

rather than the values themselves. Error measures should be normalized. Thus, for an approximate solution  $\hat{x}$  to  $Ax = b$ , the relative residual  $\|b - A\hat{x}\|/(\|A\|\|\hat{x}\| + \|b\|)$  is more meaningful than the scale-dependent quantity  $\|b - A\hat{x}\|$ . If you are measuring the speed of a numerical algorithm it is important to show that the right answers are being produced (otherwise the algorithm "the answer is 42" is hard to beat).

You may also wish to state the programming language, the version of the compiler used, and the compiler options and optimizations that were selected, as these can all have a significant influence on run-times. In areas where attaining full precision is not the aim (such as the numerical solution of ordinary differential equations), it is appropriate to consider both speed and accuracy (e.g., by plotting cost versus requested accuracy).

One of the difficulties in designing experiments is finding good test problems—ones which reveal extremes of behaviour, cover a wide range of difficulty, are representative of practical problems, and (ideally) have known solutions. In many areas of computational mathematics good test problems have been identified, and several collections of such problems have been published. For example, collections are available in the areas of nonlinear optimization, linear programming, ordinary differential equations and partial differential equations. Several collections of test matrices are available and there is a book devoted entirely to test matrices. For references to test problem collections see [140].

In your conclusions you should make a clear distinction between objective statements and opinions and speculation. It is very tempting to extrapolate from results, but this is dangerous. As you analyse the results you may begin to formulate conclusions that are not fully supported by the data, perhaps because they were not anticipated when the experiments were designed. If so, further experimentation will be needed. When evaluating numerical algorithms I have found that it pays to print out every statistic that could conceivably be of interest; if I decide not to print out a residual or relative error, for example, I often find a need for it later on.

Further guidelines on how to report the results of computational experiments can be found in the journals *ACM Transactions on Mathematical Software* [65] and *Mathematical Programming* [146], and in an article by Bailey [12]. Perhaps the best advice is to read critically the presentation of experimental results in papers in your area of interest—and learn from them.

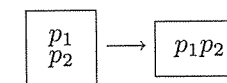
## 6.10. Tables

Many of the principles that apply to writing apply also to the construction of tables and graphs. However, some particular points should be considered

when designing a table.

To maximize readability the table design should be as simple as possible. Repetition should be avoided; for example, units of measurement or descriptions common to each entry in a column should go in the column header. Compare Tables 6.1 and 6.2. It is best to minimize the number of rules in the table. Two busy examples are shown in Table 6.3 and Table 6.5. The simplified versions, Table 6.4 and Table 6.6, are surely more aesthetically pleasing. Table 6.3 is taken from [159, p. 366],<sup>11</sup> where it is given without any rules and is still perfectly readable.

It is easier to compare like quantities if they are arranged in columns rather than rows. Research reported by Hartley [132], [134] supports this fact, and Tables 6.7 and 6.8 provide illustration, Table 6.8 being the easier to read. The difference between row and column orientation is more pronounced in complex tables. Of course, the orientation may be determined by space considerations, as a horizontal orientation usually takes less space on the page. If a vertical table is too tall, but is narrow, it can be broken into two pieces side by side:



It is also helpful to put columns or rows that need to be compared next to each other.

Only essential information should be included in a table. Omit data whose presence cannot be justified and state only as many digits as are needed (this number is often surprisingly small). In particular, do not state numerical results to more significant figures than are known for the data. As an example, in Table 6.1 there is no need to quote the timings and speedups to six significant figures, so Table 6.2 gives just one decimal place. Note that there is justification for showing so many digits in Tables 6.3 and 6.4:  $\pi(x)$  in this table is the number of primes less than or equal to  $x$ , and nearly all the digits of  $\pi(10^9)$  are needed to show the error in Riemann's formula. Displaying the first one or two digits of the fractional parts of the approximations emphasizes that the approximations are not integers.

If you need to present a large amount of data in tabular form, consider displaying it in an appendix, to avoid cluttering the main text. You could give smaller tables in the text that summarize the data. Large sets of data are often better displayed as graphs, however, particularly if it is the trends rather than the numerical values that are of interest. Tufte [275,

<sup>11</sup>Donald E. Knuth, *The Art of Computer Programming*, vol. 2, ©1981 by Addison-Wesley Publishing Co. Reprinted by permission of Addison-Wesley Publishing Co., Inc., Reading, MA. The form of the original table is not reproduced exactly here.

Table 6.1. Timings for a parallel algorithm.

# processors	Time	Speedup
$p = 1$	28.352197 secs	—
$p = 4$	7.218812 secs	3.9275
$p = 8$	3.634951 secs	7.7999
$p = 16$	1.929347 secs	14.6952

Table 6.2. Timings for a parallel algorithm.

No. of processors	Time (secs)	Speedup
1	28.4	—
4	7.2	4.0
8	3.6	7.8
16	1.9	14.7

Table 6.3. Approximations to  $\pi(x)$ .

$x$	$\pi(x)$	$x/\ln x$	$L(x)$	Riemann's formula
$10^3$	168	144.8	176.6	168.36
$10^6$	78498	72382.4	78626.5	78527.40
$10^9$	50847534	48254942.4	50849233.9	50847455.43

Table 6.4. Approximations to  $\pi(x)$ .

$x$	$\pi(x)$	$x/\ln x$	$L(x)$	Riemann's formula
$10^3$	168	144.8	176.6	168.36
$10^6$	78498	72382.4	78626.5	78527.40
$10^9$	50847534	48254942.4	50849233.9	50847455.43

Table 6.5. Results for inverting a lower triangular matrix on a Cray 2.

$n$	Mflops			
	128	256	512	1024
Method 1 ( $n_b = 1$ )	95	162	231	283
Method 2 ( $n_b = 1$ )	114	211	289	330
$k$ variant ( $n_b = 1$ )	114	157	178	191
Method 1B ( $n_b = 64$ )	125	246	348	405
Method 2C ( $n_b = 64$ )	129	269	378	428
$k$ variant ( $n_b = 64$ )	148	263	344	383

Table 6.6. Mflop rates for inverting a lower triangular matrix on a Cray 2.

$n$		128	256	512	1024
Unblocked:	Method 1	95	162	231	283
	Method 2	114	211	289	330
	$k$ variant	114	157	178	191
Blocked: ( $n_b = 64$ )	Method 1B	125	246	348	405
	Method 2C	129	269	378	428
	$k$ variant	148	263	344	383

Table 6.7. SI prefixes ( $10^{-1}$ – $10^{12}$ ). Row orientation.

Multiple	$10^{12}$	$10^9$	$10^6$	$10^3$	$10^{-1}$
Prefix	tera	giga	mega	kilo	deci
Symbol	T	G	M	K	d

Table 6.8. SI prefixes ( $10^{-1}$ – $10^{12}$ ). Column orientation.

Multiple	Prefix	Symbol
$10^{12}$	tera	T
$10^9$	giga	G
$10^6$	mega	M
$10^3$	kilo	K
$10^{-1}$	deci	d

p. 56] advises that tables are usually better than graphs at reporting small data sets of twenty numbers or less.

The caption should be informative and should not merely repeat information contained in the table. Notice the simplification obtained by moving the word “Mflops” from the table to the caption in Table 6.6.

Give a clear reference to the table at an appropriate place in the text—you cannot rely on the reader to refer to the table automatically. It is helpful if you explain the salient features of the table in words. The reader will appreciate this guidance, especially if the table contains a lot of data. However, you should not summarize the whole table—if you do, the table might as well be omitted.

### Further Reading

*The Chicago Manual of Style* [58] devotes a whole chapter to tables and offers much useful advice. Another good reference is *A Manual for Writers* [278, Chap. 6]. Bentley [21, Chap. 10] gives a good example of how to redesign a table. References that discuss the preparation of graphs include Bentley [21, Chaps. 10, 11], Hartley [132], [134], MacGregor [187], [188], and Tufte [275], [276], [277].

## 6.11. Citations

The two main styles of citation in mathematics journals are by number (as used in this book) and by name and year, which is the Harvard system. Examples of the Harvard system are *These results agree with an existing study of Smith* (1990) and *These results agree with an existing study (Smith, 1990)*. If more than one paper maps to Smith (1990), the papers are distinguished by appending a letter to the year: Smith (1990a), Smith (1990b), and so on. In the number-only system, the number is usually placed in square brackets, though some styles require it to be superscripted.

The main requirement is that a citation does not intrude upon a sentence. For example, *This method was found [17] to be unstable* is better written as *This method was found to be unstable [17]*. There are circumstances, however, where a citation has to be placed part-way through a sentence to convey the correct meaning. A good test for whether a citation is well placed is to see whether the sentence reads properly when the citation is deleted. The style of citation inevitably affects how you phrase sentences, so it is worth checking in advance what style is used by the journal in which you wish to publish. Knuth [164] explains that when his paper “Structured programming with go to statements” [*Computing Surveys*, 6:261–301, 1974] was reprinted in a book, he made numerous changes

to make sentences read well with the citation style used in the book.

When you cite by number, it is good style to incorporate the author’s name if the citation is more than just a passing one. As well as saving the reader the trouble of turning to the reference list to find out who you are referring to, this practice has an enlivening effect because of the human interest it introduces. Examples:

Let  $A\Pi = QR$  be a  $QR$  factorization with column pivoting [10].  
(Passing reference to a textbook for this standard factorization.)  
The rate of convergence is quadratic, as shown by Wilkinson [27]. (Instead of “as shown in [27]”.)

The sentence “This question has been addressed by [5]” is logically incorrect and should be modified to “addressed by Jones [5]” or “addressed in [5]”.

When you cite several references together it is best to arrange them so that the citation numbers are in increasing order, e.g., “several variations have been developed [2], [7], [13].” Ordering by year of publication serves no purpose when only citation numbers appear in the text. If you want to emphasize the historical progression it is better to add names and years: “variations have been developed by Smith (1974) [13], Hall (1981) [2], and Jones (1985) [7].”

It is important to be aware that the reference list says a lot about a paper. It helps to define the area in which the paper lies and may be used by a reader to judge whether the author is aware of previous work in the area. Some readers look at the reference list immediately after reading the title, and if the references do not look sufficiently familiar, interesting or comprehensive they may decide not to read further. Therefore it is desirable that your reference list contain at least a few of the key papers in the area in which you are writing. Papers should not, however, be cited just for effect. Each citation should serve a purpose within the paper. Note also that if you cite too often (say, for several consecutive sentences) you may give the impression that you lack confidence in what you are saying.

There are several conventions for handling multiple authors using the Harvard system. One such convention is as follows [45], [68, Chap. 12]. For one or two authors, both names are given (e.g., “see Golub and Van Loan (1989)”). If there are three authors, all three are listed in the first citation and subsequent citations replace the second and third names by “et al.” (e.g., “see Knuth, Larrabee and Roberts (1989),” then “see Knuth et al. (1989)”). For four or more authors, all citations use the first author with “et al.” These conventions can also be used when naming authors in conjunction with the numbered citation system.

If you make significant use of a result from another reference you should give some indication of the difficulty and depth of the result (and give the

author's name). Otherwise, unless readers look up the reference, they will not be able to judge the weight of your contribution.

When you make reference to a specific detail from a book or long paper it helps the reader if your citation includes information that pinpoints the reference, such as a page, section, or theorem number.

For further details on the subtleties of citation consult van Leunen [283].

## 6.12. Conclusions

If there is a conclusions section (and not every paper needs one) it should not simply repeat earlier sections in the same words. It should offer another viewpoint, discuss limitations of the work, or give suggestions for further research. Often the conclusions are best worked into the introduction or the last section. It is not uncommon to see papers where the conclusions are largely sentences taken from the introduction, such as *We show that X's result can be extended to a larger class ...*; this practice is not recommended.

The conclusions section is a good place to mention further work: to outline open problems and directions for future research and to mention work in progress. Be wary of referring to your "forthcoming paper", for such papers can fail to materialize. A classic example of a justified reference to future work is the following quote from the famous paper<sup>12</sup> by Watson and Crick [290] (*Nature*, April 25, 1953) in which the double helix structure of DNA was proposed:

It has not escaped our notice that the specific pairing we have postulated immediately suggests a possible copying mechanism for the genetic material.

Full details of the structure, including the conditions assumed in building it, together with a set of co-ordinates for the atoms, will be published elsewhere.

The promise "will be published elsewhere" was fulfilled shortly afterwards, in the May 30, 1953 issue of *Nature*.<sup>13</sup>

## 6.13. Acknowledgements

Be sure to acknowledge any financial support for your work: grants, fellowships, studentships, sponsorship. A researcher might write "This re-

<sup>12</sup>For some comments by Crick on the writing of this and subsequent papers by Watson and Crick, see [64].

<sup>13</sup>Pyke [231] points out that the words "It has not escaped our notice that" can be removed.

search was supported by the National Science Foundation under grant DCR-1234567"; grant agencies like authors to be this specific. A Ph.D. student supported by the Engineering and Physical Sciences Research Council (UK) might write "This work was supported by an EPSRC Research Studentship." In SIAM journals this type of acknowledgement appears in a footnote on the first page. Other acknowledgements usually appear in a section titled *Acknowledgements* at the end of the paper. (An alternative spelling is *acknowledgments*.)

It is customary to thank anyone who read the manuscript in draft form and offered significant suggestions for improvement (as well as anyone who helped in the research), but not someone who was just doing his or her normal work in helping you (for example, a secretary). The often-used "I would like to thank" can be shortened to "I thank." Note that if you say "I thank X for pointing out an error in the proof of Theorem Y," you are saying that the proof is incorrect; "in an earlier proof" or "in an earlier attempted proof" is what you meant to say.

The concept of anonymous referee sometimes seems to confuse authors when they write acknowledgements. An anonymous referee should not be thanked, as is often the case, for *his* suggestions—it may be a *she*. One author wrote "I thank the anonymous referees, particularly Dr. J. R. Ockendon, for numerous suggestions and for the source of references." Another explained, not realising the two ways in which the sentence can be read, "I would like to thank the unknown referees for their valuable comments."

## 6.14. Appendix

An appendix contains information that is essential to the paper but does not fit comfortably into the body of the text. The most common use of an appendix is to present detailed analysis that would distract the reader if it were given at the point where the results of the analysis are needed. An appendix can also be used to give computer program listings or detailed numerical results. An appendix should not be used to squeeze inessential information into the paper (though this may be acceptable in a technical report or thesis).

## 6.15. Reference List

Preparing the reference list can be one of the most tedious aspects of writing a paper, although it is made much easier by appropriate software (see §13.3). The precise format in which references are presented varies among publishers and sometimes among different journals from the same publisher.



Here are four examples.

**SIAM journals:** J. H. WILKINSON, *Error analysis of floating-point computation*, Numer. Math., 2 (1960), pp. 319–340.

**IMA journals:** WILKINSON, J. H. 1960 Error analysis of floating-point computation. *Numer. Math.* 2, 319–340.

**Elsevier journals:** J. H. Wilkinson, Error analysis of floating-point computation, *Numer. Math.* 2:319–340 (1960).

**Springer-Verlag journals:** Wilkinson, J. H. (1960): Error analysis of floating-point computation. Numer. Math. 2, 319–340.

All journals that I am familiar with ask for the use of their own format but will accept other formats and copy edit them as necessary. All publishers have a minimum amount of information that they require for references, as defined in their instructions for authors. It is important to provide all the required information, whatever format you use for the references.

Here are some comments and suggestions on preparing reference lists. For further details I strongly recommend the book by van Leunen [283], but keep in mind that her recommendations may conflict with those of publishers in certain respects.

1. Do not rely on secondary sources to learn the contents of a reference or its bibliographic details—always check the original reference. In studies on the accuracy of citation, the percentage of references containing errors has been found to be as high as 50% [95]. A 1982 paper by Vieira and Messing in the journal *Gene* had been cited correctly 2,212 times up to 1988, but it had also been cited incorrectly 357 times under “Viera”; these errors led to the paper being placed too low in a list of most-cited papers [98]. In another well-documented case in the medical literature, the Czech title “O Úplavici” (“On Dysentery”) of an 1887 paper in a Czech medical journal was taken by one writer to be the author’s name, and the mistake propagated until it was finally exposed in 1938 [135], [239]. If a secondary source has to be used (perhaps because the reference is unavailable), it is advisable to append to the reference “cited in [ss]”, where [ss] is the secondary source.
2. Always provide the full complement of initials of an author, as given in the paper or book you are referencing.
3. Some authors are inconsistent in the name they use in their papers, sometimes omitting a middle initial, for example. In such cases, my

preference is to use the author’s full name in the reference list when it is known, to avoid ambiguities such as: Is A. Smith the same author as A. B. Smith?

4. Some sources contain typographical errors or nonstandard usage. Titles should be given unaltered. For example, the title “Van der Monde systems and numerical differentiation” [J. N. Lyness and C. B. Moler, *Numer. Math.*, 8:458–464, 1966] appears to be incorrect because the name is usually written Vandermonde, but it should not be altered (I have occasionally had to reinstate “Van der Monde” in my reference list after a copy editor has changed it). A typographical error in an author’s name is rare, but not unknown. It seems reasonable to correct such an error, but to provide some indication of the correction that has been made, such as a note at the end of the reference.
5. Copy bibliographic information of a journal article from the journal pages, not the cover of the journal. The cover sometimes contains typographical errors and you cannot deduce the final page number of the article if the journal puts blank pages between articles or begins articles part-way down a page.
6. Electronic journals do not usually cause any difficulties in referencing, since it is in the journals’ interests to make clear how papers should be referenced. For example, the journal *Electronic Transactions on Numerical Analysis* provides papers in PostScript form, and each paper has a clearly defined page range, volume and year; papers are therefore referenced just like those in a traditional journal. It may help readers if a URL for an electronic journal is appended to the reference, but the journal in which you are publishing may delete it to save space.

It is more difficult to decide how to reference email messages and unpublished documents or programs on the Web. The following suggestions are adapted from those in *Electronic Styles* [178]. I assume that an email address and a URL are both clearly identifiable as such by the @ and http, respectively, so I omit the descriptors “email” and “URL”. There are so many different types of item on the Web that no referencing scheme can cover all possibilities.

- (a) A publication available in print and online.

Nicholas J. Higham. The Test Matrix Toolbox for MATLAB (version 3.0). Numerical Analysis Report No. 276, Manchester Centre for Computational Mathematics, Manch-

- ester, England, Sept. 1995; also available from `ftp://ftp.ma.man.ac.uk/pub/narep/narep276.ps.gz`
- (b) A publication available online only.  
Melvin E. Page. A Brief Citation Guide for Internet Sources in History and the Humanities (Version 2.1), `http://h-net.msu.edu/~africa/citation.html`, 1996.
- (c) A publication on CD-ROM.  
A. G. Anderson, Immersed interface methods for the compressible equations. In Proceedings of the Eighth SIAM Conference on Parallel Processing for Scientific Computing (Minneapolis, MN, 1997), CD-ROM, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1997.
- (d) A piece of software.  
Piet van Oostrum. L<sup>A</sup>T<sub>E</sub>X package fancyhdr. CTAN archive (e.g., `http://www.tex.ac.uk/tex-archive`), macros/latex/contrib/supported/fancyhdr.
- (e) An item in a discussion list, digest or newsgroup.  
David Hough. Random story. NA Digest, 89 (1), 1989. `na.help@na-net.ornl.gov`, `http://www.netlib.org/index.html`
- (f) A standard email message. The title is taken from the Subject: line.  
Desmond J. Higham (`aas96106@ccsun.strath.ac.uk`). Comments on your paper. Email message to Nicholas J. Higham (`higham@ma.man.ac.uk`), August 18, 1997.
- (g) A forwarded email message.  
Susan Ciambrano (`ciambran@siam.org`). Reader's comments on HWMS. (Original message A. Reader, Handbook of Writing.) Forwarded email message to Nicholas J. Higham (`higham@ma.man.ac.uk`), October 20, 1995.
7. When you reference a manuscript or technical report that is more than a few months old, check to see if it has appeared in a journal. An author will usually be happy to inform you of its status. If it has appeared, check whether the title has changed. The referees may have asked for a better title, or the copy editor may have added a hyphen, combined a hyphenated pair of words, or changed British spelling to American or vice versa.

8. Take care to respect letters and accented characters from other languages. Examples: Å, å, ß, é, ö, ø, Ø.
9. If you maintain a database of references (for example, in BibT<sub>E</sub>X format—see §13.3), it is worth recording full details of a reference, even if not all of them are needed for journal reference lists. For a journal article, record the part (issue) number as well as the volume number; this extra information can speed the process of looking up a reference, especially if the journal issues are unbound. For a technical report, the month of publication is useful to know.
10. Be sure you are using the correct journal name and watch out for journals that change their names. For example, the *SIAM Journal on Scientific and Statistical Computing* (1980–1992) became the *SIAM Journal on Scientific Computing* in 1993.
11. In book titles, van Leunen recommends that a colon be added if it is needed to separate a title from a subtitle, and an awkward comma or colon separating a title from a subtitle should be removed. Thus *On Writing Well An Informal Guide to Writing Nonfiction* (as copied from the title page of [304]) needs a colon added after *Well*, and the colon should be removed from *Interpolation Theory*: 5.
12. Van Leunen recommends simplifying the names of major publishers to the bare bones, so that *John Wiley & Sons* becomes *Wiley*, and *Penguin Books* becomes *Penguin*. She also recommends omitting the city for a major publisher; I usually include it because many journals require it. For obscure publishers it is best to give as complete an address as possible.
13. For a book, the International Standard Book Number (ISBN) is worth recording, as it can be used to search library and publishers' catalogues. (Note, though, that hardback and softback editions of a book usually have different ISBNs.) An ISBN consists of ten digits, arranged in four groups whose size can vary. The first group specifies the language group of the publisher (0, 1 = English speaking countries, 2 = French speaking, 3 = German speaking, etc.). The second group (2–7 digits) identifies the publisher (e.g., Oxford University Press is 19) and the third group (1–6 digits) identifies the particular title. The last digit is a checksum. If the ISBN is expressed as  $d_1 d_2 \dots d_{10}$  then

$$d_{10} = [s/11] * 11 - s, \quad \text{where} \quad s = \sum_{i=1}^9 (11 - i) d_i$$

( $\lceil x \rceil$  denotes the smallest integer greater than or equal to  $x$ ). A value  $d_{10} = 10$  is written as "X". This book has the ISBN 0-89871-420-6: 89871 identifies SIAM as the publisher and 420 is the book's individual number. An International Standard Serial Number (ISSN) identifies a serial publication such as a journal, yearbook or institutional report. An ISSN has eight digits.

14. The date to quote for a book is the latest copyright date (excluding copyright renewals)—ignore dates of reprinting. Always state the edition number if it is not the first.
15. Make sure that every reference is actually cited in the paper. Some copy editors check this, as you may see from their pencilled marks on the manuscript when you receive the proofs.
16. Most mathematics journals require the reference list to be ordered alphabetically by author. Many science journals order by citation, so that the  $n$ th paper to be cited is  $n$ th in the reference list.
17. A list of standard abbreviations for mathematics journals can be found in *Mathematical Reviews* (see §14.3).

## 6.16. Specifics and Deprecated Practices

### Capitalization

References to proper nouns should be capitalized: *See Theorem 1.5, the proof of Lemma 3.4 and the discussion in Section 6.* References to common nouns (generic objects) should not: *Next we prove the major theorems of the paper.*

### Dangling Theorem

The term *dangling theorem* [147] (or *hanging theorem* [121]) refers to a construction such as the following one, where a theorem dangles or hangs from the end of a sentence.

This result is proved in the following

**Theorem 3.13.** If  $f$  is a twice continuously differentiable function ...

Halmos argues that while the practice can be defended, some readers dislike it, and it is not worth risking annoying them for the sake of avoiding the extra word *theorem*.

The following example does not strictly dangle, but is even more irritating.

**5.1. Accuracy of the Computed Solution.** It depends on the machine precision and the conditioning of the problem.

Section headings stand alone and should not be taken as part of the text. The obvious solution

**5.1. Accuracy of the Computed Solution.** The accuracy of the computed solution depends on the machine precision and the conditioning of the problem.

is inelegant in its repetition, but this could be avoided by rewriting the sentence or the title.

### Footnotes

Footnotes are used sparingly nowadays in mathematical writing, and some journals do not allow them (see page 78 for an example of a footnote). It is bad practice to use them to squeeze more into a sentence than it can happily take. Their correct use is to add a note or comment that would deflect from the main message of the sentence. Donald W. Marquardt, the author of the 92nd most-cited paper in the Science Citation Index 1945–1988 [An algorithm for least-squares estimation of nonlinear parameters. *J. Soc. Indust. Appl. Math.*, 11(2):431–441, 1963], has stated that a critical part of the algorithm he proposed was described in a footnote and has sometimes been overlooked by people who have programmed the algorithm [97].

### Numbering Mathematical Objects

Generally, you should number only those equations that are referenced within the text. This avoids the clutter of extraneous equation numbers and focuses the reader's attention on the important equations. Occasionally it is worth numbering key equations that are not referenced but which other authors might want to quote when citing your paper. Except in very short papers it is best to number equations by section rather than globally (equation (2.3) instead of equation (14)), for this makes referenced equations easier to find. The same applies to the numbering of theorems and other mathematical objects. Whether equation numbers appear on the left or the right of the page depends on the journal.

Two possible numbering sequences are illustrated by

Definition 1, Lemma 1, Theorem 1, Remark 1, Definition 2, Lemma 2, ...

Definition 1, Lemma 2, Theorem 3, Remark 4, Lemma 5, ...

Opinions differ as to which is the best scheme. The last has the advantage that it makes it easier to locate a particular numbered item, and the equation numbers themselves can even be included in the sequence for complete uniformity. The disadvantage is that the scheme mixes structures of a different character, which makes it difficult to focus on one particular set of structures (say, all the definitions); and on reading Remark 24 (say), the reader may wonder how many previous remarks there have been. A compromise between the two schemes is to number all lemmas, theorems and corollaries in one sequence, and definitions, remarks and so on in another. Some typesetting systems control the numbering of mathematical objects automatically.  $\text{\LaTeX}$  does so, for example, and the numbering sequence for definitions, lemmas, theorems, etc., can be specified by  $\text{\LaTeX}$  commands.

### Plagiarism

Plagiarism is the act of publishing borrowed ideas or words as though they are your own. It is a major academic sin. In writing, if you copy a sentence or more you should either place it in quotes and acknowledge the source via a citation, or give an explicit reference such as "As Smith observed ...". In the case of a theorem statement it is acceptable to copy it word for word if you cite the source, but before copying it you should see whether you can improve the wording or make it fit better into your notation and style.

Regarding when to quote and when to paraphrase, van Leunen [283] advises "Quote what is memorable or questionable, strange or witty. Paraphrase the rest." When you wish to paraphrase, it is best to put the source aside, wait a reasonable period, and then rewrite what you want to say in your own words.

If you rework what you yourself have previously published without citing the source, thus passing it off as new, that is self-plagiarism, which is no less a sin than plagiarism.

Plagiarism has led to the downfall of many a career, in academia and elsewhere. Some notable cases are described in Mallon's *Stolen Words* [192] and LaFollette's *Stealing into Print* [169]. The former includes the ironic news, quoted from the *New York Times* of June 6, 1980, that

Stanford University said today it had learned that its teaching assistant's handbook section on plagiarism had been plagiarized by the University of Oregon. ... Oregon officials apologized and said they would revise their guidebook.

Fraud is another serious malpractice, though apparently and understandably rare in mathematical research. Numerous cases of scientific fraud through history are catalogued in *Betrayers of the Truth* by Broad and Wade [39], while allegations that the psychologist Cyril Burt acted fraudulently are examined carefully in [189].

### The Invalid Theorem

Avoid the mistake of calling a theorem into question through sentences such as the following:

The theorem holds for any continuously differentiable function  $f$ .  
Unfortunately, the theorem is invalid because  $S$  is not path connected.

A theorem holds and is valid, by definition. A theorem might be *applicable* to any continuously differentiable function or its invocation may be *invalid* because  $S$  is not path connected.

### "This Paper Proves ..."

In the abstract and introduction it is tempting to use wording such as "this paper proves" or "Section 3 shows" in place of "we prove" or "we show". This usage grates on the ear of some readers, as it is logically incorrect (though "Theorem 2 gives" cannot be criticized). The grating can be avoided by rewriting, but care is required to avoid a succession of sentences beginning "we".