

International Journal of Geomechanics
Numerical integration scheme for coupled elastoplastic-viscoplastic constitutive law for tunnels
--Manuscript Draft--

Manuscript Number:	GMENG-7493R2
Full Title:	Numerical integration scheme for coupled elastoplastic-viscoplastic constitutive law for tunnels
Manuscript Region of Origin:	BRAZIL
Article Type:	Technical Paper
Manuscript Classifications:	Elasticity and Inelasticity; Nonlinear analysis; Rock mechanics; Tunnels; Underground structures
Funding Information:	
Abstract:	The paper presents an efficient numerical integration scheme for coupled elastoplasticity-viscoplasticity constitutive behavior with internal-state variables standing for irreversible processes. In most quasi-static structural analyses, the solution to boundary value problems involving materials that exhibit time-dependent constitutive behavior proceeds from the equations integration handled at two distinct levels. On the one hand, the first or local level refers to the numerical integration at each Gaussian point of the rate constitutive stress/strain relationships. For a given strain increment, the procedure of local integration is iterated for stresses and associated internal variables until convergence of the algorithm. On the other hand, the second or global level is related to structure equilibrium between internal and external forces achieved by the Newton-Raphson iterative scheme. A review of the elastoplastic and viscoplastic model will be shown, following the coupling between these models. Particular emphasis is given in this contribution to address the first level integration procedure, also referred to as algorithm for stress and internal variable update, considering a general elastoplastic-viscoplastic constitutive behavior. The formulation is described for semi-implicit Euler schemes. The efficacy of the numerical formulation is assessed by comparison with analytical and numerical solutions derived for deep tunnels in coupled elastoplasticity-viscoplasticity. Finally, a parametric analysis was performed to show the importance that this model can have, in the long-term convergence profile, against other models. For the considered flow surfaces, potential functions and properties, differences order of 23% to 52% were found in the long-term convergence.
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Suggested Reviewers:	Albert Giraud Paper: Effet du comportement différé sur le creusement des tunnels, Revue Française de Géotechnique 118(118), 2007 Giovanni Barla Paper: Time-dependent modelling of tunnels in squeezing conditions. International Journal of Geomechanics, 12(6), 2012. F.I. Shalabi Paper: FE analysis of time-dependent behaviour of tunnelling in squeezing ground using two different creep models. Tunnelling and Underground Space Technology, 30, 2013.

	20(3), 2005. Donatella Sterpi Paper: Visco-plastic behaviour around advancing tunnels in squeezing rock. Rock Mechanics and Rock Engineering, 42(2), 2009.
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1 **Numerical integration scheme for coupled elastoplastic-viscoplastic**
2 **constitutive law for tunnels**

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10 **ABSTRACT**

11 The paper presents an efficient numerical integration scheme for coupled elastoplasticity-
12 viscoplastiticy constitutive behavior with internal-state variables standing for irreversible processes.
13 In most quasi-static structural analyses, the solution to boundary value problems involving materials
14 that exhibit time-dependent constitutive behavior proceeds from the equations integration handled
15 at two distinct levels. On the one hand, the first or local level refers to the numerical integration at
16 each Gaussian point of the rate constitutive stress/strain relationships. For a given strain increment,
17 the procedure of local integration is iterated for stresses and associated internal variables until
18 convergence of the algorithm. On the other hand, the second or global level is related to structure
19 equilibrium between internal and external forces achieved by the Newton-Raphson iterative scheme.
20 A review of the elastoplastic and viscoplastic model will be shown, following the coupling between
21 these models. Particular emphasis is given in this contribution to address the first level integration
22 procedure, also referred to as algorithm for stress and internal variable update, considering a gen-
23 eral elastoplastic-viscoplastic constitutive behavior. The formulation is described for semi-implicit

24 Euler schemes. The efficacy of the numerical formulation is assessed by comparison with ana-
25 lytical and numerical solutions derived for deep tunnels in coupled elastoplasticity-viscoplasticity.
26 Finally, a parametric analysis was performed to show the importance that this model can have,
27 in the long-term convergence profile, against other models. For the considered flow surfaces,
28 potential functions and properties, differences order of 23% to 52% were found in the long-term
29 convergence.

30 **INTRODUCTION**

31 Deep tunnels are those whose deformation field, induced by excavation, does not significantly
32 reaches the surface. The field of strain and stresses around the cavity of deep tunnels depends on
33 several interrelated factors, such as the depth of the tunnel, the geometry of the cross section, the
34 anisotropy of stresses in situ, the heterogeneity of the rock mass, the coupling between the rock
35 mass and the lining during the construction of the tunnel and the mechanical behavior of the rock
36 mass and lining. In general, for both the rock mass and the lining, several developments in literature
37 of the rheological models are found whose parameters are adjusted based on samples tests.

38 An elastoplastic-viscoplastic constitutive law becomes important when the material behavior
39 can't be describe by the usual models like elastoplasticity or viscoplastic. For example, this problem
40 is characteristic of deep tunnels excavated in clay rock mass as described by [Rousset \(1988\)](#). In these
41 cases plastification around the rock mass, gradual closing of the tunnel section, extrusion of the
42 excavation face and overloading on the lining can develop over the construction time (short term), or
43 even months and years after the construction of the tunnel (long term), which can lead to excessive
44 deformations ([Barla et al. 2008](#)), entrapment of the machine ([Ramoni and Anagnostou 2010](#)) and
45 damage to the lining. In addition to the present work, elastoplastic-viscoplastic models applied to
46 the problem of deep tunnels can be found in: [Rousset \(1988\)](#), [Piepi \(1995\)](#), [Purwodihardjo and](#)
[Cambou \(2003\)](#), [Kleine \(2007\)](#), [Shafiq et al. \(2008\)](#), [Debernardi and Barla \(2009\)](#), [Souley et al.](#)
[\(2011\)](#), [Manh et al. \(2015\)](#). This work presents a numerical integration scheme for the general
47 elastoplastic-viscoplastic constitutive behavior. For that, a brief bibliographical review will be made
48 about each model separately and later its coupling. The validation of this model will be presented,
49
50

51 comparing its numerical solution with the analytical and numerical solutions obtained by Piepi
 52 (1995) of an excavated tunnel under axisymmetric conditions. Finally a parametric analysis was
 53 performed to show the importance that this model can have, in the long-term convergence profile,
 54 against other models.

55 **ELASTOPLASTIC CONSTITUTIVE MODEL**

56 For problems with isothermal evolution, quasi-static in small transformations, the elastoplastic
 57 constitutive model can be described through the decomposition of the total strain tensor, the flow
 58 surface, the plastic flow rule, the hardening-softening law and the conditions of loading-unloading.

59 **Decomposition of the total strain tensor**

60 Considering the hypothesis of small transformations (which includes the hypothesis of small
 61 strains) the total strain rate can be decomposed into an elastic and a plastic component:

$$62 \dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p , \quad (1)$$

63 and following constitutive relationship is used:

$$64 \dot{\boldsymbol{\sigma}} = \mathbf{D}^{ep} : \dot{\boldsymbol{\varepsilon}} = \mathbf{D} : \dot{\boldsymbol{\varepsilon}}^e = \mathbf{D} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p) , \quad (2)$$

65 where $\boldsymbol{\sigma}$ is the stress tensor, \mathbf{D} and \mathbf{D}^{ep} are fourth-order tensors representing the elastic and elasto-
 66 plastic modulus, respectively. Sign convention of positive stress in tension is adopted throughout
 67 the paper.

68 **Flow surface**

69 A phenomenological characteristic observed in elastoplastic materials is the existence of a limit
 70 within which the material behaves elastically. In isotropic materials, this domain is delimited by a
 71 hypersurface in the space of principal stresses $\partial\Gamma = \{\boldsymbol{\sigma} \mid f = 0\}$ where f is the flow function. This
 72 surface delimits the set of stresses that are elastoplastically admissible $\Gamma = \{\boldsymbol{\sigma} \mid f \leq 0\}$.

73 The flow function is commonly described as a function of the stress tensor invariants and the
 74 forces q associated with the internal variables α related to the hardening-softening phenomenon
 75 $f(\sigma, q) = f(I_1, J_2, \theta, q)$ with

$$\begin{aligned} I_1 &= \text{tr}(\sigma) = \sigma_{11} + \sigma_{22} + \sigma_{33} \\ J_2 &= \frac{1}{2}\text{tr}(s^2) = \frac{1}{6} [(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2] + \sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2, \\ J_3 &= \frac{1}{3}\text{tr}(s^3) = \det(s) = s_{11}s_{22}s_{33} - s_{11}\sigma_{23}^2 - s_{22}\sigma_{13}^2 - s_{33}\sigma_{12}^2 + 2\sigma_{12}\sigma_{23}\sigma_{13}, \\ \theta &= \frac{1}{3}\arcsin\left(\frac{-3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}\right), \quad -\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6}, \quad \text{and} \quad s = \sigma - \frac{p}{3}\mathbf{1}. \end{aligned} \quad (3)$$

77 In Eq. (3), I_1 is the first invariant of the stress tensor, J_2, J_3 are the second and the third invariant
 78 of the deviator tensor s and θ is the Lode's angle ([Chen and Han 1988](#)). When the flow function
 79 does not depend of I_1 , it is said that the plasticity is independent of pressure, being determined only
 80 by the state of stresses along the deviator plane. Several flow functions can be found in the literature
 81 as in [Chen and Han \(1988\)](#), [Souza Neto et al. \(2008\)](#) and [Zienkiewicz and Cormeau \(1974\)](#). Here
 82 is used, for example, the Drucker-Prager flow surface:

$$f(\sigma, q) = f(I_1, J_2, q) = \beta_1 I_1 + \beta_2 \sqrt{J_2} - q(\alpha), \quad (4)$$

84 where β_1, β_2 and $q(\alpha)$ are strength parameters related to the friction angle ϕ and cohesion c ,
 85 respectively. The later $c = c(\alpha)$ evolves with the internal parameter controlling hardening of the
 86 flow surface. The following expressions for the case of Drucker-Prager surface inscribed on the
 87 Mohr-Coulomb surface are ([Bernaud 1991](#)):

$$\beta_1 = \frac{(k-1)}{3}, \quad \beta_2 = \frac{(2k+1)}{\sqrt{3}}, \quad q(\alpha) = 2\sqrt{k} c(\alpha), \quad (5)$$

88 where $k = (1 + \sin \phi)/(1 - \sin \phi)$. The Lode angle does not appear because the Drucker-Prager
 89 surface does not depend on this angle in the deviator plane.

91 **Plastic flow rule**

92 The law of evolution of plastic deformation (known as plastic flow rule) is postulated as

$$93 \quad \dot{\boldsymbol{\varepsilon}}^p = \lambda \mathbf{g}_\sigma \quad \text{with} \quad \mathbf{g}_\sigma = \frac{\partial g}{\partial \boldsymbol{\sigma}}, \quad (6)$$

94 where λ is the positive plastic multiplier and \mathbf{g}_σ is the tensor that gives the direction of plastic flow
 95 through the gradient of a potential function g analogous to f . Like the flow function, the plastic
 96 potential is usually described using the invariants of the stress tensor and \mathbf{g}_σ can be determined
 97 using the chain rule. For example, if $g(I_1, \sqrt{J_2}, J_3, q)$ we have:

$$98 \quad \begin{aligned} \mathbf{g}_\sigma &= C_1 \mathbf{g}_1 + C_2 \mathbf{g}_2 + C_3 \mathbf{g}_3, \\ \mathbf{g}_1 &= \frac{\partial I_1}{\partial \boldsymbol{\sigma}} = \mathbf{1}, \quad \mathbf{g}_2 = \frac{\partial \sqrt{J_2}}{\partial \boldsymbol{\sigma}} = \frac{1}{2\sqrt{J_2}} \mathbf{s}, \quad \mathbf{g}_3 = \frac{\partial J_3}{\partial \boldsymbol{\sigma}} = \mathbf{s}^2 - \frac{2}{3} J_2 \mathbf{1}, \\ C_1 &= \frac{\partial g}{\partial I_1}, \quad C_2 = \frac{\partial g}{\partial \sqrt{J_2}} - \frac{\tan(3\theta)}{\sqrt{J_2}} \frac{\partial g}{\partial \theta}, \quad C_3 = -\frac{\sqrt{3}}{2 \cos(3\theta)} \frac{1}{J_2^{3/2}} \frac{\partial g}{\partial \theta} \end{aligned} \quad (7)$$

99 Expressions for \mathbf{g}_1 , \mathbf{g}_2 and \mathbf{g}_3 for given particular stress-strain states (plane strain, axisymmetry or
 100 3D) can be found in [Owen and Hinton \(1980\)](#). As can be seen in Eq. (7), the constants C_1 , C_2 and C_3
 101 are particularities of each type of potential function. In [Viladkar et al. \(1995\)](#) it is possible to obtain
 102 the value of these constants for several functions, such von-Mises, Tresca, Drucker-Prager, Mohr-
 103 Coulomb, Cap Models, etc. In the particular case of Drucker-Prager potential flow $g = \beta_1 I_1 + \beta_2 \sqrt{J_2}$
 104 corresponding to $C_1 = \beta_1$, $C_2 = \beta_2$, $C_3 = 0$. From the numerical viewpoint, the main advantage
 105 of using such a potential function lies in the fact it is a smooth function. Another advantage is that
 106 it can simulate the volume variation during the evolution of plastic deformations (dilation). This
 107 effect is commonly introduced through non-associated plasticity, adopting, instead of the friction
 108 angle a dilatancy angle $0 < \psi < \phi$ in the potential function g .

109 **Hardening-softening law**

110 The hardening-softening law characterizes the dependence of internal variables α during the
 111 evolution of plastic deformations. This law is postulated as follows:

$$112 \quad \dot{q} = \lambda h_q(\sigma, q) = -\lambda \frac{\partial h}{\partial q}, \quad (8)$$

113 where $h_q = -\partial h / \partial q$ is a gradient of a potential function h with respect to the associated thermodynamic forces q . As the flow function f is dependent on q , the plastic deformation will change the position and/or shape of the flow surface. In this paper, the isotropic associated hardening-softening law ($h = f$) is considered. The associated thermodynamic force is given exclusively by the portion that does not depend on the hydrostatic (I_1) and deviating (J_2) invariants. For Drucker-Prager surface, this force is controlled by the cohesive parameter, i.e. $q(\alpha) = 2\sqrt{k}c(\alpha)$, where the equivalent plastic deformation is the internal variable $\alpha = \bar{\varepsilon}^p$. For cohesion, the present approach considers a piecewise linear function to represent the hardening-softening of the rock mass, as shown in Eq. (9) and Fig. 1.

$$122 \quad c(\bar{\varepsilon}^p) = \begin{cases} c_i + \frac{(c_p - c_i)}{\bar{\varepsilon}_I^p} \bar{\varepsilon}^p & \text{for zone I} \\ c_p, & \text{for zone II} \\ c_p + \frac{(c_r - c_p)}{\bar{\varepsilon}_r^p - \bar{\varepsilon}_{II}^p} (\bar{\varepsilon}^p - \bar{\varepsilon}_{II}^p), & \text{for zone III} \\ c_r, & \text{for zone IV} \end{cases}, \quad (9)$$

123 where c_i , c_p and c_r is the initial, plateau or peak and residual cohesion, respectively. Using the
124 chain rule in Eq. (8) we obtain the following expression:

$$125 \quad \dot{q} = -\lambda \frac{\partial h}{\partial q} = -\lambda \frac{\partial f}{\partial q} = \lambda \left(-\frac{\partial f}{\partial q} \frac{\partial q}{\partial c} \frac{\partial c}{\partial \bar{\varepsilon}^p} \right) = \lambda \left(2\sqrt{k} \frac{\partial c}{\partial \bar{\varepsilon}^p} \right), \quad (10)$$

$$126 \quad \frac{\partial c}{\partial \bar{\varepsilon}^p} = \begin{cases} \frac{(c_p - c_i)}{\bar{\varepsilon}_I^p} & \text{for zone I} \\ 0, & \text{for zone II} \\ \frac{(c_r - c_p)}{\bar{\varepsilon}_r^p - \bar{\varepsilon}_{II}^p}, & \text{for zone III} \\ 0, & \text{for zone IV} \end{cases}. \quad (11)$$

¹²⁸ The equivalent plastic deformation is calculated from the following expression:

¹²⁹ $\dot{\bar{\epsilon}}^p = C||\dot{\epsilon}^p||,$ (12)

¹³⁰ where

¹³¹ $||\dot{\epsilon}^p|| = \sqrt{(\dot{\epsilon}_{11}^p)^2 + (\dot{\epsilon}_{22}^p)^2 + (\dot{\epsilon}_{33}^p)^2 + 2(\dot{\epsilon}_{12}^p)^2 + 2(\dot{\epsilon}_{23}^p)^2 + 2(\dot{\epsilon}_{13}^p)^2}$ (13)

¹³² and

¹³³ $C = \frac{\beta_1 + 1/\sqrt{3}}{\sqrt{3\beta_1^2 + 1/2}}.$ (14)

¹³⁴ When $\phi = 0$ the rock mass is independent of the hydrostatic pressure: $\beta_1 = 0$ and $C = \sqrt{2/3}.$

¹³⁵ Loading and unloading conditions

¹³⁶ The evolution of Eq. (6) and Eq. (8) are subject to three conditions (conditions of Kuhn-Tucker),
¹³⁷ which are:

¹³⁸ $f \leq 0, \quad \lambda \geq 0, \quad \lambda f = 0.$ (15)

¹³⁹ These conditions establish that plastic flow only occurs when the state of stresses is on the flow
¹⁴⁰ surface (know as consistency condition) and, in this case, there is no variation of the flow function
¹⁴¹ in relation to the stresses, that is:

¹⁴² $\dot{f} = \frac{\partial f}{\partial \sigma} : \dot{\sigma} + \frac{\partial f}{\partial q} \cdot \dot{q} = f_\sigma : \dot{\sigma} + f_q \cdot \dot{q} = 0.$ (16)

¹⁴³ Plastic multiplier and elastoplastic module

¹⁴⁴ Placing Eq. (2), Eq. (8) and Eq. (6) into Eq. (16) the follow plastic multiplier writes:

¹⁴⁵ $\lambda = \frac{f_\sigma : \mathbf{D} : \dot{\epsilon}}{f_\sigma : \mathbf{D} : \mathbf{g}_\sigma - f_q \cdot h_q}.$ (17)

146 Introducing Eq. (17) in the constitutive relation (Eq. (2)) leads to:

$$\small \mathbf{D}^{ep} = \mathbf{D} - \frac{(\mathbf{D} : \mathbf{g}_\sigma) \otimes (\mathbf{f}_\sigma : \mathbf{D})}{f_\sigma : \mathbf{D} : \mathbf{g}_\sigma - f_q \cdot h_q} \quad (18)$$

148 where \otimes is the tensorial product.

149 VISCOPLASTIC CONSTITUTIVE MODEL

150 Decomposition of the strain tensor

151 The viscoplastic constitutive model has a rationale similar to that of elastoplasticity, which leads
152 to the following relationship:

$$\small \dot{\boldsymbol{\sigma}} = \mathbf{D}^{vp} : \dot{\boldsymbol{\epsilon}} = \mathbf{D} : \dot{\boldsymbol{\epsilon}}^e = \mathbf{D} : (\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{vp}) \quad (19)$$

154 where \mathbf{D}^{vp} is the viscoplastic fourth order tensor.

155 Flow surface

156 Viscoplasticity does not always have an elastic domain, for example, at high temperatures
157 certain materials can always flow under stress, that is, the flow function is zero. For these materials
158 there are explicit functions. However, in problems involving deep tunnels, the phenomenon occurs
159 from a certain level of stress, as described by [Rousset \(1988\)](#). For these cases, surfaces f^{vp}
160 similar to those of elastoplasticity are used. Here is used the Drucker-Prager viscoplastic flow
161 surface similar to Eq. (4), however, with ϕ^{vp} in β_1 and β_2 , and $q^{vp} = 2\sqrt{k^{vp}} - c^{vp}$ where $k^{vp} =$
162 $(1 + \sin \phi^{vp})/(1 - \sin \phi^{vp})$ and c^{vp} is a constant parameter.

163 Viscoplastic flow rule and hardening-softening law

164 Analogous to elastoplasticity, the viscoplastic flow rule and the hardening-softening law are
165 postulated as follows:

$$\begin{aligned} \dot{\boldsymbol{\epsilon}}^{vp} &= \dot{\lambda}^{vp} \mathbf{g}_\sigma^{vp} \quad \text{with} \quad \mathbf{g}_\sigma = \frac{\partial g^{vp}}{\partial \boldsymbol{\sigma}}, \\ \dot{q}^{vp} &= \dot{\lambda}^{vp} h_q(\boldsymbol{\sigma}, q^{vp}) = -\dot{\lambda}^{vp} \frac{\partial h^{vp}}{\partial q^{vp}} \quad \text{and} \quad \bar{\boldsymbol{\epsilon}}^{vp} = \int_0^t \dot{\lambda}^{vp} dt. \end{aligned} \quad (20)$$

167 However, in this work , the viscoplasticity is perfect, that is, $\dot{q}^{vp} = 0$.

168 **Viscoplastic multiplier**

169 Unlike elastoplasticity, viscoplastic deformation occur even when $f^{vp} > 0$, and therefore, the
170 consistency condition is not imposed. Thus, the rate of the viscoplastic multiplier λ^{vp} cannot be
171 obtained from a condition like $\dot{f}^{vp} = 0$. Therefore, there are models that provide an explicit
172 expression for λ^{vp} and in this paper Perzyna model ([Perzyna 1966](#)) will be adopted, as described
173 in [Zienkiewicz and Cormeau \(1974\)](#):

174
$$\lambda^{vp} = \frac{\Phi(\sigma, q^{vp})}{\eta} \quad \text{and} \quad \Phi = \left\langle \frac{f^{vp}(\sigma, q^{vp})}{f_0} \right\rangle^n \quad (21)$$

175 where Φ is the overstress function, η is the dynamic viscosity constant, n is the dimensionless
176 parameter that gives the form of the power law , f_0 a parameter conveniently adopted and $\langle * \rangle$ is
177 the McCauley function which is null when $* < 0$, i.e. viscoplastic flow will only occur when the
178 overstress function is positive.

179 **ELASTOPLASTIC-VISCOPLASTIC CONSTITUTIVE MODEL**

180 The proposed elastoplastic-viscoplastic model is formulated from the serial association of the
181 constitutive models described in the preceding sections, i.e. $\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^p + \dot{\epsilon}^{vp}$, which leads to the
182 following constitutive relationship:

183
$$\dot{\sigma} = D^{epvp} : \dot{\epsilon} = D : \dot{\epsilon}^e = D : (\dot{\epsilon} - \dot{\epsilon}^p - \dot{\epsilon}^{vp}) . \quad (22)$$

184 This association can be seen in the one-dimensional representation of Fig. 2. An important
185 observation is that the flow surfaces and the internal variables that define the elastoplastic and
186 viscoplastic components of this model can be different from each other, including the association
187 of their respective potential functions with their flow surfaces.

188 An interesting aspect of the coupled model, using the Drucker-Prager criteria f and f^{vp} for
189 plasticity and viscoplasticity with $\phi = \phi^{vp}$, is that the evolution of local mechanical fields are

entirely controlled by the cohesion. In particular, when $c \rightarrow \infty$ and $c^{vp} \rightarrow \infty$ the solution is purely elastic. Besides, the purely elastoviscoplastic solution is retrieved when $c \rightarrow \infty$, whereas purely elastoplastic solution is obtained when $c^{vp} \rightarrow \infty$.

ALGORITHM FOR UPDATING THE STRESS AND INTERNAL VARIABLES

The algorithms for updating stress and internal variables propose to solve the system of differential equations involving the constitutive relations through some integration scheme (generally Runge-Kutta). The algorithm occurs for each Gauss point of each element during the global equilibrium iterations and, given a known set of $\{\boldsymbol{\epsilon}_n, \boldsymbol{\epsilon}_n^{in}, \boldsymbol{\sigma}_n, q_n\}$ in the substep n and the increment of total strain $\Delta\boldsymbol{\epsilon}$ the values of the next substep $\{\boldsymbol{\epsilon}_{n+1}, \boldsymbol{\epsilon}_{n+1}^{in}, \boldsymbol{\sigma}_{n+1}, q_{n+1}\}$ where $\boldsymbol{\epsilon}_{n+1}^{in}$ is the inelastic strain (plastic or viscoplastic).

Integration of elastoplastic constitutive equations

Using the first order Runge-Kutta method, the following scheme of integration of the constitutive equations are:

$$\begin{cases} \boldsymbol{\epsilon}_{n+1} = \boldsymbol{\epsilon}_n + \Delta\boldsymbol{\epsilon} \\ \boldsymbol{\epsilon}_{n+1}^p = \boldsymbol{\epsilon}_n^p + [(1 - \Theta)\Delta\lambda_n \mathbf{g}_{\sigma_n} + \Theta\Delta\lambda_{n+1} \mathbf{g}_{\sigma_{n+1}}] \\ q_{n+1} = q_n + [(1 - \Theta)\Delta\lambda_n h_{q_n} + \Theta\Delta\lambda_{n+1} h_{q_{n+1}}] \\ \boldsymbol{\sigma}_{n+1} = \mathbf{D} \boldsymbol{\epsilon}_{n+1}^e = \mathbf{D}(\boldsymbol{\epsilon}_{n+1} - \boldsymbol{\epsilon}_{n+1}^p) \\ f_{n+1} = f(\boldsymbol{\sigma}_{n+1}, q_{n+1}) = 0 \end{cases} \quad (23)$$

where $\Delta\lambda = \dot{\lambda}\Delta t$ and $0 \leq \Theta \leq 1$ provides the generalized trapezoidal rule for plastic flow and the evolution of internal variables. When $\Theta = 0$ and $\Theta = 1$ the fully explicit and implicit form are obtained, respectively. Semi-implicit algorithms adopt $0 \leq \Theta < 1$ or a combination of implicit and explicit of the $\Delta\lambda$, \mathbf{g}_{σ} and h_q .

The fully explicit schemes did not satisfy the consistency condition $f_{n+1} = f(\boldsymbol{\sigma}_{n+1}, q_{n+1}) = 0$ at the end of the substep, since the plastic multiplier and the flow vectors were calculated with the stress of the previous substep n . Currently, fully implicit or semi-implicit algorithms that satisfy $f_{n+1} = 0$ are used and some semi-implicit ones avoid the need to calculate the second order gradients of flow vectors \mathbf{g}_{σ} and h_q , but need more equilibrium iterations in relation to the fully

213 implicit scheme. Several integration schemes for elastoplasticity can be found in Souza Neto et al.
 214 (2008), Belytschko et al. (2000) and Simo and Hughes (1998). In this work, we use a semi-implicit
 215 two-step integration scheme, presented in Belytschko et al. (2000), in which the plastic multiplier
 216 is integrated through an implicit scheme and the flow vectors are explicitly integrated.

217 The integration schemes have two steps: first, the elastic predictor is calculated and, if necessary,
 218 the plastic corrector. Defining $\Delta\boldsymbol{\epsilon}^p = \boldsymbol{\epsilon}_{n+1}^p - \boldsymbol{\epsilon}_n^p$, the elastic predictor can be explained from Eq. (23)₄
 219 as:

$$220 \quad \boldsymbol{\sigma}_{n+1} = \mathbf{D}(\boldsymbol{\epsilon}_{n+1} - \boldsymbol{\epsilon}_{n+1}^p) = \mathbf{D}(\boldsymbol{\epsilon}_n + \Delta\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_n^p - \Delta\boldsymbol{\epsilon}^p) = \boldsymbol{\sigma}_{n+1}^{trial} + \Delta\boldsymbol{\sigma}_{n+1} \quad (24)$$

221 where $\boldsymbol{\sigma}_{n+1}^{trial} = \mathbf{D}(\boldsymbol{\epsilon}_n + \Delta\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_n^p)$ is the elastic predictor (also known as trial stress). Thus, in the
 222 first step, the trial stress is calculated and the correspondent flow function f is tested. If $f < 0$ the
 223 stress state is in the elastic domain and there is no need to apply the plastic corrector. However, if
 224 $f > 0$ the stress state is outside the plastically admissible domain and it is necessary to apply the
 225 plastic corrector step.

226 The plastic corrector is the system solution procedure of the Eq. (23)_{2,3,5} which will determine
 227 the increments $\Delta\boldsymbol{\sigma}_{n+1}$ and $\Delta\boldsymbol{q}_{n+1}$. When it is not possible to obtain an analytical solution for this
 228 system, the Newton-Raphson solution procedure is used, which iterates k times through the space
 229 of the stresses and internal variables until the stress state returns on the flow surface. This is why
 230 these schemes are also known as return mapping algorithms. Fig. 3 geometrically illustrates this
 231 solution.

232 Therefore, to solve the plastic corrector using the Newton-Raphson scheme, the Eq. (23)_{2,3,5}
 233 are written as (omitting the index $n + 1$):

$$234 \quad \begin{cases} \mathbf{a} = -\boldsymbol{\epsilon}^p + \boldsymbol{\epsilon}_n^p + \Delta\lambda\mathbf{g}_{\boldsymbol{\sigma}_n} = \mathbf{0} \\ \mathbf{b} = -\boldsymbol{q} + \boldsymbol{q}_n + \Delta\lambda\mathbf{h}_{\boldsymbol{q}_n} = \mathbf{0} \\ f = f(\boldsymbol{\sigma}, \boldsymbol{q}) = 0 \end{cases} \quad . \quad (25)$$

235 Linearizing the Eq. (25) in relation to $\Delta\lambda$, with $\Delta\boldsymbol{\epsilon}^p = -\mathbf{D}^{-1}\Delta\boldsymbol{\sigma}$, we obtain:

$$\begin{aligned}
& \left\{ \begin{array}{l} \mathbf{a}_k + \mathbf{D}^{-1} \Delta \boldsymbol{\sigma}_k + \delta \lambda_k \mathbf{g}_{\sigma_n} = \mathbf{0} \\ \mathbf{b}_k - \Delta q_k + \delta \lambda_k h_{q_n} = \mathbf{0} \\ f_k + \mathbf{f}_{\sigma_k}^T \Delta \boldsymbol{\sigma}_k + f_{q_k}^T \Delta q_k = 0 \end{array} \right. . \quad (26)
\end{aligned}$$

Eq. (26) is a system of three equations with three unknowns: $\Delta \boldsymbol{\sigma}_k$, Δq_k and $\delta \lambda_k$ and as the flow vectors \mathbf{g}_σ and h_q are calculated in the initial step n , their gradients did not appear in the formulations. Reorganizing this system we obtain the following solution for the plastic corrector:

$$\begin{aligned}
& \left\{ \begin{array}{l} \Delta \boldsymbol{\sigma}_k \\ \Delta q_k \end{array} \right\} = -\delta \lambda_k [\mathbf{A}] \left\{ \begin{array}{l} \mathbf{g}_{\sigma_n} \\ h_{q_n} \end{array} \right\}, \quad (27)
\end{aligned}$$

$$[\mathbf{A}] = \begin{bmatrix} \mathbf{D} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{bmatrix}, \quad (28)$$

$$\delta \lambda_k = \frac{f_k}{[\mathbf{f}_{\sigma_k}^T \quad f_{q_k}^T] [\mathbf{A}] \left\{ \begin{array}{l} \mathbf{g}_{\sigma_n} \\ h_{q_n} \end{array} \right\}}. \quad (29)$$

Due to this explicit treatment of the flow vectors in Eq. (28) $[\mathbf{A}]$ presents a closed expression involving only the elastic constitutive modulus. In addition, as the system Eq. (26) is composed of linear functions in relation to $\Delta \lambda$ the residuals \mathbf{a}_k and \mathbf{b}_k will automatically be null, dispensing its verification in the convergence criterion, as given by [Belytschko et al. \(2000\)](#).

After updating the stresses and internal variables, it is possible to update the constitutive module. It is possible to use the tangent modulus consistent with the linearization made during the algorithm for integrating the constitutive laws (that is why this modulus is also known as the algorithmic modulus). Its general expression is defined by:

$$\mathbf{D}^{alg} = \left(\frac{d\boldsymbol{\sigma}}{d\boldsymbol{\varepsilon}} \right)_{n+1}. \quad (30)$$

To derive the expression \mathbf{D}^{alg} we resolve the system of Eq. (23). In elastoplasticity, the following

255 relationship are (omitting the $n + 1$ index) (Belytschko et al. 2000):

$$256 \quad D^{alg} = D - \frac{D g_{\sigma_n} f_{\sigma}^T D}{f_{\sigma}^T D g_{\sigma_n} - f_q h_{q_n}} \quad (31)$$

257 In non-associated plasticity, the constitutive modulus matrix is not symmetric and its update
 258 leads to a non-symmetric global stiffness matrix, thus requiring a non-symmetric solver for the
 259 global equilibrium iterations. Symmetrization techniques of the constitutive stiffness matrix, such
 260 that proposed in Pande and Pietruszczak (1986) or in Deb et al. (2013), can be used. It is observed
 261 that the algorithm converges even not updating the constitutive modulus. Although it is optional,
 262 the calculations performed in this paper make use of this update.

263 Integration of viscoplastic constitutive equations

264 Different integration algorithms can be found in the literature and some of the most used are
 265 in Belytschko et al. (2000), Souza Neto et al. (2008), Huang and Griffiths (2009) and Smith et al.
 266 (2014). In the present work, a scheme introduced by Pierce et al. (1984), known as the Rate Tangent
 267 Modulus Method, which used an explicit Euler scheme for all variables, excepted for $\Delta\lambda^{vp}$ that is
 268 integrated according to the generalized trapezoidal rule. So the following scheme is deduced:

$$269 \quad \left\{ \begin{array}{l} \boldsymbol{\varepsilon}_{n+1} = \boldsymbol{\varepsilon}_n + \Delta\boldsymbol{\varepsilon} \\ \boldsymbol{\varepsilon}_{n+1}^{vp} = \boldsymbol{\varepsilon}_n^{vp} + \Delta\lambda^{vp} \mathbf{g}_{\sigma_n}^{vp} \\ q_{n+1}^{vp} = q_n^{vp} + \Delta\lambda^{vp} h_{q_n}^{vp} \\ \boldsymbol{\sigma}_{n+1} = \mathbf{D} \boldsymbol{\varepsilon}_{n+1}^e = \mathbf{D}(\boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}_{n+1}^{vp}) \\ \Delta\lambda = \frac{\Delta t}{\eta} [(1 - \Theta)\Phi_n + \Theta\Phi_{n+1}] \end{array} \right. . \quad (32)$$

270 Linearizing the overstress of Eq. (21) the follow expression is obtained:

$$271 \quad \Phi_{n+1} = \Phi_n + \boldsymbol{\Phi}_{\sigma_n}^T \Delta\boldsymbol{\sigma} + \boldsymbol{\Phi}_{q_n}^T \Delta q^{vp} \quad (33)$$

272 where $\boldsymbol{\Phi}_{\sigma} = \partial\Phi/\partial\boldsymbol{\sigma}$ and $\boldsymbol{\Phi}_q = \partial\Phi/\partial q$. Replacing Eq. (33) into Eq. (32)₅ we get:

273

$$\Delta\lambda^{vp} = \frac{\Delta t}{\eta}\Phi_n + \frac{\theta\Delta t}{\eta}(\Phi_{\sigma n}^T\Delta\sigma + \Phi_{q n}^T\Delta q). \quad (34)$$

274 And placing Eq. (32)₂ into Eq. (32)₄ and rewriting Eq. (32)₃ we obtain:

275

$$\begin{Bmatrix} \Delta\sigma \\ \Delta q^{vp} \end{Bmatrix} = \begin{Bmatrix} D(\Delta\varepsilon - \Delta\lambda^{vp}g_{\sigma n}^{vp}) \\ \Delta\lambda^{vp}h_{q n}^{vp} \end{Bmatrix}. \quad (35)$$

276 Finally, replacing (35) in (34) we can isolate $\Delta\lambda^{vp}$:

277

$$\Delta\lambda^{vp} = \frac{\Phi_n + \Theta\Phi_{\sigma n}^T D \Delta\varepsilon}{\frac{\eta}{\Delta t} + \Theta(\Phi_{\sigma n}^T D g_{\sigma n}^{vp} - \Phi_{q n}^T h_{q n}^{vp})}. \quad (36)$$

278 When $0 < \Theta < 1$ a semi-implicit algorithm is obtained and when $\Theta = 0$ a totally explicit
 279 algorithm, with $\Delta\lambda^{vp} = \Delta t \frac{\Phi_n}{\eta}$. As can be seen from this deduction, unlike the integration of the
 280 constitutive relationship in elastoplasticity, there is no need to solve the system iteratively. And,
 281 in addition, all variables are taken from the previous n substep. This fact, as will be seen in the
 282 next section, will facilitate the coupling between the viscoplasticity and elastoplasticity algorithm.
 283 Furthermore, in this work, for viscoplasticity, hardening-softening laws are not used, simplifying
 284 the expression (35) to:

285

$$\Delta\lambda^{vp} = \frac{\Phi_n + \Theta\Phi_{\sigma n}^T D \Delta\varepsilon_n}{\frac{\eta}{\Delta t} + \Theta\Phi_{\sigma n}^T D g_{\sigma n}^{vp}}. \quad (37)$$

286 Substituting (37) into (35)₁ scheme gives a closed expression for the stress update:

287

$$\Delta\sigma = D^{alg}\Delta\varepsilon - p, \quad (38)$$

288 with

289

$$D^{alg} = D - \frac{\Theta D g_{\sigma n}^{vp} \Phi_{\sigma n}^T D}{\eta/\Delta t + \Theta\Phi_{\sigma n}^T D g_{\sigma n}^{vp}}, \quad p = \frac{\Phi_n D g_{\sigma n}^{vp}}{\eta/\Delta t + \Theta\Phi_{\sigma n}^T D g_{\sigma n}^{vp}} \quad (39)$$

290 where D^{alg} is the algorithmic constitutive modulus and p a pseudo-stress that does not depend on

291 the increment of total strain.

292 The integration scheme represented by (32)₅ is unconditionally stable for a value of $\Theta \geq 1/2$.
293 However, this does not guarantee the accuracy of the solution. Thus, as for values $\Theta < 1/2$, a
294 limit value must be used for the time increment $\Delta t \leq \Delta t_{\text{lim}}$. This limit can generally be achieved
295 by reducing the time increment until the solution does not change. Strictly it depends on the
296 material parameters, the integration scheme, the flow surface and the flow rule and there are some
297 analytical solutions for classical surfaces. To avoid this precision problem, in this work, the limits
298 of Drucker-Prager surface was used ([Cormeau 1975](#)):

$$299 \quad \Delta t_{\text{lim}} \leq \begin{cases} \frac{4}{3} \frac{\eta}{\Phi} \frac{(1+\nu)}{E} \sqrt{3J_2} \\ \frac{\eta f_0}{\Phi'} \frac{(1+\nu)(1-2\nu)}{E} \frac{(3-\sin\phi)^2}{\frac{3}{4}(1-2\nu)(3-\sin\phi)^2 + 6(1+\nu)\sin\phi^2} \end{cases} \quad (40)$$

300 where $\Phi' = \frac{d\Phi}{d(f/f_0)} = n(f/f_0)^{n-1}$. In the case of the von-Mises surface $\phi = 0$. This limit is
301 deduced considering the associated viscoplasticity and a fully explicit integration scheme.

302 **Integration of elastoplastic-viscoplastic constitutive equations**

303 As viscoplastic strains are integrated using a semi-implicit rule in which all variables are
304 calculated with known stresses (from substep n), the viscoplastic strain increment is computed
305 first. Subsequently, it is deducted from the total strain increment evaluated by elastic predictor in
306 the elastoplasticity algorithm. This is actually the correct sequence for computation modeling of
307 the coupling, since elastoplasticity algorithm update the stress state for substep $n+1$ in order to
308 verify the criterion $f_{n+1} = 0$. If the plastic strain were calculated before the viscoplastic strain,
309 the criterion value would be $f_{n+1} \neq 0$. The algorithm for integrating the elastoplastic-viscoplastic
310 constitutive equations can be seen in the flowchart of Fig. 4. The implementation in the ANSYS
311 USERMAT subroutine in FORTRAN77 can be found in [Quevedo \(2021\)](#).

312 In elastoplastic analyses, time is a kinematical parameter that marks the load history and con-
313 trol the incremental solution, but it does not influence the constitutive relationships. In coupled
314 elastoplastic-viscoplastic analyses, the elastoplastic component of constitutive relationships be-

315 comes time-dependent since it explicitly depends on the viscoplasticity behavior. In particular, the
316 stress corrections in the elastoplastic step of the solution procedure shall a priori depend on Δt

317 **VALIDATION OF THE MODEL**

318 Before proceeding to the model validation, it is emphasized that the momentum balance equa-
319 tions in quasi-static conditions read in incremental form $\text{div}\Delta\sigma + \Delta F = \mathbf{0}$, where $\Delta\sigma$ and ΔF are
320 respectively the stress increment and the increment of external load. In the case of tunnel simulation
321 considered in the sequel, the increment of external load is time dependent since it is associated with
322 the process of ground excavation and tunnel advancement (rock elements are removed).

323 As validation, a comparison will be made with the analytical and numerical solutions deduced
324 by (Piepi 1995) for a perfect elastoplastic-viscoplastic model with Tresca's criterion applied to deep
325 clay rock mass. This analytical solution was chosen because it uses the same association principle
326 as in Fig. 2. Furthermore, it is considered the same surface for plasticity and viscoplasticity, and
327 their flow vectors are fully associated $f^p = g^p = f^{vp} = g^{vp}$. For this analysis, the Tresca surface
328 was approximated through the Drucker-Prager surface without friction and dilatance angle.

329 The mesh (Fig. 5) comprises 1431 linear elements of four nodes, two degrees of freedom per
330 node and four integration points. The system size, after applying the boundary conditions is of
331 3024 equations. The excavation method consisted of the technique of deactivating the elements
332 to be excavated by multiplying the modulus of elasticity by 10^{-16} (eliminating its contribution to
333 the stiffness matrix) and making the stresses to zero in the Gauss points during the integration of
334 internal forces. The geometric parameters are in Table 1. The FEM model in ANSYS APDL script
335 can be found in Quevedo (2021).

336 The following constitutive parameters are used: $E = 1500\text{MPa}$ and 2000MPa , $\nu = 0.498$,
337 $c^i = c^p = c^r = 4\sqrt{3}/2\text{MPa}$, $c^{vp} = 3\sqrt{3}/2\text{MPa}$, $\eta = 4 \cdot 10^4\text{day}$, $n = 1$, $f_0 = 1\text{MPa}$ and $p_v = p_h = 9$
338 MPa. The viscous phenomenon evolves over time between excavation steps. The excavation
339 step time is calculated as the ratio between the step size of the excavation L_p and the excavation
340 speed $V_p = 10\text{m/day}$. After the last excavation the model continues incrementing the time until
341 the deformation increment is in the order of 10^{-8} . Note that the elastoplastic part has greater

cohesion than the viscoplastic part. In this way, viscoplastic deformations start even before the solid plasticize. The long-term convergence profile, that is, after the end of viscous effects, can be seen in Fig. 6. We can observe an excellent agreement between the numerical solution of this work and the analytical/numerical solutions given by Piepi (1995).

PARAMETRIC ANALYSIS, RESULTS AND DISCUSSIONS

In this section, a parametric analysis is presented to investigate the influence of this elastoplastic-viscoplastic model on the long-term convergence profile, with and without lining. The rock mass properties are the same as the previous analysis with $E = 1500\text{MPa}$. A variable lining modulus of $E_{rev} = 3000\text{MPa}$ to $E_{rev} = 30000\text{MPa}$ with $\nu_{rev} = 0.3$ and the unsupported distance of $d_0 = 0$ and $d_0 = 4L_p$ was adopted. Convergence at equilibrium is measured in $y/R_i = 6$. Fig. 7 shows this results with elastic (E), viscoplastic (VP) and elastoplastic (EP) behavior. The stiffness of the lining is calculated by the expression:

$$K_{rev} = \frac{E_{rev}}{(1 + \nu_{rev})} \frac{R_i^2 - (R_i - e_{rev})^2}{[(1 - 2\nu_{rev})R_i^2 + (R_i - e_{rev})^2]}. \quad (41)$$

When the lining stiffness is high, the model (VP) approaches the (EL). This is because the viscous deformations of the viscoplastic model are resisted by the lining and prevented to appear. In the other hand, the model (EPVP) approaches the model (EP). This is because during excavation and placement of the lining, some plastification occurs, but the viscous deformations of the model (EPVP) are resisted by the lining. Comparing the elastoplastic-viscoplastic model with the viscoplastic model, in the long term, without lining, the (EPVP) had a 52% higher convergence. With lining applied at $d_0 = 0$ a convergence of 23% greater appears, and for $d_0 = 4L_p$ it is about 31%. This is a big difference in relation to viscoplastic model that try to estimate the convergence in the long term, but do not consider the plastification of the rock mass in the instantaneous behavior.

The Fig. 8 shows the effect of using non-associative plastic flow on the convergence profile (at end of excavation and long-term) without lining. For the parameters used in the study, only a slight difference is observed between non-associated and associated plasticity. More parametric studies

367 are needed to assess this particular aspect of the constitutive behavior.

368 **CONCLUSIONS**

369 This work have presented a numerical integration scheme for the elastoplastic-viscoplastic
370 constitutive behavior. A brief review of each model was performed separately and then their
371 coupling. The numerical solution of the model was verified with the analytical and numerical
372 solution obtained by [Piepi \(1995\)](#) under axisymmetric conditions, demonstrating an excellent
373 agreement.

374 Finally, a parametric analysis was performed to show the importance of this model in the long-
375 term convergence profile comparing with anotheres models that don't consider the instantaneous
376 elastoplastic-viscoplastic behavior. For the considered properties, differences about of 23% to 52%
377 were founded.

378 **DATA AVAILABILITY STATEMENT**

379 Some or all data, models, or code that support the findings of this study are available from
380 the corresponding author upon reasonable request. (ANSYS APDL script for FEM model and
381 USERMAT subroutine in FORTRAN77 for constitutive rock mass model)

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⁴³⁸ 821–845.

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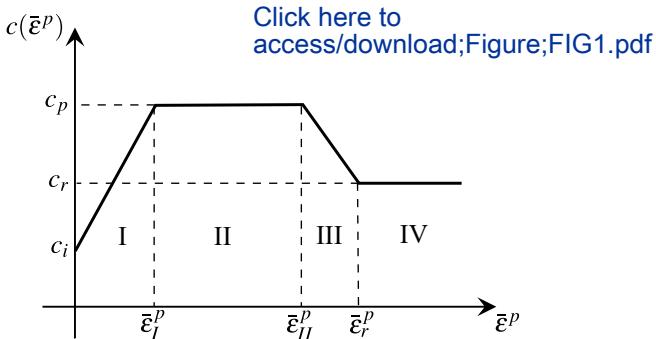
TABLE 1. Geometric parameters of the mesh

PARAMETERS	SYMBOL	UNIT	VALUES
GEOMETRIC DOMAIN			
Radius of interface between tunnel and rock mass	R_i	m	1
Lining thickness	e_{rev}	m	0.1
Radius of the region near the tunnel R_1	R_1	m	$10R_i$
Total domain lenght	L_x	m	$20R_i$
Length beyond R_1	L_{x_1}	m	$L_x - R_1$
EXCAVATION AND PLACEMENT OF LINING			
Number of excavation steps	n_p	un	38
Number of steps in first excavation	n_{p_i}	un	3
Length of the excavated part	Ly_1	m	$n_p L_p$
Length of unexcavated part	Ly_2	m	$25L_p$
Longitudinal length of domain	L_y	m	$Ly_1 + Ly_2$
Excavation step size	L_p	m	$1/3R_i$
Unsupported dimension	d_0	m	0, $2L_p$, $4L_p$
Lining face coordinate	y_r	m	$(i_p - 1)L_p + n_{p_i}L_p - (L_p + d_0)$
Excavation face coordinate	y_f	m	$(i_p - 1)L_p + n_{p_i}L_p$
DISCRETIZATION			
Elements along R_i	nR_i	un	5
Elements in lining thickness	n_{rev}	un	2
Elements along R_1	nR_1	un	15
First and last element ration of R_1	mR_1	adm	15
Elements along L_{x_1}	nL_{x_1}	un	5
First and last element ration of L_{x_1}	mL_{x_1}	adm	1.2
Elements in excavated step	nL_p	un	1
Number of elements along Ly_2	nL_{y_2}	un	15
First and last element ratio of Ly_2	mL_{y_2}	adm	5

441 **List of Figures**

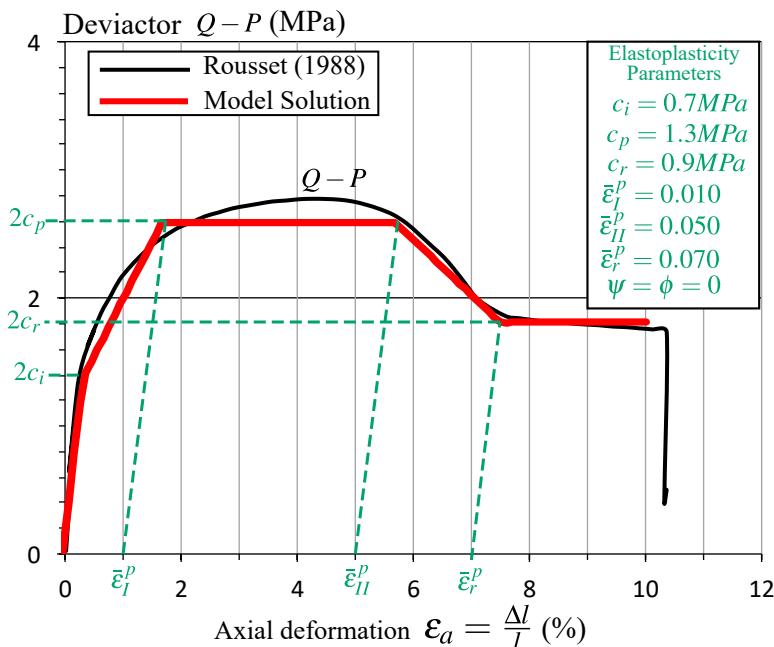
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Figure 1



b)

TRIAXIAL TEST - DUCTIL CLAY ROCK MASS



Boom Clay Rock Mass

Triaxial Test
20.06.86
Depth = 243.40 m
 $H = 71 \text{ mm}$ $D = 36 \text{ mm}$
 $W = 23.4\%$ $d = 2.05$

Hydrostacit Pressure
 $P = 0.5 \text{ MPa}$

Test Velocity
1 mm/min

Elasticity
 $E = 403 \text{ MPa}$ $v = 0.39$

Rupture
Deviator = 2.7 MPa
Strain = 4.39%
Residual deviator = 1.7 MPa

Figure 2

Click here to [f^{vp}\(σ, q^{vp}\)](#)
access/download;Figure;FIG2.p

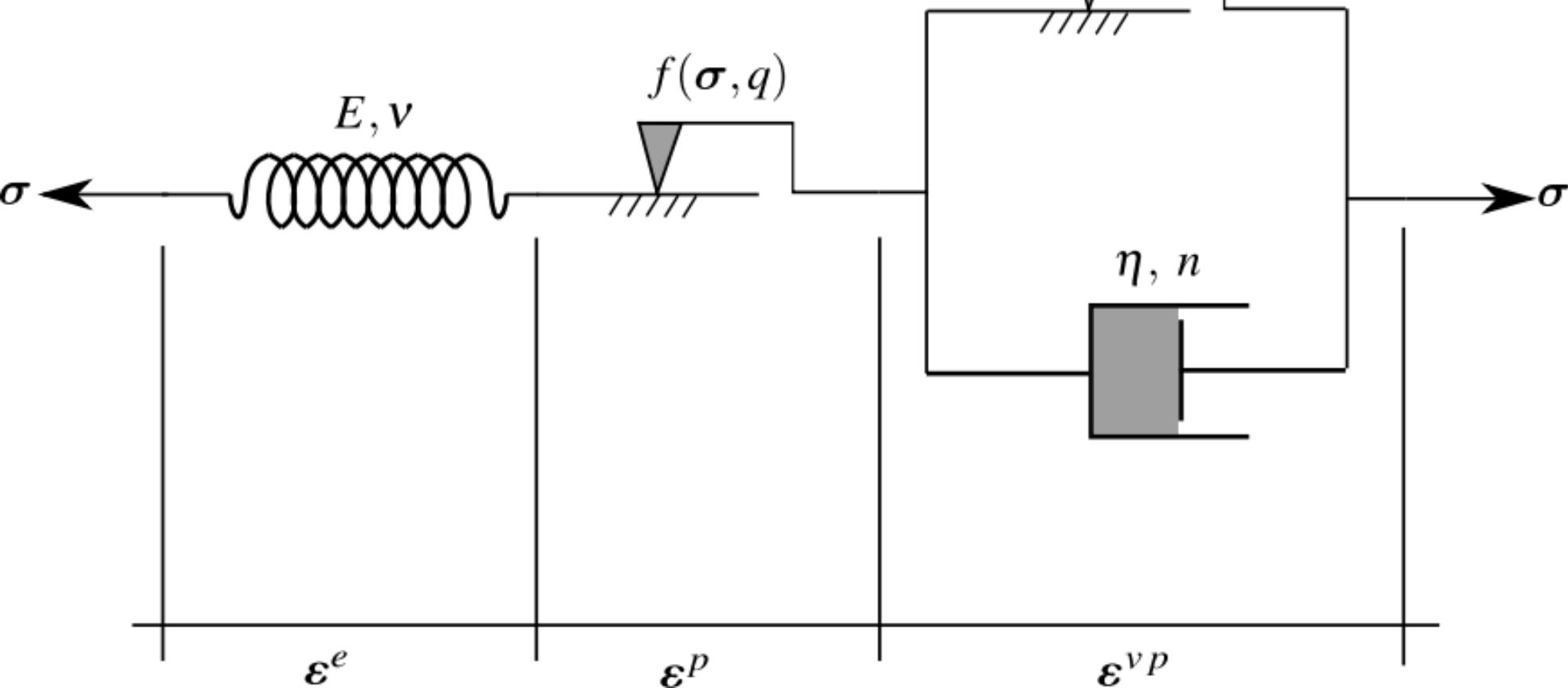


Figure 3

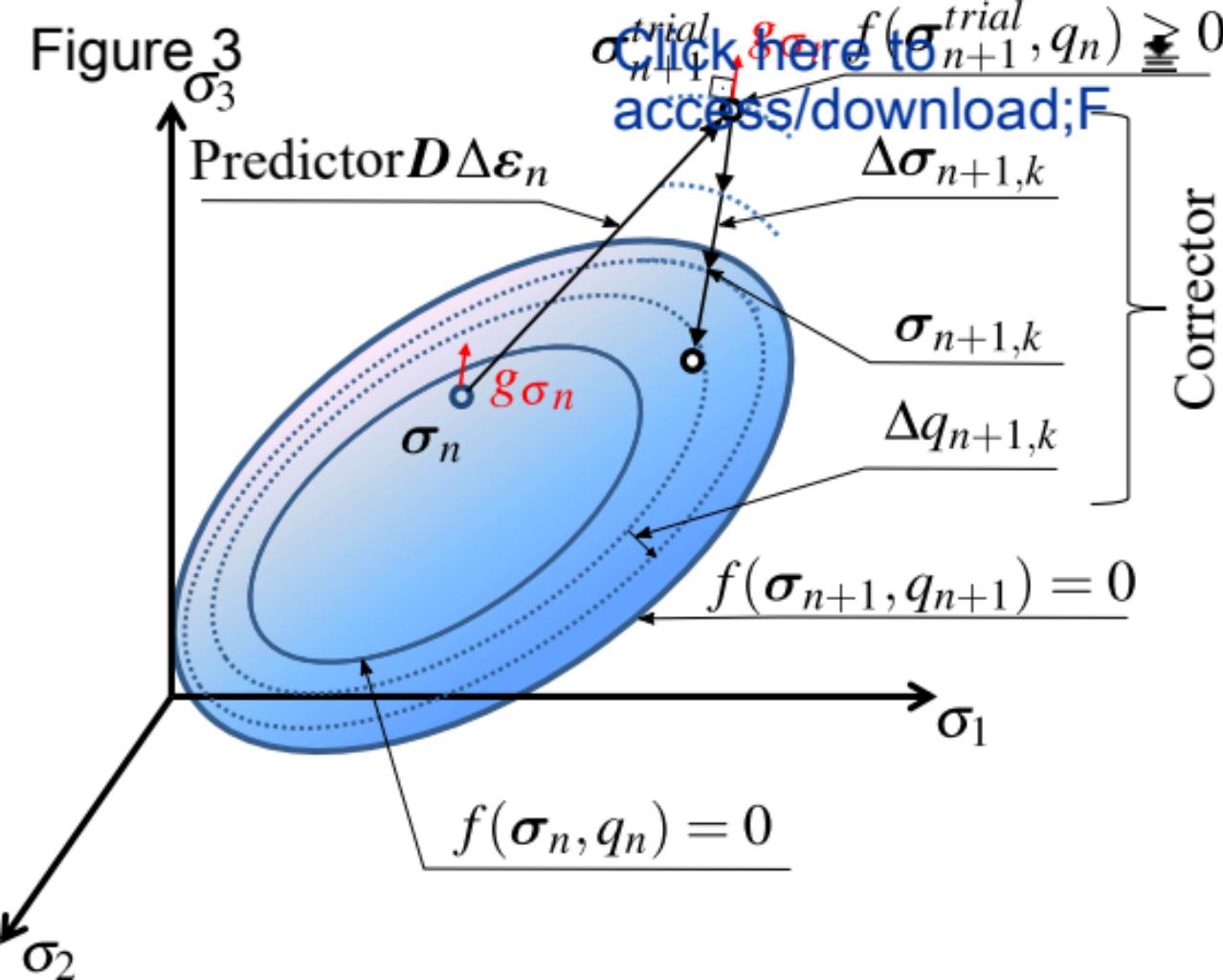


Figure 4

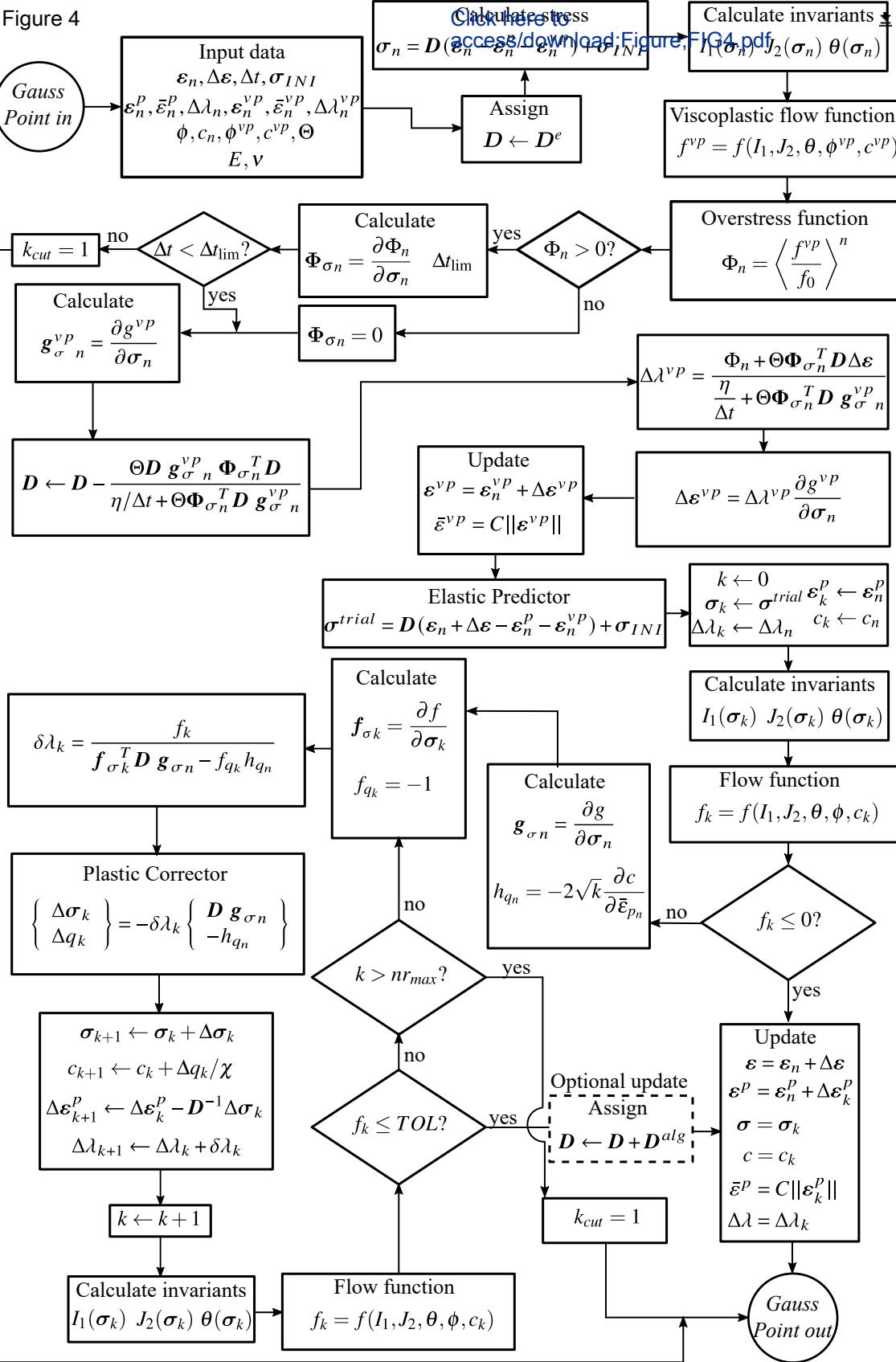
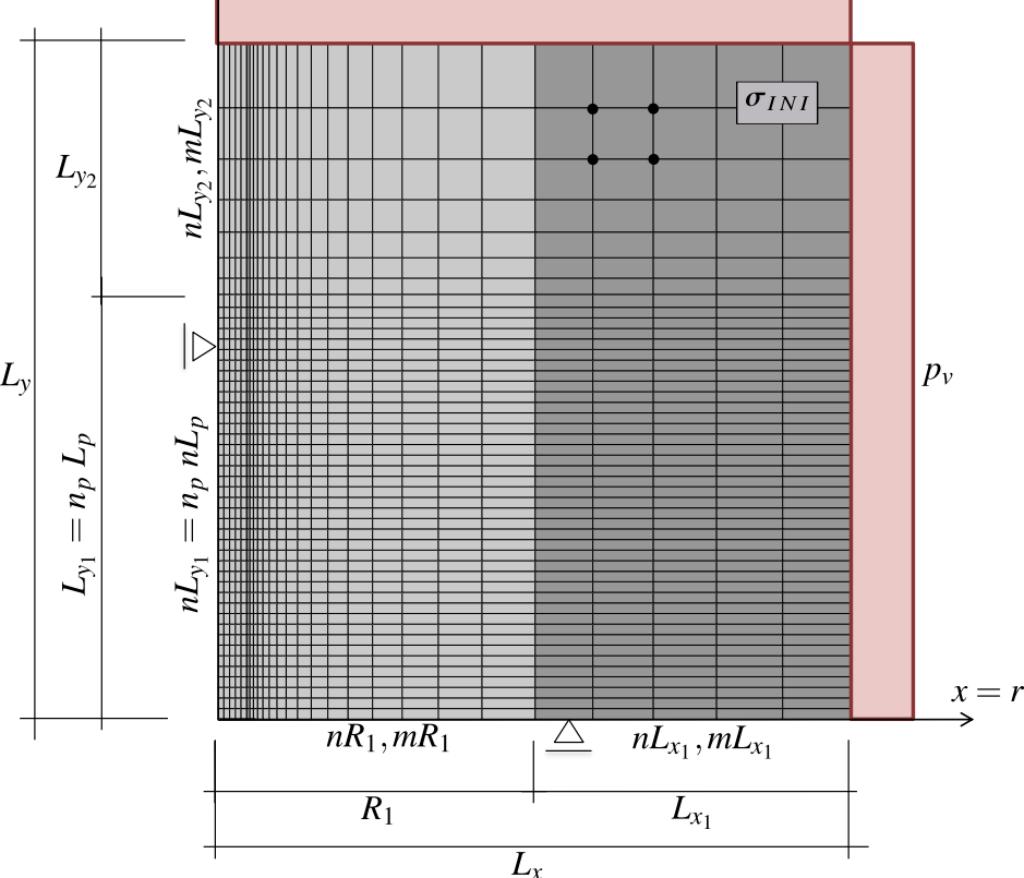


Figure 5

$\theta \uparrow^y$ (tunnel axis)

Click here to
[p_b](#) access/download;Figure;FIG5.pdf



$\theta \uparrow^y$ (tunnel axis)

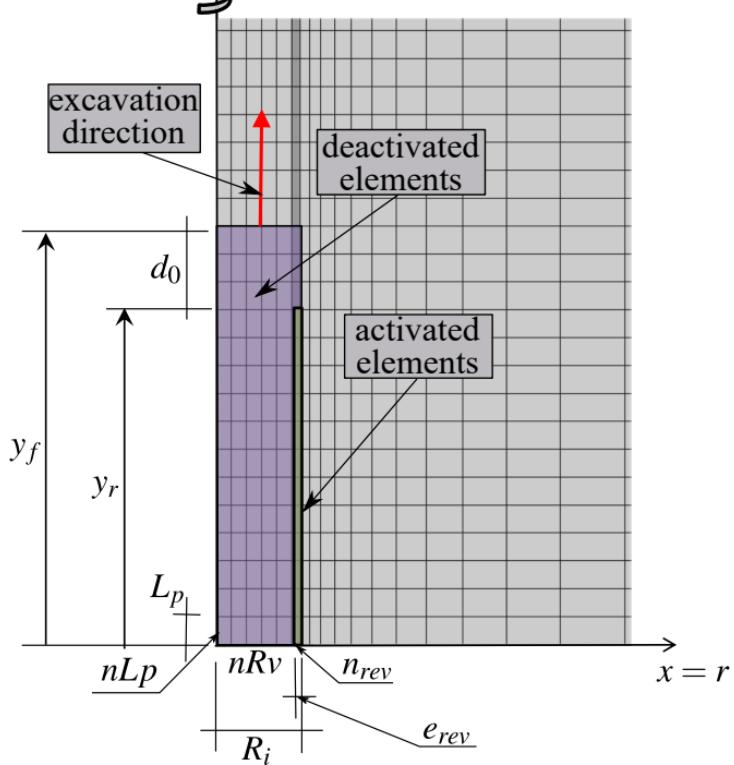
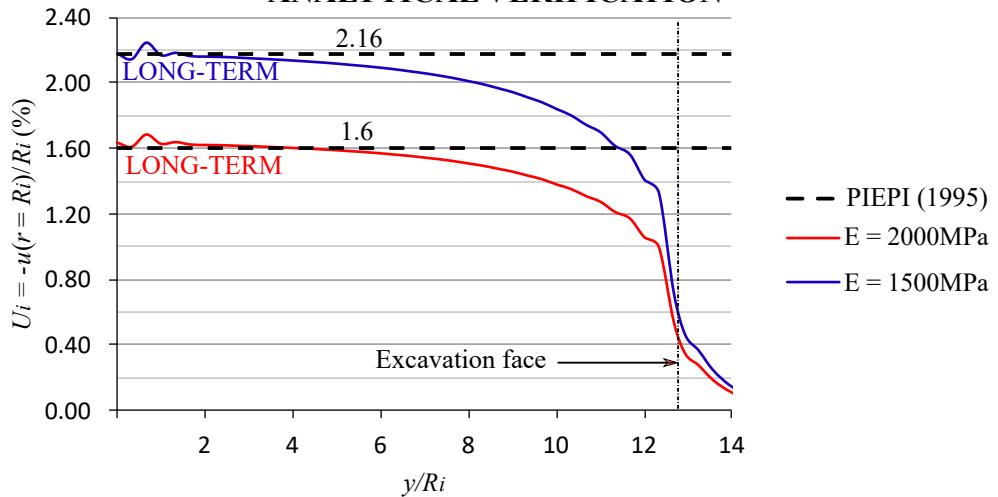


Figure 6

CONVERGENCE PROFILE WITHOUT LINING ANALYTICAL VERIFICATION



CONVERGENCE PROFILE WITH LINING NUMERICAL VERIFICATION

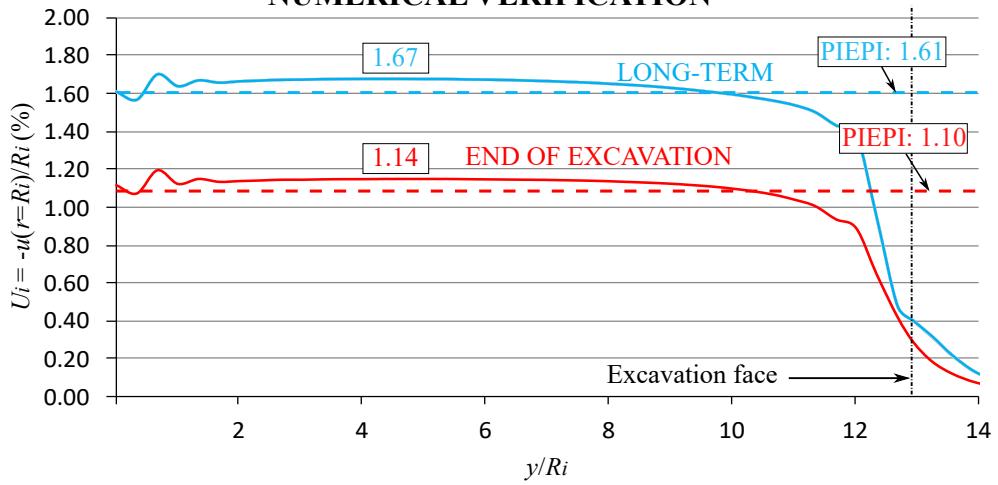
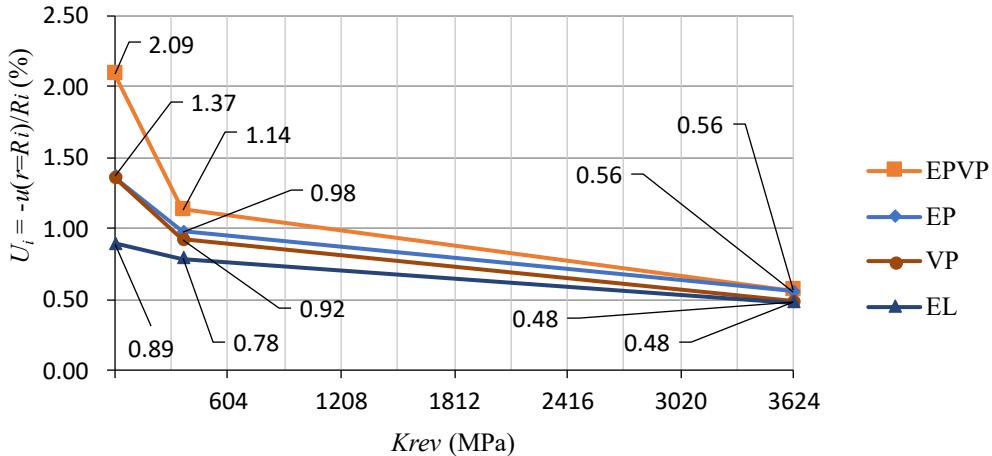


Figure 7

LONG-TERM CONVERGENCE x LINING MODULE

$d_0 = 0L_p$

[Click here to access/download Figure PIC7.pdf](#)



LONG-TERM CONVERGENCE x LINING MODULE

$$d_0 = 4L_p$$

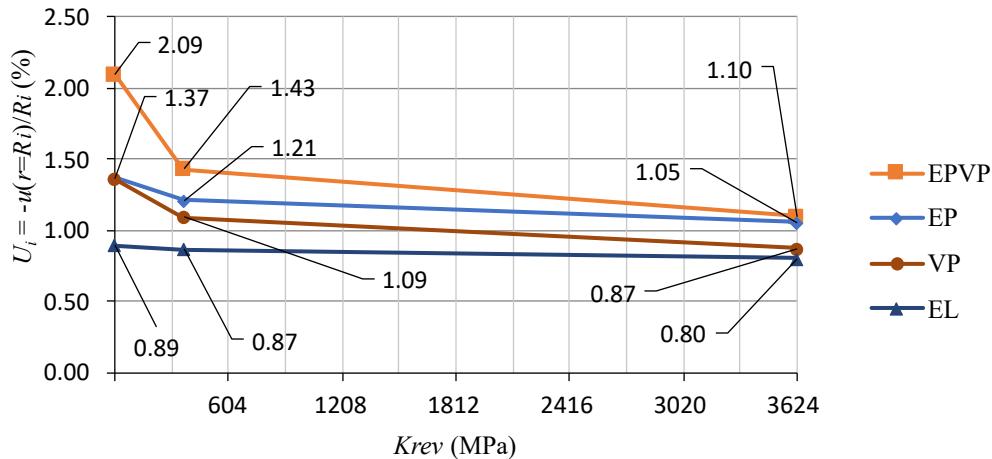
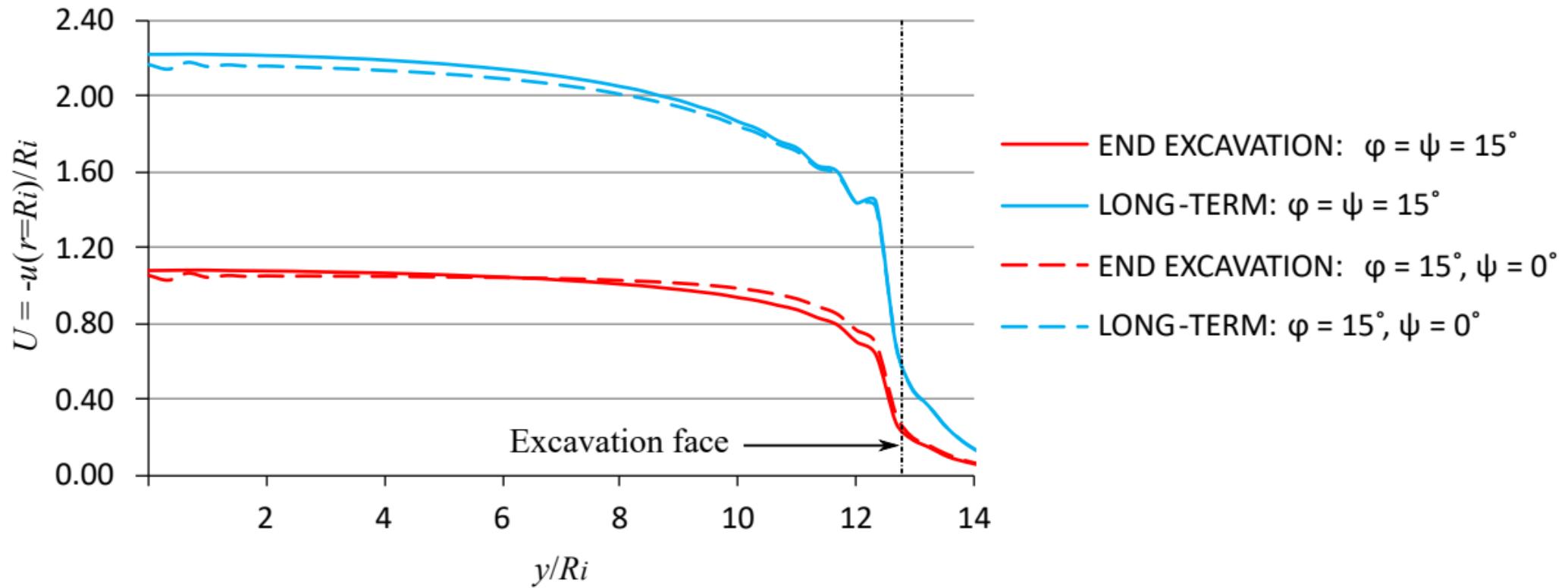


Figure 8

[Click here to access/download;Figure;FIG8.pdf](#)

CONVERGENCE PROFILE ASSOCIATED AND NON-ASSOCIATED PLASTICITY





Dear Editor,

Thank you for giving us the opportunity to submit a revised draft of the manuscript "Numerical integration scheme for coupled elastoplastic-viscoplastic constitutive law for tunnels" for publication in the International Journal of Geomechanics. We appreciate the time and effort that you and the reviewers dedicated to providing feedback on our manuscript and are grateful for the insightful comments on and valuable improvements to our paper. We have incorporated most of the suggestions made by the reviewers. Please see below, in blue, for a point-by-point response to the reviewers' comments and concerns. All page numbers refer to the revised manuscript file.

REPLIES TO EDITOR'S E-MAIL

- Please note the byline on your manuscript text doesn't match the manuscript data entered (add/edit/delete authors step). Please correct this and resubmit.

The author's name was changed in the Editorial Management system from Felipe Quevedo to Felipe Pinto da Motta Quevedo to match the manuscript data.

- Please upload your revised manuscript file in Microsoft Word or LaTex format. If you using LaTex, you may submit a PDF file for review. Please see our LaTex instructions on the Author main page for more information.

The revised manuscript file is submitted in pdf format together with the project Latex files.

- Remove the figures from your manuscript text and upload them separately (one figure per file) in TIFF, EPS or PDF format. Also, please make sure to reference the figure number in each file name.

The figures were removed from the manuscript text and uploaded as separate files.

- Double-spaced list of figure captions. Please provide a double-spaced list of figure captions with your submission. This can be at the end of your manuscript text or uploaded as a separate Word file. Also, please make sure if you have figures labeled as Figure 1a, 1b, etc. that the captions for these parts of the figure are included in your Figure Caption List.

The list of figure captions is now provided at the end of the revised manuscript.

- Embedded Tables. Please remove tables from within the text of your paper and place them at the end of your manuscript after the references . If you upload them separately, please make sure they are uploaded in Microsoft Word/LaTex format.

Tables are at provided the end of the revised manuscript after the references.

Also, please note in order to clarify math for copyeditors, please ensure that you use boldface for matrices, vectors, and tensors; italics for all variables and lowercase Greek letters; and roman for all numerals, uppercase Greek characters, and mathematical operators.

The manuscript has been carefully checked to clarify math for copyeditors, meeting the above recommendations.

Please submit the revised manuscript and a detailed response to the reviewers' criticisms by logging onto the Editorial Management system at <https://www.editorialmanager.com/jrnmeng/> and clicking on the "Submissions Needing Revision" link.

The revised version of the manuscript has been submitted onto the Editorial Management system together with a detailed response to the reviewers' comments.

RESPONSES TO REVIEWER 1

This is very interesting research work to apply the hyperelasticity to the geo-material. In addition, this is the suitable method to employ this constitutive law to the integral of the implicit method. However, it is difficult to understand it. The authors should friendly revise some explanation in each model's part.

1. The reviewer does not think that it is necessary to explain according to Hyperelasticity. However, the authors should explain it more concisely so that the reader can understand it. Why did the authors employ Hyperelasticity to the constitutive model of the geo-material. The authors should explain the reason in detail.

The constitutive model adopted in the paper to describe the behavior of the geo-material is not formulated within the context of hyperelasticity. It actually consists of a coupled elastoplastic-viscoplastic model , which is formulated in the framework of infinitesimal strains. For the sake of clarity, the equations that express the specific free energy were therefore removed from the manuscript text.

2. The potential function of hyperelasticity, ψ_e , should be described.

As stated in the above comment, the constitutive model of the geo-material does not refer to hyperelasticity, but to an infinitesimal coupled elastoplastic-viscoplastic model. For the sake of clarity, the equations that express the specific free energy were therefore removed from the manuscript text.

3. The authors should declare that the cohesion, c , is a variable when it is employed at the yield function.

Following the reviewer's recommendation, the cohesion c is declared as a variable when it is introduced to define the yield function.

4. The authors should show the concrete form of the plastic potential function. In addition, they should explain why they use such a function.

Referring to the Reviewer's comment regarding the plastic potential function, the last paragraph of Sub-Section "Plastic flow rule" has been reformulated (lines 103 to 108) :

"[...] In the particular case of Drucker-Prager potential flow, $g = \beta_1 I_1 + \beta_2 \sqrt{J_2}$ corresponding to $C_1 = \beta_1$, $C_2 = \beta_2$, $C_3 = 0$. From the numerical viewpoint, the main advantage of using such a potential function lies in the fact it is a smooth function. Another advantage is that it can simulate the volume variation during the evolution of plastic deformations (dilation). This

effect is commonly introduced through non-associated plasticity, adopting, instead of the friction angle a dilatancy angle $0 < \psi < \phi$ in the potential function g ."

5. There are many variables in the hardening rule, and the authors should explain them such as the magnitude relationship of c_p , c_i , c_r , etc., in detail. In addition, the authors should also explain the difference between zones. It should also show a comparison with the response of the actual material.

The hardening rule described by the variations of the material cohesion with the state variable (equivalent plastic deformation) is now better described by introducing new Figure 1. The latter illustrates the variations of the piecewise linear function $c = c(\bar{\varepsilon}^p)$, as well as the different zones I to IV. Figure 1 also provides a comparison between the model hardening law and triaxial rock tests.

6. The reviewer thinks it is better to explain the validity of the constitutive rule by showing some analysis examples (stress-strain relations) such as triaxial tests. At that time, it is better to compare it with the actual experimental data.

As explained in the response to the preceding Reviewer's comment, new Figure 1 shows a comparison of the model predictions with experimental data from triaxial tests on the Boom Clay Rock Mass (Rousset, 1988).

7. Equations (2), (3), (4): The parameter, q , in the proposed model may be variable. In essence, it is a model that the cohesion, c , changes due to plastic deformation. However, it is difficult to understand. The authors should describe and explain the yield function and the cohesion in the yield function.

The yield function and related parameters are better defined in the revised draft (see comment after Equation (4)).

8. Equation (5): The reviewer thinks that it is better to remove equation (5) since it is difficult to understand it.

Following the Reviewer's suggestion, Equation (5) of the original submission has been removed from the manuscript text.

9. Equation 6: The authors indicated the Load angle, θ . However, it was not employed in the yield function. On the other hand, it was employed in the plastic potential. The authors should explain it.

In a first step, expression (3) (Equation 6 in the original submission) describing the flow function of isotropic materials is written in the general form as a function of all the stress invariants. In the sequence, θ that stands for the effects of third stress invariant J_3 does not appear because the Drucker-Prager surface used for the numerical simulations is independent of this stress invariant. At that respect, the following sentence has been introduced in line 89 after Equation (5):

"The Lode angle does not appear because the Drucker-Prager surface does not depend on this angle in the deviator plane".

The potential function is also described in its general form, however, the Drucker-Prager potential function used in the analyses does not depend on this term: $C_3 = 0$ (as stated in line 104).

10. In the geomechanics, the compression is often implicitly positive. At the first appearance, σ , the authors may declare the tension is positive.

Convention of positive stress in tension is now clearly stated in the revised draft. The following sentence has been added after Equation (2) – line 66:

“[...] Sign convention of positive stress in tension is adopted throughout the paper.”

11. Equation 10: The formula of the function, g , should be described.

As indicated in the response to comment 4, function g adopted in the analyses is now better described in the revised draft.

12. Equation 11: g_3 should be solved and the formula of $dJ_3/d\sigma$ should be also described.

Expression $g_3 = \partial J_3 / \partial \sigma$ is now provided in Equation (7) of the revised manuscript.

13. Equation 24: The first term of the left hand side is the strain rate. Therefore, "dot" is necessary.

The first term of the left hand side in Equation (20) (Equation 24 in the original submission) has been corrected.

14. Equation 26: It is normally use to $\varepsilon = \varepsilon_e + \varepsilon_{vp}$. It is rear to separate between plastic strain, ε_p , and ε_{vp} . The authors should explain how ε_p and ε_{vp} were calculated individually.

From a physical viewpoint, the model shall account for both instantaneous irreversible strains (plasticity) and delayed irreversible strains (viscoplasticity). In this context, additive form for the total strain rate is considered in the modeling. The latter consists in three elementary rheological models associated in series. The individual contribution of each component is described separately.

The first paragraph of Section “ELASTOPLASTIC-VISCOPLASTIC CONSTITUTIVE MODEL” (line 180) has been reformulated as:

“The proposed elastoplastic-viscoplastic model is formulated from the serial association of the constitutive models described in the preceding sections, i.e., $\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p + \dot{\varepsilon}^{vp}$, which leads to the following constitutive relationship:

$$\dot{\sigma} = \mathbf{D}^{epvp} : \dot{\varepsilon} = \mathbf{D} : \dot{\varepsilon}^e = \mathbf{D} : (\dot{\varepsilon} - \dot{\varepsilon}^p - \dot{\varepsilon}^{vp})$$

This association can be seen in the one-dimensional representation of Fig. 2. [...]”

RESPONSES TO REVIEWER 2

The concepts presented in the paper are available in standard texts. May focus on the work done by the Authors using UPF.

Elastoplastic-viscoplastic constitutive formulations are presented in the paper. Generally, such coupled analysis is required if rate-independent (instantaneous) analysis needs to be augmented with time-dependent yielding analysis such as the effect of creep. The concept presented in the paper is well documented in standard literature (both rate-independent and time-dependent). Coupling of these two methods are also presented in several literatures as also mentioned by the authors. In that sense, the paper does not provide any new information.

From a physical viewpoint, the model shall account for both instantaneous irreversible strains (plasticity) and delayed irreversible strains (viscoplasticity). In this context, additive form for the total strain rate is generally considered in coupled analyses. Although each separate component of the geo-material deformation is classically well documented in standard literature, few models specifically devised for dealing with coupled plasticity-viscoplasticity are available and they address very specific situations regarding the plastic and viscoplastic flow surfaces. A flowchart for a general scheme, such as that presented in Figure 4, is rarely presented for these coupled models.

However, a coupling integration algorithm is presented in UPF platform of ANSYS which may have some interest to the readers of this journal. It is encouraged that the authors focus their paper in that direction rather than elaborating the concept already available in the standard books and literature. While revising the paper, authors may highlight the following:

1. Dilation of rock materials is neglected in the present study. Dep in elasto-plastic analysis will be unsymmetrical in non-associative flow. Please make comments on symmetrisation techniques or comment whether one needs a non-symmetrical solver. Authors may refer to the following symmetrisation papers.

Pande et al., 1986, "Symmetric tangential stiffness formulation for non-associative plasticity", Compu Geotech, 2(2) 89-99

Deb et al., 2013, "Generalized symmetric formulation of tangential stiffness for nonassociative plasticity", J. of Engg. Mech, Vol 139, issue 2.

The effects of dilation of rock materials are accounted for in the constitutive model as well in the numerical algorithm. As a matter of fact, the plastic and viscoplastic models (Drucker-Prager) involve irreversible volume variation (dilation) material deformation.

Regarding the non-symmetric solver, the following paragraph has been added after Equation (31) – line 257:

"In non-associated plasticity, the constitutive modulus matrix is not symmetric and its update leads to a non-symmetric global stiffness matrix, thus requiring a non-symmetric solver for the global equilibrium iterations. Symmetrization techniques of the constitutive stiffness matrix, such that proposed in Pande and Pietruszczak. (1986) or in Deb et al. (2013), can be used. It is observed that the algorithm converges even not updating the constitutive modulus. Although it is optional, the calculations performed in this paper make use of this update."

2. Express momentum balance equation in static condition and make comments on the increment of external load, especially whether it will be time dependent or not. Generally, for elasto-plastic analysis $\delta(t)$ is a pseudo-parameter, however, it is an important parameter for visco-plasticity. Will the stress corrections in elasto-plastic analysis now be depended on $\delta(t)$?

Referring to the momentum balance equation, the following text has been added at the beginning of Section “VALIDATION OF THE MODEL” – Line 318:

“Before proceeding to the model validation, it is emphasized that the momentum balance equations in quasi-static conditions read in incremental form $\text{div } \Delta\sigma + \Delta F = 0$, where $\Delta\sigma$ and ΔF are respectively the stress increment and the increment of external load. In the case of tunnel simulation considered in the sequel, the increment of external load is time dependent since it is associated with the process of ground excavation and tunnel advancement (rock elements are removed).”

Regarding the time dependence of the mechanical fields in the coupled elastoplastic-viscoplastic analysis, the following paragraph has been added at the end of Sub-section “Integration of elastoplastic-viscoplastic constitutive equations” - line 312:

“In elastoplastic analyses, time is a kinematical parameter that marks the load history and controls the incremental solution, but it does not influence the constitutive relationships. In coupled elastoplastic-viscoplastic analyses, the elastoplastic component of constitutive relationships becomes time-dependent since it explicitly depends on the viscoplasticity behavior. In particular, the stress corrections in the elastoplastic step of the solution procedure shall a priori depend on Δt ”

3. Authors have assumed that increment of strain(vp) will be estimated first and stress will be updated before elasto-plastic analysis starts. One would think it may happen in the reverse way.

From a physical viewpoint, the plastic strain increment refers to instantaneous component of deformation increment whereas the viscoplastic strain increment refers to the delayed one. However, the procedure for numerical solution evaluates first the viscoplastic strain increment and then implements in a subsequent analysis the elastoplastic integration algorithm. For the sake of clarity, the first paragraph of Sub-section “Integration of elastoplastic-viscoplastic constitutive equations” – line 303 has been reformulated as:

“As viscoplastic strains are integrated using a semi-implicit rule in which all variables are calculated with known stresses (from substep n), the viscoplastic strain increment is computed first. Subsequently, it is deducted from the total strain increment evaluated by elastic predictor in the elastoplasticity algorithm. This is actually the correct sequence for computation modeling of the coupling, since elastoplasticity algorithm updates the stress state for substep $n+1$ in order to verify the criterion $f_{n+1} = 0$. If the plastic strain were calculated before the viscoplastic strain, the criterion value would be $f_{n+1} \neq 0$. [...]”

4. Define ci, cp, cr in equation 13.

The hardening rule described by the variations of the material cohesion with the state variable (equivalent plastic deformation) is now better described by introducing new Figure 1. The latter illustrates the variations of the piecewise linear function $c = c(\bar{\varepsilon}^p)$, as well as the different zones I to IV. Figure 1 also provides a comparison between the model hardening law and triaxial rock tests.

5. Elaborate on the UPF code in ANSYS for the benefit of the readers. This is probably the novelty of this paper. Authors may schematically present the code block in Fortran. Title of the paper may be changed accordingly.

The procedure for computation implementation is described by the flowchart in Figure 4. The idea of presenting the flowchart instead of focusing on the Fortran code implementation is that the model can be implemented in any software and any programming language. In this way, it will not be restricted to ANSYS users and Fortran 77 readers. However, reference to Quevedo PhD thesis (2021), which provides the Fortran code and related implementation details was added at line 310 of the revised draft:

"[...] The implementation in the USERMAT subroutine in FORTRAN77 can be found in Quevedo (2021)"

6. Is superscript p valid in equation given in line 214?

Subscript p has been introduced in the Equation of line 218 (line 214 in the original submission).

7. The example problem is solved considering associative flow. Analyze the same example considering non-associative flow rule.

Following the Reviewer's suggestion, additional simulations have been performed considering non-associative flow rule. The results of this analysis are shown in Figure 8.

A comment regarding this aspect has been added at the end of Sub-section "PARAMETRIC ANALYSIS, RESULTS AND DISCUSSION"

8. Line 318: ratio not ration.

The term has been corrected (in line 339).

The paper needs major revision as mentioned above.

RESPONSES TO REVIEWER 3

1. Authors should review the consistency and notation of all equations.

The manuscript has been carefully checked and corrected, the consistency and notation have been reviewed.

2. The "zones" in equations 13 and 15 are not well explained (Authors could make a graphic showing these different zones)

The hardening rule described by the variations of the material cohesion with the state variable (equivalent plastic deformation) is now better described by introducing new Figure 1. The latter illustrates the variations of the piecewise linear function $c = c(\bar{\varepsilon}^p)$, as well as the different zones I to IV. Figure 1 also provides a comparison between the model hardening law and triaxial rock tests.

3. The literature review should be reduced and more details should be given on the coupling of the constitutive models

The coupled elastoplastic-viscoplastic model is the junction of two components (elastoplastic and viscoplastic). For the sake of clarity, it is therefore essential to first describe the main features of both the plastic and viscoplastic models involved in the formulation of the coupled

one. In that respect, several additional comments referring to the formulation of coupled model and related numerical aspects have been added in the revised drafts.

A) Last paragraph of Section “ELASTOPLASTIC-VISCOPLASTIC CONSTITUTIVE MODEL” (line 188):

“An interesting aspect of the coupled model, using the Drucker-Prager criteria f and f^{vp} for plasticity and viscoplasticity with $\phi = \phi^{vp}$, is that the evolution of local mechanical fields are entirely controlled by the cohesion. In particular, when $c \rightarrow \infty$ and $c^{vp} \rightarrow \infty$ the solution is purely elastic. Besides, the purely elastoviscoplastic solution is retrieved when $c \rightarrow \infty$, whereas purely elastoplastic solution is obtained when $c^{vp} \rightarrow \infty$.”

B) First paragraph of Sub-section “Integration of elastoplastic-viscoplastic constitutive equations” – line 303 has been reformulated as:

“As viscoplasticity strains are integrated using a semi-implicit rule in which all variables are calculated with known stresses (from substep n), the viscoplastic strain increment is computed first. Subsequently, it is deducted from the total strain increment evaluated by elastic predictor in the elastoplasticity algorithm. This is actually the correct sequence for computation modeling of the coupling, since elastoplasticity algorithm updates the stress state for substep $n+1$ in order to verify the criterion $f_{n+1} = 0$. If the plastic strain were calculated before the viscoplastic strain, the criterion value would be $f_{n+1} \neq 0$. [...]”

C) The following paragraph has been added at the end of Sub-section “Integration of elastoplastic-viscoplastic constitutive equations” - line 312:

“In elastoplastic analyses, time is a kinematical parameter that marks the load history and control the incremental solution, but it does not influence the constitutive relationships. In coupled elastoplastic-viscoplastic analyses, the elastoplastic component of constitutive relationships becomes time-dependent since it explicitly depends on the viscoplasticity behavior. In particular, the stress corrections in the elastoplastic step of the solution procedure shall a priori depend on Δt ”

4- The ANSYS APDL script for the FEM model and the USERMAT subroutine in FOTRAN for the rock constituent model should be publicly available datasets. It is recommended to use for example "Datasets related to this article can be found at [INSERT PERMANENT URL(s) TO BE LINKED TO DATASET], hosted at [NAME OF HOSTING REPOSITORY] ([CITATION TO DATASET])".

The procedure for computation implementation is described by the flowchart in Figure 4. The idea of presenting the flowchart instead of focusing on the Fortran code implementation is that the model can be implemented in any software and any programming language. In this way, it will not be restricted to ANSYS users and Fortran 77 readers. However, reference to Quevedo PhD thesis (2021), which provides the Fortran code and related implementation details was added at line 310 of the revised draft:

“[...] The implementation in the USERMAT subroutine in FORTRAN77 can be found in Quevedo (2021)”

and the following sentence has been added at line 334:

“[...] The FEM model in ANSYS APDL script can be found in Quevedo (2021).”

It is recalled the following sentence regarding the following data availability statement is provided at line 378 of the original submission:

"Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request. (ANSYS APDL script for FEM model and USERMAT subroutine in FORTRAN 77 for constitutive rock mass model)."