt{s}\left(\frac{t_1}{s'} + \frac{1}{2}\right) \Rightarrow \quad dp_{1,||} = \frac{\sqrt{s}}{s'}dt_1 \tag{23}$$

$$\Rightarrow PS_2 = \frac{1}{2s'\Gamma((n-2)/2)(4\pi)^{(n-2)/2}} \int dt_1 \left(\frac{(t_1(u_1-q^2) - s'm^2)s' - q^2t_1^2}{s'^2}\right)^{(n-4)/2} \tag{24}$$

2 2 to 3 phase space

process:

$$\gamma^*(q) + q(k_1) \to Q(p_1) + \bar{Q}(p_2) + q(k_2) \tag{25}$$

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2.1 kinematic constraints

definitions of kinematic variables:

$$\begin{aligned}
s &= (q+k_1)^2 & \Rightarrow && 2qk_1 &= s - q^2 & (26)\\
s_3 &= (k_2+p_2)^2 - m^2 & \Rightarrow && 2k_2p_2 &= s_3 & (27)\\
s_4 &= (k_2+p_1)^2 - m^2 & \Rightarrow && 2k_2p_1 &= s_4 & (28)\\
s_5 &= (p_1+p_2)^2 = -u_5 & \Rightarrow && 2p_1p_2 &= s_5 - 2m^2 & (29)\\
t_1 &= (k_1-p_2)^2 - m^2 = t - m^2 & \Rightarrow && 2k_1p_2 &= -t_1 & (30)\\
t' &= (k_1-k_2)^2 & \Rightarrow && 2k_1k_2 &= -t' & (31)\\
u_1 &= (q-p_2)^2 - m^2 = u - m^2 & \Rightarrow && 2qp_2 &= -u_1 + q^2 & (32)\\
u_6 &= (k_1-p_1)^2 - m^2 & \Rightarrow && 2k_1p_1 &= -u_6 & (33)\\
u_7 &= (q-p_1)^2 - m^2 & \Rightarrow && 2qp_1 &= -u_7 + q^2 & (34)\\
u' &= (q-k_2)^2 & \Rightarrow && 2qk_2 &= -u' + q^2 & (35)
\end{aligned}$$

impose momentum conservation:

$$q + k_1 = p_1 + p_2 + k_2 \tag{36}$$

contract with 2 times momentum:

$$\begin{aligned}
2q^2 \quad +s - q^2 &= -u_7 + q^2 \quad -u_1 + q^2 \quad -u' + q^2 &&\Leftrightarrow \quad 0 = s + u_1 + u_7 + u' - 2q^2 && (37)\\
s - q^2 \quad +0 &= -u_6 \quad -t_1 \quad -t' &&\Leftrightarrow \quad 0 = s + t_1 + t' + u_6 - q^2 && (38)\\
-u_7 + q^2 \quad -u_6 &= 2m^2 \quad +s_5 - 2m^2 \quad +s_4 &&\Leftrightarrow \quad 0 = s_4 + s_5 + u_6 + u_7 - q^2 && (39)\\
-u_1 + q^2 \quad -t_1 &= s_5 - 2m^2 \quad +2m^2 \quad +s_3 &&\Leftrightarrow \quad 0 = s_3 + s_5 + t_1 + u_1 - q^2 && (40)\\
-u' + q^2 \quad -t' &= s_4 \quad +s_3 \quad +0 &&\Leftrightarrow \quad 0 = s_3 + s_4 + t' + u' - q^2 && (41)
\end{aligned}$$

$$\frac{1}{2}\left((37) + (38) + (40) - (39) - (41)\right) = 0 = s - q^2 + t_1 + u_1 - s_4 \tag{42}$$

2.2 choose framework

use c.m.s. of recoiling heavy and light quark ($Q(p_1)$ and $q(k_2)$):

$$\begin{aligned}
k_2 &= (\omega_2, k_{2,x}, \omega_2 \sin\theta_1 \cos\theta_2, \omega_2 \cos\theta_1, \hat{k}_2) && (43)\\
p_1 &= (E_1, -k_{2,x}, -\omega_2 \sin\theta_1 \cos\theta_2, -\omega_2 \cos\theta_1, -\hat{k}_2) && (44)\\
k_1 &= (\omega_1, 0, 0, \omega_1, \hat{0}) && (45)\\
q &= (q_0, 0, |\vec{p}_2| \sin\psi, |\vec{p}_2| \cos\psi - \omega_1, \hat{0}) && (46)\\
p_2 &= (E_2, 0, |\vec{p}_2| \sin\psi, |\vec{p}_2| \cos\psi, \hat{0}) && (47)
\end{aligned}$$

light quark masses are already fixed: $k_1^2 = 0 = k_2^2$

constraints:

$$\begin{aligned}
q_0 + \omega_1 &= E_1 + E_2 + \omega_2 && (48)\\
m^2 = p_1^2 &= E_1^2 - \omega_2^2 && (49)\\
m^2 = p_2^2 &= E_2^2 - |\vec{p}_2|^2 && (50)\\
q^2 &= q_0^2 - |\vec{p}_2|^2 + 2\,|\vec{p}_2|\,\omega_1 \cos\psi - \omega_1^2 && (51)\\
s = (q + k_1)^2 &= (q_0 - \omega_1)^2 - |\vec{p}_2|^2 && (52)\\
t = (k_1 - p_2)^2 &= (\omega_1 - E_2)^2 - |\vec{p}_2|^2 + 2\,|\vec{p}_2|\,\omega_1 \cos\psi - \omega_1^2 && (53)\\
u = (q - p_2)^2 &= (q_0 - E_2)^2 - \omega_1^2 && (54)
\end{aligned}$$

solve:

$$(52) - (51) + (53) - (50) + (54) = s - q^2 + t - m^2 + u \tag{55}$$

$$= s_4 + m^2 = (E_1 + \omega_2)^2 \tag{56}$$

$$(53) + (54) - (51) = t + u - q^2 = -2(E_1 + \omega_2)E_2 \tag{57}$$

$$\Rightarrow E_2 = -\frac{t + u - q^2}{2\sqrt{s_4 + m^2}} = \frac{s - s_4 - 2m^2}{2\sqrt{s_4 + m^2}} \tag{58}$$

$$(56) \wedge (49) \Rightarrow \omega_2 = \frac{s_4}{2\sqrt{s_4 + m^2}} \tag{59}$$

$$(56) \Rightarrow E_1 = \frac{s_4 + 2m^2}{2\sqrt{s_4 + m^2}} \tag{60}$$

$$(52) + (54) - (50) = s + u - m^2 = 2q_0(E_1 + \omega_2) \tag{61}$$

$$\Rightarrow q_0 = \frac{s + u - m^2}{2\sqrt{s_4 + m^2}} \tag{62}$$

$$(53) - (51) = t - q^2 = (\omega_1 - E_2)^2 - q_0^2 \tag{63}$$

$$\Rightarrow \omega_1 = \frac{s_4 + m^2 - u}{2\sqrt{s_4 + m^2}} \tag{64}$$

$$(50) \Rightarrow |\vec{p}_2| = \sqrt{E_2^2 - m^2} = \frac{\sqrt{(s - s_4)^2 - 4sm^2}}{2\sqrt{s_4 + m^2}} \tag{65}$$

$$(51) \Rightarrow \cos\psi = \frac{q^2 - q_0^2 + |\vec{p}_2|^2 + \omega_1^2}{2\,|\vec{p}_2|\,\omega_1} \tag{66}$$

$$= \frac{2u(q^2 - s - m^2 + t) - (2m^2 - q^2 - t)(s_4 + m^2 - u)}{(s_4 + m^2 - u)\sqrt{(s - s_4)^2 - 4sm^2}} \tag{67}$$

$$t' = -2k_1k_2 = -2\omega_1\omega_2(1 - \cos\theta_1) \tag{68}$$

$$u_6 = -2k_1p_1 = -2\omega_1(E_1 + \omega_2\cos\theta_1) \tag{69}$$

$$(38): \qquad 0 = s + t_1 + t' + u_6 - q^2 \quad \checkmark \tag{70}$$

$$s_3 = 2k_2p_2 = 2\omega_2(E_2 - |\vec{p}_2|\,(\sin\psi\sin\theta_1\cos\theta_2 + \cos\psi\cos\theta_1)) \tag{71}$$

$$s_5 = (p_1 + p_2)^2 = 2m^2 + 2p_1p_2 \tag{72}$$

$$= 2(m^2 + E_1E_2 + \omega_2\,|\vec{p}_2|\,(\sin\psi\sin\theta_1\cos\theta_2 + \cos\psi\cos\theta_1)) \tag{73}$$

$$(40): \qquad 0 = s_3 + s_5 + t_1 + u_1 - q^2 \tag{74}$$

$$u' = (q - k_2)^2 = q^2 - 2qk_2 \tag{75}$$

$$= q^2 - 2(q_0\omega_2 - \omega_2\,|\vec{p}_2|\,(\sin\psi\sin\theta_1\cos\theta_2 + \cos\psi\cos\theta_1) - \omega_1\omega_2\cos\theta_1) \tag{76}$$

$$(37): \qquad 0 = s + u_1 + u_7 + u' - 2q^2 \tag{77}$$