1 Introduction

This work is mainly based on the paper "Complete $O(\alpha_S)$ corrections to heavy-flavour structure functions in electroproduction" by Laenen et. al.[1] - that is, it recalculates all properties and formulas. It extends then the application to the equivalent *polarized* processes. The treating of the polarized processes can for example be found in [2] and we will use many ideas and technices from there. **FiXme Error: more** FiXme Error!

1.1 Motivation

FiXme Error!

FiXme Error: why do we do this

1.2 Notation

To collect all soft and collinear poles we have to calculate in $n = 4 + \epsilon$ dimension. Unfortunaly the extension for *polarized* processes is nontrivial, because the occuring Levi-Civita tensors $\varepsilon_{\mu\nu\rho\sigma}$ and γ_5. A common choice to deal with these objects is the HVBM prescription[3] that keeps those two objects four dimensional at the price for splitting the full n-dimensional space into a $(n-4)$-dimensional space, called "hat-space", and a four-dimensional space (that is actually never used).

In leading order (LO) we have to consider the following processes

$$\gamma^*(q;\sigma_q) + \mathrm{g}(k_1;\sigma_{k_1}) \to Q(p_1) + \overline{Q}(p_2) \tag{1}$$

The corresponding parton structure tensor $W^{(0)}_{\mu\mu'}$ can then be written as **FiXme Error: avoid all order expr?** FiXme Error!

$$\begin{aligned}
&W^{(0)}_{\mu\mu'}(k_1, q; s, t_1, u_1, q^2; \sigma_{k_1}\sigma_q) \\
&= \frac{1}{2} E_\epsilon K_{\mathrm{g}\gamma} \int \frac{d^{n-1}p_1}{2E_1(2\pi)^{n-1}} \int \frac{d^{n-1}p_2}{2E_2(2\pi)^{n-1}} \delta(p_1^2 - m^2)\delta(p_2^2 - m^2) \\
&\quad (2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2)\, \mathcal{M}^{(0)}_{\mu}(\sigma_{k_1}, \sigma_q)\, \mathcal{M}^{(0)}_{\mu'}(\sigma_{k_1}, \sigma_q)
\end{aligned} \tag{2}$$

where the initial $1/2$ is the initial state spin average, $K_{\mathrm{g}\gamma}$ is the color average,

$$E_\epsilon := \begin{cases} 1/(1+\epsilon/2) & \text{unpolarized} \\ 1 & \text{polarized} \end{cases} \tag{3}$$

accounts for initial freedom in n dimensions for bosons and we defined the following Mandelstam variables:

$$s = (q + k_1)^2, \quad t_1 = t - m^2 = (k_1 - p_2)^2 - m^2, \quad u_1 = u - m^2 = (q - p_2)^2 - m^2 \tag{4}$$

$$s' = s - q^2, \quad u_1' = u_1 - q^2 \tag{5}$$

FiXme Error: move to LO? The Lorentz indices μ and μ' refer to the virtual photon that is exchanged with the scattering lepton. FiXme Error!

By using Lorentz covariance, hermiticity, parity invariance and current conservation the parton structure tensor can be decomposed into several parts:

$$\begin{aligned} W_{\mu\mu'}(k_1, q; s, t_1, u_1, q^2; \sigma_{k_1}, \sigma_q) = &\left(-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2}\right) \frac{d^2\sigma_T(s, t_1, u_1, q^2)}{dt_1 du_1} \\ &+ \left(k_{1,\mu} - \frac{k_1 \cdot q}{q^2} q_\mu\right) \left(k_{1,\mu'} - \frac{k_1 \cdot q}{q^2} q_{\mu'}\right) \left(\frac{-4q^2}{s'^2}\right) \\ &\cdot \left(\frac{d^2\sigma_T(s, t_1, u_1, q^2)}{dt_1 du_1} + \frac{d^2\sigma_L(s, t_1, u_1, q^2)}{dt_1 du_1}\right) \end{aligned} \tag{6}$$

FiXme Error: extend We can then define appropiate projection operators[1, 4] FiXme Error!

$$\hat{\mathcal{P}}^\gamma_{G,\mu\mu'} = -g_{\mu\mu'} \qquad b_G(\epsilon) = \frac{1}{2(1+\epsilon/2)} \tag{7}$$

$$\hat{\mathcal{P}}^\gamma_{L,\mu\mu'} = -\frac{4q^2}{s'^2} k_{1,\mu} k_{1,\mu'} \qquad b_L(\epsilon) = 1 \tag{8}$$

$$\hat{\mathcal{P}}^\gamma_{P,\mu\mu'} = i\varepsilon_{\mu\mu'\rho\rho'} \frac{q^\rho k_1^{\rho'}}{s'} \qquad b_P(\epsilon) = 1 \tag{9}$$

FiXme Error: justify avoidance of $\boldsymbol{\Delta}$? and we then find FiXme Error!

$$\frac{d^2\sigma_k(s, t_1, u_1, q^2)}{dt_1 t u_1} = b_k(\epsilon) \hat{\mathcal{P}}^\gamma_{k,\mu\mu'} W^{\mu\mu'} \tag{10}$$

with $k \in \{G, L, P\}$ denoting (here and mostly ever after) the projection type. The transverse partonic cross section $d\sigma_T$ can be reconstructed from the above definitions by using

$$d\sigma_T = d\sigma_G + b_G(\epsilon) d\sigma_L \tag{11}$$

We also define accordingly

$$E_G(\epsilon) = E_L(\epsilon) = \frac{1}{1+\epsilon/2} \qquad E_P(\epsilon) = 1 \tag{12}$$

The final state spins are always summed over, but the initial spins have to be treated seperately: for unpolarized projections $k \in \{G, L\}$ they are also summed over, but for the polarized projection $k = P$ they are projected on their asymmetric part:

$$\hat{\mathcal{P}}^{\mathrm{g}}_G \varepsilon^{(\sigma_k)}_\nu(k_1) \varepsilon^{*(\sigma_k)}_{\nu'}(k_1) = \hat{\mathcal{P}}^{\mathrm{g}}_L \varepsilon^{(\sigma_k)}_\nu(k_1) \varepsilon^{*(\sigma_k)}_{\nu'}(k_1) = -g_{\nu\nu'} \tag{13}$$

$$\hat{\mathcal{P}}^{\mathrm{g}}_P \varepsilon^{(\sigma_k)}_\nu(k_1) \varepsilon^{*(\sigma_k)}_{\nu'}(k_1) = 2i\epsilon_{\nu\nu'\rho\rho'} \frac{k_1^\rho q^\rho}{s'} \tag{14}$$

By writing eq. (13) we decided to introduce Fadeev-Popov ghosts[2] as we got a single diagram in next-to-leading order with a triple-gluon vertex**FiXme Error: explain ghosts?**. As we can consider all quarks in the initial state as massless, we further find FiXme Error!

$$\hat{\mathcal{P}}_G^q u(k_1)\bar{u}(k_1) = \hat{\mathcal{P}}_L^q u(k_1)\bar{u}(k_1) = \not{k}_1 \qquad \hat{\mathcal{P}}_P^q u(k_1)\bar{u}(k_1) = -\gamma_5 \not{k}_1 \tag{15}$$

$$\hat{\mathcal{P}}_G^{\bar{q}} v(k_1)\bar{v}(k_1) = \hat{\mathcal{P}}_L^{\bar{q}} v(k_1)\bar{v}(k_1) = \not{k}_1 \qquad \hat{\mathcal{P}}_P^{\bar{q}} v(k_1)\bar{v}(k_1) = \gamma_5 \not{k}_1 \tag{16}$$

We define a set of partonic variables:

$$0 \leq \rho = \frac{4m^2}{s} \leq 1 \qquad 0 \leq \beta = \sqrt{1-\rho} \leq 1 \qquad 0 \leq \chi = \frac{1-\beta}{1+\beta} \leq 1 \tag{17}$$

$$\rho_q = \frac{4m^2}{q^2} \leq 0 \qquad 1 \leq \beta_q = \sqrt{1-\rho_q} \qquad 0 \leq \chi_q = -\frac{1-\beta_q}{1+\beta_q} \leq 1 \tag{18}$$

When computing Feynman diagrams a computer algebra system (CAS) is almost obligatory: common choices are `FORM`[5] or `Mathematica`[6] - for the later the most common choice is `TRACER`[7], but we have chosen `HEPMath`[8]. We used the Feynman rules given by [9].

2 Leading Order Calculations

In leading order we have to consider photon-gluon-fusion (PGF), that is

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) \tag{19}$$

with two contributing diagrams depicted in figure **FiXme Error: todo**. The result can then be written as FiXme Error!

$$\hat{\mathcal{P}}_k^{\gamma,\mu\mu'} \hat{\mathcal{P}}_k^{\mathrm{g}} \sum_{j=1}^{2} \mathcal{M}_{j,\mu}^{(0)} \mathcal{M}_{j,\mu'}^{(0)*} = 8 g^2 \mu_D^{-\epsilon} e^2 e_H^2 N_C C_F B_{k,QED} \tag{20}$$

where g and e are the strong and electromagnetic coupling constants respectively, μ_D is an arbitray mass parameter introduced to keep the couplings dimensionless and e_H is the magnitude of the heavy quark in units of e. Further N_C corresponds to the gauge group $SU(N_C)$ and the color factor $C_F = (N_C^2 - 1)/(2N_C)$ refers to the second Casimir constant of the fundamental representation for the quarks. We then find:

$$
\begin{aligned}
B_{G,QED} &= \frac{t_1}{u_1} + \frac{u_1}{t_1} + \frac{4m^2 s'}{t_1 u_1}\left(1 - \frac{m^2 s'}{t_1 u_1}\right) + \frac{2s'q^2}{t_1 u_1} + \frac{2q^4}{t_1 u_1} + \frac{2m^2 q^2}{t_1 u_1}\left(2 - \frac{s'^2}{t_1 u_1}\right) \\
&\quad + \epsilon\left\{-1 + \frac{s'^2}{t_1 u_1} + \frac{s' q^2}{t_1 u_1} - \frac{q^4}{t_1 u_1} - \frac{m^2 q^2 s'^2}{t_1^2 u_1^2}\right\} + \epsilon^2 \frac{s'^2}{4 t_1 u_1} \qquad (21) \\
B_{L,QED} &= -\frac{4q^2}{s'}\left(\frac{s}{s'} - \frac{m^2 s'}{t_1 u_1}\right) \qquad (22) \\
B_{P,QED} &= \frac{1}{2}\left(\frac{t_1}{u_1} + \frac{u_1}{t_1}\right)\left(\frac{2m^2 s'}{t_1 u_1} - 1 - \frac{2q^2}{s'}\right) \qquad (23) \\
B_{k,QED} &= B^{(0)}_{k,QED} + \epsilon B^{(1)}_{k,QED} + \epsilon^2 B^{(2)}_{k,QED} \qquad (24)
\end{aligned}
$$

By using eq. (2) we can derive the n-dimensional $2 \to 2$ phase space

$$
dPS_2 = \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_2}{(2\pi)^{n-1}} \Theta(p_{1,0})\delta(p_1^2 - m^2)\Theta(p_{2,0})\delta(p_2^2 - m^2)(2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2) \qquad (25)
$$

that can be solved by using the center-of-mass system (CMS) of the incoming particles[2]

$$
q = \left(\frac{s + q^2}{2\sqrt{s}}, 0, 0, -\frac{s - q^2}{2\sqrt{s}}, \hat{0}\right) \qquad k_1 = \frac{s - q^2}{2\sqrt{s}}\left(1, 0, 0, 1, \hat{0}\right) \qquad (26)
$$

such that $q + k_1 = (\sqrt{s}, \vec{0})$ and $k_1^2 = 0$. For the outgoing particles it follows

$$
p_1 = \frac{\sqrt{s}}{2}\left(1, 0, \beta\sin\theta, \beta\cos\theta, \hat{0}\right) \qquad p_2 = \frac{\sqrt{s}}{2}\left(1, 0, -\beta\sin\theta, -\beta\cos\theta, \hat{0}\right) \qquad (27)
$$

such that $p_1 + p_2 = (\sqrt{s}, \vec{0})$ and $p_1^2 = p_2^2 = m^2$. Finally we have to use the n-sphere

$$
d^n x = \frac{2\pi^{n/2}}{\Gamma(n/2)} x^{n-1} dx = \frac{\pi^{n/2}}{\Gamma(n/2)} (x^2)^{(n-2)/2} dx^2 \qquad (28)
$$

and arrive at the well known result[1]

$$
\begin{aligned}
dPS_2 &= \frac{\delta(s' + t_1 + u_1)}{2s'\Gamma((n-2)/2)(4\pi)^{(n-2)/2}} \left(\frac{(t_1 u_1' - s' m^2)s' - q^2 t_1^2}{s'^2}\right)^{(n-4)/2} dt_1 du_1 \qquad (29) \\
&= \delta(s' + t_1 + u_1) h_2(n)\, dt_1 du_1 \qquad (30) \\
h_2(4+\epsilon) &= \frac{2\pi S_\epsilon}{s'\Gamma(1 + \epsilon/2)} \left(\frac{(t_1 u_1' - s' m^2)s' - q^2 t_1^2}{s'^2}\right)^{\epsilon/2} \qquad (31)
\end{aligned}
$$

with $S_\epsilon = (4\pi)^{(-2-\epsilon/2)}$.

The final double differential LO partonic cross section can then be written as

$$
\begin{aligned}
s'^2 \frac{d^2\sigma^{(0)}_{k,\mathrm{g}}(s',t_1,u_1,q^2)}{dt_1 du_1} &= 2^6 \alpha\alpha_s e_H^2 K_{\mathrm{g}\gamma} N_C C_F E_k(\epsilon) b_k(\epsilon) \delta(s'+t_1+u_1) \frac{\pi^3 S_\epsilon}{\Gamma(1+\epsilon/2)} \\
&\qquad \left(\frac{(t_1 u_1' - s' m^2)s' - q^2 t_1^2}{m^2 s'^2}\right)^{\epsilon/2} \left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2} B_{k,QED} \qquad (32)
\end{aligned}
$$

where we used $e^2 = 4\pi\alpha$ and $g^2 = 4\pi\alpha_s$. The color average is given by $K_{\mathrm{g}\gamma} = 1/(N_C^2 - 1)$.

From the results above we can easily obtain the *total* LO partonic cross sections

$$
\sigma_G^{(0)}(s,q^2,m^2) = -4\pi\alpha\alpha_s e_H^2 K_{\mathrm{g}\gamma} N_C C_F \frac{1}{s'^3} \Big((s^2+q^4+4m^2 s)\beta + (s^2+q^4-4m^2(2m^2-s'))\ln(\chi)\Big) \qquad (33)
$$

$$
\sigma_L^{(0)}(s,q^2,m^2) = 16\pi\alpha\alpha_s e_H^2 K_{\mathrm{g}\gamma} N_C C_F \left(\frac{-q^2 s}{s'^3}\right)\left(\beta + \frac{2m^2}{s}\ln(\chi)\right) \qquad (34)
$$

$$
\sigma_P^{(0)}(s,q^2,m^2) = 4\pi\alpha\alpha_s e_H^2 K_{\mathrm{g}\gamma} N_C C_F \frac{1}{s'^2}\Big((3s+q^2)\beta + (s+q^2)\ln(\chi)\Big) \qquad (35)
$$

from which we also find

$$
\lim_{s\to 4m^2} \sigma_T^{(0)}(s',q^2) = 4\pi\alpha\alpha_s e_H^2 K_{\mathrm{g}\gamma} N_C C_F \frac{\beta}{4m^2-q^2} + O(\beta^3) = \lim_{s\to 4m^2} \sigma_P^{(0)}(s',q^2) \qquad (36)
$$

$$
\lim_{s\to 4m^2} \sigma_L^{(0)}(s',q^2) = -\frac{128}{3}\pi\alpha\alpha_s e_H^2 K_{\mathrm{g}\gamma} N_C C_F \frac{m^2 q^2 \beta^3}{(4m^2-q^2)^3} + O(\beta^5) \qquad (37)
$$

(Note the missing factor of 2 in [1, eq. (5.9)].) **FiXme Error: shift to partonic?** FiXme Error!

3 Next-To-Leading Order Calculations

Next-to-leading order contributions can be split into three parts: one loop virtual contributions, one gluon radiation and the light quark processes. **FiXme Error: more?** FiXme Error!

3.1 One Loop Virtual Contributions

Virtual contributions have the same initial and final state as the Born process, but have a looping particle. All contributing Feynman diagrams are depicted in figure **FiXme Error: do**. The result can then be written as FiXme Error!

$$
\begin{aligned}
&\hat{\mathcal{P}}_k^{\gamma,\mu\mu'} \hat{\mathcal{P}}_k^{\mathrm{g}} \sum_j \left[\mathcal{M}^{(1),V}_{j,\mu}\left(\mathcal{M}^{(0)}_{1,\mu'} + \mathcal{M}^{(0)}_{2,\mu'}\right)^* + c.c.\right] \\
&= 8 g^4 \mu_D^{-\epsilon} e^2 e_H^2 N_C C_F C_\epsilon \left(C_A V_{k,OK} + 2 C_F V_{k,QED}\right) \qquad (38)
\end{aligned}
$$

where $C_\epsilon = \exp(\epsilon/2(\gamma_E - \ln(4\pi)))/(16\pi^2)$ and C_A is the second Casimir constant of the adjoint representation for the gluon (that introduces a non-abelian part).

For the computation of the loops the Passarino-Veltman-decomposition[10] in $n = 4+\epsilon$ dimension is used as far as possible. The decomposition is based on Lorentz invariance and a good explanation is for example given in [2]. The needed scalar integrals are given in [11] and [1], but there is also one wrong integral: we find with [12, Box 16]:

$$\begin{aligned}
&D_0(m^2, 0, q^2, m^2, t, s, 0, m^2, m^2, m^2) \\
&= \frac{iC_\epsilon}{\beta s t_1}\Big[-\frac{2\ln(\chi)}{\epsilon} - 2\ln(\chi)\ln(-t_1/m^2) + \mathrm{Li}_2(1-\chi^2) - 4\zeta(2) + \ln^2(\chi_q) + 2\,\mathrm{Li}_2(-\chi\chi_q) \\
&\qquad + 2\,\mathrm{Li}_2(-\chi/\chi_q) + 2\ln(\chi\chi_q)\ln(1+\chi\chi_q) + 2\ln(\chi/\chi_q)\ln(1+\chi/\chi_q)\Big]
\end{aligned} \tag{39}$$

where we used the argument ordering of `LoopTools`[13, 14] (and also checked it against `LoopTools`).

As the short example above shows, the full expressions for the $V_{k,OK}, V_{k,QED}$ are quite complicated and too long to be presented here, nevertheless the arising poles are quite compact:

$$V_{k,OK} = -2B_{k,QED}\left(\frac{4}{\epsilon^2} + \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) - \frac{2m^2-s}{s}\ln(\chi)\right)\frac{2}{\epsilon}\right) + O(\epsilon^0) \tag{40}$$

$$V_{k,QED} = -2B_{k,QED}\left(1 - \frac{2m^2-s}{s}\ln(\chi)\right)\frac{2}{\epsilon} + O(\epsilon^0) \tag{41}$$

The above results already include the mass renormalization that we have performed *on-shell*, so all ultra-violet poles have been removed. For the renormalization of the strong coupling we use the $\overline{\mathrm{MS}}_m$ scheme defined in [2] and so the full (remaining) renormalization can be achieved by

$$\begin{aligned}
\frac{d^2\sigma_k^{(1),V,ren.}}{dt_1du_1} &= \frac{d^2\sigma_k^{(1),V}}{dt_1du_1} + \frac{\alpha_s(\mu_R^2)}{4\pi}\left[\left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln(\mu_R^2/m^2) - \ln(\mu_D^2/m^2)\right)\beta_0^f\right. \\
&\qquad \left. + \frac{2}{3}\ln(\mu_R^2/m^2)\right]\frac{d^2\sigma_k^{(0)}}{dt_1du_1} \qquad (42) \\
&= \frac{d^2\sigma_k^{(1),V}}{dt_1du_1} + 4\pi\alpha_s(\mu_R^2)C_\epsilon\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}\left[\left(\frac{2}{\epsilon} + \ln(\mu_R^2/m^2)\right)\beta_0^f\right. \\
&\qquad \left. + \frac{2}{3}\ln(\mu_R^2/m^2)\right]\frac{d^2\sigma_k^{(0)}}{dt_1du_1} \qquad (43)
\end{aligned}$$

with μ_R the renormalization scale introduced by the RGE, $\beta_0^f = (11C_A - 2n_f)/3$ the first coefficient of the beta function and n_f the number of *total* flavours (i.e. $n_{lf} = n_f - 1$ active (light) flavours and one heavy flavour). The double poles occuring in $V_{k,OK}$ are introduced by the diagrams **FiXme Error: do** when the soft and collinear singularities FiXme Error!

coincide.

The double differential partonic cross section is given by

$$s'^2\frac{d^2\sigma^{(1),V}_{k,\mathrm{g}}(s',t_1,u_1,q^2)}{dt_1du_1} = 2^8\alpha\alpha_s^2e_H^2K_{\mathrm{g}\gamma}N_CC_FE_k(\epsilon)b_k(\epsilon)\delta(s'+t_1+u_1)\frac{\pi^4S_\epsilon}{\Gamma(1+\epsilon/2)}$$
$$\left(\frac{(t_1u_1'-s'm^2)s'-q^2t_1^2}{m^2s'^2}\right)^{\epsilon/2}C_\epsilon\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}$$
$$\left(C_AV_{k,OK}+2C_FV_{k,QED}\right) \tag{44}$$

The results agree in the photo-production limit ($q^2\to 0$) with [15] **FiXme Error: Matrix elements available upon request**. FiXme Error!

3.2 Single Gluon Radiation

In next-to-leading order we have to consider the following process:

$$\gamma^*(q)+\mathrm{g}(k_1)\to Q(p_1)+\overline{Q}(p_2)+\mathrm{g}(k_2) \tag{45}$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as FiXme Error!

$$\hat{\mathcal{P}}_k^{\gamma,\mu\mu'}\hat{\mathcal{P}}_k^{\mathrm{g}}\sum_{j,j'}\mathcal{M}^{(1),g}_{j,\mu}\mathcal{M}^{(1),g^*}_{j',\mu'} = 8g^4\mu_D^{-2\epsilon}e^2e_H^2N_CC_F\left(C_AR_{k,OK}+2C_FR_{k,QED}\right) \tag{46}$$

and it will depend on ten kinematical invariants:

$$s=(q+k_1)^2 \qquad t_1=(k_1-p_2)^2-m^2 \qquad u_1=(q-p_2)^2-m^2 \tag{47}$$
$$s_3=(k_2+p_2)^2-m^2 \qquad s_4=(k_2+p_1)^2-m^2 \qquad s_5=(p_1+p_2)^2=-u_5 \tag{48}$$
$$t'=(k_1-k_2)^2 \tag{49}$$
$$u'=(q-k_2)^2 \qquad u_6=(k_1-p_1)^2-m^2 \qquad u_7=(q-p_1)^2-m^2 \tag{50}$$

from which only five are independent as can be seen from momentum conservation $k_1+q=p_1+p_2+k_2$ and s,t_1,u_1 match to their leading order definition.

The $2\to 3$ n-dimensional phase space is given by

$$dPS_3=\int\frac{d^np_1}{(2\pi)^{n-1}}\frac{d^np_2}{(2\pi)^{n-1}}\frac{d^nk_2}{(2\pi)^{n-1}}(2\pi)^n\delta^{(n)}(k_1+q-p_1-p_2-k_2)$$
$$\Theta(p_{1,0})\delta(p_1^2-m^2)\Theta(p_{2,0})\delta(p_2^2-m^2)\Theta(k_{2,0})\delta(k_2^2) \tag{51}$$

This can be solved by writing eq. (51) as product of a $2 \to 2$ decay and a subsequent $1 \to 2$ decay[11]. We find

$$dPS_3 = \frac{1}{(4\pi)^n \Gamma(n-3) s'} \frac{s_4^{n-3}}{(s_4+m^2)^{n/2-1}} \left(\frac{(t_1 u_1' - s' m^2)s' - q^2 t_1^2}{s'^2} \right)^{(n-4)/2} dt_1 du_1 d\Omega_n d\hat{\mathcal{I}} \tag{52}$$

$$= h_3(n)\, dt_1 du_1 d\Omega_n d\hat{\mathcal{I}} \tag{53}$$

with $d\Omega_n = \sin^{n-3}(\theta_1) d\theta_1 \sin^{n-4}(\theta_2) d\theta_2$ and $d\hat{\mathcal{I}}$ taking care of all occuring hat momenta:

$$d\hat{\mathcal{I}} = \frac{1}{B(1/2,(n-4)/2)} \frac{x^{(n-6)/2}}{\sqrt{1-x}} dx \qquad \text{with } x = \hat{p}_1^2/\hat{p}_{1,max} \tag{54}$$

$$\hat{p}_{1,max} = \frac{s_4^2}{4(s_4+m^2)} \sin^2(\theta_1)\sin^2(\theta_2) \tag{55}$$

$$\Rightarrow \int d\hat{\mathcal{I}} = 1 \qquad \int d\hat{\mathcal{I}}\, \hat{p}_1^2 = \epsilon \hat{p}_{1,max} + O(\epsilon^2) \tag{56}$$

The needed phase space integrals for θ_1 and θ_2 can be found in [11] and [2]. We find for the difference to the $2 \to 2$ phase space

$$\frac{h_3(4+\epsilon)}{h_2(4+\epsilon)} = \frac{S_\epsilon}{2\pi} \frac{\Gamma(1+\epsilon/2)}{\Gamma(1+\epsilon)} \frac{s_4^{1+\epsilon}}{(s_4+m^2)^{1+\epsilon/2}} \tag{57}$$

$$= \frac{C_\epsilon}{2\pi} \left(1 - \frac{3}{8}\zeta(2)\epsilon^2\right) \frac{s_4^{1+\epsilon}}{(s_4+m^2)^{1+\epsilon/2}} + O(\epsilon^3) \tag{58}$$

where ζ is Riemanns zeta function. **FiXme Error: introduce psLogs? in appendix?** FiXme Error!

Again when integrating the phase space angles the expressions become quite lengthy, but the (collinear) pole parts are compact:

$$\frac{s_4}{4\pi(s_4+m^2)} \int d\Omega_n d\hat{\mathcal{I}}\, C_A R_{k,OK} = -\frac{1}{u_1} B_{k,QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} P^H_{k,\mathrm{gg}}(x_1) \frac{2}{\epsilon} + O(\epsilon^0) \tag{59}$$

with $x_1 = -u_1/(s'+t_1)$ and the hard part of the Altarelli-Parisi splitting functions $P^H_{k,\mathrm{gg}}$[16, 17]:

$$P^H_{G,\mathrm{gg}}(x) = P^H_{L,\mathrm{gg}}(x) = C_A \left(\frac{2}{1-x} + \frac{2}{x} - 4 + 2x - 2x^2 \right) \tag{60}$$

$$P^H_{P,\mathrm{gg}}(x) = C_A \left(\frac{2}{1-x} - 4x + 2 \right) \tag{61}$$

The $R_{k,QED}$ do not contain poles. **FiXme Error: shift to factorization?** FiXme Error!

The double differential partonic cross section is given by

$$
\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),R}_{k,\mathrm{g}}(s',t_1,u_1,q^2)}{dt_1 du_1} &= 2^7 \alpha \alpha_s^2 e_H^2 K_{\mathrm{g}\gamma} N_C C_F E_k(\epsilon) b_k(\epsilon) \frac{\pi^3 S_\epsilon^2}{\Gamma(1+\epsilon)} \frac{s_4}{s_4+m^2} \\
&\quad \left(\frac{(t_1 u_1' - s'm^2)s' - q^2 t_1^2}{m^2 s'^2}\right)^{\epsilon/2} \left(\frac{s_4^2}{m^2(s_4+m^2)}\right)^{\epsilon/2} \left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon} \\
&\quad \int d\Omega_n d\hat{\mathcal{I}} \left(C_A R_{k,OK} + 2C_F R_{k,QED}\right) \qquad (62)
\end{aligned}
$$

From the above expression we can obtain the soft limit $k_2 \to 0$ and seperate their calculations:

$$
\lim_{k_2\to 0} \left(C_A R_{k,OK} + 2C_F R_{k,QED}\right) = \left(C_A S_{k,OK} + 2C_F S_{k,QED}\right) + O(1/s_4, 1/s_3, 1/t') \qquad (63)
$$

$$
S_{k,OK} = 2\left(\frac{t_1}{t's_3} + \frac{u_1}{t's_4} - \frac{s-2m^2}{s_3 s_4}\right) B_{k,QED} \qquad (64)
$$

$$
S_{k,QED} = 2\left(\frac{s-2m^2}{s_3 s_4} - \frac{m^2}{s_3^2} - \frac{m^2}{s_4^2}\right) B_{k,QED} \qquad (65)
$$

Note that the einkonal factors multiplying the Born functions $B_{k,QED}$ neither depend on q^2 nor on the projection k. We can then split the phase space by introducing an infrared cut-off Δ and distinguish then between soft $s_4 \leq \Delta$ and hard $s_4 > \Delta$ contributions. Let $\mathcal{R}(s_4)$ be a function with a soft pole $s_4^{-1+\epsilon}\mathcal{S}(s_4)$ and a finite part $\mathcal{F}(s_4)$, we then find [2]:

$$
\int_0^{s_{4,max}} \mathcal{R}(s_4) = \int_0^{s_{4,max}} \left(s_4^{-1+\epsilon}\mathcal{S}(s_4) + \mathcal{F}(s_4)\right) \qquad (66)
$$

$$
\simeq \frac{\Delta^\epsilon}{\epsilon}\mathcal{S}(0) + \int_\Delta^{s_{4,max}} \mathcal{R}(s_4) \qquad (67)
$$

This expansion is valid for Δ being small, i.e. smaller then any leading order scale or m^2;

a typical choice is $\Delta/m^2 \sim 10^{-6}$. We then find

$$\frac{s_4^2}{4\pi(s_4+m^2)}\left(1-\frac{3}{8}\zeta(2)\epsilon^2\right)\int d\Omega_n d\hat{\mathcal{I}}\, S_{k,QED}$$
$$= B_{k,QED}\left[-\frac{2}{\epsilon}\left(1+\frac{s-2m^2}{s\beta}\ln(\chi)\right)+1-\frac{s-m^2}{s\beta}\left(\ln(\chi)\left(1+\ln(\chi)\right)+\mathrm{Li}_2(1-\chi^2)\right)\right] \tag{68}$$

$$\frac{s_4^2}{4\pi(s_4+m^2)}\left(1-\frac{3}{8}\zeta(2)\epsilon^2\right)\int d\Omega_n d\hat{\mathcal{I}}\, S_{k,OK}$$
$$= B_{k,QED}\left[\frac{4}{\epsilon^2}+\frac{2}{\epsilon}\left(\ln(t_1/u_1)+\frac{s-2m^2}{s\beta}\ln(\chi)\right)-\ln^2(\chi)-\frac{3}{2}\zeta(2)+\frac{1}{2}\ln^2(t_1/(u_1\chi))\right.$$
$$\left.+\mathrm{Li}_2(1-t_1/(u_1\chi))-\mathrm{Li}_2(1-u_1/(t_1\chi))+\frac{s-2m^2}{s\beta}\left(\mathrm{Li}_2(1-\chi^2)+\ln^2(\chi)\right)\right] \tag{69}$$

(Note the mistyped sign of $\ln(\chi)^2$ in [1, eq. (3.25)]) The additional factors on the left hand sides originate from the difference between the $2 \to 3$ phasespace of R_k and the $2 \to 2$ phasespace needed for S_k.

The double differential partonic cross section is given by

$$s'^2\frac{d^2\sigma^{(1),S}_{k,\mathrm{g}}(s',t_1,u_1,q^2)}{dt_1du_1}$$
$$= 2^8\alpha\alpha_s^2e_H^2K_{\mathrm{g}\gamma}N_CC_FE_k(\epsilon)b_k(\epsilon)\delta(s'+t_1+u_1)\frac{\pi^4S_\epsilon}{\Gamma(1+\epsilon/2)}$$
$$\left(\frac{(t_1u_1'-s'm^2)s'-q^2t_1^2}{m^2s'^2}\right)^{\epsilon/2}C_\epsilon\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon}\left(\frac{\Delta}{m^2}\right)^{\epsilon}$$
$$\frac{s_4^2}{4\pi(s_4+m^2)}\left(1-\frac{3}{8}\zeta(2)\epsilon^2\right)\int d\Omega_n d\hat{\mathcal{I}}\,\left(C_AS_{k,OK}+2C_FS_{k,QED}\right) \tag{70}$$

The results agree in the photo-production limit ($q^2 \to 0$) with [15].

3.3 Light Quark Processes

In next-to-leading order a new production mechanism enters that is induced by a light quark, so we have to consider the process

$$\gamma^*(q)+q(k_1)\to Q(p_1)+\overline{Q}(p_2)+q(k_2) \tag{71}$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be FiXme Error!

written as

$$\hat{\mathcal{P}}_k^{\gamma,\mu\mu'}\hat{\mathcal{P}}_k^q \sum_{j,j'} \mathcal{M}_{j,\mu}^{(1),q} \mathcal{M}_{j',\mu'}^{(1),q*} = 8g^4\mu_D^{-2\epsilon}e^2 N_C C_F \left(e_H^2 A_{k,1} + e_L^2 A_{k,2} + e_L e_H A_{k,3}\right) \tag{72}$$

where e_L denotes the charge of the light quark q in units of e.

The needed $2 \to 3$ phase space has already been calculated in section 3.2, so we can immediately quote the (collinear) poles:

$$\frac{s_4}{2\pi(s_4+m^2)}\int d\Omega_n d\hat{\mathcal{I}}\, C_F A_{k,1} = -\frac{1}{u_1} B_{k,QED}\begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} P_{k,gq}(x_1)\frac{2}{\epsilon} + O(\epsilon^0) \tag{73}$$

with $x_1 = -u_1/(s'+t_1)$ and the Altarelli-Parisi splitting functions $P_{k,gq}$[16, 17]:

$$P_{G,gq}(x) = P_{L,gq}(x) = C_F\left(\frac{1}{x} + \frac{(1-x)^2}{x}\right) \tag{74}$$

$$P_{P,gq}(x) = C_F\,(2-x) \tag{75}$$

$A_{k,2}$ does not contain poles and we find $\int dt_1 du_1 \int d\Omega_n d\hat{\mathcal{I}}\, A_{k,3} = 0$. Note that in the limit $q^2 \to 0$ $A_{k,2}$ will also get collinear poles.

The double differential partonic cross section is given by

$$\begin{aligned} s'^2\frac{d^2\sigma_{k,q}^{(1)}(s',t_1,u_1,q^2)}{dt_1 du_1} &= 2^7\alpha\alpha_s^2 K_{q\gamma} N_C C_F b_k(\epsilon)\frac{\pi^3 S_\epsilon^2}{\Gamma(1+\epsilon)}\frac{s_4}{s_4+m^2} \\ &\quad \left(\frac{(t_1u_1' - s'm^2)s' - q^2t_1^2}{m^2 s'^2}\right)^{\epsilon/2}\left(\frac{s_4^2}{m^2(s_4+m^2)}\right)^{\epsilon/2}\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon} \\ &\quad \int d\Omega_n d\hat{\mathcal{I}}\,\left(e_H^2 A_{k,1} + e_L^2 A_{k,2} + e_H e_L A_{k,3}\right) \end{aligned} \tag{76}$$

with the color average $K_{q\gamma} = 1/N_C$.

The results agree in the photo-production limit ($q^2 \to 0$) with [15].

4 Mass Factorization

All collinear poles in the gluonic subprocess can be removed by mass factorization in the following way:

$$s'^2 \frac{d^2\sigma^{(1),fin}_{k,\mathrm{g}}(s',t_1,u_1,q^2,\mu_F)}{dt_1du_1} = \lim_{\epsilon\to 0}\Bigg[s'^2 \frac{d^2\sigma^{(1)}_{k,\mathrm{g}}(s',t_1,u_1,q^2,\epsilon)}{dt_1du_1}$$

$$- \int_0^1 \frac{dx_1}{x_1}\Gamma^{(1)}_{k,\mathrm{gg}}(x_1,\mu_F^2,\mu_D,\epsilon) \tag{77}$$

$$(x_1s')^2 \frac{d^2\sigma^{(0)}_{k,\mathrm{g}}(x_1s',x_1t_1,u_1,q^2,\epsilon)}{d(x_1t_1)du_1}\Bigg] \tag{78}$$

$$\Gamma^{(1)}_{k,ij}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_s}{2\pi}\left(P_{k,ij}(x)\frac{2}{\epsilon} + f_{k,ij}(x,\mu_F^2,\mu_D^2)\right) \tag{79}$$

where $\Gamma^{(1)}_{k,ij}$ is the first order correction to the transition functions $\Gamma_{k,ij}$ for *incoming* particle j and *outgoing* particle i in projection k. In the $\overline{\mathrm{MS}}$-scheme the $f_{k,ij}$ take their usual form and we find

$$\Gamma^{(1),\overline{\mathrm{MS}}}_{k,ij}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_s}{2\pi}P_{k,ij}(x)\left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln(\mu_F^2/m^2) - \ln(\mu_D^2/m^2)\right) \tag{80}$$

$$= 8\pi\alpha_s P_{k,ij}(x) C_\epsilon \left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}\left(\frac{2}{\epsilon} + \ln(\mu_F^2/m^2)\right) \tag{81}$$

The $P_{k,ij}(x)$ are the Altarelli-Parisi splitting functions for which we find[16, 17]

$$P_{k,\mathrm{gg}}(x) = \Theta(1-\delta-x)P^H_{k,\mathrm{gg}}(x) + \delta(1-x)\left(2C_A\ln(\delta) + \frac{\beta_0}{2}\right) \tag{82}$$

where we introduced another infrared cut-off δ to seperate soft ($x \geq 1-\delta$) and hard ($x < 1-\delta$) gluons that is connected to Δ via $\delta = \Delta/(s'+t_1)$. The structure here explains why we were able to write the equation (59).

The light quark process can be regularized in a complete analogous way:

$$s'^2 \frac{d^2\sigma^{(1),fin}_{k,q}(s',t_1,u_1,q^2,\mu_F)}{dt_1du_1} = \lim_{\epsilon\to 0}\Bigg[s'^2 \frac{d^2\sigma^{(1)}_{k,q}(s',t_1,u_1,q^2,\epsilon)}{dt_1du_1}$$

$$- \int_0^1 \frac{dx_1}{x_1}\Gamma^{(1)}_{k,\mathrm{g}q}(x_1,\mu_F^2,\mu_D,\epsilon)$$

$$(x_1s')^2 \frac{d^2\sigma^{(0)}_{k,\mathrm{g}}(x_1s',x_1t_1,u_1,q^2,\epsilon)}{d(x_1t_1)du_1}\Bigg] \tag{83}$$

The needed splitting functions $P_{k,gq}$ have been already quoted in equations (74) and (75). Note that $K_{q\gamma} = 1/(N_C) = 2C_F K_{g\gamma}$.

The final finite cross sections are then

$$
\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),H,fin}_{k,g}}{dt_1 du_1} = \frac{1}{2\pi} K_{g\gamma}\alpha\alpha_S e_H^2 N_C C_F b_k(0) \Bigg[&-\frac{1}{u_1} P^H_{k,gg}(x_1) \\
&\left\{ 4\pi B^{(0)}_{k,QED}\begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)}\right) - \ln(\mu_F^2/m^2)\right) \right. \\
&\left. -8\pi B^{(1)}_{k,QED}\begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \right\} \\
&+ C_A \frac{s_4}{s_4+m^2} \left(\int d\Omega_n d\hat{\mathcal{I}}\, R_{k,OK} \right)^{finite} \\
&+ 2C_F \frac{s_4}{s_4+m^2} \int d\Omega_4 d\hat{\mathcal{I}}\, R_{k,QED} \Bigg]
\end{aligned} \tag{84}
$$

$$
\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),S+V,fin}_{k,g}}{dt_1 du_1} = 4K_{g\gamma}\alpha\alpha_S e_H^2 N_C C_F b_k(0) B^{(0)}_{k,QED} \delta(s'+t_1+u_1) \Big[&C_A \ln^2(\Delta/m^2) \\
+ \ln(\Delta/m^2) \Big(\Big(\ln(-t_1/m^2) - \ln(-u_1/m^2) - \ln(\mu_F^2/m^2) \Big) &C_A \\
- \frac{2m^2 - s}{s\beta} \ln(\chi)(C_A - 2C_F) - 2C_F \Big) & \\
+ \frac{\beta_0^{lf}}{4} \Big(\ln(\mu_R^2/m^2) - \ln(\mu_F^2/m^2) \Big) + f_k(s', u_1, t_1, q^2) \Big] &
\end{aligned} \tag{85}
$$

where f_k contains lots of logarithms and dilogarithms, but does not depend on Δ, μ_F^2, μ_R^2 nor n_f and $\beta_0^{lf} = (11C_A - 2n_{lf})/3$.

$$
\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),fin}_{k,q}}{dt_1 du_1} = \frac{1}{2\pi} K_{q\gamma}\alpha\alpha_S N_C b_k(0) \Bigg[&-\frac{1}{u_1} e_H^2 P_{k,gq}(x_1) \\
&\left\{ 2\pi B^{(0)}_{k,QED}\begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)}\right) - \ln(\mu_F^2/m^2) + 1 - \delta_{k,P}\right) \right. \\
&\left. -4\pi B^{(1)}_{k,QED}\begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \right\} \\
&+ C_F \frac{s_4}{s_4+m^2} \left(\int d\Omega_n d\hat{\mathcal{I}}\, e_H^2 A_{k,1} \right)^{finite} \\
&+ C_F \frac{s_4}{s_4+m^2} \int d\Omega_4 d\hat{\mathcal{I}}\, e_L^2 A_{k,2} + C_F \frac{s_4}{s_4+m^2} \int d\Omega_4 d\hat{\mathcal{I}}\, e_H e_L A_{k,3} \Bigg]
\end{aligned} \tag{86}
$$

where $1 - \delta_{k,P}$ may also be written as $-2\partial_\epsilon E_k(\epsilon = 0)$ as it originates from the additional factor of $E_k(\epsilon)$ in the subtraction part of equation (83).

5 Partonic Results

The *total* partonic cross sections can be computed by

$$\sigma^{(n)}_{k,j}(s,q^2,m^2) = \int\limits_{-s'(1+\beta)/2}^{-s'(1-\beta)/2} dt_1 \int\limits_0^{s_{4,max}} ds_4 \frac{d^2\sigma^{(n),fin}_{k,j}(s',t_1,u_1,q^2)}{dt_1 ds_4} \tag{87}$$

$$s_{4,max} = \frac{s}{s't_1}\left(t_1 + \frac{s'(1-\beta)}{2}\right)\left(t_1 + \frac{s'(1+\beta)}{2}\right) \tag{88}$$

where k denotes as usual projection, $j \in \{\mathrm{g}, q, \bar{q}\}$ is a parton index and we used $s_4 = s' + t_1 + u_1$. The result can then be parametrised as

$$\begin{aligned}
&\sigma_{k,j}(s,q^2,m^2) \\
&= \sigma^{(0)}_{k,j}(s,q^2,m^2) + \sigma^{(1)}_{k,j}(s,q^2,m^2) \qquad (89)\\
&= \frac{\alpha\alpha_S}{m^2}\left(f^{(0)}_{k,j}(\eta,\xi) + 4\pi\left(f^{(1)}_{k,j}(\eta,\xi) + \ln(\mu_F^2/m^2)\bar{f}^{F,(1)}_{k,j}(\eta,\xi) + \ln(\mu_R^2/m^2)\bar{f}^{R,(1)}_{k,j}(\eta,\xi)\right)\right) \qquad (90)
\end{aligned}$$

where each function $f_{k,j}$ depends on the scaling variables $\eta = 1/\rho - 1$ and $\xi = -q^2/m^2$ and can be further decomposed by the electric charges

$$f_{k,\mathrm{g}}(\eta,\xi) = e_H^2 c_{k,\mathrm{g}}(\eta,\xi) \tag{91}$$

$$f_{k,q}(\eta,\xi) = e_H^2 c_{k,q}(\eta,\xi) + e_L^2 d_{k,q}(\eta,\xi) \tag{92}$$

As already mentioned the interference term proportional to $e_H e_L$ vanishes for total cross sections. FiXme Error!

FiXme Error: shift to appendix? FiXme Error: compare T and P? FiXme Error: how much to comment? FiXme Error! FiXme Error!

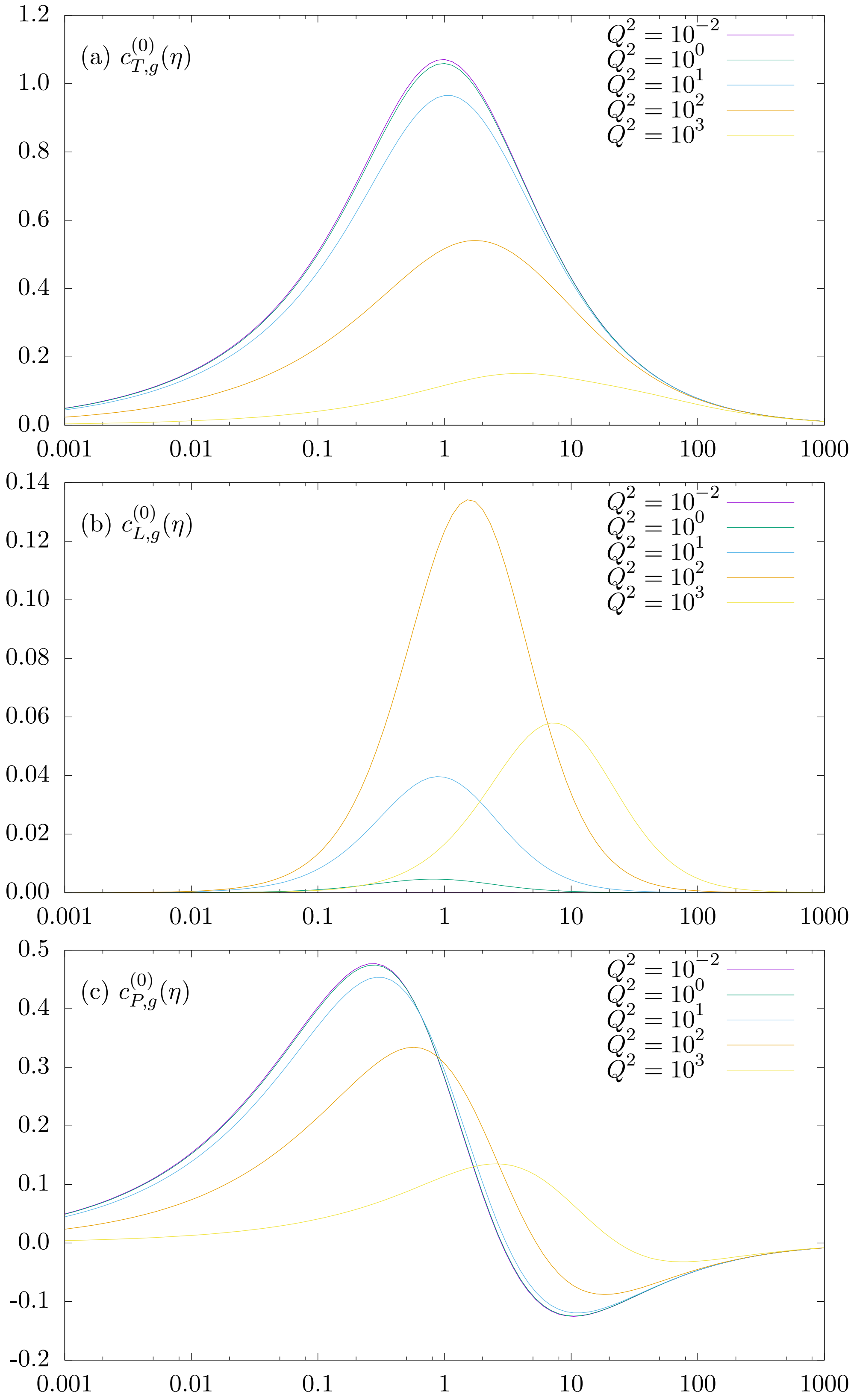

Figure 1: leading order scaling functions $c^{(0)}_{k,g}(\eta, \xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,$GeV (i.e. different values of $\xi = Q^2/m^2$)

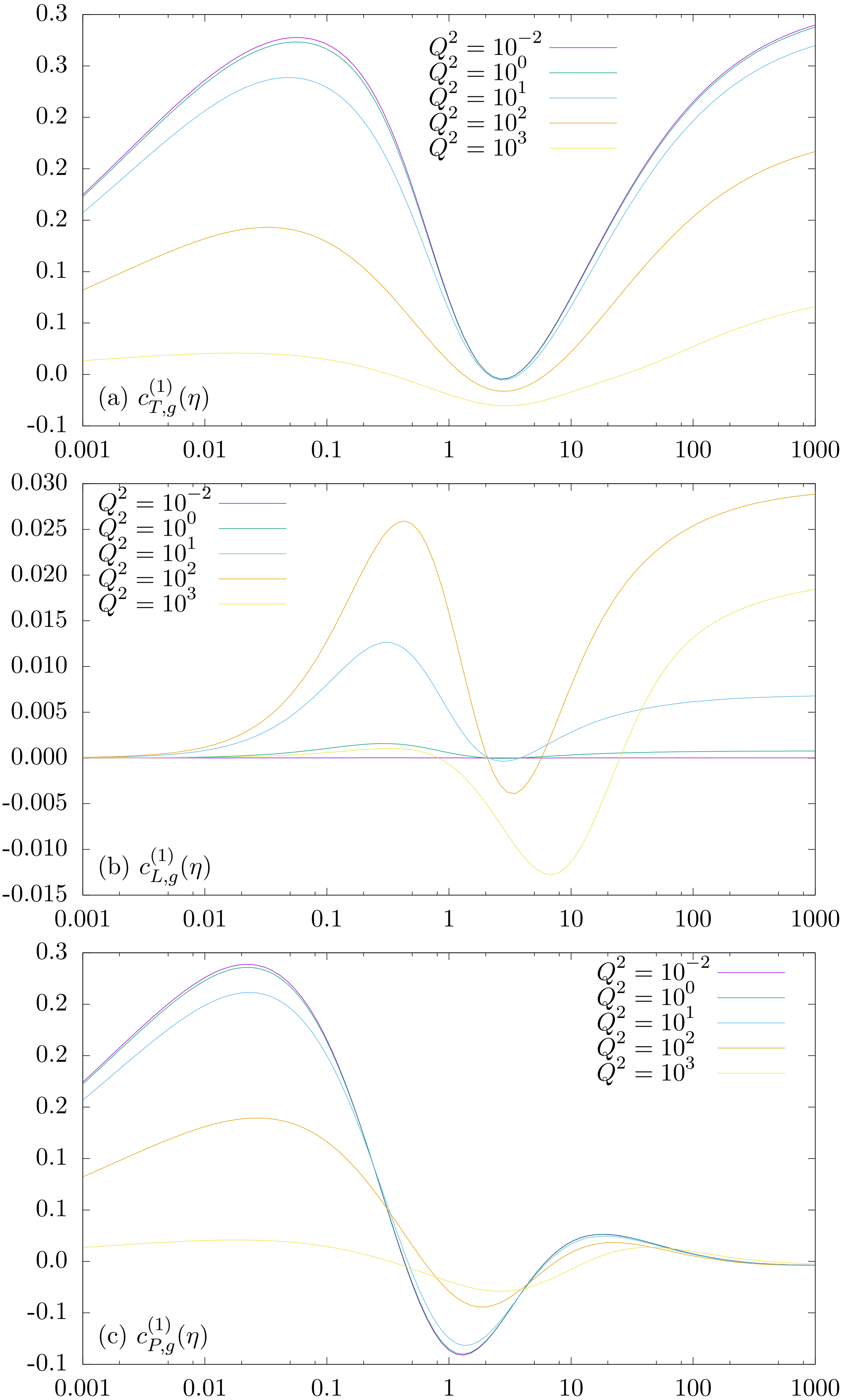

Figure 2: next-to-leading order scaling functions $c^{(1)}_{k,g}(\eta, \xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\text{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

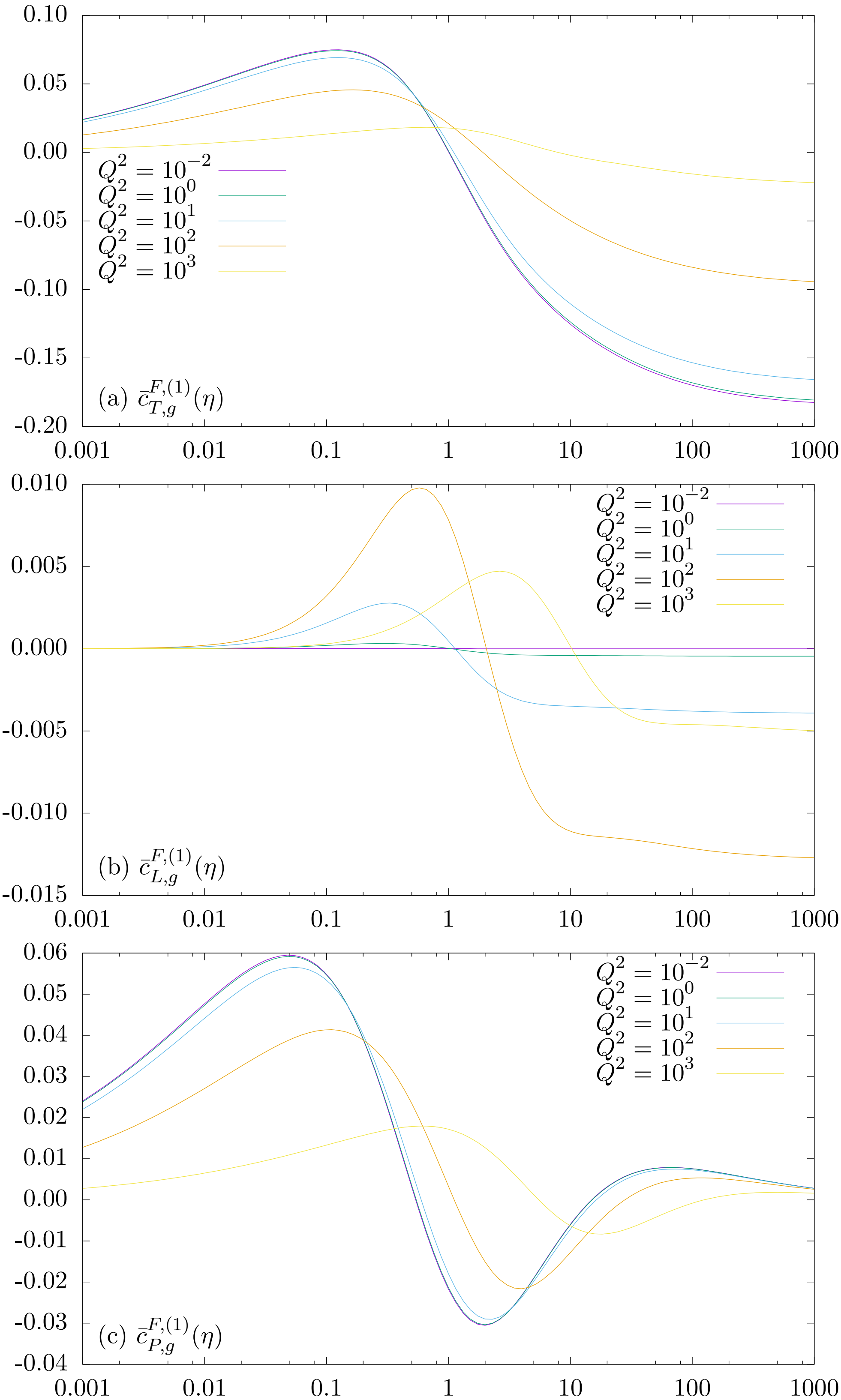

Figure 3: next-to-leading order scaling functions $\bar{c}^{F,(1)}_{k,g}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$) and $n_{lf} = 4$

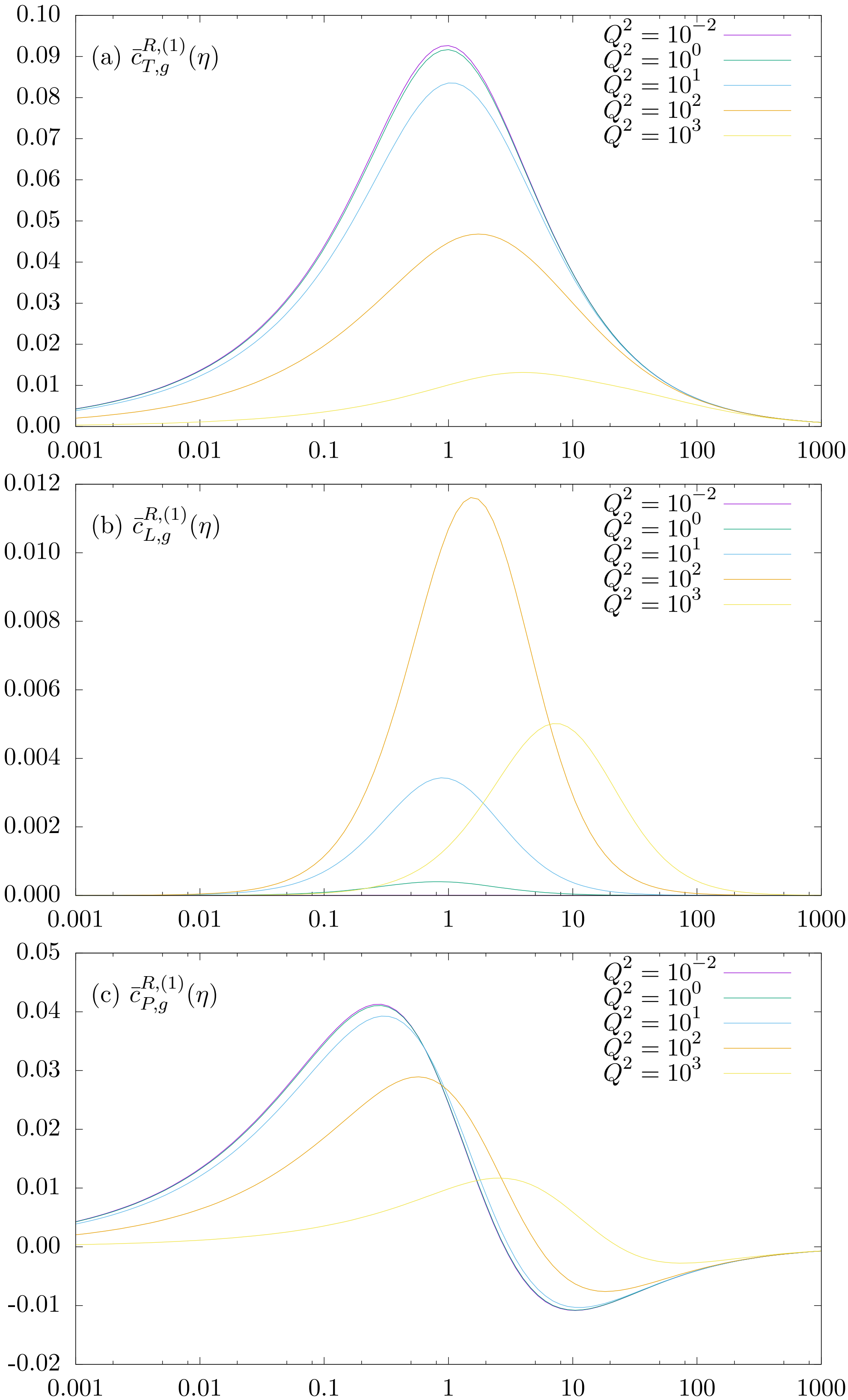

Figure 4: next-to-leading order scaling functions $\bar{c}^{R,(1)}_{k,g}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

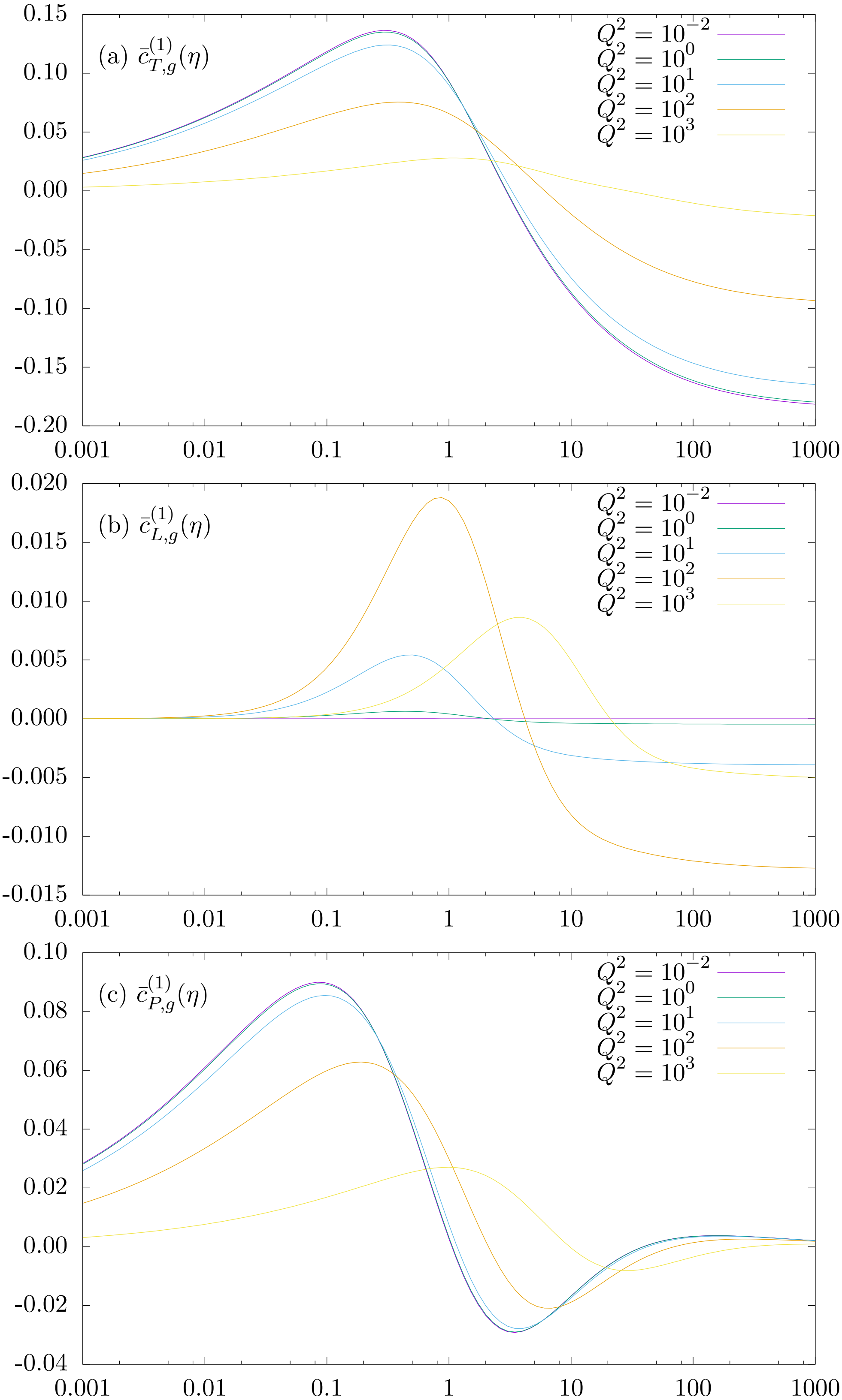

Figure 5: next-to-leading order scaling functions $\bar{c}^{(1)}_{k,g}(\eta,\xi) = \bar{c}^{R,(1)}_{k,g}(\eta,\xi) + \bar{c}^{F,(1)}_{k,g}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\text{GeV}$ (i.e. different values of $\xi = Q^2/m^2$) and $n_{lf} = 4$

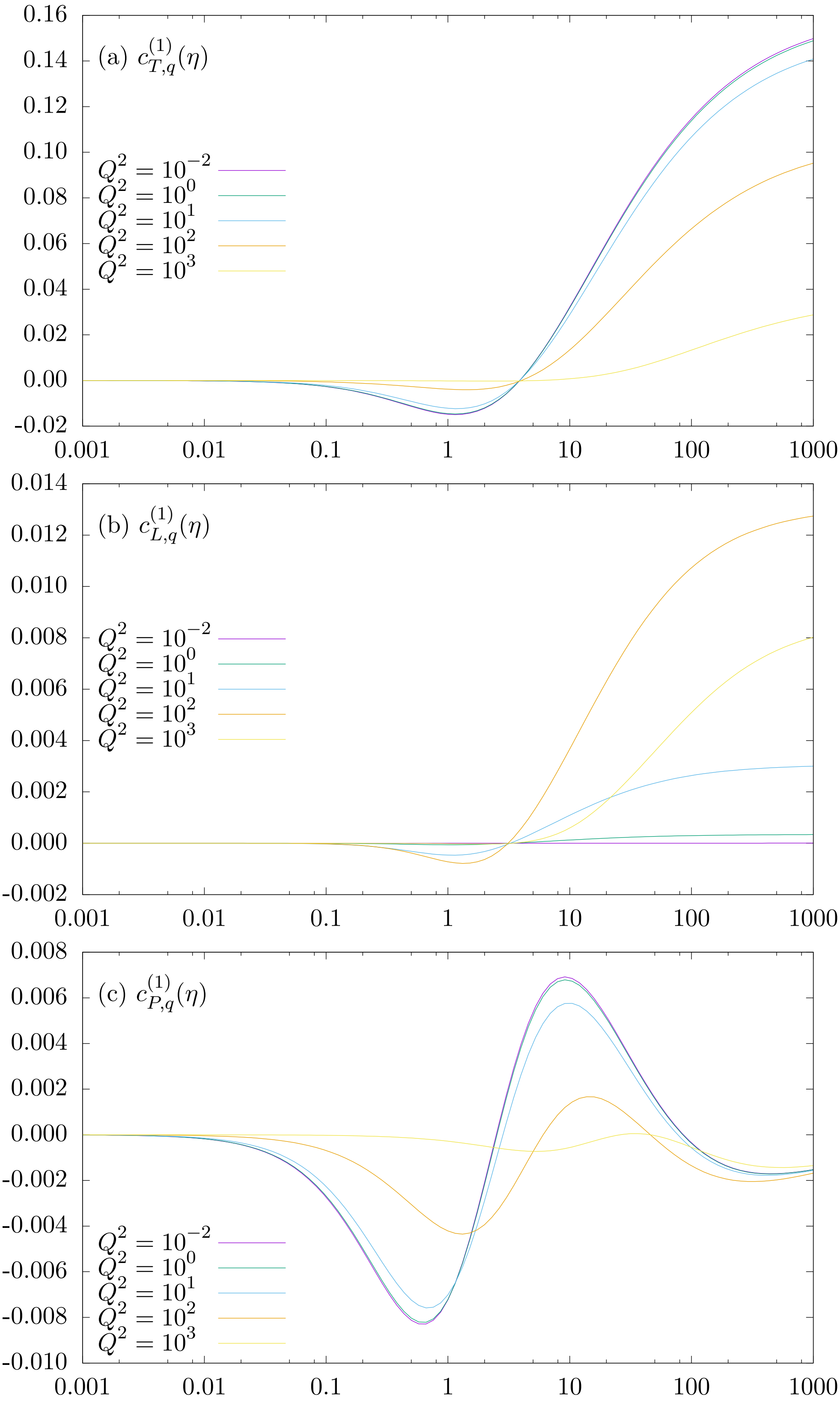

Figure 6: next-to-leading order scaling functions $c^{(1)}_{k,q}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$). Note that [1, Fig. 9 (b)] is wrong.

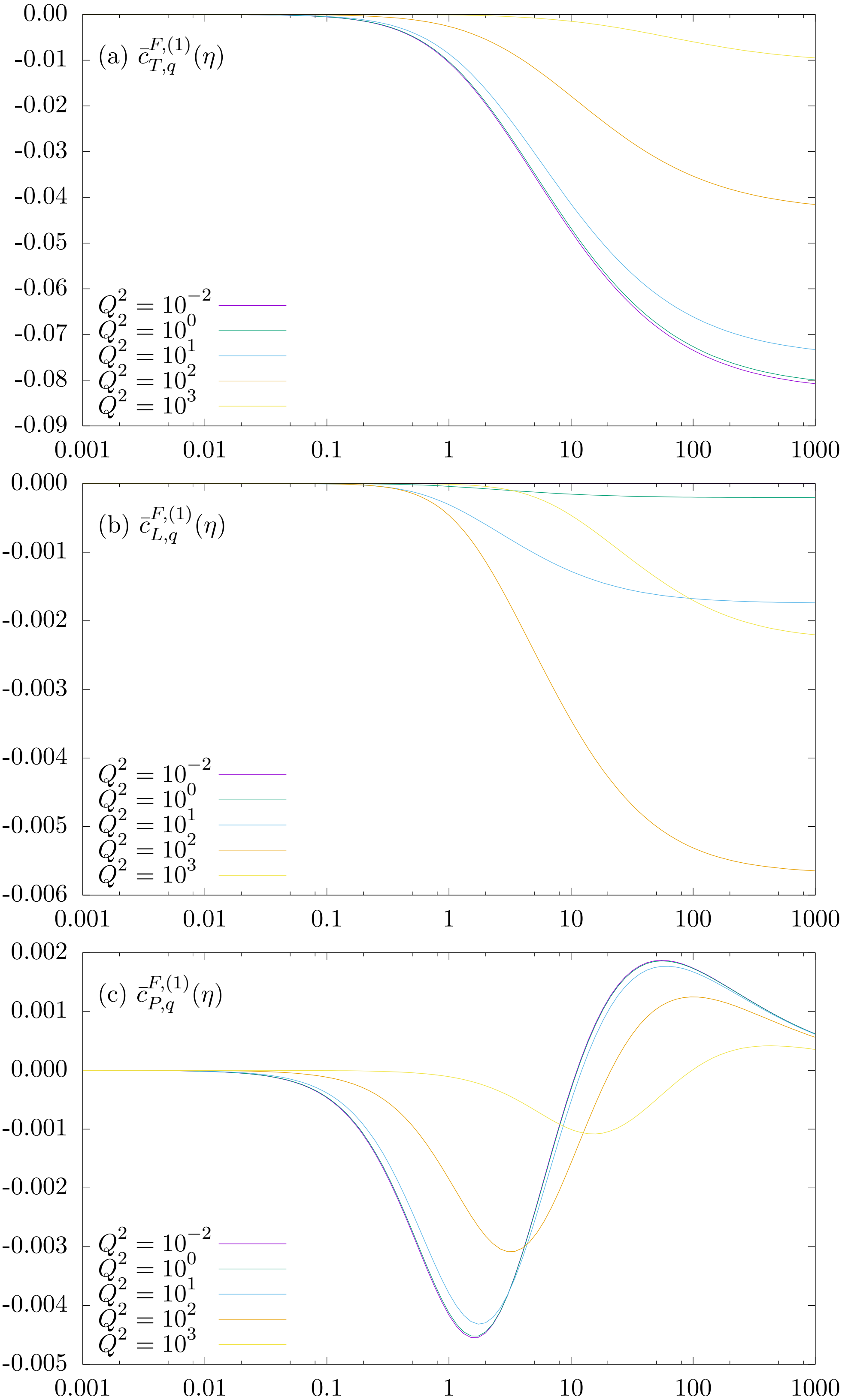

Figure 7: next-to-leading order scaling functions $\bar{c}^{F,(1)}_{k,q}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\text{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

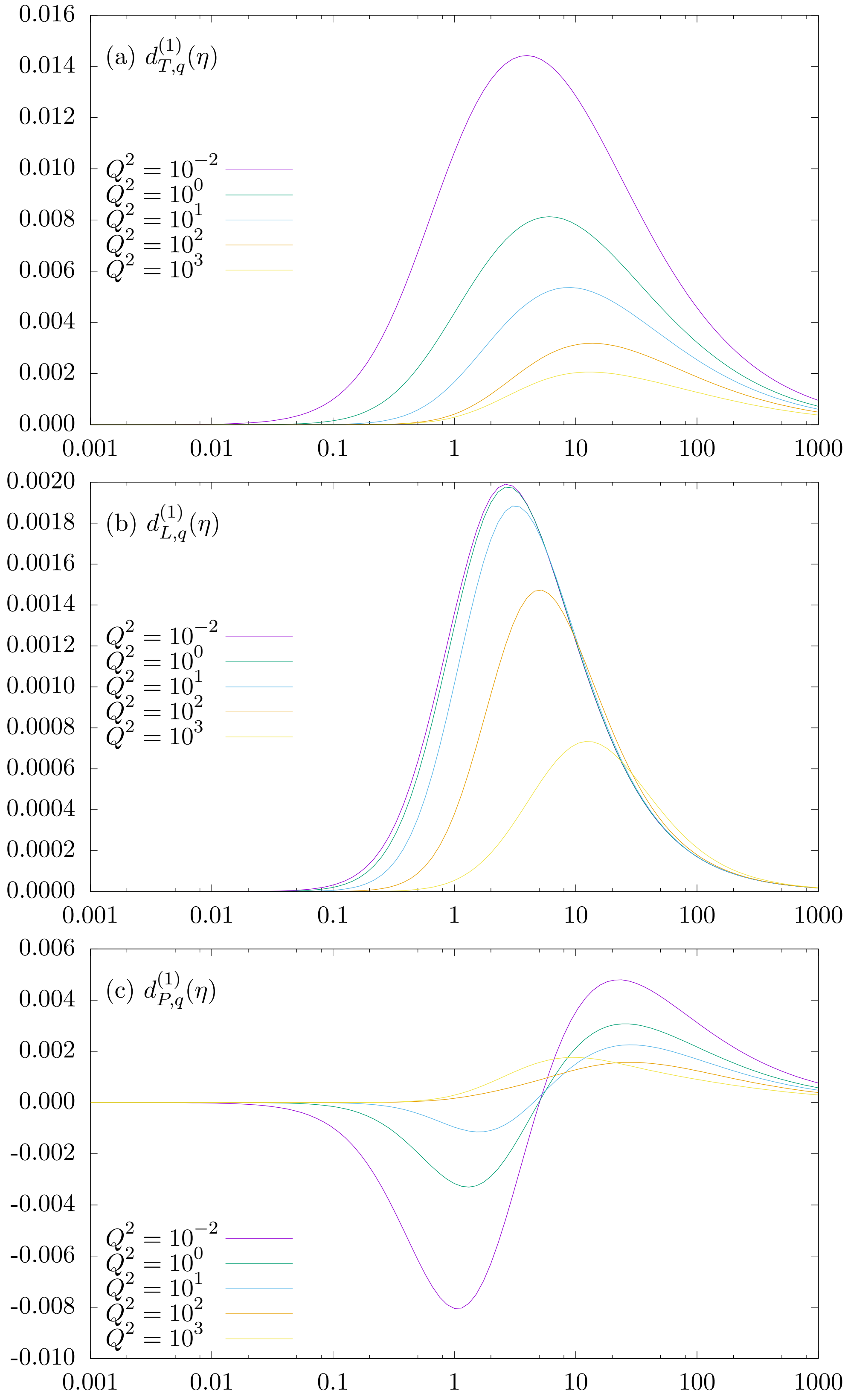

Figure 8: next-to-leading order scaling functions $d^{(1)}_{k,q}(\eta, \xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

6 Hadronic Results

The hadronic reaction to study is deep-inelastic lepton-proton scattering:

$$\ell^-(l_1) + \mathrm{p}(p) \to \ell^-(l_2) + Q(p_1)(\overline{Q}(p_2)) + X \tag{93}$$

where one either detects the heavy quark Q *or* the heavy antiquark $\overline{Q}$ and X stands for any final hadronic state allowed by quantum-number conservation. We define then the hadronic Bjorken variables

$$q = l_1 - l_2, \quad Q^2 = -q^2, \quad x = \frac{Q^2}{2p\cdot q}, \quad z = \frac{p\cdot q}{p\cdot l_1} \tag{94}$$

We can then define the measurable deep-inelastic hadron structure functions

$$F_k(x, Q^2, m^2) = \sum_{n=0}^{\infty} F_k^{(n)}(x, Q^2, m^2) \tag{95}$$

$$F_k^{(n)}(x, Q^2, m^2) = \frac{Q^2}{4\pi^2\alpha} \sum_{j\in\{\mathrm{g},q,\bar{q}\}} \int_x^{z_{max}} \frac{dz}{z} f_j(x/z, \mu_F^2)\sigma_{k,j}^{(n)}(s, q^2, m^2) \tag{96}$$

where $k \in \{G, L, P\}$ denotes as usual projection, $z = Q^2/s'$, $z_{max} = Q^2/(4m^2 + Q^2)$ and $f_j(x/z, \mu_F^2)$ denotes parton momentum density functions[18, 19]. We can then split the contributions whether there is a gluon in the initial state $F_{k,g}$ or a (anti)quark $F_{k,q}$. In leading order we find:

$$F_{k,g}^{(0)}(x, Q^2, m^2) = \frac{\alpha_s Q^2}{4\pi^2 m^2} e_H^2 \int_x^{z_{max}} \frac{dz}{z} f_g(x/z, \mu_F^2) c_{k,g}^{(0)}(\eta, \xi) \tag{97}$$

We find for the gluonic part in next-to-leading order:

$$\begin{aligned} &F_{k,g}^{(1)}(x, Q^2, m^2) \\ &= \frac{\alpha_s^2 Q^2}{\pi m^2} e_H^2 \int_x^{z_{max}} \frac{dz}{z} f_g(x/z, \mu_F^2) \left(c_{k,g}^{(1)}(\eta, \xi) + \ln(\mu_F^2/m^2)\bar{c}_{k,g}^{F,(1)}(\eta, \xi) + \ln(\mu_R^2/m^2)\bar{c}_{k,g}^{R,(1)}(\eta, \xi)\right) \end{aligned} \tag{98}$$

We find for the quark part in next-to-leading order:

$$\begin{aligned} &F_{k,q}^{(1)}(x, Q^2, m^2) \\ &= \frac{\alpha_s^2 Q^2}{\pi m^2} e_H^2 \int_x^{z_{max}} \frac{dz}{z} \left(\sum_{j=1}^{n_{lf}} f_{q(j)}(x/z, \mu_F^2) + f_{q(-j)}(x/z, \mu_F^2)\right) \\ &\qquad \cdot \left(c_{k,q}^{(1)}(\eta, \xi) + \ln(\mu_F^2/m^2)\bar{c}_{k,q}^{F,(1)}(\eta, \xi)\right) \\ &\quad + \frac{\alpha_s^2 Q^2}{\pi m^2} \int_x^{z_{max}} \frac{dz}{z} \left(\sum_{j=1}^{n_{lf}} e_{q(j)}^2 \left(f_{q(j)}(x/z, \mu_F^2) + f_{q(-j)}(x/z, \mu_F^2)\right)\right) d_{k,q}(\eta, \xi) \end{aligned} \tag{99}$$

where we used the PDG particle labeling[20]: $q(1) = u, q(-1) = \bar{u}, q(2) = d, q(-2) = \bar{d}$, $\ldots$ and $e_u = e_c = e_t = 2/3, e_d = e_s = e_b = -1/3$.

We can then also define some more practical functions:

$$
\begin{aligned}
F_2(x, Q^2, m^2) &= F_T(x, Q^2, m^2) + F_L(x, Q^2, m^2) && (100)\\
&= F_G(x, Q^2, m^2) + \frac{3}{2} F_L(x, Q^2, m^2) && (101)\\
F_1(x, Q^2, m^2) &= (F_2(x, Q^2, m^2) - F_L(x, Q^2, m^2))/(2x) && (102)\\
&= \left(F_G(x, Q^2, m^2) + \frac{1}{2} F_L(x, Q^2, m^2)\right)/(2x) && (103)\\
g_1(x, Q^2, m^2) &= F_P(x, Q^2, m^2)/(2x) && (104)
\end{aligned}
$$

and we define

$$
R_{k'}(x, Q^2, m^2) = \frac{F^{(0)}_{k'}(x, Q^2, m^2) + F^{(1)}_{k'}(x, Q^2, m^2)}{F^{(0)}_{k'}(x, Q^2, m^2)} \tag{105}
$$

with $k' \in \{2, L, P\}$ to better observe next-to-leading order effects.

We define the spin asymmetry by

$$
A_1(x, Q^2, m^2) = \frac{g_1(x, Q^2, m^2)}{F_1(x, Q^2, m^2)} = \frac{F_P(x, Q^2, m^2)}{F_2(x, Q^2, m^2) - F_L(x, Q^2, m^2)} \tag{106}
$$

For the plots we focused on charm production ($n_{lf} = 3$) with $m_c = 1.5\,\mathrm{GeV}$ and we used the two-loop running coupling of [21]:

$$
\alpha_s(\mu_R^2) = \frac{1}{\beta_0^4 \ln(\mu_R^2/\Lambda_4)} \left(1 - \frac{\beta_1^4}{(\beta_0^4)^2} \ln(\ln(\mu_R^2/\Lambda_4))\right) \tag{107}
$$

with the first two coefficients of the QCD beta function $\beta_0^f = (33 - 2f)/(12\pi)$ and $\beta_1^f = (306 - 38f)/(48\pi^2)$**FiXme Error: find CA?** and $\Lambda_4 = 0.194\,\mathrm{GeV}^2$. We set $\mu_F^2 = \mu_R^2 = 4m^2 - q^2$ in analogy to [1]. We used the PDF set MSTW2008nlo90cl[18, 22, 23] provided by LHAPDF[24] for the unpolarized structure functions (F_2, F_1, F_G, F_L) and DSSV2014[19] for the polarized structure function (g_1, F_G).

FiXme Error!

FiXme Error!

FiXme Error: shift images to appendix?

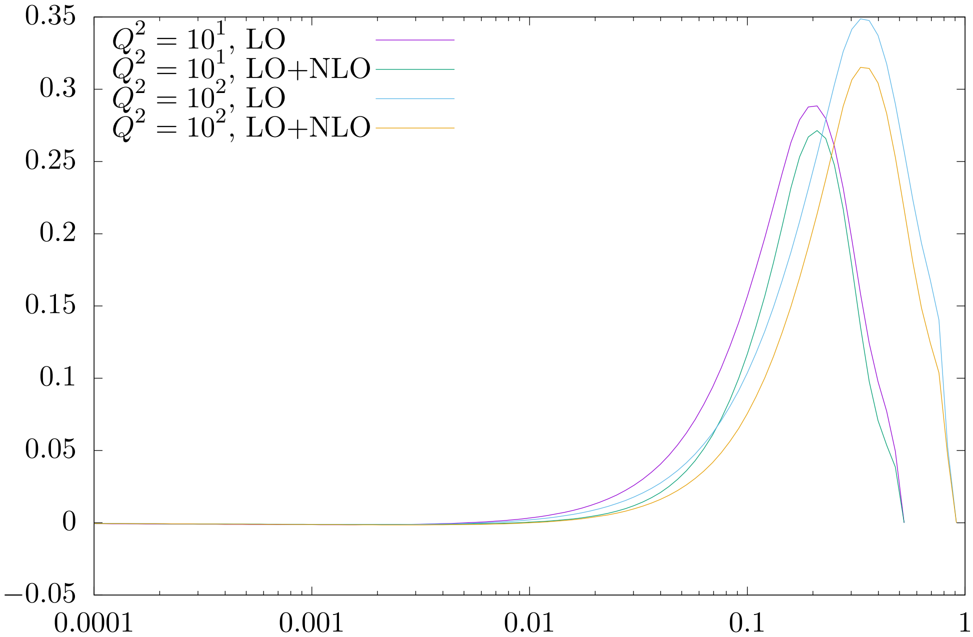

Figure 9: spin asymmetry $A_1(x, Q^2, m_c^2)$ plotted as function of x for different values of Q^2 in units of GeV^2

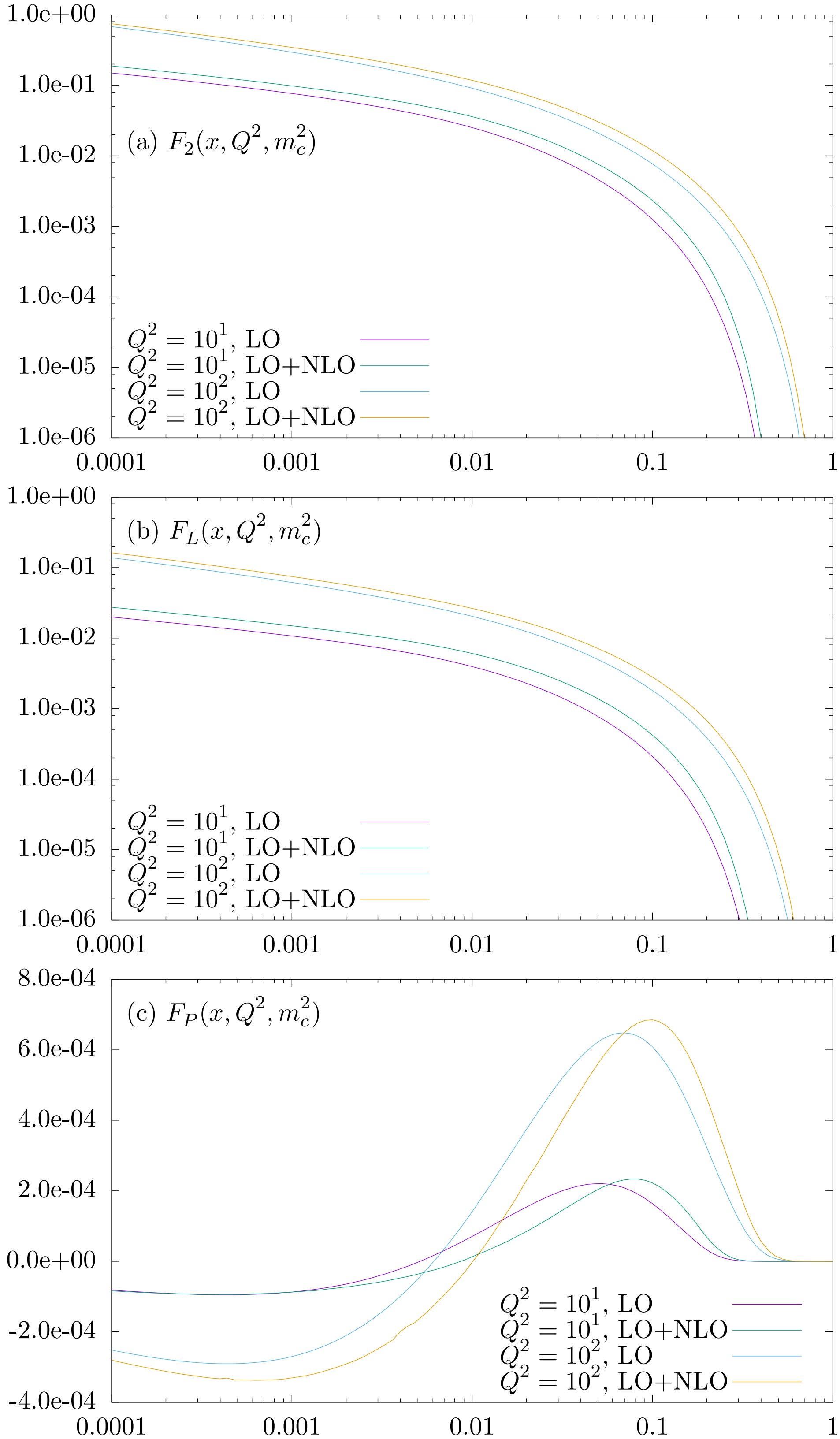

Figure 10: hadronic structure functions $F_k(x,Q^2,m_c^2)$ plotted as function of x for different values of Q^2 in units of GeV^2

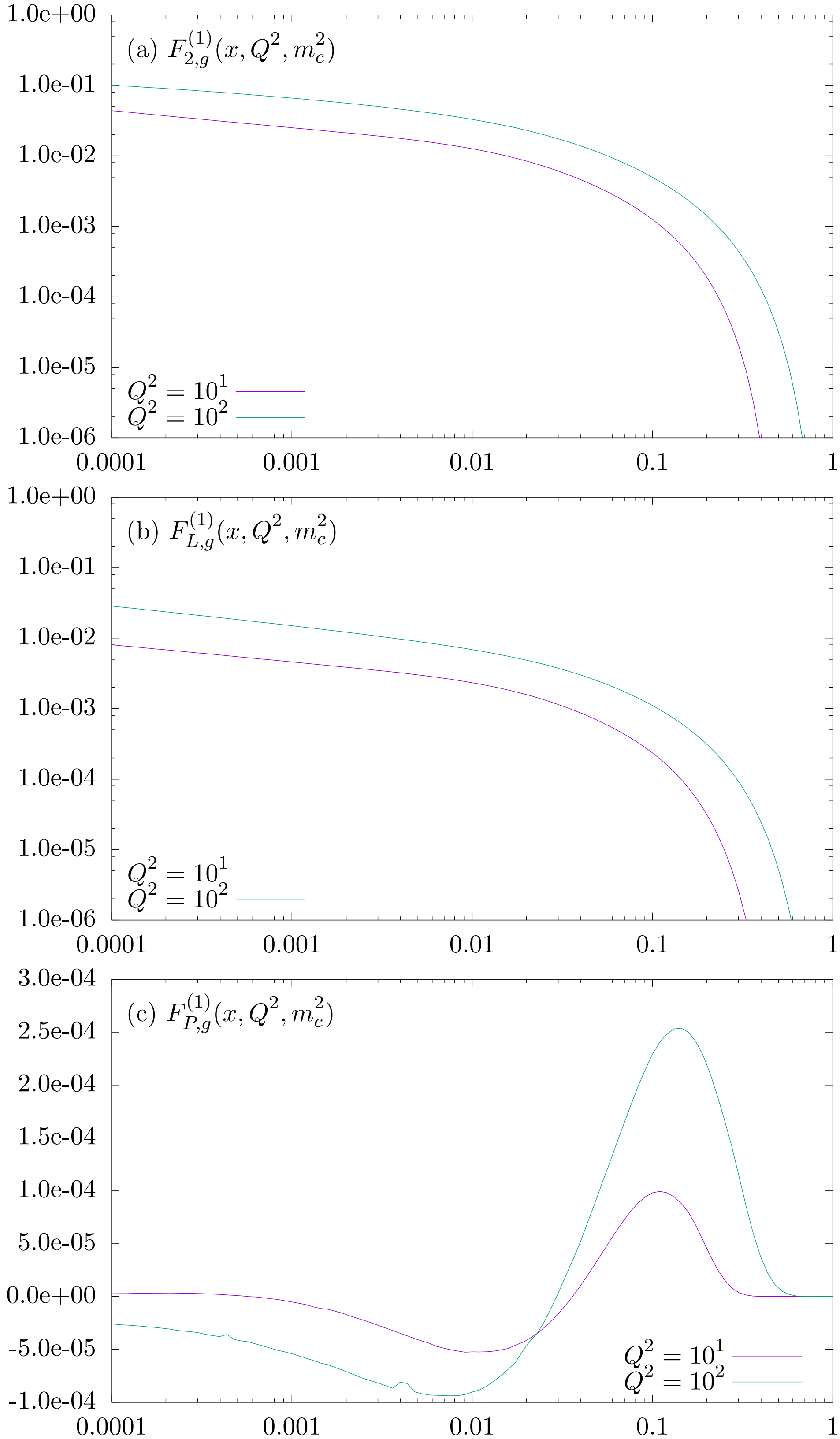

Figure 11: next-to-leading order hadronic structure functions $F_{k,g}^{(1)}(x, Q^2, m_c^2)$ plotted as function of x for different values of Q^2 in units of GeV^2

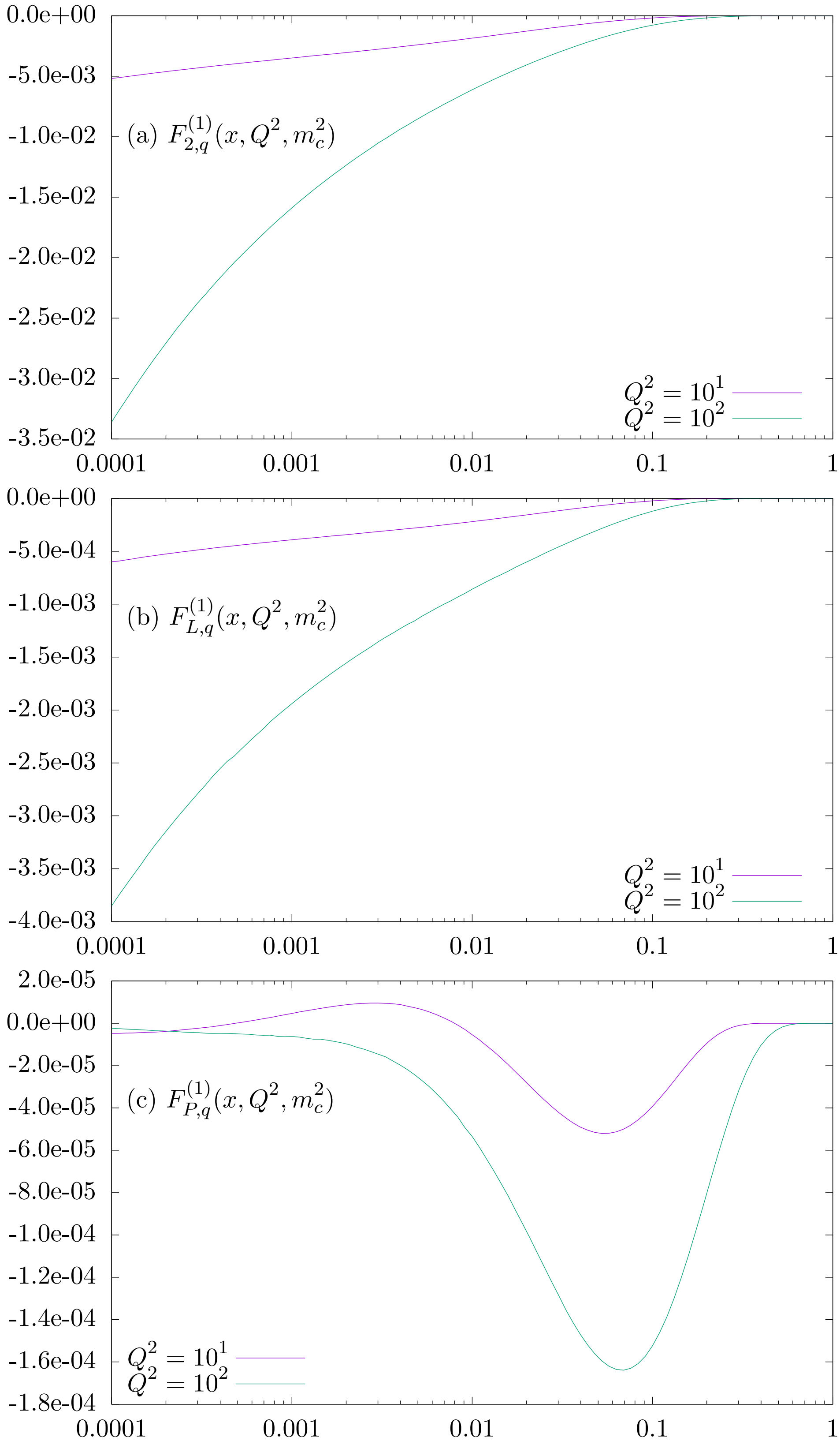

Figure 12: next-to-leading order hadronic structure functions $F^{(1)}_{k,q}(x,Q^2,m_c^2)$ plotted as function of x for different values of Q^2 in units of GeV^2

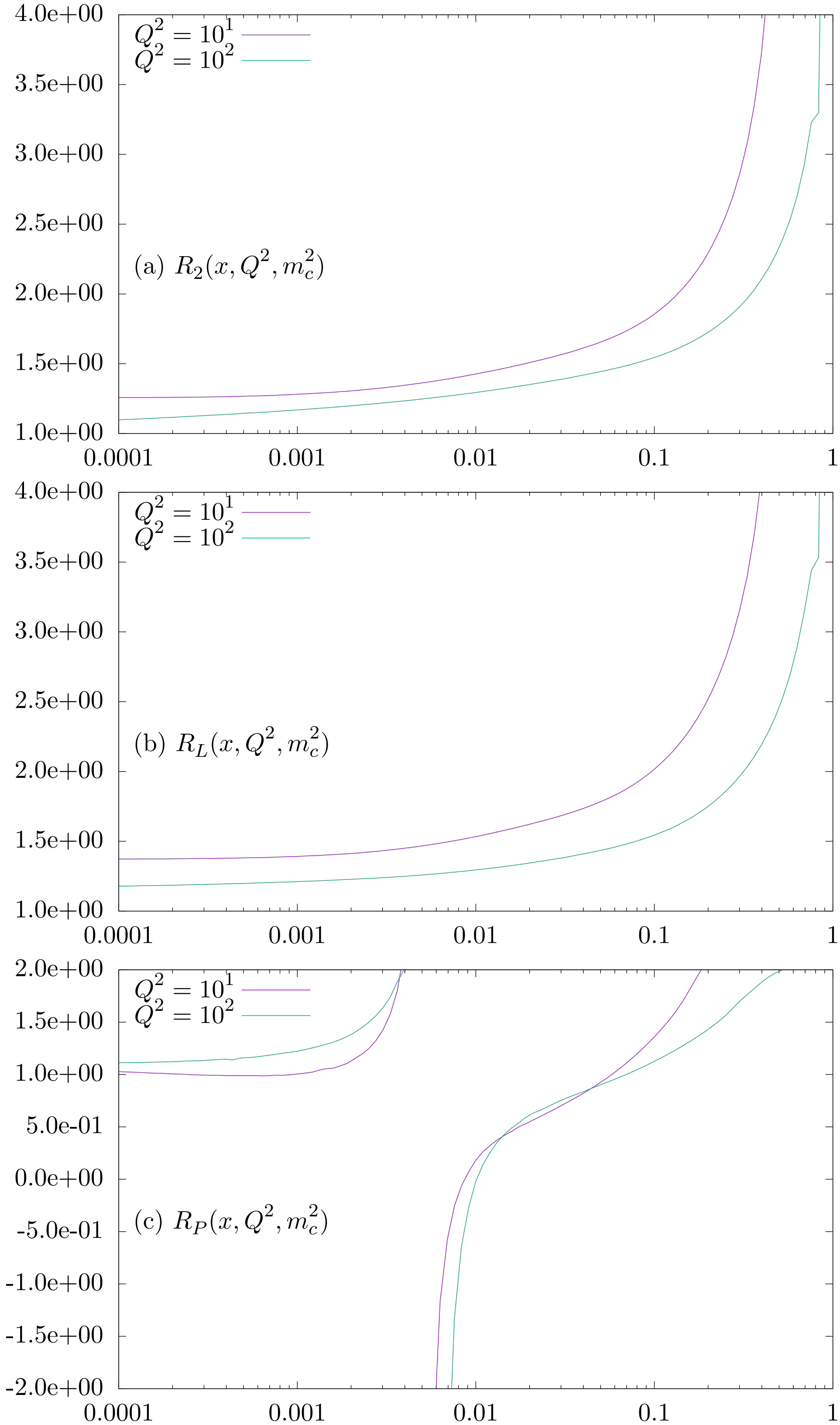

Figure 13: ratio of hadronic structure functions $R_{k'}^{(1)}(x, Q^2, m_c^2)$ plotted as function of x for different values of Q^2 in units of GeV^2

7 Summary

FiXme Error!

FiXme Error: do

A References

[1] E. Laenen, S. Riemersma, J. Smith, and W. van Neerven, "Complete $O(\alpha_S)$ corrections to heavy-flavour structure functions in electroproduction," Nuclear Physics B **392** no. 1, (1993) 162 – 228. http://www.sciencedirect.com/science/article/pii/055032139390201Y.

[2] I. Bojak, NLO QCD corrections to the polarized photoproduction and hadroproduction of heavy quarks. PhD thesis, Dortmund U., 2000. arXiv:hep-ph/0005120 [hep-ph].

[3] P. Breitenlohner and D. Maison, "Dimensional renormalization and the action principle," Comm. Math. Phys. **52** no. 1, (1977) 11–38. http://projecteuclid.org/euclid.cmp/1103900439.

[4] W. Vogelsang, Tests and signatures of spin dependent parton distributions in leading and next-to-leading order of QCD. PhD thesis, Dortmund U., 1993. http://alice.cern.ch/format/showfull?sysnb=0171841.

[5] J. A. M. Vermaseren, "New features of FORM," arXiv:math-ph/0010025 [math-ph].

[6] S. Wolfram, "Mathematica." Wolfram research, 1997. Ver. 3 or higher.

[7] M. Jamin and M. E. Lautenbacher, "TRACER version 1.1: A mathematica package for γ-algebra in arbitrary dimensions," Computer Physics Communications **74** no. 2, (1993) 265 – 288. http://www.sciencedirect.com/science/article/pii/001046559390097V.

[8] M. Wiebusch, "HEPMath 1.4: A Mathematica Package for Semi-Automatic Computations in High Energy Physics," Computer Physics Communications **195** (Oct., 2015) 172–190. http://arxiv.org/abs/1412.6102. arXiv: 1412.6102.

[9] E. Leader and E. Predazzi, An introduction to Gauge theories and modern particle physics. Univ. Pr., Cambridge.

[10] G. Passarino and M. J. G. Veltman, "One Loop Corrections for e+ e- Annihilation Into mu+ mu- in the Weinberg Model," Nucl. Phys. **B160** (1979) 151.

[11] Beenakker, W. and Kuijf, H. and van Neerven, W. L. and Smith, J., "Qcd corrections to heavy-quark production in $p\overline{p}$ collisions," Phys. Rev. D **40** (Jul, 1989) 54–82. http://link.aps.org/doi/10.1103/PhysRevD.40.54.

[12] R. K. Ellis and G. Zanderighi, "Scalar one-loop integrals for QCD," JHEP **02** (2008) 002, arXiv:0712.1851 [hep-ph].

[13] T. Hahn and M. Perez-Victoria, "Automatized one loop calculations in four-dimensions and D-dimensions," Comput. Phys. Commun. **118** (1999) 153–165, arXiv:hep-ph/9807565 [hep-ph].

[14] T. Hahn, "LoopTools 2.12 User's Guide." http://www.feynarts.de/looptools/, 2014.

[15] I. Bojak and M. Stratmann, "Photoproduction of heavy quarks in next-to-leading order QCD with longitudinally polarized initial states," Nucl. Phys. **B540** (1999) 345–381, `arXiv:hep-ph/9807405 [hep-ph]`. [Erratum: Nucl. Phys.B569,694(2000)].

[16] G. Altarelli and G. Parisi, "Asymptotic Freedom in Parton Language," Nucl. Phys. **B126** (1977) 298–318.

[17] W. Vogelsang, "A Rederivation of the spin dependent next-to-leading order splitting functions," Phys. Rev. **D54** (1996) 2023–2029, `arXiv:hep-ph/9512218 [hep-ph]`.

[18] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, "Parton distributions for the LHC," Eur. Phys. J. **C63** (2009) 189–285, `arXiv:0901.0002 [hep-ph]`.

[19] D. de Florian, R. Sassot, M. Stratmann, and W. Vogelsang, "Evidence for polarization of gluons in the proton," Phys. Rev. Lett. **113** (Jul, 2014) 012001. `http://link.aps.org/doi/10.1103/PhysRevLett.113.012001`.

[20] **Particle Data Group** Collaboration, K. Hagiwara et al., "Review of particle physics. Particle Data Group," Phys. Rev. **D66** (2002) 010001.

[21] G. Altarelli, M. Diemoz, G. Martinelli, and P. Nason, "Total Cross-Sections for Heavy Flavor Production in Hadronic Collisions and QCD," Nucl. Phys. **B308** (1988) 724–752.

[22] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, "Uncertainties on alpha(S) in global PDF analyses and implications for predicted hadronic cross sections," Eur. Phys. J. **C64** (2009) 653–680, `arXiv:0905.3531 [hep-ph]`.

[23] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, "Heavy-quark mass dependence in global PDF analyses and 3- and 4-flavour parton distributions," Eur. Phys. J. **C70** (2010) 51–72, `arXiv:1007.2624 [hep-ph]`.

[24] A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht, M. Schönherr, and G. Watt, "LHAPDF6: parton density access in the LHC precision era," Eur. Phys. J. **C75** (2015) 132, `arXiv:1412.7420 [hep-ph]`.

List of Corrections