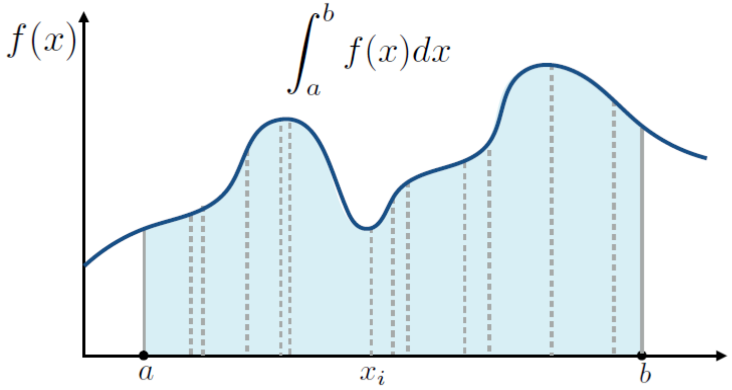
Machine Learning for Computer Graphics

Monte Carlo Integration for the Illumination Integral

Last Class



Monte Carlo Methods

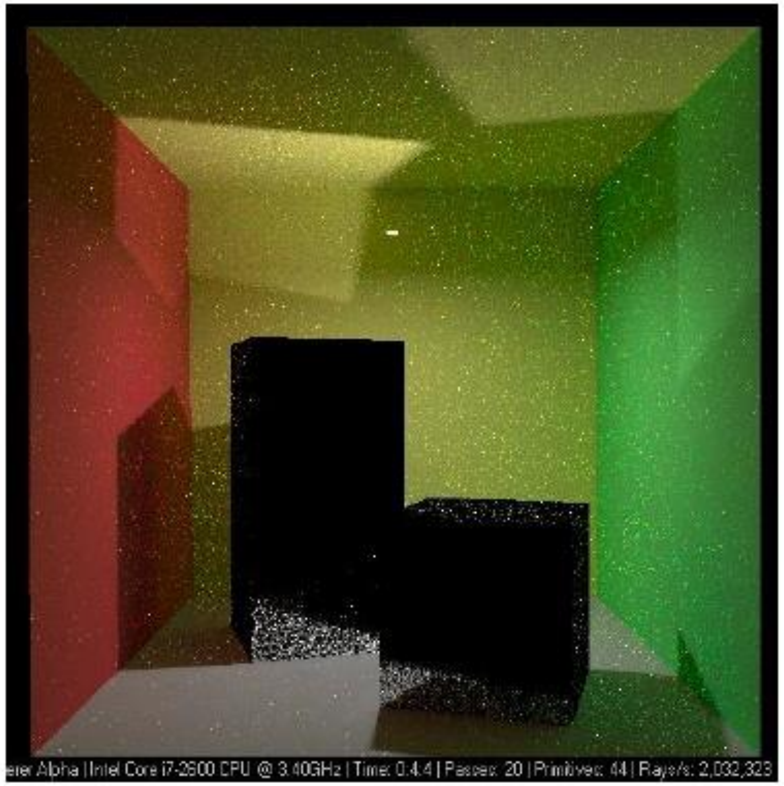
- History
- Details and Mathematical Grounding

Motivation

- Solve the Rendering Integral

This Class

Monte Carlo Integration for the Illumination Integral



Course Roadmap

Week no.	Class Date	Theory	Labs	Notes
Week 1	22/02/2022	1.1 Intro	-	-
Week 2	02/03/2022	1.2 Ray-Tracing Phong	Lab 1	P1 Phong
Week 3	08/03/2022	1.3 Radiometry and BRDF	Lab 2	
Week 4	15/03/2022	2.1 Monte Carlo (MC)	Lab 3	P2 MC
Week 5	**22/03/2022**	**2.2 MC Rendering**	**Lab 4**	
Week 6	29/03/2022	2.3 Bayesian Monte Carlo (BMC)	Lab 5	P3 BMC
	05/04/2022	PARTIAL EXAMS (NO CLASS)		
	12/04/2022	EASTER HOLYDAYS (NO CLASS)		
Week 7	19/04/2022	2.4 BMC Rendering	Lab 6	
Week 8	26/04/2022	3.1 MC Importance Sampling	Lab 7	P4 AMC
Week 9	03/05/2022	3.2 BMC Importance Sampling	Lab 8	Papers Out
	10/05/2022	AI SEMINAR MAI (NO CLASS)		
Week 10	17/05/2022	Presentations I		
Week 11	24/05/2022	Presentations II		

From the BRDF to the Illumination Integral

Summary of Basic Radiometric Quantities

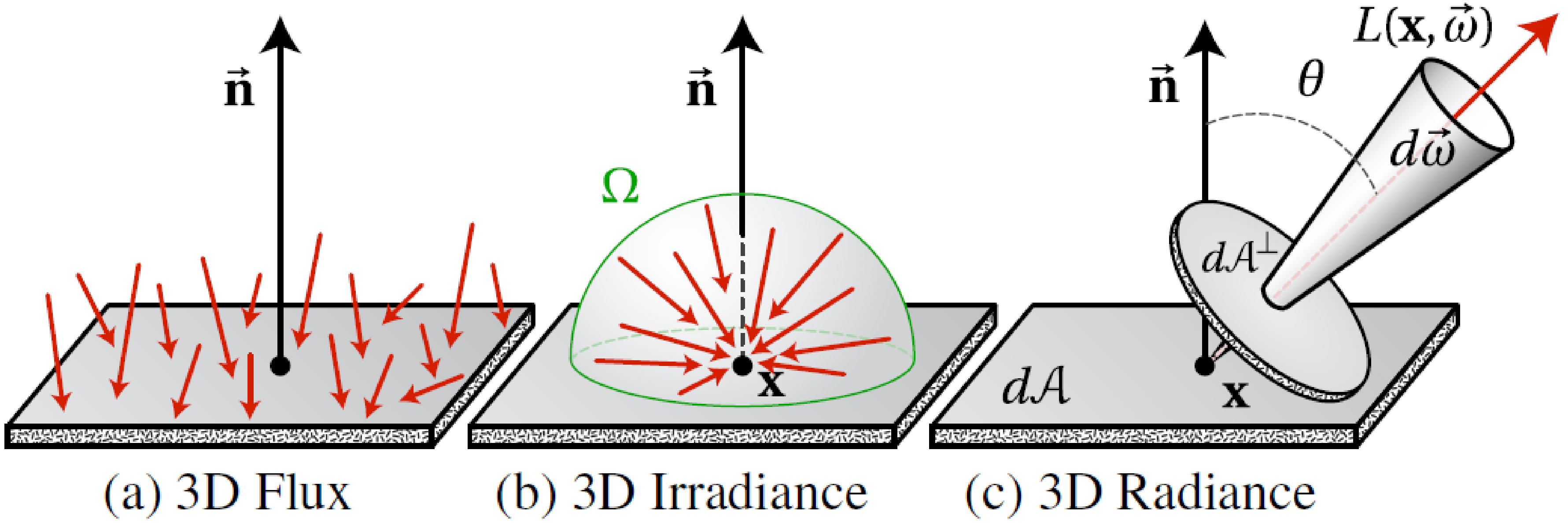
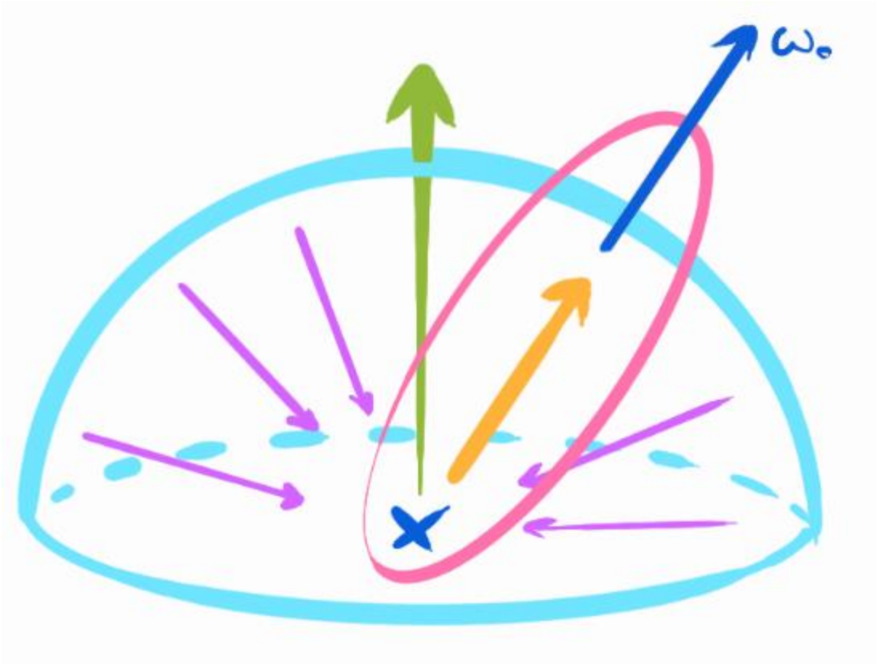


Image: Wojciech Jarosz

Illumination Integral (or Reflection Equation)

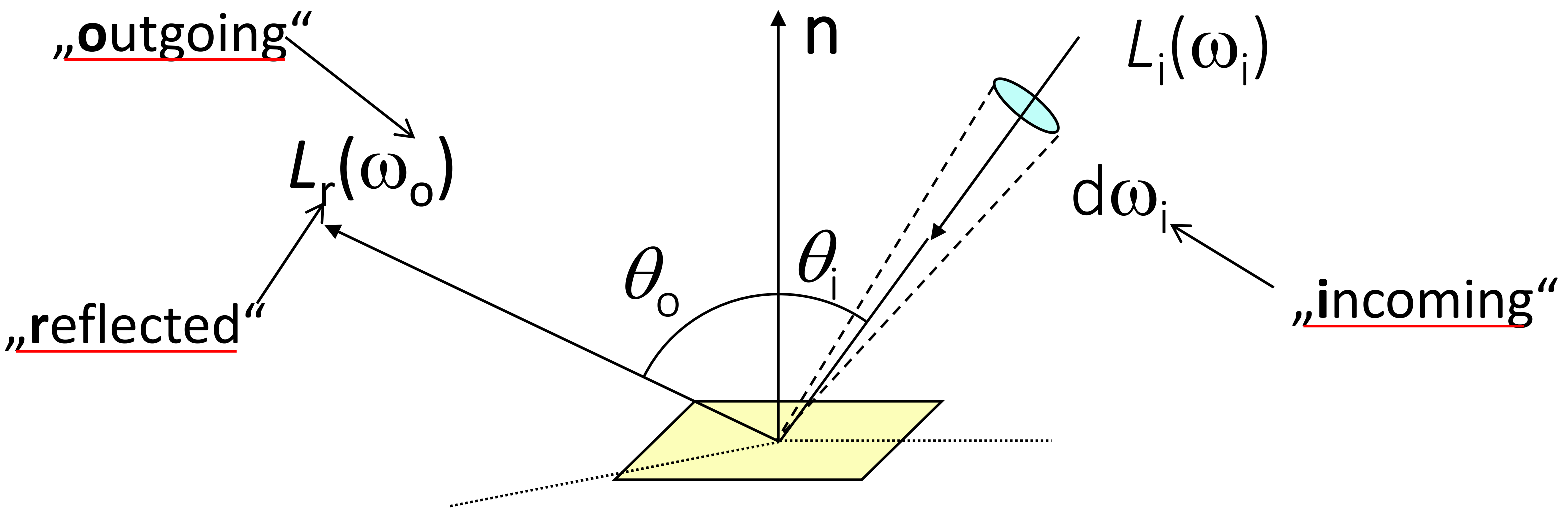
Objective:

- Compute how much radiance arrives to each pixel of our image
- Determine the reflected radiance at a given point x in direction ω_o



$$L_r(\omega_o) = ?$$

BRDF – Formal Definition



Bidirectional Reflectance Distribution Function (BRDF) [F. Nicodemus (1965)]

$$f_r(\omega_i, \omega_o) = \frac{\mathrm{d}L_r(\omega_o)}{\mathrm{d}E_i(\omega_i)} = \frac{\mathrm{d}L_r(\omega_o)}{L_i(\omega_i)\cos(\theta_i)\mathrm{d}\omega_i} \quad [1/sr]$$

Illumination Integral (or Reflection Equation)

Recall the BRDF equation:

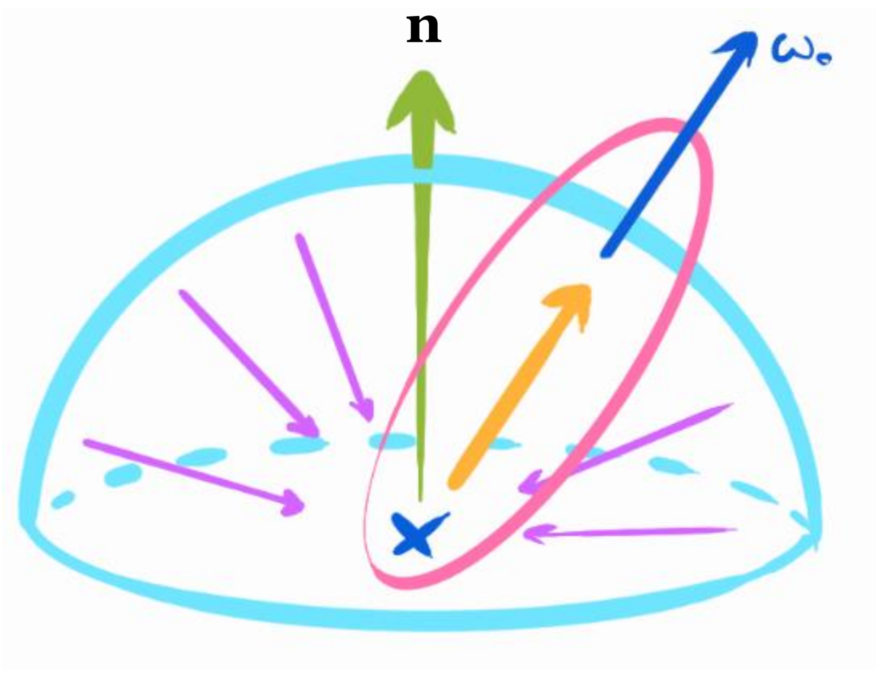
$$f_r(\omega_i, \omega_o) = \frac{\mathrm{d}\color{red}{L_r(\omega_o)}}{L_i(\omega_i)\cos(\theta_i)\mathrm{d}\omega_i}$$

From where:

$$\mathrm{d}\color{red}{L_r(\omega_o)} = f_r(\omega_i, \omega_o) L_i(\omega_i)\cos(\theta_i)\mathrm{d}\omega_i$$

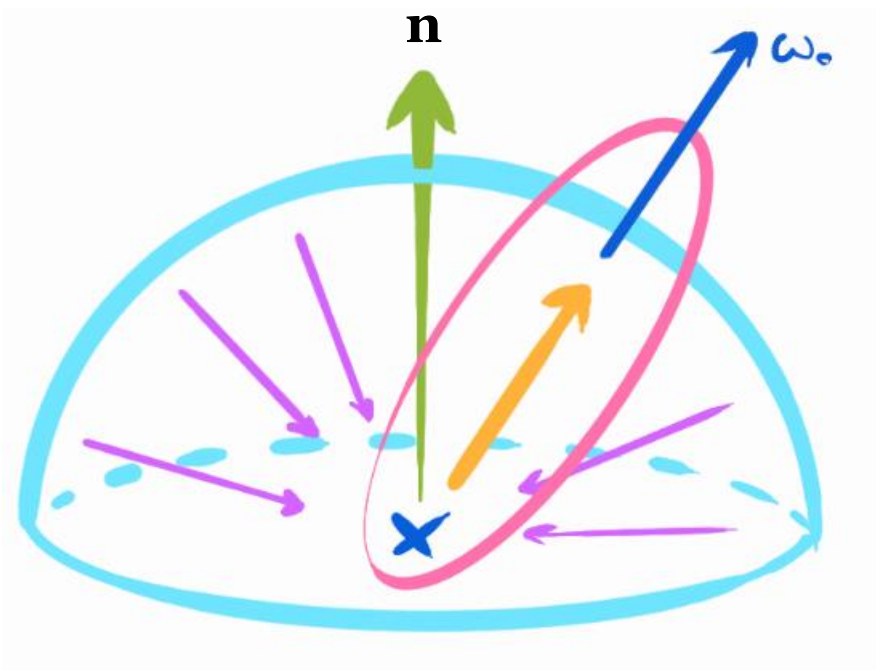
And then:

$$\color{red}{L_r(\omega_o)} = \int_{\Omega(x)} L_i(\omega_i) f_r(\omega_i, \omega_o)\cos\theta_i \,\mathrm{d}\omega_i$$



The different terms of the Illumination Integral

$$L_r(\omega_o) = \int_{\Omega(x)} L_i(\omega_i) f_r(\omega_i, \omega_o) \cos\theta_i \, \mathrm{d}\omega_i$$



$\Omega(x)$ is the hemisphere around the point x

$L_i(\omega_i)$ is the incident radiance from direction ω_i

$f_r(\omega_i, \omega_o)$ is the BRDF

$\cos\theta_i$ is the cosine of the angle between n and ω_i

Illumination Integral in Practice

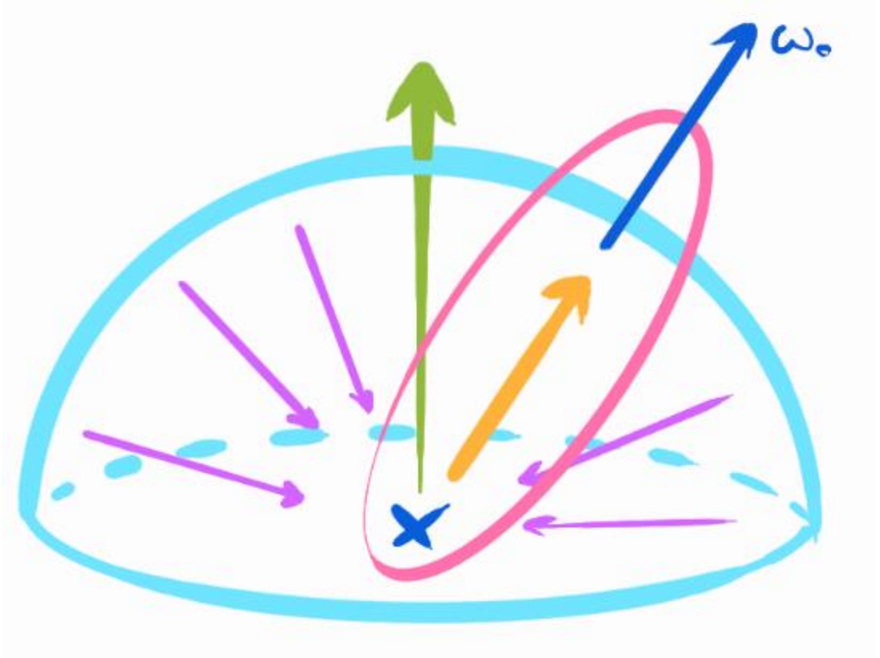
$$L_r(\omega_o) = \int_{\Omega(x)} L_i(\omega_i) f_r(\omega_i, \omega_o) \cos\theta_i \, \mathrm{d}\omega_i$$

No analytic solution in the general case

- $L_i(\omega_i)$ does not have an analytic expression
- $L_i(\omega_i)$ can only be known through sampling

Need to ressort to numerical methods

- Most common: *Monte Carlo Integration*

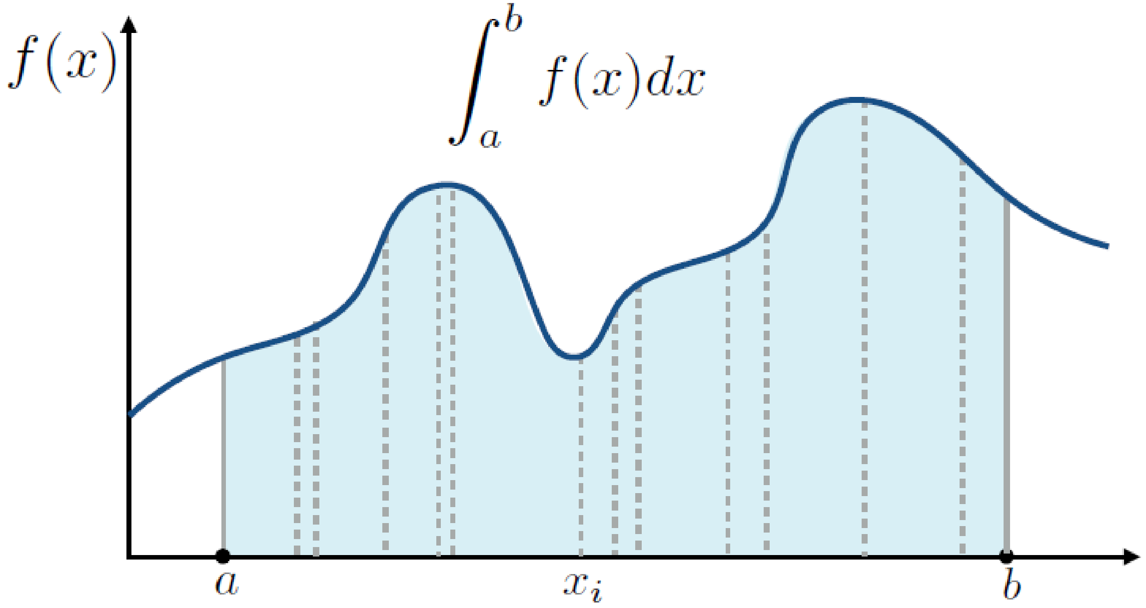


Recall: Monte Carlo Estimator

Estimate the integral by ***randomly*** sampling the integrand

General formula

$$\hat{I} = \frac{1}{N}\sum_{j=1}^{N}\frac{f(x_j)}{p(x_j)}$$



N is the number of used samples
x_j is the sample location
$f(x_j)$ is the sample value (or observation)
$p(x_j)$ is the probability of choosing x_j

Monte Carlo Integration for the Illumination Integral

$$\hat{I} = \frac{1}{N}\sum_{j=1}^{N}\frac{f(x_j)}{p(x_j)}$$

Problem Set-Up

Recall the Illumination Integral (what we want to estimate):

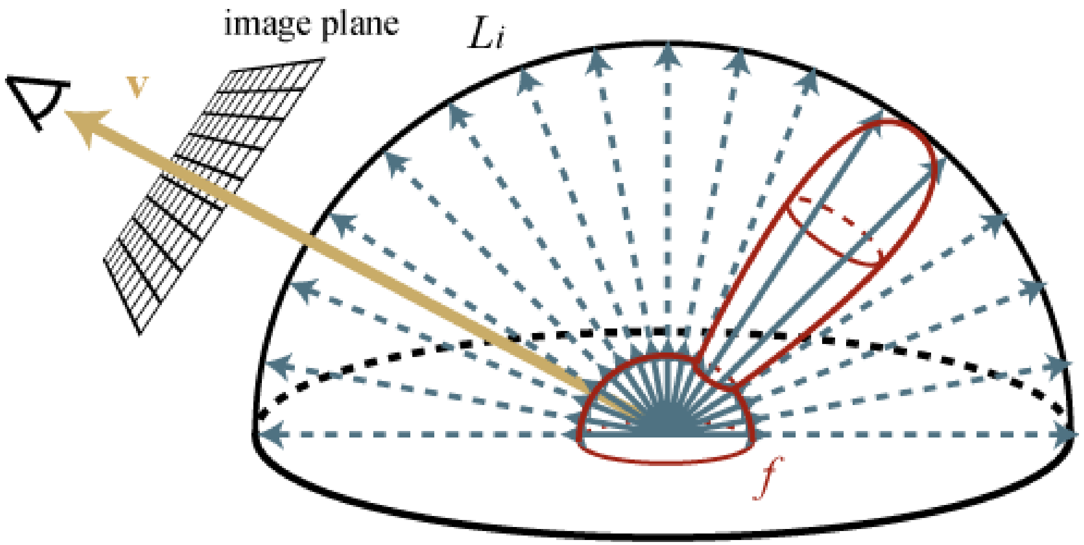
$$L_r(\omega_o) = \int_{\Omega(x)} L_i(\omega_i) f_r(\omega_i, \omega_o) \cos\theta_i \, \mathrm{d}\omega_i$$

Monte Carlo Estimator for the Illumination Integral:

$$\hat{L}_r(\omega_o) = \frac{1}{N}\sum_{n=1}^{N}\frac{L_i(\omega_n) f_r(\omega_n, \omega_o)\cos\theta_n}{p(\omega_n)}$$

Solving the Illumination Integral

$$L_r(\omega_o) \approx \hat{L}_r(\omega_o) = \frac{1}{N}\sum_{n=1}^{N} \frac{L_i(\omega_n) f_r(\omega_n, \omega_o) \cos\theta_n}{p(\omega_n)}$$

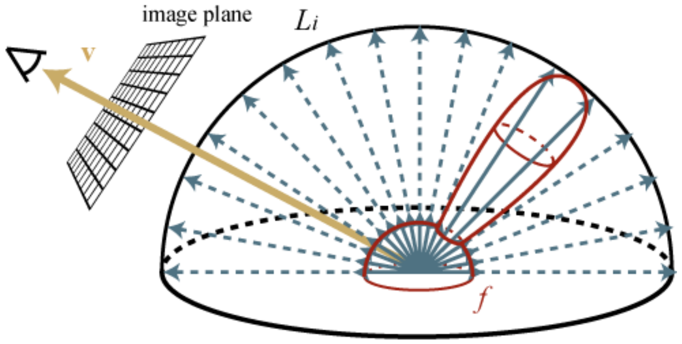


[Video]

Solving the Illumination Integral

$$L_r(\omega_o) \approx \hat{L}_r(\omega_o) = \frac{1}{N}\sum_{n=1}^{N}\frac{L_i(\omega_n) f_r(\omega_n,\omega_o)\cos\theta_n}{p(\omega_n)}$$

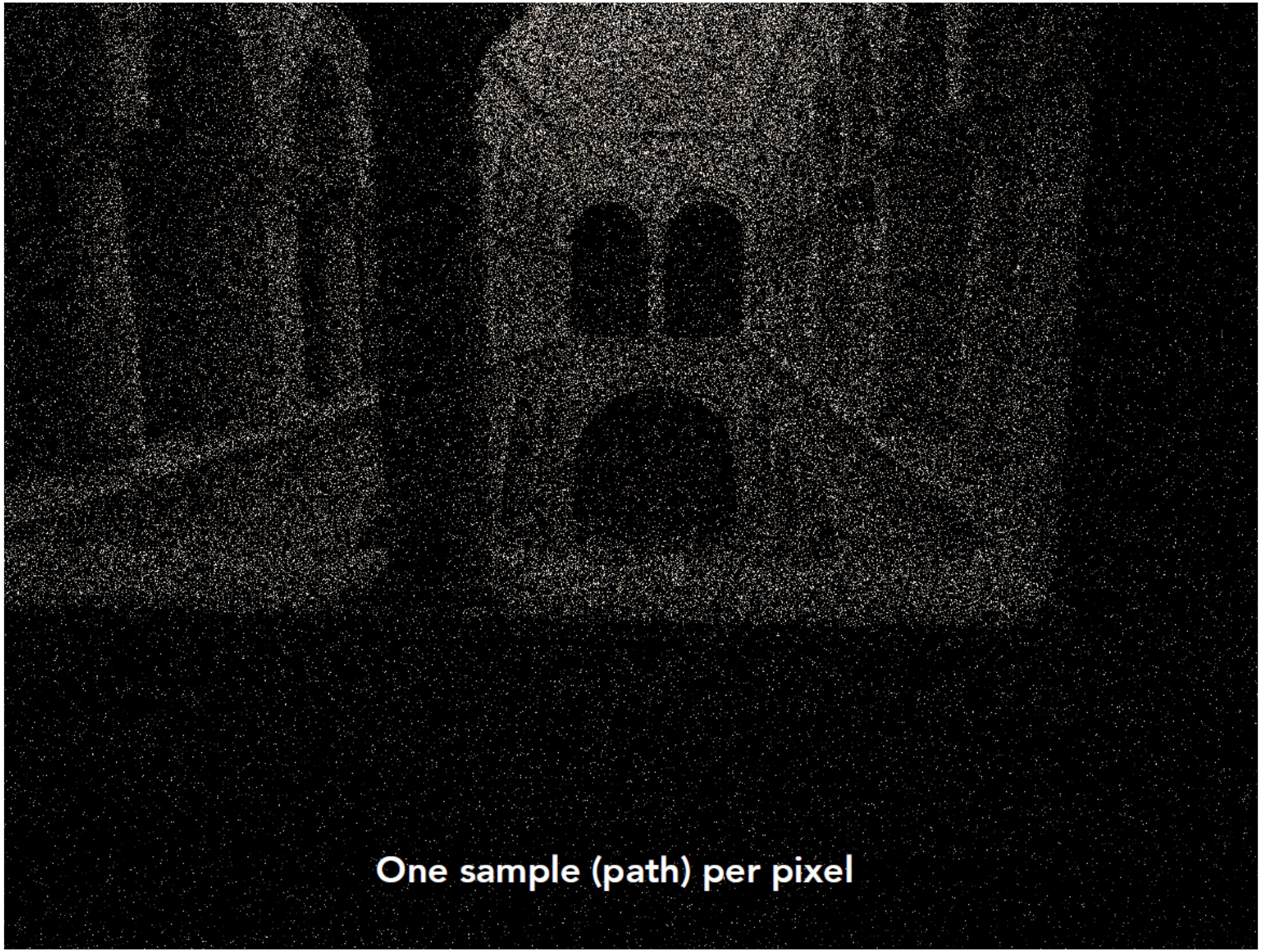
Monte Carlo Estimator:



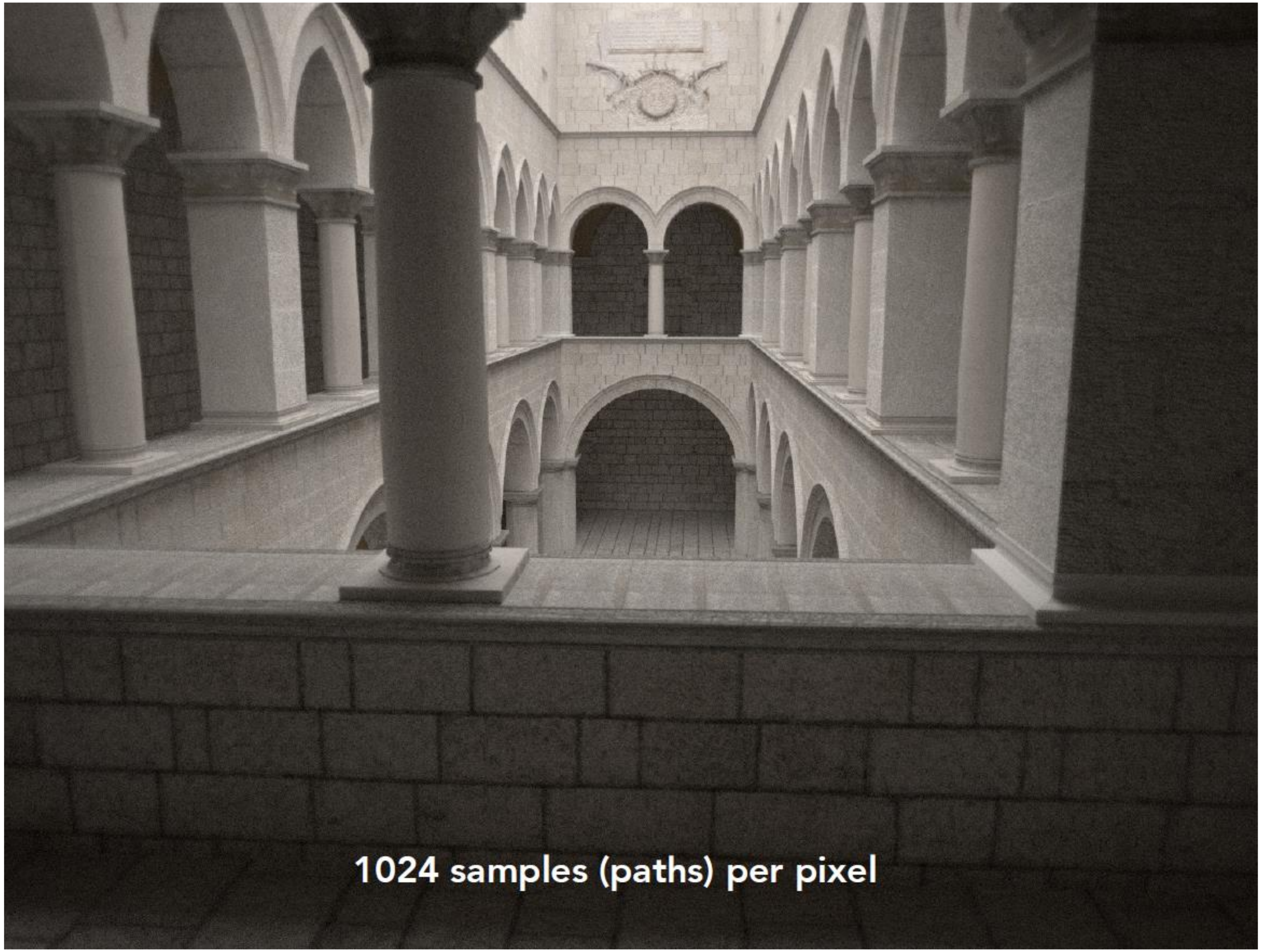
- Weighted average of integrand samples
- $p(\omega_n)$ is the probability of sampling direction ω_n
- Different choices for $p(\omega_n)$ are possible…
  - ... but some work much better than others
  - Can we choose/learn the best pdf? More details later.
- Requires a very large number of samples to converge to the correct solution

Expected Value of $\hat{L}_r(\omega_o)$

$$
\begin{aligned}
\mathrm{E}\left[\hat{L}_r(\omega_o)\right] &= \mathrm{E}\left[\frac{1}{N}\sum_{n=1}^{N}\frac{L_i(\omega_n)f_r(\omega_n,\omega_o)\cos\theta_n}{p(\omega_n)}\right] \\
&= \frac{1}{N}\mathrm{E}\left[\sum_{n=1}^{N}\frac{L_i(\omega_n)f_r(\omega_n,\omega_o)\cos\theta_n}{p(\omega_n)}\right] \\
&= \frac{1}{N}N\mathrm{E}\left[\frac{L_i(\omega_n)f_r(\omega_n,\omega_o)\cos\theta_n}{p(\omega_n)}\right] \\
&\stackrel{\text{def}}{=} \int_{\Omega(x)}\frac{L_i(\omega_i)f_r(\omega_i,\omega_o)\cos\theta_i}{p(\omega_n)}p(\omega_n)\,\mathrm{d}\omega_i \\
&= L_r(\omega_o)
\end{aligned}
$$


One sample (path) per pixel


32 samples (paths) per pixel


1024 samples (paths) per pixel

Practical Application to Rendering

A Practical Application to Rendering

$$L_r(\omega_o) \approx \hat{L}_r(\omega_o) = \frac{1}{N}\sum_{n=1}^{N}\frac{L_i(\omega_n) f_r(\omega_n, \omega_o)\cos\theta_n}{p(\omega_n)}$$

What we need:

- Generate random directions over the hemisphere following a uniform pdf
  - Simplest case (uniform pdf):

$$p(\omega) = \frac{1}{2\pi}$$

- Be able to evaluate the integrand for any direction ω_n

Sampling Hemisphere with Equal Probability

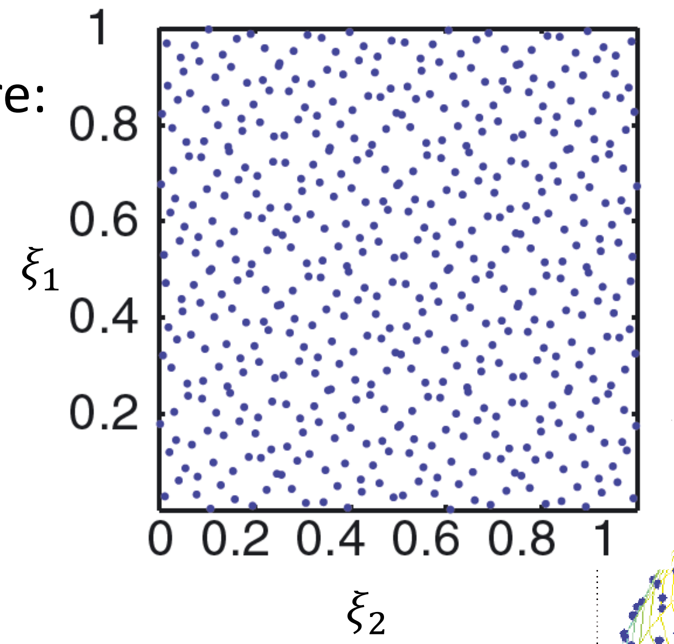
Consider a uniform pdf over the hemisphere:

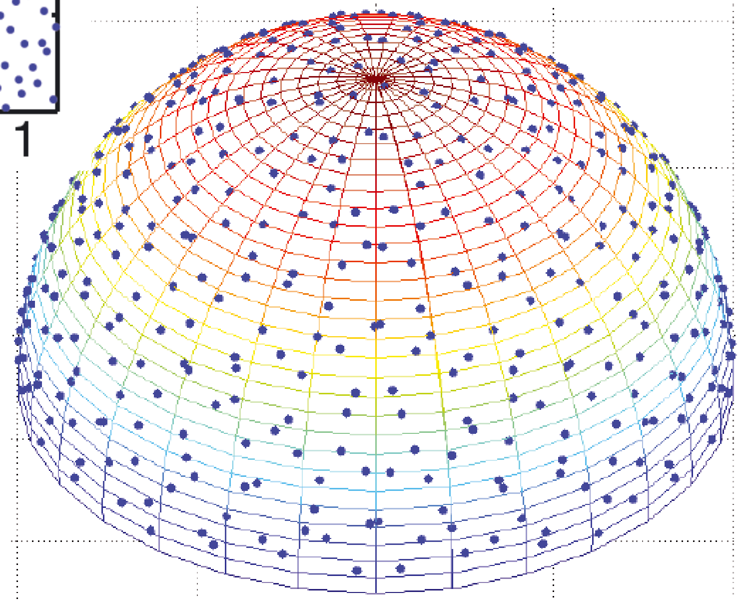
$$p(\omega) = \frac{1}{2\pi}$$

Consider $\xi_1 \sim \mathrm{U}(0,1)$ and $\xi_2 \sim \mathrm{U}(0,1)$

A random direction $\omega = (\theta, \phi)$ on the hemisphere is given by:

$$\begin{cases} \theta = 2\pi\xi_1 \\ \phi = \operatorname{acos}\xi_2 \end{cases}$$





Evaluating the Integrand

$$L_r(\omega_o) \approx \hat{L}_r(\omega_o) = \frac{1}{N}\sum_{n=1}^{N} \frac{L_i(\omega_n) f_r(\omega_n, \omega_o) \cos\theta_n}{p(\omega_n)}$$

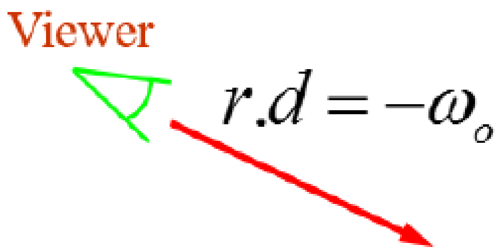
- Given ω_o and a random direction ω_n, we want to evaluate:

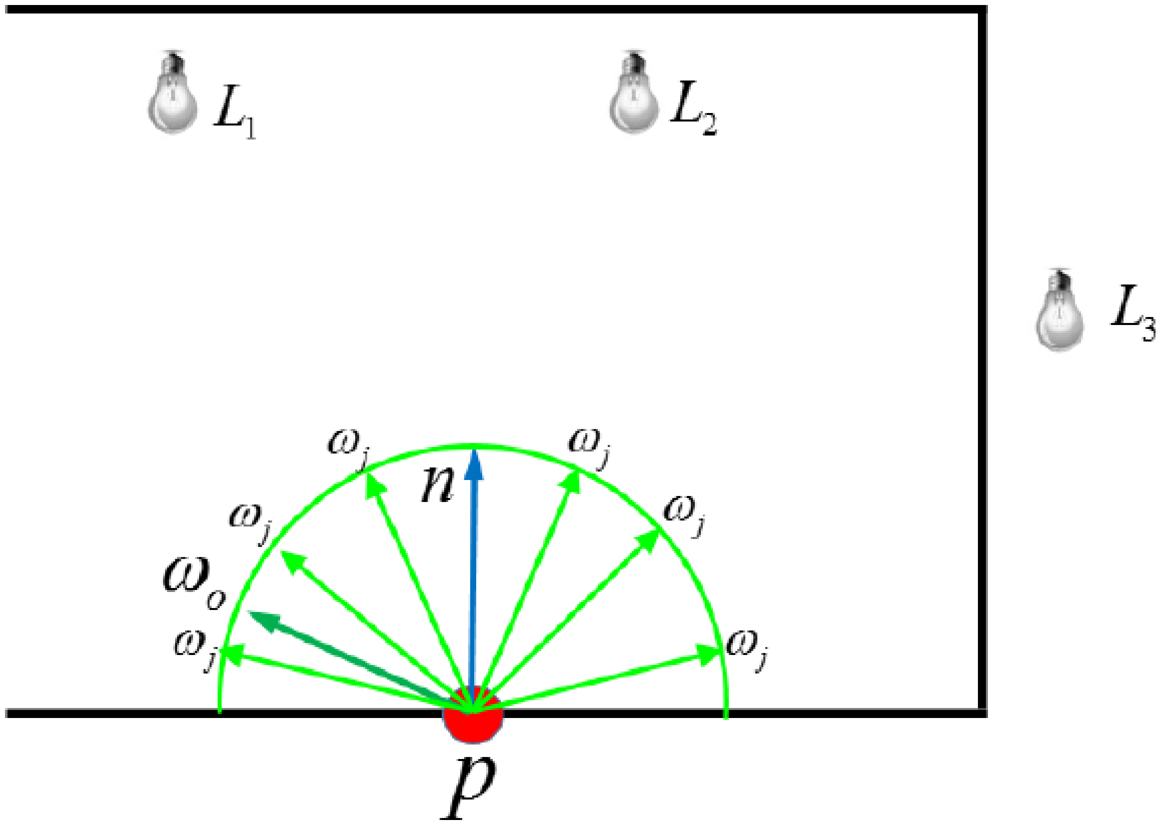
$$L_i(\omega_n) f_r(\omega_n, \omega_o) \cos\theta_n$$

- BRDF $f_r(\omega_n, \omega_o)$ can be easily evaluated if we have an analytic model (e.g., Phong)

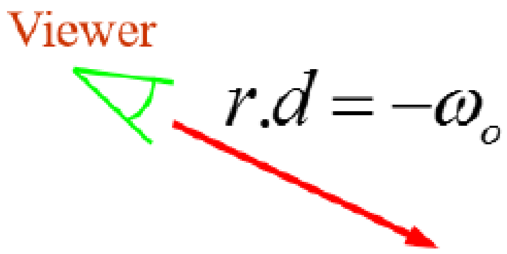
- As regards $L_i(\omega_n)$, things are not so simple…

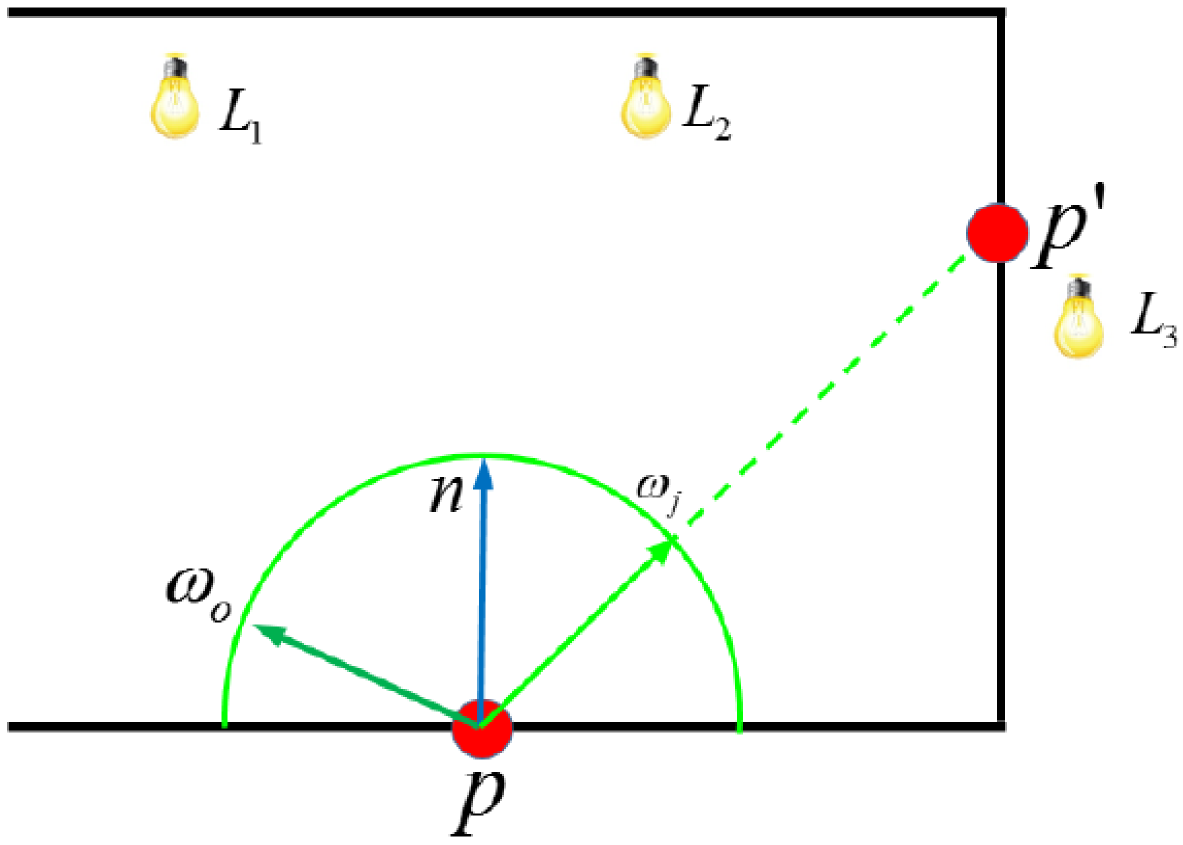
The Recursive Character of L_i



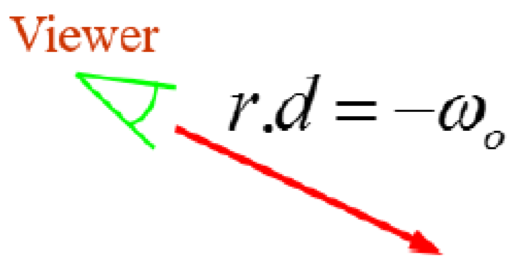


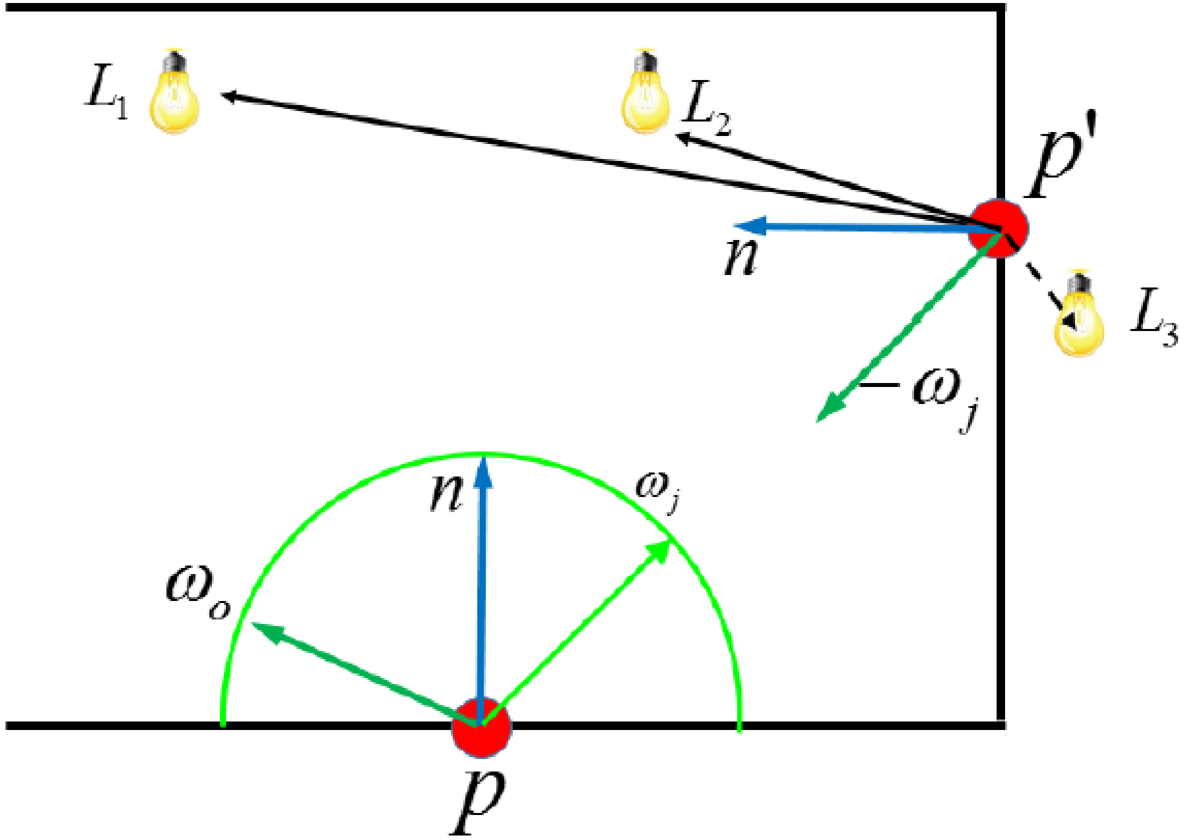
The Recursive Character of L_i



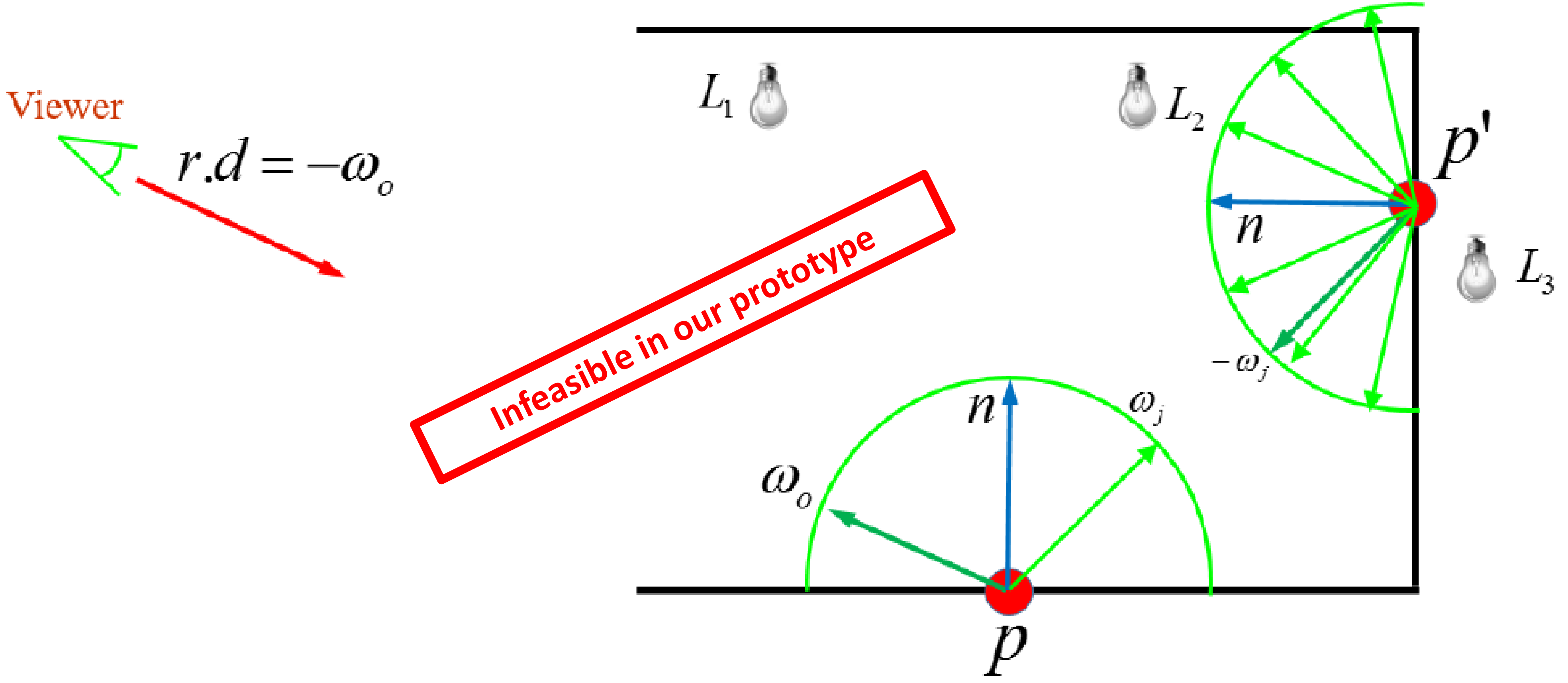


The Recursive Character of L_i





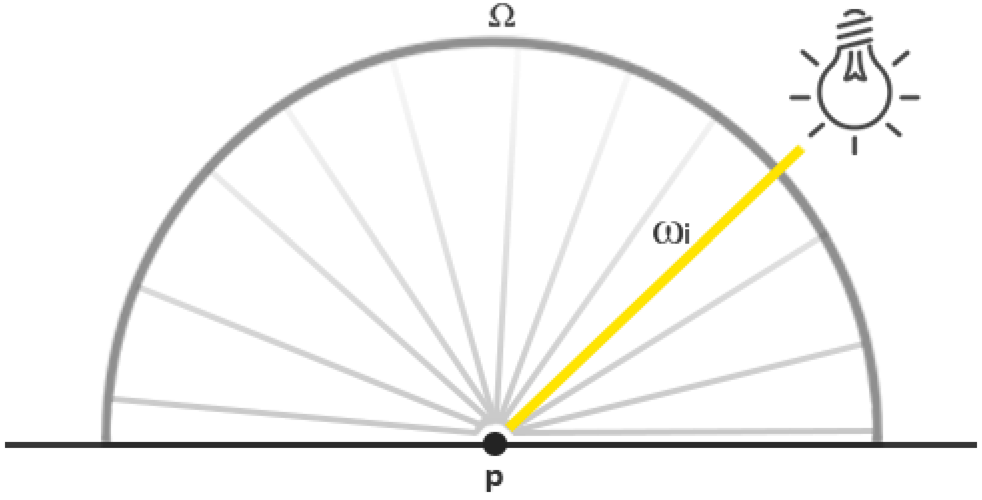
The Recursive Character of L_i



Evaluating the Integrand

$$L_r(\omega_o) \approx \hat{L}_r(\omega_o) = \frac{1}{N}\sum_{n=1}^{N}\frac{L_i(\omega_n) f_r(\omega_n, \omega_o)\cos\theta_n}{p(\omega_n)}$$

Practical option 1: Direct Illumination only



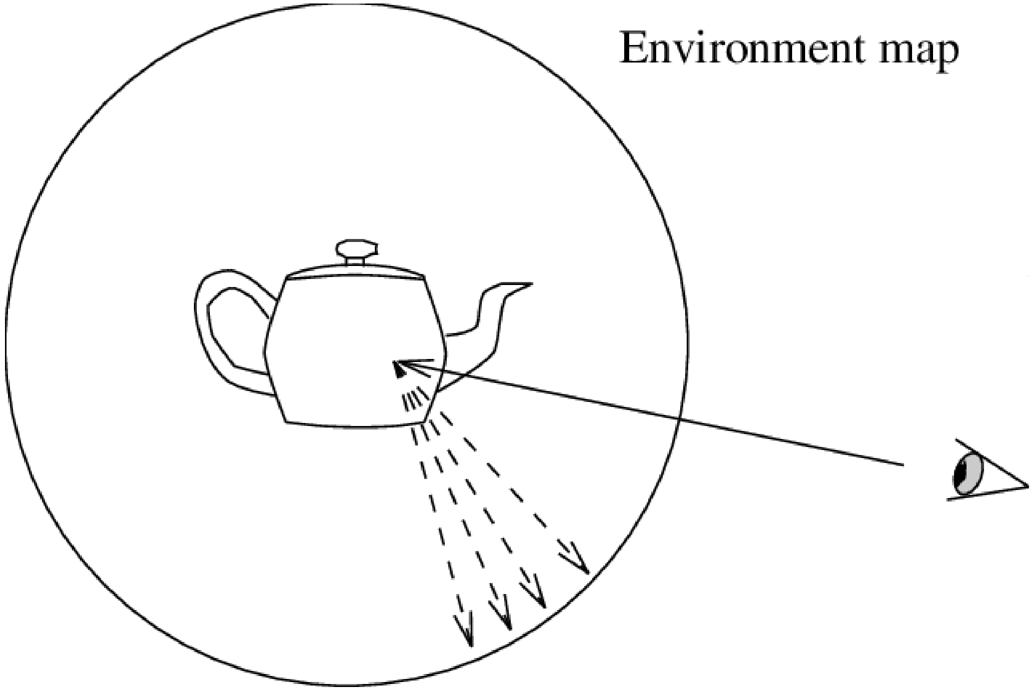
Leads to poor results

Evaluating the Integrand

$$L_r(\omega_o) \approx \hat{L}_r(\omega_o) = \frac{1}{N}\sum_{n=1}^{N}\frac{L_i(\omega_n) f_r(\omega_n, \omega_o)\cos\theta_n}{p(\omega_n)}$$

Practical option 2:

- Simulate complex illumination using environment maps



Good results at low computational cost

Environment Mapping

Also known as Image-based Lighting (IBL)

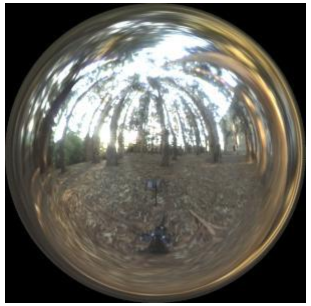
Miller and Hoffman, 1984
Later, Greene 86, Cabral et al, Debevec 97, …

Environment Mapping

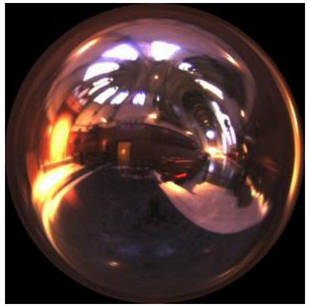
Simple idea:

- Use light probes to capture real light conditions
- Illuminate 3D virtual objects using measurements of real light

Eucaliptus grove



Grace cathedral



Uffizi gallery

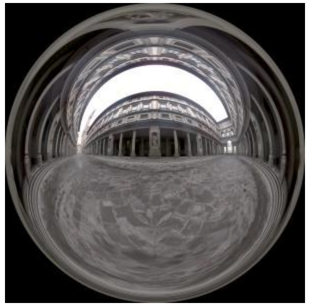
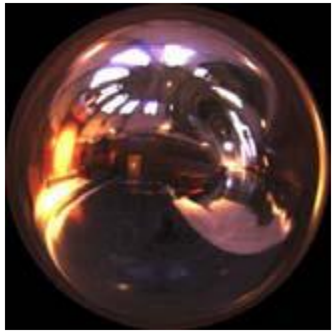
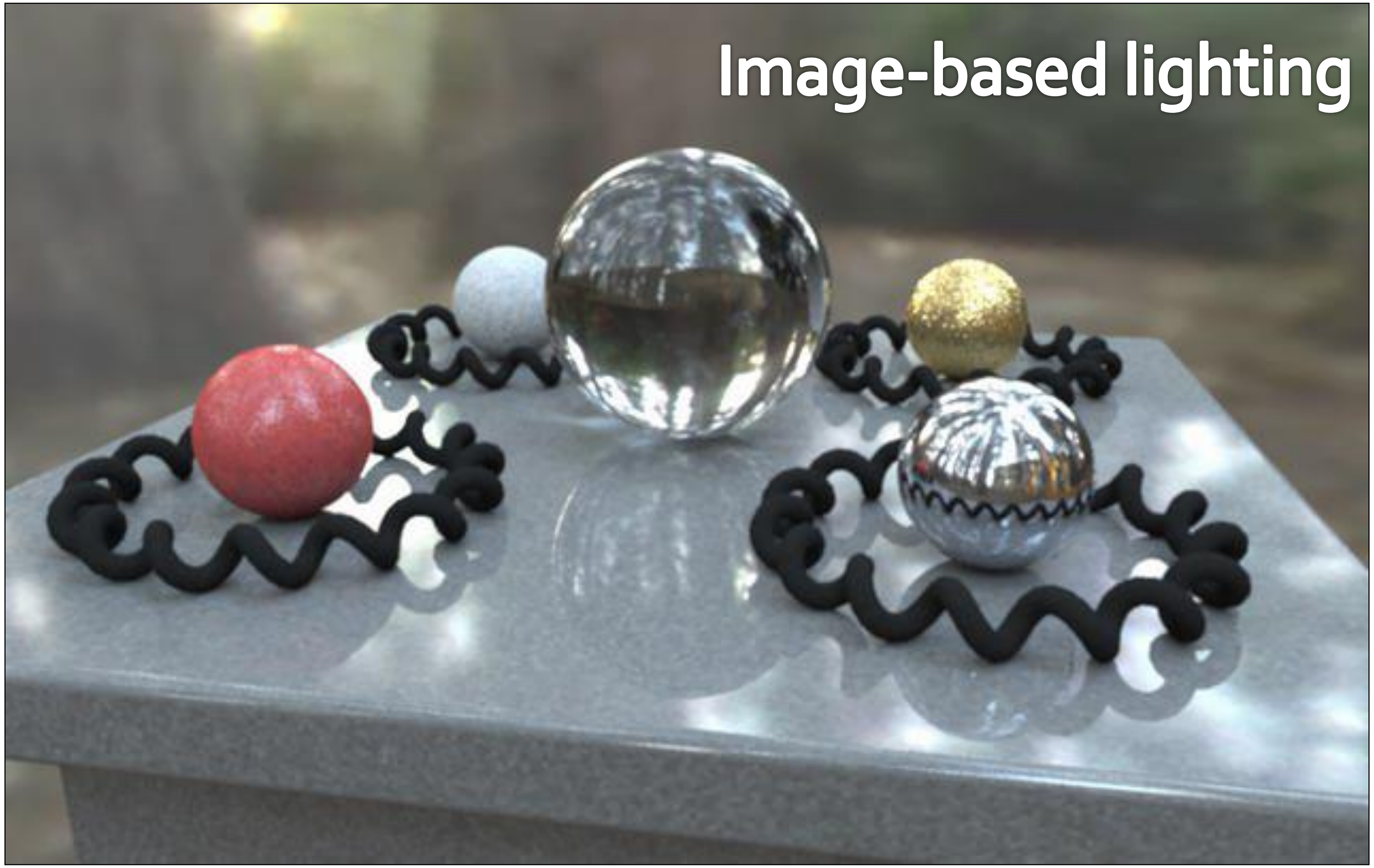


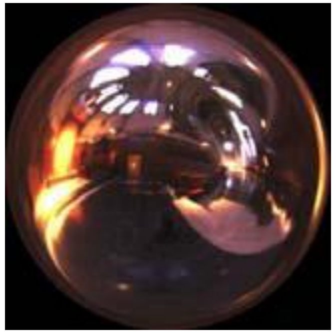
Image-based lighting



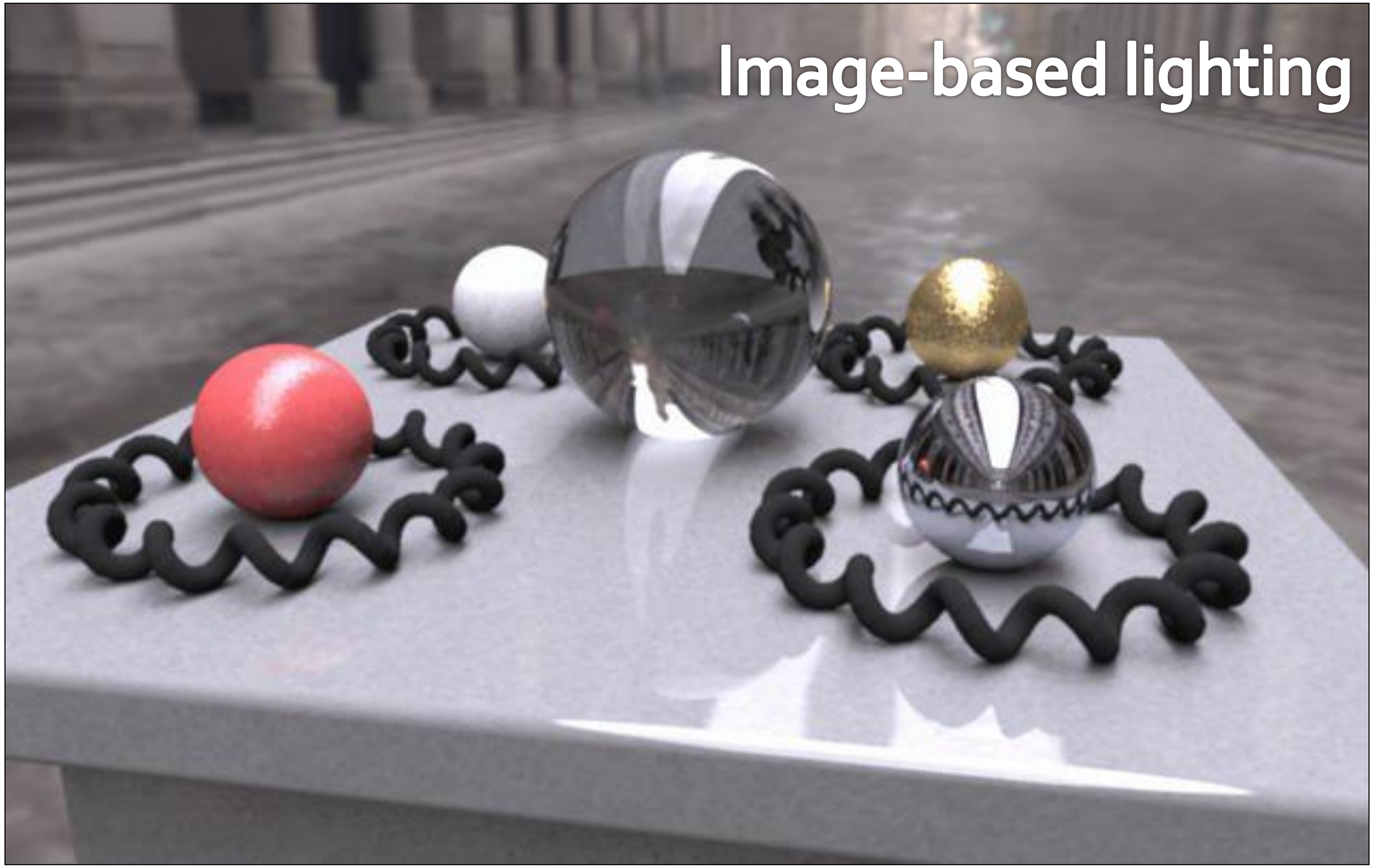
© Paul Debevec

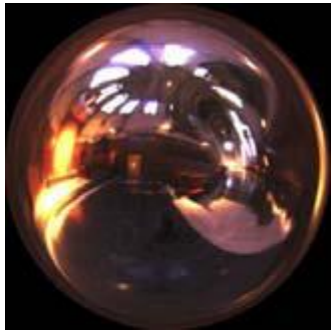




© Paul Debevec

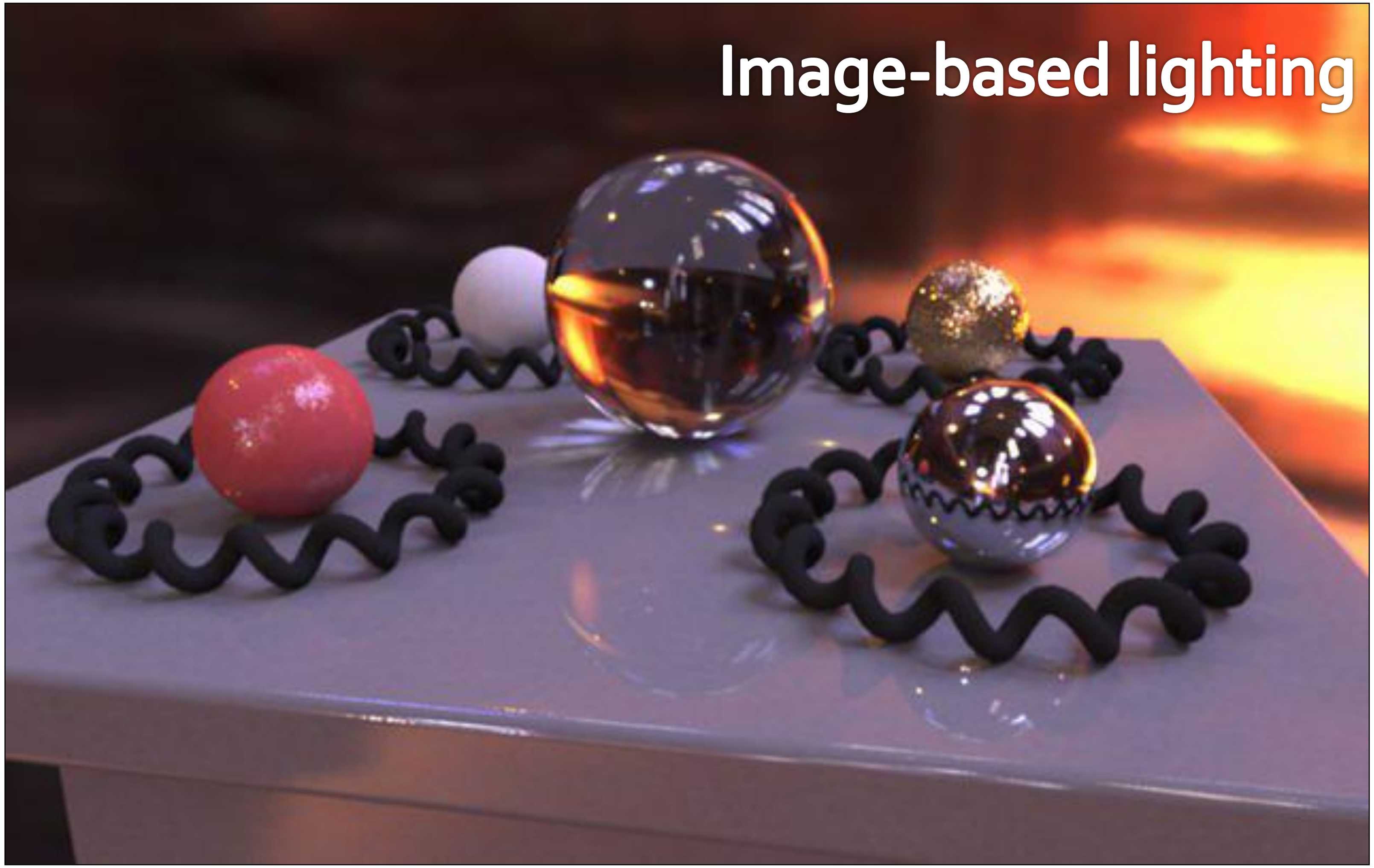
Image-based lighting





© Paul Debevec

Image-based lighting



© Paul Debevec

Lecture Highlights

Consolidated knowledge on MC methods

Monte Carlo for solving the Illumination Integral

A practical application to rendering

Next class

- Dive into a machine-learning technique for estimating the Illumination Integral
- Bayesian Monte Carlo (Gaussian Process-based method)

Course Roadmap

Week no.	Class Date	Theory	Labs	Notes
Week 1	22/02/2022	1.1 Intro	-	-
Week 2	02/03/2022	1.2 Ray-Tracing Phong	Lab 1	P1 Phong
Week 3	08/03/2022	1.3 Radiometry and BRDF	Lab 2	
Week 4	15/03/2022	2.1 Monte Carlo (MC)	Lab 3	P2 MC
Week 5	**22/03/2022**	**2.2 MC Rendering**	**Lab 4**	
Week 6	29/03/2022	2.3 Bayesian Monte Carlo (BMC)	Lab 5	P3 BMC
	05/04/2022	PARTIAL EXAMS (NO CLASS)		
	12/04/2022	EASTER HOLYDAYS (NO CLASS)		
Week 7	19/04/2022	2.4 BMC Rendering	Lab 6	
Week 8	26/04/2022	3.1 MC Importance Sampling	Lab 7	P4 AMC
Week 9	03/05/2022	3.2 BMC Importance Sampling	Lab 8	Papers Out
	10/05/2022	AI SEMINAR MAI (NO CLASS)		
Week 10	17/05/2022	Presentations I		
Week 11	24/05/2022	Presentations II		

Hands on the Code!

Time to finish Practice 2