Next-to-Leading Order QCD Corrections to Inclusive Heavy-Flavor Production in Polarized Deep-Inelastic Scattering

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Outline

Introduction - Heavy Quarks (HQ)

- Heavy Quarks (HQ): c($m_c = 1.5\,\text{GeV}$), b($m_b = 4.75\,\text{GeV}$), t($m_t = 175\,\text{GeV}$)
- EIC will reach region with HQ relevant to structure functions
- compare unpolarized case @HERA: at small $x \sim 30\%$ charm contributions

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- need improved charm tagging
- full inclusive cross section is complicated to reconstruct
- no hadronization here

- scale of hard process is in a pertubative regime
  $m > \Lambda_{QCD}$
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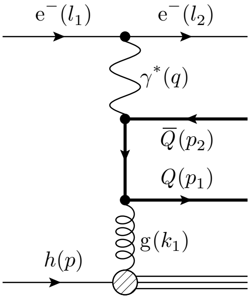
- scale of hard process is in a pertubative regime $m > \Lambda_{QCD}$
- finite mass m ensures full inclusive cross sections
- full m^2 dependence makes computations complicated: phase space + matrix elements
- 2-scale problem: $\ln\left(\frac{s-4m^2}{4m^2}\right)$ and/or $\ln(Q^2/m^2)$
- keep analytic expressions

Introduction - DIS Setup

$$e^-(l_1) + h(p) \to e^-(l_2) + \bar{Q}(p_2) + X[Q]$$



- $S_h = (p + l_1)^2 = x\,y\,Q^2$, x, y,
  $Q^2 = -q^2 = -(l_1 - l_2)^2 \ll M_Z^2$
- unpolarized cross section:
  $$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2}{xyQ^2}\left(Y_+ F_2(x,Q^2) - y^2 F_L(x,Q^2)\right)$$
  $$2xF_1(x,Q^2) = F_2(x,Q^2) - F_L(x,Q^2)$$
- polarized cross section:
  $$\frac{d^2\Delta\sigma}{dxdy} = \frac{4\pi\alpha^2}{xyQ^2} Y_- \cdot 2xg_1(x,Q^2)$$
- with $Y_\pm = 1 \pm (1-y)^2$
- $[k = T] \to 2xF_1$, $[k = L] \to F_L$ and $[k = P] \to 2xg_1$

Computation Review

- use factorisation theorem: PDF and $s = \xi S_h$
- PGF: $\mathrm{g}(k_1) + \gamma^*(q) \to \bar{Q}(p_2) + Q(p_1)$
- three massive particles: $2 \cdot m^2 > 0$, $q^2 = -Q^2 < 0$

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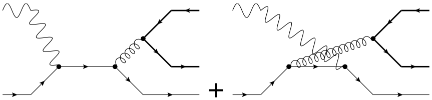
- compute 2-to-3-phase space: e.g. $dPS_3 \sim dt_1 du_1 d\Omega_n d\hat{\mathcal{I}}$
- compute diagrams: 

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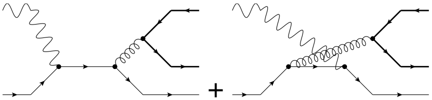
- compute diagrams: 
- $\Rightarrow 2xg_1(x) \sim e_u^2 \cdot \xi \Delta u(\xi) \otimes d_{P,q}^{(1)}(\chi, \chi')$
- $d_{P,q}^{(1)}(\chi, \chi') = c_1(\chi, \chi') \ln(\chi) + c_2(\chi, \chi') \operatorname{Li}_2\left(\frac{1+\chi'}{1+\chi}\right) + \dots$ ✓
- $\frac{m^2}{s} = \frac{\chi}{(1+\chi)^2}$ and $\frac{m^2}{s+Q^2} = \frac{m^2}{s'} = \frac{\chi'}{(1+\chi')^2}$ and $\frac{m^2}{Q^2} = \frac{\chi_q}{(1-\chi_q)^2}$

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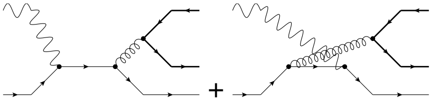
- compute diagrams: 

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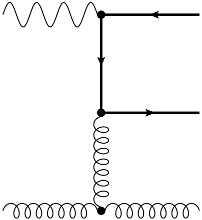
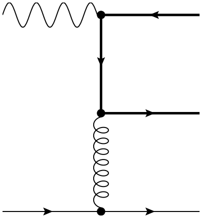
- $\frac{m^2}{s} = \frac{\chi}{(1+\chi)^2}$ and $\frac{m^2}{s+Q^2} = \frac{m^2}{s'} = \frac{\chi'}{(1+\chi')^2}$ and $\frac{m^2}{Q^2} = \frac{\chi_q}{(1-\chi_q)^2}$
- γ_5 and $\varepsilon_{\mu\nu\rho\sigma}$ in n-dimension? $\to$ HVBM scheme

Computation Review - Collinear Poles

collinear poles appear in, e.g.,



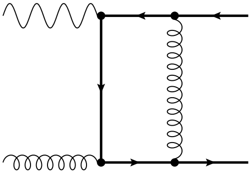
or



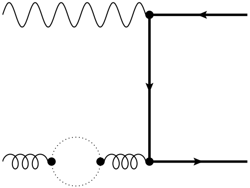
- remove by mass factorization $\to \overline{\text{MS}}$
- $\Rightarrow 2xg_1(x) \sim e_H^2 \cdot \xi\Delta g(\xi) \otimes \ln(\mu_F^2/m^2)\bar{c}_{P,g}^{F,(1)}(\chi,\chi_q)$
- $\bar{c}_{P,g}^{F,(1)}(\chi,\chi_q) = c_1(\chi,\chi_q)\ln(\chi) + c_2(\chi,\chi_q)\,\mathrm{Li}_2\left(\frac{1-\chi_q}{1+\chi}\right) + \ldots$ (✓ for $Q^2 \gg m^2$)

Computation Review - UV and IR Poles

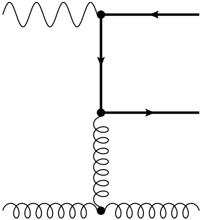
virtual diagrams are, e.g.,



or



soft poles appear in the limit of a soft gluon $k_2 \to 0$, e.g.,



soft + virtual + renormalization ($\overline{\text{MS}}_m$) + factorization is finite!

Computation Review - Analytic Expressions

$$D_0(m^2,0,q^2,m^2,t,s,0,m^2,m^2,m^2) = \frac{iC_\epsilon}{\beta s t_1} \times \left[-\frac{2}{\epsilon}\ln(\chi) - 2\ln(\chi)\ln\left(\frac{-t_1}{m^2}\right) \right.$$
$$+ \mathrm{Li}_2(1-\chi^2) - 4\zeta(2) + \ln^2(\chi_q) + 2\,\mathrm{Li}_2(-\chi\chi_q) + 2\,\mathrm{Li}_2\left(\frac{-\chi}{\chi_q}\right)$$
$$\left. + 2\ln(\chi\chi_q)\ln(1+\chi\chi_q) + 2\ln\left(\frac{\chi}{\chi_q}\right)\ln\left(1+\frac{\chi}{\chi_q}\right) \right]$$

$$\int \frac{d\Omega_n}{t' u_7{}^2} = -\frac{2\pi(m^2+s_4)(s'+t_1)}{s_4 t_1{}^2 u_1{}^2} \left[-2 + \frac{t_1 u_1 (-q^2 s_4 + (2m^2+s_4)(s'+u_1))}{(s'+t_1)\left(q^2 s_4 t_1 + m^2(s'+u_1)^2\right)} \right.$$
$$\left. + \frac{2}{\epsilon} + \ln\left(\frac{t_1{}^2 u_1{}^2 (m^2+s_4)}{(s'+t_1)^2\left(m^2(s'+u_1)^2 + q^2 t_1 s_4\right)} \right) \right]$$

Computation Review - Analytic Expressions

$$D_0(m^2,0,q^2,m^2,t,s,0,m^2,m^2,m^2) = \frac{iC_\epsilon}{\beta s t_1} \times \Bigg[-\frac{2}{\epsilon}$$

$$+ \mathrm{Li}_2(1-\chi^2) - 4\zeta(2) + \ln^2(\chi_q) + 2\,\mathrm{Li}_2(-\chi\chi_q) + 2\,\mathrm{Li}$$

$$+ 2\ln(\chi\chi_q)\ln(1+\chi\chi_q) + 2\ln\left(\frac{\chi}{\chi_q}\right)\ln\left(1+\frac{\chi}{\chi_q}\right)\Bigg]$$

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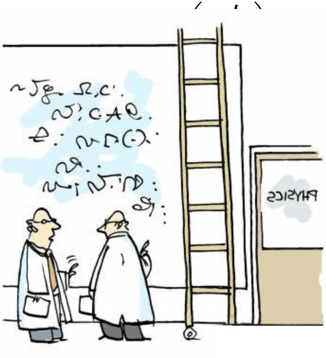
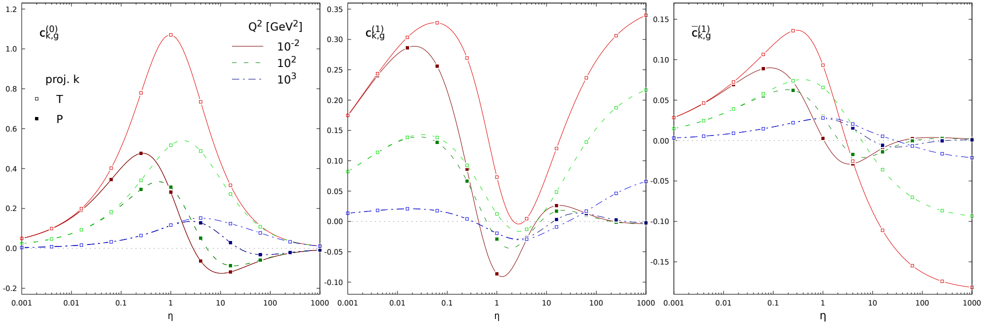
$$+\frac{2}{\epsilon}+\ln\left(\frac{{t_1}^2{u_1}^2(m^2+s_4)}{(s'+t_1)^2\left(m^2(s'+u_1)^2+q^2 t_1 s_4\right)}\right)\Bigg]$$



OOO, I'VE THOUGHT OF A NEW ONE!
TWO SQUIGGLES AND A BACKWARDS G!

Partonic Results - Gluon Channel

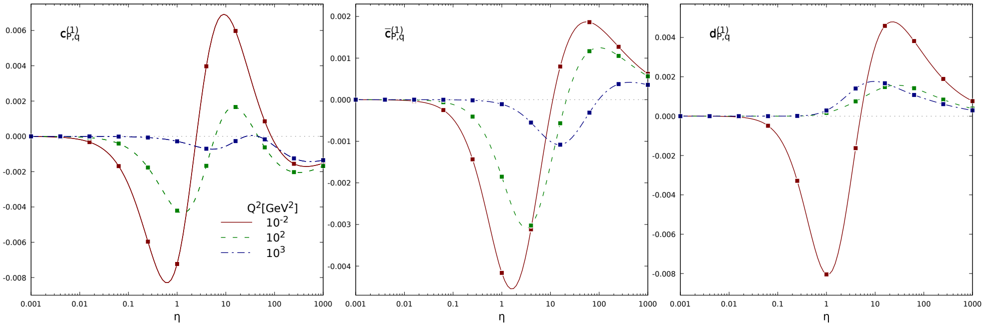
$$2xg_1(x) \sim \alpha_s \cdot \xi \Delta g(\xi) \otimes \left(c^{(0)}_{P,g} + 4\pi\alpha_s \left[c^{(1)}_{P,g} + \ln\left(\frac{\mu^2}{m^2}\right) \bar{c}^{(1)}_{P,g} \right] \right)$$



$$\eta = \frac{s - 4m^2}{4m^2}, \quad m = m_b = 4.75\,\mathrm{GeV}$$

Partonic Results - Light Quark Channel

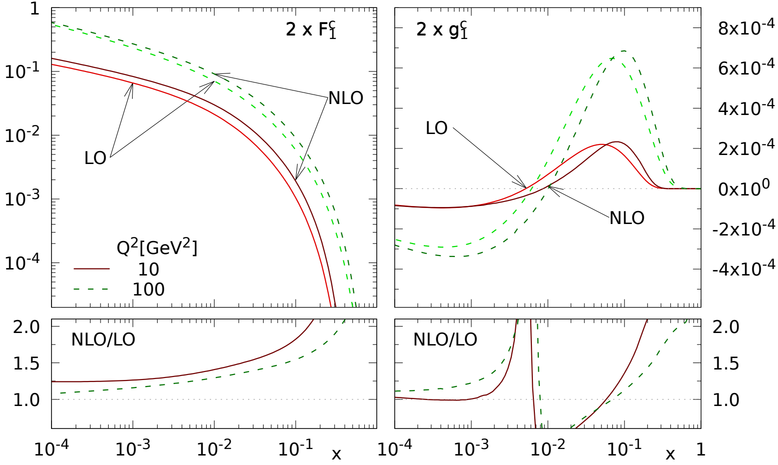
$$2xg_1(x) \sim \alpha_s^2 \sum_{q\in\{u,d,s\}} \xi\left(\Delta q(\xi) + \Delta\bar{q}(\xi)\right) \otimes \left(e_H^2 \left[c_{P,q}^{(1)} + \ln\left(\frac{\mu^2}{m^2}\right)\bar{c}_{P,q}^{(1)}\right] + e_q^2 d_{P,q}^{(1)}\right)$$



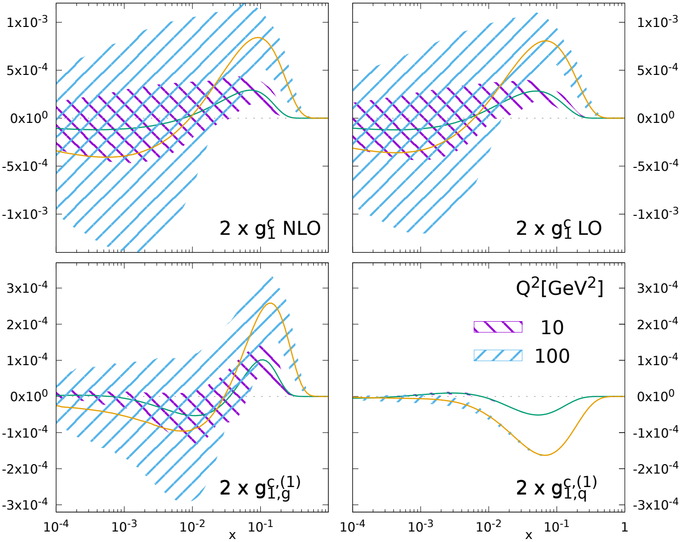
$$\eta = \frac{s - 4m^2}{4m^2}, \quad m = m_b = 4.75\,\mathrm{GeV}$$

Hadronic Results - Unpolarized vs. Polarized

unpolarized $\sim$ MSTW2008 $\leftrightarrow$ polarized $\sim$ DSSV2014

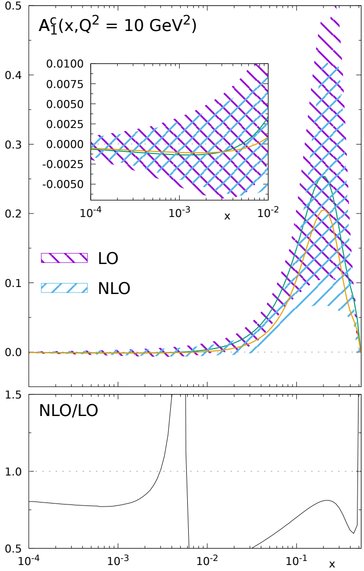


Hadronic Results - PDF Uncertainties DSSV (I)



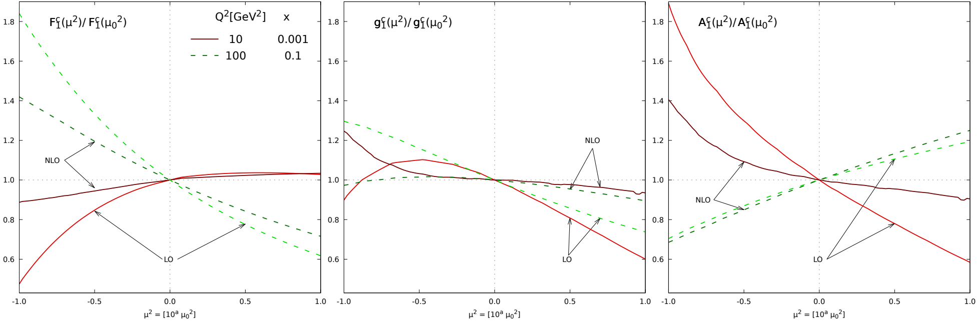
- gluon dominates
- quarks only make small contributions
- sign unclear
- quark band small
- default gluon is small

Hadronic Results - PDF Uncertainties DSSV (II)



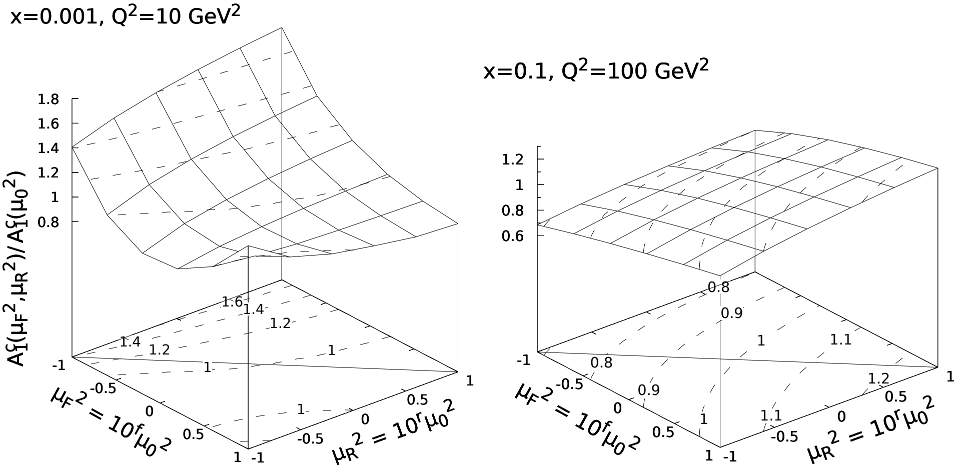
- $A_1^c(x,Q^2) = \frac{g_1^c(x,Q^2)}{F_1^c(x,Q^2)}$
- error band are only due to DSSV uncertainties (no correlations!)
- sign unclear
- need measurement of $\mathcal{O}(10^{-3})$
- NLO $\lessapprox$ LO

Hadronic Results - Scale Uncertainties (I)



$$\mu_F^2 = \mu_R^2 = 10^a \mu_0^2 \text{ with } \mu_0^2 = 4m^2 + Q^2$$

Hadronic Results - Scale Uncertainties (II)



$$\mu_0^2 = 4m^2 + Q^2$$

Outlook

- inclusive distributions: $\frac{dg_1}{dp_{T,\bar{Q}}}, \frac{dg_1}{dy_{\bar{Q}}}$
- correlated distributions: $\frac{dg_1}{dM^2_{Q\bar{Q}}}, \frac{dg_1}{d\phi_{Q\bar{Q}}}$

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- correlated distributions: $\frac{dg_1}{dM^2_{Q\bar{Q}}}$, $\frac{dg_1}{d\phi_{Q\bar{Q}}}$
- full neutral current (NC) contributions: $F_3^{Z\gamma}, g_4^{Z\gamma}, g_5^{Z\gamma}$ and F_2^Z, F_L^Z, g_1^Z
- distributions of full NC structure functions: $\frac{dg_1^{NC}}{dp_{T,\bar{Q}}}$, $\frac{dg_1^{NC}}{dM^2_{Q\bar{Q}}}$

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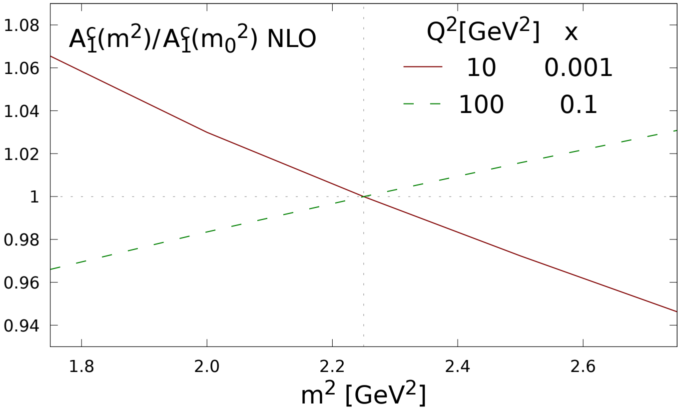
Thank you for your attention!

Backup: Hadronic Results - Mass Variation



$$m_0 = 1.5\,\text{GeV}$$