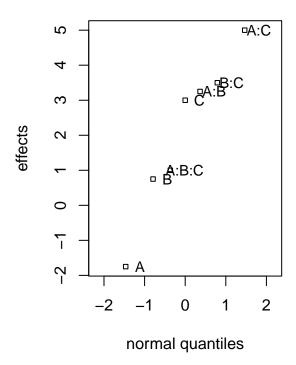
101B - HW5

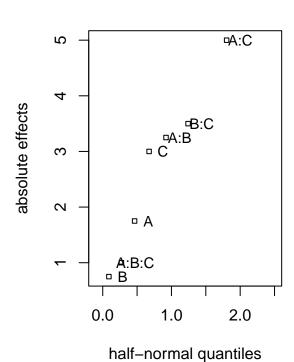
```
library(phia)
## Loading required package: car
## Warning: package 'car' was built under R version 4.1.2
## Loading required package: carData
## Warning: package 'carData' was built under R version 4.1.2
A \leftarrow c(-1,1,-1,1,-1,1,-1,1)
B \leftarrow c(-1,-1,1,1,-1,-1,1,1)
C \leftarrow c(-1,-1,-1,-1,1,1,1,1)
response \leftarrow c(50,54,
               44,42,
               46,48,
               42,43,
               49,46,
               48,45,
               47,48,
               56,54)
data <- data.frame("A" = A, "B" = B, "C" = C, "Y" = response)
dat = data.frame(response=response, A=rep(A, each=2), B=rep(B, each=2), C=rep(C, each=2))
normalplot <- function(y, label=F, n=length(y), fac.names=NULL, xlim=c(-2.2, 2.2), main="Normal Plot",
{ # label the most singificant n effects
m <- length(y)
x \leftarrow seq(0.5/m, 1.0-0.5/m, by=1/m)
x <- qnorm(x)
y <- sort(y)</pre>
qqplot(x, y,
xlab="normal quantiles", ylab="effects",
xlim=xlim, main=main, ...)
if(is.null(fac.names)) fac.names <- names(y)</pre>
else fac.names <- rev( c(fac.names, rep("", length(y)-length(fac.names)) ) )</pre>
ord=order(abs(y))
if(label) for(i in ord[(m-n+1):m]) text(x[i]+.35,y[i], fac.names[i], cex=0.9)
}
```

```
halfnormalplot <- function(y, label=F, n=length(y), fac.names=NULL, xlim=c(-.1, 2.5), main="Half-Normal
{ # label the most singificant n effects
m <- length(y)
x \leftarrow seq(0.5+0.25/m, 1.0-0.25/m, by=0.5/m)
x \leftarrow qnorm(x)
y <- sort(abs(y))
qqplot(x, y,
xlab="half-normal quantiles", ylab="absolute effects",
xlim=xlim, main=main, ...)
if(is.null(fac.names)) fac.names <- names(y)</pre>
else fac.names <- rev( c(fac.names, rep("", length(y)-length(fac.names)) ) )</pre>
if(label) for(i in (m-n+1):m) text(x[i]+.2,y[i], fac.names[i],cex=0.9)
res.lm = lm(response~A*B*C, data = dat)
summary(res.lm)
##
## Call:
## lm(formula = response ~ A * B * C, data = dat)
##
## Residuals:
##
      Min
              1Q Median
                            3Q
                                   Max
##
              -1
                             1
                                     2
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
                 47.625
                             0.433 109.985 5.22e-14 ***
## (Intercept)
## A
                 -0.875
                             0.433 -2.021 0.077971 .
## B
                  0.375
                             0.433
                                    0.866 0.411694
## C
                             0.433
                                    3.464 0.008516 **
                  1.500
## A:B
                  1.625
                             0.433
                                     3.753 0.005602 **
                                     5.774 0.000418 ***
## A:C
                  2.500
                             0.433
## B:C
                  1.750
                             0.433
                                     4.041 0.003728 **
## A:B:C
                  0.500
                             0.433
                                     1.155 0.281537
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Residual standard error: 1.732 on 8 degrees of freedom
## Multiple R-squared: 0.911, Adjusted R-squared: 0.8332
## F-statistic: 11.7 on 7 and 8 DF, p-value: 0.001223
par(mfrow=c(1,2))
effects = 2*coef(res.lm)[-1]
normalplot(effects, label=T, type="p", pch=22, cex=0.7)
halfnormalplot(effects,label=T,type="p",pch=22,cex=0.7)
```

Normal Plot

Half-Normal Plot



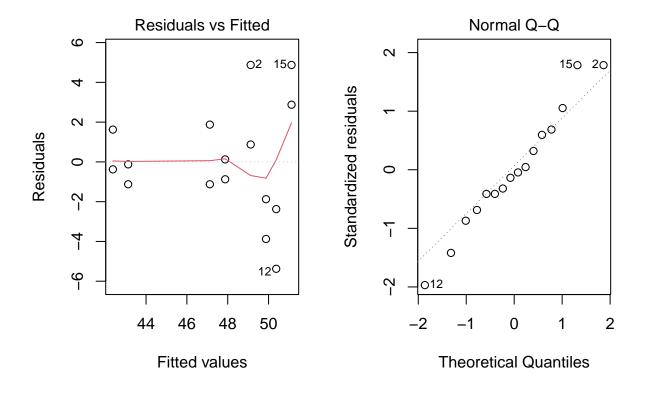


#Looking at the half normal probability plot, we can conclude the main effects are A and C, and the int

```
res.anova = anova(res.lm)
res.anova
## Analysis of Variance Table
##
## Response: response
##
             Df Sum Sq Mean Sq F value
## A
                12.25
                         12.25 4.0833 0.0779708 .
## B
                  2.25
                          2.25 0.7500 0.4116944
                 36.00
                         36.00 12.0000 0.0085163 **
## C
                 42.25
                         42.25 14.0833 0.0056019 **
## A:B
                        100.00 33.3333 0.0004176 ***
## A:C
              1 100.00
## B:C
                 49.00
                         49.00 16.3333 0.0037282 **
## A:B:C
                  4.00
                          4.00
                               1.3333 0.2815369
              1
              8
                 24.00
                          3.00
## Residuals
                  0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Signif. codes:
res.lm2 = lm(response~A+B+C+A:C, data = dat)
summary(res.lm2)
```

Call:

```
## lm(formula = response ~ A + B + C + A:C, data = dat)
##
## Residuals:
      Min
##
              1Q Median
                            3Q
                                  Max
  -5.375 -1.312 -0.250
##
                         1.688
                                4.875
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
               47.6250
                            0.8231
                                    57.858 5.08e-15 ***
## A
                -0.8750
                            0.8231
                                    -1.063
                                              0.3106
## B
                 0.3750
                            0.8231
                                      0.456
                                              0.6576
## C
                 1.5000
                            0.8231
                                      1.822
                                              0.0957 .
## A:C
                 2.5000
                            0.8231
                                      3.037
                                              0.0113 *
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
## Residual standard error: 3.293 on 11 degrees of freedom
## Multiple R-squared: 0.5579, Adjusted R-squared: 0.3972
## F-statistic: 3.471 on 4 and 11 DF, p-value: 0.04567
par(mfrow=c(1,2))
plot(res.lm2,1:2)
```



#Nothing unusual about residuals

```
aov <- aov(response ~ factor(A) * factor(B) * factor(C), data = dat)
factorial <- interactionMeans(aov, factors = c("factor(A)", "factor(B)", "factor(C)"))
plot(factorial)</pre>
```

adjusted mean 52 20 ф. φ. factor(A) 48 <u>ф</u> 46 4 5242 20 factor(B) 48 ⊖- 1 46 4 20 車 factor(C) 8 46 重 42 -1 factor(A) 1 ₋₁ factor(B) ₁ -1 factor(C) 1

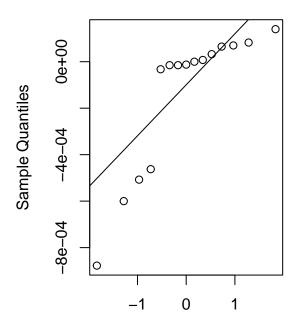
```
B1 <- c(-1,-1,1,1,-1,-1,1,1,-1,-1,1,1,-1,-1,1,1)
C1 \leftarrow c(-1,-1,-1,-1,1,1,1,-1,-1,-1,-1,1,1,1,1)
response1 <- c(0.00340,
          0.00362,
          0.00301,
          0.00182,
          0.00280,
          0.00290,
          0.00252,
          0.00160,
          0.00336,
          0.00344,
          0.00308,
          0.00184,
          0.00269,
          0.00284,
```

```
0.00253.
                                  0.00163)
data <- data.frame("A" = A1, "B" = B1, "C" = C1, "D" = D1, "response" = response)
m <- lm(response1 ~ A*B*C*D, data = data)</pre>
summary(m)
##
## Call:
## lm(formula = response1 ~ A * B * C * D, data = data)
## Residuals:
## ALL 16 residuals are 0: no residual degrees of freedom!
##
## Coefficients:
##
                                         Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2.692e-03
                                                                                 {\tt NaN}
                                                                                                    NaN
                                    -2.312e-04
                                                                                 NaN
                                                                                                    NaN
                                                                                                                           NaN
## A
## B
                                    -4.387e-04
                                                                                 NaN
                                                                                                    NaN
                                                                                                                           NaN
## C
                                    -2.537e-04
                                                                                 {\tt NaN}
                                                                                                    NaN
                                                                                                                           NaN
## D
                                    -1.625e-05
                                                                                 {\tt NaN}
                                                                                                   {\tt NaN}
                                                                                                                           NaN
## A:B
                                    -3.000e-04
                                                                                 {\tt NaN}
                                                                                                   {\tt NaN}
                                                                                                                          \mathtt{NaN}
## A:C
                                     3.500e-05
                                                                                 {\tt NaN}
                                                                                                   {\tt NaN}
                                                                                                                          {\tt NaN}
                                                                                 {\tt NaN}
                                                                                                   NaN
## B:C
                                    7.000e-05
                                                                                                                          NaN
## A:D
                                    -7.500e-06
                                                                                 \mathtt{NaN}
                                                                                                    NaN
                                                                                                                           NaN
## B:D
                                     3.250e-05
                                                                                 {\tt NaN}
                                                                                                    {\tt NaN}
                                                                                                                           NaN
## C:D
                                    -8.868e-20
                                                                                 \mathtt{NaN}
                                                                                                    NaN
                                                                                                                           NaN
## A:B:C
                                    4.125e-05
                                                                                 {\tt NaN}
                                                                                                    {\tt NaN}
                                                                                                                           NaN
## A:B:D
                                    3.750e-06
                                                                                 {\tt NaN}
                                                                                                    {\tt NaN}
                                                                                                                           NaN
## A:C:D
                                    1.625e-05
                                                                                 {\tt NaN}
                                                                                                    {\tt NaN}
                                                                                                                           NaN
## B:C:D
                                    -6.250e-06
                                                                                 NaN
                                                                                                    NaN
                                                                                                                           NaN
## A:B:C:D
                                    -7.500e-06
                                                                                 {\tt NaN}
                                                                                                    NaN
                                                                                                                           NaN
## Residual standard error: NaN on O degrees of freedom
## Multiple R-squared:
                                                                    1, Adjusted R-squared:
## F-statistic: NaN on 15 and 0 DF, p-value: NA
par(mfrow=c(1,2))
effects = 2*coef(m)[-1]
qqnorm(effects);qqline(effects)
\#normalplot \leftarrow function(y, label=F, n=length(y), fac.names=NULL, xlim=c(-2.2, 2.2), main="Normal Plot", label=F, n=length(y), label
#{ # label the most singificant n effects
\#m \leftarrow length(y)
\#x \leftarrow seq(0.5/m, 1.0-0.5/m, by=1/m)
\#x \leftarrow qnorm(x)
#y <- sort(y)
#qqplot(x, y,
#xlab="normal quantiles", ylab="effects",
#xlim=xlim, main=main, ...)
#if(is.null(fac.names)) fac.names <- names(y)</pre>
#else fac.names <- rev( c(fac.names, rep("", length(y)-length(fac.names)) ) )</pre>
```

```
#ord=order(abs(y))
\#if(label) for(i in ord[(m-n+1):m]) text(x[i]+.35,y[i], fac.names[i],cex=0.9)
model1 <- lm(response ~ A*B*C, data)</pre>
summary(model1)
##
## lm(formula = response ~ A * B * C, data = data)
## Residuals:
   Min
           1Q Median
                          3Q
                                Max
## -7.000 -1.625 0.000 1.625 7.000
##
## Coefficients:
             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 47.625
                       1.262 37.725 2.68e-10 ***
## A
               -0.125
                          1.262 -0.099
                                           0.924
## B
                          1.262 -0.693
                                          0.508
               -0.875
## C
                0.375
                          1.262
                                  0.297
                                          0.774
## A:B
                -0.625
                           1.262 -0.495
                                           0.634
## A:C
               0.375
                          1.262
                                  0.297
                                         0.774
## B:C
                1.625
                          1.262 1.287 0.234
## A:B:C
                 0.125
                           1.262 0.099 0.924
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Residual standard error: 5.05 on 8 degrees of freedom
## Multiple R-squared: 0.2437, Adjusted R-squared: -0.418
## F-statistic: 0.3683 on 7 and 8 DF, p-value: 0.8971
res1 <- model1$residuals</pre>
```

par(mfrow=c(1,2))

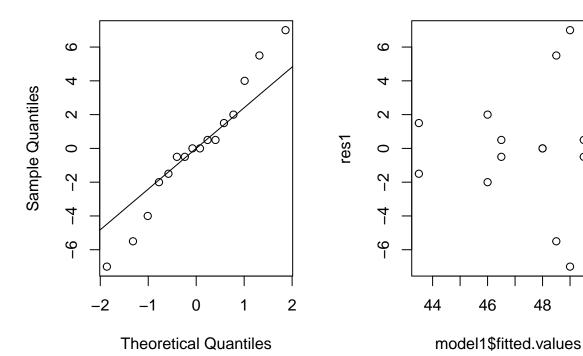
Normal Q-Q Plot



Theoretical Quantiles

```
qqnorm(res1); qqline(res1)
plot(model1$fitted.values, res1)
```

Normal Q-Q Plot



```
dat$new <- 1/dat$response</pre>
\#model2 \leftarrow lm(new \sim A*B*C*D, dat)
#summary(model2)
```

0

0

0

0

0

50

0

0

0

48

0

0

0

46

0

0

```
#Q3
#There can be 16 effects in a 2^4 full factorial model
#i.e.
x \leftarrow choose(4,0) + choose(4,1) + choose(4,2) + choose(4,3) + choose(4,4)
```

[1] 16

```
#Q4
#2^3 replicated twice - (2^k)(n-1)
(2^3)*(2-1)
```

[1] 8

```
#Q5
#2^3 replicated twice:
n \leftarrow (2^3)*2 #Total number of replicates
```

[1] 16

```
#3 Factors
d <- 3
#residual df
f <- 1
#number of residual degrees of freedom for reduced
total = n - d - f
total</pre>
```

[1] 12