

Next-to-Leading Order QCD Corrections to Inclusive Heavy-Flavor Production in Polarized Deep-Inelastic Scattering

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- 2 Computation Review
- 3 Partonic Results
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- 5 Outlook

- Heavy Quarks (HQ): $c(m_c = 1.5 \text{ GeV})$, $b(m_b = 4.75 \text{ GeV})$, $t(m_t = 175 \text{ GeV})$
- EIC will reach region with HQ relevant to structure functions
- compare unpolarized case HERA@DESY: at small $x \sim 30\%$ charm contributions [Laenen,Riemersma,Smith,van Neerven]

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- need improved charm tagging
- fully inclusive cross section is complicated to reconstruct
- no hadronization here

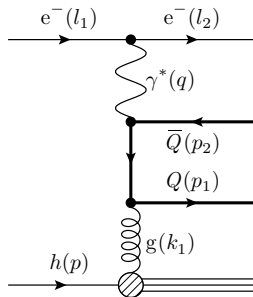
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- scale of hard process is in a perturbative regime
 $m > \Lambda_{QCD}$
- finite mass m provides total cross sections
- full m^2 dependence makes computations complicated: phase space + matrix elements
- 2-scale problem: $\ln\left(\frac{s-4m^2}{4m^2}\right)$ and/or $\ln(Q^2/m^2)$
- keep analytic expressions



$$e^-(l_1) + h(p) \rightarrow e^-(l_2) + \bar{Q}(p_2) + X[Q]$$

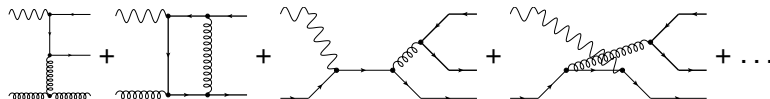


- $S_h = (p + l_1)^2 = x y Q^2$, x, y ,
 $Q^2 = -q^2 = -(l_1 - l_2)^2 \ll M_Z^2$
- unpolarized cross section: [PDG]
 $\frac{d^2\sigma}{dx dy} = \frac{2\pi\alpha^2}{xyQ^2} \left(Y_+ F_2(x, Q^2) - y^2 F_L(x, Q^2) \right)$
 $2x F_1(x, Q^2) = F_2(x, Q^2) - F_L(x, Q^2)$
- polarized cross section: [PDG]
 $\frac{d^2\Delta\sigma}{dx dy} = \frac{4\pi\alpha^2}{xyQ^2} Y_- \cdot 2x g_1(x, Q^2)$
- with $Y_{\pm} = 1 \pm (1 - y)^2$

- use factorisation theorem: PDF and $s = \xi S_h$
- PGF: $g(k_1) + \gamma^*(q) \rightarrow \bar{Q}(p_2) + Q(p_1)$
- three massive particles: $m^2 > 0, q^2 = -Q^2 < 0$

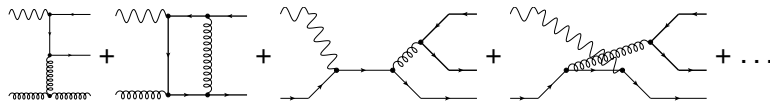
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- compute 2-to-3-phase space: e.g. $dPS_3 \sim dt_1 du_1 d\Omega_n d\hat{\mathcal{I}}$ [Laenen, Bojak]

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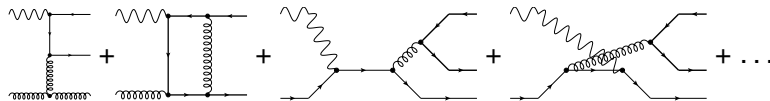
- diagrams:



- $\Rightarrow g_1 \sim e_u^2 \cdot \Delta u \otimes d_{P,q}^{(1)}$
- $d_{P,q}^{(1)}(\chi, \chi') = c_1(\chi, \chi') \ln(\chi) + c_2(\chi, \chi') \text{Li}_2\left(\frac{1+\chi'}{1+\chi}\right) + \dots \checkmark$ [Blümlein, Falcioni, De Freitas]
- $\frac{m^2}{s} = \frac{\chi}{(1+\chi)^2}$ and $\frac{m^2}{s+Q^2} = \frac{m^2}{s'} = \frac{\chi'}{(1+\chi')^2}$ and $\frac{m^2}{Q^2} = \frac{\chi_q}{(1-\chi_q)^2}$

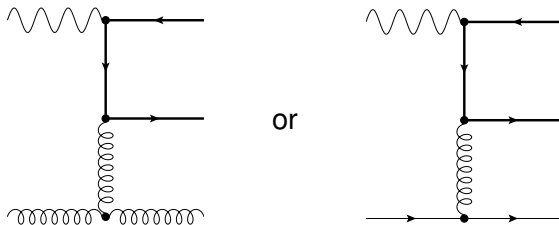
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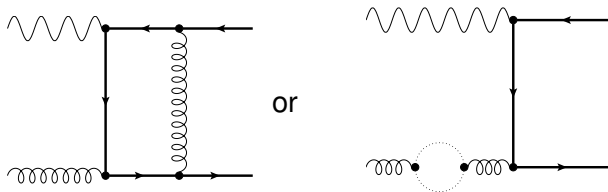
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- γ_5 and $\varepsilon_{\mu\nu\rho\sigma}$ in n -dimension? \rightarrow HVBM scheme [t Hooft, Veltman, Breitenlohner, Maison]

collinear poles appear in, e.g.,

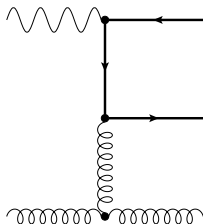


- remove by mass factorization $\rightarrow \overline{\text{MS}}$
- $\Rightarrow g_1 \sim e_H^2 \cdot \Delta g \otimes \ln(\mu_F^2/m^2) \bar{c}_{P,g}^{F,(1)}$
- $\bar{c}_{P,g}^{F,(1)}(\chi, \chi_q) = c_1(\chi, \chi_q) \ln(\chi) + c_2(\chi, \chi_q) \text{Li}_2\left(\frac{1-\chi_q}{1+\chi}\right) + \dots$ (✓ for $Q^2 \gg m^2$ [Buza,Matiounine,Smith,van Neerven])

virtual diagrams are, e.g.,



soft poles appear in the limit of a soft gluon, e.g.,



soft + virtual + renormalization ($\overline{\text{MS}}_m$) + factorization is finite! [Laenen,Bojak]

$$\begin{aligned}
 D_0(m^2, 0, q^2, m^2, t, s, 0, m^2, m^2, m^2) = & \frac{iC_\epsilon}{\beta s t_1} \times \left[-\frac{2}{\epsilon} \ln(\chi) - 2 \ln(\chi) \ln\left(\frac{-t_1}{m^2}\right) \right. \\
 & + \text{Li}_2(1 - \chi^2) - 4\zeta(2) + \ln^2(\chi_q) + 2 \text{Li}_2(-\chi\chi_q) + 2 \text{Li}_2\left(\frac{-\chi}{\chi_q}\right) \\
 & \left. + 2 \ln(\chi\chi_q) \ln(1 + \chi\chi_q) + 2 \ln\left(\frac{\chi}{\chi_q}\right) \ln\left(1 + \frac{\chi}{\chi_q}\right) \right]
 \end{aligned}$$

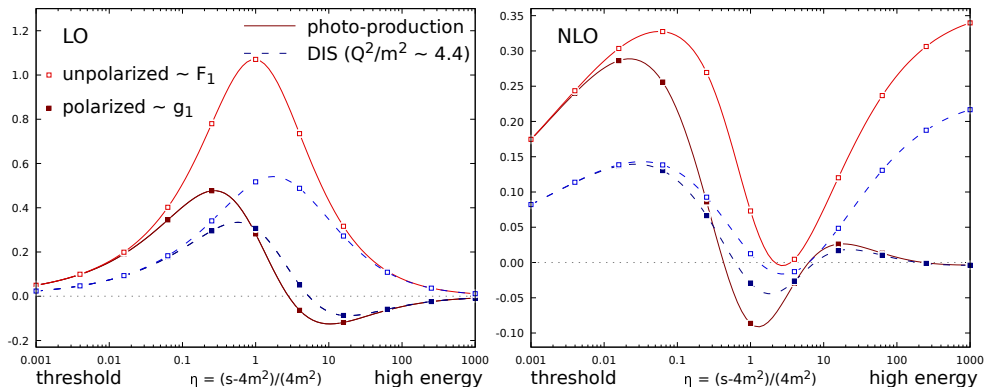
$$\begin{aligned}
 \int \frac{d\Omega_n}{t' u_7^2} = & -\frac{2\pi(m^2 + s_4)(s' + t_1)}{s_4 t_1^2 u_1^2} \left[-2 + \frac{t_1 u_1 (-q^2 s_4 + (2m^2 + s_4)(s' + u_1))}{(s' + t_1)(q^2 s_4 t_1 + m^2(s' + u_1)^2)} \right. \\
 & \left. + \frac{2}{\epsilon} + \ln\left(\frac{t_1^2 u_1^2 (m^2 + s_4)}{(s' + t_1)^2 (m^2(s' + u_1)^2 + q^2 t_1 s_4)}\right) \right]
 \end{aligned}$$

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Partonic Results - Gluon Channel

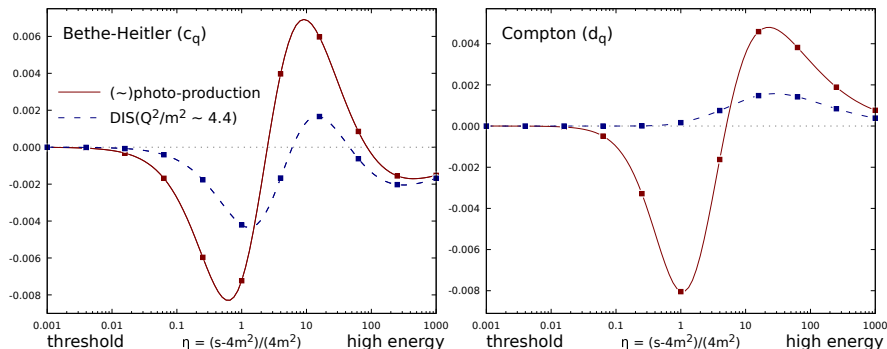
$$g_1 \sim \alpha_s \cdot \Delta g \otimes \left(c_{P,g}^{(0)} + 4\pi\alpha_s \left[c_{P,g}^{(1)} + \ln\left(\frac{\mu^2}{m^2}\right) \bar{c}_{P,g}^{(1)} \right] \right)$$



■ polarized \sim unpolarized near threshold, but not at high energy

Partonic Results - Light Quark Channel

$$g_1 \sim \alpha_s^2 \sum_q (\Delta q + \Delta \bar{q}) \otimes \left(e_H^2 \left[c_{P,q}^{(1)} + \ln \left(\frac{\mu_F^2}{m^2} \right) \bar{c}_{P,q}^{(1)} \right] + e_q^2 d_{P,q}^{(1)} \right)$$

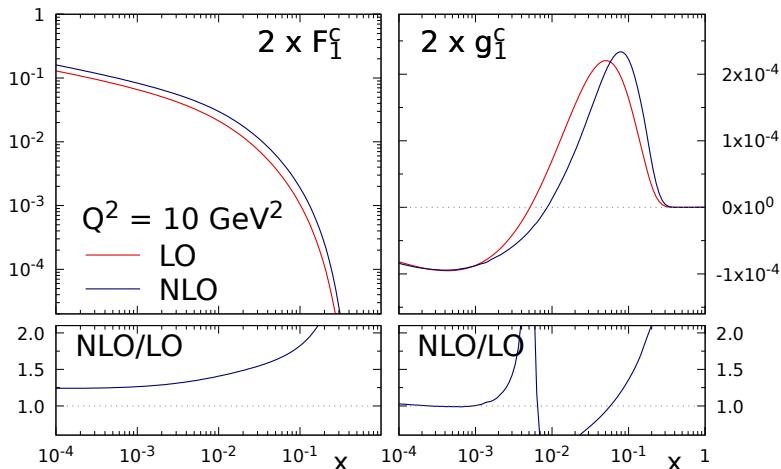


- no interference term $\sim e_H e_q$
- Compton subprocess contains $\ln(Q^2/m^2)$

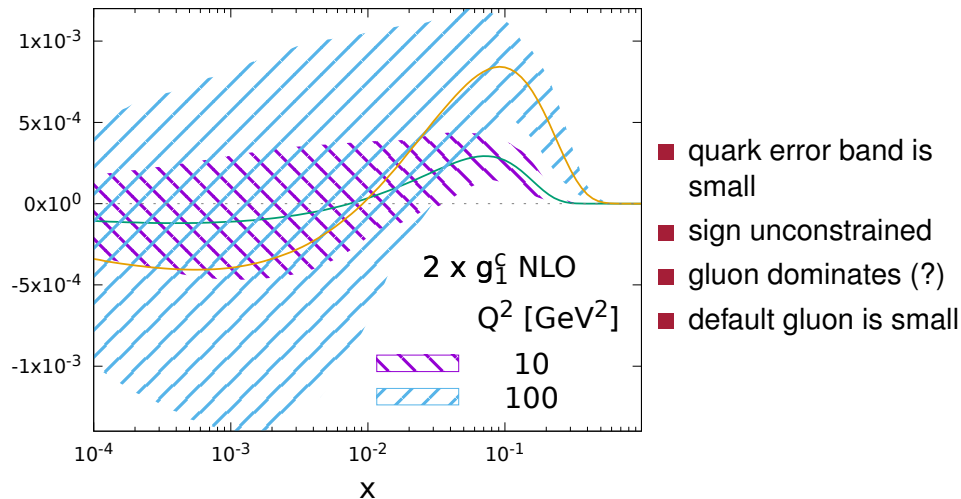
Hadronic Results - Unpolarized vs. Polarized

unpolarized \sim MSTW2008 \leftrightarrow polarized \sim DSSV2014

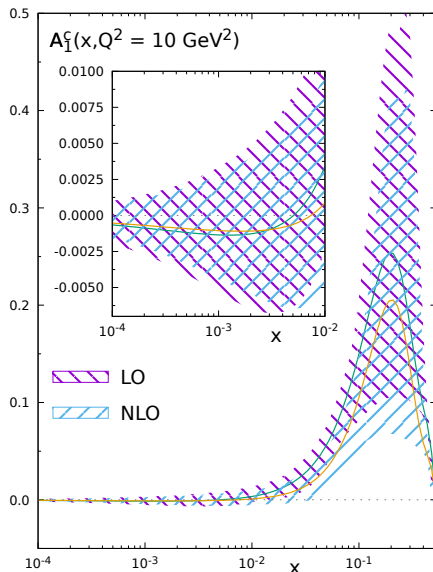
[Martin,Stirling,Thorne,Watt] \leftrightarrow [de Florian,Sassot,Stratmann,Vogelsang]



Hadronic Results - PDF Uncertainties DSSV (I)

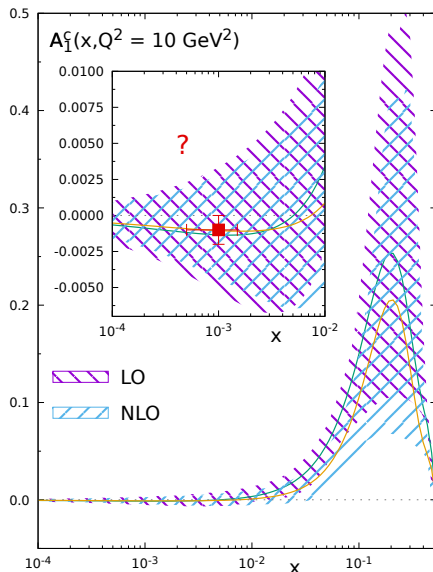


Hadronic Results - PDF Uncertainties DSSV (II)



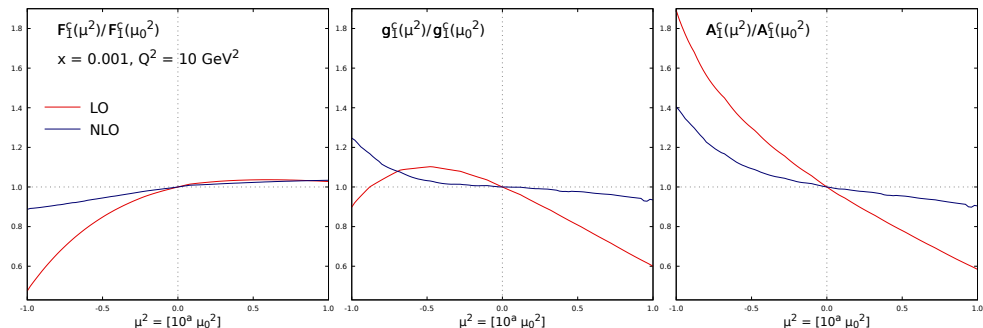
- $A_1^c(x, Q^2) = \frac{g_1^c(x, Q^2)}{F_1^c(x, Q^2)}$
- error band are only due to DSSV uncertainties (no correlations!)

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- $A_1^c(x, Q^2) = \frac{g_1^c(x, Q^2)}{F_1^c(x, Q^2)}$
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- sign unconstrained
- need measurement of $\mathcal{O}(10^{-3})$
- $\text{NLO} \approx \text{LO}$

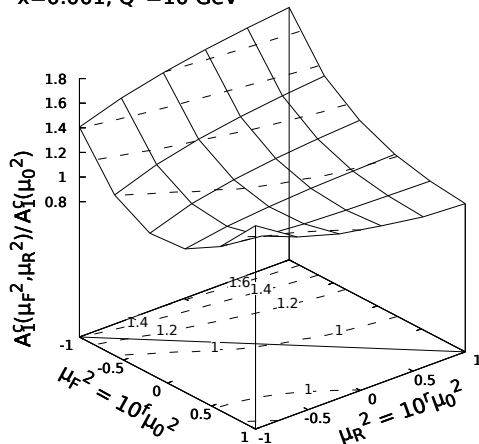
Hadronic Results - Scale Uncertainties (I)



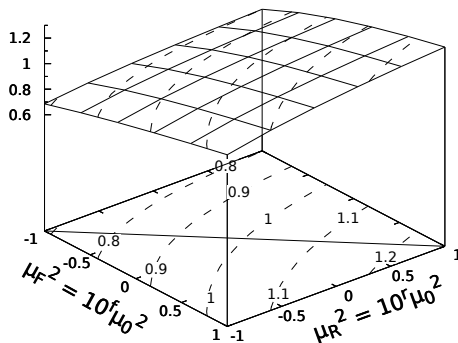
$$\mu_F^2 = \mu_R^2 = 10^a \mu_0^2 \text{ with } \mu_0^2 = 4m^2 + Q^2$$

Hadronic Results - Scale Uncertainties (II)

$x=0.001, Q^2=10 \text{ GeV}^2$



$x=0.1, Q^2=100 \text{ GeV}^2$



$$\mu_0^2 = 4m^2 + Q^2$$

- inclusive distributions: $\frac{dg_1}{dp_{T,\bar{Q}}}, \frac{dg_1}{dy_{\bar{Q}}}, \dots$ [Laenen,Riemersma,Smith,van Neerven]
- correlated distributions: $\frac{dg_1}{dM_{Q\bar{Q}}^2}, \frac{dg_1}{d\phi_{Q\bar{Q}}}, \text{TMD}, \dots$ [Harris,Smith]

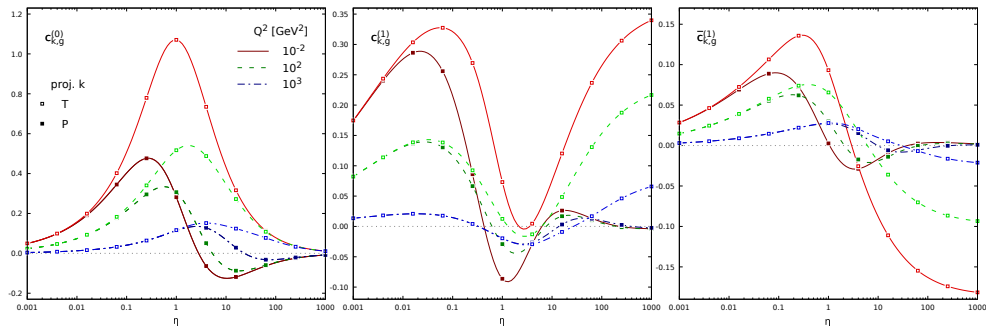
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- full neutral current (NC) contributions: $F_3^{Z\gamma}, g_4^{Z\gamma}, g_5^{Z\gamma}$ and F_2^Z, F_L^Z, g_1^Z
- distributions of full NC structure functions: $\frac{dg_1^{NC}}{dp_{T,\bar{Q}}}, \frac{dg_1^{NC}}{dM_{Q\bar{Q}}^2}, \dots$

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Thank you for your attention!

Backup: Partonic Results - Gluon Channel

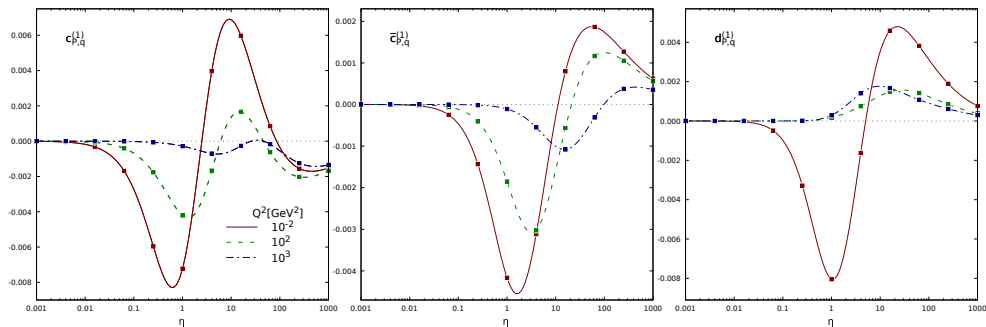
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$$\eta = \frac{s - 4m^2}{4m^2}, \quad m = m_b = 4.75 \text{ GeV}$$

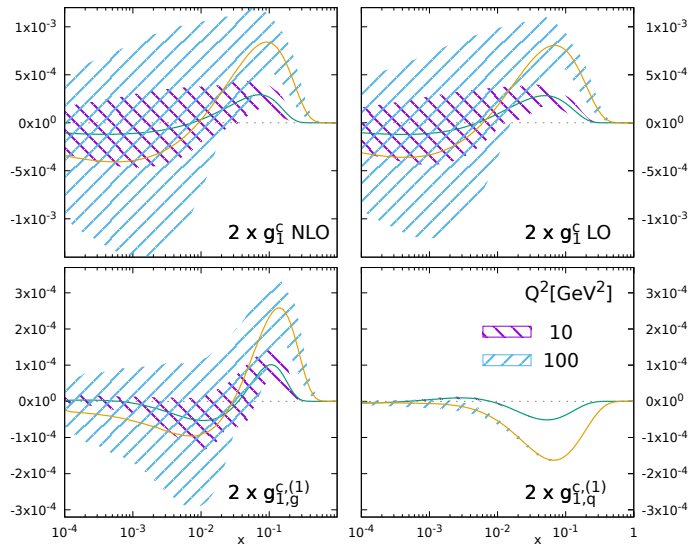
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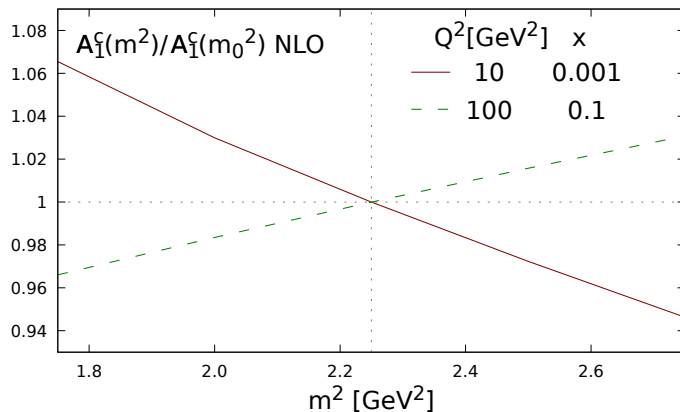
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Backup: Hadronic Results - PDF Uncertainties DSSV



- gluon dominates
- sign unconstrained
- quark band small
- default gluon is small

Backup: Hadronic Results - Mass Variation



$$m_0 = 1.5 \text{ GeV}$$