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Calculating the CMB power spectrum from simulated cosmic structure formation initiated by primordial fluctuations

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ABSTRACT

An abstract for the paper. Describe the paper. What is the paper about, what are the main results, etc.

Key words. cosmic microwave background – large-scale structure of Universe

1. Introduction

Write an introduction here. Give context to the paper. Citations to relevant papers. You only need to do this in the end for the last milestone.

2. Milestone I: Background cosmology

To calculate the CMB power spectrum, we evolve primordial perturbations from the early Universe to the present day. Thus we must begin by constructing the background cosmology, into which we then can introduce perturbations. Our model Universe will have the possibility to contain two relativistic components (photons and neutrinos), two non-relativistic matter components (baryonic and CDM), a general (possibly non-zero) curvature density parameter $-1 \leq \Omega_{k0} \leq 1$, and a cosmological constant.

We first develop the theoretical framework describing the temporal evolution of density components in the Universe, along with derived quantities for model validation. We then employ an MCMC algorithm to constrain cosmological parameters using luminosity distance measurements from supernova observations.

Equality times
Acceleration time
Age of the Universe; spline $t(x)$
Plot $d_L(z)/z$ vs. supernova data
Compare $\mathcal{H}'/\mathcal{H}$ and $\mathcal{H}''/\mathcal{H}$ to expectations
Posterior for h

2.1. Theory

Einstein field eqs.
FLRW metric and the Friedmann eqs.
Hubble function
Evolution of density parameters
Cosmic time
Conformal time
Luminosity distance

$$\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}x} = \mathcal{H}\left[1 - \frac{3}{2}\Omega_m - 2\Omega_r - \Omega_k\right]\Bigg|_{a=e^x}$$

$$\frac{\mathrm{d}^2\mathcal{H}}{\mathrm{d}x^2} = \mathcal{H}\Bigg\{\left[1 - \frac{3}{2}\Omega_m - 2\Omega_r - \Omega_k\right]^2 - \frac{1}{2}\Big(3\Omega_m + 4\Omega_r + 2\Omega_k\Big)^2 + \frac{1}{2}\Big(9\Omega_m + 16\Omega_r + 4\Omega_k\Big)\Bigg\}\Bigg|_{a=e^x}$$

$$x_{\mathrm{eq}}^{mr} = \ln\left(\frac{\Omega_{r0}}{\Omega_{m0}}\right), \qquad x_{\mathrm{eq}}^{\Lambda m} = \frac{1}{3}\ln\left(\frac{\Omega_{m0}}{\Omega_{\Lambda 0}}\right)$$

2.2. Implementation details

2.2.1. MCMC parameter fitting

$$\chi^2(h, \Omega_{m0}, \Omega_{k0}) = \sum_{i=1}^{N} \frac{[d_L(z_i, h, \Omega_{m0}, \Omega_{k0}) - d_L^{\mathrm{obs}}(z_i)]^2}{\sigma_i^2}$$

Something about the numerical work Minimum chi2 found: chi 29.2799, h 0.701711, OmegaM0 0.255027, OmegaK0 0.0789514 t0 = 13.6781 Gyr

2.3. Results

Show and discuss the results.

Acknowledgements. I thank my mom for financial support!

References

Baker, N. 1966, in Stellar Evolution, ed. R. F. Stein,& A. G. W. Cameron (Plenum, New York) 333

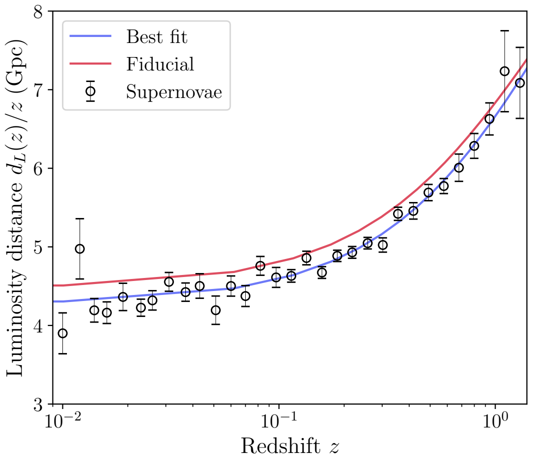

Fig. 1. Luminosity distance as a function of redshift z. The data points represent the luminosity distances deduced from observations of supernovae at different redshifts. The two curves show the luminosity distance functions $d_L(z)/z$ for the best fit parameters and for the fiducial cosmological parameters.

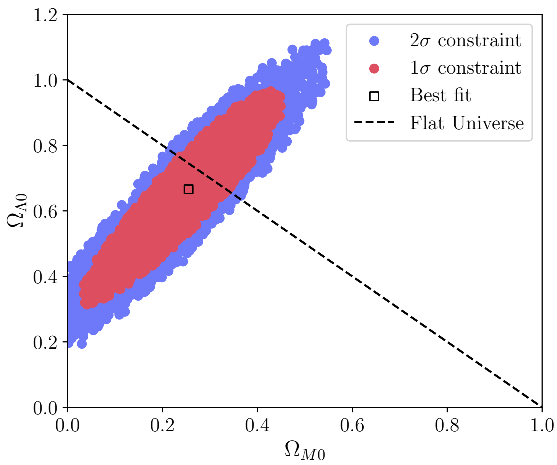

Fig. 2. The regions of MCMC sampled points in the $(\Omega_{M0}, \Omega_{\Lambda 0})$ parameter space within 1σ and 2σ of the pair associated with the minimal χ^2. The diagonal dashed line shows the cosmologies giving a flat Universe; $\Omega_{K0} = 1 - \Omega_{M0} - \Omega_{\Lambda 0} = 0$.

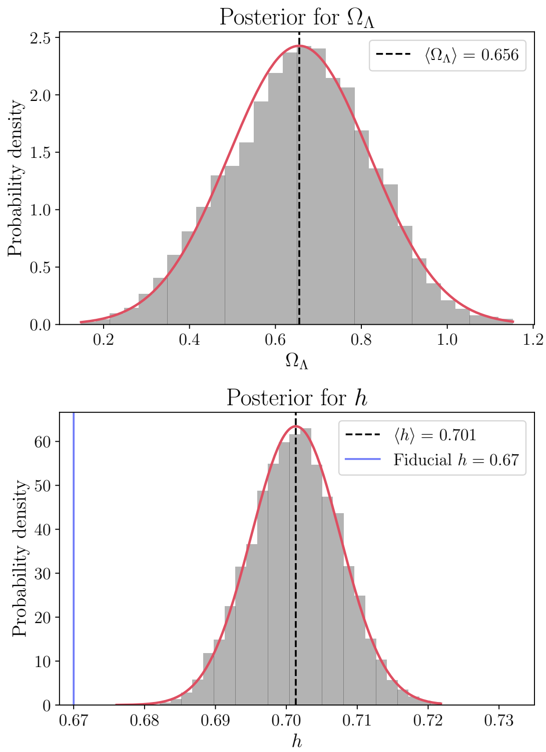

Fig. 3. k

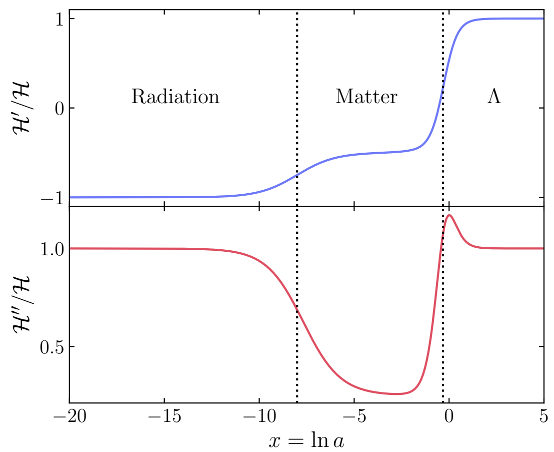

Fig. 4. yes

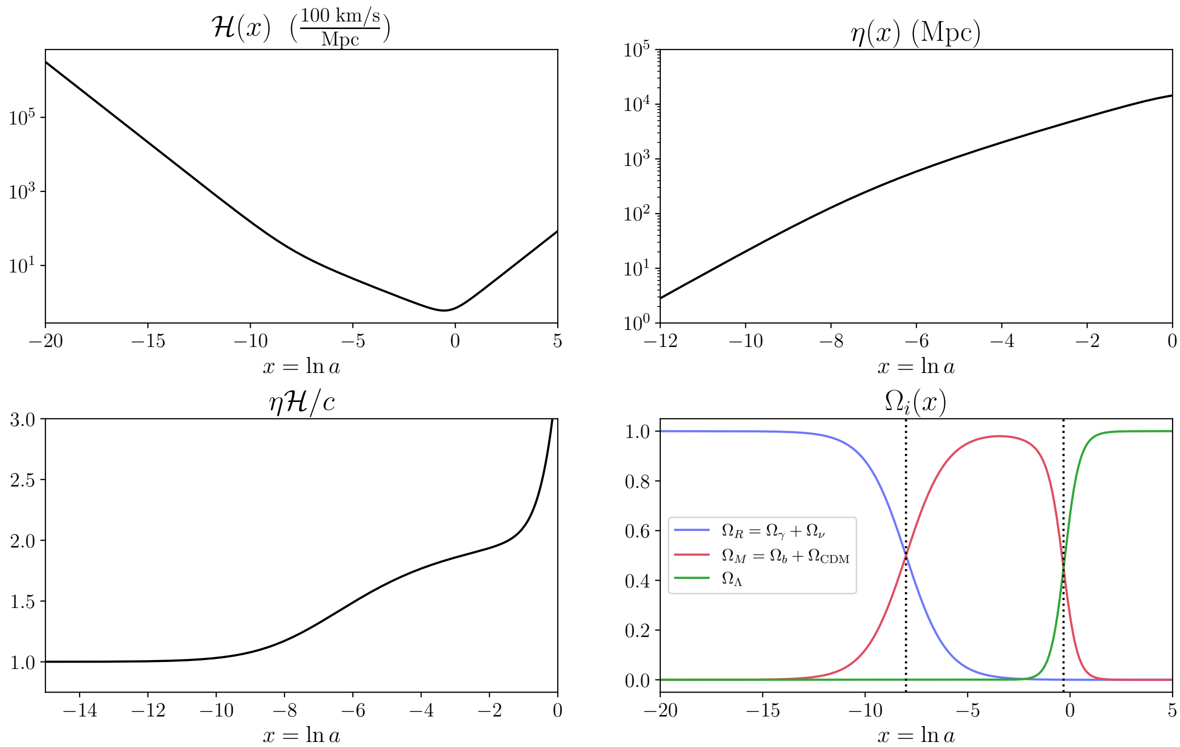
$\mathcal{H}(x)$ $(\frac{100\ \mathrm{km/s}}{\mathrm{Mpc}})$
$\eta(x)$ (Mpc)
$\eta\mathcal{H}/c$
$\Omega_i(x)$
$x = \ln a$
$\Omega_R = \Omega_\gamma + \Omega_\nu$
$\Omega_M = \Omega_b + \Omega_{\mathrm{CDM}}$
Ω_Λ

Fig. 5. uhm

Appendix A: Derivatives of the conformal Hubble function

The Hubble function is given by

$$H = H_0 \sqrt{(\Omega_{b0} + \Omega_{\mathrm{CDM0}})a^{-3} + (\Omega_{\gamma 0} + \Omega_{\nu 0})a^{-4} + \Omega_{k0}a^{-2} + \Omega_{\Lambda 0}},$$

where we can define $\Omega_{m0} \equiv \Omega_{b0} + \Omega_{\mathrm{CDM0}}$ and $\Omega_{r0} \equiv \Omega_{\gamma 0} + \Omega_{\nu 0}$. The first derivative of the conformal Hubble function $\mathcal{H} \equiv aH(a)$ with respect to $x \equiv \ln a$ is

$$\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}x} = \frac{\mathrm{d}a}{\mathrm{d}x}H + a\frac{\mathrm{d}H}{\mathrm{d}x} = \mathcal{H} + a^2\frac{\mathrm{d}H}{\mathrm{d}a}.$$

Let us compute the derivative of $H(a)$:

$$\frac{\mathrm{d}H}{\mathrm{d}a} = -\frac{H_0^2}{2H}\Big(3\Omega_{m0}a^{-4} + 4\Omega_{r0}a^{-5} + 2\Omega_{k0}a^{-3}\Big).$$

This gives

$$\begin{aligned}\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}x} &= \mathcal{H} - a^2\frac{H_0^2}{2H}\Big(3\Omega_{m0}a^{-4} + 4\Omega_{r0}a^{-5} + 2\Omega_{k0}a^{-3}\Big)\\ &= \mathcal{H}\Big[1 - \frac{3}{2}\Omega_m - 2\Omega_r - \Omega_k\Big].\end{aligned}$$

Here the Ω_i's refer to the density parameters as functions of scale factor, i.e. $\Omega_i(a)$, which should be evaluated at $a = e^x$.

The second derivative is a bit more cumbersome to calculate; from the product rule it follows that

$$\begin{aligned}\frac{\mathrm{d}^2\mathcal{H}}{\mathrm{d}x^2} &= \mathcal{H}\Big[1 - \frac{3}{2}\Omega_m - 2\Omega_r - \Omega_k\Big]^2\\ &\quad + \mathcal{H}\frac{\mathrm{d}}{\mathrm{d}x}\Big[1 - \frac{3}{2}\Omega_m - 2\Omega_r - \Omega_k\Big].\end{aligned}$$

Let B be the expression inside the big bracket. We thus need $\mathrm{d}B/\mathrm{d}x$, which is most easily calculated by factoring out $H_0^2/H(a)^2$ from the last three terms of B;

$$\begin{aligned}\frac{\mathrm{d}B}{\mathrm{d}x} &= -\frac{a}{2}\frac{\mathrm{d}}{\mathrm{d}a}\left(\frac{H_0^2}{H^2}\right)\Big(3\Omega_{m0}a^{-3} + 4\Omega_{r0}a^{-4} + 2\Omega_{k0}a^{-2}\Big)\\ &\quad - \frac{a}{2}\frac{H_0^2}{H^2}\frac{\mathrm{d}}{\mathrm{d}a}\Big(3\Omega_{m0}a^{-3} + 4\Omega_{r0}a^{-4} + 2\Omega_{k0}a^{-2}\Big)\\ &= -\frac{1}{2}\Big(3\Omega_m + 4\Omega_r + 2\Omega_k\Big)^2 + \frac{1}{2}\Big(9\Omega_m + 16\Omega_r + 4\Omega_k\Big).\end{aligned}$$

From this we end up with second derivative

$$\begin{aligned}\frac{\mathrm{d}^2\mathcal{H}}{\mathrm{d}x^2} &= \mathcal{H}\bigg\{\Big[1 - \frac{3}{2}\Omega_m - 2\Omega_r - \Omega_k\Big]^2\\ &\quad - \frac{1}{2}\Big(3\Omega_m + 4\Omega_r + 2\Omega_k\Big)^2 + \frac{1}{2}\Big(9\Omega_m + 16\Omega_r + 4\Omega_k\Big)\bigg\}\bigg|_{a=e^x}.\end{aligned}$$