4.14. Distinctions

Affect, Effect. *Affect* is a verb meaning to produce a change. *Effect* is a noun meaning the result of a change. Examples: "Multiple roots affect the convergence rate of Newton's method", "One effect of multiple roots is to reduce the convergence rate of Newton's method to linear." Effect is also a verb meaning to bring about (as in "to effect a change"), but in this form it is rarely needed in mathematical writing.

Alternate, Alternative. *Alternate* implies changing repeatedly from one thing to another. An *alternative* is one of several options. Compare "While writing his thesis the student alternated between elation and misery", with "The first attempt to prove the theorem failed, so an alternative method of proof was tried."

Compare with, Compare to. *Compare with* analyses similarities and differences between two things, whereas *compare to* states a resemblance between them. Examples: "We now compare Method A with Method B", "Shakespeare compared the world to a stage", "Shall I compare thee to a summer's day?" As Bryson [41] explains, "Unless you are writing poetry or love letters, *compare with* is usually the expression you want." *Compare and* is an alternative to *compare with*: "We now compare Method A and Method B" or, better, "We now compare Methods A and B."

Compose, Comprise, Constitute. *Compose* means to make up, *comprise* means to consist of. "Comprised of" is always incorrect. Thus, "the course is composed of three topics" or "the course comprises three topics", but not "the course is comprised of three topics." *Constitute* is a transitive verb used in the reverse sense: "these three topics constitute the course."

Due to, Owing to. These two expressions are not interchangeable, though writers frequently use *due to* in place of *owing to*. Use *due to* where you could use "caused by", or "attributable to"; use *owing to* where you could use "because of". Thus "The instability is due to a rank deficient submatrix" but "Owing to a rank deficient submatrix the computed result was inaccurate."

Fewer, Less. *Less* refers to quantity, amount or size, *fewer* to number. Thus "the zeros of $f(x)$ are less than those of $g(x)$" means that if x is a zero of f and y a zero of g then $x < y$, whereas "the zeros of $f(x)$ are fewer than those of $g(x)$" means that g has more zeros than f. Bryson [41] states the rule of thumb that *less* should be used with singular nouns and *fewer* with plural nouns: less research, less computation, fewer graduates, fewer papers.

Practice, Practise. In British English, *practice* is the noun and *practise* the verb (as with advice and advise). Thus "in practice", "practice

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Practice, Practise. In British English, *practice* is the noun and *practise* the verb (as with advice and advise). Thus "in practice", "practice session", "practise the technique", "practised speaker". But in American English both verb and noun are spelt *practice*.

Which, That. A "wicked which"[5] is an instance of the word *which* that should be *that* (example: replace the word before *should*, earlier in this sentence, by *which*). The rule is that *that* defines and restricts, whereas *which* informs and does not restrict.[6] (Mathematicians should be good at spotting definitions.) Note the difference between the following two examples.

"Consider the Pei matrix, which is positive definite." We are being told additional information about the Pei matrix: that it is positive definite.

"Consider the Pei matrix that is positive definite." Now we are being asked to focus on a particular Pei matrix from among several: the one that is positive definite.

A useful guide is that which-clauses are surrounded by commas, or preceded by a comma if at the end of a sentence. If you're not sure whether to use *which* or *that*, see whether your sentence looks right with commas around the relevant clause. Sometimes it pays to introduce a wicked which to avoid ugly repetition, as has been done in the sentence "This approach is similar to that which we used in our earlier paper" (though "the one we used" is better). A rule of thumb discussed in [164, pp. 96–97] is to replace *which* by *that* whenever it sounds right to do so.

4.15. Elegant Variation

Elegant variation is defined by the Fowlers [84] as "substitution of one word for another for the sake of variety". It is a tempting way to avoid repetition, but is often unnecessary and can introduce ambiguity. Consider the sentence "The eigenvalue estimate from Gershgorin's theorem is a crude bound, but it is easy to compute." Does Gershgorin's theorem yield an estimate or a strict bound? We cannot tell from the sentence. In fact, the answer is that it can yield either, depending on how you interpret the theorem. A rewrite of the sentence (with knowledge of the theorem) produces "The eigenvalue inclusion regions provided by Gershgorin's theorem are crude, but easy to evaluate", where *inclusion regions* can be replaced by *estimates* or *bounds*, depending on the desired emphasis.

The opposite of elegant variation is when the same word is repeated in different forms or with different meanings. Here are two examples.

> The performance is impressive and gives the impression that

[5] A term coined by Knuth [164].

[6] Some authorities permit *which* to be used in a defining clause (e.g., Gowers [115]), but, as Bryson [41] puts it, "the practice is on the whole better avoided."

> the blocksize is nearly optimal. [impressive, impression]
> In the remainder of this chapter we examine the remainder in Euler's summation formula. [remainder, remainder]

Such echoing of words is distracting and is easily avoided by choosing a synonym for one of them.

4.16. Enumeration

Consider the extract

> The Basic Linear Algebra Subprograms (BLAS) have several advantages. They
>
> - Lead to shorter and clearer codes.
> - Improve modularity.
> - Machine dependent optimizations can be confined to the BLAS, aiding portability, and
> - Tuned BLAS have been provided by several manufacturers.

This explanation reads badly because the entries in the list are not grammatically parallel: the preceding "they" applies only to the first two entries of the list, and the third entry is not a complete sentence, unlike the others. This is an example of bastard enumeration, so-named by Fowler [83, p. 28], who explains that in an enumeration "there must be nothing common to two or more of the items without being common to all."

4.17. False If

The if–then construct is a vital tool in expressing technical arguments, but it is sometimes used incorrectly. Consider the sentence

> If we wish to compare the solutions of $f - \lambda k(f) = 0$ and $f_n - \lambda k_n(f_n) = 0$, then Jones shows that for a wide class of nonlinear $k(f)$, $\|f - f_n\| \leq c(\lambda)\|k_n(f) - k(f)\|$.

Jones's demonstration is independent of whether or not we wish to compare solutions, so the *if* is misleading: it falsely heralds a logical condition. False ifs can always be removed by rewriting:

> To compare the solutions of $f - \lambda k(f) = 0$ and $f_n - \lambda k_n(f_n) = 0$, we can use Jones's result that for a wide class of nonlinear $k(f)$, $\|f - f_n\| \leq c(\lambda)\|k_n(f) - k(f)\|$.

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A more confusing example is

> If we assume that rational fractions behave like almost all real numbers, a theorem of Khintchine states that the sum of the first k partial quotients will be approximately $k \log_2 k$.

The *if* appears to be a false one, because the statement of Khintchine's theorem must be independent of what the writer assumes. In fact, with knowledge of the theorem, it can be seen that the main error is in the word "states". If we change "states" to "implies" and delete "we assume that", then the sentence is correct.

The next example is an unnecessary *if*, rather than a false *if*.

> We show that if $\widehat{x}$ is the computed solution to $Lx = b$ then $(L + \Delta L)\widehat{x} = b$, where $\|\Delta L\| \leq \alpha(n)\epsilon\|L\|$.

This type of construction is acceptable if used sparingly. I prefer

> We show that the computed solution $\widehat{x}$ to $Lx = b$ satisfies $(L + \Delta L)\widehat{x} = b$, where $\|\Delta L\| \leq \alpha(n)\epsilon\|L\|$.

4.18. Hyphenation

As Turabian [278, p. 44] notes, the trend is not to hyphenate compound words beginning with prefixes such as multi, pre, post, non, pseudo and semi. In mathematical writing it is common to write *nonsingular*, *semidefinite* (but *semi-infinite* to avoid a double i) and *pseudorandom*. However, a hyphen is retained before a proper noun, as in *non-Euclidean*. In deciding whether to hyphenate or to combine two words as one, it is worth bearing in mind that the hyphenated form tends to be easier to read because the prefix can be seen at a glance. Readers whose first language is not English may appreciate the hyphenated form.

Compound words involving *ill* and *well* occur frequently in mathematical writing and opinions differ about their hyphenation. *The Chicago Manual of Style* [58] recommends hyphenation when a compound with *ill*, *well*, *better*, *best*, *little*, *lesser*, etc., appears before a noun, unless the compound is itself modified. The purpose of this hyphenation rule is to avoid ambiguity. Examples:

> This is an ill-posed problem *but* This problem is ill posed.
> The well-known theorem *but* The theorem is well known.
> An ill-conditioned function *but* A very ill conditioned function.
> The second-order term has a constant 2 *but* This term is of second order.

The second example is widely accepted, but many writers always hyphenate compounds involving *ill*, such as ill-conditioned, and it is hard to argue against this practice. There are some common phrases that some writers hyphenate and others do not. An example is *floating point arithmetic* (*floating-point arithmetic*).

In the phrase "we use the 1, 2 and ∞-norms", a *suspensive hyphen* is required after "1" and "2" since they are prefixes to "norm" and we need to show that they are to be linked to this word. Thus the phrase should be rewritten "we use the 1-, 2- and ∞-norms".

Notice the hyphen in the title of Halmos's best-seller *Finite-Dimensional Vector Spaces* [122]. Halmos explains in [128, p. 146] that in the original 1942 edition the hyphen was omitted, but it was added for the 1958 edition.

4.19. Linking Words

If a piece of writing is to read well there must be no abrupt changes in mood or direction from sentence to sentence within a paragraph (unless such changes are used deliberately for effect). One way to achieve a smooth flow is to use linking words. Notice how the following paragraph would be improved by adding "In particular" to the start of the second sentence and "Furthermore" to the start of the third.

> Once we move from a convex program to a general nonlinear program, matters become far more complicated. Certain topological assumptions are required to avoid pathological cases. The results apply only in a neighbourhood of a constrained minimizer, and involve convergence of subsequences of global minimizers of the barrier function.

Of course, a sequence of sentences of the form "adverb, fact" quickly becomes tiresome, so linking words should not be overused.

Here is a list of linking words and phrases, arranged according to sense. For examples of use see §5.8.

combinations. also, and, as well as, besides, both, furthermore, in addition to, likewise, moreover, similarly.

implications or explanations. as, because, conversely, due to, for example, given, in other words, in particular, in view of, it follows that, otherwise, owing to, since, specifically, that is, thus, unlike.

modifications and restrictions. although, alternatively, but, despite, except, however, in contrast, in spite of, nevertheless, of course, on the contrary, on the other hand, though, unfortunately, whereas, yet.

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emphasis. actually, certainly, clearly, in fact, indeed, obviously, surely.

consequences. accordingly, consequently, hence, therefore, thus.

4.20. Misused and Ambiguous Words

Both. A common misuse of *both* is illustrated by "In Gaussian elimination we can order the inner loops 'ij' or 'ji'. Both orderings are equivalent, mathematically." *Both* means "the two together" and is redundant when the sentence already carries this implication, as in this example. It would also be incorrect to say "Both orderings produce identical results." Correct versions are "These orderings are equivalent, mathematically", "Both orderings yield the same result", or "The two orderings produce identical results." Another common misuse of *both* is misplacement when it is used with prepositional phrases. For example,

> Incorrect: "Solutions are found both in the left and right quadrants." (*Both* is followed by a preposition, *in the left*, but *and* is followed by a noun.)
>
> Correct: "'Solutions are found both in the left and in the right quadrants." (Prepositional phrases follow *both* and *and*.)
>
> Correct: "'Solutions are found in both the left and the right quadrants." (Nouns follow *both* and *and*.)

Like. Consider the sentence

> Solving triangular systems is such a common operation that it has been standardized as a subroutine, along with many other common linear algebra operations like matrix multiplication.

The word *like* incorrectly limits the choice of linear algebra operations rather than serving as an example. Replacing *like* by *such as* conveys the intended meaning. The correct use of *like* is illustrated by "The Schulz iteration is quadratically convergent, like the Newton iteration."

Problem. An overused and, at times, ambiguous word in mathematical writing is *problem*, which can refer to both the focus of a piece of work and the difficulties encountered in carrying out the work. Sentences such as "In solving this problem we encountered a number of problems" can be avoided by using a synonym for the second occurrence of "problem", or rewriting. The sentence "We describe the special problems arising when solving stiff differential equations" is ambiguous: "problems" could refer to

classes of sub-problems produced by the solution process (such as nonlinear equations), or particular difficulties faced when solving the differential equations. Again, a rewrite is necessary.

Reason. In the phrase "the reason ... is because" the word *because* is redundant, since it means "for the reason that". Therefore in the sentence "The reason for the slow convergence is because α is a double root" *because* should be replaced by *that*. Similarly, in the phrase "the reason why" the word *why* can often be omitted. Thus "The reason why this question is important is that" is better written as "The reason this question is important is that" or "This question is important because".

Significant. Be careful if you use the word *significant* in mathematical writing, because to some readers it is synonymous with *statistically significant*, which carries a precise statistical meaning.

Try and, try to. The expression *try and* is frequently used in spoken English, but it is colloquial and should be replaced by *try to* in written English. Thus in the sentence "We sum the numbers in ascending order to try and minimize the effect of rounding errors" *try and* should be replaced by *try to*.

4.21. Numbers

Small integers should be spelled out when used as adjectives ("The three lemmas"), but not when used as names or numbers ("The median age is 43" or "This follows from Theorem 3"). The number 1 is a special case, for often "one" or "unity" reads equally well or better: "z has modulus one".

4.22. Omit These Words?

Here are some words and phrases whose omission often improves a sentence:

actually, very, really, currently, in fact, thing, without doubt.

The phrase "we have" can often be omitted. "Hence we have $x = 0$" should be replaced by "Hence $x = 0$." "Hence we have the following theorem" should be deleted or replaced by a sentence conveying some information (e.g., "Hence we have proved the following theorem").

4.23. Paragraphs

A standard device for making text more appetizing is to break it into small paragraphs, as is done in newspapers. The short paragraph principle is worth following in technical writing, where the complexity of the ideas

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makes it more important than usual for the reader to be given frequent rests. Furthermore, long paragraphs tend to give a page a heavy image that can be a visual deterrent to the potential reader. A mix of different lengths is best. Ideally, each paragraph contains a main idea or thought that separates it from its neighbours. A long paragraph that is hard to break may be indicative of convoluted thinking.

The best writers occasionally slip in one-sentence paragraphs.

4.24. Punctuation

Much can be said about punctuation, and for thorough treatments of the topic I refer the reader to the references mentioned in Chapter 2. It is worth keeping in mind Carey's explanation [52] that "the main function of punctuation is *to make perfectly clear the construction* of the written words." A few common mistakes and difficulties deserve mention here.

- In "This result is well known, see [9]" the comma should be a semicolon, which conveys a slightly longer pause. Even better is to say "This result is well known [9]." A common mistake is the let-comma-then construction: "Let x^* be a local maximum of $F(x)$, then a Taylor series expansion gives ...." The comma should be a full stop.

  Similarly, the comma should be a semicolon, or even a full stop, in this sentence: "This bound has the disadvantage that it uses a norm of X, moreover the multiplicative constant can be large when X is not a normal matrix." These errors are called "comma splices" by Gordon [111].

- In this sentence the semicolon should be a comma: "The secant method can also be used; its lack of need for derivatives being an advantage." A rough guide is to use a semicolon only where you could also use a full stop.

- In the sentence "If we use iterative refinement solutions are computed to higher accuracy", a comma is needed after "refinement", otherwise the reader may take "iterative refinement" as modifying "solutions". Another example where a comma is needed to avoid ambiguity is the sentence "However, the singularity can be removed by a change of variable."

- In sentences such as

  > Fortran 77 contains the floating point data types real, complex and double precision.
  > The output can be rotated, stretched, reduced or magnified.

we have a choice of whether or not to place a comma (called a serial comma) before the *and* or *or* that precedes the final element of the list. In some sentences a serial comma is needed to avoid ambiguity, as in the sentence "A dictionary is used to check spelling, shades of meaning, and usage", where the absence of the comma makes "shades" modify usage. Opinion differs on whether a serial comma should always be used. It is a matter of personal preference. The house styles of some publishers require serial commas.

If a list contains commas within the items, ambiguity can be avoided by using a semicolon as the list separator. Example:

> The test collection includes matrices with known inverses or known eigenvalues; ill-conditioned or rank deficient matrices; and symmetric, positive definite, orthogonal, defective, involutary, and totally positive matrices.

- The exclamation mark should be used with extreme caution in technical writing. If you are tempted to exclaim, read "!" as "shriek"; nine times out of ten you will decide a full stop is adequate. An example of correct usage is, from [217, p. 46], "When A is tridiagonal the computation of $A^{-1}u$ costs little more than the computation of Au!" The exclamation mark could be omitted, but then the reader might not realize that this is a surprising fact. Another example is, from [159, p. 42], "The chi-square table tells us, in fact, that V_2 *is much too low*: the observed values are so close to the expected values, we cannot consider the result to be random!"

- In the US, standard practice is to surround quotes by double quotation marks, with single quotation marks being used for a quote within a quote. In the UK, the reverse practice is generally used. The placement of final punctuation marks also differs: in US usage, the final punctuation is placed inside the closing quotation marks (except for "!" and "?" when they are not part of the quotation), while in UK usage it goes outside (except for "!" and "?" when they are part of the quotation). In this book, for quotations that end sentences, the end of sentence period appears outside quotation marks unless the quotation is itself a valid sentence.

- An apostrophe denotes possession for nouns (*the proof's length*) but not for pronouns (*this book is yours*). An apostrophe is used with the plural of letters and of words when the words are used without regard to their meaning: "there are three l's in the word parallel", "his prose contains too many however's." For plurals of numbers the

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4.25. Say It Better, Think It Gooder

The title of this section combines the titles of two papers by George Piranian [227] and Paul Halmos [126] that appeared in *The Mathematical Intelligencer* in 1982. As Gillman explains in [105], "George said good English is important. Paul said, what do you mean, good English is important?—good mathematics is more important. They are both right." While correct English usage is important, it must not be allowed to deflect you from the language-independent tasks of planning and organizing your writing.

4.26. Saying What You Mean

In technical writing you need to take great care to say what you mean. A hastily constructed sentence can have a meaning very different from the one intended. In a book review in *SIAM Review* [vol. 34, 1992, pp. 330–331] the reviewer quotes the statement "According to Theorem 1.1, a single trajectory $X(t, x)$ passes through almost every point in phase space ...." The book's author meant to say that for every point in phase space there is a unique trajectory that goes through it.

4.27. Sentence Opening

Try not to begin a sentence with *there is* or *there are*. These forms of the verb *be* make a weak start to a sentence, because they delay the appearance of the main verb (see the quote by Dixon on page 77). Sometimes these phrases can simply be deleted, as in the sentence "There are several methods that are applicable" ("Several methods are applicable"). Also worth avoiding, if possible, are "It is" openers, such as "It is clear that" and "It is interesting to note that". If you can find alternative wordings, your writing will be more fresh and lively.

4.28. Simplification

Each word or phrase in the left column below can (or, if marked with an asterisk, should) be replaced by the corresponding one in the right column. This is not an exhaustive list (see [69, Appendix 4] or [14] for many

Table 4.2. The origins of some synonyms.

Anglo-Saxon	French	Latin
ask	question	interrogate
rise	mount	ascend
good	marvellous	superior
show	establish	exhibit, demonstrate
need	requirement	necessity
think	ponder	cogitate

more examples) but comprises periphrastic phrases that I have spotted in mathematical writing.

by means of	by
conduct an investigation	investigate
due to the fact that*	since/because
firstly	first
in a position to	can
in order to	to
in the case that*	if
in the course of	during
in the event that*	if
in the first place*	first
it is apparent that	apparently
it may happen that	there may/might
most unique*	unique
take into consideration	consider
that is to say	in other words/that is
to begin with*	to begin

4.29. Synonym Selection

English is one of the most synonym-rich languages, thanks to the words it has adopted from other languages, and each member of a set of synonyms can have a different tone and shade of meaning. A dictionary and a thesaurus are vital aids to choosing the right word (see §2.1). When you have a choice of words, use a short, concrete one in preference to a long or abstract one. Often, this means using an Anglo-Saxon word instead of one of French or Latin origin, as Table 4.2 illustrates.

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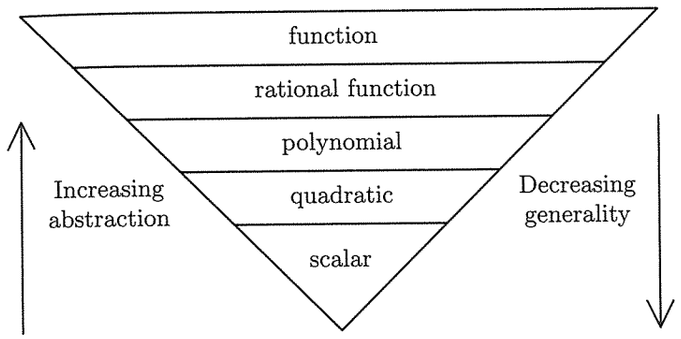

Figure 4.1. Word pyramid.

Don't be afraid of using a long or unusual word if it is just the right one, or if it adds spice to your writing or conveys an interesting image. Acton [3, p. 145] writes "The second term ... is obstreperous." No other word quite so vividly describes the uncontrollable term to which he refers (with the possible exception of *recalcitrant*, another word used by Acton in the same book).

I chose the title "Otiose Symbols" on page 29 in preference to "Unnecessary Symbols" for two reasons. First, the former is shorter and more likely to catch the reader's eye. Second, *otiose* means "serving no practical purpose", so "otiose symbols" has a stronger meaning than "unnecessary symbols".

It helps to think of word pyramids showing levels of abstraction [281], such as the one in Figure 4.1. As Turk and Kirkman [280] point out, "Abstract words are less easy to decode because the reader has to 'scan' all the possibilities subconsciously before deciding on a specific meaning." Generally, it is best to use the least abstract, most specific word possible. This leads to more lively and precise prose. Here are some examples of chains of words in order of increasing specificity:

result–theorem–inequality
statistic–error–relative error
optimum–minimum–global minimum
random–normally distributed–normal (0,1)

4.30. Tense

One of the decisions the writer of a technical paper must make is what tense to use. There are no hard and fast rules, except to be consistent in the use of tense, but I offer some guidelines.

I prefer the present tense to the future tense for referring to later parts of the paper. Thus I write "This is proved in Theorem 3, below" rather than "This will be proved in Theorem 3", and "We discuss these matters in detail in Section 5" rather than "We will discuss these matters in detail." Note that since the latter example does not contain a section reference it could be construed as referring to a future paper; this potential ambiguity is a danger of using the future tense.

The present tense can also be used for backward references, as in "The analysis of Section 1 proves the following ...", though the past tense is more common: "We showed/saw in Section 1 that ...." Tables and figures exist in the present, so the present tense should be used to refer to them: "Table 4 shows that", "Figure 1 displays". Similarly, in a citation where the reference is the subject of the sentence, the present tense is the one to use: "Reference [4] contains the interesting result that ...."

To refer to work in earlier papers either the past or the present tense can be used: "Banach shows that" or "Banach showed that". I use the present tense unless I want to emphasize the historical aspects of the reference, as, for example, when summarizing a number of earlier results in a survey.

A summary or conclusions section should use the past tense when specifying actions: "We have shown that" or "We showed that" rather than "We show that". But if a simple statement of results is given, the present tense is appropriate: "Asymptotic expansions are useful for ...", "Our conclusion is that ...."

In summary, I recommend the rule "if in doubt use the present tense."

4.31. What to Call Yourself

All technical writers face the question of how to refer to themselves in their papers. For example, if you refer to a previous result of yours you may have to choose between

(1) I showed in [3] that
(2) We showed in [3] that
(3) The author showed in [3] that
(4) It was shown in [3] that

Sentence 4 should be avoided because it is in the passive voice, which is unnecessary in this case. Sentence 3 has a formal, stilted air and is again

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best avoided. Some authorities on technical writing advise against using "we" to refer to a single author, but are happy for it to be used when there are two or more authors. Nevertheless, in mathematical writing "we" is by far the most common choice of personal pronoun; I frequently use it myself (in papers, but not in this book). "I" creates a more frank, personal contact with the reader and is less commonly used than "we". Zinsser [304, p. 24] suggests "If you aren't allowed to use 'I', at least think 'I' while you write, or write the first draft in the first person and then take the 'I's out. It will warm up your impersonal style."

"We" can be used in the sense of "the reader and I", as in the sentence

> We saw earlier that $f_n \to 0$ as $n \to \infty$.

Whether you choose "I" or "we", you should not mix the two in a single document, except, possibly, when using the "reader and I" form of "we".

Other personal pronouns to consider are "one" and "us". "One", as in "one can show that ..." is often used, but is perhaps best avoided because of its vague, impersonal nature. "Us" is useful in sentences such as

> Let us now see how the results can be extended to non-smooth functions.

where it means "the reader and I". It is generally quite easy to remove both "one" and "us" by phrasing sentences in an alternative form. For example, the sentence above can be rewritten as "How can the results be extended to non-smooth functions? One approach is ...", or "To see how the results can be extended to non-smooth functions, we ...."

4.32. Word Order

The first words of a sentence are usually regarded by the reader as the most important. Therefore, the word order of sentences should be chosen to reflect the desired emphasis. Compare the first sentence of this paragraph with "The reader usually regards the first words of a sentence as the most important", which places the emphasis on the reader rather than "the first words of a sentence".

Reordering the words of a sentence can strengthen it and remove ambiguity. Here is an example of a misplaced *only*.

> Bad: The limit point is only a stationary point when the regularity conditions are satisfied. (Might be taken to mean that the limit point is expected to be more than just a stationary point.)

Good: The limit point is a stationary point only when the regularity conditions are satisfied.

Here is an example of an incorrect *either*.

Bad: Mathematically, either we have an integral equation of the first kind or one of the second kind.

Good: Mathematically, we have an integral equation of either the first kind or the second kind. (Or simply delete *either* from the bad example.)