

1 Introduction

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1.1 Motivation

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1.2 Notation

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FiXme Error: write more notation We use the definition of [1] for the hadronic tensor:

$$W_{\mu\mu'} = (-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2})F_1(x, Q^2) + \frac{\hat{P}_\mu \hat{P}_{\mu'}}{P \cdot q} F_2(x, Q^2) \quad (1)$$

2 Leading Order Calculations

In leading order we have to consider photon-gluon-fusion (PGF), that is

$$\gamma^*(q) + g(k_1) \rightarrow Q(p_1) + \bar{Q}(p_2) \quad (2)$$

with two contributing diagrams depicted in figure 1.

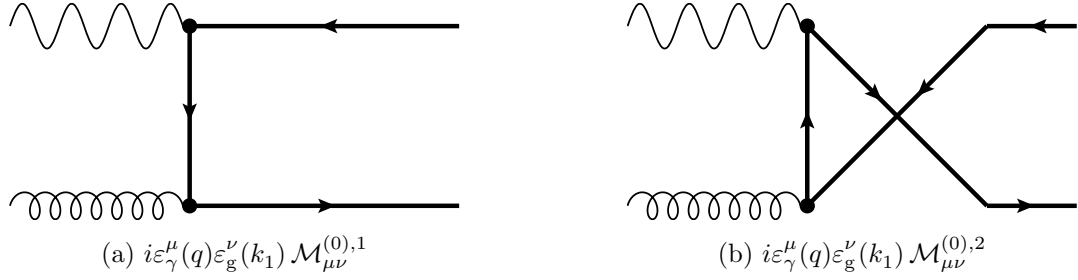


Figure 1: leading order Feynman diagrams

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The result can then be written as

$$\hat{\mathcal{P}}_k^{\gamma,\mu\mu'}\hat{\mathcal{P}}_k^{g,\nu\nu'}\sum_{j,j'=1}^2\mathcal{M}_{\mu\nu}^{(0),j}\left(\mathcal{M}_{\mu'\nu'}^{(0),j'}\right)^*=8g^2\mu_D^{-\epsilon}e^2e_H^2N_C C_F B_{k,QED} \quad (3)$$

where g and e are the strong and electromagnetic coupling constants respectively, μ_D is an arbitrary mass parameter introduced to keep the couplings dimensionless and e_H is the magnitude of the heavy quark in units of e . Further N_C corresponds to the gauge group $SU(N_C)$ and the color factor $C_F = (N_C^2 - 1)/(2N_C)$ refers to the second Casimir constant of the fundamental representation for the quarks. We then find:

$$B_{QED, VV, F, P_g} = -2 + \frac{2(2m^2 + q^2)s + s'^2}{t_1 u_1} - \frac{2(2m^2 + q^2)m^2 s'^2}{(t_1 u_1)^2} + \epsilon \left\{ -1 + \frac{s^2 - q^2 s'}{t_1 u_1} - \frac{m^2 q^2 s'^2}{t_1^2 u_1^2} \right\} + \epsilon^2 \frac{s'^2}{4t_1 u_1} \quad (4)$$

$$B_{QED, VV, F, P_{k1k1}} = -\frac{4q^2}{s'} \left(\frac{s}{s'} - \frac{m^2 s'}{t_1 u_1} \right) \quad (5)$$

$$B_{P, QED} = \frac{1}{2} \left(\frac{t_1}{u_1} + \frac{u_1}{t_1} \right) \left(\frac{2m^2 s'}{t_1 u_1} - 1 - \frac{2q^2}{s'} \right) \quad (6)$$

$$B_{k, QED} = B_{k, QED}^{(0)} + \epsilon B_{k, QED}^{(1)} + \epsilon^2 B_{k, QED}^{(2)} \quad (7)$$

3 Next-To-Leading Order Calculations

3.1 One Loop Virtual Contributions

$$\begin{aligned} M_k^{(1), V} &= \hat{\mathcal{P}}_k^{\gamma, \mu \mu'} \hat{\mathcal{P}}_k^g \sum_j \left[\mathcal{M}_{j, \mu}^{(1), V} \left(\mathcal{M}_{1, \mu'}^{(0)} + \mathcal{M}_{2, \mu'}^{(0)} \right)^* + c.c. \right] \\ &= 8g^4 \mu_D^{-\epsilon} e^2 e_H^2 N_C C_F C_\epsilon (C_A V_{k, OK} + 2C_F V_{k, QED}) \end{aligned} \quad (8)$$

where $C_\epsilon = \exp(\epsilon/2(\gamma_E - \ln(4\pi)))/(16\pi^2)$ and C_A is the second Casimir constant of the adjoint representation for the gluon (that introduces a non-abelian part).

As the short example above shows, the full expressions for the $V_{k, OK}$, $V_{k, QED}$ are quite complicated and too long to be presented here, nevertheless the arising poles are quite compact:

$$V_{k, OK} = -2B_{k, QED} \left(\frac{4}{\epsilon^2} + \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) - \frac{2m^2 - s}{s} \ln(\chi) \right) \frac{2}{\epsilon} \right) + O(\epsilon^0) \quad (9)$$

$$V_{k, QED} = -2B_{k, QED} \left(1 + \frac{2m^2 - s}{s} \ln(\chi) \right) \frac{2}{\epsilon} + O(\epsilon^0) \quad (10)$$

The above results already include the mass renormalization that we have performed *on-shell*, so all ultra-violet poles have been removed. For the renormalization of the strong

coupling we use the $\overline{\text{MS}}_m$ scheme defined in [2] and so the full (remaining) renormalization can be achieved by

$$\frac{d^2\sigma_k^{(1),V,ren.}}{dt_1 du_1} = \frac{d^2\sigma_k^{(1),V}}{dt_1 du_1} + \frac{\alpha_s(\mu_R^2)}{4\pi} \left[\left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln(\mu_R^2/m^2) - \ln(\mu_D^2/m^2) \right) \beta_0^f \right. \\ \left. + \frac{2}{3} \ln(\mu_R^2/m^2) \right] \frac{d^2\sigma_k^{(0)}}{dt_1 du_1} \quad (11)$$

$$= \frac{d^2\sigma_k^{(1),V}}{dt_1 du_1} + 4\pi\alpha_s(\mu_R^2)C_\epsilon \left(\frac{\mu_D^2}{m^2} \right)^{-\epsilon/2} \left[\left(\frac{2}{\epsilon} + \ln(\mu_R^2/m^2) \right) \beta_0^f \right. \\ \left. + \frac{2}{3} \ln(\mu_R^2/m^2) \right] \frac{d^2\sigma_k^{(0)}}{dt_1 du_1} \quad (12)$$

with μ_R the renormalization scale introduced by the RGE, $\beta_0^f = (11C_A - 2n_f)/3$ the first coefficient of the beta function and n_f the number of *total* flavours (i.e. $n_{lf} = n_f - 1$ active (light) flavours and one heavy flavour). The double poles occurring in $V_{k,OK}$ are introduced by the diagrams **FiXme Error: do** when the soft and collinear singularities coincide. FiXme Error!

The partonic cross section is given by

$$d\sigma_{k,g}^{(1),V} = \frac{1}{2s'} \frac{1}{2} E_k(\epsilon) b_k(\epsilon) M_k^{(1),V} dPS_2 \quad (13)$$

3.2 Single Gluon Radiation

In next-to-leading order we have to consider the following process:

$$\gamma^*(q) + g(k_1) \rightarrow Q(p_1) + \bar{Q}(p_2) + g(k_2) \quad (14)$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as FiXme Error!

$$\hat{\mathcal{P}}_k^{\gamma,\mu\mu'} \hat{\mathcal{P}}_k^g \sum_{j,j'} \mathcal{M}_{j,\mu}^{(1),g} \mathcal{M}_{j',\mu'}^{(1),g*} = 8g^4 \mu_D^{-2\epsilon} e^2 e_H^2 N_C C_F (C_A R_{k,OK} + 2C_F R_{k,QED}) \quad (15)$$

and it will depend on ten kinematical invariants:

$$s = (q + k_1)^2 \quad t_1 = (k_1 - p_2)^2 - m^2 \quad u_1 = (q - p_2)^2 - m^2 \quad (16)$$

$$s_3 = (k_2 + p_2)^2 - m^2 \quad s_4 = (k_2 + p_1)^2 - m^2 \quad s_5 = (p_1 + p_2)^2 = -u_5 \quad (17)$$

$$t' = (k_1 - k_2)^2 \quad (18)$$

$$u' = (q - k_2)^2 \quad u_6 = (k_1 - p_1)^2 - m^2 \quad u_7 = (q - p_1)^2 - m^2 \quad (19)$$

from which only five are independent as can be seen from momentum conservation $k_1 + q = p_1 + p_2 + k_2$ and s, t_1, u_1 match to their leading order definition.

The $2 \rightarrow 3$ n -dimensional phase space is given by

$$dPS_3 = \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_2}{(2\pi)^{n-1}} \frac{d^n k_2}{(2\pi)^{n-1}} (2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2 - k_2) \Theta(p_{1,0}) \delta(p_1^2 - m^2) \Theta(p_{2,0}) \delta(p_2^2 - m^2) \Theta(k_{2,0}) \delta(k_2^2) \quad (20)$$

This can be solved by writing eq. (20) as product of a $2 \rightarrow 2$ decay and a subsequent $1 \rightarrow 2$ decay[3]. We find

$$dPS_3 = \frac{1}{(4\pi)^n \Gamma(n-3) s'} \frac{s_4^{n-3}}{(s_4 + m^2)^{n/2-1}} \left(\frac{(t_1 u'_1 - s' m^2) s' - q^2 t_1^2}{s'^2} \right)^{(n-4)/2} dt_1 du_1 d\Omega_n d\hat{\mathcal{I}} \quad (21)$$

$$= h_3(n) dt_1 du_1 d\Omega_n d\hat{\mathcal{I}} \quad (22)$$

with $d\Omega_n = \sin^{n-3}(\theta_1) d\theta_1 \sin^{n-4}(\theta_2) d\theta_2$ and $d\hat{\mathcal{I}}$ taking care of all occuring hat momenta:

$$d\hat{\mathcal{I}} = \frac{1}{B(1/2, (n-4)/2)} \frac{x^{(n-6)/2}}{\sqrt{1-x}} dx \quad \text{with } x = \hat{p}_1^2 / \hat{p}_{1,max} \quad (23)$$

$$\hat{p}_{1,max} = \frac{s_4^2}{4(s_4 + m^2)} \sin^2(\theta_1) \sin^2(\theta_2) \quad (24)$$

$$\Rightarrow \int d\hat{\mathcal{I}} = 1 \quad \int d\hat{\mathcal{I}} \hat{p}_1^2 = \epsilon \hat{p}_{1,max} + O(\epsilon^2) \quad (25)$$

Again when integrating the phase space angles the expressions become quite lengthy, but the (collinear) pole parts are compact:

$$\frac{s_4}{4\pi(s_4 + m^2)} \int d\Omega_n d\hat{\mathcal{I}} C_A R_{k,OK} = -\frac{1}{u_1} B_{k,QED} \left(\frac{s' \rightarrow x_1 s'}{t_1 \rightarrow x_1 t_1} \right) P_{k,gg}^H(x_1) \frac{2}{\epsilon} + O(\epsilon^0) \quad (26)$$

with $x_1 = -u_1/(s' + t_1)$ and the hard part of the Altarelli-Parisi splitting functions $P_{k,gg}^H$ [4, 5]:

$$P_{G,gg}^H(x) = P_{L,gg}^H(x) = C_A \left(\frac{2}{1-x} + \frac{2}{x} - 4 + 2x - 2x^2 \right) \quad (27)$$

$$P_{P,gg}^H(x) = C_A \left(\frac{2}{1-x} - 4x + 2 \right) \quad (28)$$

The $R_{k,QED}$ do not contain poles. **FiXme Error: shift to factorization?**

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From the above expression we can obtain the soft limit $k_2 \rightarrow 0$ and separete their calculations:

$$\lim_{k_2 \rightarrow 0} (C_A R_{k,OK} + 2C_F R_{k,QED}) = (C_A S_{k,OK} + 2C_F S_{k,QED}) + O(1/s_4, 1/s_3, 1/t') \quad (29)$$

$$S_{k,OK} = 2 \left(\frac{t_1}{t' s_3} + \frac{u_1}{t' s_4} - \frac{s - 2m^2}{s_3 s_4} \right) B_{k,QED} \quad (30)$$

$$S_{k,QED} = 2 \left(\frac{s - 2m^2}{s_3 s_4} - \frac{m^2}{s_3^2} - \frac{m^2}{s_4^2} \right) B_{k,QED} \quad (31)$$

Note that the einkonal factors multiplying the Born functions $B_{k,QED}$ neither depend on q^2 nor on the projection k .

3.3 Light Quark Processes

In next-to-leading order a new production mechanism enters that is induced by a light quark, so we have to consider the process

$$\gamma^*(q) + q(k_1) \rightarrow Q(p_1) + \bar{Q}(p_2) + q(k_2) \quad (32)$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as **FiXme Error!**

$$\hat{\mathcal{P}}_k^{\gamma, \mu \mu'} \hat{\mathcal{P}}_k^{q, aa'} \sum_{j, j'=1}^4 \mathcal{M}_{j, \mu a}^{(1), q} \left(\mathcal{M}_{j', \mu' a'}^{(1), q} \right)^* = 8g^4 \mu_D^{-2\epsilon} e^2 N_C C_F \left(e_H^2 A_{k,1} + e_L^2 A_{k,2} + e_L e_H A_{k,3} \right) \quad (33)$$

where e_L denotes the charge of the light quark q in units of e .

The needed $2 \rightarrow 3$ phase space has already been calculated in section 3.2, so we can immediately quote the (collinear) poles:

$$\frac{s_4}{2\pi(s_4 + m^2)} \int d\Omega_n d\hat{\mathcal{L}} C_F A_{k,1} = -\frac{1}{u_1} B_{k,QED} \left(\frac{s' \rightarrow x_1 s'}{t_1 \rightarrow x_1 t_1} \right) P_{k,gq}(x_1) \frac{2}{\epsilon} + O(\epsilon^0) \quad (34)$$

with $x_1 = -u_1/(s' + t_1)$ and the Altarelli-Parisi splitting functions $P_{k,gq}$ [4, 5]:

$$P_{G,gq}(x) = P_{L,gq}(x) = C_F \left(\frac{1}{x} + \frac{(1-x)^2}{x} \right) \quad (35)$$

$$P_{P,gq}(x) = C_F (2 - x) \quad (36)$$

$A_{k,2}$ does not contain poles and we find $\int dt_1 du_1 \int d\Omega_n d\hat{\mathcal{L}} A_{k,3} = 0$. Note that in the limit $q^2 \rightarrow 0$ $A_{k,2}$ will also get collinear poles.

4 Mass Factorization

All collinear poles in the gluonic subprocess can be removed by mass factorization in the following way:

$$s'^2 \frac{d^2 \sigma_{\vec{k},g}^{(1),fin}(s', t_1, u_1, q^2, \mu_F)}{dt_1 du_1} = \lim_{\epsilon \rightarrow 0} \left[s'^2 \frac{d^2 \sigma_{\vec{k},g}^{(1)}(s', t_1, u_1, q^2, \epsilon)}{dt_1 du_1} - \int_0^1 \frac{dx_1}{x_1} \Gamma_{\vec{k},gg}^{(1)}(x_1, \mu_F^2, \mu_D, \epsilon) \right. \quad (37)$$

$$\left. (x_1 s')^2 \frac{d^2 \sigma_{\vec{k},g}^{(0)}(x_1 s', x_1 t_1, u_1, q^2, \epsilon)}{d(x_1 t_1) du_1} \right] \quad (38)$$

$$\Gamma_{\vec{k},ij}^{(1)}(x, \mu_F^2, \mu_D, \epsilon) = \frac{\alpha_s}{2\pi} \left(P_{\vec{k},ij}(x) \frac{2}{\epsilon} + f_{\vec{k},ij}(x, \mu_F^2, \mu_D^2) \right) \quad (39)$$

where $\Gamma_{\vec{k},ij}^{(1)}$ is the first order correction to the transition functions $\Gamma_{\vec{k},ij}$ for *incoming* particle j and *outgoing* particle i in projection k . In the $\overline{\text{MS}}$ -scheme the $f_{\vec{k},ij}$ take their usual form and we find

$$\Gamma_{\vec{k},ij}^{(1),\overline{\text{MS}}}(x, \mu_F^2, \mu_D, \epsilon) = \frac{\alpha_s}{2\pi} P_{\vec{k},ij}(x) \left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln(\mu_F^2/m^2) - \ln(\mu_D^2/m^2) \right) \quad (40)$$

$$= 8\pi\alpha_s P_{\vec{k},ij}(x) C_\epsilon \left(\frac{\mu_D^2}{m^2} \right)^{-\epsilon/2} \left(\frac{2}{\epsilon} + \ln(\mu_F^2/m^2) \right) \quad (41)$$

The $P_{\vec{k},ij}(x)$ are the Altarelli-Parisi splitting functions for which we find [4, 5]

$$P_{\vec{k},gg}(x) = \Theta(1 - \delta - x) P_{\vec{k},gg}^H(x) + \delta(1 - x) \left(2C_A \ln(\delta) + \frac{\beta_0}{2} \right) \quad (42)$$

where we introduced another infrared cut-off δ to separate soft ($x \geq 1 - \delta$) and hard ($x < 1 - \delta$) gluons that is connected to Δ via $\delta = \Delta/(s' + t_1)$. The structure here explains why we were able to write the equation (26).

The light quark process can be regularized in a complete analogous way:

$$s'^2 \frac{d^2 \sigma_{\vec{k},q}^{(1),fin}(s', t_1, u_1, q^2, \mu_F)}{dt_1 du_1} = \lim_{\epsilon \rightarrow 0} \left[s'^2 \frac{d^2 \sigma_{\vec{k},q}^{(1)}(s', t_1, u_1, q^2, \epsilon)}{dt_1 du_1} - \int_0^1 \frac{dx_1}{x_1} \Gamma_{\vec{k},gq}^{(1)}(x_1, \mu_F^2, \mu_D, \epsilon) \right. \quad (43)$$

$$\left. (x_1 s')^2 \frac{d^2 \sigma_{\vec{k},g}^{(0)}(x_1 s', x_1 t_1, u_1, q^2, \epsilon)}{d(x_1 t_1) du_1} \right]$$

The needed splitting functions $P_{\vec{k},gq}$ have been already quoted in equations (35) and (36). Note that $K_{q\gamma} = 1/(N_C) = 2C_F K_{g\gamma}$.

The final finite cross sections are then

$$\begin{aligned}
s'^2 \frac{d^2 \sigma_{\vec{k},g}^{(1),H,fin}}{dt_1 du_1} &= \frac{1}{2\pi} K_{g\gamma} \alpha \alpha_S e_H^2 N_C C_F \left[-\frac{1}{u_1} P_{\vec{k},gg}^H(x_1) \right. \\
&\quad \left\{ 4\pi B_{\vec{k},QED}^{(0)} \left(\begin{matrix} s' \rightarrow x_1 s' \\ t_1 \rightarrow x_1 t_1 \end{matrix} \right) \left(\ln \left(\frac{s_4^2}{m^2(s_4 + m^2)} \right) - \ln(\mu_F^2/m^2) \right) \right. \\
&\quad \left. \left. - 8\pi B_{\vec{k},QED}^{(1)} \left(\begin{matrix} s' \rightarrow x_1 s' \\ t_1 \rightarrow x_1 t_1 \end{matrix} \right) \right\} \right. \\
&\quad \left. + C_A \frac{s_4}{s_4 + m^2} \left(\int d\Omega_n d\hat{\mathcal{I}} R_{\vec{k},OK} \right)^{finite} \right. \\
&\quad \left. + 2C_F \frac{s_4}{s_4 + m^2} \int d\Omega_4 d\hat{\mathcal{I}} R_{\vec{k},QED} \right] \quad (44)
\end{aligned}$$

$$\begin{aligned}
s'^2 \frac{d^2 \sigma_{\vec{k},g}^{(1),S+V,fin}}{dt_1 du_1} &= 4K_{g\gamma} \alpha \alpha_S e_H^2 N_C C_F B_{\vec{k},QED}^{(0)} \delta(s' + t_1 + u_1) \left[C_A \ln^2(\Delta/m^2) \right. \\
&\quad \left. + \ln(\Delta/m^2) \left(\left(\ln(-t_1/m^2) - \ln(-u_1/m^2) - \ln(\mu_F^2/m^2) \right) C_A \right. \right. \\
&\quad \left. \left. - \frac{2m^2 - s}{s\beta} \ln(\chi)(C_A - 2C_F) - 2C_F \right) \right. \\
&\quad \left. + \frac{\beta_0^{lf}}{4} \left(\ln(\mu_R^2/m^2) - \ln(\mu_F^2/m^2) \right) + f_{\vec{k}}(s', u_1, t_1, q^2) \right] \quad (45)
\end{aligned}$$

where $f_{\vec{k}}$ contains lots of logarithms and dilogarithms, but does not depend on Δ, μ_F^2, μ_R^2 nor n_f and $\beta_0^{lf} = (11C_A - 2n_{lf})/3$.

$$\begin{aligned}
s'^2 \frac{d^2 \sigma_{\vec{k},q}^{(1),fin}}{dt_1 du_1} &= \frac{1}{2\pi} K_{q\gamma} \alpha \alpha_S N_C \left[-\frac{1}{u_1} e_H^2 P_{\vec{k},gq}(x_1) \right. \\
&\quad \left\{ 2\pi B_{\vec{k},QED}^{(0)} \left(\begin{matrix} s' \rightarrow x_1 s' \\ t_1 \rightarrow x_1 t_1 \end{matrix} \right) \left(\ln \left(\frac{s_4^2}{m^2(s_4 + m^2)} \right) - \ln(\mu_F^2/m^2) - 2\partial_\epsilon E_{\vec{k}}(\epsilon=0) \right) \right. \\
&\quad \left. \left. - 4\pi B_{\vec{k},QED}^{(1)} \left(\begin{matrix} s' \rightarrow x_1 s' \\ t_1 \rightarrow x_1 t_1 \end{matrix} \right) \right\} \right. \\
&\quad \left. + C_F \frac{s_4}{s_4 + m^2} \left(\int d\Omega_n d\hat{\mathcal{I}} e_H^2 A_{\vec{k},1} \right)^{finite} \right. \\
&\quad \left. + C_F \frac{s_4}{s_4 + m^2} \int d\Omega_4 d\hat{\mathcal{I}} e_L^2 A_{\vec{k},2} + C_F \frac{s_4}{s_4 + m^2} \int d\Omega_4 d\hat{\mathcal{I}} e_H e_L A_{\vec{k},3} \right] \quad (46)
\end{aligned}$$

5 Partonic Results

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$$\bar{c}_{\text{VA},xF_3,g}^{(1)} = \bar{c}_{\text{VA},g_4,g}^{(1)} = \bar{c}_{\text{VA},g_L,g}^{(1)} = 0 \quad (47)$$

Near threshold, we find

$$\bar{c}_{\vec{k},g}^{(1),\text{thr}} = c_{\vec{k},g}^{(0),\text{thr}} \frac{1}{\pi^2} C_A \left(\bar{a}_{\vec{k}}^{(1,1)} \ln(\beta) + \bar{a}_{\vec{k}}^{(1,0)} \right), \quad (48)$$

with

$$\bar{a}_{\vec{k}}^{(1,1)} = -\frac{1}{2} \quad (49)$$

$$\bar{a}_{\text{VV},F_2}^{(1,0)} = -\frac{1}{4} \ln \left(\frac{16\chi_q}{(1+\chi_q)^2} \right) + \frac{1}{2} \quad (50)$$

$$\bar{a}_{\text{VV},F_2}^{(1,0)} = \bar{a}_{\text{VV},F_2}^{(1,0)} + \frac{1}{6} \quad (51)$$

$$\bar{a}_{\text{VV},2xg_1}^{(1,0)} = \bar{a}_{\text{AA},F_2}^{(1,0)} = \bar{a}_{\text{AA},F_L}^{(1,0)} = \bar{a}_{\text{AA},2xg_1}^{(1,0)} = \bar{a}_{\text{VV},F_2}^{(1,0)} \quad (52)$$

6 Hadronic Results

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7 Summary

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A References

- [1] **Particle Data Group** Collaboration, C. Patrignani et al., “Review of Particle Physics,” Chin. Phys. **C40** no. 10, (2016) 100001.
- [2] I. Bojak, NLO QCD corrections to the polarized photoproduction and hadroproduction of heavy quarks. PhD thesis, Dortmund U., 2000. [arXiv:hep-ph/0005120](#) [[hep-ph](#)].
- [3] Beenakker, W. and Kuijf, H. and van Neerven, W. L. and Smith, J., “Qcd corrections to heavy-quark production in $p\bar{p}$ collisions,” Phys. Rev. D **40** (Jul, 1989) 54–82. <http://link.aps.org/doi/10.1103/PhysRevD.40.54>.
- [4] G. Altarelli and G. Parisi, “Asymptotic Freedom in Parton Language,” Nucl. Phys. **B126** (1977) 298–318.
- [5] W. Vogelsang, “A Rederivation of the spin dependent next-to-leading order splitting functions,” Phys. Rev. **D54** (1996) 2023–2029, [arXiv:hep-ph/9512218](#) [[hep-ph](#)].

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