1 Feynman Rules

 $\begin{array}{c} \text{following [1]} \\ \text{Error:} \end{array}$

To perform the calculation of Dirac traces in n dimensions use HEPMath[2] or TRACER[3].

TODO

2 Leading Order: $O(\alpha \alpha_s)$

diagramatic:

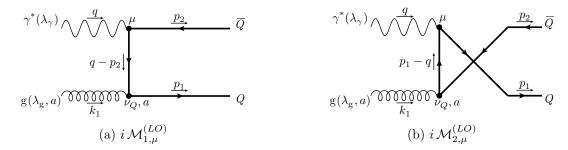


Figure 1: LO contributions

formula:

$$i\,\mathcal{M}_{1,\mu}^{(LO)} = \bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not q - \not p_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_Q}^{(\lambda_{\rm g})}(k_1) \tag{1}$$

$$i\mathcal{M}_{2,\mu}^{(LO)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not p_1 - \not q + m)}{t_1}(igT_a\gamma^{\nu_Q})v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)$$
 (2)

color space:

$$\left| \mathcal{M}_{1,\mu}^{(LO)} + \mathcal{M}_{2,\mu}^{(LO)} \right|^2 \sim \text{tr}(T_a T_a) = N_c C_F$$
 (3)

3 Next-to-leading Order: $O(\alpha \alpha_S^2)$

3.1 Light Quark Contributions

$$\gamma^*(q) + q(k_1) \to \overline{Q}(p_2) + Q(p_1) + q(k_2)$$
 (4)

diagramatic:

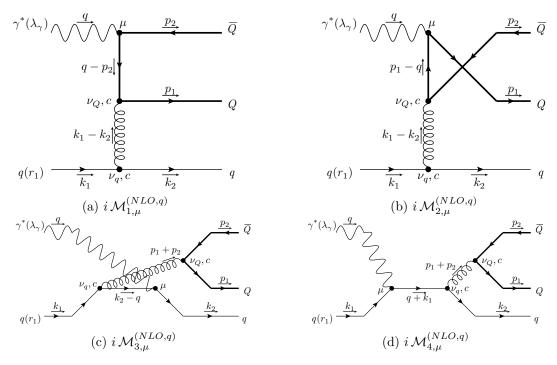


Figure 2: NLO contributions by light quarks

formula:

$$i\mathcal{M}_{1,\mu}^{(NLO,q)} = \bar{u}_{Q}(p_{1})(igT_{c}\gamma^{\nu_{Q}})\frac{i(\not q - \not p_{2} + m)}{u_{1}}(-iee_{H}\gamma_{\mu})v_{Q}(p_{2})\cdot \frac{-ig_{\nu_{Q},\nu_{q}}}{t'} \cdot \bar{u}_{q}(k_{2})(igT_{c}\gamma^{\nu_{q}})u_{q}^{(r_{1})}(k_{1}) \tag{5}$$

$$i\mathcal{M}_{2,\mu}^{(NLO,q)} = \bar{u}_{Q}(p_{1})(-iee_{H}\gamma_{\mu})\frac{i(\not p_{1} - \not q + m)}{u_{7}}(igT_{c}\gamma^{\nu_{Q}})v_{Q}(p_{2})\cdot \frac{-ig_{\nu_{Q},\nu_{q}}}{t'} \cdot \bar{u}_{q}(k_{2})(igT_{c}\gamma^{\nu_{q}})u_{q}^{(r_{1})}(k_{1}) \tag{6}$$

$$i\mathcal{M}_{3,\mu}^{(NLO,q)} = \bar{u}_{Q}(p_{1})(igT_{c}\gamma^{\nu_{Q}})v_{Q}(p_{2}) \cdot \frac{-ig_{\nu_{Q},\nu_{q}}}{s_{5}}\cdot \frac{\bar{u}_{q}(k_{2})(-iee_{L}\gamma_{\mu})\frac{i(\not k_{2} - \not q)}{u'}(igT_{c}\gamma^{\nu_{q}})u_{q}^{(r_{1})}(k_{1}) \tag{7}$$

$$i\mathcal{M}_{4,\mu}^{(NLO,q)} = \bar{u}_{Q}(p_{1})(igT_{c}\gamma^{\nu_{Q}})v_{Q}(p_{2}) \cdot \frac{-ig_{\nu_{Q},\nu_{q}}}{s_{5}}\cdot \frac{\bar{u}_{q}(k_{2})(igT_{c}\gamma^{\nu_{Q}})v_{Q}(p_{2}) \cdot \frac{\bar{u}_{q}(k_{2})(igT_{c}\gamma^{\nu_{Q}})v_{Q}(p_{2})}{s_{5}}\cdot \frac{\bar{u}_{q}(k_{2})(igT_{c}\gamma^{\nu_{Q}})v_{Q}(p_{2}) \cdot \frac{\bar{u}_{q}(k_{2})(igT_{c}\gamma^{\nu_{Q}})v_{Q}(p_{2}) \cdot \frac{\bar{u}_{q}(k_{2})(igT_{c}\gamma^{\nu_{Q}})v_{Q}(p_{2})}{s_{5}}\cdot \frac{\bar{u}_{q}(k_{2})(igT_{c}\gamma^{\nu_{Q}})v_{Q}(p_{2})}{s_{5}}\cdot \frac{\bar{u}_{q}(k_{2})(igT_{c}\gamma^{\nu_{Q}})v_{Q}(p_{2})}{s_{5}}\cdot \frac{\bar{u}_{q}(k_{2})(igT_{c}\gamma^{\nu_{Q}})v_{Q}(p_{2})}{s_{5}}\cdot \frac{\bar{u}_{q}(k_{2})(igT_{$$

color space:

$$\left| \mathcal{M}_{1,\mu}^{(NLO,q)} + \mathcal{M}_{2,\mu}^{(NLO,q)} + \mathcal{M}_{3,\mu}^{(NLO,q)} + \mathcal{M}_{4,\mu}^{(NLO,q)} \right|^2 \sim \operatorname{tr}(T_c T_d) \operatorname{tr}(T_c T_d) = \frac{1}{2} N_c C_F \quad (9)$$

3.2 Gluon Bremsstrahlung

$$\gamma^*(q) + g(k_1) \to \overline{Q}(p_2) + Q(p_1) + g(k_2)$$
 (10)

diagramatic:

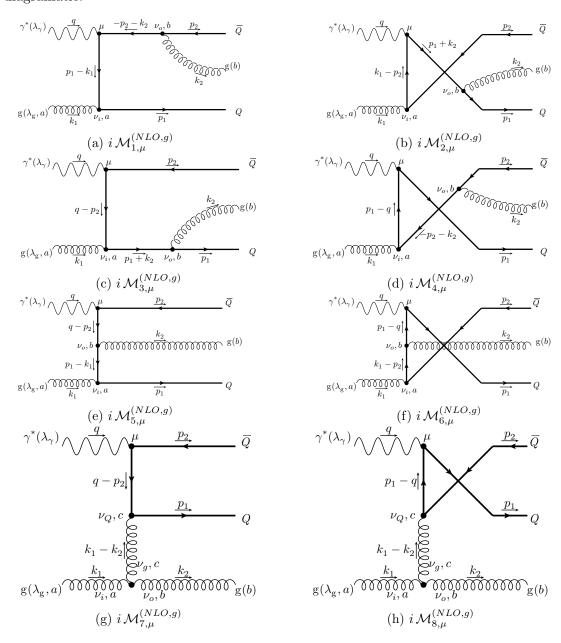


Figure 3: NLO contributions by gluon bremsstrahlung

formula:

color space:

$$\begin{split} &\sum_{j=1}^{6} \left| \mathcal{M}_{j,\mu}^{(NLO,g)} \right|^{2} + \mathcal{M}_{1,\mu}^{(NLO,g)} \left(\mathcal{M}_{4,\mu'}^{(NLO,g)} + \mathcal{M}_{5,\mu'}^{(NLO,g)} \right)^{*} + \mathcal{M}_{3,\mu}^{(NLO,g)} \left(\mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^{*} + \\ &\mathcal{M}_{2,\mu}^{(NLO,g)} \left(\mathcal{M}_{3,\mu'}^{(NLO,g)} + \mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^{*} + \mathcal{M}_{4,\mu}^{(NLO,g)} \left(\mathcal{M}_{5,\mu'}^{(NLO,g)} \right)^{*} \\ &\sim \operatorname{tr}(T_{a}T_{a}T_{b}T_{b}) = N_{C}C_{F}^{2} & (19) \\ &\mathcal{M}_{1,\mu}^{(NLO,g)} \left(\mathcal{M}_{2,\mu'}^{(NLO,g)} + \mathcal{M}_{3,\mu'}^{(NLO,g)} + \mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^{*} + \\ &\left(\mathcal{M}_{2,\mu}^{(NLO,g)} + \mathcal{M}_{3,\mu}^{(NLO,g)} \right) \left(\mathcal{M}_{4,\mu'}^{(NLO,g)} + \mathcal{M}_{5,\mu'}^{(NLO,g)} \right)^{*} + \\ &\left(\mathcal{M}_{4,\mu}^{(NLO,g)} + \mathcal{M}_{5,\mu}^{(NLO,g)} \right) \left(\mathcal{M}_{6,\mu'}^{(NLO,g)} \right)^{*} \\ &\sim \operatorname{tr}(T_{a}T_{b}T_{a}T_{b}) = N_{C}C_{F} \left(C_{F} - \frac{C_{A}}{2} \right) & (20) \\ &\left(\mathcal{M}_{2,\mu}^{(NLO,g)} + \mathcal{M}_{3,\mu}^{(NLO,g)} + \mathcal{M}_{6,\mu}^{(NLO,g)} \right) \left(\mathcal{M}_{7,\mu'}^{(NLO,g)} + \mathcal{M}_{8,\mu'}^{(NLO,g)} \right)^{*} \\ &\sim -if_{bda} \operatorname{tr}(T_{a}T_{b}T_{d}) = \frac{1}{2}N_{C}C_{F}C_{A} & (21) \\ &\left(\mathcal{M}_{1,\mu}^{(NLO,g)} + \mathcal{M}_{4,\mu}^{(NLO,g)} + \mathcal{M}_{5,\mu}^{(NLO,g)} \right) \left(\mathcal{M}_{7,\mu'}^{(NLO,g)} + \mathcal{M}_{8,\mu'}^{(NLO,g)} \right)^{*} \\ &\sim -if_{bda} \operatorname{tr}(T_{b}T_{a}T_{d}) = if_{bda} \operatorname{tr}(T_{a}T_{b}T_{d}) = -\frac{1}{2}N_{C}C_{F}C_{A} & (22) \\ &\left| \mathcal{M}_{7,\mu}^{(NLO,g)} + \mathcal{M}_{8,\mu}^{(NLO,g)} \right|^{2} \\ &\sim f_{acb}f_{adb} \operatorname{tr}(T_{c}T_{d}) = N_{C}C_{F}C_{A} & (23) \\ \end{split}$$

To get the polarisation sums right, one has to subtract the contributions of the Faddeev-Popov ghosts[4, 5]:

diagramatic:

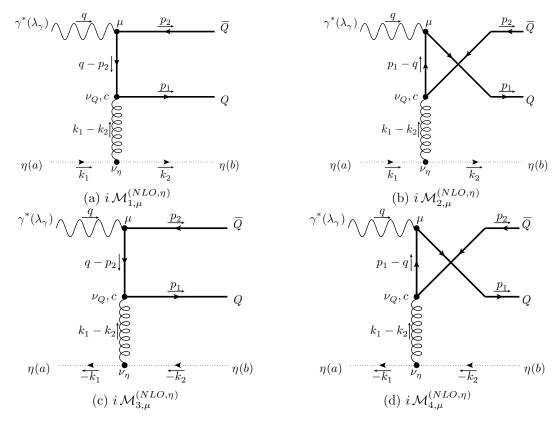


Figure 4: NLO contributions by ghosts

formula:

$$i\,\mathcal{M}_{1,\mu}^{(NLO,\eta)} = \bar{u}(p_{1})(igT_{c}\gamma^{\nu_{Q}})\frac{i(\not q - \not p_{2} + m)}{u_{1}}(-iee_{H}\gamma_{\mu})v(p_{2}) \cdot \frac{-ig_{\nu_{Q},\nu_{\eta}}}{t'} \cdot (gf^{acb}k_{2}^{\nu_{\eta}}) \quad (24)$$

$$i\,\mathcal{M}_{2,\mu}^{(NLO,\eta)} = \bar{u}(p_{1})(-iee_{H}\gamma_{\mu})\frac{i(\not p_{1} - \not q + m)}{u_{7}}(igT_{c}\gamma^{\nu_{Q}})v(p_{2}) \cdot \frac{-ig_{\nu_{Q},\nu_{\eta}}}{t'} \cdot (gf^{acb}k_{2}^{\nu_{\eta}}) \quad (25)$$

$$i\,\mathcal{M}_{3,\mu}^{(NLO,\eta)} = \bar{u}(p_{1})(igT_{c}\gamma^{\nu_{Q}})\frac{i(\not q - \not p_{2} + m)}{u_{1}}(-iee_{H}\gamma_{\mu})v(p_{2}) \cdot \frac{-ig_{\nu_{Q},\nu_{\eta}}}{t'} \cdot (gf^{cab}(-k_{1})^{\nu_{\eta}})$$

$$(26)$$

$$i\,\mathcal{M}_{4,\mu}^{(NLO,\eta)} = \bar{u}(p_{1})(-iee_{H}\gamma_{\mu})\frac{i(\not p_{1} - \not q + m)}{u_{7}}(igT_{c}\gamma^{\nu_{Q}})v(p_{2}) \cdot \frac{-ig_{\nu_{Q},\nu_{\eta}}}{t'} \cdot (gf^{cab}(-k_{1})^{\nu_{\eta}})$$

color space:

$$\left| \mathcal{M}_{1,\mu}^{(NLO,\eta)} + \mathcal{M}_{2,\mu}^{(NLO,\eta)} \right|^2 \sim f_{acb} f_{adb} \operatorname{tr}(T_c T_d) = N_C C_F C_A \tag{28}$$

$$\left| \mathcal{M}_{1,\mu}^{(NLO,\eta)} + \mathcal{M}_{2,\mu}^{(NLO,\eta)} \right|^{2} \sim f_{acb} f_{adb} \operatorname{tr}(T_{c} T_{d}) = N_{C} C_{F} C_{A}$$

$$\left| \mathcal{M}_{3,\mu}^{(NLO,\eta)} + \mathcal{M}_{4,\mu}^{(NLO,\eta)} \right|^{2} \sim f_{cab} f_{dab} \operatorname{tr}(T_{c} T_{d}) = f_{acb} f_{adb} \operatorname{tr}(T_{c} T_{d}) = N_{C} C_{F} C_{A}$$
(28)

3.3 Virtual Contributions

$$\gamma^*(q) + g(k_1) \to \overline{Q}(p_2) + Q(p_1) \tag{30}$$

3.3.1 Loops

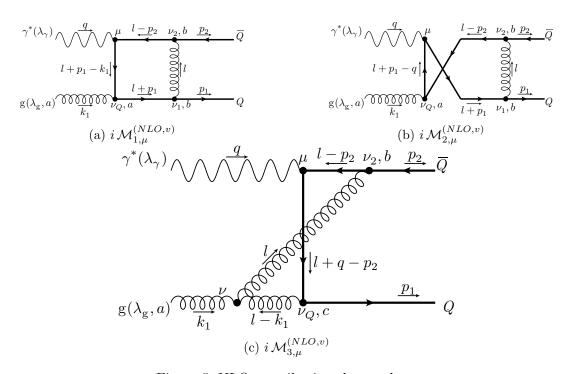


Figure 5: NLO contributions by one loop

$$i\,\mathcal{M}_{1,\mu}^{(NLO,v)} = \mu_D^{4-n} \int \frac{d^n l}{(2\pi)^n} \,\bar{u}(p_1) (igT_b \gamma^{\nu_1}) \frac{i(\not l + \not p_1 + m)}{(l + p_1)^2 - m^2} (igT_a \gamma^{\nu_Q}) \frac{i(\not l + \not p_1 - \not k_1 + m)}{(l + p_1 - k_1)^2 - m^2} \cdot \frac{i(\not l - \not p_2 + m)}{(l - p_2)^2 - m^2} (igT_b \gamma^{\nu_2}) \frac{-ig_{\nu_1,\nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)$$
(31)
$$i\,\mathcal{M}_{2,\mu}^{(NLO,v)} = \mu_D^{4-n} \int \frac{d^n l}{(2\pi)^n} \,\bar{u}(p_1) (igT_b \gamma^{\nu_1}) \frac{i(\not l + \not p_1 + m)}{(l + p_1)^2 - m^2} (igT_a \gamma^{\mu}) \frac{i(\not l + \not p_1 - \not q + m)}{(l + p_1 - q)^2 - m^2} \cdot \frac{(-iee_H \gamma_{\nu_Q})}{(l - p_2)^2 - m^2} (igT_b \gamma^{\nu_2}) \frac{-ig_{\nu_1,\nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)$$
(32)
$$i\,\mathcal{M}_{3,\mu}^{(NLO,v)} = \mu_D^{4-n} \int \frac{d^n l}{(2\pi)^n} \,\bar{u}(p_1) (igT_c \gamma^{\nu_Q}) \frac{i(\not l + \not q - \not p_2 + m)}{(l + q - p_2)^2 - m^2} (-iee_H \gamma_{\mu}) \frac{i(\not l - \not p_2 + m)}{(l - p_2)^2 - m^2} \cdot \frac{(igT_b \gamma^{\nu_2}) \frac{(-i)^2}{l^2 (l - k_1)^2} v(p_2) \varepsilon^{\nu,(\lambda_g)}(k_1) \cdot \frac{(gf_{abc} \left(g_{\nu_2\nu_Q}(k_1 - 2l)_{\nu} + g_{\nu_Q\nu}(l - 2k_1)_{\nu_2} + g_{\nu\nu_2}(k_1 + l)_{\nu_Q}\right)\right)$$
(33)

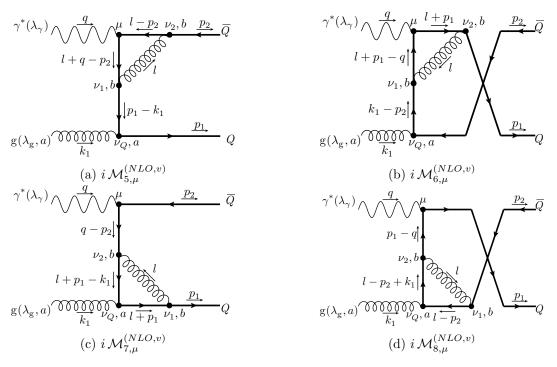


Figure 6: NLO contributions by one loop (cont'ed)

$$i\,\mathcal{M}_{5,\mu}^{(NLO,v)} = \mu_D^{4-n} \int \frac{d^n l}{(2\pi)^n} \,\bar{u}(p_1) (igT_a \gamma^{\nu_Q}) \frac{i(\not p_1 - \not k_1 + m)}{u_1} (igT_b \gamma^{\nu_1}) \frac{i(\not l + \not q - \not p_2 + m)}{(l + q - p_2)^2 - m^2} \cdot \\ (-iee_H \gamma_\mu) \frac{i(\not l - \not p_2 + m)}{(l - p_2)^2 - m^2} (igT_b \gamma^{\nu_2}) \frac{-ig_{\nu_1,\nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \qquad (34)$$

$$i\,\mathcal{M}_{6,\mu}^{(NLO,v)} = \mu_D^{4-n} \int \frac{d^n l}{(2\pi)^n} \,\bar{u}(p_1) (igT_b \gamma^{\nu_2}) \frac{i(\not l + \not p_1 + m)}{(l + p_1)^2 - m^2} (-iee_H \gamma_\mu) \frac{i(\not l + \not p_1 - \not q + m)}{(l + p_1 - q)^2 - m^2} \cdot \\ (igT_b \gamma^{\nu_1}) \frac{i(\not k_1 - \not p_2 + m)}{t_1} (igT_a \gamma^{\nu_Q}) \frac{-ig_{\nu_1,\nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \qquad (35)$$

$$i\,\mathcal{M}_{7,\mu}^{(NLO,v)} = \mu_D^{4-n} \int \frac{d^n l}{(2\pi)^n} \,\bar{u}(p_1) (igT_b \gamma^{\nu_1}) \frac{i(\not l + \not p_1 + m)}{(l + p_1)^2 - m^2} (igT_a \gamma^{\nu_Q}) \frac{i(\not l + \not p_1 - \not k_1 + m)}{(l + p_1 - k_1)^2 - m^2} \cdot \\ (igT_b \gamma^{\nu_2}) \frac{i(\not q - \not p_2 + m)}{u_1} (-iee_H \gamma_\mu) \frac{-ig_{\nu_1,\nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \qquad (36)$$

$$i\,\mathcal{M}_{8,\mu}^{(NLO,v)} = \mu_D^{4-n} \int \frac{d^n l}{(2\pi)^n} \,\bar{u}(p_1) (-iee_H \gamma_\mu) \frac{i(\not p_1 - \not q + m)}{t_1} (igT_b \gamma^{\nu_2}) \frac{i(\not l - \not p_2 + \not k_1 + m)}{(l - p_2 + k_1)^2 - m^2} \cdot \\ (igT_a \gamma^{\nu_Q}) \frac{i(\not l - \not p_2 + m)}{(l - p_2)^2 - m^2} (igT_b \gamma^{\nu_1}) \frac{-ig_{\nu_1,\nu_2}}{l^2} v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1) \qquad (37)$$

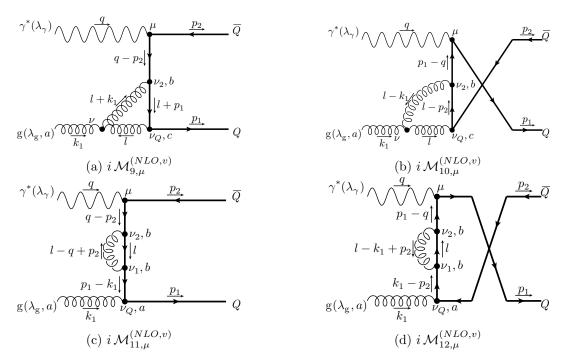


Figure 7: NLO contributions by one loop (cont'ed)

$$\begin{split} i\,\mathcal{M}_{9,\mu}^{(NLO,v)} &= \mu_D^{4-n} \! \int \! \frac{d^n l}{(2\pi)^n} \, \bar{u}(p_1) (igT_c \gamma^{\nu_Q}) \frac{i(\not l + \not p_1 + m)}{(l + p_1)^2 - m^2} (igT_b \gamma^{\nu_2}) \frac{i(\not l - \not p_2 + m)}{u_1} \cdot \\ &\qquad \qquad (-iee_H \gamma_\mu) \frac{(-i)^2}{l^2 (l + k_1)^2} v(p_2) \varepsilon^{\nu, (\lambda_g)} (k_1) \cdot \\ &\qquad \qquad \left(gf_{abc} \left(g_{\nu\nu_2} (2k_1 + l)_{\nu_Q} + g_{\nu_2\nu_Q} (-2l - k_1)_{\nu} + g_{\nu_Q\nu} (l - k_1)_{\nu_2} \right) \right) \quad (38) \\ i\,\mathcal{M}_{10,\mu}^{(NLO,v)} &= \mu_D^{4-n} \! \int \! \frac{d^n l}{(2\pi)^n} \, \bar{u}(p_1) (-iee_H \gamma_\mu) \frac{i(\not p_1 - \not p + m)}{t_1} (igT_b \gamma^{\nu_2}) \frac{i(\not l - \not p_2 + m)}{(l - p_2)^2 - m^2} \cdot \\ &\qquad \qquad (igT_c \gamma^{\nu_Q}) \frac{(-i)^2}{l^2 (l - k_1)^2} v(p_2) \varepsilon^{\nu, (\lambda_g)} (k_1) \cdot \\ &\qquad \qquad \left(gf_{abc} \left(g_{\nu\nu_2} (2k_1 - l)_{\nu_Q} + g_{\nu_2\nu_Q} (2l - k_1)_{\nu} + g_{\nu_Q\nu} (-l - k_1)_{\nu_2} \right) \right) \quad (39) \\ i\,\mathcal{M}_{11,\mu}^{(NLO,v)} &= \mu_D^{4-n} \! \int \! \frac{d^n l}{(2\pi)^n} \, \bar{u}(p_1) (igT_a \gamma^{\nu_Q}) \frac{i(\not p_1 - \not k_1 + m)}{u_1} (igT_b \gamma^{\nu_1}) \frac{i(\not l + m)}{l^2 - m^2} \cdot \\ &\qquad \qquad (igT_b \gamma^{\nu_2}) \frac{i(\not p - \not p_2 + m)}{u_1} (-iee_H \gamma_\mu) \frac{-ig_{\nu_1,\nu_2}}{(l - q + p_2)^2} v(p_2) \varepsilon^{(\lambda_g)}_{\nu_Q} (k_1) \quad (40) \\ i\,\mathcal{M}_{12,\mu}^{(NLO,v)} &= \mu_D^{4-n} \! \int \! \frac{d^n l}{(2\pi)^n} \, \bar{u}(p_1) (-iee_H \gamma_\mu) \frac{i(\not p_1 - \not p + m)}{t_1} (igT_b \gamma^{\nu_2}) \frac{i(\not l + m)}{l^2 - m^2} \cdot \\ &\qquad \qquad (igT_b \gamma^{\nu_1}) \frac{i(\not k_1 - \not p_2 + m)}{t_1} (igT_a \gamma^{\nu_Q}) \frac{-ig_{\nu_1,\nu_2}}{(l - k_1 + p_2)^2} v(p_2) \varepsilon^{(\lambda_g)}_{\nu_Q} (k_1) \quad (41) \end{split}$$

Color space:

$$\left(\mathcal{M}_{1,\mu}^{(NLO,v)} + \mathcal{M}_{2,\mu}^{(NLO,v)} + \mathcal{M}_{7,\mu}^{(NLO,v)} + \mathcal{M}_{8,\mu}^{(NLO,v)}\right)
\cdot \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)}\right)^* \sim -i \operatorname{tr}(T_a T_b T_a T_b) = -i N_C C_F \left(C_F - \frac{C_A}{2}\right)
\left(\mathcal{M}_{3,\mu}^{(NLO,v)} + \mathcal{M}_{9,\mu}^{(NLO,v)} + \mathcal{M}_{10,\mu}^{(NLO,v)}\right)
\cdot \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)}\right)^* \sim f_{abc} \operatorname{tr}(T_c T_b T_a) = -\frac{i}{2} N_C C_F C_A
\left(\mathcal{M}_{5,\mu}^{(NLO,v)} + \mathcal{M}_{6,\mu}^{(NLO,v)} + \mathcal{M}_{11,\mu}^{(NLO,v)} + \mathcal{M}_{12,\mu}^{(NLO,v)}\right)
\cdot \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)}\right)^* \sim -i \operatorname{tr}(T_a T_a T_b T_b) = -i N_C C_F^2$$
(44)

3.3.2 Counter terms

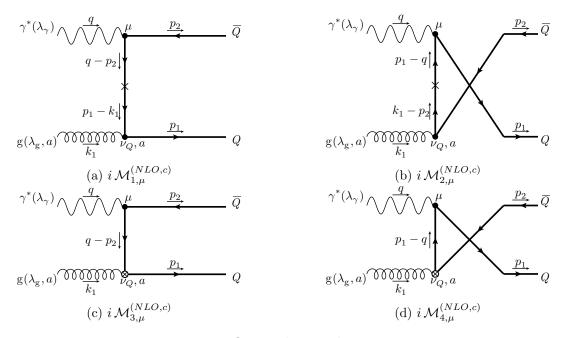


Figure 8: NLO contributions by counter terms

$$-i\,\mathcal{M}_{1,\mu}^{(NLO,c)} = \bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not p_1 - \not k_1 + m)}{u_1}\left(i((Z_2 - 1)(\not q - \not p_2 - m) - (Z_m - 1)m)\right)$$

$$\frac{i(\not q - \not p_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon^{(\lambda_g)}_{\nu_Q}(k_1) \tag{45}$$

$$-i\,\mathcal{M}_{2,\mu}^{(NLO,c)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not p_1 - \not q + m)}{t_1}\left(i((Z_2 - 1)(\not p_1 - \not q - m) - (Z_m - 1)m)\right)$$

$$\frac{i(\cancel{k}_1 - \cancel{p}_2 + m)}{t_1} (igT_a \gamma^{\nu_Q}) v(p_2) \varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)$$

$$\tag{46}$$

$$-i\mathcal{M}_{3,\mu}^{(NLO,c)} = \bar{u}(p_1)(-i(Z_{1f} - 1)gT_a\gamma^{\nu_Q})\frac{i(\not q - \not p_2 + m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)$$
(47)

$$-i\mathcal{M}_{4,\mu}^{(NLO,c)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not p_1 - \not q + m)}{t_1}(-i(Z_{1f} - 1)gT_a\gamma^{\nu_Q})v(p_2)\varepsilon_{\nu_Q}^{(\lambda_g)}(k_1)$$
(48)

Color space:

$$\left(\mathcal{M}_{1,\mu}^{(NLO,c)} + \mathcal{M}_{2,\mu}^{(NLO,c)}\right) \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)}\right)^* \sim \operatorname{tr}(T_a T_a)(Z_2 - 1) = N_C C_F^2 \tag{49}$$

$$\left(\mathcal{M}_{3,\mu}^{(NLO,c)} + \mathcal{M}_{4,\mu}^{(NLO,c)}\right) \left(\mathcal{M}_{1,\mu'}^{(LO)} + \mathcal{M}_{2,\mu'}^{(LO)}\right)^* \sim \operatorname{tr}(T_a T_a)(Z_{1f} - 1) = N_C C_F(C_F + C_A)$$
(50)

3.3.3 Quark Self Energy

To compute self energies, we follow [6]. It is

$$\{\gamma_{\mu}, \gamma_{\nu}\} = 2g_{\mu\nu} \tag{51}$$

$$\gamma_{\mu}\gamma^{\mu} = g^{\mu}_{\mu} = n \tag{52}$$

$$\gamma_{\mu}\gamma_{\nu}\gamma^{\mu} = (2-n)\gamma_{\nu} \tag{53}$$

Figure 9: NLO contributions by quark self energy

$$-i\Sigma(p) = \mu_D^{4-n} \int \frac{d^n l}{(2\pi)^n} (igT_b \gamma_{\nu_1}) \frac{i(l+\not p+m)}{(l+p)^2 - m^2} (igT_b \gamma_{\nu_2}) \frac{-ig^{\nu_1,\nu_2}}{l^2}$$
 (54)

$$= -\mu_D^{4-n} g^2 C_F \int \frac{d^n l}{(2\pi)^n} \frac{n \cdot m + (2-n) \not p + (2-n) \not l}{l^2 ((l+p)^2 - m^2)}$$
(55)

$$= -g^2 C_F \left(\left(n \cdot m + (2 - n) \not p \right) B_0(p^2, 0, m^2) + (2 - n) \not p B_1(p^2, 0, m^2) \right)$$
 (56)

$$=-g^{2}C_{F}\left(B_{0}(p^{2},0,m^{2})\left(n\cdot m+(2-n)p\frac{p^{2}+m^{2}}{2p^{2}}\right)-(2-n)p\frac{1}{2p^{2}}A_{0}(m^{2})\right)$$
(57)

Using [6] we find

$$C_{\epsilon} = \frac{1}{16\pi^2} \exp\left(\left(\gamma_E - \log(4\pi)\right) \frac{\epsilon}{2}\right) \left(m^2/\mu_D^2\right)^{\epsilon/2}$$
 (58)

$$A_0(m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 1 \right) \tag{59}$$

$$B_0(p^2, 0, m^2) = iC_{\epsilon} \left(-\frac{2}{\epsilon} + 2 + \frac{m^2 - p^2}{p^2} \ln \left(\frac{m^2 - p^2}{m^2} \right) \right)$$
 (60)

$$\Rightarrow -i\Sigma(p) = -ig^2 C_F C_\epsilon \left[\frac{2\not p - 8m}{\epsilon} + 2m \left(3 - 2\left(1 - \frac{m^2}{p^2} \right) \ln\left(1 - \frac{p^2}{m^2} \right) \right) - \not p \left(1 + \frac{m^2}{p^2} \right) \left(1 - \left(1 - \frac{m^2}{p^2} \right) \ln\left(1 - \frac{p^2}{m^2} \right) \right) \right]$$
(61)

$$\stackrel{!}{=} -i(Am + B(\not p - m)) \tag{62}$$

$$\Rightarrow A = \frac{1}{m} \left. \Sigma(p) \right|_{p=m} \tag{63}$$

$$= -g^2 C_F C_{\epsilon} \left(\frac{6}{\epsilon} - 5 + \frac{m^2}{p^2} + \left(3 - 4 \frac{m^2}{p^2} + \frac{m^4}{p^4} \right) \ln \left(1 - \frac{p^2}{m^2} \right) \right)$$
 (64)

$$\Rightarrow B = \frac{1}{m} \left. \frac{d\Sigma(p)}{dp} \right|_{p=m} \tag{65}$$

$$= \frac{g^2}{16\pi^2} C_F \left(\frac{2}{\hat{\epsilon}_m} - 1 - \frac{m^2}{p^2} + \left(1 - \frac{m^4}{p^4} \right) \ln \left(1 - \frac{p^2}{m^2} \right) \right)$$
 (66)

Counterterm:

$$-i\Sigma_C(p) = i((Z_2 - 1)\not p - (Z_2 Z_m - 1)m)$$
(67)

$$= i((Z_2 - 1)(\not p - m) - (Z_m - 1)m) + O(\alpha_S^2)$$
(68)

Use on-shell renormalization:

$$0 \stackrel{!}{=} (-i\Sigma(p) - i\Sigma_C(p))|_{p=m}$$
(69)

$$= i \left(((Z_m - 1) + A)m + (B - (Z_2 - 1))(\not p - m) \right) \Big|_{\not p = m}$$
 (70)

$$\Rightarrow (Z_m - 1) = -A|_{p = m} \tag{71}$$

$$=\frac{g^2}{16\pi^2}C_F\left(\frac{6}{\hat{\epsilon}_m}-4\right) \tag{72}$$

such that $m \to Z_m m$.

We also choose

$$Z_2 - 1 = \frac{g^2}{16\pi^2} C_F \frac{2}{\hat{\epsilon}_m} \tag{73}$$

$$Z_{1f} = Z_2 + \frac{g^2}{16\pi^2} C_A \frac{2}{\hat{\epsilon}} \tag{74}$$

3.3.4 Gluon Self Energy (Off shell)

Again, we follow [6].

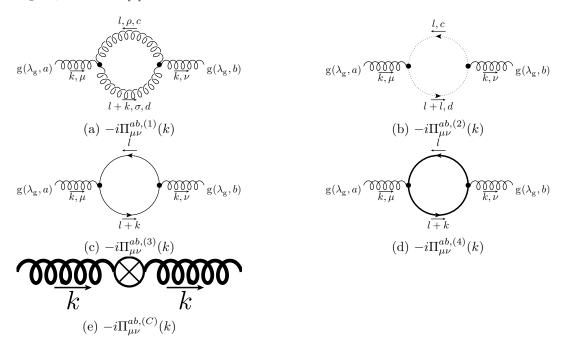


Figure 10: NLO contributions by quark self energy

$$-i\Pi_{\mu\nu}^{ab,(1)}(k) = \frac{1}{2!}\mu_R^{4-n}\int \frac{d^n l}{(2\pi)^n} g f_{cda} \left(g_{\rho\sigma}(2l+k)_{\mu} + g_{\sigma\mu}(-l-2k)_{\rho} + g_{\mu\rho}(k-l)_{\sigma}\right)$$

$$\cdot g f_{cbd} \left(g^{\rho}_{\ \nu}(k-l)^{\sigma} + g^{\sigma}_{\ \nu}(-l-2k)^{\rho} + g^{\sigma\rho}(2l+k)_{\nu}\right) \cdot \frac{(-i)^2}{l^2(l+k)^2}$$

$$-i\Pi_{\mu\nu}^{ab,(2)}(k) = -\mu_R^{4-n}\int \frac{d^n l}{(2\pi)^n} g^2 f_{adc} f_{bcd} l_{\mu}(l+k)_{\nu} \frac{i^2}{l^2(l+k)^2}$$

$$(75)$$

$$-i\Pi_{\mu\nu}^{ab,(3)}(k) = -\mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{\text{tr}((ig\gamma_\mu)(il)(ig\gamma_\nu)(i(l+k))) \text{tr}(T_a T_b)}{l^2 (l+k)^2}$$
(77)

$$-i\Pi_{\mu\nu}^{ab,(4)}(k) = -\mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{\operatorname{tr}((ig\gamma_\mu)(i(l+m))(ig\gamma_\nu)(i(l+k+m)))\operatorname{tr}(T_a T_b)}{(l^2 - m^2)((l+k)^2 - m^2)}$$
(78)

$$-i\Pi_{\mu\nu}^{ab,(C)}(k) = i(Z_3 - 1)\delta^{ab}(k_{\mu}k_{\nu} - k^2g_{\mu\nu})$$
(79)

Color space:

$$f_{acd}f_{bdc} = -\delta_{ab}C_A = f_{dca}f_{dbc} \tag{80}$$

$$tr(T_a T_b) = \frac{1}{2} \delta_{ab} \tag{81}$$

By Slavnov-Taylor we know:

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$$\Pi_{\mu\nu}^{ab}(k) = \delta^{ab}(k_{\mu}k_{\nu} - k^2g_{\mu\nu})\Pi(k^2)$$
(82)

$$\Rightarrow \Pi(k^2) = -\frac{\delta_{ab}}{N_c^2 - 1} \frac{1}{(3+\epsilon)k^2} g^{\mu\nu} \Pi_{\mu\nu}^{ab}(k)$$
 (83)

Gluon + Ghost loop:

$$\Pi^{(1+2)}(k^2) = -ig^2 C_A \frac{1}{(3+\epsilon)k^2} \mu_R^{-\epsilon} \int \frac{d^n l}{(2\pi)^n} \frac{(8+3\epsilon)k \cdot l + (9+3\epsilon)k^2 + (8+3\epsilon)l^2}{l^2(l+k)^2}$$
(84)

$$=-ig^{2}C_{A}\frac{10+3\epsilon}{2(3+\epsilon)}\mu_{R}^{-\epsilon}\int \frac{d^{n}l}{(2\pi)^{n}}\frac{1}{l^{2}(l+k)^{2}}$$
(85)

with

$$B_0(k^2, 0, 0) = \mu_R^{-\epsilon} \int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2 (l+k)^2}$$
(86)

$$= \frac{i}{16\pi^2} \left(-\frac{2}{\hat{\epsilon}} - \ln(-k^2/\mu_R^2) + 2 \right) \tag{87}$$

We find

$$\Rightarrow \Pi^{(1+2)}(k^2) = g^2 C_A \left(-\frac{10}{3\hat{\epsilon}} - \frac{5}{3} \ln(-k^2/\mu_R^2) + \frac{31}{9} \right)$$
 (88)

$$= -C_A \frac{g^2}{16\pi^2} \frac{5}{3} \left(\frac{2}{\hat{\epsilon}} + \ln(-k^2/\mu_R^2) - \frac{31}{15} \right)$$
 (89)

Heavy quark loop:

$$\Pi^{(4)}(k^2) = ig^2 \frac{1}{2(3+\epsilon)k^2} \mu_R^{-\epsilon} \int \frac{d^n l}{(2\pi)^n} \frac{4((2+\epsilon)(m^2 - k \cdot l - l^2) + 2m^2)}{(l^2 - m^2)((l+k)^2 - m^2)}$$
(90)

$$= ig^{2} \frac{2}{(3+\epsilon)k^{2}} \left(2m^{2}B_{0}(k^{2}, m^{2}, m^{2}) - (2+\epsilon)A_{0}(m^{2})\right)$$

$$-(2+\epsilon)k^2B_1(k^2, m^2, m^2)$$
(91)

with

$$A_0(m^2) = \frac{im^2}{16\pi^2} \left(-\frac{2}{\hat{\epsilon}_m} + 1 \right) \tag{92}$$

$$B_1(k^2, m^2, m^2) = -\frac{1}{2}B_0(k^2, m^2, m^2)$$
(93)

$$B_0(k^2, m^2, m^2) = \frac{i}{16\pi^2} \left(-\frac{2}{\hat{\epsilon}_m} + 2 + \beta_k \ln(\chi_k) \right)$$
 (94)

and $\beta_k = \sqrt{1 - 4m^2/k^2}$, $\chi_k = (\beta_k - 1)/(1 + \beta_k)$. We then find

$$\Rightarrow \Pi^{(4)}(k^2) = \frac{2g^2}{16\pi^2} \left(\frac{2}{3\hat{\epsilon}_m} - \frac{5}{9} - \frac{4m^2}{3k^2} - \frac{1}{3} \left(1 + 2\frac{m^2}{k^2} \right) \beta_k \ln(\chi_k) \right)$$
(95)

$$= \frac{g^2}{16\pi^2} \frac{2}{3} \left(\frac{2}{\hat{\epsilon}_m} - \frac{5}{3} - 4\frac{m^2}{k^2} - \left(1 + 2\frac{m^2}{k^2} \right) \beta_k \ln(\chi_k) \right)$$
(96)

Light quark loop:

$$\Pi^{(3)}(k^2) = -ig^2 \frac{2(2+\epsilon)}{(3+\epsilon)} B_1(k^2, 0, 0)$$
(97)

$$=ig^{2}\frac{(2+\epsilon)}{(3+\epsilon)}B_{0}(k^{2},0,0)$$
(98)

$$=\frac{g^2}{16\pi^2}\frac{2}{3}\left(\frac{2}{\hat{\epsilon}} + \ln(-k^2/\mu_R^2) - \frac{5}{3}\right) \tag{99}$$

We choose the counterterm to be:

$$-\Pi^{(C)}(k^2) = Z_3 - 1 = \frac{g^2}{16\pi^2} \left(-\frac{5}{3} C_A \frac{2}{\hat{\epsilon}} + n_{lf} \frac{2}{3} \frac{2}{\hat{\epsilon}} + \frac{2}{3} \frac{2}{\hat{\epsilon}_m} \right)$$
(100)

$$= \frac{g^2}{16\pi^2} \left((2C_A - \beta_0^{lf}) \frac{2}{\hat{\epsilon}} + \frac{2}{3} \frac{2}{\hat{\epsilon}_m} \right)$$
 (101)

$$= \frac{g^2}{16\pi^2} \left((2C_A - \beta_0^f) \frac{2}{\hat{\epsilon}} + \frac{2}{3} \ln(m^2/\mu_R^2) \right)$$
 (102)

with $\beta_0^{lf} = (11C_A - 2n_{lf})/3$, $\beta_0^f = (11C_A - 2n_f)/3$ and $n_f = n_{lf} + 1$.

We also find

$$Z_g - 1 = \frac{g^2}{16\pi^2} \frac{1}{2} \left(\beta_0^f \frac{2}{\hat{\epsilon}} - \frac{2}{3} \ln(m^2/\mu_R^2) \right)$$
 (103)

such that $g \to Z_g g$.

3.3.5 Gluon Self Energy (On shell)

We have [7, (A.18)-(A.20)]:

$$I_0 = \int \frac{d^n p}{(2\pi)^n} \frac{1}{(p^2 + 2kp + M^2 + i\eta)^{\alpha}}$$
 (104)

$$= i \frac{(-\pi)^{n/2}}{(2\pi)^n} \frac{\Gamma(\alpha - n/2)}{\Gamma(\alpha)} \frac{1}{(M^2 - k^2 + in)^{\alpha - n/2}}$$
(105)

$$I_{\mu} = \int \frac{d^{n}p}{(2\pi)^{n}} \frac{p_{\mu}}{(p^{2} + 2kp + M^{2} + i\eta)^{\alpha}}$$
(106)

$$= -k_{\mu}I_0 \tag{107}$$

$$I_{\mu\nu} = \int \frac{d^n p}{(2\pi)^n} \frac{p_{\mu} p_{\nu}}{(p^2 + 2kp + M^2 + i\eta)^{\alpha}}$$
 (108)

$$=I_0\left(k_{\mu}k_{\nu}+\frac{1}{2}g_{\mu\nu}(M^2-k^2)\frac{1}{\alpha-n/2-1}\right)$$
(109)

Heavy Quark loop:

$$-i\Pi_{\mu\nu}^{ab,(4)}(k) = -\mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{\operatorname{tr}((ig\gamma_\mu)(i(l+m))(ig\gamma_\nu)(i(l+k+m)))\operatorname{tr}(T_a T_b)}{(l^2 - m^2)((l+k)^2 - m^2)}$$
(110)
$$= -\frac{\delta^{ab}}{2} g^2 \mu_R^{4-n} \int_0^1 dx \int \frac{d^n l}{(2\pi)^n} 4 \frac{2l_\mu l_\nu + (l_\mu k_\nu + l_\nu k_\mu) + (m^2 - k \cdot l - l^2)g_{\mu\nu}}{(l^2 + 2xk \cdot l - m^2 + k^2 x)^2}$$
(111)
$$= -2\delta^{ab} g^2 \mu_R^{4-n} \int_0^1 dx \left(2I_{\mu\nu} + \left(k_\nu I_\nu + k_\nu I_\mu \right) + g_{\mu\nu} \left(m^2 I_0 - k^\rho I_\rho - g_\rho^\sigma I_{\rho\sigma} \right) \right)$$
(112)
$$= -2\delta^{ab} g^2 \mu_R^{4-n} \int_0^1 dx \, I_0 \left[k_\mu k_\nu (2x^2 - 2x) + g_{\mu\nu} \left((k^2 x (1-x) - m^2) \frac{1 - n/2}{1 - n/2} + m^2 + k^2 x (1-x) \right) \right]$$
(113)
$$= 4\delta^{ab} g^2 (k_\mu k_\nu - g_{\mu\nu} k^2) \mu_R^{4-n} \int_0^1 dx \, x (1-x) I_0$$
(114)

where

$$\mu_R^{4-n} I_0 = i \frac{(-\pi)^{n/2}}{(2\pi)^n} \frac{\Gamma(2-n/2)}{\Gamma(2)} \left(\frac{\mu_R^2}{k^2 x (1-x) - m^2}\right)^{2-n/2}$$
(115)

$$= -\frac{i}{16\pi^2} \left(\frac{2}{\hat{\epsilon}} - \ln((k^2 x (1 - x) - m^2) / \mu_R^2) \right) + O(\epsilon)$$
 (116)

so for $k^2 = 0$ (on shell) we end up with

$$-i\Pi_{\mu\nu}^{ab,(4)}(k) = -i\delta^{ab} \frac{g^2}{16\pi^2} (k_{\mu}k_{\nu} - g_{\mu\nu}k^2) \frac{4}{3\hat{\epsilon}_m}$$
(117)

Light Quark loop:

$$-i\Pi_{\mu\nu}^{ab,(3)}(k) = -\mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{\text{tr}((ig\gamma_\mu)(il)(ig\gamma_\nu)(i(l+k))) \text{tr}(T_a T_b)}{l^2(l+k)^2}$$
(118)

$$=4\delta^{ab}g^{2}(k_{\mu}k_{\nu}-g_{\mu\nu}k^{2})\mu_{R}^{4-n}\int_{0}^{1}dx\,x(1-x)I_{0}$$
(119)

where

$$\mu_R^{4-n} I_0 = \mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{1}{(l^2 + 2xk \cdot l + k^2 x)^2}$$
(120)

$$=\mu_R^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{1}{((l+xk)^2 + k^2 x(1-x))^2}$$
 (121)

so for $k^2 = 0$ (on shell) we get $I_0 = 0$ and end up with

$$-i\Pi_{\mu\nu}^{ab,(3)}(k) = 0 (122)$$

Gluon + Ghost loop vanish by the same arguments as the light quark loop:

$$-i\Pi_{\mu\nu}^{ab,(1+2)}(k) = 0 (123)$$

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List of Corrections

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