

1 Passarino-Veltman decomposition

1.1 Definitions

[1]:

$$A(m) = \frac{1}{i\pi^2} \int d^n q \frac{1}{q^2 + m^2} \quad (1)$$

$$B_0(p, m_1, m_2) = \frac{1}{i\pi^2} \int d^n q \frac{1}{(q^2 + m_1^2)((q + p)^2 + m_2^2)} \quad (2)$$

and apart from their pole term (called Δ - see [1, eq. D.1]), they keep $n = 4$.

[2, 3]:

$$A(m) = \mu^{-\epsilon} \int \frac{d^n q}{(2\pi)^n} \frac{1}{q^2 - m^2} \quad (3)$$

$$B(q_1, m_1, m_2) = \mu^{-\epsilon} \int \frac{d^n q}{(2\pi)^n} \frac{1}{(q^2 - m_1^2)((q + q_1)^2 - m_2^2)} \quad (4)$$

and $n = 4 + \epsilon$. ([2] writes “The notations for the one-, two-, three-, and four-point functions have been taken over from Ref. [1].” - obviously they do not.)

HEPMath[4] and FeynCalc[5, 6] refer to LoopTools[7, 8]. [8, eq. (1.1)] and [9, eq. (2.6)], QCDLoop[10]:

$$T_{\mu_1 \dots \mu_P}^N = \frac{\mu^{4-D}}{i\pi^{D/2} r_\Gamma} \int d^D q \frac{q_{\mu_1} \dots q_{\mu_P}}{[q^2 - m_1^2] [(q + k_1)^2 - m_2^2] \dots [(q + k_{N-1})^2 - m_N^2]} \quad (5)$$

$$r_\Gamma = \frac{\Gamma^2(1 - \epsilon)\Gamma(1 + \epsilon)}{\Gamma(1 - 2\epsilon)}, \quad D = 4 - 2\epsilon$$

later in the code they use a different signature (to avoid any vector structure):

$$\begin{aligned} A(m^2), B_0(p^2, m_1^2, m_2^2), C_0(p_1^2, p_2^2, (p_1 + p_2)^2, m_1^2, m_2^2, m_3^2) \\ D_0(p_1^2, p_2^2, p_3^2, p_4^2, (p_1 + p_2)^2, (p_2 + p_3)^2, m_1^2, m_2^2, m_3^2, m_4^2) \end{aligned} \quad (6)$$

[11]:

$$T_{\mu_1 \dots \mu_P}^N(p_1, \dots, p_{N-1}, m_0, \dots, m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \frac{q_{\mu_1} \dots q_{\mu_P}}{L_0 L_1 \dots L_{N-1}} \quad (7)$$

$$L_0 = q^2 - m_0^2 + i\epsilon \quad (8)$$

$$L_i = (q + p_i)^2 - m_i^2 + i\epsilon \quad i = 1, \dots, N-1 \quad (9)$$

I will stick to the integrals of [3] as it is the most natural form, I think, and to the non-vector signature, if possible.

The transformation of the *analytic* results from the notation in [10] is given by

$$[10]: \frac{\mu^{4-n}}{i\pi^{n/2}r_\Gamma} \mathcal{I} \leftrightarrow [3]: \frac{\mu^{4-n}}{(2\pi)^n} \mathcal{I} \quad (10)$$

with \mathcal{I} denoting the *raw* integral. We then need to solve (B=Bojak[3],E=Ellis[10]):

$$\begin{aligned} &\Rightarrow iC_\epsilon \left(\frac{a_2^B}{(n-4)^2} + \frac{a_1^B}{n-4} + a_0^B + O(n-4) \right) \\ &\stackrel{!}{=} \frac{\mu^{4-n}}{(2\pi)^n} \frac{i\pi^{n/2}r_\Gamma}{\mu^{4-n}} \left(\frac{a_2^E}{(n-4)^2} + \frac{a_1^E}{n-4} + a_0^E + O(n-4) \right) \end{aligned} \quad (11)$$

$$\Rightarrow a_2^B = a_2^E \quad (12)$$

$$a_1^B = a_1^E - \frac{1}{2}a_2^E \ln(m^2/\mu^2) \quad (13)$$

$$a_0^B = a_0^E - \frac{a_2^E}{8}\zeta(2) + \frac{a_2^E}{8}\ln^2(m^2/\mu^2) - \frac{a_1^E}{2}\ln(m^2/\mu^2) \quad (14)$$

To compare *numeric* results from **LoopTools** or **QCDLoop** one need to solve

$$\begin{aligned} &\Rightarrow \left(\frac{b_2^B}{(n-4)^2} + \frac{b_1^B}{n-4} + b_0^B + O(n-4) \right) \\ &\stackrel{!}{=} \frac{\mu^{4-n}}{(2\pi)^n} \frac{i\pi^{n/2}r_\Gamma}{\mu^{4-n}} \left(\frac{b_2^E}{(n-4)^2} + \frac{b_1^E}{n-4} + b_0^E + O(n-4) \right) \end{aligned} \quad (15)$$

$$\Rightarrow b_2^B = \frac{i}{16\pi^2} b_2^E \quad (16)$$

$$b_1^B = \frac{i}{16\pi^2} \left(b_1^E + \frac{b_2^E}{2}(\gamma_E - \ln(4\pi)) \right) \quad (17)$$

$$b_0^B = \frac{i}{16\pi^2} \left(b_0^E + \frac{b_1^E}{2}(\gamma_E - \ln(4\pi)) + \frac{b_2^E}{8} \left((\gamma_E - \ln(4\pi))^2 - \zeta(2) \right) \right) \quad (18)$$

$$(19)$$

1.2 Decomposition Labeling

[1, 3]:

$$B_\mu(p, m_1, m_2) = p_\mu B_1(p, m_1, m_2) \quad (20)$$

$$B_{\mu\nu} = p_\mu p_\nu B_{21} + g_{\mu\nu} B_{22} \quad (21)$$

$$C_\mu(p_1, p_2, m_1, m_2, m_3) = p_{1,\mu} C_{11} + p_{2,\mu} C_{12} \quad (22)$$

$$C_{\mu\nu} = p_{1,\mu} p_{1,\nu} C_{21} + p_{2,\mu} p_{2,\nu} C_{22} + (p_{1,\mu} p_{2,\nu} + p_{1,\nu} p_{2,\mu}) C_{23} + g_{\mu\nu} C_{24} \quad (23)$$

The arguments of the functions are always inherited.

`HEPMath`, `FeynCalc`, `LoopTools`, [9]:

$$B_\mu(p, m_1, m_2) = p_\mu B_1(p, m_1, m_2) \quad (24)$$

$$B_{\mu\nu} = g_{\mu\nu} B_{00} + p_\mu p_\nu B_{11} \quad (25)$$

$$C_\mu(p_1, p_2, m_1, m_2, m_3) = p_{1,\mu} C_1 + p_{2,\mu} C_2 = \sum_{j=1}^2 p_{j,\mu} C_j \quad (26)$$

$$C_{\mu\nu} = p_{1,\mu} p_{1,\nu} C_{11} + p_{2,\mu} p_{2,\nu} C_{22} + (p_{1,\mu} p_{2,\nu} + p_{1,\nu} p_{2,\mu}) C_{12} + g_{\mu\nu} C_{00} \quad (27)$$

$$= g_{\mu\nu} C_{00} + \sum_{j,k=1}^2 p_{j,\mu} p_{k,\nu} C_{jk} \quad (28)$$

The arguments of the functions are always inherited.

I will stick to `HEPMath` as it is the more generic and extensible form, I think.

1.3 B Decomposition

define

$$f_1 = m_1^2 - m_0^2 - p^2 \quad (29)$$

then one finds easily

$$B_1(p^2, m_0^2, m_1^2) = \frac{1}{2p^2} \left(f_1 B_0(p^2, m_0^2, m_1^2) + A_0(m_0^2) - A_0(m_1^2) \right) \quad (30)$$

$$B_{00}(p^2, m_0^2, m_1^2) = \frac{1}{2(n-1)} \left(2m_0^2 B_0(p^2, m_0^2, m_1^2) + A_0(m_1^2) - f_1 B_1(p^2, m_0^2, m_1^2) \right) \quad (31)$$

$$B_{11}(p^2, m_0^2, m_1^2) = \frac{1}{2p^2} \left(f_1 B_0(p^2, m_0^2, m_1^2) + A_0(m_1^2) - 2B_{00}(p^2, m_0^2, m_1^2) \right) \quad (32)$$

in accordance with [3, 9].

Concering B_1 [1] and `LoopTools` use the following identity

$$A_0(m_0^2) - A_0(m_1^2) = (m_0^2 - m_1^2) B_0(0, m_0^2, m_1^2) \quad (33)$$

that might help away with

In case m_1 and/or m_2 are very large the expression on the right-hand side of eq. (30) suffers very strong cancellations: the total is very much smaller than the individual terms. For this reason we have not used these algebraic relations, except to rewrite self-energy diagrams as much as possible in a form most suitable for numerical evaluation. ([1, below eq. D.6])

To compare the other results to [1] and **LoopTools** one has to use the *strict* $n \rightarrow 4$ limit and the following identities[12]:

$$(n-4)B_{00}(p^2, m_0^2, m_1^2) = \frac{1}{6}(p^2 - 3m_0^2 - 3m_1^2) \quad (34)$$

$$(n-4)B_{11}(p^2, m_0^2, m_1^2) = -\frac{2}{3} \quad (35)$$

2 Scalar Integrals

We focus on:

$$\gamma^*(q) + g(k_1) \rightarrow Q(p_1) + \bar{Q}(p_2) \quad (36)$$

$$k_1^2 = 0 \quad p_1^2 = p_2^2 = m^2 \quad (p_1 + p_2)^2 = s \quad (p_2 - q)^2 = t \quad (p_1 - q)^2 = u \quad (37)$$

define some shortcuts

$$0 \leq \rho = \frac{4m^2}{s} \leq 1 \quad 0 \leq \beta = \sqrt{1 - \rho} \leq 1 \quad 0 \leq \chi = \frac{1 - \beta}{1 + \beta} \leq 1 \quad (38)$$

$$\rho_q = \frac{4m^2}{q^2} \leq 0 \quad 1 \leq \beta_q = \sqrt{1 - \rho_q} \quad 0 \leq \chi_q = -\frac{1 - \beta_q}{1 + \beta_q} \leq 1 \quad (39)$$

2.1 One-Point Function A_0

[11]:

$$A_0(m^2) = -\frac{i}{16\pi^2} m^2 \left(\frac{m^2}{4\pi\mu^2} \right)^{(n-4)/2} \Gamma(1 - n/2) \quad (40)$$

$$= \frac{im^2}{16\pi^2} \left(\Delta - \log(m^2/\mu^2) + 1 \right) + O(n-4) \quad (41)$$

$$= iC_\epsilon m^2 \left(-\frac{2}{\epsilon} + 1 \right) + O(n-4) \quad (42)$$

$$\Delta = \frac{2}{4-n} - \gamma_E + \log(4\pi) \quad (43)$$

$$C_\epsilon = \frac{1}{16\pi^2} \exp \left(\left(\gamma_E - \log(4\pi) + \log(m^2/\mu^2) \right) \frac{\epsilon}{2} \right) \quad (44)$$

this is *up to order* $O(n-4)$ in accordance with [3][13], but NOT beyond - see also [3, eq. (A.12)]. So we can treat C_ϵ and Δ as equal. We also find with [3]

$$A_0(0) = 0 \quad (45)$$

2.2 Two-Point Function B_0

In [11, eq. (4.23)] is a generic function given and we end up with

$$B_0(s, m^2, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 + \beta \log(\chi) \right) \quad (46)$$

$$B_0(q^2, m^2, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 + \beta_q \log(\chi_q) \right) \quad (47)$$

$$B_0(0, m^2, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} \right) \quad (48)$$

$$B_0(m^2, 0, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 \right) \quad (49)$$

$$B_0(t, 0, m^2) = iC_\epsilon \left(-\frac{2}{\epsilon} + 2 - \frac{t - m^2}{t} \ln \left(-\frac{t - m^2}{m^2} \right) \right) \quad (50)$$

focussing on imaginary part *only*; this in accordance with [3][13]. We also find with [3]

$$B_0(0, 0, 0) = 0 \quad (51)$$

2.3 Three-Point Function C_0

Again, in [14, eq. (4.26)] is a generic function given.

First, we compute $C_0(s, q^2, 0, m^2, m^2, m^2)$ and by taking the limit $k_1^2 \rightarrow 0$ (or equivalently $s_4 \rightarrow 0$) we end up with:

$$C_0(s, q^2, 0, m^2, m^2, m^2) = \frac{i}{16\pi^2} \cdot \frac{1}{s - q^2} \left(\text{Li}_2 \left(\frac{2}{1 + \beta_q} \right) + \text{Li}_2 \left(\frac{2}{1 - \beta_q} \right) - \text{Li}_2 \left(\frac{2}{1 + \beta} \right) - \text{Li}_2 \left(\frac{2}{1 - \beta} \right) \right) \quad (52)$$

with [15] we find:

$$\text{Li}_2 \left(\frac{2}{1 + b} \right) + \text{Li}_2 \left(\frac{2}{1 - b} \right) = 3\zeta(2) + \frac{1}{2} \ln^2 \left(\frac{1 - b}{1 + b} \right) - \ln \left(\frac{1 - b}{1 + b} \right) \ln \left(-\frac{1 - b}{1 + b} \right) \quad (53)$$

and if we focus on real part *only*, we find:

$$\text{Li}_2 \left(\frac{2}{1 + \beta} \right) + \text{Li}_2 \left(\frac{2}{1 - \beta} \right) = 3\zeta(2) - \frac{1}{2} \ln^2(\chi) \quad (54)$$

$$\text{Li}_2 \left(\frac{2}{1 + \beta_q} \right) + \text{Li}_2 \left(\frac{2}{1 - \beta_q} \right) = -\frac{1}{2} \ln^2(\chi_q) \quad (55)$$

Additionally, we find

$$\lim_{q^2 \rightarrow 0} \left[\text{Li}_2 \left(\frac{2}{1 + \beta_q} \right) + \text{Li}_2 \left(\frac{2}{1 - \beta_q} \right) \right] = 0 \quad (56)$$

So we get:

$$C_0(s, q^2, 0, m^2, m^2, m^2) = iC_\epsilon \frac{1}{s - q^2} \left(\frac{1}{2} \ln^2(\chi) - \frac{1}{2} \ln^2(\chi_q) - 3\zeta(2) \right) \quad (57)$$

$$C_0(s, 0, 0, m^2, m^2, m^2) = iC_\epsilon \frac{1}{s} \left(\frac{1}{2} \ln^2(\chi) - 3\zeta(2) \right) \quad (58)$$

in accordance with [3][13][16]. These results can also be obtained by the methods described in [3, chap. 3].

Next, we compute $C_0(m^2, 0, t, 0, m^2, m^2)$ again by taking the limit $k_1^2 \rightarrow 0$ we end up with:

$$C_0(m^2, 0, t, 0, m^2, m^2) = \frac{i}{16\pi^2} \cdot \frac{1}{t - m^2} \left(2\text{Li}_2(2) + \text{Li}_2(m^2/t) - \frac{\pi^2}{6} - \text{Li}_2((t + m^2)/m^2) - \text{Li}_2((m^2 + t)/t) \right) \quad (59)$$

Using [15] and focussing on real part, we find

$$\text{Li}_2(2) = \frac{\pi^2}{4} - i\pi \ln(2) \quad (60)$$

$$2\text{Li}_2(2) + \text{Li}_2(1/z) - \frac{\pi^2}{6} - \text{Li}_2(1 + z) - \text{Li}_2(1 + 1/z) = \frac{\pi^2}{6} - \text{Li}_2(z) \quad (61)$$

So we get:

$$C_0(m^2, 0, t, 0, m^2, m^2) = iC_\epsilon \frac{1}{t - m^2} \left(\zeta(2) - \text{Li}_2(t/m^2) \right) \quad (62)$$

in accordance with [3][13].

To compute $C_0(m^2, s, m^2, 0, m^2, m^2)$ we use [3] and find

$$C_0(m^2, s, m^2, 0, m^2, m^2) = \frac{iC_\epsilon}{s\beta} \left(-\frac{2}{\epsilon} \ln(\chi) - \frac{\pi^2}{2} + \frac{1}{2} \ln^2(\chi) - \ln(\chi) \ln(1 - \chi) - \text{Li}_2(1/(1 - \chi)) + \text{Li}_2(\chi/(\chi - 1)) \right) \quad (63)$$

Using [15] and focussing on real part, we find

$$-\text{Li}_2(1/(1 - \chi)) + \text{Li}_2(\chi/(\chi - 1)) = -2\text{Li}_2(\chi) - \ln(\chi) \ln(1 - \chi) - \frac{\pi^2}{6} \quad (64)$$

So we get:

$$C_0(m^2, s, m^2, 0, m^2, m^2) = iC_\epsilon \frac{1}{s\beta} \left(-\frac{2}{\epsilon} \ln(\chi) - 2\ln(\chi) \ln(1 - \chi) - 2\text{Li}_2(\chi) + \frac{1}{2} \ln^2(\chi) - 4\zeta(2) \right) \quad (65)$$

in accordance with [3][13].

To compute $C_0(t, m^2, q^2, 0, m^2, m^2)$ we use [14] and find immediatly:

$$\begin{aligned}
C_0(t, m^2, q^2, 0, m^2, m^2) = \frac{iC_\epsilon}{\alpha} & \left[-\zeta(2) + 2 \operatorname{Li}_2 \left(\frac{t_1 + \alpha}{t_1} \right) + \operatorname{Li}_2 \left(\frac{q^2 - t - m^2 + \alpha}{q^2 - t - m^2 - \alpha} \right) \right. \\
& \operatorname{Li}_2 \left(\frac{t_1 - q^2 \beta_q^2 + \alpha}{t_1 - q^2 \beta_q^2 - \beta_q \alpha} \right) - \operatorname{Li}_2 \left(\frac{t_1 - q^2 \beta_q^2 - \alpha}{t_1 - q^2 \beta_q^2 + \beta_q \alpha} \right) \\
& \operatorname{Li}_2 \left(\frac{t_1 - q^2 \beta_q^2 + \alpha}{t_1 + q^2 \beta_q^2 - \beta_q \alpha} \right) - \operatorname{Li}_2 \left(\frac{t_1 - q^2 \beta_q^2 - \alpha}{t_1 + q^2 \beta_q^2 + \beta_q \alpha} \right) \\
& - \operatorname{Li}_2 \left(\frac{t_1(q^2 - t - m^2 - \alpha) - 2m^2 \alpha}{t_1(q^2 - t - m^2 + \alpha)} \right) \\
& \left. - \operatorname{Li}_2 \left(\frac{t_1(q^2 - t - m^2 - \alpha) - 2m^2 \alpha}{t_1(q^2 - t - m^2 - \alpha)} \right) \right] \quad (66)
\end{aligned}$$

with $\alpha = \kappa(t, q^2, m^2)$ and the Källén function (as defined in [14, eq. (4.27)])

$$\kappa(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 2(xy + xz + yx)} \quad (67)$$

This is in accordance with [16, eq. (A.8)](Note the typo there!).

Additionally, we find

$$\lim_{q^2 \rightarrow 0} C_0(t, m^2, q^2, 0, m^2, m^2) = C_0(t, m^2, 0, 0, m^2, m^2) = C_0(m^2, 0, t, 0, m^2, m^2) \quad (68)$$

To compute $C_0(0, m^2, t, 0, 0, m^2)$ we use [3] and find

$$C_0(0, m^2, t, 0, 0, m^2) = \frac{iC_\epsilon}{t_1} \left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln(-t_1/m^2) + \ln^2(-t_1/m^2) + \operatorname{Li}_2(t/m^2) + \frac{\zeta(2)}{4} \right) \quad (69)$$

2.4 Four-Point Function D_0

To compute $D_0(m^2, 0, q^2, m^2, t, s, 0, m^2, m^2, m^2)$ we follow Ingos way[3] of computing his $D_0(p_1, -k_1, -k_2, 0, m, m, m) = D_0(m^2, 0, 0, m^2, t, s, 0, m^2, m^2, m^2)$ and find

$$\tilde{t} = -\frac{t_1}{m^2} \quad (70)$$

$$K = \frac{x}{\rho \rho_q} [4x(-1+y)yz\rho + yz\rho \rho_q \tilde{t} + x(-4(-1+y)y(-1+z) + \rho - \tilde{t}yz\rho)\rho_q] \quad (71)$$

$$I_{xy} = \frac{2x^{\epsilon/2} \rho \rho_q^{2-\epsilon/2} [\tilde{t}y\rho_q + x(\rho_q + y(4(y-1) - \tilde{t}\rho_q))]}{(-2+\epsilon) [4x(-1+y)\rho + \tilde{t}\rho \rho_q - x(4(y-1) + \tilde{t}\rho)\rho_q]}^{-1+\epsilon/2} \quad (72)$$

$$II_{xy} = -\frac{2x^{-1+\epsilon} \rho^{2-\epsilon/2} \rho_q [4(-1+y)y + \rho]}{(-2+\epsilon) [4x(-1+y)\rho + \tilde{t}\rho \rho_q - x(4(y-1) + \tilde{t}\rho)\rho_q]}^{-1+\epsilon/2} \quad (73)$$

“The integration of I_{xy} does not diverge and one easily gets upon setting $\epsilon \rightarrow 0$ ”

$$I = \frac{m^4}{st_1\beta} \left[\ln^2(\chi) + 4 \text{Li}_2(-\chi) + \frac{\pi^2}{3} + 2 \ln(\chi_q) \ln \left(\frac{\beta_q + \beta}{\beta_q - \beta} \right) - 2 \ln(\chi) \ln(1 - q^2/s) \right. \\ \left. + 2 \text{Li}_2 \left(\frac{\beta_q - 1}{\beta_q - \beta} \right) - 2 \text{Li}_2 \left(\frac{\beta_q + 1}{\beta_q - \beta} \right) + 2 \text{Li}_2 \left(\frac{\beta_q + 1}{\beta_q + \beta} \right) - 2 \text{Li}_2 \left(\frac{\beta_q - 1}{\beta_q + \beta} \right) \right] \quad (74)$$

$$= \frac{m^4}{st_1\beta} \left[\ln^2(\chi) + 4 \text{Li}_2(-\chi) + \frac{\pi^2}{3} + \ln \left(\frac{\beta_q^2 - \beta^2}{(\beta_q - 1)^2} \right) \ln \left(\frac{\beta_q - \beta}{\beta_q + \beta} \right) - 2 \ln(\chi) \ln(1 - q^2/s) \right. \\ \left. + 2 \text{Li}_2 \left(\frac{\beta_q - 1}{\beta_q - \beta} \right) + 2 \text{Li}_2 \left(\frac{\beta_q - \beta}{\beta_q + 1} \right) - 2 \text{Li}_2 \left(\frac{\beta_q + \beta}{\beta_q + 1} \right) - 2 \text{Li}_2 \left(\frac{\beta_q - 1}{\beta_q + \beta} \right) \right] \quad (75)$$

with

$$\lim_{q^2 \rightarrow 0} \left[\ln \left(\frac{\beta_q^2 - \beta^2}{(\beta_q - 1)^2} \right) \ln \left(\frac{\beta_q - \beta}{\beta_q + \beta} \right) - 2 \ln(\chi) \ln(1 - q^2/s) \right. \\ \left. + 2 \text{Li}_2 \left(\frac{\beta_q - 1}{\beta_q - \beta} \right) + 2 \text{Li}_2 \left(\frac{\beta_q - \beta}{\beta_q + 1} \right) - 2 \text{Li}_2 \left(\frac{\beta_q + \beta}{\beta_q + 1} \right) - 2 \text{Li}_2 \left(\frac{\beta_q - 1}{\beta_q + \beta} \right) \right] = 0 \quad (76)$$

“Integrating II_{xy} over x gives”

$$II_y = -\frac{2}{\tilde{t}(-2 + \epsilon)\epsilon} \left(\frac{\rho - 4y(1 - y)}{\rho} \right)^{-1 + \epsilon/2} {}_2F_1 \left(1, \epsilon; 1 + \epsilon; 1 - \frac{4(1 - y)(\rho_q - \rho)}{\tilde{t}\rho\rho_q} \right) \quad (77)$$

“The integration over y does not give an additional pole, so we can expand to $O(1)$ using ([3, eq. B.5]) and then integrate to obtain”

$$II = -\frac{m^4}{\beta st_1} \left(\frac{2 \ln(\chi)}{\epsilon} + \ln(\chi) \left(1 + 2 \ln(\beta \tilde{t}) + \ln(\chi) - 2 \ln(1 - q^2/s) \right) + \text{Li}_2(\chi^2) + \frac{5\pi^2}{6} \right) \quad (78)$$

with

$$\lim_{q^2 \rightarrow 0} \ln(1 - q^2/s) = 0 \quad (79)$$

“The final result is then”

$$D_0(m^2, 0, q^2, m^2, t, s, 0, m^2, m^2, m^2) \\ = \frac{iC_\epsilon}{\beta st_1} \left[-\frac{2 \ln(\chi)}{\epsilon} - 2 \ln(\chi) \ln(\beta \tilde{t}) + 2 \text{Li}_2(-\chi) - 2 \text{Li}_2(\chi) - 3\zeta(2) \right. \\ \left. + \ln \left(\frac{\beta_q^2 - \beta^2}{(\beta_q - 1)^2} \right) \ln \left(\frac{\beta_q - \beta}{\beta_q + \beta} \right) - 2 \ln(\chi) \ln(1 - q^2/s) \right. \\ \left. + 2 \text{Li}_2 \left(\frac{\beta_q - 1}{\beta_q - \beta} \right) + 2 \text{Li}_2 \left(\frac{\beta_q - \beta}{\beta_q + 1} \right) - 2 \text{Li}_2 \left(\frac{\beta_q + \beta}{\beta_q + 1} \right) - 2 \text{Li}_2 \left(\frac{\beta_q - 1}{\beta_q + \beta} \right) \right] \quad (80)$$

This is NOT in accordance with [16, eq. (A.3)] - but I suspect a bunch of typos there.

We get the match to [3] and [13] by using

$$\lim_{q^2 \rightarrow 0} \left[\ln \left(\frac{\beta_q^2 - \beta^2}{(\beta_q - 1)^2} \right) \ln \left(\frac{\beta_q - \beta}{\beta_q + \beta} \right) - 2 \ln(\chi) \ln(1 - q^2/s) \right. \\ \left. + 2 \operatorname{Li}_2 \left(\frac{\beta_q - 1}{\beta_q - \beta} \right) + 2 \operatorname{Li}_2 \left(\frac{\beta_q - \beta}{\beta_q + 1} \right) - 2 \operatorname{Li}_2 \left(\frac{\beta_q + \beta}{\beta_q + 1} \right) - 2 \operatorname{Li}_2 \left(\frac{\beta_q - 1}{\beta_q + \beta} \right) \right] = 0 \quad (81)$$

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Error: fix

Nevertheless this is probably wrong - because we find with [10, Box 16]:

$$D_0(m^2, 0, q^2, m^2, t, s, 0, m^2, m^2, m^2) \\ = \frac{iC_\epsilon}{\beta s t_1} \left[-\frac{2 \ln(\chi)}{\epsilon} - 2 \ln(\chi) \ln(\tilde{t}) + \operatorname{Li}_2(1 - \chi^2) - 4\zeta(2) + \ln^2(\chi_q) + 2 \operatorname{Li}_2(-\chi\chi_q) \right. \\ \left. + 2 \operatorname{Li}_2(-\chi/\chi_q) + 2 \ln(\chi\chi_q) \ln(1 + \chi\chi_q) + 2 \ln(\chi/\chi_q) \ln(1 + \chi/\chi_q) \right] \quad (82)$$

We get the match to [3] and [13] by using

$$\lim_{q^2 \rightarrow 0} \left[\operatorname{Li}_2(1 - \chi^2) - 4\zeta(2) + \ln^2(\chi_q) + 2 \operatorname{Li}_2(-\chi\chi_q) + 2 \operatorname{Li}_2(-\chi/\chi_q) \right. \\ \left. + 2 \ln(\chi\chi_q) \ln(1 + \chi\chi_q) + 2 \ln(\chi/\chi_q) \ln(1 + \chi/\chi_q) \right] \\ = -2 \ln(\chi) \ln(\beta) - 3\zeta(2) + 2 \operatorname{Li}_2(2, -\chi) - 2 \operatorname{Li}_2(2, \chi) \quad (83)$$

To compute $D_0(0, m^2, q^2, m^2, t, u, 0, 0, m^2, m^2)$ I neither succeeded with [3] nor [14], but one can use [10, Box 11] (transformation see sec. 1.1) So we get

$$D_0(0, m^2, q^2, m^2, t, u, 0, 0, m^2, m^2) = \frac{iC_\epsilon}{t_1 u_1} \left(\frac{4}{\epsilon^2} + \frac{2}{\epsilon} \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) \right) \right. \\ \left. + 2 \ln(-t_1/m^2) \ln(-u_1/m^2) - \frac{7}{2} \zeta(2) - \ln^2(\chi_q) \right) \quad (84)$$

This is in accordance with [16, eq. (A.4)] using [15] (as above):

$$2 \operatorname{Li}_2 \left(\frac{q^2(1 + \beta_q)}{2m^2} \right) + 2 \operatorname{Li}_2 \left(\frac{q^2(1 - \beta_q)}{2m^2} \right) = 2 \operatorname{Li}_2 \left(\frac{2}{1 - \beta_q} \right) + 2 \operatorname{Li}_2 \left(\frac{2}{1 + \beta_q} \right) \quad (85)$$

$$= -\ln^2(\chi_q) \quad (86)$$

(The question, why they use a complicated dilogarithm remains open ...)

We get the match to [3] and [13] by using

$$\lim_{q^2 \rightarrow 0} \ln^2(\chi_q) = 0 \quad (87)$$

to find

$$D_0(0, m^2, 0, m^2, t, u, 0, 0, m^2, m^2) = \frac{iC_\epsilon}{t_1 u_1} \left(\frac{4}{\epsilon^2} + \frac{2}{\epsilon} \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) \right) \right. \\ \left. + 2 \ln(-t_1/m^2) \ln(-u_1/m^2) - \frac{7}{2} \zeta(2) \right) \quad (88)$$

A References

- [1] G. Passarino and M. J. G. Veltman, “One Loop Corrections for $e^+ e^-$ Annihilation Into $\mu^+ \mu^-$ in the Weinberg Model,” Nucl. Phys. **B160** (1979) 151.
- [2] Beenakker, W. and Kuijf, H. and van Neerven, W. L. and Smith, J., “Qcd corrections to heavy-quark production in $p\bar{p}$ collisions,” Phys. Rev. D **40** (Jul, 1989) 54–82. <http://link.aps.org/doi/10.1103/PhysRevD.40.54>.
- [3] I. Bojak,
NLO QCD corrections to the polarized photoproduction and hadroproduction of heavy quarks.
PhD thesis, Dortmund U., 2000. [arXiv:hep-ph/0005120](https://arxiv.org/abs/hep-ph/0005120) [hep-ph].
- [4] M. Wiebusch, “HEPMath 1.4: A Mathematica Package for Semi-Automatic Computations in High Energy Physics,” Computer Physics Communications **195** (Oct., 2015) 172–190. <http://arxiv.org/abs/1412.6102>. [arXiv: 1412.6102](https://arxiv.org/abs/1412.6102).
- [5] R. Mertig, M. Bohm, and A. Denner, “FEYN CALC: Computer algebraic calculation of Feynman amplitudes,” Comput. Phys. Commun. **64** (1991) 345–359.
- [6] V. Shtabovenko, R. Mertig, and F. Orellana, “New Developments in FeynCalc 9.0,” [arXiv:1601.01167](https://arxiv.org/abs/1601.01167) [hep-ph].
- [7] T. Hahn and M. Perez-Victoria, “Automatized one loop calculations in four-dimensions and D-dimensions,” Comput. Phys. Commun. **118** (1999) 153–165, [arXiv:hep-ph/9807565](https://arxiv.org/abs/hep-ph/9807565) [hep-ph].
- [8] T. Hahn, “LoopTools 2.12 User’s Guide.” <http://www.feynarts.de/looptools/>, 2014.
- [9] R. K. Ellis, Z. Kunszt, K. Melnikov, and G. Zanderighi, “One-loop calculations in quantum field theory: from Feynman diagrams to unitarity cuts,” Phys. Rept. **518** (2012) 141–250, [arXiv:1105.4319](https://arxiv.org/abs/1105.4319) [hep-ph].
- [10] R. K. Ellis and G. Zanderighi, “Scalar one-loop integrals for QCD,” JHEP **02** (2008) 002, [arXiv:0712.1851](https://arxiv.org/abs/0712.1851) [hep-ph].
- [11] A. Denner, “Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200,” Fortsch. Phys. **41** (1993) 307–420, [arXiv:0709.1075](https://arxiv.org/abs/0709.1075) [hep-ph].
- [12] A. Denner and S. Dittmaier, “Reduction schemes for one-loop tensor integrals,” Nucl. Phys. **B734** (2006) 62–115, [arXiv:hep-ph/0509141](https://arxiv.org/abs/hep-ph/0509141) [hep-ph].
- [13] W. Beenakker, H. Kuijf, W. L. van Neerven, and J. Smith, “QCD Corrections to Heavy Quark Production in p anti- p Collisions,” Phys. Rev. **D40** (1989) 54–82.
- [14] A. Denner, U. Nierste, and R. Scharf, “A Compact expression for the scalar one loop four point function,” Nucl. Phys. **B367** (1991) 637–656.

- [15] D. Zagier, Frontiers in Number Theory, Physics, and Geometry II: On Conformal Field Theories, Discrete Groups and Renormalization, ch. The Dilogarithm Function, pp. 3–65. Springer Berlin Heidelberg, Berlin, Heidelberg, 2007.
http://dx.doi.org/10.1007/978-3-540-30308-4_1.
- [16] E. Laenen, S. Riemersma, J. Smith, and W. van Neerven, “Complete $O(\alpha_S)$ corrections to heavy-flavour structure functions in electroproduction,” Nuclear Physics B **392** no. 1, (1993) 162 – 228.
<http://www.sciencedirect.com/science/article/pii/055032139390201Y>.

List of Corrections

Error: fix	9
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