

1 Introduction

This work is mainly based on the paper “Heavy quark correlations in deep inelastic electroproduction” by Harris et. al.[1] - that is, it recalculates all properties and formulas. It extends then the application to the equivalent *polarized* processes. The treating of the polarized processes can for example be found in [2] and we will use many ideas and techniques from there. **FiXme Error: more**

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1.1 Motivation

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1.2 Notation

We calculate in $n = 4 + \epsilon$ dimension to regularize all soft, collinear and ultraviolet poles. Unfortunaly this extension for *polarized* processes is nontrivial, due to the occurance of the Levi-Civita tensors $\varepsilon_{\mu\nu\rho\sigma}$ and γ_5 . A common choice to deal with these objects is the HVBM prescription[3] that keeps those two objects four dimensional at the price of splitting the full n -dimensional space into a $(n-4)$ -dimensional space, called “hat-space”, and a four-dimensional space (that is actually never used).

In leading order (LO) we have to consider the following process:

$$\gamma^*(q) + g(k_1) \rightarrow Q(p_1) + \bar{Q}(p_2) \quad (1)$$

The corresponding parton structure tensor $W_{\mu\mu'}^{(0)}$ can then be written as **FiXme Error: avoid all order expr? FiXme Error: remove?**

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$$\begin{aligned} W_{\mu\mu'}^{(0)}(k_1, q; s, t_1, u_1, q^2; \sigma_q) \\ = \frac{1}{2} E_\epsilon K_{g\gamma} \frac{1}{2s'} \int \frac{d^{n-1}p_1}{2E_1(2\pi)^{n-1}} \int \frac{d^{n-1}p_2}{2E_2(2\pi)^{n-1}} \delta(p_1^2 - m^2) \delta(p_2^2 - m^2) \\ (2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2) \mathcal{M}_\mu^{(0)} \mathcal{M}_{\mu'}^{(0)} \end{aligned} \quad (2)$$

where the initial $1/2$ is the initial state spin average, $K_{g\gamma}$ is the color average,

$$E_\epsilon := \begin{cases} 1/(1 + \epsilon/2) & \text{unpolarized} \\ 1 & \text{polarized} \end{cases} \quad (3)$$

accounts for additional degrees of freedom in n dimensions for initial bosons. The Lorentz indices μ and μ' refer to the virtual photon that is exchanged with the scattering lepton. We have chosen to detect the heavy *antiquark* $\bar{Q}(p_2)$ and so we define the following Mandelstam variables:

$$s = (q + k_1)^2, \quad t_1 = t - m^2 = (k_1 - p_2)^2 - m^2, \quad u_1 = u - m^2 = (q - p_2)^2 - m^2 \quad (4)$$

For convenience we also define $s' = s - q^2$ and $u'_1 = u_1 - q^2$. If the heavy quark $\bar{Q}(p_1)$ is detected, p_2 in eq. (4) has to be replaced by p_1 which effectively interchanges $t_1 \leftrightarrow u_1$.
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By using Lorentz covariance, hermiticity, parity invariance and current conservation the parton structure tensor can be decomposed into several parts:

$$W_{\mu\mu'}(k_1, q; s, t_1, u_1, q^2; \sigma_q) = \left(-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2} \right) \frac{d^2 \sigma_T(s, t_1, u_1, q^2)}{dt_1 du_1} + \left(k_{1,\mu} - \frac{k_1 \cdot q}{q^2} q_\mu \right) \left(k_{1,\mu'} - \frac{k_1 \cdot q}{q^2} q_{\mu'} \right) \left(\frac{-4q^2}{s'^2} \right) \cdot \left(\frac{d^2 \sigma_T(s, t_1, u_1, q^2)}{dt_1 du_1} + \frac{d^2 \sigma_L(s, t_1, u_1, q^2)}{dt_1 du_1} \right) \quad (5)$$

FiXme Error: extend We can then define appropriate projection operators[4, 5]

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$$\hat{\mathcal{P}}_{G,\mu\mu'}^\gamma = -g_{\mu\mu'} \quad b_G(\epsilon) = \frac{1}{2(1 + \epsilon/2)} \quad (6)$$

$$\hat{\mathcal{P}}_{L,\mu\mu'}^\gamma = -\frac{4q^2}{s'^2} k_{1,\mu} k_{1,\mu'} \quad b_L(\epsilon) = 1 \quad (7)$$

$$\hat{\mathcal{P}}_{P,\mu\mu'}^\gamma = i\epsilon_{\mu\mu'\rho\rho'} \frac{q^\rho k_1^{\rho'}}{s'} \quad b_P(\epsilon) = 1 \quad (8)$$

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$$\frac{d^2 \sigma_k(s, t_1, u_1, q^2)}{dt_1 tu_1} = b_k(\epsilon) \hat{\mathcal{P}}_{k,\mu\mu'}^\gamma W^{\mu\mu'} \quad (9)$$

FiXme Error: how to insert 2nd projector? with $k \in \{G, L, P\}$ denoting (here and mostly ever after) the projection type. The transverse partonic cross section $d\sigma_T$ can be reconstructed from the above definitions by using

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$$d\sigma_T = d\sigma_G + b_G(\epsilon) d\sigma_L \quad (10)$$

We also define accordingly

$$E_G(\epsilon) = E_L(\epsilon) = \frac{1}{1 + \epsilon/2} \quad E_P(\epsilon) = 1 \quad (11)$$

The final state spins are always summed over, but the initial spins have to be treated separately: for unpolarized projections $k \in \{G, L\}$ they are also summed over, but for the polarized projection $k = P$ they are projected on their asymmetric part:

$$\hat{\mathcal{P}}_{G,\nu\nu'}^g = \hat{\mathcal{P}}_{L,\nu\nu'}^g = -g_{\nu\nu'} \quad \hat{\mathcal{P}}_{P,\nu\nu'}^g = 2i\epsilon_{\nu\nu'\rho\rho'} \frac{k_1^\rho q^\rho}{s'} \quad (12)$$

where ν and ν' refer to the initial gluon. By writing $\hat{\mathcal{P}}_{G,\nu\nu'}^g$ in eq. (12) we decided to introduce Fadeev-Popov ghosts[2] as we got a single diagram in next-to-leading order with

a triple-gluon vertex **FiXme Error: explain ghosts?**. As we can consider all quarks in the initial state as massless, we further find

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$$\hat{\mathcal{P}}_{G,aa'}^q = \hat{\mathcal{P}}_{L,aa'}^q = (\not{k}_1)_{aa'} \quad \hat{\mathcal{P}}_{P,aa'}^q = -(\gamma_5 \not{k}_1)_{aa'} \quad (13)$$

$$\hat{\mathcal{P}}_{G,bb'}^{\bar{q}} = \hat{\mathcal{P}}_{L,bb'}^{\bar{q}} = (\not{k}_1)_{bb'} \quad \hat{\mathcal{P}}_{P,bb'}^{\bar{q}} = (\gamma_5 \not{k}_1)_{bb'} \quad (14)$$

where a and a' refer to the Dirac-index of the initial quark spinor in next-to-leading order - analogous for b and the antiquark.

We further define a set of partonic variables:

$$0 \leq \rho = \frac{4m^2}{s} \leq 1 \quad 0 \leq \beta = \sqrt{1 - \rho} \leq 1 \quad 0 \leq \chi = \frac{1 - \beta}{1 + \beta} \leq 1 \quad (15)$$

$$\rho_q = \frac{4m^2}{q^2} \leq 0 \quad 1 \leq \beta_q = \sqrt{1 - \rho_q} \quad 0 \leq \chi_q = \frac{\beta_q - 1}{\beta_q + 1} \leq 1 \quad (16)$$

When computing Feynman diagrams a computer algebra system (CAS) is almost obligatory: common choices are FORM[6] or Mathematica[7] - for the later the most common choice is TRACER[8], but we have chosen HEPMath[9]. We used the Feynman rules given by [10] and [2].

2 Leading Order Calculations

In leading order we have to consider photon-gluon-fusion (PGF), that is

$$\gamma^*(q) + g(k_1) \rightarrow Q(p_1) + \bar{Q}(p_2) \quad (17)$$

with two contributing diagrams depicted in figure 1.

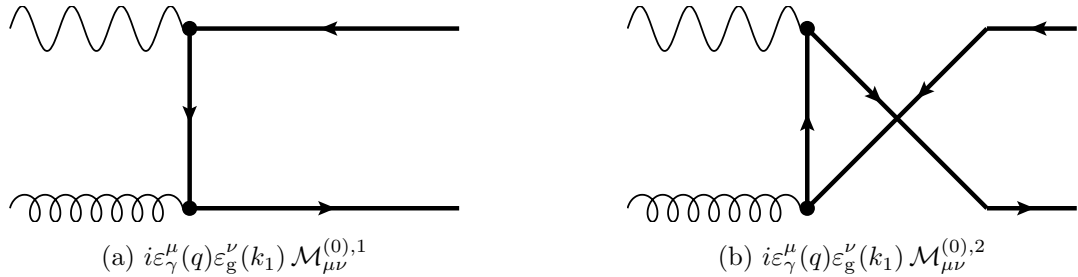


Figure 1: leading order Feynman diagrams

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The result can then be written as

$$M_k^{(0)} = \hat{\mathcal{P}}_k^{\gamma,\mu\mu'} \hat{\mathcal{P}}_k^{g,\nu\nu'} \sum_{j,j'=1}^2 \mathcal{M}_{\mu\nu}^{(0),j} \left(\mathcal{M}_{\mu'\nu'}^{(0),j'} \right)^* = 8g^2 \mu_D^{-\epsilon} e^2 e_H^2 N_C C_F B_{k,QED} \quad (18)$$

where g and e are the strong and electromagnetic coupling constants respectively, μ_D is an arbitrary mass parameter introduced to keep the couplings dimensionless and e_H is the magnitude of the heavy quark in units of e . Further N_C corresponds to the gauge group $SU(N_C)$ and the color factor $C_F = (N_C^2 - 1)/(2N_C)$ refers to the second Casimir constant of the fundamental representation for the quarks. We then find:

$$B_{G,QED} = \frac{t_1}{u_1} + \frac{u_1}{t_1} + \frac{4m^2 s'}{t_1 u_1} \left(1 - \frac{m^2 s'}{t_1 u_1}\right) + \frac{2s' q^2}{t_1 u_1} + \frac{2q^4}{t_1 u_1} + \frac{2m^2 q^2}{t_1 u_1} \left(2 - \frac{s'^2}{t_1 u_1}\right) + \epsilon \left\{ -1 + \frac{s'^2}{t_1 u_1} + \frac{s' q^2}{t_1 u_1} - \frac{q^4}{t_1 u_1} - \frac{m^2 q^2 s'^2}{t_1^2 u_1^2} \right\} + \epsilon^2 \frac{s'^2}{4t_1 u_1} \quad (19)$$

$$B_{L,QED} = -\frac{4q^2}{s'} \left(\frac{s}{s'} - \frac{m^2 s'}{t_1 u_1} \right) \quad (20)$$

$$B_{P,QED} = \frac{1}{2} \left(\frac{t_1}{u_1} + \frac{u_1}{t_1} \right) \left(\frac{2m^2 s'}{t_1 u_1} - 1 - \frac{2q^2}{s'} \right) \quad (21)$$

$$B_{k,QED} = B_{k,QED}^{(0)} + \epsilon B_{k,QED}^{(1)} + \epsilon^2 B_{k,QED}^{(2)} \quad (22)$$

By using eq. (2) we can derive the n -dimensional $2 \rightarrow 2$ phase space

$$dPS_2 = \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_2}{(2\pi)^{n-1}} \Theta(p_{1,0}) \delta(p_1^2 - m^2) \Theta(p_{2,0}) \delta(p_2^2 - m^2) (2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2) \quad (23)$$

that can be solved by using the center-of-mass system (CMS) of the incoming particles[2]

$$q = \left(\frac{s + q^2}{2\sqrt{s}}, 0, 0, -\frac{s - q^2}{2\sqrt{s}}, \hat{0} \right) \quad k_1 = \frac{s - q^2}{2\sqrt{s}} (1, 0, 0, 1, \hat{0}) \quad (24)$$

such that $q + k_1 = (\sqrt{s}, \vec{0})$ and $k_1^2 = 0$. For the outgoing particles it follows

$$p_1 = \frac{\sqrt{s}}{2} (1, 0, -\beta \sin \theta_1, -\beta \cos \theta_1, \hat{0}) \quad p_2 = \frac{\sqrt{s}}{2} (1, 0, \beta \sin \theta_1, \beta \cos \theta_1, \hat{0}) \quad (25)$$

such that $p_1 + p_2 = (\sqrt{s}, \vec{0})$ and $p_1^2 = p_2^2 = m^2$ and

$$t_1 = -\frac{s'}{2}(1 - \beta \cos(\theta_1)), \quad u_1 = -\frac{s'}{2}(1 + \beta \cos(\theta_1)) \quad (26)$$

We then arrive at the well known result[1]:

$$dPS_2 = \frac{\beta \sin(\theta_1)}{16\pi \Gamma(1 + \epsilon/2)} \left(\frac{s \beta^2 \sin^2(\theta_1)}{16\pi} \right)^{\epsilon/2} d\theta_1 \quad (27)$$

The cross sections are then given by:

$$d\sigma_k^{(0)} = \frac{1}{2s} \frac{K_{\gamma g} E_k(\epsilon)}{2} b_k(\epsilon) M_k^{(0)} dPS_2 \quad (28)$$

The procedure is completely analogous to the inclusive case **FiXme Error: cite** and the results agree to there.

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3 Next-To-Leading Order Calculations

Next-to-leading order contributions can be split into three parts: one loop virtual contributions, one gluon radiation and the light quark processes. **FiXme Error: more?**

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3.1 One Loop Virtual Contributions

Virtual contributions have the same initial and final state as the Born process, but have a looping particle. All contributing Feynman diagrams are depicted in figure **FiXme Error: do**. The result can then be written as

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$$\begin{aligned} M_k^{(1),V} &= \hat{\mathcal{P}}_k^{\gamma,\mu\mu'} \hat{\mathcal{P}}_k^{g,\nu\nu'} \sum_j \left[\mathcal{M}_{j,\mu\nu}^{(1),V} \left(\mathcal{M}_{1,\mu'\nu'}^{(0)} + \mathcal{M}_{2,\mu'\nu'}^{(0)} \right)^* + c.c. \right] \\ &= 8g^4 \mu_D^{-\epsilon} e^2 e_H^2 N_C C_F C_\epsilon (C_A V_{k,OK} + 2C_F V_{k,QED}) \end{aligned} \quad (29)$$

where $C_\epsilon = \exp(\epsilon/2(\gamma_E - \ln(4\pi)))/(16\pi^2)$ and $C_A = N_C$ is the second Casimir constant of the adjoint representation for the gluon (that introduces a non-abelian part).

For the computation of the loops the Passarino-Veltman-decomposition[11] in $n = 4 + \epsilon$ dimension is used as far as possible. The decomposition is based on Lorentz invariance and a good explanation is for example given in [2]. The needed scalar integrals are given in [12] and [4], but there is also one wrong integral: we find with [13, Box 16]:

$$\begin{aligned} D_0(m^2, 0, q^2, m^2, t, s, 0, m^2, m^2, m^2) \\ = \frac{iC_\epsilon}{\beta s t_1} \left[-\frac{2\ln(\chi)}{\epsilon} - 2\ln(\chi) \ln(-t_1/m^2) + \text{Li}_2(1 - \chi^2) - 4\zeta(2) + \ln^2(\chi_q) + 2\text{Li}_2(-\chi\chi_q) \right. \\ \left. + 2\text{Li}_2(-\chi/\chi_q) + 2\ln(\chi\chi_q) \ln(1 + \chi\chi_q) + 2\ln(\chi/\chi_q) \ln(1 + \chi/\chi_q) \right] \end{aligned} \quad (30)$$

where we used the argument ordering of `LoopTools`[14, 15] (and also checked it against `LoopTools`).

As the short example above shows, the full expressions for the $V_{k,OK}$, $V_{k,QED}$ are quite complicated and too long to be presented here, nevertheless the arising poles are quite compact:

$$V_{k,OK} = -2B_{k,QED} \left(\frac{4}{\epsilon^2} + \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) - \frac{2m^2 - s}{s\beta} \ln(\chi) \right) \frac{2}{\epsilon} \right) + O(\epsilon^0) \quad (31)$$

$$V_{k,QED} = -2B_{k,QED} \left(1 + \frac{2m^2 - s}{s\beta} \ln(\chi) \right) \frac{2}{\epsilon} + O(\epsilon^0) \quad (32)$$

The above results already include the mass renormalization that we have performed *on-shell*, so all ultra-violet poles have been removed. For the renormalization of the strong coupling we use the $\overline{\text{MS}}_m$ scheme defined in [2] and so the full (remaining) renormalization can be achieved by

$$d\sigma_k^{(1),V,ren.} = d\sigma_k^{(1),V} + \frac{\alpha_s(\mu_R^2)}{4\pi} \left[\left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln(\mu_R^2/m^2) - \ln(\mu_D^2/m^2) \right) \beta_0^f + \frac{2}{3} \ln(\mu_R^2/m^2) \right] d\sigma_k^{(0)} \quad (33)$$

$$= d\sigma_k^{(1),V} + 4\pi\alpha_s(\mu_R^2)C_\epsilon \left(\frac{\mu_D^2}{m^2} \right)^{-\epsilon/2} \left[\left(\frac{2}{\epsilon} + \ln(\mu_R^2/m^2) \right) \beta_0^f + \frac{2}{3} \ln(\mu_R^2/m^2) \right] d\sigma_k^{(0)} \quad (34)$$

with μ_R the renormalization scale introduced by the renormalization group equation (RGE), $\beta_0^f = (11C_A - 2n_f)/3$ the first coefficient of the beta function and n_f the number of *total* flavours (i.e. $n_{lf} = n_f - 1$ active (light) flavours and one heavy flavour). The double poles occuring in $V_{k,OK}$ are introduced by the diagrams **FiXme Error: do** when the soft and collinear singularities coincide. FiXme Error!

The results agree in the photo-production limit ($q^2 \rightarrow 0$) with [16] **FiXme Error: Matrix elements available upon request.** FiXme Error!

The procedure is completely analogous to the inclusive case **FiXme Error: cite.** FiXme Error!

3.2 2-to-3 particle phase space

In next-to-leading order we have to consider processes which involve an additional particle in the final state. The matrix elements will then depend on ten kinematical invariants:

$$s = (q + k_1)^2 \quad t_1 = (k_1 - p_2)^2 - m^2 \quad u_1 = (q - p_2)^2 - m^2 \quad (35)$$

$$s_3 = (k_2 + p_2)^2 - m^2 \quad s_4 = (k_2 + p_1)^2 - m^2 \quad s_5 = (p_1 + p_2)^2 = -u_5 \quad (36)$$

$$t' = (k_1 - k_2)^2 \quad (37)$$

$$u' = (q - k_2)^2 \quad u_6 = (k_1 - p_1)^2 - m^2 \quad u_7 = (q - p_1)^2 - m^2 \quad (38)$$

from which only five are independent as can be seen from momentum conservation $k_1 + q = p_1 + p_2 + k_2$ and s, t_1, u_1 match to their leading order definition.

The $2 \rightarrow 3$ n -dimensional phase space is given by

$$dPS_3 = \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_2}{(2\pi)^{n-1}} \frac{d^n k_2}{(2\pi)^{n-1}} (2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2 - k_2) \Theta(p_{1,0}) \delta(p_1^2 - m^2) \Theta(p_{2,0}) \delta(p_2^2 - m^2) \Theta(k_{2,0}) \delta(k_2^2) \quad (39)$$

This can be solved by writing eq. (39) as product of a $2 \rightarrow 2$ decay and a subsequent $1 \rightarrow 2$ decay[12]. We choose the following decomposition[1]:

$$q = (q^0, 0, |\vec{q}|) \quad (40)$$

$$k_1 = k_0(1, 0, \sin \psi, \cos \psi) \quad (41)$$

$$p_1 = \frac{\sqrt{s_5}}{2}(1, \beta_5 \sin \theta_2 \sin \theta_1, \beta_5 \sin \theta_2 \cos \theta_1, \beta_5 \cos \theta_1) \quad (42)$$

$$p_2 = \frac{\sqrt{s_5}}{2}(1, -\beta_5 \sin \theta_2 \sin \theta_1, -\beta_5 \sin \theta_2 \cos \theta_1, -\beta_5 \cos \theta_1) \quad (43)$$

$$k_2 = (k_2^0, 0, k_1 \sin \psi, |\vec{q}| + k_1^0 \cos \psi) \quad (44)$$

where

$$q_0 = \frac{s + u'}{2\sqrt{s_5}}, \quad |\vec{q}| = \frac{1}{2\sqrt{s_5}} \sqrt{(s + u')^2 - 4s_5 q^2}, \quad (45)$$

$$k_1^0 = \frac{s_5 - u'}{2\sqrt{s_5}}, \quad \cos \psi = \frac{2k_1^0 q^0 - s'}{2k_1^0 |\vec{q}|}, \quad \beta_5 = \sqrt{1 - 4m^2/s_5}, \quad (46)$$

$$k_2^0 = \frac{s - s_5}{2\sqrt{s_5}} \quad (47)$$

We further introduce $\rho^* = \frac{4m^2 - q^2}{s - q^2} \leq x = \frac{s_5 - q^2}{s - q^2} \leq 1$ and $-1 \leq y \leq 1$ where y is the cosine of the angle between \vec{q} and \vec{k}_2 in the system with $\vec{q} + \vec{k}_2 = 0$. We then find[1]:

$$dPS_3 = \frac{T_\epsilon}{2\pi} \left(\frac{s'^2}{s} \right)^{1+\epsilon/2} (1-x)^{1+\epsilon} (1-y^2)^{\epsilon/2} dPS_2^{(5)} dy \sin^\epsilon(\theta_1) d\theta_1 d\theta_2 \quad (48)$$

with $0 \leq \theta_1 \leq \pi, 0 \leq \theta_2 \leq \pi, \rho^* \leq x \leq 1, -1 \leq y \leq 1$ and

$$S_\epsilon = (4\pi)^{-2-\epsilon/2} \quad (49)$$

$$T_\epsilon = \frac{\Gamma(1+\epsilon/2)}{\Gamma(1+\epsilon)} S_\epsilon = \frac{1}{16\pi^2} \left(1 + \frac{\epsilon}{2} (\gamma_E - \ln(4\pi)) + O(\epsilon^2) \right) \quad (50)$$

$$dPS_2^{(5)} = \frac{\beta_5 \sin(\theta_1)}{16\pi \Gamma(1+\epsilon/2)} \left(\frac{s_5 \beta_5^2 \sin^2(\theta_1)}{16\pi} \right)^{\epsilon/2} d\theta_1 dx = dPS_2(s \rightarrow s_5) dx \quad (51)$$

3.3 Single Gluon Radiation

In next-to-leading order we have to consider the following process:

$$\gamma^*(q) + g(k_1) \rightarrow Q(p_1) + \bar{Q}(p_2) + g(k_2) \quad (52)$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can

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be written as

$$M_k^{(1),g} = \hat{\mathcal{P}}_k^{\gamma,\mu\mu'} \hat{\mathcal{P}}_k^{g,\nu\nu'} \sum_{j,j'} \mathcal{M}_{j,\mu\nu}^{(1),g} \mathcal{M}_{j',\mu'\nu'}^{(1),g*} \quad (53)$$

$$= 8g^4 \mu_D^{-2\epsilon} e_H^2 N_C C_F (C_A R_{k,OK} + 2C_F R_{k,QED}) \quad (54)$$

The partonic cross section is then given by:

$$d\sigma_{k,g}^{(1)} = \frac{1}{2s'} \frac{K_{\gamma g} E_k(\epsilon)}{2} b_k(\epsilon) M_k^{(1),g} dPS_3 \quad (55)$$

For the gluonic part we shift the occuring soft ($x \rightarrow 1$) and collinear ($y \rightarrow -1$) poles from the matrix elements to the phase space by dividing by $t' \propto (1+y)(1-x)$ and $u' - q^2 s_5/s \propto (1-x)$:

$$dPS'_{3,g} = \frac{dPS_3}{t'(u' - q^2 s_5/s)} = dPS_3 \cdot \left(\frac{2s}{s'^2} \right)^2 \frac{1}{(1-x)^2(1-y)(1+y)} \quad (56)$$

$$= \frac{2T_\epsilon}{\pi} \left(\frac{s'^2}{s} \right)^{-1+\epsilon/2} (1-x)^{-1+\epsilon} (1-y^2)^{-1+\epsilon/2} dPS_2^{(5)} dy \sin^\epsilon(\theta_2) d\theta_2 \quad (57)$$

$$M_k^{(1),g'} = t'(u' - q^2 s_5/s) M_k^{(1),g} \quad (58)$$

$$\Rightarrow d\sigma_{k,g}^{(1)} = \frac{1}{2s'} \frac{K_{\gamma g} E_k(\epsilon)}{2} b_k(\epsilon) M_k^{(1),g'} dPS'_{3,g} \quad (59)$$

The soft and collinear factors $(1-x)^{-1+\epsilon}$ and $(1-y^2)^{-1+\epsilon/2}$ can be replaced by generalized plus distributions[1]

$$(1-x)^{-1+\epsilon} \sim \left(\frac{1}{1-x} \right)_{\tilde{\rho}} + \epsilon \left(\frac{\ln(1-x)}{1-x} \right)_{\tilde{\rho}} + \delta(1-x) \left(\frac{1}{\epsilon} + 2\ln \tilde{\beta} + 2\epsilon \ln^2(\tilde{\beta}) \right) + O(\epsilon^2) \quad (60)$$

$$(1-y^2)^{-1+\epsilon} \sim \frac{1}{2} \left(\left(\frac{1}{1+y} \right)_\omega + \left(\frac{1}{1-y} \right)_\omega \right) + (\delta(1+y) + \delta(1-y)) \left(\frac{1}{2\epsilon} + \frac{1}{2} \ln(2\omega) \right) + O(\epsilon) \quad (61)$$

$$(1+y)^{-1+\epsilon} \sim \left(\frac{1}{1+y} \right)_\omega + \delta(1+y) \left(\frac{1}{\epsilon} + \ln \omega \right) + O(\epsilon) \quad (62)$$

inside integration over smooth functions with $\tilde{\beta} = \sqrt{1-\tilde{\rho}}$. The distributions are defined

by

$$\int_{\tilde{\rho}}^1 dx f(x) \left(\frac{1}{1-x} \right)_{\tilde{\rho}} = \int_{\tilde{\rho}}^1 dx \frac{f(x) - f(1)}{1-x} \quad (63)$$

$$\int_{\tilde{\rho}}^1 dx f(x) \left(\frac{\ln(1-x)}{1-x} \right)_{\tilde{\rho}} = \int_{\tilde{\rho}}^1 dx \frac{f(x) - f(1)}{1-x} \ln(1-x) \quad (64)$$

$$\int_{-1}^{-1+\omega} dy f(y) \left(\frac{1}{1+y} \right)_{\omega} = \int_{-1}^{-1+\omega} dy \frac{f(y) - f(-1)}{1+y} \quad (65)$$

$$\int_{1-\omega}^1 dy f(y) \left(\frac{1}{1-y} \right)_{\omega} = \int_{1-\omega}^1 dy \frac{f(y) - f(1)}{1-y} \quad (66)$$

with $\rho^* \leq \tilde{\rho} < 1$ and $0 < \omega \leq 2$. If the integration does not include a singularity the distribution sign can be dropped. From an analytical point of view the results may not depend on the specific choice of the regularisation parameters $\tilde{\rho}$ and ω but for any numerical purpose they may influence the rate of convergence or stability. For numerical computations we must also cut the poles out of the integrations

$$\int_{\rho^*}^1 dx \rightarrow \int_{\rho^*}^{1-\delta_x} dx \quad \int_{-1}^1 dy \rightarrow \int_{-1+\delta_y}^1 dy \quad (67)$$

If not stated otherwise we use as a default setup:

$$\tilde{\rho} = \rho^* + \tilde{x}(1 - \rho^*) \text{ with } \tilde{x} = 0.8 \quad \omega = 1.0 \quad (68)$$

$$\delta_x = 1 \times 10^{-6} \quad \delta_y = 7 \times 10^{-6} \quad (69)$$

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With the given distribution we can split the gluonic NLO part into three pieces[1]:

$$d\sigma_{k,g}^{(1)} = d\sigma_{k,g}^{(1),s} + d\sigma_{k,g}^{(1),c-} + d\sigma_{k,g}^{(1),f} \quad (70)$$

corresponding to the soft ($d\sigma_{k,g}^{(1),s} \sim \delta(1-x)$), the collinear ($d\sigma_{k,g}^{(1),c-} \sim \delta(1+y)$) and the finite parts ($d\sigma_{k,g}^{(1),f} \sim \left(\frac{1}{1-x} \right)_{\tilde{\rho}} \left(\frac{1}{1+y} \right)_{\omega}$). Note that for $q^2 < 0$ there is only a single collinear contribution ($y \rightarrow -1$).

The soft matrix elements can be obtained from the above expressions by taking the soft limit $k_2 \rightarrow 0$:

$$\lim_{k_2 \rightarrow 0} (C_A R_{k,OK} + 2C_F R_{k,QED}) = (C_A S_{k,OK} + 2C_F S_{k,QED}) + O(1/s_4, 1/s_3, 1/t') \quad (71)$$

with

$$S_{k,OK} = 2 \left(\frac{t_1}{t' s_3} + \frac{u_1}{t' s_4} - \frac{s - 2m^2}{s_3 s_4} \right) B_{k,QED} \quad (72)$$

$$S_{k,QED} = 2 \left(\frac{s - 2m^2}{s_3 s_4} - \frac{m^2}{s_3^2} - \frac{m^2}{s_4^2} \right) B_{k,QED} \quad (73)$$

Note that the einkonal factors multiplying the Born functions $B_{k,QED}$ neither depend on q^2 nor on the projection k . But the integrated expressions do depend on the regularization scheme, i.e. will here depend on $\tilde{\rho}$ rather than on a phasespace slicing parameter Δ as for inclusive calculations[4] **FiXme Error: add my cite**. We find for the integrated expressions FiXme Error!

$$d\sigma_{k,g}^{(1),s} = \frac{1}{2s'} \frac{1}{2} E_k(\epsilon) b_k(\epsilon) M_k^{(1),S} dPS_2 \quad (74)$$

$$M_k^{(1),S} = 8g^4 \mu_D^{-\epsilon} e^2 e_H^2 N_C C_F C_\epsilon \left(C_A \tilde{S}_{OK} + 2C_F \tilde{S}_{QED} \right) B_{k,QED} \quad (75)$$

where the full expressions for \tilde{S}_k can be found in [1] and the poles are given by

$$\tilde{S}_{OK} = 2 \left(\frac{4}{\epsilon^2} + \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) - \frac{2m^2 - s}{s\beta} \ln(\chi) + 4 \ln(\tilde{\beta}) \right) \frac{2}{\epsilon} \right) + O(\epsilon^0) \quad (76)$$

$$\tilde{S}_{QED} = -2 \cdot \left(1 - \frac{2m^2 - s}{s\beta} \ln(\chi) \right) \frac{2}{\epsilon} + O(\epsilon^0) \quad (77)$$

The results agree in the photo-production limit ($q^2 \rightarrow 0$) with [16].

3.4 Light Quark Processes

In next-to-leading order a new production mechanism enters that is induced by a light quark, so we have to consider the process

$$\gamma^*(q) + q(k_1) \rightarrow Q(p_1) + \bar{Q}(p_2) + q(k_2) \quad (78)$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as FiXme Error!

$$\hat{\mathcal{P}}_k^{\gamma, \mu \mu'} \hat{\mathcal{P}}_k^q \sum_{j, j'} \mathcal{M}_{j, \mu}^{(1), q} \mathcal{M}_{j', \mu'}^{(1), q*} = 8g^4 \mu_D^{-2\epsilon} e^2 N_C C_F \left(e_H^2 A_{k,1} + e_L^2 A_{k,2} + e_L e_H A_{k,3} \right) \quad (79)$$

where e_L denotes the charge of the light quark q in units of e .

The results agree in the photo-production limit ($q^2 \rightarrow 0$) with [16].

4 Mass Factorization

All collinear poles in the gluonic subprocess can be removed by mass factorization in the following way: **FiXme Error: todo**

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The light quark process can be regularized in a complete analogous way: **FiXme Error: todo**

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The final finite cross sections are then

$$\begin{aligned}
 d\sigma_{k,g}^{(1),c-} = & 16 \cdot \frac{1}{2xs'} \frac{K_{\gamma g}}{2} b_k 0 \cdot \alpha_S^2 e^2 e_H^2 N_C C_F B_{k,QED}(xk_1) dPS_2^{(5)} \\
 & \cdot \left((1-x) P_{k,gg}^{H,(0)}(x) \left(\frac{1}{(1-x)_{\tilde{\rho}}} \left(\ln(s'/\mu_F^2) + \ln(s'/s) + \ln(\omega/2) \right) \right. \right. \\
 & \left. \left. + 2 \left(\frac{\ln(1-x)}{1-x} \right)_{\tilde{\rho}} \right) + 2 P_{k,gg}^{H,(1)}(x) \right) \quad (80)
 \end{aligned}$$

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5 Partonic Results

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6 Hadronic Results

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7 Summary

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List of Corrections

Error: more	1
Error: why do we do this	1
Error: avoid all order expr?	1
Error: remove?	1
Error: move to LO?	2
Error: extend	2
Error: justify avoidance of Δ ?	2
Error: how to insert 2nd projector?	2
Error: explain ghosts?	3
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Error: more?	5
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Error: do	10
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