1 Introduction

FiXme Error!

FiXme Error: write intro

1.1 Motivation

FiXme Error!

FiXme Error: write motivation

1.2 Notation

FiXme Error!

FiXme Error: write more notation We use the definition of [1] for the hadronic tensor:

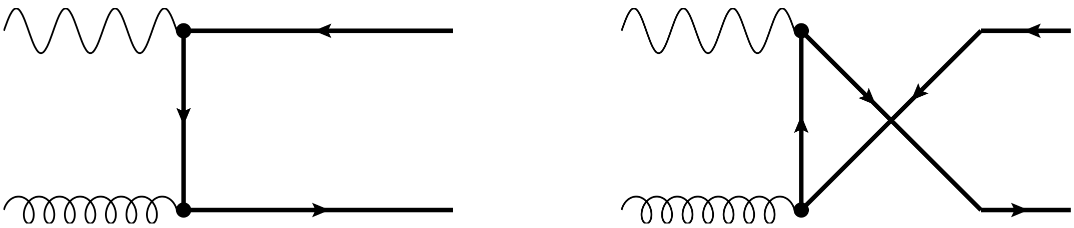
$$W_{\mu\mu'} = (-g_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{q^2})F_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_{\mu'}}{P \cdot q} F_2(x,Q^2) \tag{1}$$

2 Leading Order Calculations

In leading order we have to consider photon-gluon-fusion (PGF), that is

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) \tag{2}$$

with two contributing diagrams depicted in figure 1.



(a) $i\varepsilon_\gamma^\mu(q)\varepsilon_\mathrm{g}^\nu(k_1)\,\mathcal{M}_{\mu\nu}^{(0),1}$ (b) $i\varepsilon_\gamma^\mu(q)\varepsilon_\mathrm{g}^\nu(k_1)\,\mathcal{M}_{\mu\nu}^{(0),2}$

Figure 1: leading order Feynman diagrams
FiXme Error: shift to appendix?

The result can then be written as

$$\hat{\mathcal{P}}_{\vec{k}}^{\gamma,\mu\mu'} \hat{\mathcal{P}}_{\vec{k}}^{\mathrm{g},\nu\nu'} \sum_{j,j'=1}^{2} \mathcal{M}_{\mu\nu}^{(0),j} \left(\mathcal{M}_{\mu'\nu'}^{(0),j'}\right)^* = 8g^2\mu_D^{-\epsilon}e^2e_H^2 N_C C_F B_{\vec{k},QED} \tag{3}$$

where g and e are the strong and electromagnetic coupling constants respectively, μ_D is an arbitray mass parameter introduced to keep the couplings dimensionless and e_H is the magnitude of the heavy quark in units of e. Further N_C corresponds to the gauge group $SU(N_C)$ and the color factor $C_F = (N_C^2 - 1)/(2N_C)$ refers to the second Casimir constant of the fundamental representation for the quarks. We then find:

$$
\begin{aligned}
B_{\mathrm{VV},F_2,\mathrm{QED}} &= \left[-1 - \frac{6q^2}{s'} - \frac{6q^4}{s'^2} + \frac{q^2(6m^2+s) + 2m^2 s + s'^2/2}{t_1 u_1} - \frac{(2m^2+q^2)m^2 s'^2}{(t_1 u_1)^2}\right] \\
&\quad + \frac{\epsilon}{2}\left[-1 + \frac{s^2 - q^2 s'}{t_1 u_1} - \frac{m^2 q^2 s'^2}{t_1^2 u_1^2}\right] + \epsilon^2 \frac{s'^2}{8 t_1 u_1}
\end{aligned} \tag{4}
$$

$$
B_{\mathrm{VV},F_L,\mathrm{QED}} = -\frac{4q^2}{s'}\left(\frac{s}{s'} - \frac{m^2 s'}{t_1 u_1}\right) \tag{5}
$$

$$
B_{\mathrm{VV},2xg_1,\mathrm{QED}} = \left\{1 + \frac{2q^2}{s'} - \frac{s'(2(2m^2+q^2)+s')}{2t_1 u_1} + \frac{m^2 s'^3}{(t_1 u_1)^2} + \epsilon\left(-\frac{1}{2} + \frac{s'^2}{4 t_1 u_1}\right)\right\}(1+\epsilon) \tag{6}
$$

$$
\begin{aligned}
B_{\mathrm{AA},F_2,\mathrm{QED}} &= \frac{m^2 s'^2 (1+\epsilon)(2+\epsilon)(12m^2(-1+\epsilon) + q^2(-6+(-3+\epsilon)\epsilon))}{12(t_1 u_1)^2} - \\
&\quad \frac{(1+\epsilon)\left(8s'^3\epsilon + 12q^6(2+\epsilon) + 12q^4 s'(2+\epsilon) + q^2 s'^2(4+\epsilon(20-(-3+\epsilon)\epsilon))\right)}{4q^2 s'^2} - \\
&\quad \frac{(1+\epsilon)}{48q^2(t_1 u_1)}\left(q^2(2+\epsilon)(-6+(-3+\epsilon)\epsilon)\left(4q^4 + 4q^2 s' + s'^2(2+\epsilon)\right)\right. \\
&\qquad \left. +48m^2\left(-s'^2(-2+\epsilon) + q^4(-4+\epsilon)(2+\epsilon) + q^2 s'\left(-2+\epsilon+\epsilon^2\right)\right)\right)
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
B_{\mathrm{AA},F_2,\mathrm{QED}} &= -\frac{m^2 s'^2(1+\epsilon)(2+\epsilon)(12m^2 + q^2\epsilon)}{6(t_1 u_1)^2} - \\
&\quad \frac{(1+\epsilon)\left(4s'^3\epsilon + 4q^6(2+\epsilon) + 4q^4 s'(2+\epsilon) + q^2 s'^2\epsilon(6+\epsilon)\right)}{2q^2 s'^2} + \\
&\quad \frac{(1+\epsilon)}{24q^2(t_1 u_1)}\left(24m^2\left(s'^2(-2+\epsilon) + 4q^4(2+\epsilon) + 2q^2 s'(2+\epsilon)\right) + \right. \\
&\qquad \left. q^2\epsilon(2+\epsilon)\left(4q^4 + 4q^2 s' + s'^2(2+\epsilon)\right)\right)
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
B_{\mathrm{AA},2xg_1,\mathrm{QED}} &= \frac{(1+\epsilon)^2(2-\epsilon)}{2}\left[1 + \frac{2q^2}{s'} - \frac{2s'(2m^2+q^2) + s'^2}{2t_1 u_1} + \frac{m^2 s'^3}{(t_1 u_1)^2}\right. \\
&\qquad \left. + \left(-1 + \frac{s'^2}{2 t_1 u_1}\right)\frac{\epsilon}{2}\right]
\end{aligned} \tag{9}
$$

$$B_{\text{VA},xF_3,\text{QED}} = \frac{s'(1+\epsilon)(2+\epsilon)}{t_1-u_1}\left\{-1-\frac{\epsilon}{2}-2\frac{q^2}{s'}-2\frac{q^4}{s'}-\frac{m^2q^2s'^2}{2(t_1u_1)^2}\right.$$
$$\left.+\frac{4q^2(4m^2+q^2+s')+s'^2(2+\epsilon)}{t_1u_1}\right\} \tag{10}$$

$$B_{\text{VA},g_4,\text{QED}} = \frac{s'(1+\epsilon)}{t_1-u_1}\left\{-2+\epsilon-4\frac{q^2}{s'}-\frac{m^2s'^3}{(t_1u_1)^2}+\right.$$
$$\left.\frac{s'(16m^2+4q^2+s'(2-\epsilon))}{4t_1u_1}\right\} \tag{11}$$

$$B_{\text{VA},g_L,\text{QED}} = 0 \tag{12}$$

$$B_{\vec{k},\text{QED}} = B^{(0)}_{\vec{k},\text{QED}} + \epsilon B^{(1)}_{\vec{k},\text{QED}} + \epsilon^2 B^{(2)}_{\vec{k},\text{QED}} \tag{13}$$

3 Next-To-Leading Order Calculations

3.1 One Loop Virtual Contributions

$$\begin{aligned} M^{(1),V}_{\vec{k}} &= \hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\vec{k}}\hat{\mathcal{P}}^{\text{g}}_{\vec{k}}\sum_j\left[\mathcal{M}^{(1),V}_{j,\mu}\left(\mathcal{M}^{(0)}_{1,\mu'}+\mathcal{M}^{(0)}_{2,\mu'}\right)^* + c.c.\right] \\ &= 8g^4\mu_D^{-\epsilon}e^2e_H^2N_CC_FC_\epsilon\left(C_AV_{\vec{k},OK}+2C_FV_{\vec{k},QED}\right) \end{aligned} \tag{14}$$

where $C_\epsilon = \exp(\epsilon/2(\gamma_E - \ln(4\pi)))/(16\pi^2)$ and C_A is the second Casimir constant of the adjoint representation for the gluon (that introduces a non-abelian part).

As the short example above shows, the full expressions for the $V_{k,OK}, V_{k,QED}$ are quite complicated and too long to be presented here, nevertheless the arising poles are quite compact:

$$V_{\vec{k},OK} = -2B_{\vec{k},QED}\left(\frac{4}{\epsilon^2}+\left(\ln(-t_1/m^2)+\ln(-u_1/m^2)-\frac{2m^2-s}{s}\ln(\chi)\right)\frac{2}{\epsilon}\right)+O(\epsilon^0) \tag{15}$$

$$V_{\vec{k},QED} = -2B_{\vec{k},QED}\left(1+\frac{2m^2-s}{s}\ln(\chi)\right)\frac{2}{\epsilon}+O(\epsilon^0) \tag{16}$$

The above results already include the mass renormalization that we have performed *on-shell*, so all ultra-violet poles have been removed. For the renormalization of the strong coupling we use the $\overline{\text{MS}}_m$ scheme defined in [2] and so the full (remaining) renormalization can be achieved by

$$\frac{d^2\sigma^{(1),V,ren.}_{\vec{k}}}{dt_1du_1} = \frac{d^2\sigma^{(1),V}_{\vec{k}}}{dt_1du_1} + \frac{\alpha_s(\mu_R^2)}{4\pi}\left[\left(\frac{2}{\epsilon}+\gamma_E-\ln(4\pi)+\ln(\mu_R^2/m^2)-\ln(\mu_D^2/m^2)\right)\beta_0^f + \frac{2}{3}\ln(\mu_R^2/m^2)\right]\frac{d^2\sigma^{(0)}_{\vec{k}}}{dt_1du_1} \tag{17}$$

$$= \frac{d^2\sigma^{(1),V}_{\vec{k}}}{dt_1du_1} + 4\pi\alpha_s(\mu_R^2)C_\epsilon\left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}\left[\left(\frac{2}{\epsilon}+\ln(\mu_R^2/m^2)\right)\beta_0^f + \frac{2}{3}\ln(\mu_R^2/m^2)\right]\frac{d^2\sigma^{(0)}_{\vec{k}}}{dt_1du_1} \tag{18}$$

with μ_R the renormalization scale introduced by the RGE, $\beta_0^f = (11C_A - 2n_f)/3$ the first coefficient of the beta function and n_f the number of *total* flavours (i.e. $n_{lf} = n_f - 1$ active (light) flavours and one heavy flavour). The double poles occuring in $V_{\vec{k},OK}$ are introduced by the diagrams **FiXme Error: do** when the soft and collinear singularities coincide. FiXme Error!

The partonic cross section is given by

$$d\sigma^{(1),V}_{\vec{k},\mathrm{g}} = \frac{1}{2s'}\frac{1}{2}E_{k_2}(\epsilon)M^{(1),V}_{\vec{k}}dPS_2 \tag{19}$$

3.2 Single Gluon Radiation

In next-to-leading order we have to consider the following process:

$$\gamma^*(q) + \mathrm{g}(k_1) \to Q(p_1) + \overline{Q}(p_2) + \mathrm{g}(k_2) \tag{20}$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as FiXme Error!

$$\hat{\mathcal{P}}^{\gamma,\mu\mu'}_{\vec{k}}\hat{\mathcal{P}}^{\mathrm{g}}_{\vec{k}}\sum_{j,j'}\mathcal{M}^{(1),g}_{j,\mu}\mathcal{M}^{(1),g^*}_{j',\mu'} = 8g^4\mu_D^{-2\epsilon}e^2e_H^2N_CC_F\left(C_AR_{\vec{k},OK}+2C_FR_{\vec{k},QED}\right) \tag{21}$$

and it will depend on ten kinematical invariants:

$$s = (q+k_1)^2 \qquad t_1 = (k_1-p_2)^2 - m^2 \qquad u_1 = (q-p_2)^2 - m^2 \tag{22}$$

$$s_3 = (k_2+p_2)^2 - m^2 \qquad s_4 = (k_2+p_1)^2 - m^2 \qquad s_5 = (p_1+p_2)^2 = -u_5 \tag{23}$$

$$t' = (k_1-k_2)^2 \tag{24}$$

$$u' = (q-k_2)^2 \qquad u_6 = (k_1-p_1)^2 - m^2 \qquad u_7 = (q-p_1)^2 - m^2 \tag{25}$$

from which only five are independent as can be seen from momentum conservation $k_1+q = p_1+p_2+k_2$ and s, t_1, u_1 match to their leading order definition.

The $2 \to 3$ n-dimensional phase space is given by

$$dPS_3 = \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_2}{(2\pi)^{n-1}} \frac{d^n k_2}{(2\pi)^{n-1}} (2\pi)^n \delta^{(n)}(k_1 + q - p_1 - p_2 - k_2)$$
$$\Theta(p_{1,0})\delta(p_1^2 - m^2)\Theta(p_{2,0})\delta(p_2^2 - m^2)\Theta(k_{2,0})\delta(k_2^2) \qquad (26)$$

This can be solved by writing eq. (26) as product of a $2 \to 2$ decay and a subsequent $1 \to 2$ decay[3]. We find

$$dPS_3 = \frac{1}{(4\pi)^n \Gamma(n-3) s'} \frac{s_4^{n-3}}{(s_4 + m^2)^{n/2-1}} \left(\frac{(t_1 u_1' - s' m^2) s' - q^2 t_1^2}{s'^2} \right)^{(n-4)/2} dt_1 du_1 d\Omega_n d\hat{\mathcal{I}} \qquad (27)$$

$$= h_3(n)\, dt_1 du_1 d\Omega_n d\hat{\mathcal{I}} \qquad (28)$$

with $d\Omega_n = \sin^{n-3}(\theta_1) d\theta_1 \sin^{n-4}(\theta_2) d\theta_2$ and $d\hat{\mathcal{I}}$ taking care of all occuring hat momenta:

$$d\hat{\mathcal{I}} = \frac{1}{B(1/2, (n-4)/2)} \frac{x^{(n-6)/2}}{\sqrt{1-x}} dx \qquad \text{with } x = \hat{p}_1^2 / \hat{p}_{1,max} \qquad (29)$$

$$\hat{p}_{1,max} = \frac{s_4^2}{4(s_4 + m^2)} \sin^2(\theta_1) \sin^2(\theta_2) \qquad (30)$$

$$\Rightarrow \int d\hat{\mathcal{I}} = 1 \qquad \int d\hat{\mathcal{I}}\, \hat{p}_1^2 = \epsilon \hat{p}_{1,max} + O(\epsilon^2) \qquad (31)$$

Again when integrating the phase space angles the expressions become quite lengthy, but the (collinear) pole parts are compact:

$$\frac{s_4}{4\pi(s_4 + m^2)} \int d\Omega_n d\hat{\mathcal{I}}\, C_A R_{\vec{k},OK} = -\frac{1}{u_1} B_{\vec{k},QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} P^H_{\vec{k},\mathrm{gg}}(x_1) \frac{2}{\epsilon} + O(\epsilon^0) \qquad (32)$$

with $x_1 = -u_1/(s' + t_1)$ and the hard part of the Altarelli-Parisi splitting functions $P^H_{k,\mathrm{gg}}$[4, 5]:

$$P^H_{F,\mathrm{gg}}(x) = C_A \left(\frac{2}{1-x} + \frac{2}{x} - 4 + 2x - 2x^2 \right) \qquad (33)$$

$$P^H_{g,\mathrm{gg}}(x) = C_A \left(\frac{2}{1-x} - 4x + 2 \right) \qquad (34)$$

The $R_{\vec{k},QED}$ do not contain poles. **FiXme Error: shift to factorization?** FiXme Error!

From the above expression we can obtain the soft limit $k_2 \to 0$ and seperate their calculations:

$$\lim_{k_2 \to 0} \left(C_A R_{\vec{k},OK} + 2C_F R_{\vec{k},QED} \right) = \left(C_A S_{\vec{k},OK} + 2C_F S_{\vec{k},QED} \right) + O(1/s_4, 1/s_3, 1/t') \qquad (35)$$

$$S_{\vec{k},OK} = 2\left(\frac{t_1}{t' s_3} + \frac{u_1}{t' s_4} - \frac{s-2m^2}{s_3 s_4}\right) B_{\vec{k},QED} \tag{36}$$

$$S_{\vec{k},QED} = 2\left(\frac{s-2m^2}{s_3 s_4} - \frac{m^2}{s_3^2} - \frac{m^2}{s_4^2}\right) B_{\vec{k},QED} \tag{37}$$

Note that the einkonal factors multiplying the Born functions $B_{\vec{k},QED}$ neither depend on q^2 nor on the projection $\vec{k}$.

3.3 Light Quark Processes

In next-to-leading order a new production mechanism enters that is induced by a light quark, so we have to consider the process

$$\gamma^*(q) + q(k_1) \to Q(p_1) + \overline{Q}(p_2) + q(k_2) \tag{38}$$

All contributing diagrams are depicted in figure **FiXme Error: do** and the result can be written as FiXme Error!

$$\hat{\mathcal{P}}_{\vec{k}}^{\gamma,\mu\mu'} \hat{\mathcal{P}}_{\vec{k}}^{q,aa'} \sum_{j,j'=1}^{4} \mathcal{M}_{j,\mu a}^{(1),q} \left(\mathcal{M}_{j',\mu' a'}^{(1),q}\right)^* = 8g^4 \mu_D^{-2\epsilon} e^2 N_C C_F \left(e_H^2 A_{\vec{k},1} + e_L^2 A_{\vec{k},2} + e_L e_H A_{\vec{k},3}\right) \tag{39}$$

where e_L denotes the charge of the light quark q in units of e.

The needed $2 \to 3$ phase space has already been calculated in section 3.2, so we can immediately quote the (collinear) poles:

$$\frac{s_4}{4\pi(s_4+m^2)} \int d\Omega_n d\hat{\mathcal{I}}\, C_F A_{\vec{k},1} = -\frac{1}{u_1} B_{\vec{k},QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} P_{k_2,gq}(x_1) \frac{2}{\epsilon} + O(\epsilon^0) \tag{40}$$

with $x_1 = -u_1/(s'+t_1)$ and the Altarelli-Parisi splitting functions $P_{k,gq}$[4, 5]:

$$P_{F,gq}(x) = C_F\left(\frac{1}{x} + \frac{(1-x)^2}{x}\right) \tag{41}$$

$$P_{g,gq}(x) = C_F\,(2-x) \tag{42}$$

$A_{k,2}$ does not contain poles and we find $\int dt_1 du_1 \int d\Omega_n d\hat{\mathcal{I}}\, A_{k,3} = 0$. Note that in the limit $q^2 \to 0$ $A_{k,2}$ will also get collinear poles.

4 Mass Factorization

All collinear poles in the gluonic subprocess can be removed by mass factorization in the following way:

$$s'^2 \frac{d^2\sigma^{(1),fin}_{\vec{k},\mathrm{g}}(s',t_1,u_1,q^2,\mu_F)}{dt_1 du_1} = \lim_{\epsilon\to 0}\left[s'^2 \frac{d^2\sigma^{(1)}_{\vec{k},\mathrm{g}}(s',t_1,u_1,q^2,\epsilon)}{dt_1 du_1} \right.$$

$$- \int_0^1 \frac{dx_1}{x_1}\Gamma^{(1)}_{\vec{k},\mathrm{gg}}(x_1,\mu_F^2,\mu_D,\epsilon) \tag{43}$$

$$\left. (x_1 s')^2 \frac{d^2\sigma^{(0)}_{\vec{k},\mathrm{g}}(x_1 s', x_1 t_1, u_1, q^2,\epsilon)}{d(x_1 t_1) du_1}\right] \tag{44}$$

$$\Gamma^{(1)}_{\vec{k},ij}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_s}{2\pi}\left(P_{\vec{k},ij}(x)\frac{2}{\epsilon} + f_{\vec{k},ij}(x,\mu_F^2,\mu_D^2)\right) \tag{45}$$

where $\Gamma^{(1)}_{\vec{k},ij}$ is the first order correction to the transition functions $\Gamma_{\vec{k},ij}$ for *incoming* particle j and *outgoing* particle i in projection k. In the $\overline{\mathrm{MS}}$-scheme the $f_{\vec{k},ij}$ take their usual form and we find

$$\Gamma^{(1),\overline{\mathrm{MS}}}_{\vec{k},ij}(x,\mu_F^2,\mu_D,\epsilon) = \frac{\alpha_s}{2\pi}P_{\vec{k},ij}(x)\left(\frac{2}{\epsilon} + \gamma_E - \ln(4\pi) + \ln(\mu_F^2/m^2) - \ln(\mu_D^2/m^2)\right) \tag{46}$$

$$= 8\pi\alpha_s P_{\vec{k},ij}(x) C_\epsilon \left(\frac{\mu_D^2}{m^2}\right)^{-\epsilon/2}\left(\frac{2}{\epsilon} + \ln(\mu_F^2/m^2)\right) \tag{47}$$

The $P_{\vec{k},ij}(x)$ are the Altarelli-Parisi splitting functions for which we find[4, 5]

$$P_{\vec{k},\mathrm{gg}}(x) = \Theta(1-\delta-x)P^H_{\vec{k},\mathrm{gg}}(x) + \delta(1-x)\left(2C_A\ln(\delta) + \frac{\beta_0}{2}\right) \tag{48}$$

where we introduced another infrared cut-off δ to seperate soft ($x \geq 1-\delta$) and hard ($x < 1-\delta$) gluons that is connected to Δ via $\delta = \Delta/(s'+t_1)$. The structure here explains why we were able to write the equation (32).

The light quark process can be regularized in a complete analogous way:

$$s'^2 \frac{d^2\sigma^{(1),fin}_{\vec{k},q}(s',t_1,u_1,q^2,\mu_F)}{dt_1 du_1} = \lim_{\epsilon\to 0}\left[s'^2 \frac{d^2\sigma^{(1)}_{\vec{k},q}(s',t_1,u_1,q^2,\epsilon)}{dt_1 du_1} \right.$$

$$- \int_0^1 \frac{dx_1}{x_1}\Gamma^{(1)}_{\vec{k},\mathrm{g}q}(x_1,\mu_F^2,\mu_D,\epsilon)$$

$$\left. (x_1 s')^2 \frac{d^2\sigma^{(0)}_{\vec{k},\mathrm{g}}(x_1 s', x_1 t_1, u_1, q^2,\epsilon)}{d(x_1 t_1) du_1}\right] \tag{49}$$

The needed splitting functions $P_{\vec{k},\mathrm{g}q}$ have been already quoted in equations (41) and (42). Note that $K_{q\gamma} = 1/(N_C) = 2C_F K_{\mathrm{g}\gamma}$.

The final finite cross sections are then

$$
\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),H,fin}_{\vec{k},\mathrm{g}}}{dt_1 du_1} = \frac{1}{2\pi} K_{\mathrm{g}\gamma} \alpha \alpha_S e_H^2 N_C C_F \Bigg[& -\frac{1}{u_1} P^H_{\vec{k},\mathrm{gg}}(x_1) \\
& \left\{ 4\pi B^{(0)}_{\vec{k},QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)} \right) - \ln(\mu_F^2/m^2) \right) \right. \\
& \left. -8\pi B^{(1)}_{\vec{k},QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \right\} \\
& + C_A \frac{s_4}{s_4+m^2} \left(\int d\Omega_n d\hat{\mathcal{I}}\, R_{\vec{k},OK} \right)^{finite} \\
& + 2C_F \frac{s_4}{s_4+m^2} \int d\Omega_4 d\hat{\mathcal{I}}\, R_{\vec{k},QED} \Bigg]
\end{aligned} \tag{50}
$$

$$
\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),S+V,fin}_{\vec{k},\mathrm{g}}}{dt_1 du_1} = 4K_{\mathrm{g}\gamma} \alpha \alpha_S e_H^2 N_C C_F B^{(0)}_{\vec{k},QED} \delta(s'+t_1+u_1) \Big[& C_A \ln^2(\Delta/m^2) \\
& + \ln(\Delta/m^2) \Big(\Big(\ln(-t_1/m^2) - \ln(-u_1/m^2) - \ln(\mu_F^2/m^2) \Big) C_A \\
& - \frac{2m^2 - s}{s\beta} \ln(\chi)(C_A - 2C_F) - 2C_F \Big) \\
& + \frac{\beta_0^{lf}}{4} \Big(\ln(\mu_R^2/m^2) - \ln(\mu_F^2/m^2) \Big) + f_{\vec{k}}(s', u_1, t_1, q^2) \Big]
\end{aligned} \tag{51}
$$

where $f_{\vec{k}}$ contains lots of logarithms and dilogarithms, but does not depend on Δ, μ_F^2, μ_R^2 nor n_f and $\beta_0^{lf} = (11C_A - 2n_{lf})/3$.

$$
\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),fin}_{\vec{k},q}}{dt_1 du_1} = \frac{1}{2\pi} K_{q\gamma} \alpha \alpha_S N_C \Bigg[& -\frac{1}{u_1} e_H^2 P_{\vec{k},\mathrm{g}q}(x_1) \\
& \left\{ 2\pi B^{(0)}_{\vec{k},QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)} \right) - \ln(\mu_F^2/m^2) - 2\partial_\epsilon E_{\vec{k}}(\epsilon=0) \right) \right. \\
& \left. -4\pi B^{(1)}_{\vec{k},QED} \begin{pmatrix} s' \to x_1 s' \\ t_1 \to x_1 t_1 \end{pmatrix} \right\} \\
& + C_F \frac{s_4}{s_4+m^2} \left(\int d\Omega_n d\hat{\mathcal{I}}\, e_H^2 A_{\vec{k},1} \right)^{finite} \\
& + C_F \frac{s_4}{s_4+m^2} \int d\Omega_4 d\hat{\mathcal{I}}\, e_L^2 A_{\vec{k},2} + C_F \frac{s_4}{s_4+m^2} \int d\Omega_4 d\hat{\mathcal{I}}\, e_H e_L A_{\vec{k},3} \Bigg]
\end{aligned} \tag{52}
$$

5 Partonic Results

FiXme Error!

FiXme Error: write partonic

5.1 $c_{\mathbf{g}}^{(0)}$

In leading order, we find

$$c^{(0)}_{\mathrm{VV},F_2,\mathrm{g}} = -\frac{\pi{\rho'}^3}{4\rho^2{\rho_q}^2}\Big[2\beta\left(\rho^2+{\rho_q}^2+\rho\rho_q(6+\rho_q)\right) + \left(2{\rho_q}^2+2\rho{\rho_q}^2+\rho^2(2-(-4+\rho_q)\rho_q)\right)\ln(\chi)\Big] \tag{53}$$

$$c^{(0)}_{\mathrm{VV},F_L,\mathrm{g}} = -\frac{\pi{\rho'}^3}{\rho\rho_q}\left[2\beta+\rho\ln(\chi)\right] \tag{54}$$

$$c^{(0)}_{\mathrm{VV},2xg_1,\mathrm{g}} = \frac{\pi{\rho'}^2}{2\rho\rho_q}\left[\beta(\rho+3\rho_q)+(\rho+\rho_q)\ln(\chi)\right] \tag{55}$$

$$c^{(0)}_{\mathrm{AA},F_2,\mathrm{g}} = \frac{\pi{\rho'}^3}{4\rho^2{\rho_q}^2}\Big[2\beta\left(\rho^2+{\rho_q}^2+\rho\rho_q(6+\rho_q)\right) - \left(-6\rho{\rho_q}^2+2(-1+\rho_q){\rho_q}^2+\rho^2(-2+(-2+\rho_q)\rho_q)\right)\ln(\chi)\Big] \tag{56}$$

$$c^{(0)}_{\mathrm{AA},F_L,\mathrm{g}} = -\frac{\pi{\rho'}^3}{2\rho^2\rho_q}\Big[2\beta\rho(2+\rho_q)-\left(\rho^2(-1+\rho_q)-4\rho\rho_q+{\rho_q}^2\right)\ln(\chi)\Big] \tag{57}$$

$$c^{(0)}_{\mathrm{AA},2xg_1,\mathrm{g}} = c^{(0)}_{\mathrm{VV},2xg_1,\mathrm{g}} \tag{58}$$

$$c^{(0)}_{\mathrm{VA},xF_3,\mathrm{g}} = c^{(0)}_{\mathrm{VA},g_4,\mathrm{g}} = c^{(0)}_{\mathrm{VA},g_L,\mathrm{g}} = 0 \tag{59}$$

Near threshold we find

$$c^{(0),\mathrm{thr}}_{\mathrm{VV},F_2,\mathrm{g}} = \frac{\pi\beta\rho_q}{2(\rho_q-1)} \tag{60}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{VV},F_L,\mathrm{g}} = \frac{4\pi\beta^3\rho_q^2}{3(1-\rho_q)^3} \tag{61}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{VV},2xg_1,\mathrm{g}} = c^{(0),\mathrm{thr}}_{\mathrm{VV},F_2,\mathrm{g}} \tag{62}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{AA},F_2,\mathrm{g}} = \frac{\pi\beta\rho_q^2}{1-\rho_q} \tag{63}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{AA},F_2,\mathrm{g}} = \frac{\pi\beta(1-2\rho_q)\rho_q}{2(\rho_q-1)} \tag{64}$$

$$c^{(0),\mathrm{thr}}_{\mathrm{AA},2xg_1,\mathrm{g}} = c^{(0),\mathrm{thr}}_{\mathrm{VV},2xg_1,\mathrm{g}} \tag{65}$$

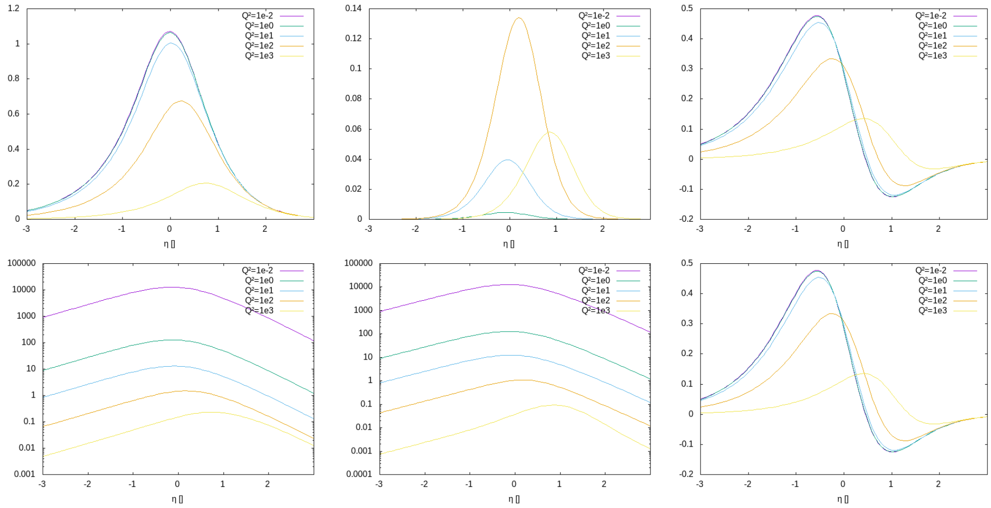


Figure 2: leading order scaling functions $c^{(0)}_{k,\mathrm{g}}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.2 $c_{\mathbf{g}}^{(1)}$

Near threshold, we find

$$c^{(1),\mathrm{thr}}_{\vec{k},\mathrm{g}} = c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{1}{\pi^2}\left[C_A\left(a^{(1,2)}_{\vec{k},\mathrm{g}}\ln^2(\beta) + a^{(1,1)}_{\vec{k},\mathrm{g}}\ln(\beta) - \frac{\pi^2}{16\beta} + a^{(1,0)}_{\vec{k},\mathrm{g},\mathrm{OK}}\right)\right.$$
$$\left. +2CF\left(\frac{\pi^2}{16\beta} + a^{(1,0)}_{\vec{k},\mathrm{g},\mathrm{QED}}\right)\right], \qquad (66)$$

with

$$a^{(1,2)}_{\vec{k},\mathrm{g}} = 1 \qquad (67)$$
$$a^{(1,1)}_{\mathrm{VV},F_2,\mathrm{g}} = -\frac{5}{2} + 3\ln(2) \qquad (68)$$
$$a^{(1,1)}_{\mathrm{VV},F_L,\mathrm{g}} = a^{(1,1)}_{\mathrm{VV},F_2,\mathrm{g}} - \frac{2}{3} \qquad (69)$$
$$a^{(1,1)}_{\mathrm{VV},2xg_1,\mathrm{g}} = a^{(1,1)}_{\mathrm{AA},F_2,\mathrm{g}} = a^{(1,1)}_{\mathrm{AA},F_L,\mathrm{g}} = a^{(1,1)}_{\mathrm{AA},2xg_1,\mathrm{g}} = a^{(1,1)}_{\mathrm{VV},F_2,\mathrm{g}} \qquad (70)$$

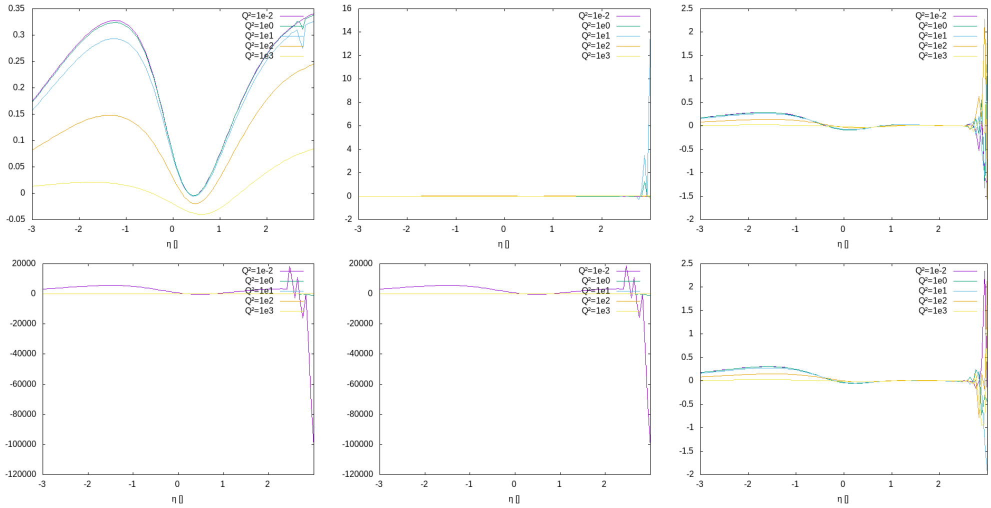


Figure 3: next-to-leading order scaling functions $c^{(1)}_{k,\mathrm{g}}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.3 $\bar{c}_{\mathrm{g}}^{(1)}$

For the scaling functions we find at this order:

$$\bar{c}^{(1)}_{\mathrm{VA},xF_3,\mathrm{g}} = \bar{c}^{(1)}_{\mathrm{VA},g_4,\mathrm{g}} = \bar{c}^{(1)}_{\mathrm{VA},g_L,\mathrm{g}} = 0 \tag{71}$$

and

$$\bar{c}^{(1)}_{\mathrm{VV},2xg_1,\mathrm{g}} = \bar{c}^{(1)}_{\mathrm{AA},2xg_1,\mathrm{g}} \tag{72}$$

and furthermore near threshold, we find

$$\bar{c}^{(1),\mathrm{thr}}_{\vec{k},\mathrm{g}} = c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{1}{\pi^2} C_A \left(\bar{a}^{(1,1)}_{\vec{k},\mathrm{g}} \ln(\beta) + \bar{a}^{(1,0)}_{\vec{k},\mathrm{g}} \right) , \tag{73}$$

with

$$\bar{a}^{(1,1)}_{\vec{k},\mathrm{g}} = -\frac{1}{2} \tag{74}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_2,\mathrm{g}} = -\frac{1}{4} \ln\left(\frac{16\chi_q}{(1+\chi_q)^2}\right) + \frac{1}{2} \tag{75}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_L,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,\mathrm{g}} + \frac{1}{6} \tag{76}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},2xg_1,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{AA},F_2,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{AA},F_L,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{AA},2xg_1,\mathrm{g}} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,\mathrm{g}} \tag{77}$$

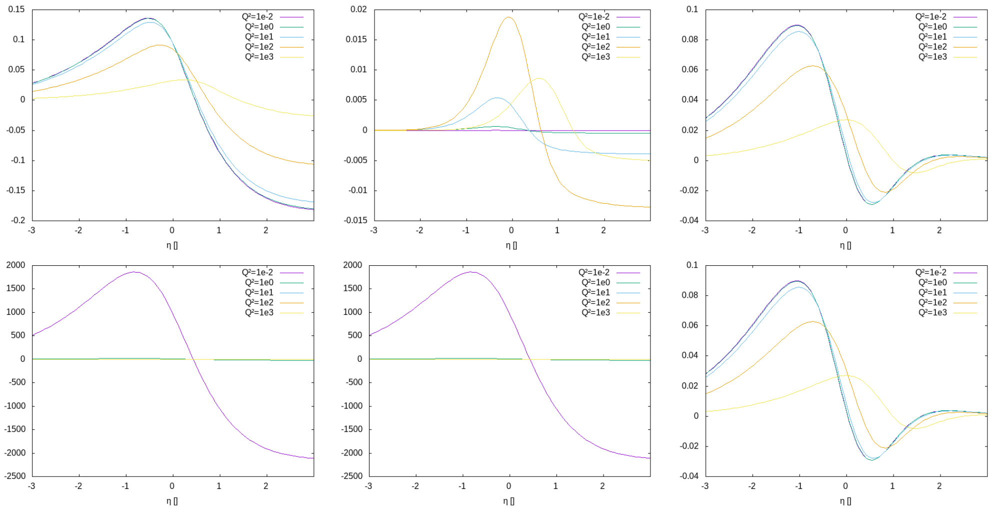


Figure 4: next-to-leading order scaling functions $\bar{c}^{(1)}_{k,\mathrm{g}}(\eta, \xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.4 $c_{\mathsf{q}}^{(1)}$

Near threshold, we find

$$c^{(1),\mathrm{thr}}_{\vec{k},q} = c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{\beta^2 \rho_q}{\pi^2(\rho_q - 1)} \frac{K_{q\gamma}}{6K_{\mathrm{g}\gamma}} \left[a^{(1,1)}_{\vec{k},q} \ln(\beta) + a^{(1,0)}_{\vec{k},q}\right], \tag{78}$$

with

$$a^{(1,1)}_{\mathrm{VV},F_2,q} = 1 \tag{79}$$

$$a^{(1,1)}_{\mathrm{VV},F_L,q} = a^{(1,1)}_{\mathrm{VV},F_2,q} - \frac{2}{3} \tag{80}$$

$$a^{(1,1)}_{\mathrm{VV},2xg_1,q} = a^{(1,1)}_{\mathrm{AA},F_2,q} = a^{(1,1)}_{\mathrm{AA},F_L,q} = a^{(1,1)}_{\mathrm{AA},2xg_1,q} = a^{(1,1)}_{\mathrm{VV},F_2,q} \tag{81}$$

$$a^{(1,0)}_{\mathrm{VV},F_2,q} = -\frac{13}{12} + \frac{3}{2}\ln(2) \tag{82}$$

$$a^{(1,0)}_{\mathrm{VV},F_L,q} = -\frac{77}{100} + \frac{9}{10}\ln(2) \tag{83}$$

$$a^{(1,0)}_{\mathrm{VV},2xg_1,q} = a^{(1,0)}_{\mathrm{VV},F_2,q} - \frac{1}{4} \tag{84}$$

$$a^{(1,0)}_{\mathrm{AA},F_2,q} = a^{(1,0)}_{\mathrm{AA},F_L,q} = a^{(1,0)}_{\mathrm{AA},2xg_1,q} = a^{(1,0)}_{\mathrm{VV},F_2,q} \tag{85}$$

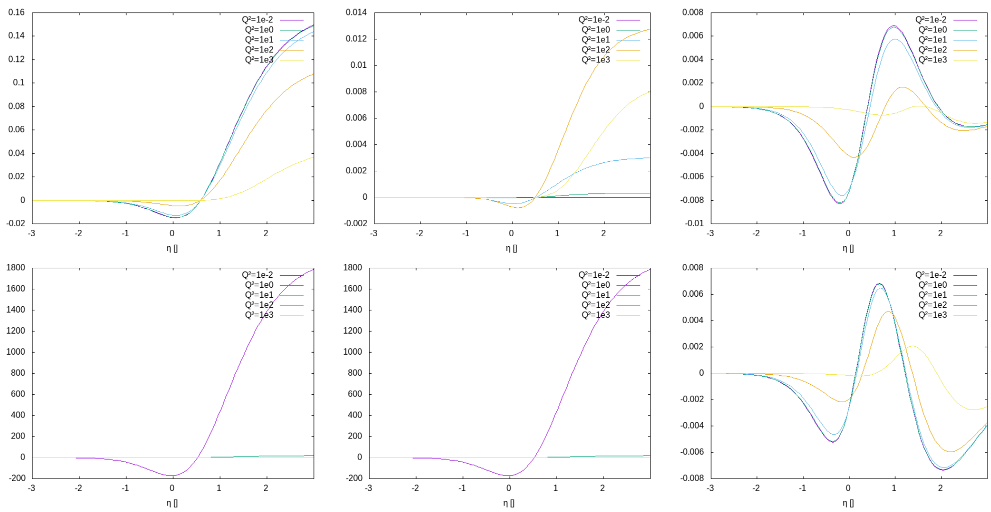


Figure 5: next-to-leading order scaling functions $c^{(1)}_{k,q}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.5 $\bar{c}_{\mathbf{q}}^{(1),F}$

For the scaling functions we find at this order:

$$\bar{c}^{(1),F}_{\mathrm{VA},xF_3,q} = \bar{c}^{(1),F}_{\mathrm{VA},g_4,q} = \bar{c}^{(1),F}_{\mathrm{VA},g_L,q} = 0 \tag{86}$$

and

$$\bar{c}^{(1),F}_{\mathrm{VV},2xg_1,q} = \bar{c}^{(1),F}_{\mathrm{AA},2xg_1,q} \tag{87}$$

and furthermore near threshold, we find

$$\bar{c}^{(1),F,\mathrm{thr}}_{\vec{k},q} = -c^{(0),\mathrm{thr}}_{\vec{k},\mathrm{g}} \frac{\beta^2 \rho_q}{\pi^2(\rho_q - 1)} \frac{K_{q\gamma}}{24 K_{\mathrm{g}\gamma}} \cdot \bar{a}^{(1,0)}_{\vec{k},q} \tag{88}$$

with

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_2,q} = 1 \tag{89}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},F_L,q} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,q} - \frac{2}{3} \tag{90}$$

$$\bar{a}^{(1,0)}_{\mathrm{VV},2xg_1,q} = \bar{a}^{(1,0)}_{\mathrm{AA},F_2,q} = \bar{a}^{(1,0)}_{\mathrm{AA},F_L,q} = \bar{a}^{(1,0)}_{\mathrm{AA},2xg_1,q} = \bar{a}^{(1,0)}_{\mathrm{VV},F_2,q} \tag{91}$$

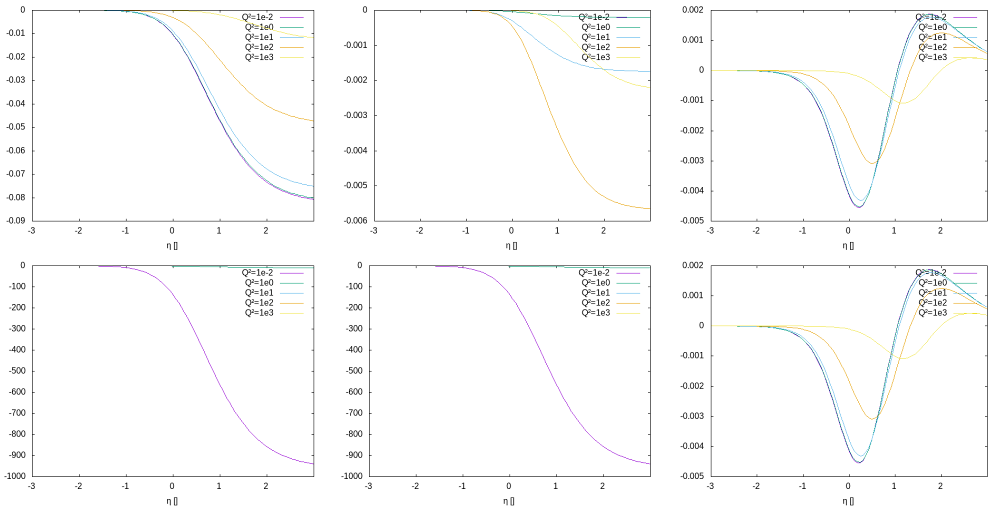


Figure 6: next-to-leading order scaling functions $\bar{c}^{(1),F}_{k,q}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

5.6 $d_{\mathbf{q}}^{(1)}$

For the scaling functions we find at this order:

$$d^{(1)}_{\mathrm{VA},g_4,q} = d^{(1)}_{\mathrm{VA},g_L,q} = 0 = d^{(1)}_{\mathrm{AA},2xg_1,q} \tag{92}$$

and

$$d^{(1)}_{\mathrm{VV},F_2,q} = d^{(1)}_{\mathrm{AA},F_2,q} \qquad d^{(1)}_{\mathrm{VV},F_L,q} = d^{(1)}_{\mathrm{AA},F_L,q} \qquad d^{(1)}_{\mathrm{VA},xF_3,q} = d^{(1)}_{\mathrm{VV},2xg_1,q} \tag{93}$$

For $\chi' \to 0$ we find:

$$\begin{aligned} d^{(1)}_{\mathrm{VV},F_2,q} &= -\frac{\rho}{9\pi}\left(\mathrm{Li}_2(-\chi) - \frac{1}{4}\ln^2(\chi) + \frac{\pi^2}{12} + \ln(\chi)\ln(1+\chi)\right) \\ &\qquad + \beta\frac{\rho(718+5\rho)}{2592\pi} + \frac{\rho(232+9\rho^2)}{1728\pi}\ln(\chi) + \mathcal{O}(\chi') \end{aligned} \tag{94}$$

$$d^{(1)}_{\mathrm{VV},F_L,q} = \left(\beta\frac{-38+23\rho}{54\pi} + \frac{-8+3\rho^2}{36\pi}\ln(\chi)\right)\chi' + \mathcal{O}(\chi'^2) \tag{95}$$

$$d^{(1)}_{\mathrm{VV},2xg_1,q} = d^{(1)}_{\mathrm{AA},F_2,q} = d^{(1)}_{\mathrm{VA},xF_3,q} \tag{96}$$

$$d^{(1)}_{\mathrm{AA},F_L,q} = d^{(1)}_{\mathrm{VV},F_L,q} \tag{97}$$

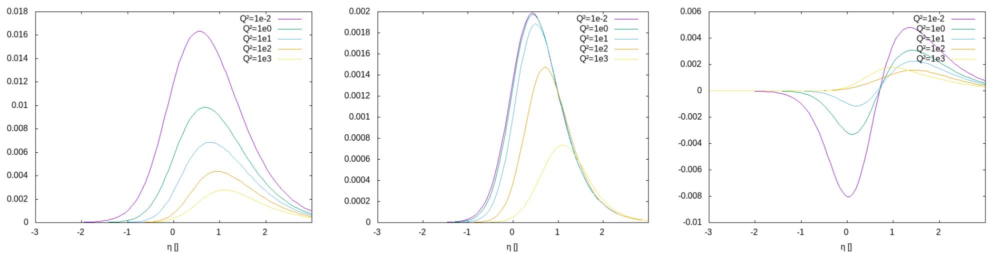


Figure 7: next-to-leading order scaling functions $\bar{c}^{(1),F}_{k,q}(\eta,\xi)$ plotted as function of $\eta = s/(4m^2) - 1$ for different values of Q^2 in units of GeV^2 at $m = 4.75\,\mathrm{GeV}$ (i.e. different values of $\xi = Q^2/m^2$)

6 Hadronic Results

FiXme Error!

FiXme Error: write hadronic

7 Summary

FiXme Error!

FiXme Error: write summary

A References

[1] **Particle Data Group** Collaboration, C. Patrignani et al., "Review of Particle Physics," Chin. Phys. **C40** no. 10, (2016) 100001.

[2] I. Bojak, NLO QCD corrections to the polarized photoproduction and hadroproduction of heavy quarks. PhD thesis, Dortmund U., 2000. `arXiv:hep-ph/0005120 [hep-ph]`.

[3] Beenakker, W. and Kuijf, H. and van Neerven, W. L. and Smith, J., "Qcd corrections to heavy-quark production in $p\bar{p}$ collisions," Phys. Rev. D **40** (Jul, 1989) 54–82. `http://link.aps.org/doi/10.1103/PhysRevD.40.54`.

[4] G. Altarelli and G. Parisi, "Asymptotic Freedom in Parton Language," Nucl. Phys. **B126** (1977) 298–318.

[5] W. Vogelsang, "A Rederivation of the spin dependent next-to-leading order splitting functions," Phys. Rev. **D54** (1996) 2023–2029, `arXiv:hep-ph/9512218 [hep-ph]`.

List of Corrections