CS70 - Lecture 15 Notes

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Review Turing and Halt

- Turing(P) includes Halt(P,P)
- If halts (infinite loop), if doesn't halt, halt (diagonalization)
- Halt program exists $\Rightarrow$ Turing program exists
- Turing("Turing") interpret program 'Turing' as text
  - Neither halts nor loops (diagonlized statement on itself)
  - Therefore, Turing program DNE
  - No Turing program $\Rightarrow$ No Halt program (Contrapositive)

Review Stars and Bars

Wikipedia: Stars and Bars

- Choose n from k with replacement, order doesn't matter
- Stars and Bars Bijection (Counting rule)
- Place n (total) objects in k bins
- k bins are distinguishable, objects are not
- $k-1$ bars represent k bins
- Thm 1 (positive nums): Each bin must contain an object, there can only be ≤ 1 bar between each star. You must choose $k-1$ bars from the $n-1$ available positions.
- Thm 2 (non-negative nums): Bins can contain any no. of objects, there can be $\geq k-1$ bars between each star. You must choose $k-1$ bars from the $n+(k-1)$ available positions.

With Replacement/Order Doesn't Matter:

$$\text{Positive Groups} = \binom{n-1}{k-1} \tag{1}$$

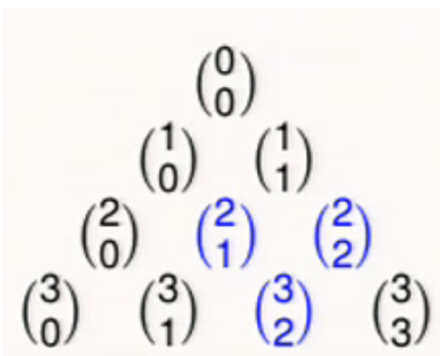
$$\text{Non-Negative Groups} = \binom{n+(k-1)}{k-1} \tag{2}$$

Combinatorial Proofs

- Define what the Left side counts and what the Right side counts, then equate them
- Ask same question for both sides and answer them using the correct approach corresponding to the side

- $\binom{n}{k} = \binom{n}{n-k}$
  - Left: Ways to choose k out of n items
  - Right: Ways to (not) choose $n-k$ out of n items, which is the same as choose k out of n items
- $\binom{n}{k} = \binom{n-1}{k-1} + \cdots + \binom{k-1}{k-1}$
  - Left: Ways to choose k out of n items
  - Right: Sum number of subsets that include the first i items and the number of subsets that do not include the first i items
- $2^n = \binom{n}{kn} + \binom{n}{n-1} + \cdots + \binom{n}{0}$: **Binomial Thm**
  - Sum of coefficients of an $(1+x)^n$ binomial (nth row in Pascal's Triangle)
  - Left: No. of subsets of n choices (element i is either in or out of the subset, 2 poss.)
  - Right: Sum of $\binom{n}{i}$ from i to n
    - $\binom{n}{i}$ ways to choose i elements of n choices

Pascal's Triangle



- Row n = coefficients of $(1+x)^n$
- Choose 2^n terms: 1 or x from $(1+x)$
  - Combine all terms corresponding to x^k
  - Coefficient of x^k is $\binom{n}{k}$: you choose k factors (products) that include x and there are n x's to choose from

Pascal's Rule:

- Left: No. of k subsets from $n+1$ choices
- Right: No. of subsets that choose the first item + No. of subsets that do not choose the first item = Left
  - $\binom{n}{k-1}$: No. of subsets of n that contain the first item. (Take away first item, left with n items and $k-1$ remaining choices)
  - $\binom{n}{k}$: No. of k subsets of n that do not contain the first item. (Take away first item, left with n items, but still need to make k choices.

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1} \tag{3}$$

Simple Inclusion/Exclusion

- **Sum Rule: For disjoint sets** S and T : $|S \cup T| = |S| + |T|$
- **Inclusion/Exclusion Rule: For any** S and T : $|S \cup T = |S| + |T| - |S \cap T|$
  - Ex. No. of 10 digit phone numbers that have 7 as the first or second digit
  - S = first digit 7. $|S| = 10^9$
  - T = second digit 7. $|T| = 10^9$
  - $S \cap T$ = first and second digit. $|S \cap T| = 10^8$
  - $|S| + |T| - |S \cap T| = 10^9 + 10^9 - 10^8$

Probability Space

- Random Experiment: Define possible outcomes and likelihoods (percentages) have Statistical regularity
- Set of Ω outcomes: (Ex. $\Omega = \{H, T\}$)
  - Probabilities assigned to each outcome: $\Pr[H] = 0.5, \Pr[T] = 0.5$
  - Elements of Ω describes one outcome of the complete experiment
- Assign probability to each outcome $\Pr[A]$
  - Probabilities assigned to each outcome: Ex. $\Pr[H] = 0.5, \Pr[T] = 0.5$

- Ω = sample space (can be countable or uncountable)
- $\omega \in \Omega$ = sample point
- probability $\Pr[\omega]$ s.t. $0 \leq \Pr[\omega] \leq 1$ and $\sum_{\omega \in \Omega} \Pr[\omega] = 1$

Uniform Probability Space

- Each outcome ω is equally probable: $\Pr[\omega] = \frac{1}{\Omega}$ for all ω
- Ω must be finite

Non-Uniform Probability Space

- Each outcome ω is any $\Pr[\omega]$ s.t. $0 \leq \Pr[\omega] \leq 1$ and $\sum_{\omega \in \Omega} \Pr[\omega] = 1$