1 Introduction

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1.1 Motivation

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1.2 Notation

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We study the reaction

$$\ell(k) + \mathrm{N}(P) \to \ell'(k') + \overline{Q}(p_2) + X[Q] \tag{1}$$

and define the usual set of kinematic variables

$$q = k - k', \qquad Q^2 = -q^2, \qquad x = \frac{Q^2}{2q \cdot P}, \qquad y = \frac{q \cdot P}{k \cdot P} \tag{2}$$

Assuming $k^2 = m_\ell^2 = 0$, we can write the hadronic tensor (using the naming convention of [1]):

$$\begin{aligned} W_{\mu\mu'} = &(-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2})F_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_{\mu'}}{P \cdot q} F_2(x,Q^2) - i\varepsilon_{\mu\mu'\alpha\beta}\frac{q^\alpha P^\beta}{2P \cdot q}F_3(x,Q^2) \\ &+ i\varepsilon_{\mu\mu'\alpha\beta}\frac{q^\alpha S^\beta}{P \cdot q}g_1(x,Q^2) + \frac{S \cdot q}{P \cdot q}\left[\frac{\hat{P}_\mu \hat{P}_{\mu'}}{P \cdot q}g_4(x,Q^2) + (-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2})g_5(x,Q^2)\right] \end{aligned} \tag{3}$$

with

$$\hat{P}_\mu = P_\mu - \frac{P \cdot q}{q^2} q_\mu \tag{4}$$

and introduce two new structure functions

$$F_L = F_2 - 2xF_1, \qquad g_L = g_4 - 2xg_5\,. \tag{5}$$

The hadronic tensor 3 can be mapped onto a partonic level with

$$k_1 = \xi P', \qquad z = \frac{Q^2}{2q \cdot k_1} = \frac{x}{\xi} \tag{6}$$

and we write

$$
\begin{aligned}
\frac{1}{2z}\hat{w}_{\mu,\mu'} = {} & (-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2})\hat{F}_1(z,Q^2) + \frac{\hat{k}_{1,\mu}\hat{k}_{1,\mu'}}{k_1\cdot q}\hat{F}_2(z,Q^2) - i\varepsilon_{\mu\mu'\alpha\beta}\frac{q^\alpha k_1^\beta}{2k_1\cdot q}\hat{F}_3(z,Q^2) \\
& + \frac{q_\mu q_{\mu'}}{q^2}\hat{F}_4(z,Q^2) + \frac{q_\mu k_{1,\mu'} + q_{\mu'}k_{1,\mu}}{2k_1\cdot q}\hat{F}_5(z,Q^2) \\
& + i\varepsilon_{\mu\mu'\alpha\beta}\frac{q^\alpha S^\beta}{k_1\cdot q}\hat{g}_1(z,Q^2) + \frac{S\cdot q}{k_1\cdot q}\left[\frac{\hat{k}_{1,\mu}\hat{k}_{1,\mu'}}{k_1\cdot q}\hat{g}_4(z,Q^2) + (-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2})\hat{g}_5(z,Q^2)\right] \\
& + \frac{S\cdot q}{k_1\cdot q}\left[\frac{q_\mu q_{\mu'}}{q^2}\hat{g}_6(z,Q^2) + \frac{q_\mu k_{1,\mu'} + q_{\mu'}k_{1,\mu}}{2k_1\cdot q}\hat{g}_7(z,Q^2)\right] \qquad (7)
\end{aligned}
$$

with

$$\hat{k}_{1,\mu} = k_{1,\mu} - \frac{k_1\cdot q}{q^2}q_\mu \qquad (8)$$

and

$$\hat{F}_L = \hat{F}_2 - 2z\hat{F}_1, \qquad \hat{g}_L = \hat{g}_4 - 2z\hat{g}_5\,. \qquad (9)$$

It is convenient to rescale the structure functions and so we focus in the following on the six structure functions

$$\hat{F}_2, \hat{F}_L, z\hat{F}_3, 2z\hat{g}_1, \hat{g}_4 \text{ and } \hat{g}_L \qquad (10)$$

or their hadronic counter parts.

Note that we have to include $\hat{F}_4, \hat{F}_5, \hat{g}_6$ and $\hat{g}_7$ into Eq. 7 as we are interested in the full neutral current case, that is, we allow for Z-bosons to be exchanged and due to this we can no longer rely on the Ward-identity, because the axial current J_5^μ does not vanish for massive particles:

$$q_\mu J_5^\mu = q_\mu \bar{\psi}\gamma^\mu\gamma^5\psi = 2m\,\bar{\psi}i\gamma^5\psi\,. \qquad (11)$$

This way we can define the projections onto the structure functions by

$$
\begin{aligned}
\hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\hat{F}_2} &= \frac{-g^{\mu\mu'}}{n-2} - \frac{n-1}{n-2}\cdot\frac{4z^2k_1^\mu k_1^{\mu'}}{q^2} - \frac{q^\mu q^{\mu'}}{q^2} - \frac{n-2}{n-1}\cdot\frac{2z(q^\mu k_1^{\mu'} + q^{\mu'}k_1^\mu)}{q^2} && (12)\\
\hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\hat{F}_L} &= -\frac{4z^2k_1^\mu k_1^{\mu'}}{q^2} - \frac{q^\mu q^{\mu'}}{q^2} - \frac{2z(q^\mu k_1^{\mu'} + q^{\mu'}k_1^\mu)}{q^2} && (13)\\
\hat{\mathcal{P}}^{\gamma,\mu\mu'}_{z\hat{F}_3} &= -\frac{iz\varepsilon^{\mu\mu'\alpha\beta}k_{1,\alpha}q_\beta}{q^2} && (14)
\end{aligned}
$$

and, due to the symmetry in the Lorentz structure,

$$\hat{\mathcal{P}}^{\gamma,\mu\mu'}_{2z\hat{g}_1} = \hat{\mathcal{P}}^{\gamma,\mu\mu'}_{z\hat{F}_3} \qquad \hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\hat{g}_4} = -\hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\hat{F}_2} \qquad \hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\hat{g}_L} = -\hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\hat{F}_L}\,. \qquad (15)$$

This way we have

$$\hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\hat{h}}\hat{w}_{\mu,\mu'} = \hat{h} \quad \text{for } \hat{h} \in \{\hat{F}_2, \hat{F}_L, z\hat{F}_3, 2z\hat{g}_1, \hat{g}_4, \hat{g}_L\} \tag{16}$$

For the unpolarized structure functions $\hat{F}_2, \hat{F}_L$ and $z\hat{F}_3$ the helicity of the parton, either gluon or (anti-)quark, has to be averaged, whereas for the polarized $2z\hat{g}_1, \hat{g}_4$ and $\hat{g}_L$ we have to consider the helicity difference. For the gluons this is achieved by

$$\hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{F} = -g^{\nu\nu'} \qquad\qquad \hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{g} = 2i\varepsilon^{\nu\nu'\alpha\beta}\frac{k_{1,\alpha}q_\beta}{2k_1\cdot q} \tag{17}$$

and by choosing just $-g^{\nu\nu'}$, we decided to include incoming external ghost to cancel all unphysical gluon polarization. All initial-state (anti-)quarks are taken as massless partons, so the relevant projection operators onto definitive helicity states are given by

$$\begin{aligned} \hat{\mathcal{P}}^{q,aa'}_{F} &= (\not{k}_1)_{aa'}, & \hat{\mathcal{P}}^{q,aa'}_{g} &= -(\gamma_5\not{k}_1)_{aa'}, \\ \hat{\mathcal{P}}^{\bar{q},bb'}_{F} &= (\not{k}_1)_{bb'}, & \hat{\mathcal{P}}^{\bar{q},bb'}_{g} &= (\gamma_5\not{k}_1)_{bb'} \end{aligned} \tag{18}$$

where a and a' (b and b') refer to the Dirac-index of the initial (anti-)quark spinor in the relevant matrix elements given below.

In order to compute the exchange of scattered vector boson $b = \{\gamma, \mathrm{Z}\}$ with a common notation, we write the coupling of b to the hadronic process by

$$\Gamma^\mu_{b,j} = g^V_{b,j}\Gamma^\mu_V + g^A_{b,j}\Gamma^\mu_A = g^V_{b,j}\gamma^\mu + g^A_{b,j}\gamma^\mu\gamma^5 \quad j \in \{q, Q\}. \tag{19}$$

Due to symmetry reasons the parity-violating structure functions $z\hat{F}_3, g_4, g_L$ can only recieve contributions from $\Gamma^\mu_V\Gamma^{\mu'}_A$, where μ refer to the Lorentz index of the boson in matrix amplitude and the μ' to the index in the complex conjungate. Likewise the parity conserving structure functions can only recieve contributions from either $\Gamma^\mu_V\Gamma^{\mu'}_V$ or $\Gamma^\mu_A\Gamma^{\mu'}_A$. To introduce a compact notation we write

$$\vec{\kappa} = (\kappa_1, \kappa_2) \quad \kappa_1 \in \{\mathrm{VV}, \mathrm{VA}, \mathrm{AA}\},\, \kappa_2 \in \{\hat{F}_2, \hat{F}_L, z\hat{F}_3, 2z\hat{g}_1, \hat{g}_4, \hat{g}_L\} \tag{20}$$

2 Leading Order Calculations

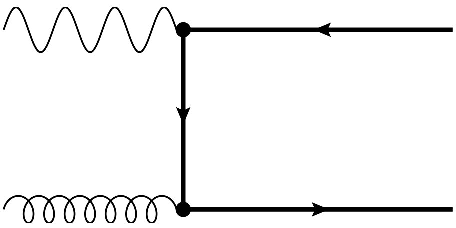
In leading order we have to consider photon-gluon-fusion (PGF), that is

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) \tag{21}$$

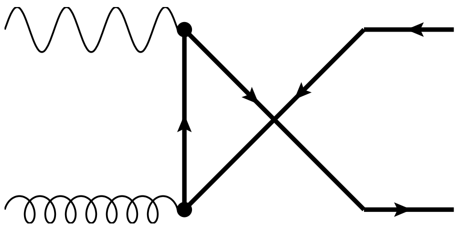
with two contributing diagrams depicted in figure 1.

The result can then be written as

$$\hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\vec{\kappa}}\hat{\mathcal{P}}^{\mathrm{g},\nu\nu'}_{\kappa_2}\sum_{j,j'=1}^{2}\mathcal{M}^{(0),j}_{\mu\nu}\left(\mathcal{M}^{(0),j'}_{\mu'\nu'}\right)^* = 8g^2\mu_D^{-\epsilon}e^2e_H^2N_CC_FB_{\vec{\kappa},\mathrm{QED}} \tag{22}$$



(a) $i\varepsilon_\gamma^\mu(q)\varepsilon_{\rm g}^\nu(k_1)\,\mathcal{M}_{\mu\nu}^{(0),1}$



(b) $i\varepsilon_\gamma^\mu(q)\varepsilon_{\rm g}^\nu(k_1)\,\mathcal{M}_{\mu\nu}^{(0),2}$

Figure 1: leading order Feynman diagrams
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where g and e are the strong and electromagnetic coupling constants respectively, μ_D is an arbitray mass parameter introduced to keep the couplings dimensionless and e_H is the magnitude of the heavy quark in units of e. Further N_C corresponds to the gauge group $SU(N_C)$ and the color factor $C_F = (N_C^2-1)/(2N_C)$ refers to the second Casimir constant of the fundamental representation for the quarks. We then find:

$$\begin{aligned} B_{\mathrm{VV},F_2,\mathrm{QED}} &= \left[-1-\frac{6q^2}{s'}-\frac{6q^4}{s'^2}+\frac{q^2(6m^2+s)+2m^2s+s'^2/2}{t_1u_1}-\frac{(2m^2+q^2)m^2s'^2}{(t_1u_1)^2}\right] \\ &\quad+\frac{\epsilon}{2}\left[-1+\frac{s^2-q^2s'}{t_1u_1}-\frac{m^2q^2s'^2}{t_1^2u_1^2}\right]+\epsilon^2\frac{s'^2}{8t_1u_1} \end{aligned} \tag{23}$$

$$B_{\mathrm{VV},F_L,\mathrm{QED}} = -\frac{4q^2}{s'}\left(\frac{s}{s'}-\frac{m^2s'}{t_1u_1}\right) \tag{24}$$

$$B_{\mathrm{VV},2xg_1,\mathrm{QED}} = \left\{1+\frac{2q^2}{s'}-\frac{s'(2(2m^2+q^2)+s')}{2t_1u_1}+\frac{m^2s'^3}{(t_1u_1)^2}+\epsilon\left(-\frac{1}{2}+\frac{s'^2}{4t_1u_1}\right)\right\}(1+\epsilon) \tag{25}$$

$$B_{\mathrm{AA},F_2,\mathrm{QED}} = \frac{m^2 s'^2(1+\epsilon)(2+\epsilon)(12m^2(-1+\epsilon)+q^2(-6+(-3+\epsilon)\epsilon))}{12(t_1u_1)^2} - \frac{(1+\epsilon)\left(8s'^3\epsilon+12q^6(2+\epsilon)+12q^4s'(2+\epsilon)+q^2s'^2(4+\epsilon(20-(-3+\epsilon)\epsilon))\right)}{4q^2s'^2} - \frac{(1+\epsilon)}{48q^2(t_1u_1)}\left(q^2(2+\epsilon)(-6+(-3+\epsilon)\epsilon)\left(4q^4+4q^2s'+s'^2(2+\epsilon)\right) + 48m^2\left(-s'^2(-2+\epsilon)+q^4(-4+\epsilon)(2+\epsilon)+q^2s'\left(-2+\epsilon+\epsilon^2\right)\right)\right) \tag{26}$$

$$B_{\mathrm{AA},F_2,\mathrm{QED}} = -\frac{m^2s'^2(1+\epsilon)(2+\epsilon)(12m^2+q^2\epsilon)}{6(t_1u_1)^2} - \frac{(1+\epsilon)\left(4s'^3\epsilon+4q^6(2+\epsilon)+4q^4s'(2+\epsilon)+q^2s'^2\epsilon(6+\epsilon)\right)}{2q^2s'^2} + \frac{(1+\epsilon)}{24q^2(t_1u_1)}\left(24m^2\left(s'^2(-2+\epsilon)+4q^4(2+\epsilon)+2q^2s'(2+\epsilon)\right) + q^2\epsilon(2+\epsilon)\left(4q^4+4q^2s'+s'^2(2+\epsilon)\right)\right) \tag{27}$$

$$B_{\mathrm{AA},2xg_1,\mathrm{QED}} = \frac{(1+\epsilon)^2(2-\epsilon)}{2}\left[1+\frac{2q^2}{s'}-\frac{2s'(2m^2+q^2)+s'^2}{2t_1u_1}+\frac{m^2s'^3}{(t_1u_1)^2} + \left(-1+\frac{s'^2}{2t_1u_1}\right)\frac{\epsilon}{2}\right] \tag{28}$$

$$B_{\mathrm{VA},xF_3,\mathrm{QED}} = \frac{s'(1+\epsilon)(2+\epsilon)}{t_1-u_1}\left\{-1-\frac{\epsilon}{2}-2\frac{q^2}{s'}-2\frac{q^4}{s'}-\frac{m^2q^2s'^2}{2(t_1u_1)^2} + \frac{4q^2(4m^2+q^2+s')+s'^2(2+\epsilon)}{t_1u_1}\right\} \tag{29}$$

$$B_{\mathrm{VA},g_4,\mathrm{QED}} = \frac{s'(1+\epsilon)}{t_1-u_1}\left\{-2+\epsilon-4\frac{q^2}{s'}-\frac{m^2s'^3}{(t_1u_1)^2}+ \frac{s'(16m^2+4q^2+s'(2-\epsilon))}{4t_1u_1}\right\} \tag{30}$$

$$B_{\mathrm{VA},g_L,\mathrm{QED}} = 0 \tag{31}$$

We will decompose the Born cross section further by their dependence on ϵ

$$B_{\vec{\kappa},\mathrm{QED}} = B^{(0)}_{\vec{\kappa},\mathrm{QED}} + \epsilon B^{(1)}_{\vec{\kappa},\mathrm{QED}} + \epsilon^2 B^{(2)}_{\vec{\kappa},\mathrm{QED}} \tag{32}$$

and do find $B^{(0)}_{\mathrm{VV},2xg_1,\mathrm{QED}} = B^{(0)}_{\mathrm{AA},2xg_1,\mathrm{QED}}$, but $B^{(1)}_{\mathrm{VV},2xg_1,\mathrm{QED}} \neq B^{(1)}_{\mathrm{AA},2xg_1,\mathrm{QED}}$.

3 Next-To-Leading Order Calculations

3.1 One Loop Virtual Contributions

$$
\begin{aligned}
M^{(1),V}_{\vec{k}} &= \hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\vec{k}} \hat{\mathcal{P}}^{\mathrm{g}}_{\vec{k}} \sum_j \left[\mathcal{M}^{(1),V}_{j,\mu} \left(\mathcal{M}^{(0)}_{1,\mu'} + \mathcal{M}^{(0)}_{2,\mu'} \right)^* + c.c. \right] \\
&= 8 g^4 \mu_D^{-\epsilon} e^2 e_H^2 N_C C_F C_\epsilon \left(C_A V_{\vec{k},OK} + 2 C_F V_{\vec{k},QED} \right)
\end{aligned} \tag{33}
$$

where $C_\epsilon = \exp(\epsilon/2(\gamma_E - \ln(4\pi)))/(16\pi^2)$ and C_A is the second Casimir constant of the adjoint representation for the gluon (that introduces a non-abelian part).

As the short example above shows, the full expressions for the $V_{k,OK}, V_{k,QED}$ are quite complicated and too long to be presented here, nevertheless the arising poles are quite compact:

$$
V_{\vec{k},OK} = -2 B_{\vec{k},QED} \left(\frac{4}{\epsilon^2} + \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) - \frac{2m^2 - s}{s} \ln(\chi) \right) \frac{2}{\epsilon} \right) + O(\epsilon^0) \tag{34}
$$

$$
V_{\vec{k},QED} = -2 B_{\vec{k},QED} \left(1 + \frac{2m^2 - s}{s} \ln(\chi) \right) \frac{2}{\epsilon} + O(\epsilon^0) \tag{35}
$$

The above results already include the mass renormalization that we have performed *on-shell*, so all ultra-violet poles have been removed. For the renormalization of the strong coupling we use the $\overline{\mathrm{MS}}_m$ scheme defined in [2] and so the full (remaining) renormalization can be achieved by

$$
\begin{aligned}
\frac{d^2\sigma^{(1),V,ren.}_{\vec{k}}}{dt_1 du_1} &= \frac{d^2\sigma^{(1),V}_{\vec{k}}}{dt_1 du_1} + \frac{\alpha_s(\mu_R^2)}{4\pi} \left[\left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln(\mu_R^2/m^2) - \ln(\mu_D^2/m^2) \right) \beta_0^f \right. \\
&\qquad \left. + \frac{2}{3} \ln(\mu_R^2/m^2) \right] \frac{d^2\sigma^{(0)}_{\vec{k}}}{dt_1 du_1} \qquad (36) \\
&= \frac{d^2\sigma^{(1),V}_{\vec{k}}}{dt_1 du_1} + 4\pi\alpha_s(\mu_R^2) C_\epsilon \left(\frac{\mu_D^2}{m^2} \right)^{-\epsilon/2} \left[\left(\frac{2}{\epsilon} + \ln(\mu_R^2/m^2) \right) \beta_0^f \right. \\
&\qquad \left. + \frac{2}{3} \ln(\mu_R^2/m^2) \right] \frac{d^2\sigma^{(0)}_{\vec{k}}}{dt_1 du_1} \qquad (37)
\end{aligned}
$$

with μ_R the renormalization scale introduced by the RGE, $\beta_0^f = (11 C_A - 2 n_f)/3$ the first coefficient of the beta function and n_f the number of *total* flavours (i.e. $n_{lf} = n_f - 1$ active (light) flavours and one heavy flavour). The double poles occuring in $V_{\vec{k},OK}$ are introduced by the diagrams **FiXme Error: do** when the soft and collinear singularities coincide. FiXme Error!

The partonic cross section is given by

$$d\sigma^{(1),V}_{\vec{k},\mathrm{g}} = \frac{1}{2s'}\frac{1}{2}E_{k_2}(\epsilon)M^{(1),V}_{\vec{k}}\,dPS_2 \tag{38}$$

3.2 Single Gluon Radiation

In next-to-leading order we have to consider the following process:

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) + \mathrm{g}(k_2) \tag{39}$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as FiXme Error!

$$\hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\vec{k}}\hat{\mathcal{P}}^{\mathrm{g}}_{\vec{k}}\sum_{j,j'}\mathcal{M}^{(1),g}_{j,\mu}\mathcal{M}^{(1),g*}_{j',\mu'} = 8g^4\mu_D^{-2\epsilon}e^2e_H^2N_CC_F\left(C_AR_{\vec{k},OK} + 2C_FR_{\vec{k},QED}\right) \tag{40}$$

and it will depend on ten kinematical invariants:

$$s = (q+k_1)^2 \qquad t_1 = (k_1-p_2)^2 - m^2 \qquad u_1 = (q-p_2)^2 - m^2 \tag{41}$$
$$s_3 = (k_2+p_2)^2 - m^2 \qquad s_4 = (k_2+p_1)^2 - m^2 \qquad s_5 = (p_1+p_2)^2 = -u_5 \tag{42}$$
$$t' = (k_1-k_2)^2 \tag{43}$$
$$u' = (q-k_2)^2 \qquad u_6 = (k_1-p_1)^2 - m^2 \qquad u_7 = (q-p_1)^2 - m^2 \tag{44}$$

from which only five are independent as can be seen from momentum conservation $k_1+q = p_1+p_2+k_2$ and s, t_1, u_1 match to their leading order definition.

The $2 \to 3$ n-dimensional phase space is given by

$$dPS_3 = \int \frac{d^np_1}{(2\pi)^{n-1}}\frac{d^np_2}{(2\pi)^{n-1}}\frac{d^nk_2}{(2\pi)^{n-1}}(2\pi)^n\delta^{(n)}(k_1+q-p_1-p_2-k_2)$$
$$\Theta(p_{1,0})\delta(p_1^2-m^2)\Theta(p_{2,0})\delta(p_2^2-m^2)\Theta(k_{2,0})\delta(k_2^2) \tag{45}$$

This can be solved by writing eq. (45) as product of a $2 \to 2$ decay and a subsequent $1 \to 2$ decay[3]. We find

$$dPS_3 = \frac{1}{(4\pi)^n\Gamma(n-3)s'}\frac{s_4^{n-3}}{(s_4+m^2)^{n/2-1}}\left(\frac{(t_1u_1'-s'm^2)s'-q^2t_1^2}{s'^2}\right)^{(n-4)/2}dt_1du_1d\Omega_nd\hat{\mathcal{I}} \tag{46}$$
$$= h_3(n)\,dt_1du_1d\Omega_nd\hat{\mathcal{I}} \tag{47}$$

with $d\Omega_n = \sin^{n-3}(\theta_1)d\theta_1\sin^{n-4}(\theta_2)d\theta_2$ and $d\hat{\mathcal{I}}$ taking care of all occuring hat momenta:

$$d\hat{\mathcal{I}} = \frac{1}{B(1/2,(n-4)/2)}\frac{x^{(n-6)/2}}{\sqrt{1-x}}dx \qquad \text{with } x = \hat{p}_1^2/\hat{p}_{1,max} \tag{48}$$
$$\hat{p}_{1,max} = \frac{s_4^2}{4(s_4+m^2)}\sin^2(\theta_1)\sin^2(\theta_2) \tag{49}$$

$$\Rightarrow \int d\hat{\mathcal{I}} = 1 \qquad \int d\hat{\mathcal{I}}\, \hat{p}_1^2 = \epsilon \hat{p}_{1,max} + O(\epsilon^2) \tag{50}$$

Again when integrating the phase space angles the expressions become quite lengthy, but the (collinear) pole parts are compact:

$$\frac{s_4}{4\pi(s_4+m^2)} \int d\Omega_n d\hat{\mathcal{I}}\, C_A R_{\vec{k},OK} = -\frac{1}{u_1} B_{\vec{k},QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} P^H_{\vec{k},\mathrm{gg}}(x_1) \frac{2}{\epsilon} + O(\epsilon^0) \tag{51}$$

with $x_1 = -u_1/(s'+t_1)$ and the hard part of the Altarelli-Parisi splitting functions $P^H_{k,\mathrm{gg}}$[4, 5]:

$$P^H_{F,\mathrm{gg}}(x) = C_A \left(\frac{2}{1-x} + \frac{2}{x} - 4 + 2x - 2x^2 \right) \tag{52}$$

$$P^H_{g,\mathrm{gg}}(x) = C_A \left(\frac{2}{1-x} - 4x + 2 \right) \tag{53}$$

The $R_{\vec{k},QED}$ do not contain poles. **FiXme Error: shift to factorization?** FiXme Error!

From the above expression we can obtain the soft limit $k_2 \to 0$ and seperate their calculations:

$$\lim_{k_2 \to 0} \left(C_A R_{\vec{k},OK} + 2C_F R_{\vec{k},QED} \right) = \left(C_A S_{\vec{k},OK} + 2C_F S_{\vec{k},QED} \right) + O(1/s_4, 1/s_3, 1/t') \tag{54}$$

$$S_{\vec{k},OK} = 2 \left(\frac{t_1}{t' s_3} + \frac{u_1}{t' s_4} - \frac{s-2m^2}{s_3 s_4} \right) B_{\vec{k},QED} \tag{55}$$

$$S_{\vec{k},QED} = 2 \left(\frac{s-2m^2}{s_3 s_4} - \frac{m^2}{s_3^2} - \frac{m^2}{s_4^2} \right) B_{\vec{k},QED} \tag{56}$$

Note that the einkonal factors multiplying the Born functions $B_{\vec{k},QED}$ neither depend on q^2 nor on the projection $\vec{k}$.

3.3 Light Quark Processes

In next-to-leading order a new production mechanism enters that is induced by a light quark, so we have to consider the process

$$\gamma^*(q) + q(k_1) \to Q(p_1) + \overline{Q}(p_2) + q(k_2) \tag{57}$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as FiXme Error!

$$\hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\vec{k}} \hat{\mathcal{P}}^{q,aa'}_{\vec{k}} \sum_{j,j'=1}^{4} \mathcal{M}^{(1),q}_{j,\mu a} \left(\mathcal{M}^{(1),q}_{j',\mu' a'} \right)^* = 8 g^4 \mu_D^{-2\epsilon} e^2 N_C C_F \left(e_H^2 A_{\vec{k},1} + e_L^2 A_{\vec{k},2} + e_L e_H A_{\vec{k},3} \right) \tag{58}$$

where e_L denotes the charge of the light quark q in units of e.

The needed $2 \to 3$ phase space has already been calculated in section 3.2, so we can immediately quote the (collinear) poles:

$$\frac{s_4}{4\pi(s_4+m^2)}\int d\Omega_n d\hat{\mathcal{I}}\, C_F A_{\vec{k},1} = -\frac{1}{u_1} B_{\vec{k},QED}\begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} P_{k_2,\mathrm{g}q}(x_1)\frac{2}{\epsilon} + O(\epsilon^0) \quad (59)$$

with $x_1 = -u_1/(s'+t_1)$ and the Altarelli-Parisi splitting functions $P_{k,\mathrm{g}q}$[4, 5]:

$$P_{F,\mathrm{g}q}(x) = C_F\left(\frac{1}{x} + \frac{(1-x)^2}{x}\right) \quad (60)$$

$$P_{g,\mathrm{g}q}(x) = C_F\,(2-x) \quad (61)$$

$A_{k,2}$ does not contain poles and we find $\int dt_1 du_1 \int d\Omega_n d\hat{\mathcal{I}}\, A_{k,3} = 0$. Note that in the limit $q^2 \to 0$ $A_{k,2}$ will also get collinear poles.

4 Mass Factorization

All collinear poles in the gluonic subprocess can be removed by mass factorization in the following way:

$$s'^2\frac{d^2\sigma^{(1),fin}_{\vec{k},\mathrm{g}}(s',t_1,u_1,q^2,\mu_F)}{dt_1 du_1} = \lim_{\epsilon\to 0}\Bigg[s'^2\frac{d^2\sigma^{(1)}_{\vec{k},\mathrm{g}}(s',t_1,u_1,q^2,\epsilon)}{dt_1 du_1}$$

$$-\int_0^1 \frac{dx_1}{x_1}\Gamma^{(1)}_{\vec{k},\mathrm{gg}}(x_1,\mu_F^2,\mu_D,\epsilon) \quad (62)$$

$$(x_1 s')^2\frac{d^2\sigma^{(0)}_{\vec{k},\mathrm{g}}(x_1 s', x_1 t_1, u_1, q^2,\epsilon)}{d(x_1 t_1) du_1}\Bigg] \quad (63)$$

$$\Gamma^{(1)}_{\vec{k},ij}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_s}{2\pi}\left(P_{\vec{k},ij}(x)\frac{2}{\epsilon} + f_{\vec{k},ij}(x,\mu_F^2,\mu_D^2)\right) \quad (64)$$

where $\Gamma^{(1)}_{\vec{k},ij}$ is the first order correction to the transition functions $\Gamma_{\vec{k},ij}$ for *incoming* particle j and *outgoing* particle i in projection k. In the $\overline{\mathrm{MS}}$-scheme the $f_{\vec{k},ij}$ take their usual form and we find

$$\Gamma^{(1),\overline{\mathrm{MS}}}_{\vec{k},ij}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_s}{2\pi}P_{\vec{k},ij}(x)\left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln(\mu_F^2/m^2) - \ln(\mu_D^2/m^2)\right) \quad (65)$$

$$= 8\pi\alpha_s P_{\vec{k},ij}(x) C_\epsilon\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}\left(\frac{2}{\epsilon} + \ln(\mu_F^2/m^2)\right) \quad (66)$$

The $P_{\vec{k},ij}(x)$ are the Altarelli-Parisi splitting functions for which we find[4, 5]

$$P_{\vec{k},\mathrm{gg}}(x) = \Theta(1-\delta-x)P^H_{\vec{k},\mathrm{gg}}(x) + \delta(1-x)\left(2C_A\ln(\delta) + \frac{\beta_0}{2}\right) \tag{67}$$

where we introduced another infrared cut-off δ to seperate soft ($x \geq 1-\delta$) and hard ($x < 1-\delta$) gluons that is connected to Δ via $\delta = \Delta/(s'+t_1)$. The structure here explains why we were able to write the equation (51).

The light quark process can be regularized in a complete analogous way:

$$s'^2\frac{d^2\sigma^{(1),fin}_{\vec{k},q}(s',t_1,u_1,q^2,\mu_F)}{dt_1du_1} = \lim_{\epsilon\to 0}\left[s'^2\frac{d^2\sigma^{(1)}_{\vec{k},q}(s',t_1,u_1,q^2,\epsilon)}{dt_1du_1}\right.$$
$$-\int_0^1\frac{dx_1}{x_1}\Gamma^{(1)}_{\vec{k},\mathrm{g}q}(x_1,\mu_F^2,\mu_D,\epsilon)$$
$$\left.(x_1s')^2\frac{d^2\sigma^{(0)}_{\vec{k},\mathrm{g}}(x_1s',x_1t_1,u_1,q^2,\epsilon)}{d(x_1t_1)du_1}\right] \tag{68}$$

The needed splitting functions $P_{\vec{k},\mathrm{g}q}$ have been already quoted in equations (60) and (61). Note that $K_{q\gamma} = 1/(N_C) = 2C_FK_{\mathrm{g}\gamma}$.

The final finite cross sections are then

$$s'^2\frac{d^2\sigma^{(1),H,fin}_{\vec{k},\mathrm{g}}}{dt_1du_1} = \alpha\alpha_Se_H^2K_{\mathrm{g}\gamma}N_CC_F\left[-\frac{2}{u_1}P^H_{\vec{k},\mathrm{gg}}(x_1)\right.$$
$$\left\{B^{(0)}_{\vec{k},QED}\begin{pmatrix}s'\to x_1s'\\ t_1\to x_1t_1\end{pmatrix}\left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)}\right)-\ln(\mu_F^2/m^2)\right)\right.$$
$$\left.-2B^{(1)}_{\vec{k},QED}\begin{pmatrix}s'\to x_1s'\\ t_1\to x_1t_1\end{pmatrix}\right\}$$
$$+C_A\frac{s_4}{2\pi(s_4+m^2)}\left(\int d\Omega_n d\hat{\mathcal{I}}\,R_{\vec{k},OK}\right)^{finite}$$
$$\left.+2C_F\frac{s_4}{2\pi(s_4+m^2)}\int d\Omega_4d\hat{\mathcal{I}}\,R_{\vec{k},QED}\right] \tag{69}$$

$$s'^2\frac{d^2\sigma^{(1),S+V,fin}_{\vec{k},\mathrm{g}}}{dt_1du_1} = 4\alpha\alpha_Se_H^2K_{\mathrm{g}\gamma}N_CC_FB^{(0)}_{\vec{k},QED}\delta(s'+t_1+u_1)\left[C_A\ln^2(\Delta/m^2)\right.$$
$$+\ln(\Delta/m^2)\left(\left(\ln(-t_1/m^2)-\ln(-u_1/m^2)-\ln(\mu_F^2/m^2)\right)C_A\right.$$
$$\left.-\frac{2m^2-s}{s\beta}\ln(\chi)(C_A-2C_F)-2C_F\right)$$
$$\left.+\frac{\beta_0^{lf}}{4}\left(\ln(\mu_R^2/m^2)-\ln(\mu_F^2/m^2)\right)+f_{\vec{k}}(s',u_1,t_1,q^2)\right] \tag{70}$$

where $f_{\vec{k}}$ contains lots of logarithms and dilogarithms, but does not depend on Δ, μ_F^2, μ_R^2 nor n_f and $\beta_0^{lf} = (11C_A - 2n_{lf})/3$.

$$\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),fin}_{\vec{k},q}}{dt_1 du_1} = \alpha\alpha_S K_{q\gamma} N_C \Bigg[& -\frac{1}{u_1} e_H^2 P_{\vec{k},\mathrm{g}q}(x_1) \\
& \quad \Bigg\{ B^{(0)}_{\vec{k},QED}\begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)}\right) - \ln(\mu_F^2/m^2) - 2\partial_\epsilon E_{\vec{k}}(\epsilon = 0)\right) \\
& \quad -2B^{(1)}_{\vec{k},QED}\begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \Bigg\} \\
& + C_F \frac{s_4}{4\pi(s_4+m^2)} \left(\int d\Omega_n d\hat{\mathcal{I}}\, e_H^2 A_{\vec{k},1}\right)^{finite} \\
& + C_F \frac{s_4}{4\pi(s_4+m^2)} \int d\Omega_4 d\hat{\mathcal{I}}\, e_L^2 A_{\vec{k},2} + C_F \frac{s_4}{4\pi(s_4+m^2)} \int d\Omega_4 d\hat{\mathcal{I}}\, e_H e_L A_{\vec{k},3} \Bigg]
\end{aligned} \tag{71}$$

5 Partonic Results

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5.1 $c_{\mathrm{g}}^{(0)}$

In leading order, we find

$$c^{(0)}_{\mathrm{VV},F_2,\mathrm{g}} = -\frac{\pi {\rho'}^3}{4\rho^2 {\rho_q}^2}\Big[2\beta\left(\rho^2+{\rho_q}^2+\rho\rho_q(6+\rho_q)\right) + \left(2{\rho_q}^2+2\rho{\rho_q}^2+\rho^2(2-(-4+\rho_q)\rho_q)\right)\ln(\chi)\Big] \tag{72}$$

$$c^{(0)}_{\mathrm{VV},F_L,\mathrm{g}} = -\frac{\pi {\rho'}^3}{\rho\rho_q}\left[2\beta+\rho\ln(\chi)\right] \tag{73}$$

$$c^{(0)}_{\mathrm{VV},2xg_1,\mathrm{g}} = \frac{\pi {\rho'}^2}{2\rho\rho_q}\left[\beta(\rho+3\rho_q)+(\rho+\rho_q)\ln(\chi)\right] \tag{74}$$

$$c^{(0)}_{\mathrm{AA},F_2,\mathrm{g}} = \frac{\pi {\rho'}^3}{4\rho^2 {\rho_q}^2}\Big[2\beta\left(\rho^2+{\rho_q}^2+\rho\rho_q(6+\rho_q)\right) - \left(-6\rho{\rho_q}^2+2(-1+\rho_q){\rho_q}^2+\rho^2(-2+(-2+\rho_q)\rho_q)\right)\ln(\chi)\Big] \tag{75}$$

$$c^{(0)}_{\mathrm{AA},F_L,\mathrm{g}} = -\frac{\pi {\rho'}^3}{2\rho^2\rho_q}\Big[2\beta\rho(2+\rho_q) - \left(\rho^2(-1+\rho_q)-4\rho\rho_q+{\rho_q}^2\right)\ln(\chi)\Big] \tag{76}$$

$$c^{(0)}_{\mathrm{AA},2xg_1,\mathrm{g}} = c^{(0)}_{\mathrm{VV},2xg_1,\mathrm{g}} \tag{77}$$

$$c^{(0)}_{\mathrm{VA},xF_3,\mathrm{g}} = c^{(0)}_{\mathrm{VA},g_4,\mathrm{g}} = c^{(0)}_{\mathrm{VA},g_L,\mathrm{g}} = 0 \tag{78}$$

Near threshold we find

$$c^{(0),\mathrm{thr}}_{\mathrm{VV},F_2,\mathrm{g}} = \frac{\pi\beta\rho_q}{2(\rho_q-1)} \tag{79}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{VV},F_L,\mathrm{g}} = \frac{4\pi\beta^3\rho_q^2}{3(1-\rho_q)^3} \tag{80}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{VV},2xg_1,\mathrm{g}} = c^{(0),\mathrm{thr}}_{\mathrm{VV},F_2,\mathrm{g}} \tag{81}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{AA},F_2,\mathrm{g}} = \frac{\pi\beta\rho_q^2}{1-\rho_q} \tag{82}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{AA},F_2,\mathrm{g}} = \frac{\pi\beta(1-2\rho_q)\rho_q}{2(\rho_q-1)} \tag{83}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{AA},2xg_1,\mathrm{g}} = c^{(0),\mathrm{thr}}_{\mathrm{VV},2xg_1,\mathrm{g}} \tag{84}$$

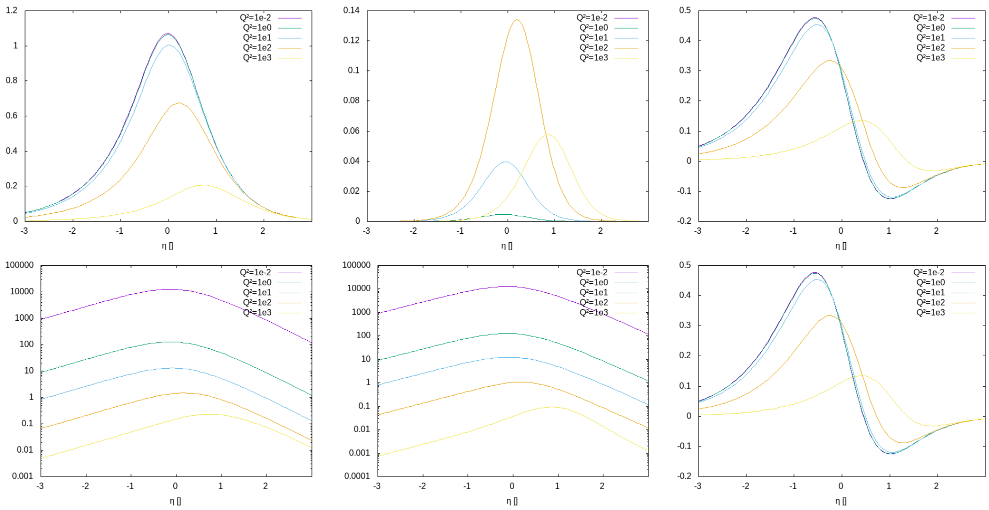

Figure 2: leading order scaling functions $c^{(0)}_{k,\mathrm{g}}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.2 $c_{\mathrm{g}}^{(1)}$

Near threshold, we find

$$c_{\vec{k},\mathrm{g}}^{(1),\mathrm{thr}} = c_{\vec{k},\mathrm{g}}^{(0),\mathrm{thr}} \frac{1}{\pi^2} \left[C_A \left(a_{\vec{k},\mathrm{g}}^{(1,2)} \ln^2(\beta) + a_{\vec{k},\mathrm{g}}^{(1,1)} \ln(\beta) - \frac{\pi^2}{16\beta} + a_{\vec{k},\mathrm{g,OK}}^{(1,0)} \right) \right.$$
$$\left. + 2CF \left(\frac{\pi^2}{16\beta} + a_{\vec{k},\mathrm{g,QED}}^{(1,0)} \right) \right], \tag{85}$$

with

$$a_{\vec{k},\mathrm{g}}^{(1,2)} = 1 \tag{86}$$

$$a_{\mathrm{VV},F_2,\mathrm{g}}^{(1,1)} = -\frac{5}{2} + 3\ln(2) \tag{87}$$

$$a_{\mathrm{VV},F_L,\mathrm{g}}^{(1,1)} = a_{\mathrm{VV},F_2,\mathrm{g}}^{(1,1)} - \frac{2}{3} \tag{88}$$

$$a_{\mathrm{VV},2xg_1,\mathrm{g}}^{(1,1)} = a_{\mathrm{AA},F_2,\mathrm{g}}^{(1,1)} = a_{\mathrm{AA},F_L,\mathrm{g}}^{(1,1)} = a_{\mathrm{AA},2xg_1,\mathrm{g}}^{(1,1)} = a_{\mathrm{VV},F_2,\mathrm{g}}^{(1,1)} \tag{89}$$

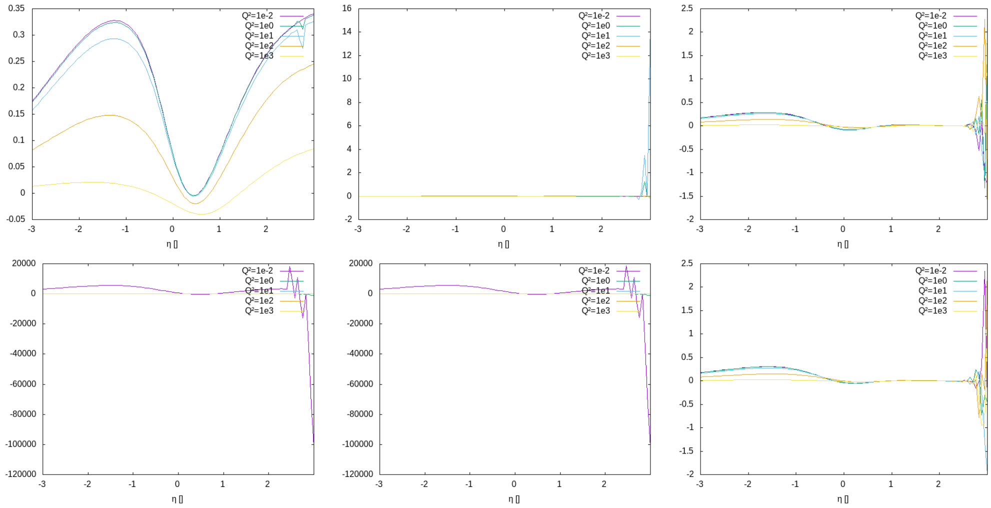

Figure 3: next-to-leading order scaling functions $c_{k,\mathrm{g}}^{(1)}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.3 $\bar{c}_{\mathrm{g}}^{(1)}$

For the scaling functions we find at this order:

$$\bar{c}^{(1)}_{\mathrm{VA},xF_3,\mathrm{g}} = \bar{c}^{(1)}_{\mathrm{VA},g_4,\mathrm{g}} = \bar{c}^{(1)}_{\mathrm{VA},g_L,\mathrm{g}} = 0 \tag{90}$$

and

$$\bar{c}^{(1)}_{\mathrm{VV},2xg_1,\mathrm{g}} = \bar{c}^{(1)}_{\mathrm{AA},2xg_1,\mathrm{g}} \tag{91}$$

and furthermore near threshold, we find

$$\bar{c}^{(1),\mathrm{thr}}_{\vec{k},\mathrm{g}} = c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{1}{\pi^2} C_A \left(\bar{a}^{(1,1)}_{\vec{k},\mathrm{g}} \ln(\beta) + \bar{a}^{(1,0)}_{\vec{k},\mathrm{g}} \right) \;, \tag{92}$$

with

$$\bar{a}^{(1,1)}_{\vec{k},\mathrm{g}} = -\frac{1}{2} \tag{93}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_2,\mathrm{g}} = -\frac{1}{4} \ln\left(\frac{16\chi_q}{(1+\chi_q)^2}\right) + \frac{1}{2} \tag{94}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_L,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,\mathrm{g}} + \frac{1}{6} \tag{95}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},2xg_1,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{AA},F_2,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{AA},F_L,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{AA},2xg_1,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,\mathrm{g}} \tag{96}$$

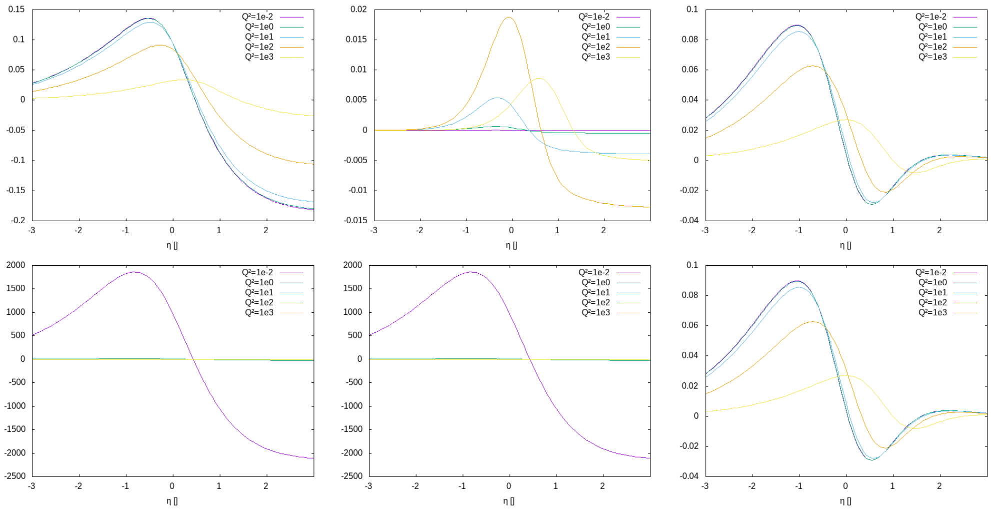

Figure 4: next-to-leading order scaling functions $\bar{c}^{(1)}_{k,\mathrm{g}}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.4 $c_{\mathsf{q}}^{(1)}$

Near threshold, we find

$$c^{(1),\mathrm{thr}}_{\vec{k},q} = c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{\beta^2 \rho_q}{\pi^2(\rho_q - 1)} \frac{K_{q\gamma}}{6K_{\mathrm{g}\gamma}} \left[a^{(1,1)}_{\vec{k},q} \ln(\beta) + a^{(1,0)}_{\vec{k},q} \right], \tag{97}$$

with

$$a^{(1,1)}_{\mathrm{VV},F_2,q} = 1 \tag{98}$$

$$a^{(1,1)}_{\mathrm{VV},F_L,q} = a^{(1,1)}_{\mathrm{VV},F_2,q} - \frac{2}{3} \tag{99}$$

$$a^{(1,1)}_{\mathrm{VV},2xg_1,q} = a^{(1,1)}_{\mathrm{AA},F_2,q} = a^{(1,1)}_{\mathrm{AA},F_L,q} = a^{(1,1)}_{\mathrm{AA},2xg_1,q} = a^{(1,1)}_{\mathrm{VV},F_2,q} \tag{100}$$

$$a^{(1,0)}_{\mathrm{VV},F_2,q} = -\frac{13}{12} + \frac{3}{2}\ln(2) \tag{101}$$

$$a^{(1,0)}_{\mathrm{VV},F_L,q} = -\frac{77}{100} + \frac{9}{10}\ln(2) \tag{102}$$

$$a^{(1,0)}_{\mathrm{VV},2xg_1,q} = a^{(1,0)}_{\mathrm{VV},F_2,q} - \frac{1}{4} \tag{103}$$

$$a^{(1,0)}_{\mathrm{AA},F_2,q} = a^{(1,0)}_{\mathrm{AA},F_L,q} = a^{(1,0)}_{\mathrm{AA},2xg_1,q} = a^{(1,0)}_{\mathrm{VV},F_2,q} \tag{104}$$

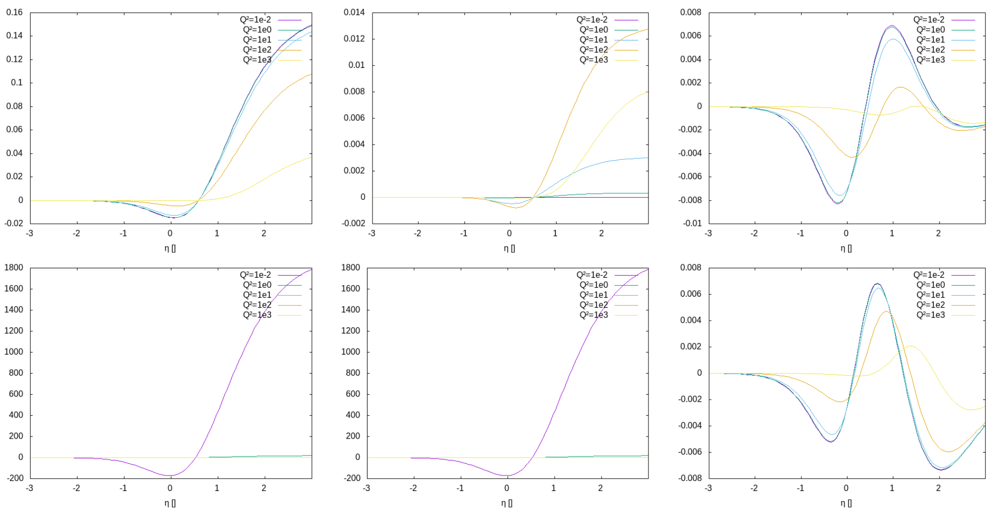

Figure 5: next-to-leading order scaling functions $c^{(1)}_{k,q}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.5 $\bar{c}_{\mathsf{q}}^{(1),F}$

For the scaling functions we find at this order:

$$\bar{c}^{(1),F}_{\mathrm{VA},xF_3,q} = \bar{c}^{(1),F}_{\mathrm{VA},g_4,q} = \bar{c}^{(1),F}_{\mathrm{VA},g_L,q} = 0 \tag{105}$$

and

$$\bar{c}^{(1),F}_{\mathrm{VV},2xg_1,q} = \bar{c}^{(1),F}_{\mathrm{AA},2xg_1,q} \tag{106}$$

and furthermore near threshold, we find

$$\bar{c}^{(1),F,\mathrm{thr}}_{\vec{k},q} = -c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{\beta^2 \rho_q}{\pi^2(\rho_q - 1)} \frac{K_{q\gamma}}{24 K_{\mathrm{g}\gamma}} \cdot \bar{a}^{(1,0)}_{\vec{k},q} \tag{107}$$

with

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_2,q} = 1 \tag{108}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_L,q} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,q} - \frac{2}{3} \tag{109}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},2xg_1,q} = \bar{a}^{(1,0)}_{\mathrm{AA},F_2,q} = \bar{a}^{(1,0)}_{\mathrm{AA},F_L,q} = \bar{a}^{(1,0)}_{\mathrm{AA},2xg_1,q} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,q} \tag{110}$$

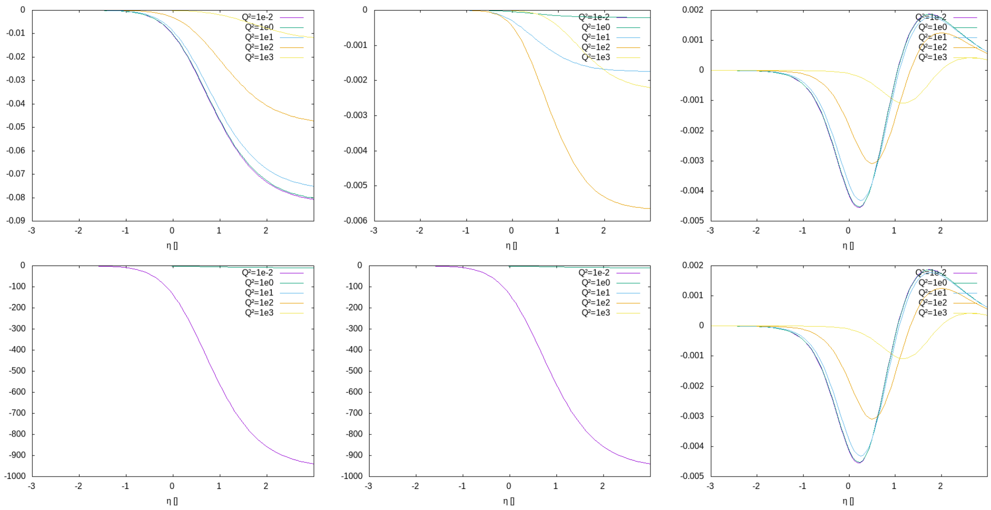

Figure 6: next-to-leading order scaling functions $\bar{c}^{(1),F}_{k,q}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.6 $d_{\mathsf{q}}^{(1)}$

For the scaling functions we find at this order:

$$d^{(1)}_{\mathrm{VA},g_4,q} = d^{(1)}_{\mathrm{VA},g_L,q} = 0 = d^{(1)}_{\mathrm{AA},2xg_1,q} \tag{111}$$

and

$$d^{(1)}_{\mathrm{VV},F_2,q} = d^{(1)}_{\mathrm{AA},F_2,q} \qquad d^{(1)}_{\mathrm{VV},F_L,q} = d^{(1)}_{\mathrm{AA},F_L,q} \qquad d^{(1)}_{\mathrm{VA},xF_3,q} = d^{(1)}_{\mathrm{VV},2xg_1,q} \tag{112}$$

For $\chi' \to 0$ we find:

$$\begin{aligned} d^{(1)}_{\mathrm{VV},F_2,q} &= -\frac{\rho}{9\pi}\left(\mathrm{Li}_2(-\chi) - \frac{1}{4}\ln^2(\chi) + \frac{\pi^2}{12} + \ln(\chi)\ln(1+\chi)\right) \\ &\qquad + \beta\frac{\rho(718+5\rho)}{2592\pi} + \frac{\rho(232+9\rho^2)}{1728\pi}\ln(\chi) + \mathcal{O}(\chi') \end{aligned} \tag{113}$$

$$d^{(1)}_{\mathrm{VV},F_L,q} = \left(\beta\frac{-38+23\rho}{54\pi} + \frac{-8+3\rho^2}{36\pi}\ln(\chi)\right)\chi' + \mathcal{O}(\chi'^2) \tag{114}$$

$$d^{(1)}_{\mathrm{VV},2xg_1,q} = d^{(1)}_{\mathrm{AA},F_2,q} = d^{(1)}_{\mathrm{VA},xF_3,q} \tag{115}$$

$$d^{(1)}_{\mathrm{AA},F_L,q} = d^{(1)}_{\mathrm{VV},F_L,q} \tag{116}$$

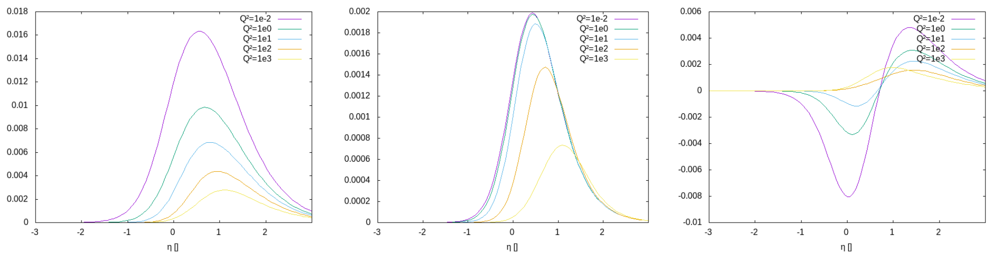

Figure 7: next-to-leading order scaling functions $\bar{c}^{(1),F}_{k,q}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

6 Hadronic Results

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7 Summary

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A References

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List of Corrections