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Question 1

Let S be the surface given by

$$x^2 + 2y^4 + 3z^6 = 1 \text{ with } z \geqslant 0.$$

Let Γ be the curve

$$\gamma(t) = \left(\cos(\pi t), \sqrt{t+1}, \frac{2t}{t^2+1}\right), \qquad \text{for} \qquad 0 \leqslant t \leqslant 1$$

and let $\boldsymbol{F}$ be the vector field

$$\boldsymbol{F}(x, y, z) = (a - 4z^2x)\boldsymbol{i} + 2by\boldsymbol{j} + cx^2z\boldsymbol{k}$$

where $a, b, c \in \mathbb{R}$

(a) Compute

$$\iint_S (\nabla \times \boldsymbol{F}) \cdot d\boldsymbol{S}.$$

Let's first find $\nabla \times \boldsymbol{F}$:

$$\nabla \times \boldsymbol{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a - 4z^2x & 2by & cx^2z \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y}(cx^2z) - \frac{\partial}{\partial z}(2by)\right)\boldsymbol{i} - \left(\frac{\partial}{\partial x}(cx^2z) - \frac{\partial}{\partial z}(a - 4z^2x)\right)\boldsymbol{j} + \left(\frac{\partial}{\partial x}(2by) - \frac{\partial}{\partial z}(a - 4z^2x)\right)\boldsymbol{k}$$

$$= (0, 2xcz + 8zx, 8zx)$$