GAUNTLET

Gauntlet Research Report

Market Risk Assessment

An analysis of the financial risk to participants in the Aave protocol.

AAVE

Contents

Disclaimer

This report is for informational purposes only and does not constitute an offer to sell, a solicitation to buy, or a recommendation for any security, nor does it constitute an offer to provide investment advisory or other services by Gauntlet Networks Inc. No reference to any specific security constitutes a recommendation to buy, sell or hold that security or any other security. Nothing in this report shall be considered a solicitation or offer to buy or sell any security, future, option or other financial instrument or to offer or provide any investment advice or service to any person in any jurisdiction. Nothing contained in this report constitutes investment advice or offers any opinion with respect to the suitability of any security, and the views expressed in this report should not be taken as advice to buy, sell or hold any security. The information in this report should not be relied upon for the purpose of investing. In preparing the information contained in this report, we have not taken into account the investment needs, objectives and financial circumstances of any particular investor. This information has no regard to the specific investment objectives, financial situation and particular needs of any specific recipient of this information and investments discussed may not be suitable for all investors. Any views expressed in this report by us were prepared based upon the information available to us at the time such views were written. Changed or additional information could cause such views to change. All information is subject to possible correction. Information may quickly become unreliable for various reasons, including changes in market conditions or economic circumstances.

Part I

Background

The Aave protocol is one of the largest liquidity protocols on Ethereum. It allows users to trustlessly supply and borrow cryptoassets on Ethereum; active borrowers who are willing to pay interest have access to pools of cryptoassets of suppliers who seek yield on their savings. The protocol has been a profitable venue for a number of market participants, such as asset suppliers and liquidators. Moreover, it was the first protocol to popularize the 'flash loan', a novel financial tool that appears to only exist within the cryptocurrency space. The protocol grew from holding less than $1 million in assets on January 2020 to having over $7 billion (notional) by April 2021. With growth of this scale, mitigating financial and market risks is increasingly important as billions in customer funds are on the line.

The Aave protocol exists in two versions on Ethereum. The first version of the Aave protocol (V1) was deployed to Ethereum mainnet in January 2020 and the second version (V2) in December 2020. Assets supplied to the V1 protocol can be migrated to the V2 protocol to take advantage of a multitude of new features, including the ability to swap collateral assets and to repay debts with a collateral asset. While V1 increased capital efficiency for arbitrageurs by introducing flash loans, V2 added improved community governance, a stake-based safety module, and batched flash loans. Community governance allows for holders of the AAVE token to vote on matters such as addition/removal of assets, adjustments of interest rate pricing curves, and risk parameters such as margin requirements. Implementation details about these smart contracts are detailed in the V1 whitepaper and V2 whitepaper.

Within decentralized finance (DeFi), borrowing protocols have four main risk factors: security risk, governance risk, oracle risk, and market risk. While this report will only cover the market risks within Aave, we describe the other risks for completeness of presentation. Security risk concerns the correct execution of smart contract code that stores supplied assets, manages borrowed assets, and liquidates uncollectable liens. Such risks are assessed by cybersecurity code auditors who focus on ensuring that the implementation of the contract exactly matches its high-level specifications. Aave V1 was assessed by a number of auditors including OpenZeppelin and Trail of Bits, while V2 also included additional coverage via formal verification from Certora as well as audits from MixBytes, Peckshield, CertiK, Consensys Diligence, and SigmaPrime. Governance risk, on the other hand, deals with management related issues such as administrator mismanagement, poor voter participation, and concentration of voting power.

Oracle and market risk are concerned with the precise pricing and liquidation mechanisms within the lending contract. Overcollateralized protocols like Aave rely on price oracles that convey asset prices external to the blockchain to the protocol. These are used for marking the value of outstanding liens and estimating which ones are in default. Manipulation of oracle price feeds can force the protocol to liquidate liens that are not in default, causing a loss of customer funds. As Aave uses Chainlink oracles, which have a variety of partial security analyses, all analysis within this report assumes negligible oracle risk. It is important to note that oracle attacks have occurred numerous times in DeFi, including at Compound, but as of the writing of this article (Apr. 2021), Chainlink has avoided catastrophic issues.

The content of this report is to assess market risks within the Aave protocol. Decentralized collateralized borrowing is exposed to a number of exogenous risks that occur when markets for borrowable assets are volatile. The safety of these systems rely on competition amongst liquidators, participants can purchase underwater collateral from the protocol at a discount. By making such a purchase, the liquidator removes a liability from the protocol's books and adds an asset such as Ethereum. However, when markets are extremely volatile, liquidators may choose not to make such purchases, leaving the protocol with more liabilities than assets. Adjusting margin requirements (e.g. the amount of collateral required to enter a loan) and the liquidation bonuses can reduce the likelihood of such events happening.

Using a rigorous definition of market risks, we construct simulation-based stress tests that evaluate the economic security of Aave as it scales to underwriting tens of billions of dollars of borrowed assets. These stress tests are trained on historical data and put through a battery of scenarios that represent the expected and worst case economic outcomes for the protocol. Our stress tests are constructed analogously to how transaction-level backtesting is done in high-frequency and algorithmic trading. These techniques are used to estimate the market risk of a systematic trading strategy before it is deployed to the market. As there are over $1 trillion US dollars of assets managed by funds that use these techniques to provide daily actuarial analyses to risk managers, we believe that these are the best methodologies for evaluating market risk.[1] By modifying these techniques to handle the idiosyncrasies of cryptocurrencies, we are able to provide similar statistical power in these actuarial analyses.

In our previous analysis of the overcollateralized protocol Compound, we focused on assessing the risk of cascading liquidations. This type of market risk captures deflationary spirals, where an increase in liquidations causes prices to crash which in turn causes more liquidations. Unlike Compound, however, Aave has a much larger variance in asset quality and behavior than Compound. Assets such as Chainlink (LINK), EnjinCoin (ENJ), and Gemini Dollar (GUSD) have extremely different risk profiles for borrowers, suppliers, and liquidators. Complicating things further, the plethora of assets on Aave also encourages many-to-many borrowing: a user can supply n assets as collateral and borrow m assets. To align the sentiment in the Aave community for balancing risk and capital efficiency our Compound model was expanded to handle multi-asset liens.

Significant model improvements were made following our Compound analysis to handle

[1] Arnott et al. (2005); Curcuru et al. (2014); Hsu (2004)

multiple assets with wide volatility ranges and return profiles. Firstly, we added in the ability for borrowers to perform many-to-many borrows where they borrow multiple assets against multiple forms of collateral. In order to model the complex risk from such borrows, we constructed a filtered covariance matrix model trained on historical price data. Our covariance construction methodology is an adaptation of those used in the traditional markets, such as those from index providers like MSCI. Secondly, we modeled more complex borrower behavior to reflect the higher-dimensional space of borrowing strategies. Finally, we include a more realistic transaction fee (gas price) model that reflects the competition for block space around large liquidations. Such competition has been empirically observed by numerous parties[2] and there are a number of services[3] that allow miners to capture liquidation profits in exchange for liquidation priority.

The first portion of this report will quantitatively define the set of market risks that Aave users face. We will break down these risks into components and identify which protocol-controlled parameters can be used to tune these risks. Then, we will describe how the principal agents of the systems affect the solvency of the protocol. Our focus will be on borrower and liquidator behavior, both of which are heavily dependent on market conditions and changes. Concluding this section will be a description of Aave's novel insurance-style protection mechanism that acts as a backstop when liquidators cannot clear the liabilities of a single market. This insurance-style protection mechanism distinctly changes the security guarantees of Aave when compared to Compound, as Compound provides implicit insurance via governance.[4] It is important to note that it is possible to use funds for insurance through the Aave protocol's governance as well.

Subsequently, we will describe the modeling methodology that we utilized, centering around the usage of agent-based simulation. This section will define the models used for generating synthetic price trajectories, computing liquidator profits, and modeling blockchain congestion delays. We will then describe the high-level market risk questions that we aim to solve and what quantitative metrics are used to answer these questions. With this framework established, we will describe the results gleaned from running large-scale simulations under a variety of tail-event scenarios in asset price and volatility.

Our simulation results suggest that both the Aave V1 and V2 protocols are safe, although both versions of the protocol could be less aggressive with liquidation bonuses. In §6.2, we describe how overincentivization can counter-intuitively be deleterious for borrowers of volatile

[2] Manuskin (2020); Robinson (2021)

[3] Obadia (2020); Kullander (2020)

[4] This implicit insurance guarantee, which is avoided with the Aave insurance model, was tested due to the oracle issues in Compound on November 26, 2020. Compound's governance eventually chose not to pay out COMP denominated remunerations to lenders. Aave's fully autonmomous insurance system which could have issued AAVE, up to 30% of total staked value in the safety module, to compensate lenders' claims if such an event were a deemed shortfall event.

assets, reducing borrower capital efficiency. To construct realistic models of borrower behavior, we analyze historical borrower data in both the V1 and V2 protocols and present this empirical study in Appendix C.5. We show that high liquidation bonuses coupled with realistic borrower behavior can lead to the protocol bearing inhomogeneous risk as a function of asset composition.

This inhomogenous, correlated risk is much stronger in Aave than in similar protocols and can cause a novel type of liquidation cascade. When liquidators choose which collateral types to liquidate, they potentially liquidate correlated assets at the same time, causing a cascade in one asset to extend to another asset. For instance, if SNX (Synthetix native token) and AAVE are highly correlated, then when a liquidator liquidates AAVE in highly volatile market conditions, they can increase the number of SNX liquidations. This occurs because when a liquidator sells AAVE — decreasing its price — they also cause SNX prices to decrease due to correlation, leading to potential SNX liquidations. We probe the likelihood of this occurring by analyzing the amount of slashing that occurs in the safety module. Slashings occur when a shortfall event takes place and liquidators are unable to clear underwater liens off of the protocols' books. In §6.4, we show that the likelihood of slash is relatively low, although non-negligible in extremely volatile conditions.

Combined, our results illustrate that with well-chosen parameters, dependent on the observed behavior of borrowers and asset volatility, the Aave protocol protects user funds while improving capital efficiency. We recommend updating parameters based off of the likelihood of depositors to hold different types of collateral. This is important as the covariance between borrower position updates is one of the most crucial variables for safety in the Aave protocol. The safety module works reasonably well to reduce the risk of correlated cascades, although further diversification of the assets in the safety module will provide enhanced tail risk coverage.

Part II

Defining Market Risk

As a decentralized protocol, the Aave protocol faces a number of risks that are more complex than those faced by their centralized counterparts. One of the main reasons for this is that the core function of liquidation, which aims to ensure that assets are always greater than liabilities, involves interactions external to the protocol. Centralized venues simply liquidate underwater collateral themselves without requiring counter-party risk, whereas decentralized protocols rely on liquidators competing to provide the service to the protocol. Smart contract audits cover endogenous risk (security risks within a contract) but do not assess market risks that concern exogenous interactions required for proper protocol function. Protocols that allow for supplying and borrowing of cryptoassets are particularly sensitive to price shocks. They require a number of different participants to be sufficiently incentivized to ensure liens are priced and liquidated correctly.

Liquidators in Aave compete for defaulted collateral and are incentivized to participate by a combination of market forces and discounts provided by the protocol. Unlike Compound, however, if no liquidators exist for an Aave market, the protocol covers its underfunded liabilities (known as shortfalls) using a safety module. Given this extra backstop for the protocol, we can enumerate the primary sources of market risk within the protocol:

1. Shocks to market prices of collateral that cause the contract to become insolvent due to under-collateralization

2. Loss of liquidity in an external market place, leading to a liquidator being disincentivized to liquidate defaulted collateral

3. Cascades of liquidations impacting external market prices which in turn lead to further liquidations (i.e. a deflationary spiral)

4. Insolvency of the safety module due to extreme events where multiple collateral types concurrently fail to be liquidated

To formally describe these market risks, we first need to be familiar with the assets, liabilities, and risk parameters within the Aave protocol. In this section, we will take care to illustrate the differences between Aave and Compound to make the risks more easily comparable.

1 Assets and Liabilities

Much like a corporation, the smart contract that executes the Aave protocol has a notion of assets and liabilities. Assets in the protocol are ERC-20 tokens that are supplied by users and the fees collected over time. Liabilities are liens that the protocol issues that are collateralized by user supplied assets. Unlike most liens in traditional finance, liens in Aave are perpetual in that a position can stay open indefinitely, accruing interest. A lien is closed when a borrower repays the borrowed quantity plus the accumulated fees.

1.1 aTokens

The Aave protocol tokenizes assets held within the protocol via a construction known as an aToken. aTokens represent the assets and accrued fees that the protocol is holding for an Aave user. The tokenization is useful for a number of reasons. First, by having a single bearer instrument that represents a user's collateral and fees, there is less accounting overhead for the end user. Secondly, this improves composability of aTokens, as users can take their aTokens and place them in other protocols. For instance, aTokens can be found on decentralized exchanges such as Uniswap, where trades between aTokens and the underlying asset are used as a form of refinancing. Finally, the tokenized asset allows for losses due to liquidations or underwater liens to be spread pro-rata amongst lenders.

A creation-redemption process is used to construct aTokens and to later receive collateral and earned fees:

- Creation: A user deposits collateral to an Aave market and mints aTokens (Aave interest bearing tokens) at a 1:1 ratio. Interest on the supplied collateral is collected by each asset's reserve and distributed directly to aToken holders wallets. This interest can also be redirected to any Ethereum public address by the aToken holder

- Redemption: A user redeems and burns their aTokens receiving their collateral plus accrued interest

There are two ways to define the token supply for a fee-wrapped tokenization of collateral. One method, which is used in Compound cTokens, is to have a fixed supply of wrapped tokens (e.g. the number of cTokens is equal to the amount of collateral deposited) whose redemption price (e.g. price of cToken to underlying) is adjusted as a function of fees accrued. Another method, which is used in Aave aTokens, is to have a dynamic supply of wrapped tokens where fees accrued cause tokens to be minted. In this method, the redemption price is fixed (e.g. 1 aToken is equal to 1 unit of the underlying collateral), but the supply balance of aTokens owned by the user fluctuates.

1.2 Risk Sensitive Parameters

In order to ensure that the protocol issues high-quality liens, there are a number of risk-sensitive parameters than can be tuned to trade-off security for capital efficiency. These parameters are similar to risk limits and stress test parameters (e.g. bounds on Value-at-Risk) that one finds in the traditional banking system. However, the autonomous, censorship resistant nature of issuance within the Aave protocol dramatically increases the importance of accurate parameter selection. In the current version of the Aave protocol (Aave V2), the protocol can adjust these parameters via governance using the AAVE token.

1.3 Loan-to-Value (LTV)

The Loan-to-Value (LTV) represents the amount of overcollateralization required for users to borrow assets. For instance, if a user deposits 1 ETH of collateral and the LTV is 0.75, then the user can borrow up to 0.75 ETH worth of another asset in the protocol. In Aave, the price of the other asset in ETH terms is provided by the Chainlink oracle, which is required to mark the loan on issuance. Within the Aave protocol, Loan-to-Values are set on a per asset basis and adjusted when market conditions change dramatically.

1.4 Liquidation Threshold

The liquidation threshold is the ratio of lien value to collateral value that leads to a loan being eligible to be liquidated. For instance, if the liquidation threshold is 0.8, then when the borrowed quantity is worth at least 80% of the collateral provided, the lien can be liquidated. Note that the liquidation threshold can be different than the LTV value. By providing a buffer, the protocol can balance between having an aggressive liquidation policy (e.g. liquidation threshold = LTV) for volatile assets and a capital efficient policy for more stable assets. For instance, if we have a stablecoin-stablecoin loan (e.g. USDT as collateral to borrow USDC) then providing a buffer (liquidation threshold > LTV) gives a borrower more time to recollateralize a loan. However, for assets where there are dramatic differences in liquidity, which would strain liquidator profitability, the protocol should prefer an aggressive liquidation policy.

1.5 Liquidation Bonus

In order to incentivize liquidators to purchase collateral from at-risk and defaulted liens, the protocol provides an incentive, termed a liquidation bonus, to liquidators. This bonus comes in the form of a discount on the collateral purchased for the liquidator. For instance, suppose that 10 ETH is liquidatable and that 1 ETH = $2000 USDT. If the protocol allows liquidators to purchase the 10 ETH for 1800 USDT each, then the liquidation bonus used is 10%. Adjusting this parameter as a function of asset volatility and liquidity is crucial for the safety of the protocol. If the bonus is too low, then during extremely volatile or illiquid conditions, liquidators might not be profitable even with the bonus. During these times, there can also be cascading, systemic effects from a lack of liquidations. For each loan that is liquidated, liquidators purchasing the collateral and selling it causes the price to further decrease causing further liquidations. On the other hand, if the bonus is too high, suppliers are losing out on profits to liquidators. Continual monitoring and adjustment of this parameter is crucial for optimizing the security vs. capital efficiency trade-off in Aave.

1.6 Reserve Factor

The Aave protocol has a reserve factor that is a percentage of the earned borrow interest that is kept by the protocol in a collector contract and not distributed to aToken holders. Currently, the funds in the collector contract are intended to promote growth and development on the protocol by paying contributors. The reserve factor is set based on the overall risk of each asset; less risky such as stablecoins have low reserve factors. The original version of the Aave protocol did not have a reserve factor, but V2 has a reserve factor that can interact with governance. One further way that the protocol can potentially increase security is via the reserve factor. If some of the reserves are used to cover risk in an asset pool, it could provide safety beyond the current safety module.

2 Safety Module

To provide further assurance to asset suppliers, the Aave protocol also has a safety module denominated in the native AAVE token. Yield seeking investors can purchase AAVE, stake (lock) it within an insurance pool, and earn a pro-rata portion of AAVE rewards (currently 550 AAVE per day). If an Aave market reaches a state where its liabilities (outstanding liens) are greater than its assets (collateral), then the AAVE safety module covers the shortfall of all markets voted by the community. Note that the AAVE-denominated pool insures each of these assets against this rare event and adds extra protection if liquidators are unable to profitably liquidate defaulted liens. Stakers in the pool receive the losses pro-rata via a similar tokenization mechanism to aTokens. When a staker redeems their staked AAVE, they have to wait 10 days (known as a cooldown period) before their redemption can be processed. This effectively acts as a withdrawal window, similar to those used in repurchase agreement markets, which aims to slow capital flight from the insurance pool and adds an additional layer of security.

We do note that there is the potential for the insurance pool to be drained when there is high correlation between AAVE and collateral assets in liens. When there is significant correlation, the protocol will effectively be selling AAVE to cover collateral losses at a time when AAVE's price relative to the collateral is decreasing. This means that distributions from the fund will need to increase in size right when an adverse market event occurs, potentially draining all of the staked assets in the safety module. Another potential concern is the lack of liquidity for AAVE token during potential slashing events. The 80/20 AAVE/ETH Balancer pool is also incentivized by the protocol to try to ameliorate the potential liquidity problems. Stakers in this pool are also receiving 550 AAVE per day in rewards.

3 Slippage and Market Impact

The main factors that affect liquidator profitability are slippage and market impact costs. Slippage represents the aggregated transaction fees that a liquidator has to pay to sell a collateral asset for a numéraire, whereas market impact refers to the price change caused by a particular sale. While slippage mainly affects individual liquidator profitability, market impact affects the likelihood of a cascading failure event. For instance, suppose that a liquidator sells 1 ETH for $1000 causing the price to move to $900. This can cause > 1 ETH of loans to be underwater, leading to another liquidation whose market impact pushes the price to $800. If this continues, the interaction between the protocol and the market impact caused by liquidators can lead to systemic failure of the protocol. Therefore, any market risk assessment requires accurate slippage and market impact models to assess the probability of systemic failure. As market conditions change, such a model needs to be continually refit and retrained to provide accurate statistics regarding such failures.

3.1 Liquidity Risk

The main source of market impact is a lack of liquidity on exchanges where liquidators sell reclaimed collateral for a numéraire. These exchanges include both on-chain, decentralized exchanges and centralized exchanges such as Binance, Coinbase, and FTX. For some assets, the primary source of liquidity is the decentralized exchange market, whereas for other assets centralized exchanges are the dominant source of liquidity. Nuances in market microstructure for a variety of assets further complicates the precise modeling of liquidator profitability. In this report, we model, on an asset-by-asset basis, the slippage and market impact costs for both centralized and decentralized exchanges.

Part III

Simulated Stress Tests

4 Background on Simulation

4.1 Agent-Based Simulation

The main tool that we use to perform simulation-based stress tests on Aave's Ethereum smart contracts is agent-based simulation (ABS). ABS has been used in a variety of stress test contexts, including to estimating censorship in cryptocurrency protocols,[5] detecting fraudulent trading activity in CFTC exchanges,[6] and stress testing frameworks from the European Central Bank[7] and the Federal Reserve.[8] These simulations, while powerful, can be difficult to make both useful and accurate as model complexity can make it hard to match experimental results.[9] Careful design, tuning, and infrastructure architecture can help avoid these pitfalls and has made ABS invaluable in industries such as algorithmic trading and self-driving car deployment.

In such industries, one takes care to ensure that the simulation environment replicates the live environment as closely as possible. Agent models interact with the deployed code in a live environment to enforce replication. This minimizes errors from mistranslations or missing minutiae. While the infrastructure overhead of simulating users interacting with a piece of complex software can be heavy, it ensures that errors are limited to those in models of agents as opposed to errors in the models of system dynamics.

As an example, interest rate curves in Aave (Appendix 1) are described via a simple mathematical formula. One can simulate agents directly interacting with this formula, without needing to host the Ethereum environment and having the agents generate transactions. However, Ethereum's 256-bit numerical system and precision differences between different ERC-20 contracts can often lead to disastrous losses due to numerical errors. These cannot be probed without running simulations directly against the Ethereum smart contract and generating the exact same transactions that an agent would if they were a liquidator interacting with the live contract. ›

[5] Chitra et al. (2019)
[6] Yang et al. (2012)
[7] Halaj (2018); Liu et al. (2017)
[8] Geanakoplos et al. (2012); Bookstaber et al. (2018)
[9] Fagiolo and Roventini (2016)

4.2 Gauntlet Simulation Environment

The Gauntlet platform, which was used for all simulations and results in this report, provides a modular, generic ABS interface for running simulations directly against Ethereum smart contracts. In this system, the agent models are specified via a Python domain-specific language (DSL), akin to Facebook's PyTorch,[10] and interact with a custom-built Ethereum virtual machine written in C++. Agents can also interact with non-blockchain modules, such as historical or synthetic market data and/or other off-chain systems. Gauntlet has made significant performance optimizations for interacting with the EVM in Python, resulting in performance gains of 50-100x over the stock tooling. The DSL hides the blockchain-level details from the analyst, allowing the end-user to develop strategies that can migrate from one smart contract to another, should they have similar interfaces. Most of the platform's design is inspired by similar platforms in algorithmic trading that allow for quantitative researchers to develop strategies that execute over multiple exchanges (with varying order books, wire protocols, slippage models, etc.) without having to know these low-level details. Moreover, the non-blockchain portions of the simulation are analogous to trading back-testing environments,[11] so that agents are interacting with realistic order books and financial data. It should be noted that the strategies emit valid EVM transactions and can be deployed to Ethereum mainnet using the same code path.

4.3 Aave Simulation Overview

For the simulations in this report, we deployed the Aave V1 and V2 contracts within the Gauntlet platform and also set up a variety of slippage models and synthetic price trajectories (see Appendix C.3). As mentioned in the introduction, we improved these models since our Compound report, allowing for a wider variety of assets to be represented. We implemented liquidator strategies in our DSL, which allowed for a variety of liquidators with different risk and time preferences to interact directly with the Aave contracts and with simulated order books (see Appendix C.7). These strategies also include optimization components so liquidators can optimize the amount of collateral purchased based on their slippage estimates (see Appendices C.7.1) We also wrote strategies for borrowers in the Aave protocol using the DSL and fit their risk preferences based on historical data (see Appendix C.5). In particular, we allow for borrowers to have dynamic strategies that involve borrowing n assets against m collateral assets. For further details on simulation methodology, please consult Appendix C.

[10] Paszke et al. (2019)
[11] Nystrup et al. (2019)

5 Questions Addressed in Stress Tests

5.1 Risk Analysis

The simulation provides insight about market insolvency and how the liquidation mechanic of the protocol will behave in high stress situations. From a liquidity supplier's perspective, the protocol is safe only if the supplied assets can be withdrawn. A functioning liquidation mechanism is critical to the safe operation of an Aave market. When an asset price drops and no liquidators have a sufficient incentive to repay the borrower's outstanding debt, the system fails and some suppliers cannot withdraw their assets. Recall that a rational liquidator's goal is to make a profit in each liquidation opportunity. Each opportunity is dependent on the liquidation bonus and slippage.

5.2 Capital Efficiency

With the abundance of collateral assets available on Aave, the yield of each collateral and the size of each pool are also important optimization inputs. Detailed in Appendix C.6, we have constructed a nonlinear programming (NLP) optimization problem which will weight the default probability of each asset, the total size of each token pool, and the total borrowed amount of each token at the end of simulation.

5.3 Metrics

We will first define metrics that will help us answer these questions in a quantitative manner. An *insolvent account* is any account where the total borrow value in USD is larger than the total collateral value in USD. The *net insolvent value* of an insolvent account is the difference between the total borrow value in USD and the total collateral value in USD. The *net insolvent value percentage* of an account is the net insolvent value of the account at the end of a simulation run divided by the total collateral value of the account at the beginning of the run, and the *net insolvent value percentage* of the entire protocol is the sum of the net insolvent value of all accounts divided by the total collateral value of all accounts. Because of the many-to-many nature of collateral and borrowed assets on the Aave protocol, attributing insolvency statistics to specific assets can be convoluted. The *asset net insolvent value percentage* is the sum of net insolvent value divided by the total collateral value pro-rata supply percentage for all accounts that supply a given asset as collateral. The *slashing run percentage* is the percentage of simulation runs that end with greater than 1% of the total value (in Aave) of the safety module slashed to cover insolvencies and with greater than 10% drop in total value (in USD) of the safety module.

5.4 Scenarios

In light of this, the main questions that we focus on answering are the following:

- Is the protocol safe when the borrowing parameters (loan-to-value and liquidation threshold) of a particular asset are adjusted?
- Is the protocol safe when the liquidation bonus of a particular asset is adjusted?
- What is the effect of slashing and selling of the safety module on the protocol?

These questions focus on the market risk and the ability of the entire protocol to remain solvent and minimize losses through slashing of the safety module and possible reserves. For each simulation run, we track the total insolvency value of the protocol, as well as the value of each collateral that is insolvent. For the entire protocol, a 1% net insolvent value percentage mark is the criteria for safety. However, for individual assets, a 5% net insolvent value percentage mark will be the failure criteria.

6 Results

We consider a number of facets of safety when answering the questions of the last section. First, we describe how to use historical market conditions to act as a baseline for stress tests of extreme, not-seen-in-the-wild conditions that we subsequently analyze. Normalizing relative to these historical conditions, we then investigate the effect of volatility on the main protocol parameters — loan-to-value ratio (LTV), liquidation threshold, and liquidation bonus. Finally, we conclude with an analysis of the efficacy of the safety module. This analysis provides a measure of the likelihood of correlated liquidation cascades, as described in Part I.

6.1 Historical Market Conditions: Black Thursday

There are two important aspects of the historical market conditions in Figure 1 of Black Thursday that are key components of our simulation. First, we try to model the volatility of the assets as closely to the realized volatility of ETH on Black Thursday. Many of the assets available on the Aave protocol today were not liquid, not heavily utilized, or not created at that time, so we rely on correlation metrics and trailing volatility to define price trajectories (See Appendix C.3) that closely resemble what could be expected during asset capitulation. In addition, network congestion (See Appendix C.4) and the inability for liquidators to close liens was prevalent during periods of time on Black Thursday.

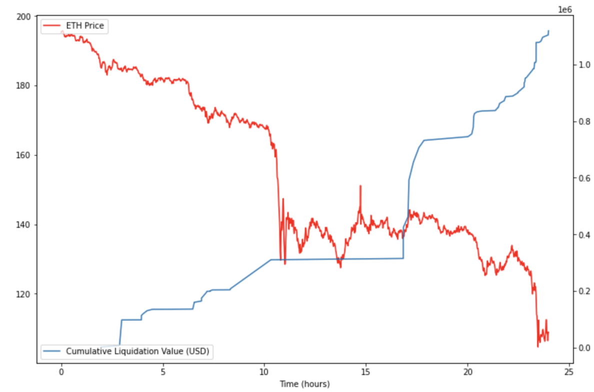


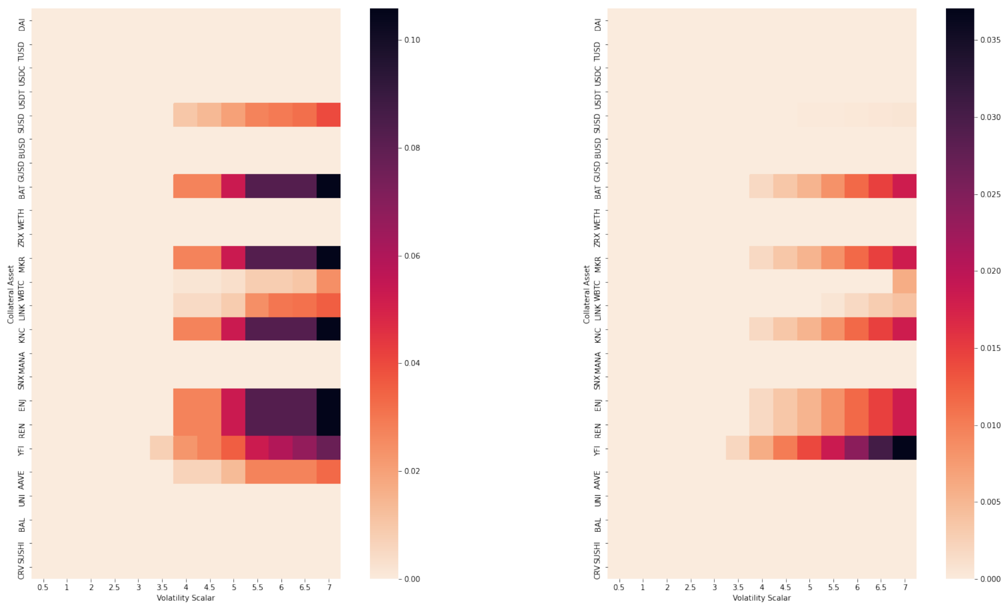
Figure 1: ETH price and the cumulative liquidated collateral value on the Aave V1 protocol on Black Thursday (March 12, 2020)

6.2 Volatility vs Asset Borrow Parameters (LTV and Liquidation Threshold)

As previously discussed, a liquidator's behavior is determined by the profit of the liquidation, e.g. the liquidation bonus net the cost of the trade. Important factors for understanding the cost of a trade include transaction fees (gas fees), trading fees, and slippage. Note that trade size and volatility have varying impacts on realized slippage per asset. This suggests that the protocol's safety heavily depends on each collateral asset's total outstanding debt, volatility, and liquidity. In our simulation, asset volatility will determine the price trajectory (c.f. Appendix C.3) of all the assets in the Aave protocol as well as the network congestion as defined in C.4. Considering that different collateral assets have different orders of magnitude of trading volume, normalizing the total outstanding debt enables us to intuitively compare the debt (relative to the collateral asset's liquidity) between different collateral assets.

We will first define normalized quantities that are utilized within the figures of this section. The volatility scalar is defined as a multiplier on the 30-day trailing volatility of each asset. For instance, a volatility scalar value of 1.5 is equal to 1.5 times the 30-day trailing volatility. Plotting figures such as Figures 2a and 2b as functions of the volatility scalar instead of raw volatility allows for simpler risk comparisons across assets with different volatility profiles.

Figure 4 depicts, on a logarithmic scale, how the daily trading volume of different assets varies as a function of annualized volatility (normalized to that of Ethereum). Similarly, Figure 5 depicts how the liquidity of an asset depends on normalized volatility. These figures clearly illustrate that the Aave protocol houses a variety of assets with dramatically different liquidity and trading volumes. Such differences are an important factor when comparing both liquidator profitability on single liquidations as well as liquidations of correlated assets that have dramatically differing trading volumes. Note that in these figures, the annualized volatil-



(a) Percentage of users with any insolvent loan per collateral

(b) Net insolvent value percentage of all insolvent accounts per collateral

Figure 2: Initial simulation results for 5880 simulation runs with no parameter changes with Aave V2 contracts

ity of ETH is 97% (e.g. the y-axis of Figures 4 and 5 is equal to 1 at 97% annualized volatility). For reference, this also implies that at a volatility scalar of 7 (the rightmost column of Figure 2b), the annualized volatility for ETH is 679%, which is a daily volatility of 42%. In order to capture changing user behavior and risk tolerance as a result of borrow parameter changes, we linearly scale the total borrowed amount as we adjust the LTV ratio of YFI. Figure 4 shows that trade-off between increasing the LTV and borrow behavior of YFI on the protocol with the asset net insolvent value. It also highlights that the protocol, with the current LTV setting, should maintain safety even in extreme volatility settings. Figure 5 demonstrates the muted impact of adjusting the liquidation threshold on YFI net asset insolvent value.

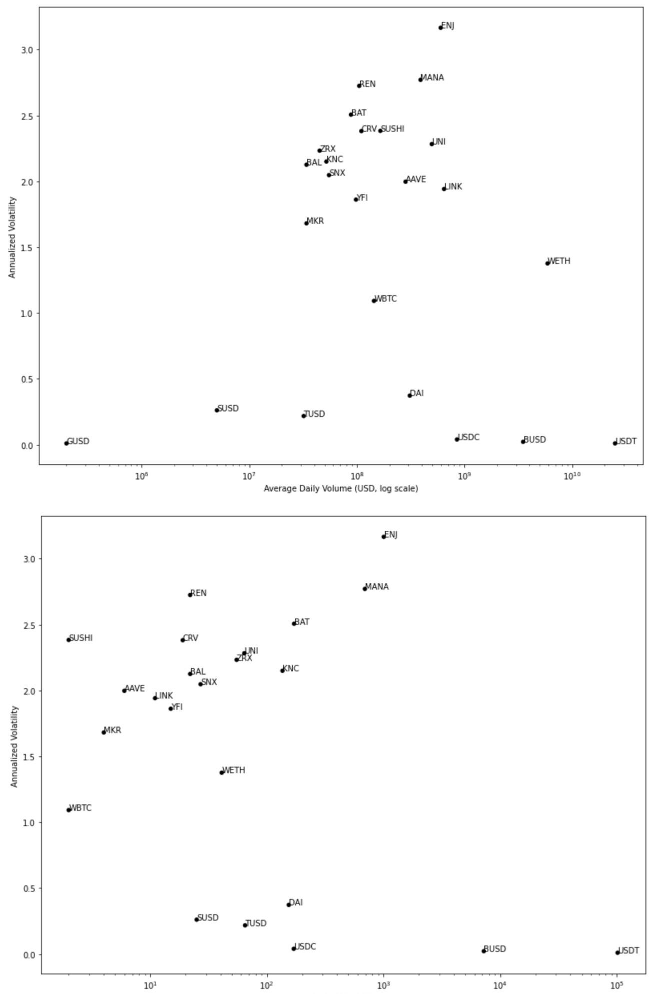


Figure 3: Distribution of market data from centralized and decentralized exchanges from January and February 2021 (top) and distribution of scaled liquidity, the ratio of average daily trading volume and total collateral used from borrower data on 28 Feb 2021 (bottom) for Aave V2 assets

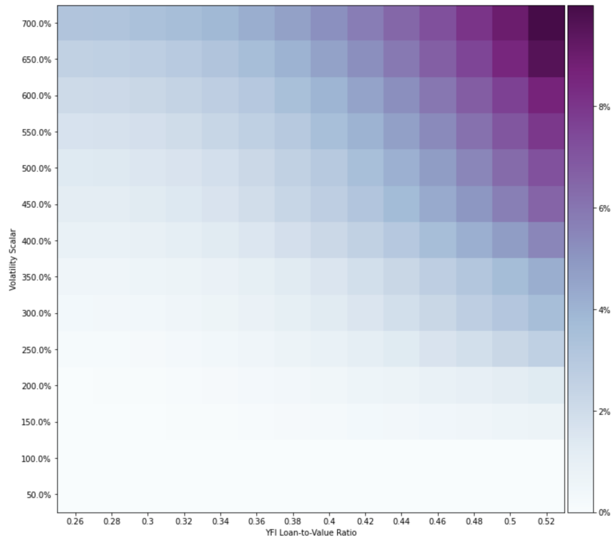


Figure 4: YFI asset net insolvent value percentage as a function of correlated asset volatility and loan-to-value ratio. Note that the insolvent value percentage is monotonic in both loan-to-value and volatility scalar, as one intuitively expects.

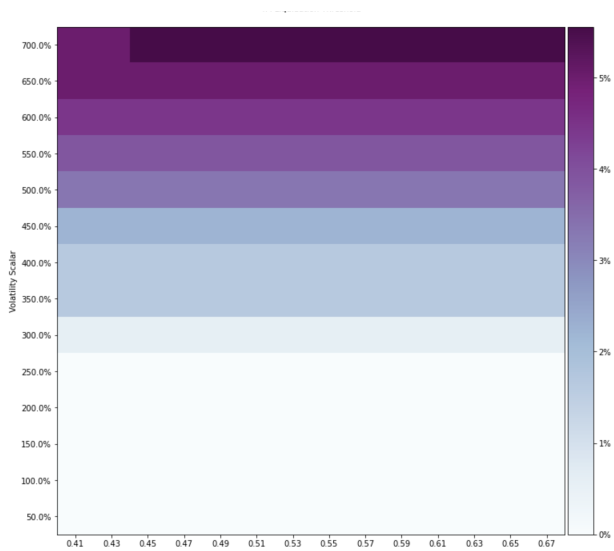


Figure 5: YFI asset net insolvent value percentage as a function of correlated asset volatility and liquidation threshold. Note that the choice of liquidation threshold is only important at high enough volatilities: at low volatilities, changes to the threshold do little to change net insolvent value. See §6.3.1 for an explanation for what happens at high volatilities.

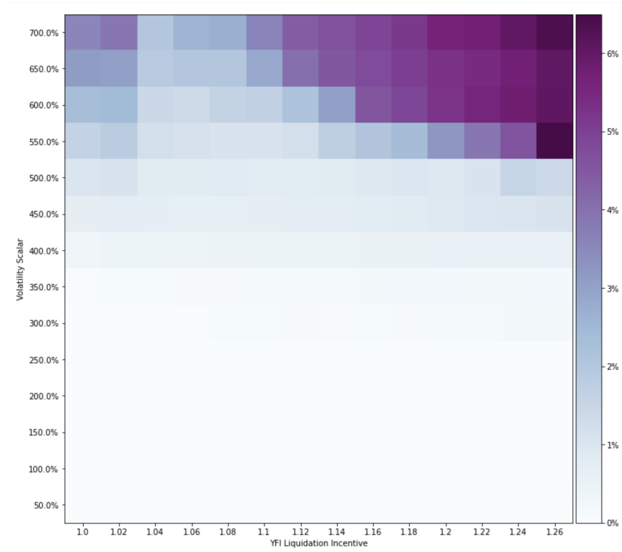


Figure 6: YFI asset net insolvent value percentage as a function of correlated asset volatility and liquidation bonus. Note that this measures the likelihood of correlated cascades (e.g. the y-axis is the volatility of an asset highly correlated to YFI with different liquidity levels). This figure illustrates that correlated assets admit a 'safe region' in terms of liquidation bonus where net insolvent value stays relatively low. Unlike Figure 5, we see that there is a much stronger dependence on liquidation threshold even at lower volatility scalars (c.f. §6.3.1)

6.3 Volatility vs Asset Liquidation Bonus

The amount of liquidity available for each asset determines how easily liquidators can sell the liquidated collateral into the market. This factor is also important for how much market impact the liquidator's behavior will have on the asset price. In our simulation, we vary the average daily trading volume for each asset as a lever for changing the amount of liquidity available per asset. This, in turn, gives us insight into how scarcity of trading in each asset will impact the probability of insolvency per asset. As previously described in Part II, the liquidation bonus enables liquidators to make more aggressive decisions and liquidate larger positions with respect to the liquidity and size available on exchange.

In Figure 6, we adjust the liquidation bonus for YFI across different asset volatility settings. There is the intuitive result that as asset volatility increases, there are more liquidatable users and there are more liquidations. Increasing the liquidation bonus to 120% and over decreased the number of liquidations, but the total value of the liquidations was larger. This leads to larger trades that have significant impact on the price of underlying asset. These results indicate that it is imperative to keep the liquidation bonus between 105% and 115% for most assets.

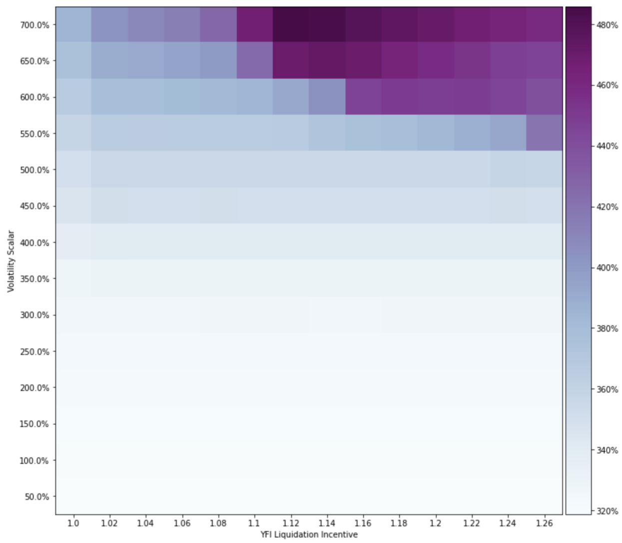


Figure 7: Total collateralization as a function of correlated asset volatility and liquidation bonus. Note that the total collateralization ratio of the system is inversely related to the net insolvency value at high volatility levels when the liquidation bonus is above 115%.

6.3.1 Why are large liquidation bonuses bad?

This seemingly anomalous dependence on liquidation bonuses is due to the following observations. The first is that large liquidation bonuses increases the regime in which the incentives are counterproductive, as outlined by the OpenZeppelin audit (H08). At specific collateralization ratios (namely at or below liquidation bonus ℓ), a liquidation can decrease the collateralization ratio. In addition, when the volatility of an asset is high, the expected number of times that the price process crosses the interval $I(p, \epsilon) = [p - \epsilon, p + \epsilon]$ increases.[12] If p is the liquidation threshold price for a lien in the protocol, then this states that the higher volatility process will spend more time in the liquidatable state, even though it may revert to a mean higher than p. Each time the price process goes below the liquidation price, liquidators have sufficient incentive to liquidate the collateral. This means that the number of partial liquidations that occur for a fixed mean is higher in the higher volatility price process.

One can view the liquidation bonus as a bound on the inverse of the number of partial liquidations needed for a loan to be completely liquidated. This is because when a loan of size L with liquidation bonus ℓ is liquidated at price p, the remaining lien has size $\approx pL(1-\ell)$.

[12] Formally, suppose that $X(t) \sim \text{GBM}(\mu, \sigma)$, $\tilde{X}(t) \sim \text{GBM}(\mu, \sigma')$ are two price paths drawn from geometric brownian distributions with mean μ and $\sigma' > \sigma > 0$. Then this statement is simply the inequality $\text{Pr}[\tilde{X}(t)\mathbf{1}(t < T) \in I(p, \epsilon)] > \text{Pr}[X(t)\mathbf{1}(t < T) \in I(p, \epsilon)]$ which is true for Geometric Brownian sample paths.

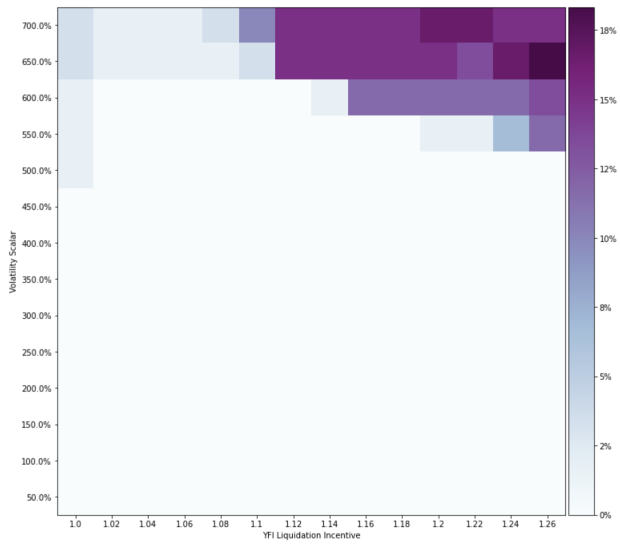


Figure 8: Percentage of simulations runs that ended with significant slashing (>1% Safety Module sold and >10% decrease in USD value). Note that we only observe significant slashing at high liquidation bonus and volatilities and that there is a 'safe region' of moderately sized liquidation bonuses.

Iterating this $O(\frac{1}{\ell})$ times will lead to remaining collateral quantity reaching zero with extremely high probability. Therefore, a high liquidation bonus corresponds to a small number of partial liquidations needs for a lien to be completely liquidated. Combining these two facts shows that a higher liquidation bonus reduces the amount of volatility needed for a lien to be completely liquidated by a sequence of partial liquidations.

6.4 Safety Module

We define the *slashing run percentage* to be the percentage of simulation runs that end with greater than 1% of the total value (in AAVE) of the safety module slashed to cover insolvencies and with greater than 10% drop in total value (in USD) of the safety module. The choice of USD percentage drop (e.g. 10%) is a coarse measure of the transaction costs (from effectively selling AAVE for USD) incurred by the safety module when slashing insurance stakers. A key aspect of this analysis is the specialized market impact model that we used to simulate slashing events. When a slashing event occurs, the expectation is that the market will be in a volatile state. In addition, if market participants anticipate the selling of AAVE to cover insolvencies, low liquidity will be a concern. We simulate this extreme market condition by removing centralized exchange bids, and implement market impact models only built on decentralized buying liquidity. In Figure 8, we see the slashing run percentage for YFI as a function of the liquidation bonus and volatility. As one intuitively might expect, an increase in asset volatility leads to a higher slashing run percentage. However, like the counter-intuitive results of previous sections, if the liquidation threshold is too high, then the system can have a high slashing run percentage. The logic of 6.3.1 applies as well when interpreting Figure 8.

7 Conclusions

In this report we conducted a market-risk assessment of the Aave protocol via agent-based simulations run against the Aave contracts. We stress-tested the liquidation mechanism under a wide range of market volatility and sizing scenarios to ensure that the protocol can prevent borrowers from becoming under-collateralized in most of these cases. We also used historical market data from centralized cryptocurrency exchanges to ensure that assumptions about volatility and slippage are representative of real-world conditions. We found that the protocol, can withstand aggressive borrowing parameters even in extreme asset volatility and network congestion situations. However, variations in the liquidation bonuses of the protocol lead to more drastic and unexpected results, and liquidation bonus should not be the primary tool used to adjust protocol-wide market risk. We also studied the relationship between borrower behavior as a result of borrowing parameter adjustments to understand the returns of both AAVE holders and lenders on the Aave protocol as a result of parameter changes. As market conditions change, the optimal parameters and suggestions will need to dynamically shift as well. Our results suggest that monitoring and adjustment of protocol parameters is crucial for reducing risk to lenders and slashing in the safety module.

8 Bibliography

Arnott, Robert D, Jason Hsu, and Philip Moore (2005) "Fundamental indexation," *Financial Analysts Journal*, 61 (2), 83–99.

Bookstaber, Richard, Mark Paddrik, and Brian Tivnan (2018) "An agent-based model for financial vulnerability," *Journal of Economic Interaction and Coordination*, 13 (2), 433–466.

Chitra, T., M. Quaintance, S. Haber, and W. Martino (2019) "Agent-Based Simulations of Blockchain protocols illustrated via Kadena's Chainweb," in *2019 IEEE European Symposium on Security and Privacy Workshops (EuroS PW)*, 386–395, June, 10.1109/EuroSPW.2019.00049.

Curcuru, Stephanie E, Charles P Thomas, Francis E Warnock, and Jon Wongswan (2014) "Uncovered equity parity and rebalancing in international portfolios," *Journal of International Money and Finance*, 47, 86–99.

Fagiolo, Giorgio and Andrea Roventini (2016) "Macroeconomic policy in dsge and agent-based models redux: New developments and challenges ahead," *Available at SSRN 2763735*.

Geanakoplos, John, Robert Axtell, J Doyne Farmer et al. (2012) "Getting at systemic risk via an agent-based model of the housing market," *American Economic Review*, 102 (3), 53–58.

Halaj, Grzegorz (2018) "Agent-based model of system-wide implications of funding risk."

Hsu, Jason C (2004) "Cap-weighted portfolios are sub-optimal portfolios," *Journal of Investment Management*, 4 (3).

Kullander, Tiantian (2020) "A keeper's guide to arbitrage mining $ROOK," Nov, https://medium.com/keeperdao/a-keepers-guide-to-arbitrage-mining-rook-d9a56373bd14.

Liu, Anqi, Mark Paddrik, Steve Y Yang, and Xingjia Zhang (2017) "Interbank contagion: An agent-based model approach to endogenously formed networks," *Journal of Banking & Finance*.

Manuskin, Alex (2020) , Dec, https://www.zengo.com/ethology-a-safari-tour-in-ethereums-dark-forest/.

Nystrup, Peter, Stephen Boyd, Erik Lindström, and Henrik Madsen (2019) "Multi-period portfolio selection with drawdown control," *Annals of Operations Research*, 282 (1-2), 245–271.

Obadia, Alex (2020) "Flashbots: Frontrunning the MEV Crisis," Nov, https://medium.com/flashbots/frontrunning-the-mev-crisis-40629a613752.

Paszke, Adam, Sam Gross, Francisco Massa et al. (2019) "PyTorch: An imperative style, high-performance deep learning library," in *Advances in Neural Information Processing Systems*, 8024–8035.

Robinson, Dan (2021) "Ethereum Is a Dark Forest," Feb, https://medium.com/@danrobinson/ethereum-is-a-dark-forest-ecc5f0505dff.

Yang, Steve, Mark Paddrik, Roy Hayes, Andrew Todd, Andrei Kirilenko, Peter Beling, and William Scherer (2012) "Behavior based learning in identifying high frequency trading strategies," in *2012 IEEE Conference on Computational Intelligence for Financial Engineering & Economics (CIFEr)*, 1–8, IEEE.

Part IV

Appendix

A Glossary

- Debt: Amount of asset borrowed from an asset pool.
- Insolvent : An account is insolvent if the value of an account's debt exceeds the value of the collateral.
- Loan-to-value (LTV) ratio: The maximum debt-to-collateral ratio of an collateral asset a user may borrow. The LTV ratio of an account is the weighted sum of the total value of each collateral and each collateral's LTV.
- Liquidation Threshold: The maximum debt-to-collateral ratio of an asset before a loan can be liquidated. The liquidation threshold of an account is the weighted sum of the total value of each collateral and each collateral's liquidation threshold.
- Collateralization ratio: The ratio of collateral-to-debt, usually reported in percentage points. For instance, a collateralization ratio of 200% means that one needs two times as much collateral deposited into the contract as the maximum borrow quantity. Concretely, this would mean that one must deposit $200 worth of ETH in order to borrow $100 of a stablecoin.

- Borrowing capacity: Current value of collateral deposited into the contract multiplied by the LTV.
- Collateral requirement: Value of debt divided by the liquidation threshold.
- Health Factor: The ratio of collateral requirement and the total borrows with fees of an account.
- Liquidatable: An account is liquidatable if the account's value of debt exceeds its borrowing capacity. In other words, an account is liquidatable if the account's collateral value falls below the collateral requirement, which is equivalent to the health factor of the account falling below 1.
- Slippage: The amount of price impact that a liquidator engenders when trying to sell collateral. Slippage is denoted $\Delta p(q)$ and is formally defined as the difference between the midpoint price at time t, $p_{\text{mid}}(t)$ and the execution price, $p_{\text{exec}}(q,t)$ for a traded quantity q at time t, $\Delta p(q,t) = p_{\text{mid}}(t) - p_{\text{exec}}(q,t)$. This quantity is usually a function of other variables, such as implied and realized volatilities. Slippage is also known as market impact within academic literature.

B Interest Rate Models

Interest rates on the Aave protocol are set by the ratio between the amount of liquidity supply L_t and the amount of borrowing demand B_t at block height t. This ratio, known as the *utilization rate*, is defined as

$$U_t = \frac{B_t}{L_t}$$

As utilization increases, the concern is that capital will become more scarce and it could possibly introduce liquidity and withdrawal problems. As utilization decreases, the concern is that suppliers will not be earning sufficient yield on their supplied assets. To maintain sufficient liquidity and borrow, a *optimal utilization* $U_{optimal}$ or target utilization is set per asset based on overall market risks including volatility and liquidity. Interest rates grow slowly between 0 and the *optimal utilization*, but grow quickly between the *optimal utilization* and 1.

$$if \;\; U_t < U_{optimal} : R_t = R_0 + \frac{U_t}{U_{optimal}} R_1 \tag{1}$$

$$if \;\; U_t \geq U_{optimal} : R_t = R_0 + R_1 + \frac{U_t - U_{optimal}}{1 - U_{optimal}} R_2 \tag{2}$$

The Aave protocol offers two different interest rate models that both utilize bonding curves to determine the supply and borrow interest rates across assets. All assets that can be borrowed have a variable interest rate model which updates per block based on the utilization . Some assets also have a stable interest rate model which only rebalances under specific conditions. A stable interest rate model rebalances up when either the asset utilization rate is above 95% or when the weighted average of all borrow rates is less than 25%. A stable interest rate model rebalances down only when the current stable rate is 20% or more above the calculated stable rate.

C Simulation Features

C.1 Environment and Sampling

The simulation environment allows for configuration of the Aave network's state, including agent distribution, agent behavior, and smart contract parameters. The simulation environment directly interacts with the Aave smart contract deployed on Gauntlet's simulated Ethereum virtual machine. At each time step, agents observe the state of environment variables and on-chain contracts. The policy defines the agent's behavior at a given time and state. When each agent performs an action, the simulation environment submits transactions to the blockchain and updates the state of the smart contract. For each set of parameters, we run simulation 20 times to make sure that the sample size is large enough to cover a wide range of borrowing distributions.

C.1.1 Simulation Duration

If the simulation time duration is too short, the price movement will be insufficient to affect the overall agent behavior. On the other hand, even if we assume that most borrowers are passive participants and won't often change their debt position, they may still adjust their position when the time duration is long. To balance both of these factors, we use one day as the time duration to run simulations.

C.1.2 Agents

There are four types of agents in the simulation setup: borrower, lender, liquidator, and oracle. Borrowers are initialized using on-chain data about supplying and borrowing behaviour per asset. The lender lends assets have net borrows larger than net lends from the borrower agents. During each time-step, the liquidator looks for borrowers who have account health factors lower than 1 and liquidates the positions by repaying the borrowed asset to receive the

Figure 9: Pearson correlation from January 2021 for all assets in the Aave V1 protocol

collateral asset. The price oracle updates the on-chain token price based on the correlated Geometric Brownian Motion model described in C.3.

C.2 Asset Price Correlation

We use the Pearson correlation coefficent of 30-day asset prices to determine the asset correlation for the multi-asset GBM model. Given two random variable X and Y with standard deviations σ_X and σ_Y, the Pearson correlation coefficient is defined as:

$$\rho_{X,Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y}$$

where the covariance of X and Y is:

$$\mathrm{cov}(X,Y) = \mathsf{E}((X - \mu_X)(Y - \mu_Y))$$

and the mean of X and Y are μ_X and μ_Y. In the graph of figures 9 and 10, we show all correlation coefficients for assets listed on Aave in January 2021. We use Cholesky decomposition with this symmetric positive definite correlation matrix to generate the lower triangular covariance matrix used in the price trajectory.

C.3 Price Trajectories

We use a multi-asset correlated Geometric Brownian motion (GBM) to simulate price trajectories. This stochastic process obeys the Ito stochastic differential equation, $dX_t = \mu S_t dt + \sigma S_t dW_t$, where dW_t is the standard Wiener measure on $\mathbb{R}^n$. GBM is also equivalent to the exponential of a randomly varying quantity follows a Brownian motion,

$$X_t = X_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right)$$

For the correlated asset price trajectory generation, the standard Wiener process in the analytical solution for asset A is replaced with Brownian motion with correlation $\rho_{A,B}$ for all other assets. For each volatility, we generate 20 different price paths to cover a wide range of variation. The graph in figure 11 shows the difference between the correlated and standard Geometric Brownian motion price trajectories.

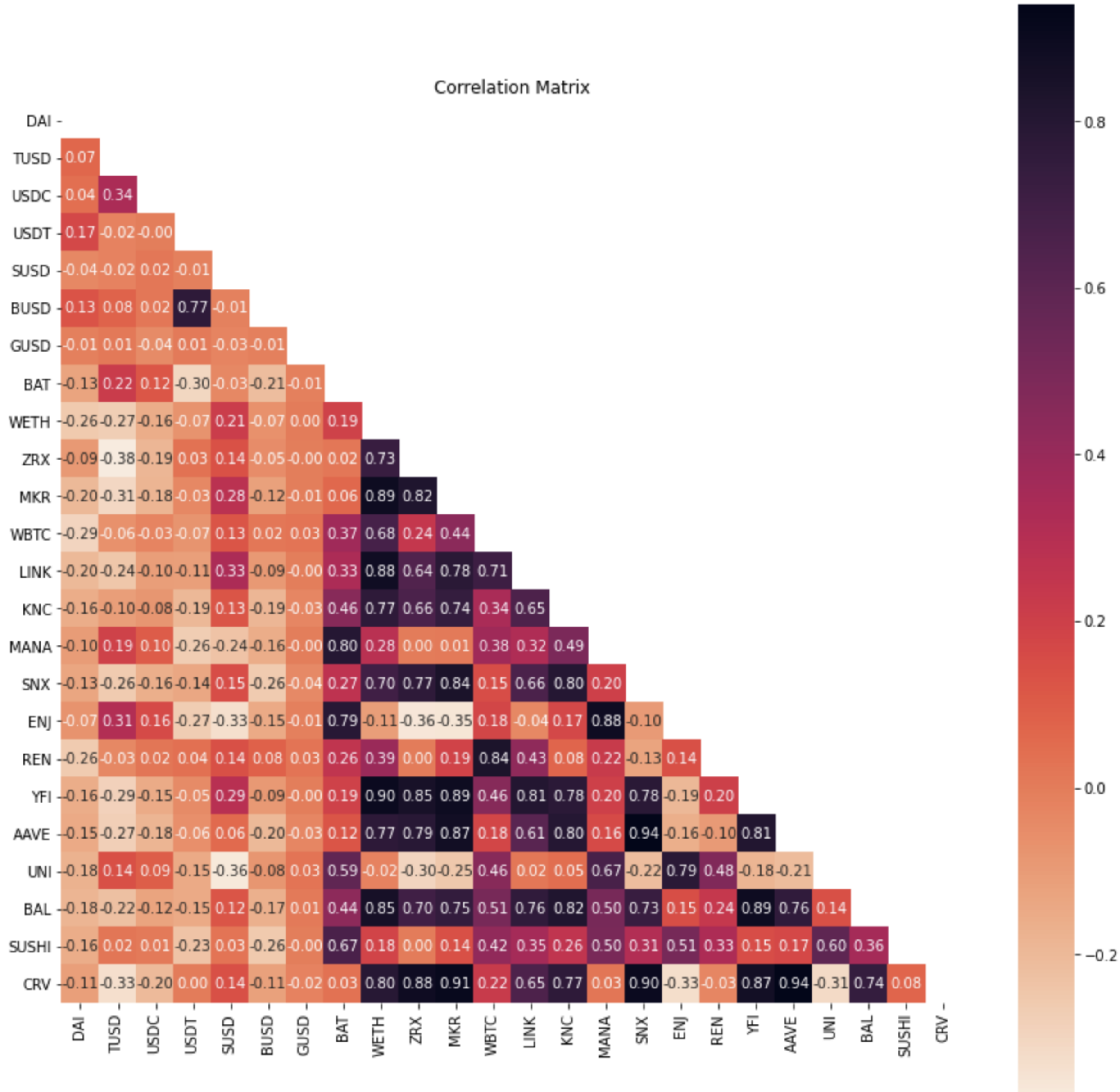


Figure 10: Pearson correlation from March 2021 for all assets in the Aave V2 protocol

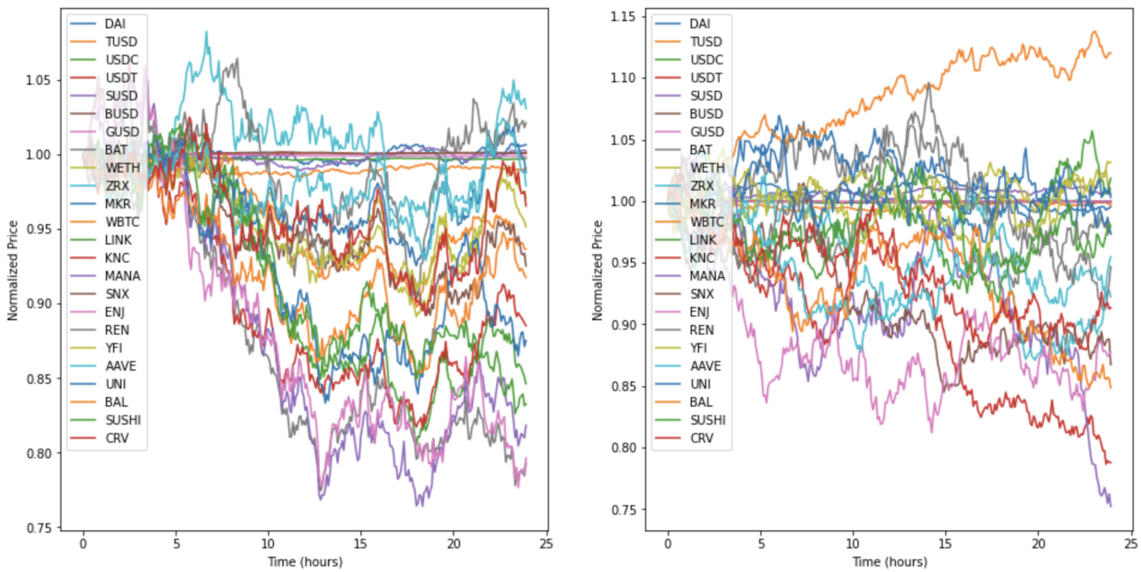


Figure 11: Sample paths from correlated GBM price trajectories (left) and the standard GBM distribution (right)

C.4 Network Congestion

In order to simulate gas congestion and transaction delay, we construct modified and truncated normal distributions from 3 month trailing gas price data. At each time step in the simulation, we sample from the probability distribution to define the liquidator behavior. As we increase ETH volatility in our simulations, we also increase the mean, standard deviation, and skew of the gas price distribution. Figure 13 depicts sample price distributions for various skew settings. The skew settings used in simulation are trained against historical ETH volatility levels. The current mapping of expected transaction delay time with respect to gas price is trained on Ethereum pending transactions ("mempool") snapshot data. This current model makes assumptions about the direct relationship between the gas price and the transaction delay. In the future, we will explore training liquidator behavior on competitive gas pricing.

C.5 Borrower Initialization

We pull Aave's onchain top 30 borrower data and use the borrowers' collateral and borrows value to initialize the borrowers. To understand the collateral-asset pair that may pose systemic risk, we visualize the borrowed amount for each pair of the assets. The borrowed amount is attributed to the pair on a pro-rata basis. e.g. If a user supplies $20 worth of WBTC and $10 worth of ETH, and borrows $3 worth of DAI, we attribute $2 to WBTC-DAI and $1 to ETH-DAI

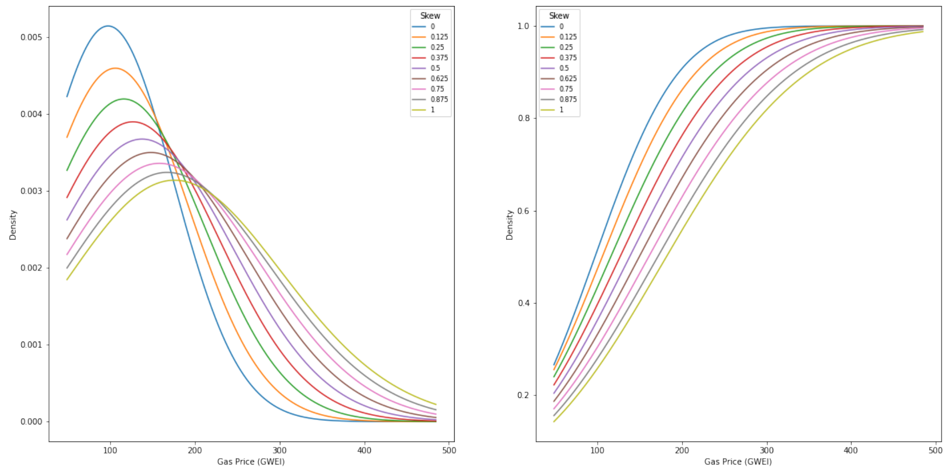


Figure 12: Gas price probability density functions (left) and cumulative distribution functions (right) for different levels of skew

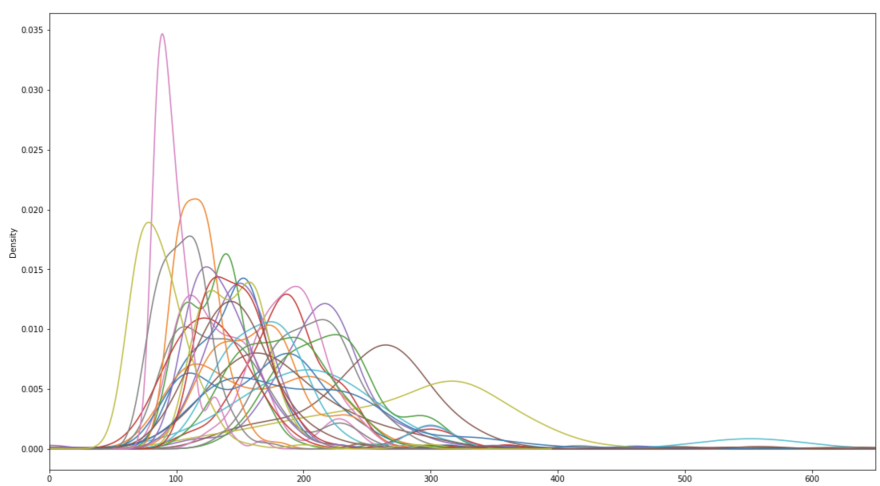


Figure 13: Smoothed historical daily gas price distributions for February 2021 from per minute percentile data

collateral-borrow pair. In addition, we compare both the overall collateralization of the ratio of the users and the collateral-borrow data, which can be seen in the graphs in figures 14 and 15.

C.5.1 Small Borrower Estimation

When using the 30 largest borrowers to initialize the borrowed positions, the impact of high gas prices on liquidations of small accounts will be lost. In order to capture this effect, additional borrowers will also be initialized per non-stablecoin from the 2000 smallest borrowers with borrow positions greater than 100 USD. These borrowers will supply USDT and supply each non-stablecoin asset in the ratio of the mean collateralization ratio for that asset. When liquidating, gas price will be modified to match the mean supply value for that asset.

C.6 Economic Optimization Function

As previously stated, the reserves on the Aave protocol do not serve the function as a safety backstop. In addition, our analysis indicates the the probability of slashing events is low. However, the coverage of each asset pool and the amount of risk that it introduces to the entire protocol isn't captured by a monolithic safety module. In this analysis, we formulate an optimization problem around safety of the protocol and economic benefits to users of the protocol Given that there is a fixed amount of AAVE paid to stakers annually, one important question to answer is how to choose reserve factors for different assets. This is because both AAVE staked in the safety module and reserve factors both serve to buffer the protocol, but provide different levels of capital buffer. There are two competing forces here: users of the protocol want to minimize the reserve factor whereas the protocol wants to maximize it as an indirect form of insurance against asset loss. We can mathematically formulate this as a constrained optimization problem that minimizes an objective function f that maps reserve factors, rf, to a cost $f(rf)$. The constraints used will ensure that the liabilities covered by the safety module less the reserve factors can still cover large scale losses.

Define $\mathcal{A}$ be the finite set of assets contained within the protocol. We will try to minimize the total value collected from the reserve factor, $V : [0,1]^{\mathcal{A}} \to \mathbb{R}$:

$$V(rf) = \sum_{a \in \mathcal{A}} rf_a \cdot \text{InterestRate}_a \cdot \text{Supply}_a$$

Given this notation, we will first write down the optimization problem and subsequently describe the constraints and constituent components. Formally, the problem we solve numerically is the following:

$$\begin{aligned}
\operatorname*{arg\,min}_{rf \in [0,1]^{\mathcal{A}}} \quad & V(rf) \\
\text{s.t.} \quad & 0 \leq SM + \sum_{a \in \mathcal{A}} DR_a \cdot \textsf{Supply}_a - RU \cdot (rf_a \cdot \textsf{InterestRate}_a \cdot \textsf{Supply}_a) \\
& 0 \leq 0.05|\mathcal{A}| - \sum_{a \in \mathcal{A}} |rf_a|
\end{aligned}$$

where SM is the target safety module utilized (approximately 9MM USD as of 2021-01-27) and DR_{asset} is the expected default percentage on aggregated high vol, high gas congestion sims, and RU is the percent of the reserve used for the safety module. The first constraint ensures that the sum of the target safety module utilized and total value collected from the reserve factor will be greater than the expected shortfall value (given high volatility and high gas congestion settings): Moreover, the second constraint encodes a restriction on the aggregate change of all reserve factors to be less than 0.05 per symbol:

$$0 \leq 0.05|\mathcal{A}| - \sum_{a \in \mathcal{A}} |rf_a|$$

Given the assumption that supply will be change based on reserve factor:

$$\textsf{Supply}_{asset} = (1 - \Delta rf_{asset}) \cdot \textsf{Supply}_o$$

The constraint on the aggregate change tries to eliminate the polarized and drastic solutions.

C.7 Liquidator

C.7.1 Slippage

A rational liquidator's main goal is to maximize profits, by ensuring that the revenue received from the liquidation bonus outweighs the costs. The slippage model assumes that a liquidator will submit a market order on a single exchange, so the cost is the worst-case estimate. Let the liquidation bonus be denoted by η, the trading fee denoted by γ, and the transaction fee denoted by α. We can then formulate the profit $\mathfrak{p}$ of each trade as

$$\mathfrak{p} = \max((\eta - \Delta p(q) - \gamma)q - \alpha, 0)$$

For each liquidation opportunity, the liquidator repays the minimum value among

- Maximum repay value: borrower's outstanding debt $\times$ close factor

- Value of borrower's liquidatable collateral
- Liquidator agent's perceived optimal repay amount

The Aave protocol defines maximum repayment amount for liquidating a borrower. If the collateral price drops too fast during periods of extreme volatility and falls below the maximum repay value, a liquidator can only repay up to the value of the borrower's liquidatable collateral. A liquidator also estimates the perceived slippage and calculates the optimal repay amount to maximize the profit. The optimal repayment amount calculation is discussed in the next section. After the liquidator acquires the collateral from the Aave protocol, she immediately sells all received ETH in exchange for USD on an open exchange to realize profit. To derive the optimal repayment amount to maximize liquidator profit, we first plug the slippage model into the profit function. For instance, in the linear slippage case, we have the following:

$$\mathfrak{p} = (\eta - I\sigma q - \gamma)q - \alpha$$

To maximize profit, we find the derivative of profit with respect to the normalized order size $\underline{q}$ and find a value $\underline{q}^*$ such that

$$\partial_q \mathfrak{p}(q^*) = 0$$

. By construction, the value $\underline{q}^*$ is the optimal order size that maximizes the net profit. Performing this calculation yields that the optimal value of $\underline{q}$ to maximize profit is, $\underline{q}^* = \frac{\eta-\gamma}{I\sigma}$. Figure 16 depicts the optimal liquidation quantity for different volatilities as a function of trade size.

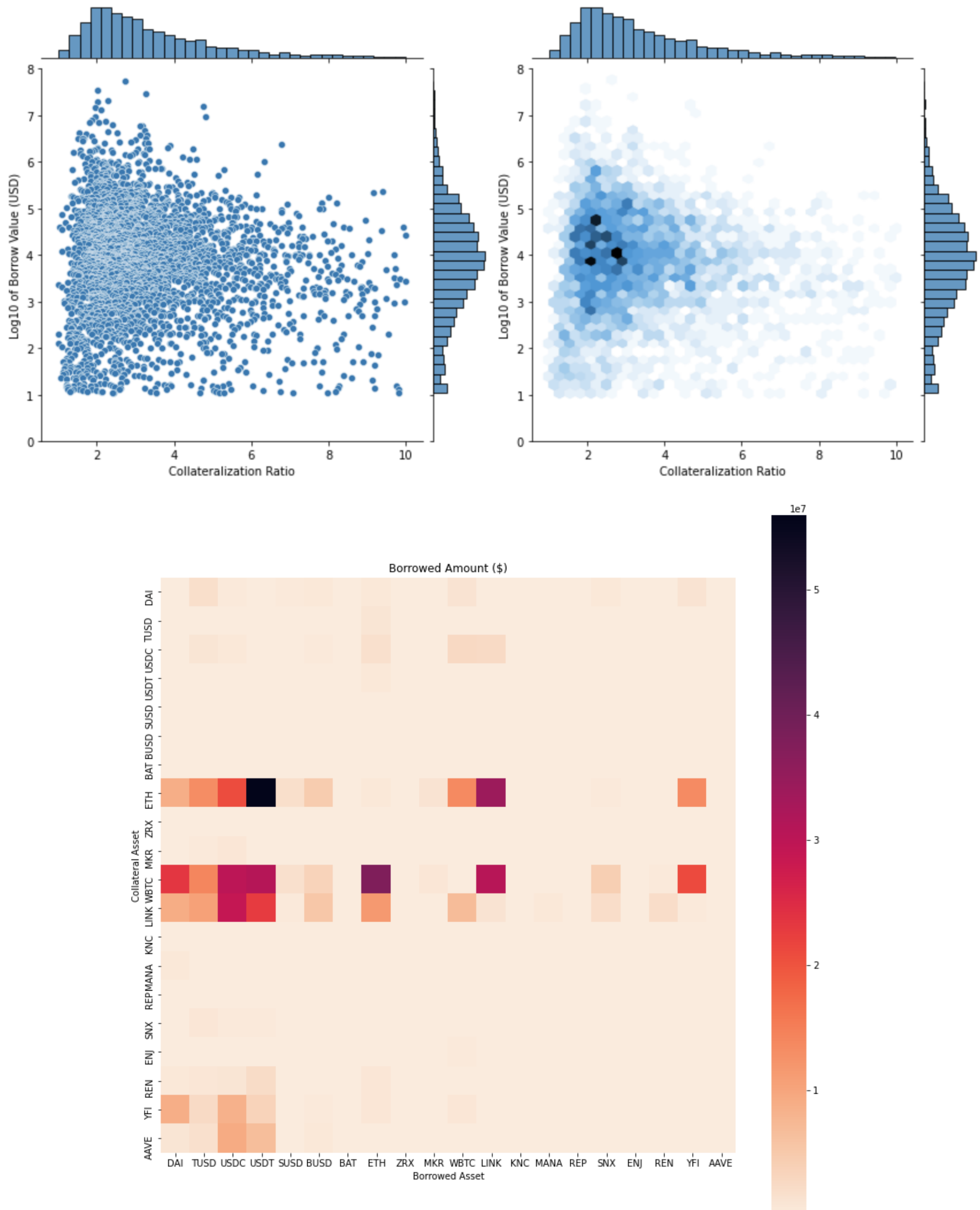


Figure 14: Distribution of collateralization ratio (top) and mean borrow size as a function of collateral input type (bottom) for Aave V1

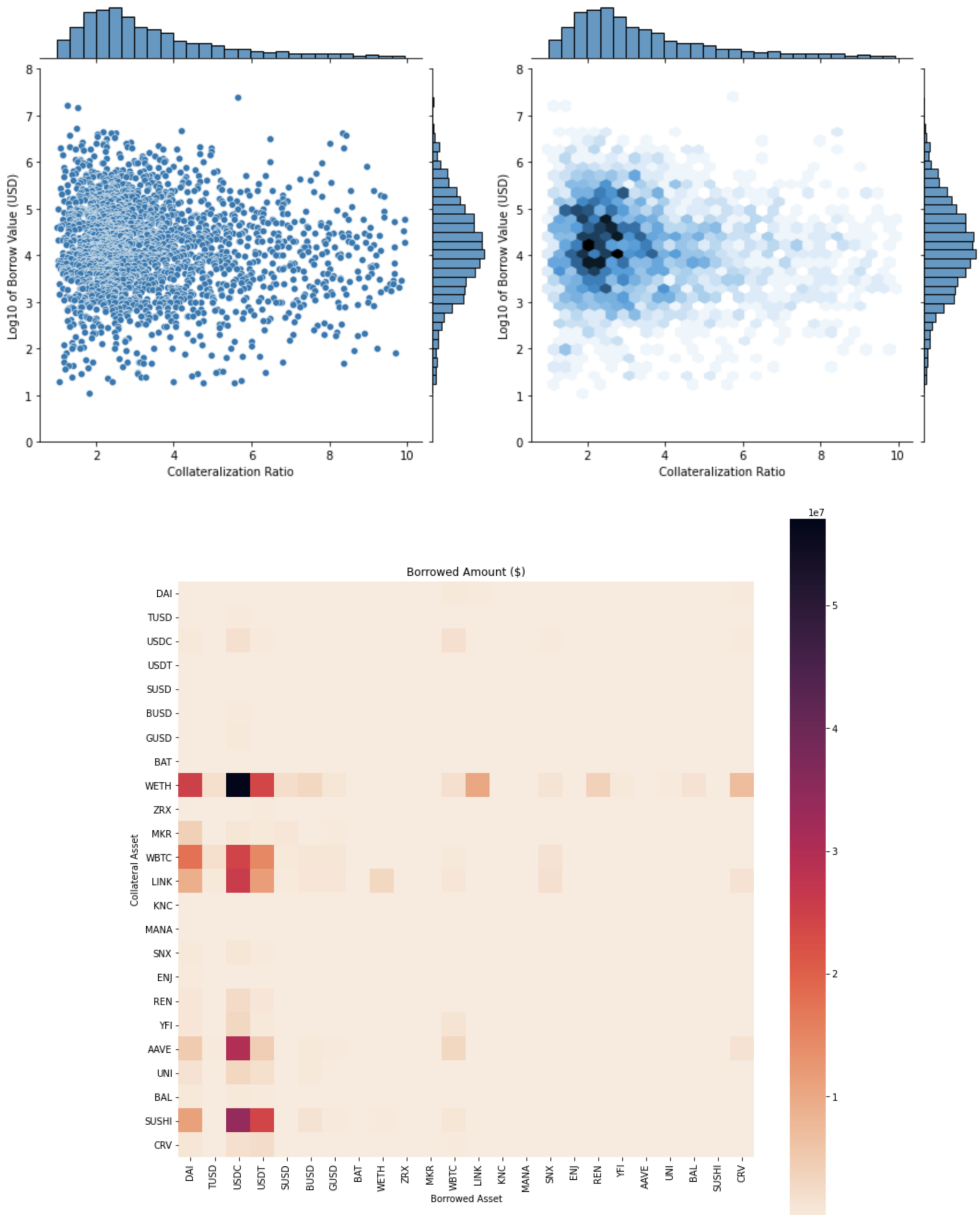


Figure 15: Distribution of collateralization ratio (top) and mean borrow size as a function of collateral input type (bottom) for Aave V2

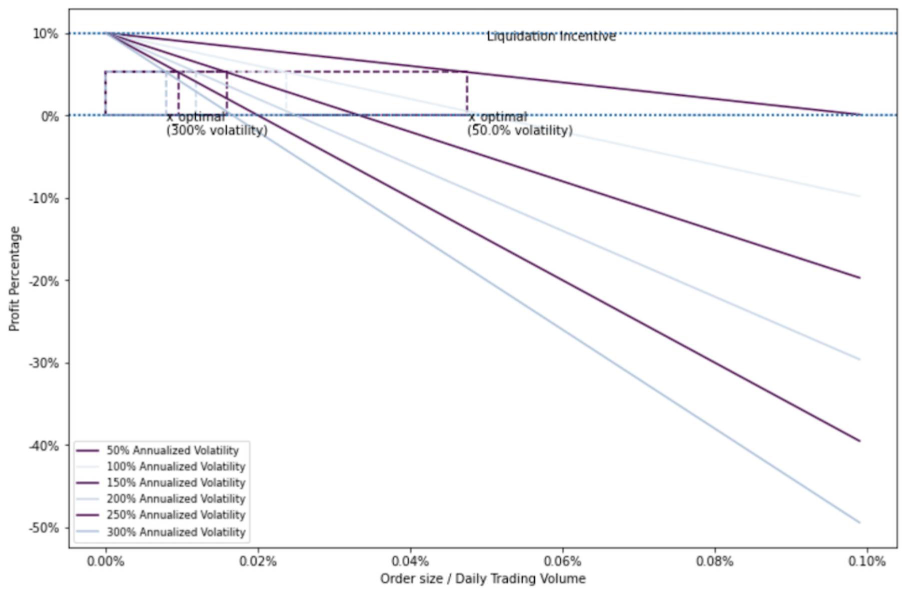


Figure 16: Liquidator profitability as a function of volatility and order size relative to daily trading volume

C.8 Raw Data

Volatility Scalar \ YFI Loan-to-Value Ratio	0.26	0.28	0.3	0.32	0.34	0.36	0.38	0.4	0.42	0.44	0.46	0.48	0.5	0.52
7.0	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.06	0.07	0.07	0.08	0.08	0.09
6.5	0.03	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.06	0.07	0.07	0.07	0.08	0.09
6.0	0.03	0.03	0.03	0.04	0.04	0.04	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07
5.5	0.02	0.03	0.03	0.03	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07
5.0	0.02	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.07
4.5	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.04	0.05	0.05	0.05	0.05	0.05
4.0	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.04	0.05
3.5	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.04
3.0	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.03
2.5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.03
2.0	0.0	0.0	0.0	0.0	0.0	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 1: Raw data of figure 4

Volatility Scalar \ YFI Liquidation Threshold	0.41	0.43	0.45	0.47	0.49	0.51	0.53	0.55	0.57	0.59	0.61	0.63	0.65	0.67
7.0	0.05	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
6.5	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
6.0	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
5.5	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
5.0	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
4.5	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
4.0	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
3.5	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
3.0	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
2.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 2: Raw data of figure 5

	YFI Liquidation Incentive													
Volatility Scalar	1.0	1.02	1.04	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.20	1.22	1.24	1.26
7.0	0.04	0.04	0.02	0.03	0.03	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.06	0.06
6.5	0.03	0.03	0.02	0.02	0.02	0.03	0.04	0.05	0.05	0.05	0.05	0.05	0.06	0.06
6.0	0.02	0.02	0.01	0.01	0.02	0.02	0.02	0.03	0.05	0.05	0.05	0.06	0.06	0.06
5.5	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.03	0.04	0.05	0.07
5.0	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01
4.5	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
4.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.01	0.01	0.01	0.01
3.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 3: Raw data of figure 6

	YFI Liquidation Incentive													
Volatility Scalar	1.0	1.02	1.04	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.20	1.22	1.24	1.26
7.0	3.85	4.03	4.1	4.14	4.27	4.67	4.86	4.85	4.79	4.75	4.71	4.67	4.63	4.59
6.5	3.76	3.9	3.92	3.95	4.0	4.25	4.7	4.73	4.7	4.63	4.58	4.53	4.48	4.46
6.0	3.67	3.78	3.78	3.81	3.82	3.85	3.92	4.05	4.46	4.49	4.49	4.49	4.45	4.39
5.5	3.58	3.66	3.66	3.66	3.66	3.66	3.67	3.73	3.76	3.77	3.83	3.88	3.93	4.2
5.0	3.5	3.56	3.56	3.56	3.56	3.56	3.56	3.56	3.56	3.56	3.56	3.56	3.58	3.57
4.5	3.46	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5
4.0	3.38	3.41	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4
3.5	3.28	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.29	3.29	3.3	3.29	3.29
3.0	3.26	3.26	3.26	3.26	3.27	3.27	3.27	3.26	3.26	3.27	3.27	3.27	3.27	3.27
2.5	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.24
2.0	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22	3.22
1.5	3.21	3.21	3.21	3.21	3.21	3.21	3.21	3.21	3.21	3.21	3.21	3.21	3.21	3.21
1.0	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2
0.5	3.19	3.19	3.19	3.19	3.19	3.19	3.19	3.19	3.19	3.19	3.19	3.19	3.19	3.19

Table 4: Raw data of figure 7

		YFI Liquidation Incentive													
		1.0	1.02	1.04	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.20	1.22	1.24	1.26
Volatility Scalar	7.0	0.07	0.03	0.03	0.03	0.07	0.10	0.13	0.13	0.13	0.13	0.17	0.17	0.13	0.13
	6.5	0.07	0.03	0.03	0.03	0.03	0.07	0.13	0.13	0.13	0.13	0.10	0.15	0.17	0.2
	6.0	0.03	0.0	0.0	0.0	0.0	0.0	0.0	0.03	0.10	0.10	0.10	0.10	0.10	0.13
	5.5	0.03	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.03	0.03	0.07	0.10
	5.0	0.03	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	4.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	4.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	2.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 5: Raw data of figure 8

D Authors

Watson Fu is a Simulation Engineer at Gauntlet. He has spent his career working in algorithmic trading at Flow Traders and the Chicago Trading Company. Prior to joining Gauntlet, he was an instructor at Hack Reactor He holds a B.S. in Chemical Engineering from Northwestern University.

Tarun Chitra is Gauntlet's CEO. Prior to Gauntlet, he spent the past decade working in simulation-based R&D roles at D. E. Shaw Research and in high-frequency futures trading. He has a B.A. in Mathematics and a B.S. in Applied Engineering Physics from Cornell University.

Rei Chiang is Gauntlet's CTO. Prior to Gauntlet, he was at Uber Advanced Technologies Group working on self-driving vehicle marketplace simulation and he was a portfolio manager and quantitative trader at GETCO and 3Red. He holds a B.S. in Mathematics and Computer Science from MIT.

John Morrow is Gauntlet's COO. Prior to Gauntlet, he worked on product and data engineering at Gigster, Microsoft, and Goldman Sachs. He also studied at Cornell where he received a B.S. in Applied Physics and Computer Science.

E Acknowledgements

The authors would like to thank Stani Kulechov, Alex Bertomeu-Gilles, Emilio Frangella, Ernesto Boado, and Jordan Gustave for helpful discussions. We would also like to thank the Gauntlet team, especially Hsien-Tang Kao for valuable help in modeling, Jeremy Batchelder and Jonathan Reem for help with the simulation infrastructure and Nicholas Cannon for proofreading and editing.