EE16A - Lecture 25 Notes

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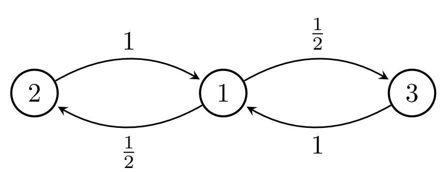
Pagerank

> 1. Webpages represented by nodes, linked to each other
> 2. Normalize Link Weights: $(\frac{\text{Node Score}}{\text{Out-Degree}})$
> 3. State at time $n+1 = \vec{s}[n+1] = A\vec{s}[n]$ where A is the **transition matrix** and $\vec{s}$ is the **state vector**
> 4. A is a Markov Matrix
>   - All entries in A are non-negative
>   - Each column of A sums to 1

- Normalize outdegrees of each node, by dividing the weight of the node by the total number of out-links from that node (Node Score/Out-Degree)
  - Node j has n_j out links
  - Weight due to that node would be $\frac{x_j}{n_j}$ where x = node's score
  - Node j's contribution to node k's score, to which it has a link, is $\frac{x_j}{n_j}$
- $A\vec{s}*\vec{s}* = 1I\vec{s}* \implies 1I\vec{s}* - A\vec{s}* = 0 \implies (I-A)\vec{s}* = 0$
- $\vec{s}*$ is non-zero and $(I-A)\vec{s}* = 0$, so $\vec{s}* \in \text{Null}((I-A))$
  - Null($(I-A)$) is non-trivial ($(I-A)$ does not have an empty null space) $\implies (I-A)$ does not have full rank

Simple Pagerank Example:



$$\vec{s}[n+1] = A\vec{s}[n] = \begin{bmatrix} 0 & 1 & 1 \\ 1/2 & 0 & 0 \\ 1/2 & 0 & 0 \end{bmatrix} \vec{s}[n] \tag{1}$$

Distribution approaches $\vec{s}*$ s.t.:

$$\vec{s}* = A\vec{s}* \tag{2}$$

From $(I-A)\vec{s}* = 0$

$$I - A = \begin{bmatrix} 1-0 & -1 & -1 \\ -1/2 & 1-0 & 0 \\ -1/2 & 0 & 1-0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ -1/2 & 1 & 0 \\ -1/2 & 0 & 1 \end{bmatrix} \tag{3}$$

Eigenvalue representation of $\vec{s}* = A\vec{s}*$ for $\lambda = 1$

$$A\vec{v}_1 = \lambda\vec{v}_1 = \vec{v}_1 \tag{4}$$

Eigenvalues and Eigenvectors

1. **Concept:** Transition matrix scales $\vec{v}_1$ in its original direction by a factor of λ
2. Every transition matrix A will have $\lambda = 1$ as an eigenvalue and a non-negative vector $\vec{v}_1$ associated with that eigenvalue, where the entries of that eigenvector sum to 1.
3. Eigenvector $\vec{v}_n$ associated with eigenvalue λ_n is the vector that solves $(I - A)\vec{v}_n = 0$

- Solution to $\vec{s}[n+1] = A\vec{s}[n]$
  - Need Initial State vector $\vec{s}[0]$
- General Solution: $\vec{s}[n] = \alpha_1 \lambda_1^n \vec{v}_1 + \alpha_2 \lambda_2^n \vec{v}_2$
  - Linear combination of eigenvalues and eigenvectors
  - $\vec{s}[n+1] = \alpha_1 \lambda_1^{n+1} \vec{v}_1 + \alpha_2 \lambda_2^{n+1} \vec{v}_2$
  - $A\vec{s}[n] = A(\alpha_1 \lambda_1^n \vec{v}_1 + \alpha_2 \lambda_2^n \vec{v}_2) = \alpha_1 \lambda_1^n A\vec{v}_1 + \alpha_2 \lambda_2^n A\vec{v}_2 = \alpha_1 \lambda_1^n \lambda_1 \vec{v}_1 + \alpha_2 \lambda_2^n \lambda_2 \vec{v}_2$
  - $= \alpha_1 \lambda_1^{n+1} \vec{v}_1 + \alpha_2 \lambda_2^{n+1} \vec{v}_2 = \vec{s}[n+1]$
- Find α_1 and α_2
  - $\vec{s}[0] = \alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} \vec{s}_1[0] \\ \vec{s}_2[0] \end{bmatrix}$
- To find the Limiting State Distribution, investigate the linear combination of the eigenvectore and eigenvalues as $n \to \infty$
  - $\lambda_2^n \to 0$ as $n \to \infty$
  - $\lambda_1^n = 1 \forall n$
  - $\lim_{n\to\infty} \vec{s}[n] = \alpha_1 \lambda_1^n \vec{v}_1 = \alpha \vec{v}_1$

Distribution Vector at State n

$$\vec{s}[n] = A^n \vec{s}[0] \tag{5}$$