Blatt 2

②

1) $17+22+45 \leq c \cdot 1$

$84 \leq c$

2) $\lim\limits_{n \to \infty} \frac{5n^3+12n^2+3n+5}{n^3} = \frac{15n^2+24n+3}{3n^2} = \frac{30n+24}{6n} = \frac{30}{6} = 5$

3) $2^{n+1} \leq c \cdot 2^n$

$2 \cdot 2^n \leq c \cdot 2^n \quad / : 2^n$

$2 \leq c$

4) $2^{2n} \leq c \cdot 2^n$

$2^n \cdot 2^n \leq c \cdot 2^n \quad / : 2^n$

$2^n \leq c$ wächst mit n

5) $\log(n!) = \Theta(n \log n)$

6) $2^n \leq \sqrt{2\pi n} \cdot \left(\frac{n}{e}\right)^n \cdot c$

7)

8) $(6^{-5}) \cdot n^{1,25} \geq n^{0,5} \cdot c$

$n^{1,25} \geq n^{0,5} \cdot c \quad / : n^{0,5}$

$n^{0,75} \geq c$ wächst mit n

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(3)

$$f(n) = \frac{\phi^n - \hat{\phi}^n}{\sqrt{5}}$$

IA: $n = 1 \wedge n = 2$

$f(1) = 1$
$f(2) = 1$

IV: Gilt für $n-1$, $n-2$

IS:

$$f(n) = f(n-1) + f(n-2)$$

$$\overset{IV}{=} \frac{\phi^{n-1} - \hat{\phi}^{n-2}}{\sqrt{5}} + \frac{\phi^{n-2} - \hat{\phi}^{n-2}}{\sqrt{5}}$$

$$= \frac{(\phi^n - \phi^n) \cdot \left(\frac{1}{\phi} + \frac{1}{\phi^2}\right)}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \phi^n - \frac{1}{\sqrt{5}} \cdot \hat{\phi}^n$$

$$= \Theta(\phi^n) \approx \Theta(1{,}6^n)$$

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4

2)

$$n^m \leq c \cdot \alpha^n \qquad \alpha > 1 \wedge m \in \mathbb{N}$$

$$n^m \leq c \cdot \alpha^n \;/ : \alpha^n$$

$$\frac{n^m}{\alpha^n} \leq c$$

$$\lim_{n \to \infty} \frac{n^m}{\alpha^n} = 0$$

3)

$$n \ln n \leq c \cdot n^{3/2} \;/ : n^{1/2}$$

$$\frac{\ln n}{n^{0,5}} \leq c$$

$$\lim_{n \to \infty} \frac{\ln n}{\sqrt{n}} = 0$$

4)

$$5^{\log_3 n} \leq c \cdot n^2 \;/ : n^2$$

$$\lim_{n \to \infty} \frac{5^{\log_3 n}}{n^2} = \frac{n^{\log_3 5}}{n^2} = 0$$

1)

$$\lim_{k \to \infty} \frac{k^2}{2^k} = 0$$

In Zusammenarbeit zwischen Daniel Heisig, Felix Scholzen und Simon Wagner.