Personal notes : DLCZ cheat sheet

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THE BASICS

Initially, we model our atomic ensemble as N atoms all prepared in the ground state $|\bar{g}\rangle = |g_1, g_2, ..., g_N\rangle$, and we consider a three level system with two ground states $|g\rangle$ and $|s\rangle$, and one excited state $|e\rangle$. A weak write pulse, with detuning Δ relative to the $|g\rangle \rightarrow |e\rangle$ transition is sent to the atomic cloud. This produces one or more Raman transitions to the state $|s\rangle$, accompanied by the emission of a write-out photon for each transfered atom. The probability to create one or more scattered photon is $\mathcal{P}$. After this first stage of the DLCZ protocol, the atom-light state can be described by a two-mode squeezed state :

$$|\psi_{\mathrm{w},at}\rangle = \sqrt{1-\mathcal{P}} \sum_{n=1}^{\infty} \mathcal{P}^{n/2} \frac{\left(a_{\mathrm{w}}^{\dagger} s_n^{\dagger}\right)^n}{n!} |0_1, \bar{g}\rangle \tag{1}$$

where $a_{\mathrm{w}}^{\dagger} = \int a_{\mathbf{k}_{\mathrm{w}}}^{\dagger} d\mathbf{k}_{\mathrm{w}}$ is the creation operator for the write-out photon and $s_i^{\dagger}$ is the operator that transfers the i-th atom in state $|s\rangle$. The next step of the DLCZ protocol consists in detecting one of this emitted write-out photon. Already, we see that if we have a perfect and number resolving detector, the detection of one write-out photon in mode $\mathbf{k}_{\mathrm{w}}$ projects the atomic ensemble into the collective spin excitation state :

$$|\bar{s}\rangle = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} |g_1, ..., s_n, ..., g_N\rangle = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \hat{s_n}^{\dagger} |g_1, ..., g_N\rangle \,. \tag{2}$$

In practice, detectors are not perfect, not number resolving, and photons can be lost before they reach out the detectors. This can lead to the generation of a mixed state in the atomic cloud as it is later discussed in this document.

The next stage consists in the read out. After a detection of a write-out photon in a specific $\mathbf{k}$ mode, a strong read pulse, resonant with the $|e\rangle \rightarrow |s\rangle$ transition, is sent after a short delay to the atomic ensemble. This pulse reads out the collective excitation by transferring back the atomic ensemble to state $|\bar{g}\rangle$ with the emission of a read-out photon. Its wavevector $\mathbf{k}_{\mathrm{r}}$ is defined by the phase matching condition $\mathbf{k}_{\mathrm{r}} = \mathbf{k}_W + \mathbf{k}_R - \mathbf{k}_{\mathrm{w}}$ where $\mathbf{k}_W(\mathbf{k}_R)$ are the write (read) laser pulse wavevectors.

If we don't herald the write-out photon, the joint field-field state can be written as a two-mode squeezed state:

$$|\psi_{\mathrm{w,r}}\rangle = \sqrt{1-\mathcal{P}} \sum_{n=1}^{\infty} \mathcal{P}^{n/2} \frac{\left(a_{\mathbf{k}_{\mathrm{w}}}^{\dagger} a_{\mathbf{k}_{\mathrm{r}}}^{\dagger}\right)^n}{n!} |0_{\mathbf{k}_{\mathrm{w}}}, 0_{\mathbf{k}_{\mathrm{r}}}\rangle \,. \tag{3}$$

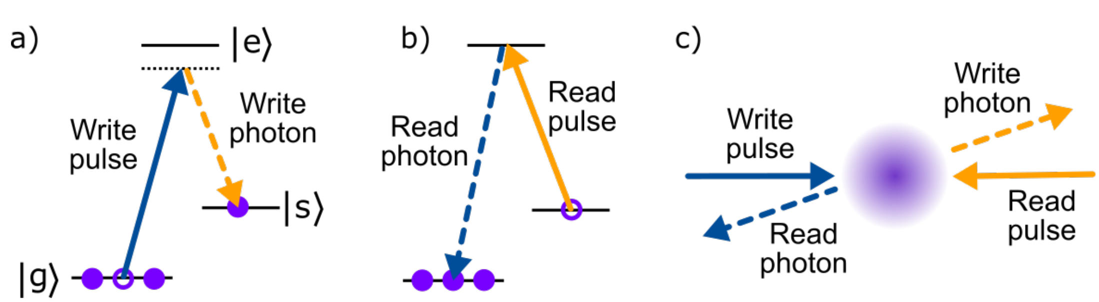

FIG. 1. DLCZ scheme in cold atoms. (a) An off-resonant write pulse scatters on the atoms, generating a spin wave. (b) The spin wave is read-out by a resonant read pulse. (c) Counterpropagating control pulses are linked to counterpropagating photons.

Note that this is true only in the ideal case of unitary read-out, meaning that each atom in state $|s\rangle$ is mapped to a read-out photon without any loss. If we set $\mathcal{P} \ll 1$, we can discard high-order terms and write the state as

$$|\psi_{\mathrm{w,r}}\rangle = \sqrt{1-\mathcal{P}}\left(|0_{\mathbf{k}_{\mathrm{w}}}, 0_{\mathbf{k}_{\mathrm{r}}}\rangle + \mathcal{P}^{1/2}\,|1_{\mathbf{k}_{\mathrm{w}}}, 1_{\mathbf{k}_{\mathrm{r}}}\rangle + \mathcal{P}\,|2_{\mathbf{k}_{\mathrm{w}}}, 2_{\mathbf{k}_{\mathrm{r}}}\rangle\right). \tag{4}$$

To satisfy $\mathcal{P} \ll 1$, it is necessary to excite the cloud with low intensity during the writing process. In this case, we see that if a write-out photon is detected, it mostly projects read-out into a single-photon Fock state with a small multiphoton component. If the write and read laser beams are counter-propagating, the read-out photon will be emitted in the opposite direction to that of write-out photon such that $\mathbf{k}_{\mathrm{w}} = -\mathbf{k}_{\mathrm{r}}$. Experimentally, this makes the read-out photon easy to collect in a fiber for later use.

CORRELATIONS

From the joint field-field state of eq. 3, a few observations can be made through calculation.

Cross correlation

The second order cross correlation function between the two photonic modes is defined as (in the following, all notations imply that correlators are evaluated at zero delay):

$$g^{(2)}_{\mathrm{w,r}} = \frac{< a^\dagger_{\mathrm{r}} a^\dagger_{\mathrm{w}} a_{\mathrm{w}} a_{\mathrm{r}} >}{< a^\dagger_{\mathrm{r}} a_{\mathrm{r}} >< a^\dagger_{\mathrm{w}} a_{\mathrm{w}} >}. \tag{5}$$

From a simple calculation, it follows that

$$< a^\dagger_{\mathrm{r}} a_{\mathrm{r}} >= \frac{\mathcal{P}}{1-\mathcal{P}} \qquad \text{and} \qquad < a^\dagger_{\mathrm{w}} a_{\mathrm{w}} >= \frac{\mathcal{P}}{1-\mathcal{P}}. \tag{6}$$

Then, we calculate the mean number of coincidences and we find that

$$< a^\dagger_{\mathrm{r}} a^\dagger_{\mathrm{w}} a_{\mathrm{w}} a_{\mathrm{r}} >= \frac{\mathcal{P}\,(\mathcal{P}+1)}{(1-\mathcal{P})^2}. \tag{7}$$

Finally, we get the following expression for the cross correlation

$$g^{(2)}_{\mathrm{w,r}} = 1 + \frac{1}{\mathcal{P}}, \tag{8}$$

where we see that high values of cross correlation are obtained for low probabilities of excitation.

Autocorrelation

Then, we study the second-order autocorrelation function of the write-out photon. To measure this quantity, we need to send the write-out photon to a 50/50 beam splitter, in such a way the photon mode is divided in two modes (we will use the notation $\mathbf{k}_{\mathrm{w}_1}$ and $\mathbf{k}_{\mathrm{w}_2}$ for them). To write the state after the Stokes photons have passed through the beam splitter, we first rewrite the two mode squeezed state of eq. 3 as a function of photons creation operators. The new state can be written as

$$|\psi_{\mathrm{w,r}}\rangle = \sqrt{1-\mathcal{P}}\sum_{n=1}^{\infty}\mathcal{P}^{n/2}\frac{\left(a^\dagger_{\mathbf{k}_{\mathrm{w}_1}} + i a^\dagger_{\mathbf{k}_{\mathrm{w}_2}}\right)^n}{\sqrt{2}}\frac{\left(a^\dagger_{\mathbf{k}_{\mathrm{r}}}\right)^n}{n!}\left|0_{\mathbf{k}_{\mathrm{w}_1}}, 0_{\mathbf{k}_{\mathrm{w}_2}}, 0_{\mathbf{k}_{\mathrm{r}}}\right\rangle. \tag{9}$$

After a long calculation, we find the second-order autocorrelation function of the write-out photon:

$$g^{(2)}_{\mathrm{w,w}} = \frac{< a^\dagger_{\mathrm{w}_1} a^\dagger_{\mathrm{w}_2} a_{\mathrm{w}_2} a_{\mathrm{w}_1} >}{< a^\dagger_{\mathrm{w}_1} a_{\mathrm{w}_1} >< a^\dagger_{\mathrm{w}_2} a_{\mathrm{w}_2} >} = 2. \tag{10}$$

We perform a similar calculation for the autocorrelation of the read-out photon and find that $g^{(2)}_{\mathrm{w,w}} = g^{(2)}_{\mathrm{r,r}} = 2$.

Antibunching

Finally, we want to compute the antibunching parameter $g^{(2)}$. This quantity measures the suppression of multiphoton components in the read-out mode, conditionned on a detection in the write-out mode. To measure this quantity we need to send the read-out photon mode to a 50/50 beam splitter. To write the two mode squeezed state after the read-out photons have passed through the beam splitter we proceed in a similar way as for the case of the write-out photon passing though the beam splitter. The antibunching parameter is defined by

$$h^{(2)} = \frac{< a_{\mathrm{w}}^{\dagger} a_{\mathrm{w}} >< a_{\mathrm{w}}^{\dagger} a_{\mathrm{r}_1}^{\dagger} a_{\mathrm{r}_2}^{\dagger} a_{\mathrm{r}_2} a_{\mathrm{r}_1} a_{\mathrm{w}} >}{< a_{\mathrm{w}}^{\dagger} a_{\mathrm{r}_1}^{\dagger} a_{\mathrm{r}_1} a_{\mathrm{w}} >< a_{\mathrm{w}}^{\dagger} a_{\mathrm{r}_2}^{\dagger} a_{\mathrm{r}_2} a_{\mathrm{w}} >}, \tag{11}$$

and we find after tedious calculation that

$$h^{(2)} = \frac{2\mathcal{P}\left(\mathcal{P}+2\right)}{\left(1+\mathcal{P}\right)^2}. \tag{12}$$

Interestingly, we see that in the limit of low excitation probability ($\mathcal{P} \ll 1$), we get

$$\lim_{\mathcal{P}\rightarrow 0}\left(h^{(2)}\right) = 4\mathcal{P} = \lim_{\mathcal{P}\rightarrow 0}\left(\frac{g_{\mathrm{w,w}}^{(2)} g_{\mathrm{w,r}}^{(2)}}{g_{\mathrm{r,r}}^{(2)}}\right) \tag{13}$$

So finally we can say that for two mode squeezed states and in the low photon pair creation probability regime, the following equation is satisfied.

$$h^{(2)} = \frac{g_{\mathrm{w,w}}^{(2)} g_{\mathrm{w,r}}^{(2)}}{g_{\mathrm{r,r}}^{(2)}} \tag{14}$$