CS70 - Lecture 14 Notes

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Counting

Tree Counting: Slow

- Build up string by bits, total amount of leaves is total possibilities

First Rule of Counting: Product Rule:

- If objects constructed from a sequence of choices $n_1, n_2, ..., n_k$
- Total number of objects = $n_1 \times n_2 \times \cdots \times n_k$

Counting Functions/Polynomials

- There are $|T|^{|s|}$ functions $f : S \to T$
  - $|T|$ choices for mapping of $f(s_i)$ (Use product rule)
- p^{d+1} polynomials of degree d mod p
  - p choices for each of the $d + 1$ coefficients

Permutations

- Derived from the first rule of counting (product rule)
- Choose from less items each step
- Permutations of n objects: number of orderings of n objects (no replacements)
  - $n \times (n-1) \times (n-2) \times \cdots \times 1 = n!$
- Number of one to one functions $|S| \to |S|$
  - Decreasing choices every step: $|S| \times |S| - 1 \times \cdots \times 1 = |S|!$

Permutation Formula

- Number of different samples of saize k from n numbers **without replacement**

$$nPk = n \times (n-1) \times (n-2) \times \cdots \times (n-(k-1)) = \frac{n!}{(n-k)!} \quad (1)$$

Counting Sets: When order doesn't matter

Second Rule of Counting: Order Doesn't Matter (Combination):

- If order doesn't matter, count the number of ordered objects (permutations) and divide by number of orderings
- Choose k out of n possibilities

$$\binom{n}{k} = nCk = \frac{n!}{k!(n-k)!} \quad (2)$$

Sampling:

- Sample k items out of n
- Without replacement:
  - If order matters (first rule): $\frac{n!}{(n-k!)}$
  - If order does not matter (second rule): $\frac{n!}{k!(n-k!)}$
- With replacement:
  - If order matters (first rule): n^k
  - **see Stars and Bars formula (3)**

Anagrams:

- First rule on total number of letters N: $N!$ total permutations
- Divide by the number of duplicate permutations generated due to D duplicate letters: First rule: $D!$
- total distinct permutations = $\frac{N!}{A!B!\cdots D!}$ (can have multiple duplicate sets of letters)

Stars and Bars:

- Ways k people split n things
- Ways to add up k numbers to sum to n
- k undordered choices from set of n possibilities
- $\binom{\text{total} + (\text{sections - 1})}{\text{sections - 1}}$

$$\binom{n+k-1}{k-1} \tag{3}$$

Summary

First Rule (Product)

- k samples
- With replacement: n^k
- Without replacement: $\frac{n!}{(n-k)!}$

Second Rule (Division)

- When order doesn't matter (sometimes): can divide
- Without replacement (order doesn't matter): $\binom{n}{k} = \frac{n!}{(n-k)!k!}$ n choose k
  - You pick a different object every time. The total amount of orderings for your k objects is $k!$, so divide sample without replacement by $k!$ because order doesn't matter

One-to-one Rule

- Equal in number if one-to-one (Bijection)
- With replacement (order doesn't matter): $\binom{k+n-1}{n-1}$