Investigating the relationship between the electric current and force exerted on a wire

May 2022

Introduction

As the world transitions from internal combustion engines to electric motors, I have begun to wonder how exactly electricity is used in vehicles. More specifically, how electrical force is transformed into physical force. Despite what I have learned from the school curriculum, there is not significant amount material covering electricity to begin with. Having worked with DC motors for robotics projects when I was younger, all I understood was that electricity made these motors spin. Later on, through IB Physics, I know that electric current in a wire can create a magnetic field and can therefore interact with other magnets, but this is described in detail. After some research, it seems that Lorentz force is at the heart of how electric motors operate.[1] There are a number of homemade experiments on the internet that demonstrate Lorentz force, particularly by using a pendulum, but only a small portion of them can quantify it. In hopes of better understanding this force, I have asked the following research question:

How does varying electric current affect the force exerted on a wire in a magnetic field?

Background

Electromagnetism

Electromagnetism can ultimately thought of as the interaction between positively and negatively charged particles. Charged particles are attracted to other particles of the opposite charge, and repelled by like charge. Force exerted between two oppositely charged particles effects a region of space where other charges will be affected. This is called an electric field and can be seen in Figure 1 as depicted with field lines, where the arrows indicate the direction of the force experienced by a positive test charge. The magnitude of the electric field at some point in space, as experienced by a positive test charge, is called the electric field strength, denoted **E**. Generally, the field strength of an electric field is greater when it is closer to a charge.[2]
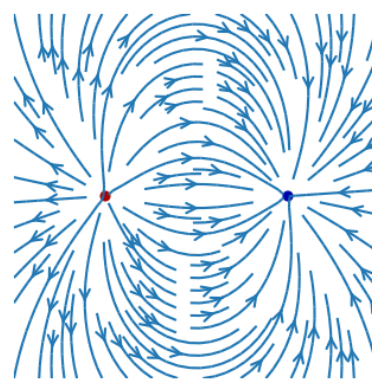

Figure 1: Electric field around two opposing charges.

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1. Martin Kramer, "The DC Motor," The DC Motor, December 13, 2012, http://large.stanford.edu/courses/2012/ph240/kramer2/.

2. T. Editors of Encyclopaedia Britannica, "electric field," Encyclopedia Britannica, January 7, 2019, https://www.britannica.com/science/electric-field.

In terms of magnetism, magnets embody this principle by having two poles, north and south, which could be thought of as positive and negative poles respectively. In this way, opposite poles attract while like poles repel. The region in which a magnetic dipole is effected by these forces is called a magnetic field and is depicted in Figure 2. The field lines of a magnetic field point from north to south. There are two nearly identical vector fields that represent a magnetic field; magnetic field strength, **H** or H-field, and magnetic flux density, **B** or B-field. While their differences are beyond the scope of this investigation, it is important to know that an H-field is concerned with the field strength of a specific point while a B-field

Figure 2: Magnetic field around a magnetic dipole.

is concerned with the density of the field lines at a specific point, also known as flux density. This investigation concerns the latter field, magnetic flux density, and will refer to it as a magnetic field.[3]
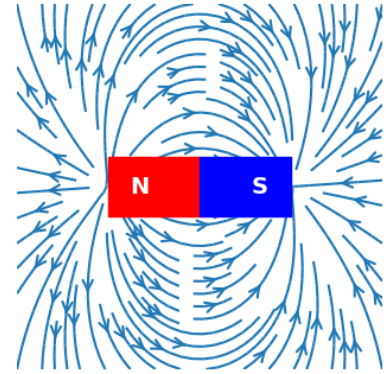
Electricity also embodies this principle in electric current, where negatively charged electrons flow towards a more positive charge. However, conventional thinking would have the direction of current as the movement from higher to lower charge, hence current flows from positive to negative. This movement of charge is what creates magnetic fields. Specifically, if current is moving through a wire, than the movement of charge creates a magnetic field whose field lines are concentric circles around the wire, perpendicular to the direction of current.[4] Similar to how magnets can attract or repel each other, these moving charges are also subject to magnetic force. This force, combined with electric force, is called Lorentz force.

Lorentz Force

Lorentz force, in terms of vectors, is defined as:

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

where q is the charge of a point particle, **v** is the velocity of the particle, **E** is the electric field, and **B** is the magnetic field. This equation represents the forces experienced by a charged particle in an electromagnetic field. The cross product of **v** and **B** indicates that the force is perpendicular to both the direction of velocity and the magnetic field.[5]
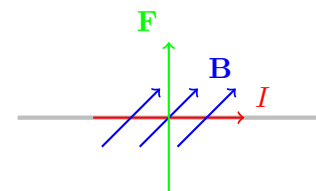

Figure 3: Force on a current-carrying wire. All vectors are perpendicular to each other.

3. T. Editors of Encyclopaedia Britannica, "magnetic field strength," Encyclopedia Britannica, June 30, 2020, https://www.britannica.com/science/magnetic-field-strength.

4. Scott Pratt and Jon Pumplin, "Magnetic field of a wire," Physics 232: Elementary Physics II, accessed April 10, 2022, https://web.pa.msu.edu/courses/1997spring/PHY232/lectures/ampereslaw/wire.html#:~:text=Magnetic%20field%20of%20a%20wire&text=Magnetic%20fields%20arise%20from%20charges,that%20generates%20a%20magnetic%20field..

5. Carl R. Nave, "Magnetic Forces," Hyperphysics, accessed April 10, 2022, http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/magfor.html#c2.

Lorentz force can be extended to describe the relationship between magnetic force and a current-carrying wire. If the wire is placed in a uniform magnetic field with no electric field, then the equation simplifies to:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}.$$

Knowing that $\mathbf{v}$ is the length of the wire $\mathbf{L}$ divided by time t, and that current I is the number of charged particles going through a point over time, the equation can then be rewritten to:[6]

$$\mathbf{F} = q\frac{\mathbf{L}}{t} \times \mathbf{B}$$
$$\mathbf{F} = \frac{q}{t}\mathbf{L} \times \mathbf{B}$$
$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}.$$

This force is illustrated in Figure 3, where current, B-field, and force are all perpendicular to each other. Assuming that the current, B-field, and force are perpendicular to begin with, then the equation can be written as:

$$F = ILB \tag{1}$$

Experimental Equation

In order to test this relationship, one of the elements of Lorentz force must be used as the independent and dependent variables. For the dependent variable, force is the most flexible variable to measure. As for the independent variable, there are three options. Looking at Equation (1), of current, length, and B-field strength, current is the easiest to vary. For B-field strength, either the type of magnet would be varied at the same distance from the wire, or the distance would be varied, of which the latter will be much more difficult since a magnetic field does not completely adhere to the inverse square law.[7] As for length, a magnet would be required whose surface area could cover the range test lengths. Current, on the other hand, can easily be varied by varying the voltage output of a power supply. However, current should be limited to under two amps to reduce the chance of burns, whether it be electrical or heat emitted by components. This means that force will inherently be quite small, even if its magnitude is scaled by using strong magnets.

In order to measure a small force, a pendulum apparatus similar to the CGI video demonstration by the National High Magnetic Field Laboratory will be used.[8] In theory, the pendulum will deflect to a specific angle in which gravity balances exerted Lorentz force. Therefore, the deflection angle can be used to determine the Lorentz force through vector analysis. Letting θ represent the deflection angle, F represent the Lorentz force, and T the tension on the pendulum wire, the relationship

6. Carl R. Nave, "Magnetic Force on a Current-Carrying Wire," Hyperphysics, accessed April 10, 2022, http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/forwir2.html.

7. Bill WW, "How Does Magnetic Field Vary With Distance?," Instructables, accessed April 10, 2022, https://www.instructables.com/How-does-magnetic-field-vary-with-distance/.

8. National MagLab, "The Lorentz Force," Youtube video, April 24, 2017, https://www.youtube.com/watch?v=nRDVm5rn_2A.
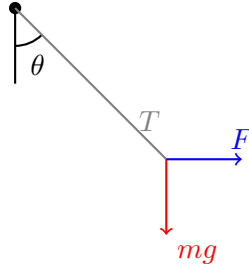
Figure 4: Vector diagram of total forces acting on the experiment apparatus.

between gravity and F is defined through the horizontal and vertical components of total force:

$$x: \quad -T\cos\theta + F = 0$$
$$y: \quad T\sin\theta - mg = 0$$

Solving these equations by isolating and substituting T for one of the equations give the following:

$$F = mg\tan\theta. \tag{2}$$

Thus, F is proportional to $\tan\theta$. Expanding F for ILB and assuming L and B are constant, then I is also proportional to $\tan\theta$.

Experiment Design

Hypothesis

As the current going through the wire increase, the force acting on the wire due to a magnetic field will linearly increase. Hence, the deflection angle of the pendulum apparatus will also increase.

Variables

Independent variable — current flowing through the wire of the pendulum, achieved through varying output voltage from the power supply. Due to the inconsistent resistance of the circuit, caused by changing contact of the wire with the supports as it swings, exact current values cannot be achieved. The target current values were 0.20A, 0.40A, 0.60A, 0.80A, 1.00A, 1.20A, and 1.40A. These variables were measured with an uncertainty of ±0.01 based off the readings from the power supply.

Dependent variable — angle of deflection of the pendulum. The angle was measured through by imaging the deflecting pendulum at a flat plane and constant distance, then using a measurement tool from GIMP, an image processing software. The measurement uncertainty was ±0.01.

Controlled variables — the mass of the wire; 4.9 ± 0.1g — length of the copper wire under the magnetic field; 4.61 ± 0.05cm — pendulum length; 5.56 ± 0.05cm — distance between magnets; 3.12 ± 0.05cm. The wire is kept perpendicular to the magnetic field lines between the magnets.

Procedure

1. Gather a breadboard, two straightened, conductive, identical paper clips, needle nose pliers, and a spool of exposed copper wire to create the pendulum.

   (a) Using needle nose pliers, take the end of a straightened paper clip and bend it into a knuckle that will act as a pivot for the pendulum wire. Create a knuckle on the other paper clip such that the length of both paper clips are the same. Ensure the knuckles overhang so that there is enough clearance between the knuckles and the wire when it is placed. Place both paper clips at least 4cm away from each other on the breadboard; they will act as supports for the wire.

   (b) Spool out an appropriate length of copper wire so that there is at least 5cm of length between the pivot and the lowest point of the wire. At least 15cm of wire should be used, with excess being used to secure the wire to the supports. Using needle nose pliers, bend the ends of the wire into a U shape hook to allow the wire to hang from the supports. Bend the remaining wire into a square U shape such that the vertical segments are of equal length and the horizontal segment is roughly equal to the distance between the supports.

   (c) Fit the wire into the supports and check if it can swing freely. If the wire does not swing freely, either minutely unbend the knuckles on the supports or widen the U shape hooks at the end of the wire.
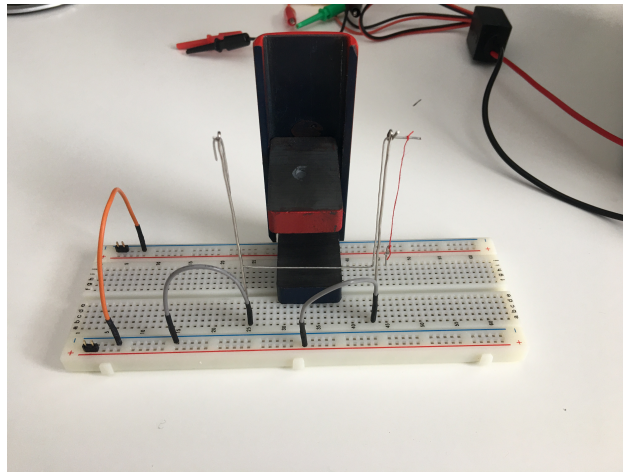

Figure 5: Image of the apparatus. The positive side of the wire is indicated by the red thread.

2. Gather two large flat surface area magnets whose poles are on the largest face and a metal bracket which the magnets will be mounted onto. The magnets must have a large enough surface area to account for the expected deflection angle range.

   (a) Take note of the poles of each magnet. Identify which face of each magnet is north and south.

(b) Mount one magnet onto the bottom of the metal bracket with its south pole facing up. Mount the other magnet so that its south pole is also facing up, with the minimum amount of distance between the two magnets so that a uniform magnetic field is created, but does not result in the two magnets breaking off the mount and attracting each other. Also ensure that the distance between the two magnets is enough to account for the movement of the wire as its deflection angle increases.

(c) Place the mount onto the breadboard and in front of the wire. The wire should be within but on the edge of the magnetic field between the two magnets.

3. Gather the power supply and necessary cables to provide current to the apparatus.

   (a) Use jumper wires to connect the lanes of the supports to the power rails on the breadboard. When viewing the apparatus perpendicular to the wire, make sure the pendulum is connect to the power rails so that conventional current flows from right to left.

   (b) Verify that there are no potential short circuits on the breadboard.

   (c) Connect the power supply to the power rails and provide 0.5V to the breadboard to test the setup. If everything is set up correctly, the wire should have deflected from the normal and away from the supports, into the magnets. If the wire deflects into the supports, switch around the direction of current in the pendulum setup. If the wire does not move, either the circuit is open somewhere, the knuckles or hooks are too small, the wire is not making contact with the supports due to insulation, or the paperclips are not conductive. Troubleshoot each potential issue.

4. Prop a camera or imaging sensor at least 15cm away from the apparatus and facing parallel to the length of the wire. Make sure the position and orientation of the camera to the apparatus remains the same throughout the experiment. The vertical segment of the wire should be on a single perpendicular plane relative to the camera and at the center of the image.

5. Use the camera to photograph the deflection angle of the wire while at rest.

6. Provide enough voltage to the breadboard such that there is about 0.20A going through the wire. Because the current may fluctuate as the wire swings, it is fine to around be within ±0.10A, so long as the current is no longer fluctuating and the wire is in a static equilibrium.

   (a) If the power supply does not measure current, use a digital multimeter and connect one end to a support and the other to its respective power rail.

   (b) Use the camera to photograph the deflection angle of the wire.

7. Repeat step 6 with 0.40A, 0.60A, 0.80A, 1.00A, 1.20A, and 1.40A.

8. Repeat steps 5, 6, and 7 four more times, ending with five trials.

Safety Considerations

As mentioned before, the current should be kept below 2.00A to reduce the chance of a burn. Parts of the apparatus may become hot, mainly the point of connection between the wire and the supports, so avoid touching those areas. Keep the overall power output of the power supply below 4.00W so that there is not enough power to induce current through your body. When handling strong magnets, avoid placing any bodily extremities between them, as they can potentially cause injury.

Results

Qualitative Observations

The current going through the wire fluctuated as the wire swung, likely due to the changing contact of the wire with the supports. There were specific ranges where the current would fluctuate, mainly at the lower currents. At around 1.40A, the wire would get close enough to the bracket that it would jump and stick to the bracket. This was mitigated by avoiding current from exceeding 1.50A. From visual observation, the angle of deflection would increase as the current increased, but their relationship was determinable from visuals.

Raw Data

The data provided below shows the measured current and respective measured angle from 5 trials. The angle measurements were done through the measurement tool in GIMP.

Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
I(A)	θ(°)	I(A)	θ(°)	I(A)	θ(°)	I(A)	θ(°)	I(A)	θ(°)
±0.01	±0.01	±0.01	±0.01	±0.01	±0.01	±0.01	±0.01	±0.01	±0.01
0	0.62	0	0.48	0	0.57	0	0.57	0	0.66
0.22	5.53	0.23	6.83	0.26	6.62	0.18	4.11	0.19	4.37
0.39	12.02	0.42	12.63	0.46	13.65	0.39	11.3	0.44	13.25
0.58	17.41	0.59	17.78	0.57	16.88	0.61	17.82	0.61	18.14
0.80	22.02	0.88	23.49	0.78	21.65	0.83	22.64	0.82	22.35
1.04	25.84	1.01	25.51	1.01	25.47	1.06	25.98	0.99	25.05
1.20	28.59	1.21	27.82	1.20	28.29	1.22	28.44	1.18	27.78
1.42	31.65	1.40	30.84	1.41	31.23	1.42	30.93	1.38	30.41

Table 1: Raw data table of measured angles θ for increasing values of I over 5 trials.

Processed Data

To account for a potential offset in angle due to the shape of the wire, the measured angle of the wire at rest is subtracted from all other measurements. This was calculated as:

$$\theta_{\text{final}} = \theta_{\text{measured}} - \theta_0.$$

For example, the for the first data point, $5.53° - 0.62° = 4.91°$. The uncertainty of the measured angle was propagated as such:

$$\Delta\theta_{\text{final}} = \Delta\theta_{\text{measured}} + \Delta\theta_0.$$

For the first data point, $\pm(0.01 + 0.01)° = \pm0.02°$. This was done for each trial.

The average deflection angle for each target current (ie. 0.20A, 0.40A, etc.) will not be taken, given the inherent variation of current.

Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
$I(A)$	$\theta(°)$	$I(A)$	$\theta(°)$	$I(A)$	$\theta(°)$	$I(A)$	$\theta(°)$	$I(A)$	$\theta(°)$
±0.01	±0.02	±0.01	±0.02	±0.01	±0.02	±0.01	±0.02	±0.01	±0.02
0.22	4.91	0.23	6.35	0.26	6.05	0.18	3.54	0.19	3.71
0.39	11.40	0.42	12.15	0.46	13.08	0.39	10.73	0.44	12.59
0.58	16.79	0.59	17.30	0.57	16.31	0.61	17.25	0.61	17.48
0.80	21.40	0.88	23.01	0.78	21.08	0.83	22.07	0.82	21.69
1.04	25.22	1.01	25.03	1.01	24.90	1.06	25.41	0.99	24.39
1.20	27.97	1.21	27.34	1.20	27.72	1.22	27.87	1.18	27.12
1.42	31.03	1.40	30.36	1.41	30.66	1.42	30.36	1.38	29.75

Table 2: Corrected angle data table for measured angles θ.

The data in Table 2 is plotted in Figure 6.

Next, the corrected angle data was used to calculate the force exerted on the wire by using Equation (2), where $m = 4.9 \pm 0.1$g and $g = 9.81$m s^{-2}. The first data point was calculated as:

$$4.9\text{g} \times 9.81\text{m s}^{-2} \times \tan 4.91° = 4.13\text{mN}.$$

Propagation of uncertainty was calculated as:

$$\Delta F = \left(\frac{1}{\tan \theta} \times \frac{\tan(\theta + \Delta\theta) - \tan(\theta - \Delta\theta)}{2} + \frac{\Delta m}{m} \right) \times F.$$

For the first data point, the uncertainty for the $\tan \theta$ calculation was determined to be:

$$\frac{\tan([4.91 + 0.02]°) - \tan([4.91 - 0.02]°)}{2} = 3.52 \times 10^{-4}.$$
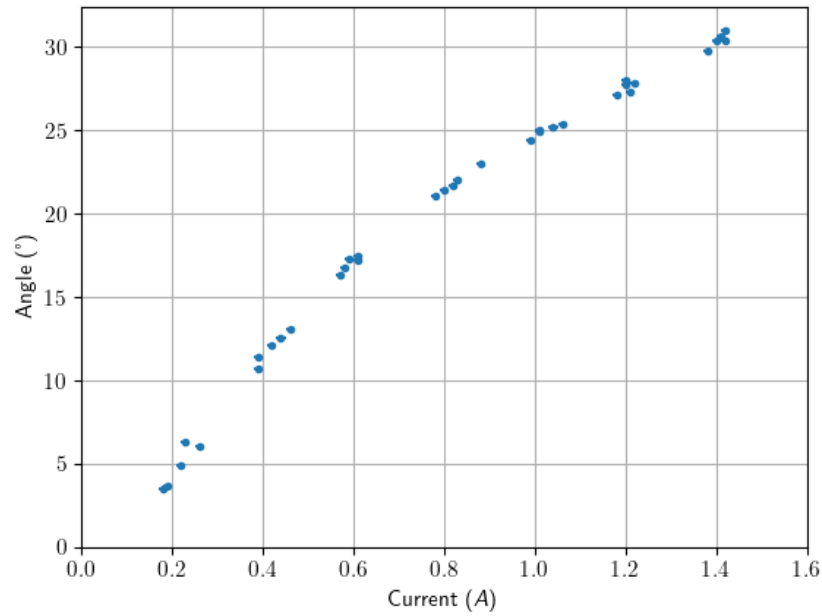
Figure 6: Graph of corrected measured angles vs. current. Note that the errorbars are so small that they are barely visible in the graph.

Then, the sum of the fractional uncertainties between $\tan \theta$ and mass were taken:

$$\frac{3.52 \times 10^{-4}}{\tan 4.91°} + \frac{0.1\text{g}}{4.9\text{g}} = 2.5 \times 10^{-2}.$$

Finally, the fractional uncertainty is converted to the absolute uncertainty of the final value of force:

$$2.5 \times 10^{-2} \times 4.13\text{mN} = \pm 0.10\text{mN}$$

The data from these calculations are displayed on Table 3 and plotted in Figure 7.

Trial 1			Trial 2			Trial 3		
I(A)	F(mN)	ΔF(mN)	I(A)	F(mN)	ΔF(mN)	I(A)	F(mN)	ΔF(mN)
±0.01			±0.01			±0.01		
0.22	4.13	±0.10	0.23	5.35	±0.13	0.26	5.09	±0.12
0.39	9.69	±0.22	0.42	10.35	±0.23	0.46	11.17	±0.25
0.58	14.50	±0.31	0.59	14.97	±0.32	0.57	14.07	±0.31
0.80	18.84	±0.40	0.88	20.41	±0.44	0.78	18.53	±0.40
1.04	22.64	±0.48	1.01	22.45	±0.48	1.01	22.31	±0.48
1.20	25.53	±0.54	1.21	24.85	±0.53	1.20	25.26	±0.54
1.42	28.92	±0.61	1.40	28.16	±0.60	1.41	28.50	±0.60

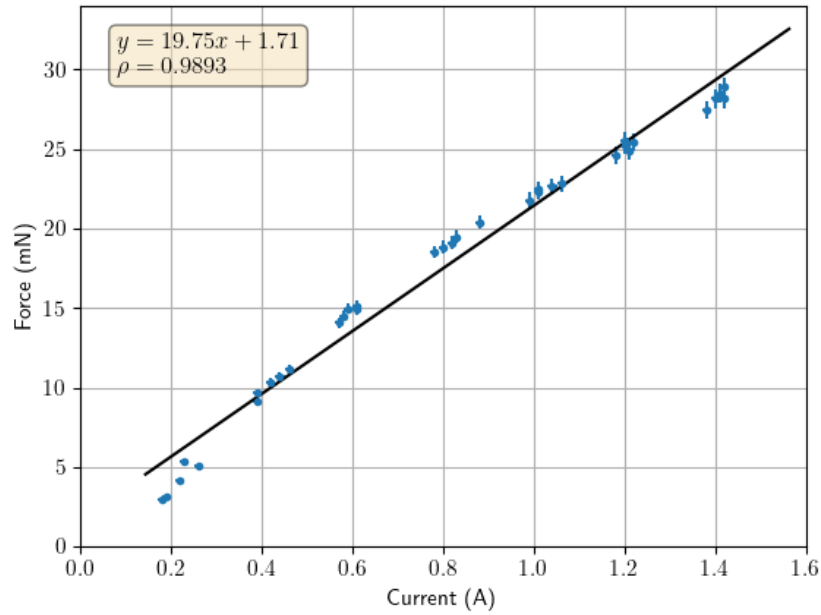| Trial 4			Trial 5		
I(A)	F(mN)	ΔF(mN)	I(A)	F(mN)	ΔF(mN)
±0.01			±0.01		
0.18	2.97	±0.08	0.19	3.12	±0.08
0.39	9.11	±0.20	0.44	10.74	±0.24
0.61	14.93	±0.32	0.61	15.14	±0.33
0.83	19.49	±0.42	0.82	19.12	±0.41
1.06	22.84	±0.49	0.99	21.79	±0.47
1.22	25.42	±0.54	1.18	24.62	±0.52
1.42	28.16	±0.60	1.38	27.47	±0.58

Table 3: Table of force in relation to current.

Figure 7: Graph of force vs. time.

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