CS70 - Lecture 12 Notes

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Berklekamp-Welsh Algorithm

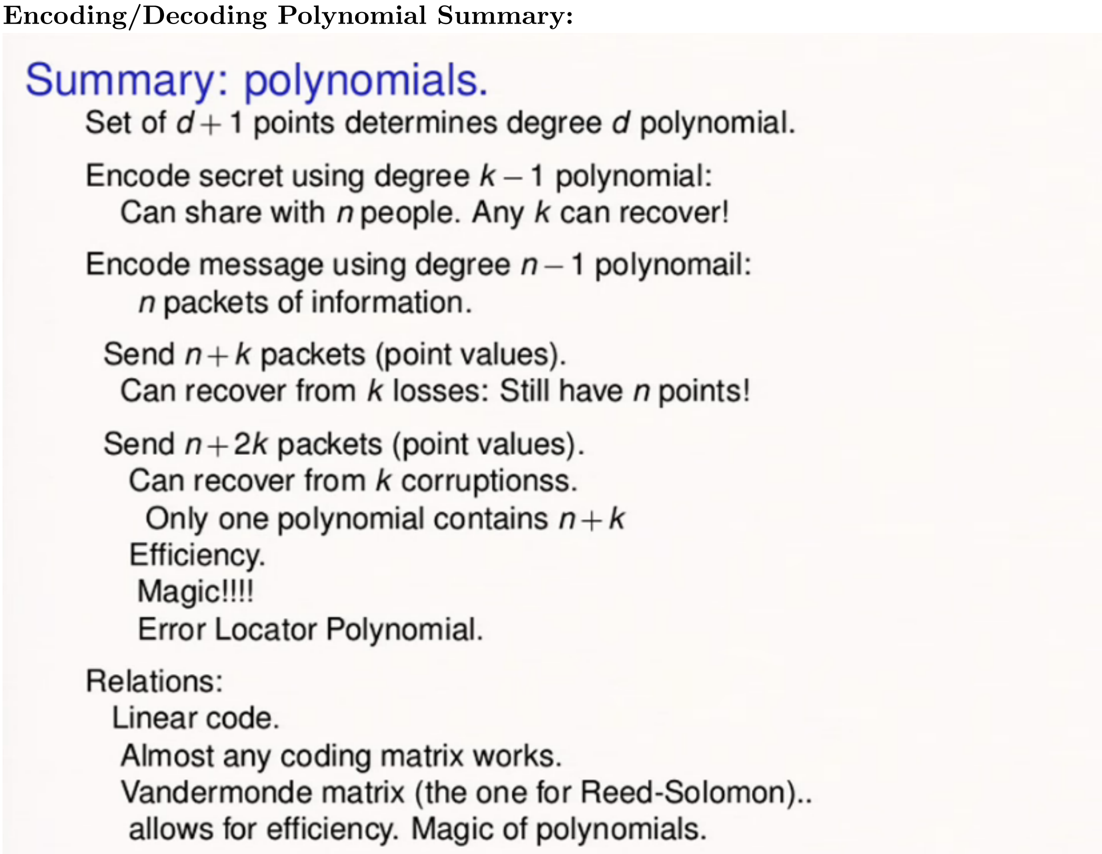
Existence:

- Exists because packets constructed using $P(x)$

Unique:

- Proved assuming $\frac{Q'(x)}{E(x)} = \frac{Q(x)}{E'(x)} = P(x$
- $E(x)$ and $E'(x)$ have at most k roots each
- Cross multiply assumption at n valid points, so claim is true for n points, which make $P(x)$ a unique $< n$ degree polynomial
- **Fact:** $Q'(x)E(x) = Q(x)E'(x)$ on $n + 2k$ values of x
  - Holds when $E(x)$ or $E'(x)$ are 0
  - Use above method of cross multiplication when not zero

Encoding/Decoding Polynomial Summary:



Counting

- Counting Numbers: 0,1,2... all Natural Numbers $\mathbb{N}$
- **Countable if there is a bijection between S and some subset of $\mathbb{N}$**
- if subset of $\mathbb{N}$ = finite, S has finite **cardinality**
- if subset of $\mathbb{N}$ = infinite, S is **countably infinite**
  - Evens are countably infinite
  - Integers are countably infinite
  - Pairs of Natural Numbers are countably infinite
  - Rationals are countably infinite (subset of pairs of natural numbers with gcd of 1)
  - Reals are uncountable
- All countably infinite sets have the same cardinality

Isomorphism Principle:

- If there is $f : D \to R$ that is one to one and onto, (bijective) then $|D| = |R|$

Listing:

- A bijection with a subset of natural numbers
- The nth item in the list is a mapping $n \in \mathbb{N} \to f(n)$
- If you can list a set you can show a bijection
- Finite List: Bijection with subset of $\mathbb{N}\{0, ..., |S| - 1\}$
- Infinite List: Bijection with $\mathbb{N}$

Enumerating $\equiv$ Countability = Listing:

- Enumerating a set $\implies$ countability
- Corollary: Any subset T of a countable set S is also countable
- Each element of $x \in S$ has a specific, finite position in a list (ex. $\mathbb{Z} = \{0, 1, -1, 2, -2, ...\}$
- Fails for integers if you list positive integers before negative integers
  - $\mathbb{Z} = \{\{0, 1, 2, ..., \}$ and then $\{-1, -2, ...\}\}$
  - -1s position is not finite, because there are ∞ positive integers
  - So.. you must **interweave**

Diagonalization:

- **Diagonal Number** Number that is not in the list of $f(n)$
- Ex. Method to create diagonal number for Reals: Digit i is 7 if number i's ith digit is not 7, 6 otherwise. For every nth position on the list, at least the nth digit will be different than the diagonal number's nth digit. Contradiction because the diagonal number is real.
- Check if this creates contradicton. If diagonal number is in the questionable set, the list could not have existed and thes et is not countable.