

1 2 to 2 phase space

process:

$$\gamma^*(q) + g(k_1) \rightarrow Q(p_1) + \bar{Q}(p_2) \quad (1)$$

use c.m.s. of incoming particles:

$$q = \left(\frac{s + q^2}{2\sqrt{s}}, 0, 0, \dots, -\frac{s - q^2}{2\sqrt{s}} \right) \quad (2)$$

$$k_1 = \frac{s - q^2}{2\sqrt{s}} (1, 0, 0, \dots, 1) \quad (3)$$

such that

$$q + k_1 = (\sqrt{s}, \vec{0}) \quad k_1^2 = 0 \quad (4)$$

for the outgoing particles it follows

$$p_1 = \frac{\sqrt{s}}{2} (1, 0, \beta \sin \theta, \dots, \beta \cos \theta) \quad (5)$$

$$p_2 = \frac{\sqrt{s}}{2} (1, 0, -\beta \sin \theta, \dots, -\beta \cos \theta) \quad (6)$$

with $\beta = \sqrt{1 - 4m^2/s}$ such that

$$p_1 + p_2 = (\sqrt{s}, \vec{0}) \quad p_1^2 = p_2^2 = m^2 \quad (7)$$

use n-sphere:

$$d^D x = \Omega_D x^{D-1} dx = \frac{2\pi^{D/2}}{\Gamma(D/2)} x^{D-1} dx = \frac{\pi^{D/2}}{\Gamma(D/2)} (x^2)^{(D-2)/2} dx^2 \quad (8)$$

compute phase space:

$$PS_2 = \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_2}{(2\pi)^{n-1}} (2\pi)^n \delta^{(n)}(q + k_1 - p_1 - p_2) \delta(p_1^2 - m^2) \delta(p_2^2 - m^2) \quad (9)$$

$$= \frac{1}{(2\pi)^{n-2}} \int d^n p_1 \delta((q + k_1 - p_1)^2 - m^2) \delta(p_1^2 - m^2) \quad (10)$$

$$= \frac{1}{(2\pi)^{n-2}} \int dp_{1,0} dp_{1,\parallel} d^2 p_{1,\perp} d^{n-4} \hat{p}_1 \delta(s - 2p_{1,0}\sqrt{s}) \delta(p_{1,0}^2 - p_{1,\parallel}^2 - p_{1,\perp}^2 - \hat{p}_1^2 - m^2) \quad (11)$$

$$= \frac{\pi}{(2\pi)^{n-2} 2\sqrt{s}} \int dp_{1,\parallel} dp_{1,\perp}^2 d^{n-4} \hat{p}_1 \delta(s/4 - p_{1,\parallel}^2 - p_{1,\perp}^2 - \hat{p}_1^2 - m^2) \quad (12)$$

$$= \frac{\pi}{(2\pi)^{n-2} 2\sqrt{s}} \int dp_{1,\parallel} d\hat{p}_1^2 \frac{\pi^{(n-4)/2}}{\Gamma((n-4)/2)} (\hat{p}_1^2)^{(n-6)/2} \quad (13)$$

$$= \frac{1}{2\sqrt{s} \Gamma((n-4)/2) (4\pi)^{(n-2)/2}} \int dp_{1,\parallel} d\hat{p}_1^2 (\hat{p}_1^2)^{(n-6)/2} \quad (14)$$

Integration borders are

$$p_{1,\parallel} \in \frac{\sqrt{s}}{2} \beta \cdot [-1, 1] \quad \hat{p}_1^2 \in \left(\frac{s\beta^2}{4} - p_{1,\parallel}^2 \right) \cdot [0, 1] \quad (15)$$

if cross section does not depend on hat-space:

$$\int d\hat{p}_1^2 (\hat{p}_1^2)^{(n-6)/2} = \frac{2}{n-4} \left(\frac{s\beta^2}{4} - p_{1,\parallel}^2 \right)^{(n-4)/2} \quad (16)$$

$$\Rightarrow PS_2 = \frac{1}{2\sqrt{s}\Gamma((n-2)/2)(4\pi)^{(n-2)/2}} \int dp_{1,\parallel} \left(\frac{s\beta^2}{4} - p_{1,\parallel}^2 \right)^{(n-4)/2} \quad (17)$$

rewrite $p_{1,\parallel}$ to $\cos \theta$:

$$p_{1,\parallel} = \frac{\sqrt{s}}{2}\beta \cos \theta \Rightarrow dp_{1,\parallel} = \frac{\sqrt{s}}{2}\beta d\cos \theta, \quad \cos \theta \in [-1, 1], \quad \hat{p}_1^2 \in \frac{s\beta^2}{4} (1 - \cos^2 \theta) \cdot [0, 1] \quad (18)$$

rewrite $\cos \theta$ to $t_1 = (k_1 - p_2)^2 - m^2$:

$$\cos \theta = t_1 \Rightarrow d\cos \theta = dt_1, \quad t_1 \in [0, 1], \quad \hat{p}_1^2 \in [0, 1] \quad (19)$$