Investigating the relationship between the electric current and force exerted on a wire

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Introduction

As the world transitions from internal combustion engines to electric motors, I have begun to wonder how exactly electricity is used in vehicles. More specifically, how electrical force is transformed into physical force. Despite what I have learned from the school curriculum, there is not significant amount material covering electricity to begin with. Having worked with DC motors for robotics projects when I was younger, all I understood was that electricity made these motors spin. Later on, through IB Physics, I know that electric current in a wire can create a magnetic field and can therefore interact with other magnets, but this is described in detail. After some research, it seems that Lorentz force is at the heart of how electric motors operate.[1] There are a number of homemade experiments on the internet that demonstrate Lorentz force, particularly by using a pendulum, but only a small portion of them can quantify it. In hopes of better understanding this force, I have asked the following research question:

How does varying electric current affect the force exerted on a wire in a magnetic field?

Background

Electromagnetism

Electromagnetism can ultimately thought of as the interaction between positively and negatively charged particles. Charged particles are attracted to other particles of the opposite charge, and repelled by like charge. Force exerted between two oppositely charged particles effects a region of space where other charges will be affected. This is called an electric field and can be seen in Figure 1 as depicted with field lines, where the arrows indicate the direction of the force experienced by a positive test charge. The magnitude of the electric field at some point in space, as experienced by a positive test charge, is called the electric field strength, denoted **E**. Generally, the field strength of an electric field is greater when it is closer to a charge.[2]

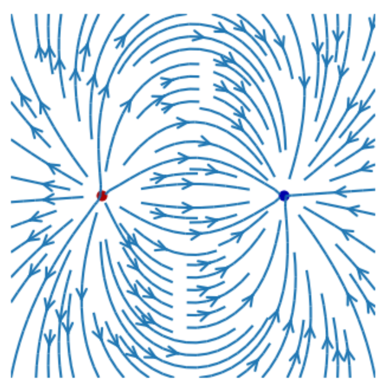

Figure 1: Electric field around two opposing charges.

1. Martin Kramer, "The DC Motor," The DC Motor, December 13, 2012, http://large.stanford.edu/courses/2012/ph240/kramer2/.
2. T. Editors of Encyclopaedia Britannica, "electric field," Encyclopedia Britannica, January 7, 2019, https://www.britannica.com/science/electric-field.

In terms of magnetism, magnets embody this principle by having two poles, north and south, which could be thought of as positive and negative poles respectively. In this way, opposite poles attract while like poles repel. The region in which a magnetic dipole is effected by these forces is called a magnetic field and is depicted in Figure 2. The field lines of a magnetic field point from north to south. There are two nearly identical vector fields that represent a magnetic field; magnetic field strength, **H** or H-field, and magnetic flux density, **B** or B-field. While their differences are beyond the scope of this investigation, it is important to know that an H-field is concerned with the field strength of a specific point while a B-field is concerned with the density of the field lines at a specific point, also known as flux density. This investigation concerns the latter field, magnetic flux density, and will refer to it as a magnetic field.[3]

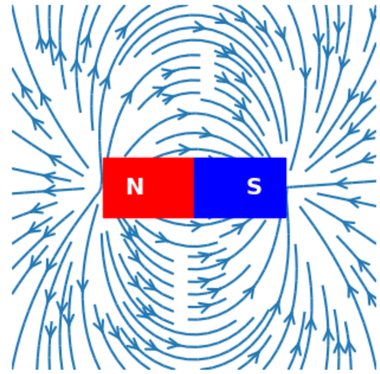

Figure 2: Magnetic field around a magnetic dipole.

Electricity also embodies this principle in electric current, where negatively charged electrons flow towards a more positive charge. However, conventional thinking would have the direction of current as the movement from higher to lower charge, hence current flows from positive to negative. This movement of charge is what creates magnetic fields. Specifically, if current is moving through a wire, than the movement of charge creates a magnetic field whose field lines are concentric circles around the wire, perpendicular to the direction of current.[4] Similar to how magnets can attract or repel each other, these moving charges are also subject to magnetic force. This force, combined with electric force, is called Lorentz force.

Lorentz Force

Lorentz force, in terms of vectors, is defined as:

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

where q is the charge of a point particle, $\mathbf{v}$ is the velocity of the particle, $\mathbf{E}$ is the electric field, and $\mathbf{B}$ is the magnetic field. This equation represents the forces experienced by a charged particle in an electromagnetic field. The cross product of $\mathbf{v}$ and $\mathbf{B}$ indicates that the force is perpendicular to both the direction of velocity and the magnetic field.[5]

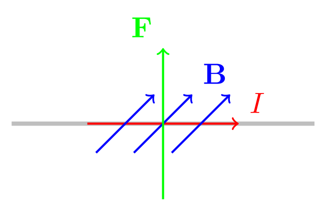

Figure 3: Force on a current-carrying wire.

3. T. Editors of Encyclopaedia Britannica, “magnetic field strength,” Encyclopedia Britannica, June 30, 2020, https://www.britannica.com/science/magnetic-field-strength.

4. Scott Pratt and Jon Pumplin, “Magnetic field of a wire,” Physics 232: Elementary Physics II, accessed April 10, 2022, https://web.pa.msu.edu/courses/1997spring/PHY232/lectures/ampereslaw/wire.html#:~:text=Magnetic%20field%20of%20a%20wire&text=Magnetic%20fields%20arise%20from%20charges,that%20generates%20a%20magnetic%20field..

5. Carl R. Nave, “Magnetic Forces,” Hyperphysics, accessed April 10, 2022, http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/magfor.html#c2.

Lorentz force can be extended to describe the relationship between magnetic force and a current-carrying wire. If the wire is placed in a uniform magnetic field with no electric field, then the equation simplifies to:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}.$$

Knowing that $\mathbf{v}$ is the length of the wire $\mathbf{L}$ divided by time t, and that current I is the number of charged particles going through a point over time, the equation can then be rewritten to:[6]

$$\mathbf{F} = q\frac{\mathbf{L}}{t} \times \mathbf{B}$$
$$\mathbf{F} = \frac{q}{t}\mathbf{L} \times \mathbf{B}$$
$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}.$$

This force is illustrated in Figure 3, where current, B-field, and force are all perpendicular to each other. Assuming that the current, B-field, and force are perpendicular to begin with, then the equation can be written as:

$$F = ILB \tag{1}$$

Experimental Equation

In order to test this relationship, an elements of Lorentz force must be used as the independent and dependent variables. For the dependent variable, force is the most flexible variable to measure. As for the independent variable, there are three options. Looking at Equation (1), of current, length, and B-field strength, current is the easiest to vary. For B-field strength, either the type of magnet would be varied at the same distance from the wire, or the distance would be varied, of which the latter will be much more difficult since a magnetic field does not completely adhere to the inverse square law.[7] As for length, a magnet would be required whose surface area could cover the range test lengths. Current, on the other hand, can easily be varied by varying the voltage output of a power supply. However, current should be limited to under two amps to reduce the chance of burns, whether it be electrical or heat emitted by components. This means that force will inherently be quite small, even if its magnitude is scaled by using strong magnets.

In order to measure a small force, a pendulum apparatus similar to the CGI video demonstration by the National High Magnetic Field Laboratory will be used.[8] In theory, the pendulum will deflect to a specific angle in which gravity balances exerted Lorentz force. Therefore, the deflection angle can be used to determine the Lorentz force through vector analysis. Letting θ represent the deflection angle, F represent the Lorentz force, and T the tension on the pendulum wire, the relationship

6. Carl R. Nave, "Magnetic Force on a Current-Carrying Wire," Hyperphysics, accessed April 10, 2022, http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/forwir2.html.

7. Bill WW, "How Does Magnetic Field Vary With Distance?," Instructables, accessed April 10, 2022, https://www.instructables.com/How-does-magnetic-field-vary-with-distance/.

8. National MagLab, "The Lorentz Force," Youtube video, April 24, 2017, https://www.youtube.com/watch?v=nRDVm5rn_2A.

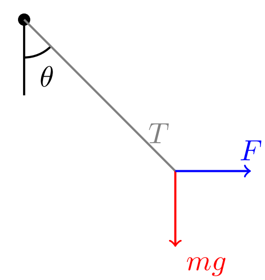

Figure 4: Vector diagram of total forces acting on the experiment apparatus.

between gravity and F is defined through the horizontal and vertical components of total force:

$$\begin{aligned} x: &\quad -T\cos\theta + F = 0 \\ y: &\quad T\sin\theta - mg = 0 \end{aligned}$$

Solving these equations by isolating and substituting T for one of the equations give the following:

$$F = mg\tan\theta. \tag{2}$$

Thus, F is proportional to $\tan\theta$. Expanding F for ILB and assuming L and B are constant, then I is also proportional to $\tan\theta$.

Experiment Design

Hypothesis

As the current going through the wire increase, the force acting on the wire due to a magnetic field will linearly increase. Hence, the deflection angle of the pendulum apparatus will also increase.

Variables

Independent variable — current flowing through the wire of the pendulum, achieved through varying output voltage from the power supply. Due to the inconsistent resistance of the circuit, caused by changing contact of the wire with the supports as it swings, exact current values cannot be achieved. The target current values were 0.20A, 0.40A, 0.60A, 0.80A, 1.00A, 1.20A, and 1.40A. These variables were measured with an uncertainty of ± 0.01 based off the readings from the power supply.

Dependent variable — angle of deflection of the pendulum. The angle was measured through by imaging the deflecting pendulum at a flat plane and constant distance, then using a measurement tool from GIMP, an image processing software. The measurement uncertainty was ± 0.01.

Controlled variables — the mass of the wire; 4.9 ± 0.1g — length of the copper wire under the magnetic field; 4.61 ± 0.05cm — pendulum length; 5.56 ± 0.05cm — distance between magnets; 3.12 ± 0.05cm. The pendulum is kept perpendicular to the magnetic field lines between the magnets.

Procedure

1. Lay out a breadboard

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