1 Feynman Rules

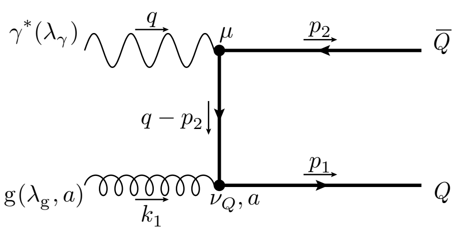
following [1]

FiXme Error: TODO

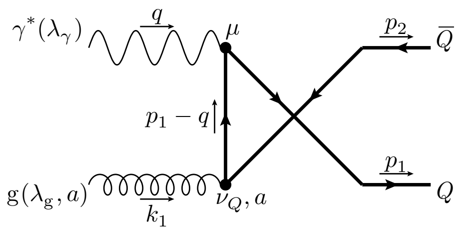
To perform the calculation of Dirac traces in n dimensions use HEPMath[2] or TRACER[3].

2 Leading Order: $O(\alpha\alpha_s)$

diagramatic:



(a) $i\,\mathcal{M}^{(LO)}_{1,\mu}$



(b) $i\,\mathcal{M}^{(LO)}_{2,\mu}$

Figure 1: LO contributions

formula:

$$i\,\mathcal{M}^{(LO)}_{1,\mu} = \bar{u}(p_1)(igT_a\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon^{(\lambda_\mathrm{g})}_{\nu_Q}(k_1) \tag{1}$$

$$i\,\mathcal{M}^{(LO)}_{2,\mu} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{t_1}(igT_a\gamma^{\nu_Q})v(p_2)\varepsilon^{(\lambda_\mathrm{g})}_{\nu_Q}(k_1) \tag{2}$$

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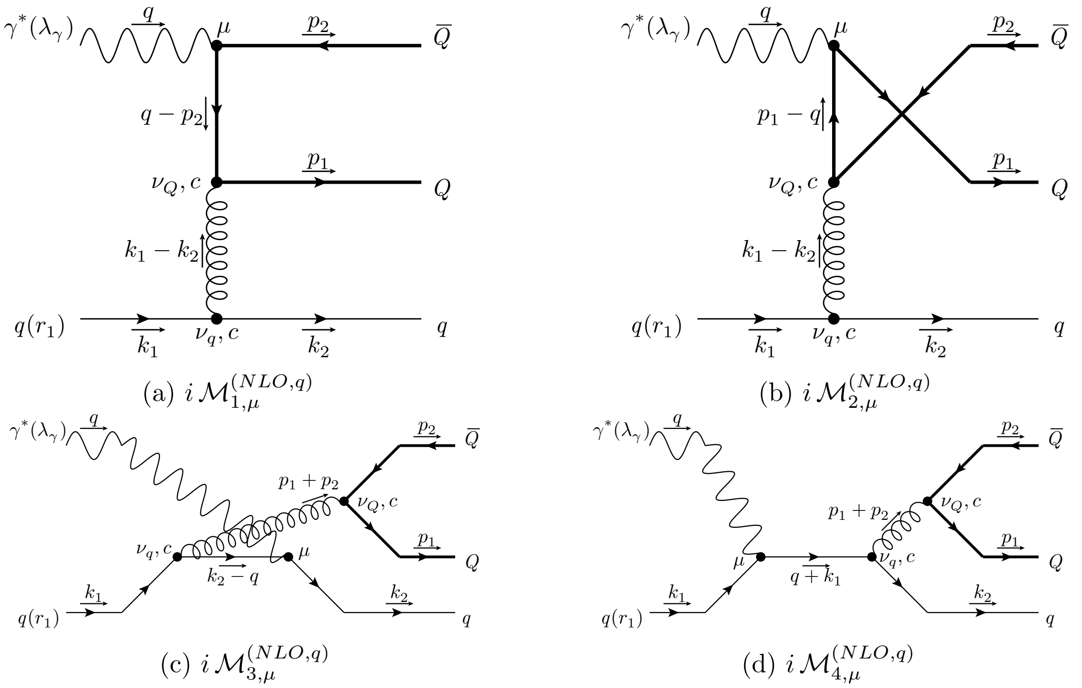
$$\left|\mathcal{M}^{(LO)}_{1,\mu}+\mathcal{M}^{(LO)}_{2,\mu}\right|^2 \sim \mathrm{tr}(T_aT_a) = N_cC_F \tag{3}$$

3 Next-to-leading Order: $O(\alpha\alpha_S^2)$

3.1 Light Quark Contributions

$$\gamma^*(q)+q(k_1)\to\overline{Q}(p_2)+Q(p_1)+q(k_2) \tag{4}$$

diagramatic:



(a) $i\,\mathcal{M}^{(NLO,q)}_{1,\mu}$ (b) $i\,\mathcal{M}^{(NLO,q)}_{2,\mu}$

(c) $i\,\mathcal{M}^{(NLO,q)}_{3,\mu}$ (d) $i\,\mathcal{M}^{(NLO,q)}_{4,\mu}$

Figure 2: NLO contributions by light quarks

formula:

$$
\begin{aligned}
i\,\mathcal{M}^{(NLO,q)}_{1,\mu} &= \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q} - \not{p}_2 + m)}{t_1}(-iee_H\gamma_\mu)v_Q(p_2)\cdot \\
&\qquad \frac{-g_{\nu_Q,\nu_q}}{t'} \cdot \bar{u}_q(k_2)(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) && (5)\\
i\,\mathcal{M}^{(NLO,q)}_{2,\mu} &= \bar{u}_Q(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1 - \not{q} + m)}{u_7}(igT_c\gamma^{\nu_Q})v_Q(p_2)\cdot \\
&\qquad \frac{-g_{\nu_Q,\nu_q}}{t'} \cdot \bar{u}_q(k_2)(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) && (6)\\
i\,\mathcal{M}^{(NLO,q)}_{3,\mu} &= \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})v_Q(p_2)\cdot\frac{-g_{\nu_Q,\nu_q}}{s_5}\cdot \\
&\qquad \bar{u}_q(k_2)(-iee_L\gamma_\mu)\frac{i(\not{k}_2 - \not{q})}{u'}(igT_c\gamma^{\nu_q})u_q^{(r_1)}(k_1) && (7)\\
i\,\mathcal{M}^{(NLO,q)}_{4,\mu} &= \bar{u}_Q(p_1)(igT_c\gamma^{\nu_Q})v_Q(p_2)\cdot\frac{-g_{\nu_Q,\nu_q}}{s_5}\cdot \\
&\qquad \bar{u}_q(k_2)(igT_c\gamma^{\nu_q})\frac{i(\not{k}_1 + \not{q})}{s}(-iee_L\gamma_\mu)u_q^{(r_1)}(k_1) && (8)
\end{aligned}
$$

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$$
\left|\mathcal{M}^{(NLO,q)}_{1,\mu} + \mathcal{M}^{(NLO,q)}_{2,\mu} + \mathcal{M}^{(NLO,q)}_{3,\mu} + \mathcal{M}^{(NLO,q)}_{4,\mu}\right|^2 \sim \mathrm{tr}(T_cT_d)\,\mathrm{tr}(T_cT_d) = \frac{1}{2}N_cC_F \quad (9)
$$

3.2 Gluon Bremsstrahlung

$$\gamma^*(q) + \mathrm{g}(k_1) \to \overline{Q}(p_2) + Q(p_1) + \mathrm{g}(k_2) \tag{10}$$

diagramatic:

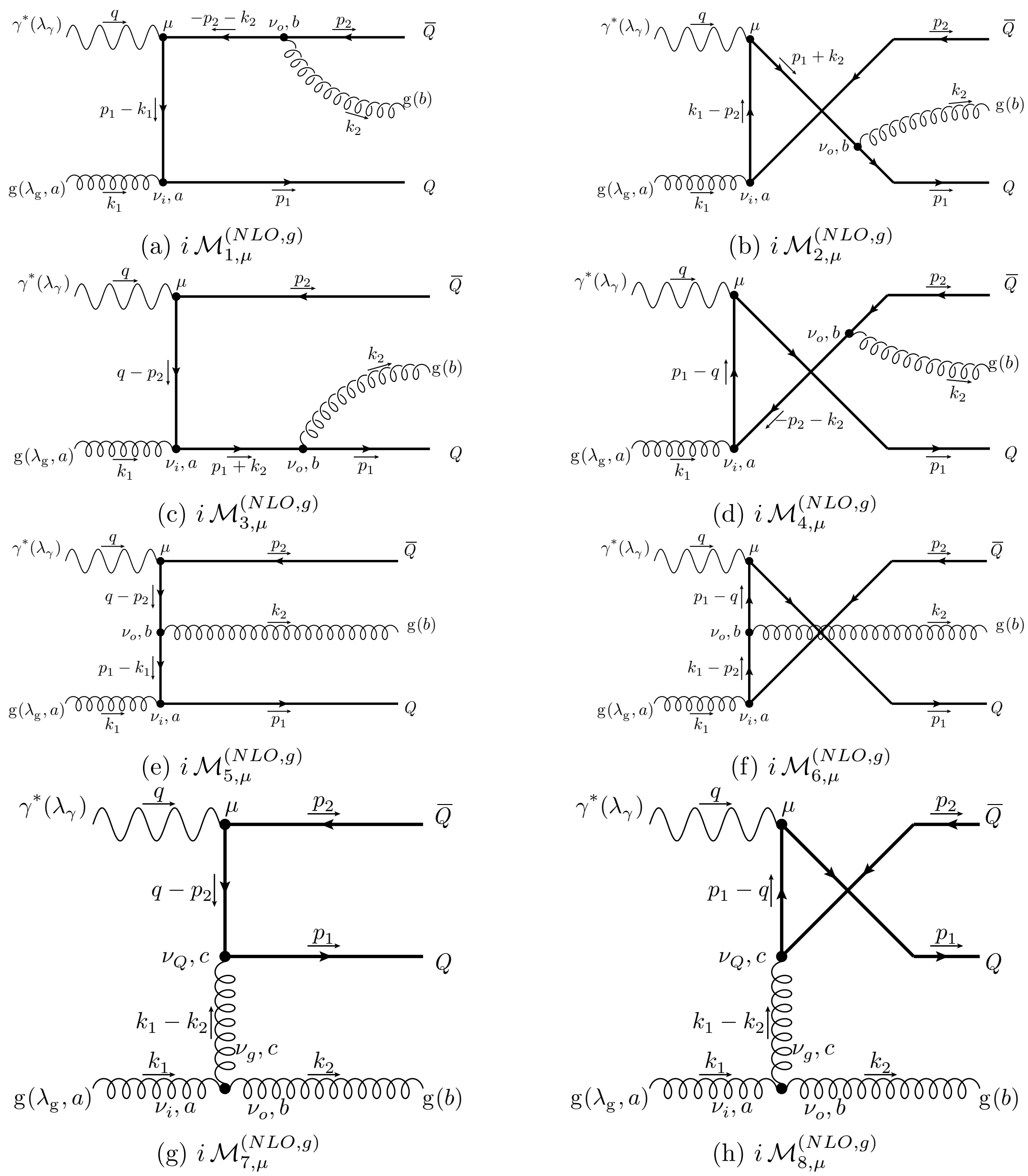

Figure 3: NLO contributions by gluon bremsstrahlung

formula:

$$
\begin{aligned}
i\,\mathcal{M}_{1,\mu}^{(NLO,g)} &= \bar{u}(p_1)(igT_a\gamma^{\nu_i})\frac{i(\not{p}_1-\not{k}_1+m)}{u_6}(-iee_H\gamma_\mu)\cdot \\
&\qquad \frac{i(-\not{p}_2-\not{k}_2+m)}{s_3}(igT_b\gamma^{\nu_o})v(p_2)\varepsilon_{\nu_i}^{(\lambda_{\rm g})}(k_1)\varepsilon_{\nu_o}(k_2) \qquad (11)\\
i\,\mathcal{M}_{2,\mu}^{(NLO,g)} &= \bar{u}(p_1)(igT_b\gamma^{\nu_o})\frac{i(\not{p}_1+\not{k}_2+m)}{s_4}(-iee_H\gamma_\mu)\cdot \\
&\qquad \frac{i(\not{k}_1-\not{p}_2+m)}{t_1}(igT_a\gamma^{\nu_i})v(p_2)\varepsilon_{\nu_i}^{(\lambda_{\rm g})}(k_1)\varepsilon_{\nu_o}(k_2) \qquad (12)\\
i\,\mathcal{M}_{3,\mu}^{(NLO,g)} &= \bar{u}(p_1)(igT_b\gamma^{\nu_o})\frac{i(\not{p}_1+\not{k}_2+m)}{s_4}(igT_a\gamma^{\nu_i})\cdot \\
&\qquad \frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_i}^{(\lambda_{\rm g})}(k_1)\varepsilon_{\nu_o}(k_2) \qquad (13)\\
i\,\mathcal{M}_{4,\mu}^{(NLO,g)} &= \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_a\gamma^{\nu_i})\cdot \\
&\qquad \frac{i(-\not{p}_2-\not{k}_2+m)}{s_3}(igT_b\gamma^{\nu_o})v(p_2)\varepsilon_{\nu_i}^{(\lambda_{\rm g})}(k_1)\varepsilon_{\nu_o}(k_2) \qquad (14)\\
i\,\mathcal{M}_{5,\mu}^{(NLO,g)} &= \bar{u}(p_1)(igT_a\gamma^{\nu_i})\frac{i(\not{p}_1-\not{k}_1+m)}{u_6}(igT_b\gamma^{\nu_o})\cdot \\
&\qquad \frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\varepsilon_{\nu_i}^{(\lambda_{\rm g})}(k_1)\varepsilon_{\nu_o}(k_2) \qquad (15)\\
i\,\mathcal{M}_{6,\mu}^{(NLO,g)} &= \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_b\gamma^{\nu_o})\cdot \\
&\qquad \frac{i(\not{k}_1-\not{p}_2+m)}{t_1}(igT_a\gamma^{\nu_i})v(p_2)\varepsilon_{\nu_i}^{(\lambda_{\rm g})}(k_1)\varepsilon_{\nu_o}(k_2) \qquad (16)\\
i\,\mathcal{M}_{7,\mu}^{(NLO,g)} &= \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\cdot\frac{-g_{\nu_Q,\nu_g}}{t'}\cdot\varepsilon_{\nu_i}^{(\lambda_{\rm g})}(k_1)\varepsilon_{\nu_o}(k_2)\cdot \\
&\qquad \Big(gf^{acb}\left(g^{\nu_o,\nu_i}(k_1+k_2)^{\nu_g}+g^{\nu_i,\nu_g}(k_2-2k_1)^{\nu_o}+g^{\nu_g,\nu_o}(k_1-2k_2)^{\nu_i}\right)\Big) \qquad (17)\\
i\,\mathcal{M}_{8,\mu}^{(NLO,g)} &= \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2)\cdot\frac{-g_{\nu_Q,\nu_g}}{t'}\cdot\varepsilon_{\nu_i}^{(\lambda_{\rm g})}(k_1)\varepsilon_{\nu_o}(k_2)\cdot \\
&\qquad \Big(gf^{acb}\left(g^{\nu_o,\nu_i}(k_1+k_2)^{\nu_g}+g^{\nu_i,\nu_g}(k_2-2k_1)^{\nu_o}+g^{\nu_g,\nu_o}(k_1-2k_2)^{\nu_i}\right)\Big) \qquad (18)
\end{aligned}
$$

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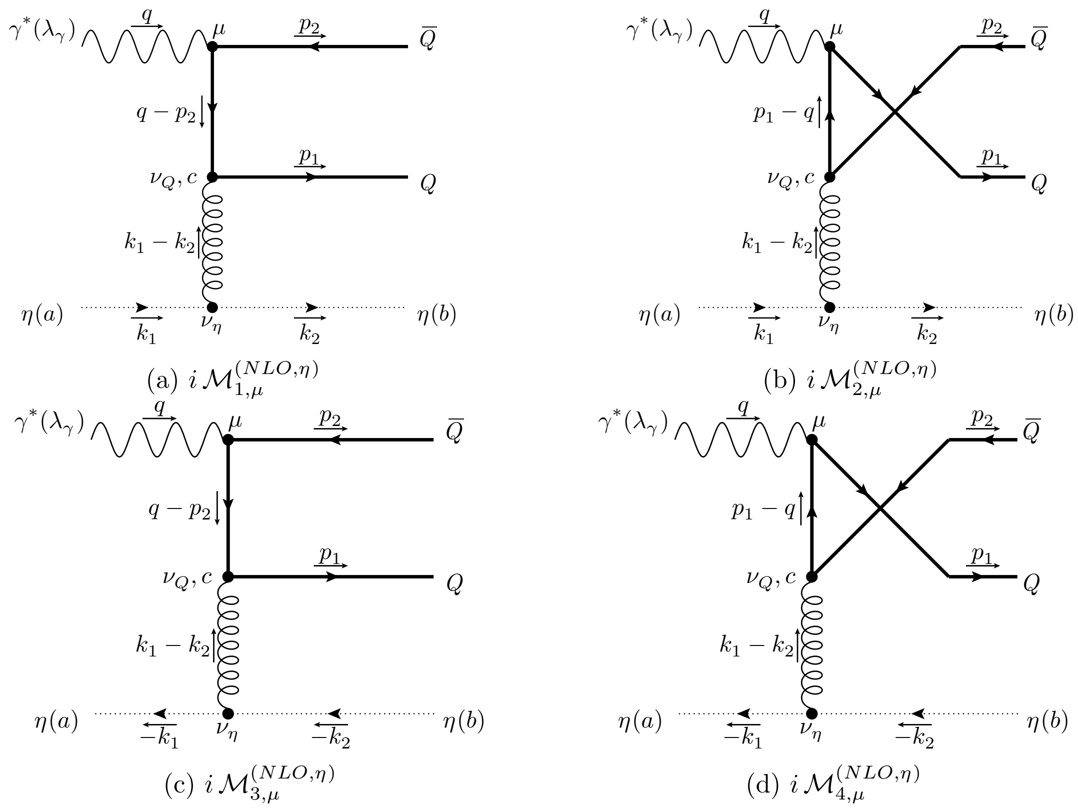
$$\sum_{j=1}^{6}\left|\mathcal{M}_{j,\mu}^{(NLO,g)}\right|^2+\mathcal{M}_{1,\mu}^{(NLO,g)}\left(\mathcal{M}_{4,\mu'}^{(NLO,g)}+\mathcal{M}_{5,\mu'}^{(NLO,g)}\right)^*+\mathcal{M}_{3,\mu}^{(NLO,g)}\left(\mathcal{M}_{6,\mu'}^{(NLO,g)}\right)^*+$$
$$\mathcal{M}_{2,\mu}^{(NLO,g)}\left(\mathcal{M}_{3,\mu'}^{(NLO,g)}+\mathcal{M}_{6,\mu'}^{(NLO,g)}\right)^*+\mathcal{M}_{4,\mu}^{(NLO,g)}\left(\mathcal{M}_{5,\mu'}^{(NLO,g)}\right)^*$$
$$\sim \mathrm{tr}(T_aT_aT_bT_b)=N_CC_F^2 \tag{19}$$
$$\mathcal{M}_{1,\mu}^{(NLO,g)}\left(\mathcal{M}_{2,\mu'}^{(NLO,g)}+\mathcal{M}_{3,\mu'}^{(NLO,g)}+\mathcal{M}_{6,\mu'}^{(NLO,g)}\right)^*+$$
$$\left(\mathcal{M}_{2,\mu}^{(NLO,g)}+\mathcal{M}_{3,\mu}^{(NLO,g)}\right)\left(\mathcal{M}_{4,\mu'}^{(NLO,g)}+\mathcal{M}_{5,\mu'}^{(NLO,g)}\right)^*+$$
$$\left(\mathcal{M}_{4,\mu}^{(NLO,g)}+\mathcal{M}_{5,\mu}^{(NLO,g)}\right)\left(\mathcal{M}_{6,\mu'}^{(NLO,g)}\right)^*$$
$$\sim \mathrm{tr}(T_aT_bT_aT_b)=N_CC_F\left(C_F-\frac{C_A}{2}\right) \tag{20}$$
$$\left(\mathcal{M}_{2,\mu}^{(NLO,g)}+\mathcal{M}_{3,\mu}^{(NLO,g)}+\mathcal{M}_{6,\mu}^{(NLO,g)}\right)\left(\mathcal{M}_{7,\mu'}^{(NLO,g)}+\mathcal{M}_{8,\mu'}^{(NLO,g)}\right)^*$$
$$\sim -if_{bda}\,\mathrm{tr}(T_aT_bT_d)=\frac{1}{2}N_CC_FC_A \tag{21}$$
$$\left(\mathcal{M}_{1,\mu}^{(NLO,g)}+\mathcal{M}_{4,\mu}^{(NLO,g)}+\mathcal{M}_{5,\mu}^{(NLO,g)}\right)\left(\mathcal{M}_{7,\mu'}^{(NLO,g)}+\mathcal{M}_{8,\mu'}^{(NLO,g)}\right)^*$$
$$\sim -if_{bda}\,\mathrm{tr}(T_bT_aT_d)=if_{bda}\,\mathrm{tr}(T_aT_bT_d)=-\frac{1}{2}N_CC_FC_A \tag{22}$$
$$\left|\mathcal{M}_{7,\mu}^{(NLO,g)}+\mathcal{M}_{8,\mu}^{(NLO,g)}\right|^2$$
$$\sim -f_{acb}f_{bda}\,\mathrm{tr}(T_cT_d)=N_CC_FC_A \tag{23}$$

To get the polarisation sums right, one has to subtract the contributions of the Faddeev-Popov ghosts[4, 5]:

diagramatic:



(a) $i\,\mathcal{M}_{1,\mu}^{(NLO,\eta)}$ (b) $i\,\mathcal{M}_{2,\mu}^{(NLO,\eta)}$ (c) $i\,\mathcal{M}_{3,\mu}^{(NLO,\eta)}$ (d) $i\,\mathcal{M}_{4,\mu}^{(NLO,\eta)}$

Figure 4: NLO contributions by ghosts

formula:

$$i\,\mathcal{M}_{1,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\cdot\frac{-g_{\nu_Q,\nu_\eta}}{t'}\cdot(gf^{acb}k_2^{\nu_\eta}) \tag{24}$$

$$i\,\mathcal{M}_{2,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2)\cdot\frac{-g_{\nu_Q,\nu_\eta}}{t'}\cdot(gf^{acb}k_2^{\nu_\eta}) \tag{25}$$

$$i\,\mathcal{M}_{3,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(igT_c\gamma^{\nu_Q})\frac{i(\not{q}-\not{p}_2+m)}{u_1}(-iee_H\gamma_\mu)v(p_2)\cdot\frac{-g_{\nu_Q,\nu_\eta}}{t'}\cdot(gf^{cab}(-k_1)^{\nu_\eta}) \tag{26}$$

$$i\,\mathcal{M}_{4,\mu}^{(NLO,\eta)} = \bar{u}(p_1)(-iee_H\gamma_\mu)\frac{i(\not{p}_1-\not{q}+m)}{u_7}(igT_c\gamma^{\nu_Q})v(p_2)\cdot\frac{-g_{\nu_Q,\nu_\eta}}{t'}\cdot(gf^{cab}(-k_1)^{\nu_\eta}) \tag{27}$$

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$$\left|\mathcal{M}_{1,\mu}^{(NLO,\eta)} + \mathcal{M}_{2,\mu}^{(NLO,\eta)}\right|^2 \sim -f_{acb} f_{bda} \,\mathrm{tr}(T_c T_d) = N_C C_F C_A \tag{28}$$

$$\left|\mathcal{M}_{3,\mu}^{(NLO,\eta)} + \mathcal{M}_{4,\mu}^{(NLO,\eta)}\right|^2 \sim -f_{cab} f_{dba} \,\mathrm{tr}(T_c T_d) = -f_{acb} f_{bda} \,\mathrm{tr}(T_c T_d) = N_C C_F C_A \tag{29}$$

A References

[1] E. Leader and E. Predazzi, An introduction to Gauge theories and modern particle physics. Univ. Pr., Cambridge.

[2] M. Wiebusch, "HEPMath 1.4: A Mathematica Package for Semi-Automatic Computations in High Energy Physics," Computer Physics Communications **195** (Oct., 2015) 172–190. `http://arxiv.org/abs/1412.6102`. arXiv: 1412.6102.

[3] M. Jamin and M. E. Lautenbacher, "TRACER version 1.1: A mathematica package for γ-algebra in arbitrary dimensions," Computer Physics Communications **74** no. 2, (1993) 265 – 288.
`http://www.sciencedirect.com/science/article/pii/001046559390097V`.

[4] L. Faddeev and V. Popov, "Feynman diagrams for the yang-mills field," Physics Letters B **25** no. 1, (1967) 29 – 30.
`http://www.sciencedirect.com/science/article/pii/0370269367900676`.

[5] W. Vogelsang, "Quantenfeldtheorie und Elementarteilchenphysik." Lecture notes, 2013.

List of Corrections

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