C183 - Project 6

```
library(dplyr)
```

```
## Warning: package 'dplyr' was built under R version 4.1.2
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
library(ggplot2)
stock <- read.csv("stockData.csv", sep=",", header=TRUE)
returns <- (stock[-1,3:ncol(stock)]-stock[-nrow(stock),3:ncol(stock)])/stock[-nrow(stock),3:ncol(stock)]
market <- var(returns$X.GSPC)

stock_names <- colnames(returns)
stock_names <- stock_names[2:length(stock_names)]


data <- data.frame()
n <- nrow(returns)


for(i in stock_names){

t <- paste(i, 'X.GSPC', sep = ' ~ ')
model <- lm(t, data = returns)
co <- as.numeric(model$coefficients)
var <- sum(model$residuals^2) / (n-2)
data <- rbind(data, c(co, var))

}


colnames(data) <- c('alpha', 'beta', 'sigma')
sigma <- var(returns$X.GSPC)
rownames(data) <- stock_names

data %>% arrange(desc(beta))
```

```
##                 alpha      beta       sigma
## TSLA   0.0274545269 1.8364717 0.026771258
## NVDA   0.0371238778 1.7397344 0.012389815
## C     -0.0064623005 1.5614556 0.004367604
## MU     0.0027510870 1.4150686 0.010227956
## GS    -0.0001222496 1.3738849 0.003146655
## DIS   -0.0057560070 1.2920864 0.003655394
## NFLX   0.0146730658 1.2890893 0.012132061
## AAPL   0.0115784779 1.2743952 0.003665194
## CRM    0.0062143302 1.2741884 0.005108393
## LULU   0.0133726178 1.1580621 0.008922870
## AXP    0.0018524266 1.1446860 0.003117340
## GOOGL  0.0068899061 1.1203555 0.002450166
## TSM    0.0115468177 1.1173894 0.005935593
## META   0.0061839656 1.1129029 0.007428385
## JPM    0.0046053430 1.1004374 0.002675371
## MA     0.0089344267 1.0967111 0.002262860
## NKE    0.0049783369 1.0205110 0.003271393
## MSFT   0.0152866567 0.9895580 0.002016417
## V      0.0071181029 0.9748402 0.001687497
## BABA   0.0004362885 0.9345299 0.012735118
## SBUX   0.0053401422 0.8685494 0.002910960
## TMO    0.0104584288 0.8682832 0.002207838
## BIDU  -0.0029954602 0.8371060 0.015080668
## UNH    0.0124201740 0.6944230 0.002514650
## CVS   -0.0037975366 0.6527675 0.004184400
## MCD    0.0093128349 0.6509412 0.001684795
## GILD  -0.0020627475 0.5657933 0.004508701
## BMY    0.0017961195 0.5226090 0.004265087
## NVO    0.0123936223 0.5223988 0.003599509
## ATVI   0.0143934518 0.4937844 0.006386527
```

```
m1 <- diag(data$sigma)
b <- as.matrix(data$beta)


m2 <- sigma * (b %*% t(b))
total <- m1 + m2

colnames(total) <- stock_names
rownames(total) <- stock_names




stock2 <- read.csv("stockData.csv", sep=",", header=TRUE)[1:60,]
data2 <- (stock2[-1,4:ncol(stock2)]-stock2[-nrow(stock2),4:ncol(stock2)])/stock2[-nrow(stock2),4:ncol(s

r <- as.matrix(colMeans(data2))
sigma_mat <- cov(data2)


i_m <- matrix(rep(1,30), 30, 1)
A <- as.numeric(t(r) %*% solve(sigma_mat) %*% i_m)
B <- as.numeric(t(r) %*% solve(sigma_mat) %*% r)
```

```r
C <- as.numeric(t(i_m) %*% solve(sigma_mat) %*% i_m)
D <- B*C - A^2

sigmas <- exp(seq(-2, -1, 0.0001))
sigmas_S <- exp(seq(-3, -1, 0.0001))
e1 <- A/C + sqrt(D * (C * sigmas^2 - 1)) / C
e2 <- A/C - sqrt(D * (C * sigmas^2 - 1)) / C

# SIM method
A_S <- as.numeric(t(r) %*% solve(total) %*% i_m)
B_S <- as.numeric(t(r) %*% solve(total) %*% r)
C_S <- as.numeric(t(i_m) %*% solve(total) %*% i_m)
D_S <- B_S*C_S - A_S^2

e1_S <- A_S/C_S + sqrt(D_S * (C_S * sigmas_S^2 - 1)) / C_S
e2_S <- A_S/C_S - sqrt(D_S * (C_S * sigmas_S^2 - 1)) / C_S
```

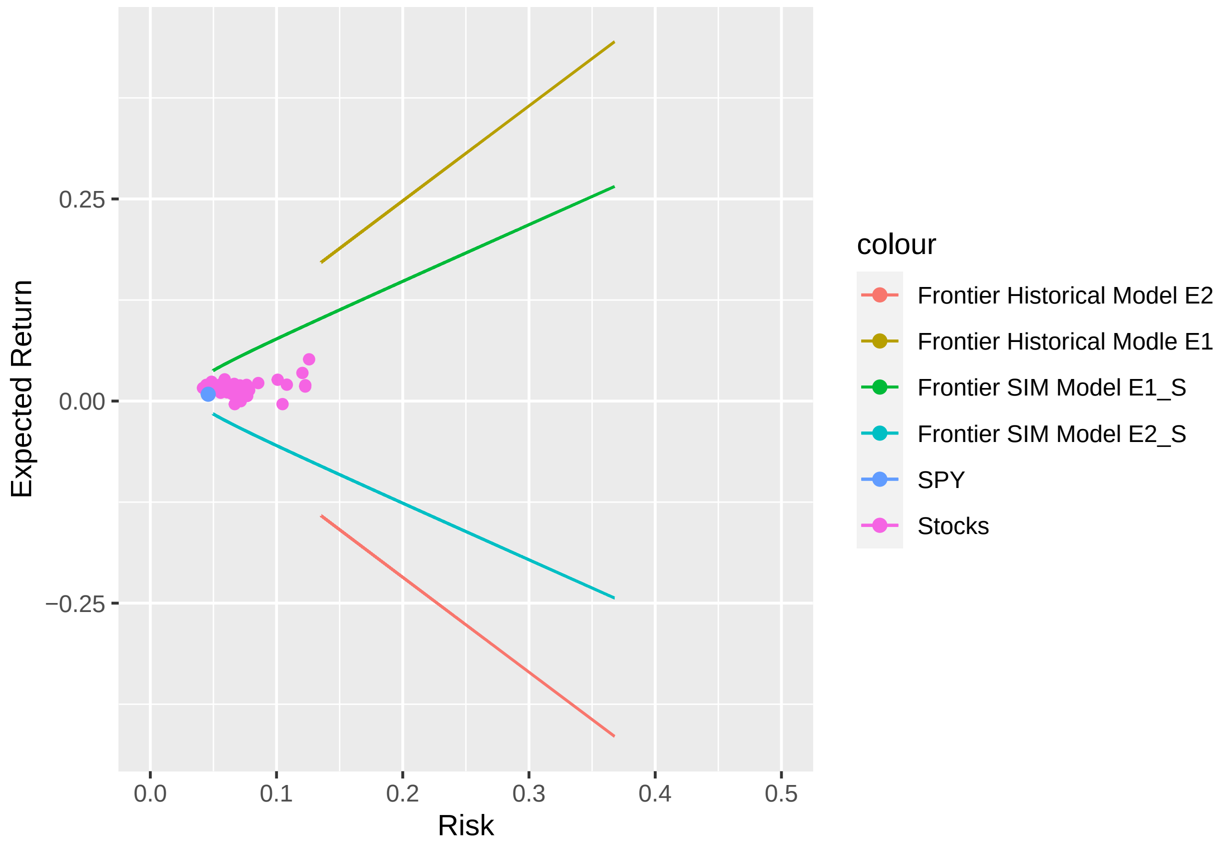
```r
library(dplyr)
r_i <- apply(returns,2,mean)[2:31]
data <- data %>% mutate(R_i = r_i) %>% mutate(stock_n = 1:30)
data_new <- data
R_f <- 0.005

data <- data %>% mutate(excess_beta = (R_i - R_f)/beta) %>%  mutate(beta_var = beta^2/sigma) %>%
mutate(C_star_num = (R_i - R_f) * beta / sigma) #(R_i - R_f) * beta / sigma^2
sort_data <- data %>% arrange(desc(excess_beta))

sort_data <- sort_data %>% mutate(sum_cstar = cumsum(C_star_num)) %>% mutate(sumbeta_var = cumsum(beta_v

C_star <- sort_data$C_i[nrow(sort_data)] #C^* = last C_i

sort_data
```

```
##              alpha      beta       sigma         R_i stock_n  excess_beta
## ATVI   0.0143934518 0.4937844 0.006386527 0.018565474      12  0.027472465
## NVDA   0.0371238778 1.7397344 0.012389815 0.051823028       3  0.026913894
## NVO    0.0123936223 0.5223988 0.003599509 0.016807410      17  0.022602291
## TSLA   0.0274545269 1.8364717 0.026771258 0.042971018      25  0.020676070
## UNH    0.0124201740 0.6944230 0.002514650 0.018287408      13  0.019134457
## MSFT   0.0152866567 0.9895580 0.002016417 0.023647509       5  0.018844280
## NFLX   0.0146730658 1.2890893 0.012132061 0.025564681       9  0.015952875
## LULU   0.0133726178 1.1580621 0.008922870 0.023157173      29  0.015678929
## MCD    0.0093128349 0.6509412 0.001684795 0.014812687      30  0.015074613
## TMO    0.0104584288 0.8682832 0.002207838 0.017794620      14  0.014735538
## TSM    0.0115468177 1.1173894 0.005935593 0.020987727       6  0.014308107
## AAPL   0.0115784779 1.2743952 0.003665194 0.022345941       1  0.013611116
## MA     0.0089344267 1.0967111 0.002262860 0.018200623      24  0.012036555
## V      0.0071181029 0.9748402 0.001687497 0.015354603      20  0.010621846
## GOOGL  0.0068899061 1.1203555 0.002450166 0.016355876       8  0.010135958
## META   0.0061839656 1.1129029 0.007428385 0.015586968      11  0.009512930
## CRM    0.0062143302 1.2741884 0.005108393 0.016980046       2  0.009402099
## SBUX   0.0053401422 0.8685494 0.002910960 0.012678583      28  0.008840698
## NKE    0.0049783369 1.0205110 0.003271393 0.013600713      27  0.008427849
```

```
## JPM    0.0046053430 1.1004374 0.002675371 0.013903023      19  0.008090440
## MU     0.0027510870 1.4150686 0.010227956 0.014707110       4  0.006859816
## AXP    0.0018524266 1.1446860 0.003117340 0.011523967      22  0.005699350
## GS    -0.0001222496 1.3738849 0.003146655 0.011485810      21  0.004720781
## BABA   0.0004362885 0.9345299 0.012735118 0.008332203      26  0.003565647
## BMY    0.0017961195 0.5226090 0.004265087 0.006211683      16  0.002318527
## C     -0.0064623005 1.5614556 0.004367604 0.006730558      23  0.001108298
## DIS   -0.0057560070 1.2920864 0.003655394 0.005160931       7  0.000124551
## BIDU  -0.0029954602 0.8371060 0.015080668 0.004077313      10 -0.001102235
## GILD  -0.0020627475 0.5657933 0.004508701 0.002717684      18 -0.004033833
## CVS   -0.0037975366 0.6527675 0.004184400 0.001717747      15 -0.005028212
##        beta_var   C_star_num sum_cstar sumbeta_var         C_i
## ATVI   38.17772  1.04883597  1.048836    38.17772 0.002043691
## NVDA  244.28743  6.57472584  7.623562   282.46514 0.010064186
## NVO    75.81604  1.71361628  9.337178   358.28119 0.011204925
## TSLA  125.97945  2.60475991 11.941938   484.26064 0.012448731
## UNH   191.76558  3.66933025 15.611268   676.02622 0.013562571
## MSFT  485.62624  9.15127686 24.762545  1161.65246 0.015129728
## NFLX  136.97189  2.18509547 26.947641  1298.62435 0.015193296
## LULU  150.30004  2.35654373 29.304184  1448.92440 0.015231234
## MCD   251.49916  3.79125262 33.095437  1700.42356 0.015213128
## TMO   341.47242  5.03177994 38.127217  2041.89598 0.015148333
## TSM   210.35120  3.00972745 41.136944  2252.24718 0.015083527
## AAPL  443.10969  6.03121733 47.168162  2695.35687 0.014877735
## MA    531.52869  6.39777420 53.565936  3226.88556 0.014469793
## V     563.14975  5.98168995 59.547626  3790.03531 0.013961718
## GOOGL 512.29029  5.19255268 64.740178  4302.32560 0.013551470
## META  166.73245  1.58611414 66.326293  4469.05804 0.013415276
## CRM   317.82132  2.98818764 69.314480  4786.87936 0.013172879
## SBUX  259.15091  2.29107491 71.605555  5046.03027 0.012969532
## NKE   318.34837  2.68299208 74.288547  5364.37865 0.012721932
## JPM   452.63338  3.66200340 77.950551  5817.01203 0.012388754
## MU    195.77901  1.34300802 79.293559  6012.79104 0.012221911
## AXP   420.32820  2.39559773 81.689156  6433.11925 0.011825044
## GS    599.86239  2.83181891 84.520975  7032.98163 0.011257440
## BABA   68.57777  0.24452410 84.765499  7101.55941 0.011187819
## BMY    64.03624  0.14846975 84.913969  7165.59565 0.011113485
## C     558.23365  0.61868924 85.532658  7723.82930 0.010432265
## DIS   456.71882  0.05688478 85.589543  8180.54812 0.009888369
## BIDU   46.46654 -0.05121703 85.538326  8227.01466 0.009829683
## GILD   71.00095 -0.28640602 85.251920  8298.01561 0.009717484
## CVS   101.83191 -0.51203243 84.739888  8399.84752 0.009548289
```

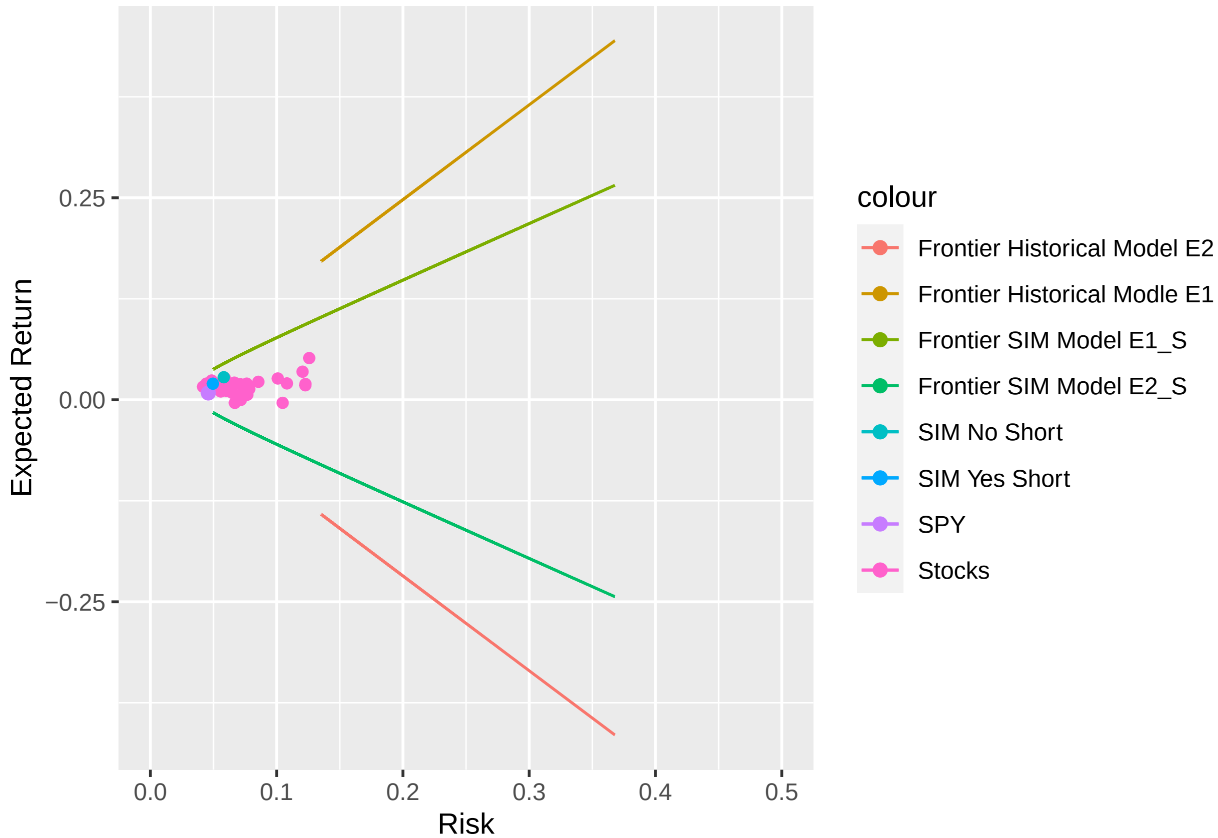
```r
x_market <- mean(returns$X.GSPC)
sd_market <- sd(returns$X.GSPC)
sd_stock <- sqrt(diag(sigma_mat))

g <- ggplot() + geom_line(aes(x = sigmas, y = e1, color = 'Frontier Historical Modle E1')) +
geom_line(aes(x = sigmas, y = e2, color = 'Frontier Historical Model E2')) +
geom_line(aes(x = sigmas_S, y = e1_S, color = 'Frontier SIM Model E1_S')) +
geom_line(aes(x = sigmas_S, y = e2_S, color = 'Frontier SIM Model E2_S')) +
geom_point(aes(x = sd_stock, y = r, color = 'Stocks')) +
geom_point(aes(x = sd_market, y = x_market, color = 'SPY'), size = 2) +
xlab('Risk') +
```

```
ylab('Expected Return') +
xlim(0, 0.5)

g
```




```
sort_data <- sort_data %>% mutate(zi_short = (beta / sigma) * (excess_beta))  %>%  mutate(x_short = zi_s

r_short <- sum(sort_data$R_i * sort_data$x_short)
total_2 <- sort_data %>% arrange(stock_n)
sd_short <- sqrt(as.numeric(t(total_2$x_short) %*% total %*% total_2$x_short))

sort_data_2 <- sort_data %>% filter(excess_beta > C_i)
C_star_short <- sort_data_2$C_i[nrow(sort_data_2)]

sort_data_2 <- sort_data_2 %>% mutate(z_i = (beta/sigma) * (excess_beta - C_star_short)) %>% mutate(x_i

stocks_2 <- rownames(sort_data_2)
Ri_2 <- r[stocks_2,]
vcov_2 <- total[stocks_2, stocks_2]
r_xshort <- sum(Ri_2 * sort_data_2$x_i)
sd_xshort <- sqrt(as.numeric(t(sort_data_2$x_i) %*% vcov_2 %*% sort_data_2$x_i))


g <- g +
  geom_point(aes(x = sd_short, y = r_short, color = 'SIM Yes Short')) +
```

```r
  geom_point(aes(x = sd_xshort, y = r_xshort, color = 'SIM No Short'))
g
```




```r
total_2$x_short
```

```
##          AAPL          CRM          NVDA            MU          MSFT
##  0.0579864050  0.0287342296  0.0463041611  0.0116285736  0.1133093530
##           TSM           DIS         GOOGL          NFLX          BIDU
##  0.0330025852  0.0005394235  0.0567871918  0.0207688638 -0.0007496513
##          META          ATVI           UNH           TMO           CVS
##  0.0174623425  0.0260252891  0.0647422858  0.0710044505 -0.0096109044
##           BMY           NVO          GILD           JPM             V
##  0.0034808591  0.0401917326 -0.0062022555  0.0407735708  0.0751823310
##            GS           AXP             C            MA          TSLA
##  0.0252545981  0.0256420561  0.0048547655  0.0714762930  0.0173783601
##          BABA           NKE          SBUX          LULU           MCD
##  0.0032059285  0.0322126925  0.0323199077  0.0249326821  0.0713618794
```

```r
sort_data_2$x_i
```

```
##        ATVI        NVDA         NVO        TSLA         UNH        MSFT
## 0.134900195 0.233816290 0.152476733 0.053237192 0.153633102 0.252725596
##        NFLX        LULU
## 0.010929097 0.008281794
```

```r
R_f <- 0.005
n <- 30

return_c <- returns[,2:31]
corr_mat <- cor(return_c)

rho = (sum(corr_mat) - n)/(n * (n-1))
sigma_i <- apply(returns,2,sd)[2:31]

results_c <- data_new %>% mutate(sigma_i = sigma_i) %>% mutate(return_sd = (R_i - R_f)/sigma_i) %>% arra
mutate(cum_excess = cumsum(return_sd)) %>% mutate(C_i = rho * cum_excess)

results_c
```

```
##               alpha      beta       sigma         R_i stock_n    sigma_i
## NVDA   0.0371238778 1.7397344 0.012389815 0.051823028       3 0.13650511
## MSFT   0.0152866567 0.9895580 0.002016417 0.023647509       5 0.06369482
## UNH    0.0124201740 0.6944230 0.002514650 0.018287408      13 0.05919350
## TMO    0.0104584288 0.8682832 0.002207838 0.017794620      14 0.06141800
## TSLA   0.0274545269 1.8364717 0.026771258 0.042971018      25 0.18328967
## AAPL   0.0115784779 1.2743952 0.003665194 0.022345941       1 0.08394238
## MCD    0.0093128349 0.6509412 0.001684795 0.014812687      30 0.05059074
## MA     0.0089344267 1.0967111 0.002262860 0.018200623      24 0.06907630
## NVO    0.0123936223 0.5223988 0.003599509 0.016807410      17 0.06431868
## TSM    0.0115468177 1.1173894 0.005935593 0.020987727       6 0.09221055
## V      0.0071181029 0.9748402 0.001687497 0.015354603      20 0.06058579
## LULU   0.0133726178 1.1580621 0.008922870 0.023157173      29 0.10795413
## NFLX   0.0146730658 1.2890893 0.012132061 0.025564681       9 0.12451987
## ATVI   0.0143934518 0.4937844 0.006386527 0.018565474      12 0.08266781
## GOOGL  0.0068899061 1.1203555 0.002450166 0.016355876       8 0.07118472
## CRM    0.0062143302 1.2741884 0.005108393 0.016980046       2 0.09205180
## JPM    0.0046053430 1.1004374 0.002675371 0.013903023      19 0.07209042
## NKE    0.0049783369 1.0205110 0.003271393 0.013600713      27 0.07368884
## SBUX   0.0053401422 0.8685494 0.002910960 0.012678583      28 0.06685071
## META   0.0061839656 1.1129029 0.007428385 0.015586968      11 0.09979541
## AXP    0.0018524266 1.1446860 0.003117340 0.011523967      22 0.07644327
## MU     0.0027510870 1.4150686 0.010227956 0.014707110       4 0.11974087
## GS    -0.0001222496 1.3738849 0.003146655 0.011485810      21 0.08418897
## BABA   0.0004362885 0.9345299 0.012735118 0.008332203      26 0.12017629
## C     -0.0064623005 1.5614556 0.004367604 0.006730558      23 0.09723785
## BMY    0.0017961195 0.5226090 0.004265087 0.006211683      16 0.06925368
## DIS   -0.0057560070 1.2920864 0.003655394 0.005160931       7 0.08445239
## BIDU  -0.0029954602 0.8371060 0.015080668 0.004077313      10 0.12806390
## GILD  -0.0020627475 0.5657933 0.004508701 0.002717684      18 0.07166673
## CVS   -0.0037975366 0.6527675 0.004184400 0.001717747      15 0.07098078
##          return_sd rank        rho cum_excess       C_i
## NVDA   0.343013001    1 0.32074382  0.3430130 0.1100193
## MSFT   0.292763342    2 0.24285090  0.6357763 0.1543989
## UNH    0.224474081    3 0.19539826  0.8602504 0.1680914
## TMO    0.208320366    4 0.16345871  1.0685708 0.1746672
## TSLA   0.207163981    5 0.14049378  1.2757348 0.1792328
## AAPL   0.206641049    6 0.12318680  1.4823758 0.1826091
## MCD    0.193962126    7 0.10967615  1.6763379 0.1838543
```

```
## MA     0.191102046    8 0.09883617  1.8674400 0.1845706
## NVO    0.183576677    9 0.08994623  2.0510167 0.1844812
## TSM    0.173382830   10 0.08252355  2.2243995 0.1835653
## V      0.170908110   11 0.07623257  2.3953076 0.1826005
## LULU   0.168193408   12 0.07083280  2.5635010 0.1815800
## NFLX   0.165151799   13 0.06614740  2.7286528 0.1804933
## ATVI   0.164096207   14 0.06204339  2.8927490 0.1794759
## GOOGL  0.159526870   15 0.05841888  3.0522759 0.1783105
## CRM    0.130144620   16 0.05519448  3.1824205 0.1756521
## JPM    0.123497995   17 0.05230740  3.3059185 0.1729240
## NKE    0.116716631   18 0.04970734  3.4226351 0.1701301
## SBUX   0.114861642   19 0.04735352  3.5374968 0.1675129
## META   0.106086716   20 0.04521255  3.6435835 0.1647357
## AXP    0.085343899   21 0.04325680  3.7289274 0.1613015
## MU     0.081067647   22 0.04146323  3.8099950 0.1579747
## GS     0.077038711   23 0.03981248  3.8870338 0.1547524
## BABA   0.027727627   24 0.03828813  3.9147614 0.1498889
## C      0.017797167   25 0.03687621  3.9325585 0.1450179
## BMY    0.017496298   26 0.03556472  3.9500548 0.1404826
## DIS    0.001905578   27 0.03434331  3.9519604 0.1357234
## BIDU  -0.007204898   28 0.03320301  3.9447555 0.1309778
## GILD  -0.031846243   29 0.03213600  3.9129093 0.1257452
## CVS   -0.046241439   30 0.03113543  3.8666678 0.1203904
```