1 Introduction

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1.1 Motivation

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1.2 Notation

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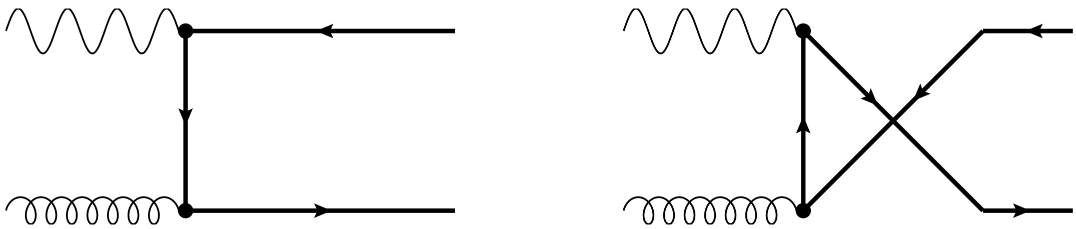
2 Leading Order Calculations

In leading order we have to consider photon-gluon-fusion (PGF), that is

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) \tag{1}$$

with two contributing diagrams depicted in figure 1.



(a) $i\varepsilon_\gamma^\mu(q)\varepsilon_\mathrm{g}^\nu(k_1)\,\mathcal{M}_{\mu\nu}^{(0),1}$ (b) $i\varepsilon_\gamma^\mu(q)\varepsilon_\mathrm{g}^\nu(k_1)\,\mathcal{M}_{\mu\nu}^{(0),2}$

Figure 1: leading order Feynman diagrams
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The result can then be written as

$$\hat{\mathcal{P}}_k^{\gamma,\mu\mu'}\hat{\mathcal{P}}_k^{\mathrm{g},\nu\nu'}\sum_{j,j'=1}^{2}\mathcal{M}_{\mu\nu}^{(0),j}\left(\mathcal{M}_{\mu'\nu'}^{(0),j'}\right)^* = 8g^2\mu_D^{-\epsilon}e^2e_H^2N_CC_FB_{k,QED} \tag{2}$$

where g and e are the strong and electromagnetic coupling constants respectively, μ_D is an arbitray mass parameter introduced to keep the couplings dimensionless and e_H is the magnitude of the heavy quark in units of e. Further N_C corresponds to the gauge group $SU(N_C)$ and the color factor $C_F = (N_C^2 - 1)/(2N_C)$ refers to the second Casimir constant of the fundamental representation for the quarks. We then find:

$$\begin{aligned}
B_{QED,VV,F,Pg} &= -2 + \frac{2(2m^2+q^2)s + s'^2}{t_1 u_1} - \frac{2(2m^2+q^2)m^2 s'^2}{(t_1 u_1)^2} \\
&\quad + \epsilon \left\{ -1 + \frac{s^2 - q^2 s'}{t_1 u_1} - \frac{m^2 q^2 s'^2}{t_1^2 u_1^2} \right\} + \epsilon^2 \frac{s'^2}{4 t_1 u_1} \qquad (3) \\
B_{QED,VV,F,Pk1k1} &= -\frac{4q^2}{s'} \left(\frac{s}{s'} - \frac{m^2 s'}{t_1 u_1} \right) \qquad (4) \\
B_{P,QED} &= \frac{1}{2} \left(\frac{t_1}{u_1} + \frac{u_1}{t_1} \right) \left(\frac{2m^2 s'}{t_1 u_1} - 1 - \frac{2q^2}{s'} \right) \qquad (5) \\
B_{k,QED} &= B^{(0)}_{k,QED} + \epsilon B^{(1)}_{k,QED} + \epsilon^2 B^{(2)}_{k,QED} \qquad (6)
\end{aligned}$$

3 Next-To-Leading Order Calculations

3.1 One Loop Virtual Contributions

$$\begin{aligned}
M_k^{(1),V} &= \hat{\mathcal{P}}_k^{\gamma,\mu\mu'} \hat{\mathcal{P}}_k^{\mathrm{g}} \sum_j \left[\mathcal{M}_{j,\mu}^{(1),V} \left(\mathcal{M}_{1,\mu'}^{(0)} + \mathcal{M}_{2,\mu'}^{(0)} \right)^* + c.c. \right] \\
&= 8 g^4 \mu_D^{-\epsilon} e^2 e_H^2 N_C C_F C_\epsilon \left(C_A V_{k,OK} + 2 C_F V_{k,QED} \right) \qquad (7)
\end{aligned}$$

where $C_\epsilon = \exp(\epsilon/2(\gamma_E - \ln(4\pi)))/(16\pi^2)$ and C_A is the second Casimir constant of the adjoint representation for the gluon (that introduces a non-abelian part).

As the short example above shows, the full expressions for the $V_{k,OK}, V_{k,QED}$ are quite complicated and too long to be presented here, nevertheless the arising poles are quite compact:

$$V_{k,OK} = -2 B_{k,QED} \left(\frac{4}{\epsilon^2} + \left(\ln(-t_1/m^2) + \ln(-u_1/m^2) - \frac{2m^2 - s}{s} \ln(\chi) \right) \frac{2}{\epsilon} \right) + O(\epsilon^0) \qquad (8)$$

$$V_{k,QED} = -2 B_{k,QED} \left(1 + \frac{2m^2 - s}{s} \ln(\chi) \right) \frac{2}{\epsilon} + O(\epsilon^0) \qquad (9)$$

The above results already include the mass renormalization that we have performed *on-shell*, so all ultra-violet poles have been removed. For the renormalization of the strong coupling we use the $\overline{\mathrm{MS}}_m$ scheme defined in [1] and so the full (remaining) renormalization can be achieved by

$$\frac{d^2\sigma_k^{(1),V,ren.}}{dt_1du_1} = \frac{d^2\sigma_k^{(1),V}}{dt_1du_1} + \frac{\alpha_s(\mu_R^2)}{4\pi}\left[\left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln(\mu_R^2/m^2) - \ln(\mu_D^2/m^2)\right)\beta_0^f \right.$$
$$\left. + \frac{2}{3}\ln(\mu_R^2/m^2)\right]\frac{d^2\sigma_k^{(0)}}{dt_1du_1} \tag{10}$$
$$= \frac{d^2\sigma_k^{(1),V}}{dt_1du_1} + 4\pi\alpha_s(\mu_R^2)C_\epsilon\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}\left[\left(\frac{2}{\epsilon} + \ln(\mu_R^2/m^2)\right)\beta_0^f \right.$$
$$\left. + \frac{2}{3}\ln(\mu_R^2/m^2)\right]\frac{d^2\sigma_k^{(0)}}{dt_1du_1} \tag{11}$$

with μ_R the renormalization scale introduced by the RGE, $\beta_0^f = (11C_A - 2n_f)/3$ the first coefficient of the beta function and n_f the number of *total* flavours (i.e. $n_{lf} = n_f - 1$ active (light) flavours and one heavy flavour). The double poles occuring in $V_{k,OK}$ are introduced by the diagrams **FiXme Error: do** when the soft and collinear singularities coincide. FiXme Error!

The partonic cross section is given by

$$d\sigma_{k,\mathrm{g}}^{(1),V} = \frac{1}{2s'}\frac{1}{2}E_k(\epsilon)b_k(\epsilon)M_k^{(1),V}dPS_2 \tag{12}$$

3.2 Single Gluon Radiation

In next-to-leading order we have to consider the following process:

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) + \mathrm{g}(k_2) \tag{13}$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as FiXme Error!

$$\hat{\mathcal{P}}_k^{\gamma,\mu\mu'}\hat{\mathcal{P}}_k^{\mathrm{g}}\sum_{j,j'}\mathcal{M}_{j,\mu}^{(1),g}\mathcal{M}_{j',\mu'}^{(1),g^*} = 8g^4\mu_D^{-2\epsilon}e^2e_H^2N_CC_F\left(C_AR_{k,OK} + 2C_FR_{k,QED}\right) \tag{14}$$

and it will depend on ten kinematical invariants:

$$s = (q+k_1)^2 \qquad t_1 = (k_1-p_2)^2 - m^2 \qquad u_1 = (q-p_2)^2 - m^2 \tag{15}$$
$$s_3 = (k_2+p_2)^2 - m^2 \qquad s_4 = (k_2+p_1)^2 - m^2 \qquad s_5 = (p_1+p_2)^2 = -u_5 \tag{16}$$
$$t' = (k_1-k_2)^2 \tag{17}$$
$$u' = (q-k_2)^2 \qquad u_6 = (k_1-p_1)^2 - m^2 \qquad u_7 = (q-p_1)^2 - m^2 \tag{18}$$

from which only five are independent as can be seen from momentum conservation $k_1+q = p_1+p_2+k_2$ and s, t_1, u_1 match to their leading order definition.

The $2 \to 3$ n-dimensional phase space is given by

$$dPS_3 = \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_2}{(2\pi)^{n-1}} \frac{d^n k_2}{(2\pi)^{n-1}} (2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2 - k_2)$$
$$\Theta(p_{1,0})\delta(p_1^2 - m^2)\Theta(p_{2,0})\delta(p_2^2 - m^2)\Theta(k_{2,0})\delta(k_2^2) \qquad (19)$$

This can be solved by writing eq. (19) as product of a $2 \to 2$ decay and a subsequent $1 \to 2$ decay[2]. We find

$$dPS_3 = \frac{1}{(4\pi)^n \Gamma(n-3) s'} \frac{s_4^{n-3}}{(s_4 + m^2)^{n/2-1}} \left(\frac{(t_1 u_1' - s' m^2) s' - q^2 t_1^2}{s'^2} \right)^{(n-4)/2} dt_1 du_1 d\Omega_n d\hat{\mathcal{I}} \qquad (20)$$

$$= h_3(n)\, dt_1 du_1 d\Omega_n d\hat{\mathcal{I}} \qquad (21)$$

with $d\Omega_n = \sin^{n-3}(\theta_1) d\theta_1 \sin^{n-4}(\theta_2) d\theta_2$ and $d\hat{\mathcal{I}}$ taking care of all occuring hat momenta:

$$d\hat{\mathcal{I}} = \frac{1}{B(1/2, (n-4)/2)} \frac{x^{(n-6)/2}}{\sqrt{1-x}} dx \qquad \text{with } x = \hat{p}_1^2 / \hat{p}_{1,max} \qquad (22)$$

$$\hat{p}_{1,max} = \frac{s_4^2}{4(s_4 + m^2)} \sin^2(\theta_1) \sin^2(\theta_2) \qquad (23)$$

$$\Rightarrow \int d\hat{\mathcal{I}} = 1 \qquad \int d\hat{\mathcal{I}}\, \hat{p}_1^2 = \epsilon \hat{p}_{1,max} + O(\epsilon^2) \qquad (24)$$

Again when integrating the phase space angles the expressions become quite lengthy, but the (collinear) pole parts are compact:

$$\frac{s_4}{4\pi(s_4 + m^2)} \int d\Omega_n d\hat{\mathcal{I}}\, C_A R_{k,OK} = -\frac{1}{u_1} B_{k,QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} P^H_{k,\mathrm{gg}}(x_1) \frac{2}{\epsilon} + O(\epsilon^0) \qquad (25)$$

with $x_1 = -u_1/(s' + t_1)$ and the hard part of the Altarelli-Parisi splitting functions $P^H_{k,\mathrm{gg}}$[3, 4]:

$$P^H_{G,\mathrm{gg}}(x) = P^H_{L,\mathrm{gg}}(x) = C_A \left(\frac{2}{1-x} + \frac{2}{x} - 4 + 2x - 2x^2 \right) \qquad (26)$$

$$P^H_{P,\mathrm{gg}}(x) = C_A \left(\frac{2}{1-x} - 4x + 2 \right) \qquad (27)$$

The $R_{k,QED}$ do not contain poles. **FiXme Error: shift to factorization?** FiXme Error!

From the above expression we can obtain the soft limit $k_2 \to 0$ and seperate their calculations:

$$\lim_{k_2 \to 0} \left(C_A R_{k,OK} + 2C_F R_{k,QED} \right) = \left(C_A S_{k,OK} + 2C_F S_{k,QED} \right) + O(1/s_4, 1/s_3, 1/t') \qquad (28)$$

$$S_{k,OK} = 2\left(\frac{t_1}{t's_3} + \frac{u_1}{t's_4} - \frac{s-2m^2}{s_3s_4}\right) B_{k,QED} \tag{29}$$

$$S_{k,QED} = 2\left(\frac{s-2m^2}{s_3s_4} - \frac{m^2}{s_3^2} - \frac{m^2}{s_4^2}\right) B_{k,QED} \tag{30}$$

Note that the einkonal factors multiplying the Born functions $B_{k,QED}$ neither depend on q^2 nor on the projection k.

3.3 Light Quark Processes

In next-to-leading order a new production mechanism enters that is induced by a light quark, so we have to consider the process

$$\gamma^*(q) + q(k_1) \to Q(p_1) + \overline{Q}(p_2) + q(k_2) \tag{31}$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as FiXme Error!

$$\hat{\mathcal{P}}_k^{\gamma,\mu\mu'}\hat{\mathcal{P}}_k^{q,aa'} \sum_{j,j'=1}^{4} \mathcal{M}_{j,\mu a}^{(1),q}\left(\mathcal{M}_{j',\mu' a'}^{(1),q}\right)^* = 8g^4\mu_D^{-2\epsilon}e^2N_CC_F\left(e_H^2A_{k,1} + e_L^2A_{k,2} + e_Le_HA_{k,3}\right) \tag{32}$$

where e_L denotes the charge of the light quark q in units of e.

The needed $2 \to 3$ phase space has already been calculated in section 3.2, so we can immediately quote the (collinear) poles:

$$\frac{s_4}{2\pi(s_4+m^2)}\int d\Omega_n d\hat{\mathcal{I}}\, C_F A_{k,1} = -\frac{1}{u_1}B_{k,QED}\begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1\end{pmatrix} P_{k,gq}(x_1)\frac{2}{\epsilon} + O(\epsilon^0) \tag{33}$$

with $x_1 = -u_1/(s'+t_1)$ and the Altarelli-Parisi splitting functions $P_{k,gq}$[3, 4]:

$$P_{G,gq}(x) = P_{L,gq}(x) = C_F\left(\frac{1}{x} + \frac{(1-x)^2}{x}\right) \tag{34}$$

$$P_{P,gq}(x) = C_F\,(2-x) \tag{35}$$

$A_{k,2}$ does not contain poles and we find $\int dt_1 du_1 \int d\Omega_n d\hat{\mathcal{I}}\, A_{k,3} = 0$. Note that in the limit $q^2 \to 0$ $A_{k,2}$ will also get collinear poles.

4 Mass Factorization

All collinear poles in the gluonic subprocess can be removed by mass factorization in the following way:

$$s'^2 \frac{d^2\sigma^{(1),fin}_{k,g}(s',t_1,u_1,q^2,\mu_F)}{dt_1 du_1} = \lim_{\epsilon\to 0}\left[s'^2 \frac{d^2\sigma^{(1)}_{k,g}(s',t_1,u_1,q^2,\epsilon)}{dt_1 du_1} \right.$$

$$- \int_0^1 \frac{dx_1}{x_1}\Gamma^{(1)}_{k,gg}(x_1,\mu_F^2,\mu_D,\epsilon) \tag{36}$$

$$\left. (x_1 s')^2 \frac{d^2\sigma^{(0)}_{k,g}(x_1 s',x_1 t_1,u_1,q^2,\epsilon)}{d(x_1 t_1) du_1}\right] \tag{37}$$

$$\Gamma^{(1)}_{k,ij}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_s}{2\pi}\left(P_{k,ij}(x)\frac{2}{\epsilon} + f_{k,ij}(x,\mu_F^2,\mu_D^2)\right) \tag{38}$$

where $\Gamma^{(1)}_{k,ij}$ is the first order correction to the transition functions $\Gamma_{k,ij}$ for *incoming* particle j and *outgoing* particle i in projection k. In the $\overline{\text{MS}}$-scheme the $f_{k,ij}$ take their usual form and we find

$$\Gamma^{(1),\overline{\text{MS}}}_{k,ij}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_s}{2\pi}P_{k,ij}(x)\left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln(\mu_F^2/m^2) - \ln(\mu_D^2/m^2)\right) \tag{39}$$

$$= 8\pi\alpha_s P_{k,ij}(x) C_\epsilon \left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}\left(\frac{2}{\epsilon} + \ln(\mu_F^2/m^2)\right) \tag{40}$$

The $P_{k,ij}(x)$ are the Altarelli-Parisi splitting functions for which we find[3, 4]

$$P_{k,gg}(x) = \Theta(1-\delta-x)P^H_{k,gg}(x) + \delta(1-x)\left(2C_A\ln(\delta) + \frac{\beta_0}{2}\right) \tag{41}$$

where we introduced another infrared cut-off δ to seperate soft ($x \geq 1-\delta$) and hard ($x < 1-\delta$) gluons that is connected to Δ via $\delta = \Delta/(s'+t_1)$. The structure here explains why we were able to write the equation (25).

The light quark process can be regularized in a complete analogous way:

$$s'^2 \frac{d^2\sigma^{(1),fin}_{k,q}(s',t_1,u_1,q^2,\mu_F)}{dt_1 du_1} = \lim_{\epsilon\to 0}\left[s'^2 \frac{d^2\sigma^{(1)}_{k,q}(s',t_1,u_1,q^2,\epsilon)}{dt_1 du_1} \right.$$

$$- \int_0^1 \frac{dx_1}{x_1}\Gamma^{(1)}_{k,gq}(x_1,\mu_F^2,\mu_D,\epsilon)$$

$$\left. (x_1 s')^2 \frac{d^2\sigma^{(0)}_{k,g}(x_1 s',x_1 t_1,u_1,q^2,\epsilon)}{d(x_1 t_1) du_1}\right] \tag{42}$$

The needed splitting functions $P_{k,\mathrm{g}q}$ have been already quoted in equations (34) and (35). Note that $K_{q\gamma} = 1/(N_C) = 2C_F K_{\mathrm{g}\gamma}$.

The final finite cross sections are then

$$\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),H,fin}_{k,\mathrm{g}}}{dt_1 du_1} = \frac{1}{2\pi} K_{\mathrm{g}\gamma} \alpha \alpha_S e_H^2 N_C C_F b_k(0) \Bigg[& -\frac{1}{u_1} P^H_{k,\mathrm{gg}}(x_1) \\
& \Bigg\{ 4\pi B^{(0)}_{k,QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)}\right) - \ln(\mu_F^2/m^2) \right) \\
& -8\pi B^{(1)}_{k,QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \Bigg\} \\
& + C_A \frac{s_4}{s_4+m^2} \left(\int d\Omega_n d\hat{\mathcal{I}}\, R_{k,OK} \right)^{finite} \\
& + 2C_F \frac{s_4}{s_4+m^2} \int d\Omega_4 d\hat{\mathcal{I}}\, R_{k,QED} \Bigg]
\end{aligned} \tag{43}$$

$$\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),S+V,fin}_{k,\mathrm{g}}}{dt_1 du_1} = 4K_{\mathrm{g}\gamma} \alpha \alpha_S e_H^2 N_C C_F b_k(0) B^{(0)}_{k,QED} \delta(s'+t_1+u_1) \Big[& C_A \ln^2(\Delta/m^2) \\
& + \ln(\Delta/m^2) \Big(\Big(\ln(-t_1/m^2) - \ln(-u_1/m^2) - \ln(\mu_F^2/m^2) \Big) C_A \\
& - \frac{2m^2 - s}{s\beta} \ln(\chi)(C_A - 2C_F) - 2C_F \Big) \\
& + \frac{\beta_0^{lf}}{4} \Big(\ln(\mu_R^2/m^2) - \ln(\mu_F^2/m^2) \Big) + f_k(s', u_1, t_1, q^2) \Big]
\end{aligned} \tag{44}$$

where f_k contains lots of logarithms and dilogarithms, but does not depend on Δ, μ_F^2, μ_R^2 nor n_f and $\beta_0^{lf} = (11C_A - 2n_{lf})/3$.

$$\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),fin}_{k,q}}{dt_1 du_1} = \frac{1}{2\pi} K_{q\gamma} \alpha \alpha_S N_C b_k(0) \Bigg[& -\frac{1}{u_1} e_H^2 P_{k,\mathrm{g}q}(x_1) \\
& \Bigg\{ 2\pi B^{(0)}_{k,QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)}\right) - \ln(\mu_F^2/m^2) + 1 - \delta_{k,P} \right) \\
& -4\pi B^{(1)}_{k,QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \Bigg\} \\
& + C_F \frac{s_4}{s_4+m^2} \left(\int d\Omega_n d\hat{\mathcal{I}}\, e_H^2 A_{k,1} \right)^{finite} \\
& + C_F \frac{s_4}{s_4+m^2} \int d\Omega_4 d\hat{\mathcal{I}}\, e_L^2 A_{k,2} + C_F \frac{s_4}{s_4+m^2} \int d\Omega_4 d\hat{\mathcal{I}}\, e_H e_L A_{k,3} \Bigg]
\end{aligned} \tag{45}$$

where $1 - \delta_{k,P}$ may also be written as $-2\partial_\epsilon E_k(\epsilon = 0)$ as it originates from the additional factor of $E_k(\epsilon)$ in the subtraction part of equation (42).

5 Partonic Results

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6 Hadronic Results

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7 Summary

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A References

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List of Corrections