

## Graphical Abstract

### Numerical analysis of the rock deformation in twin tunnels with transverse galleries considering plasticity and time-dependent constitutive models

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## Highlights

### **Numerical analysis of the rock deformation in twin tunnels with transverse galleries considering plasticity and time-dependent constitutive models**

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- The stiffness of the lining restricting viscous effects in the interaction of tunnels
- Interaction between the tunnels becomes significant at a span distance of 4 radii
- The viscous of the concrete lining can be important in the tunnel convergence
- The effect of the gallery extends into the tunnel up to 4 radii from its axis
- The proximity of the tunnels induces the ovalization of the tunnel wall

# Numerical analysis of the rock deformation in twin tunnels with transverse galleries considering plasticity and time-dependent constitutive models

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## ABSTRACT

Resorting a three-dimensional finite element analysis, this paper investigates the instantaneous and long-term implications induced by the time-dependent constitutive behavior of constituents on the convergence profile of twin tunnels linked with transverse galleries. Several constitutive models for rock mass mechanical behavior are examined at the material level, encompassing elastoplasticity, viscoplasticity, or coupled elastoplasticity-viscoplasticity frameworks. Plasticity state equations are based on a Drucker-Prager yield surface with an associated flow rule, while the viscoplasticity formulation relies on the Perzyna model with a Drucker-Prager flow surface. Tunnel lining behavior is modeled using either elastic or viscoelastic constitutive models. The viscoelastic behavior is described by a Generalized Kelvin rheological model based on Bazant and Prasanann's Solidification Theory, with model parameters derived from CEB-FIP MC90 formulations. From a computational viewpoint, the deactivation-activation method is employed to simulate the excavation process and lining installation. The accuracy of finite element predictions is assessed through comparisons with available analytical solutions formulated in a simplified setting for the twin tunnels' configuration. A parametric study delves into the mutual interaction induced by tunnels proximity, emphasizing the crucial role of concrete lining stiffness in twin tunnels' deformation. Numerical simulations indicate a highly localized influence of a transverse gallery on twin tunnels deformation, extending up to four radii from each side of the gallery axis. Finally, the paper investigates the effects of twin tunnels proximity and those induced by an interconnecting gallery on the instantaneous and long-term convergence of tunnels, contrasting these outcomes with the convergence of a single tunnel.

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## 1. Introduction

Many design methods often focus on single tunnels, but twin tunnels are a common occurrence. The interaction between tunnels can be significant, especially when the spacing between them is minimal. Additionally, many twin tunnels incorporate transverse galleries, introducing a localized effect on displacements and stresses. Also, the rheological behavior of the rock mass and lining plays a crucial role in how stress and displacements fields evolve over time. Some recent studies on deep twin tunnels can be found at [1, 2, 3, 4, 5, 6, 7, 8].

Chortis and Kavvadas [4] considered the calculation of the axial forces acting on the primary support in the intersection zone before, during, and after the construction of a perpendicular tunnel intersection. The results of the analysis indicated that the zone of influence extends approximately two diameters from the main tunnel to each side from the center of the intersection and that the interaction effects are practically eliminated when they exceed this influence zone.

In another study, Chortis and Kavvadas [5] carried out parametric 3D finite element analyses to verify the interaction between deep twin tunnel, with circular and non-circular cross-section, supported by a shotcrete elastic linear lining. Was considering the rock mass with linear elastic behavior and perfectly plastic, with Mohr-Coulumb failure criteria. The study investigates the axial forces that develop in the primary lining of the twin tunnels as a function

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of the main geometric and geomaterial parameters, but without considering the potential time-dependent deformations (creep effect) that occur in some types of rock masses.

Chen et al. [2], through analytical solutions in elasticity using complex variables, Fourier transformation, and the alternating Schwarz method, demonstrate that the mutual interaction between twin tunnels disappears if the spacing between the tunnels is greater than six times the tunnel radius. The lining effectively reduces the stress concentration, especially at high lateral stress coefficients.

Guo et al. [6] develop an elastic analytical solution for the stress field around twin circular tunnels with hydrostatic pressure using the complex variable and the superposition principle. They found that stress concentration in tunnel wall increased as the distance between the parallel tunnels decreased and the supporting pressure leads to the radial stress increasing and the tangential stress decreasing.

Ma et al. [9] proposed an analytical method, verified by a numerical solution using FLAC3D software for determining the plastic zones around deep circular twin tunnels without linings, restricting themselves where there is no overlap between the two plastic zones. In this case, the authors adopted the elastoplastic perfectly constitutive model for the homogeneous and isotropic rock mass, with the Mohr-Coulomb criterion. Also carried out parametric studies to understand the influence of the distance between the twin tunnels, cohesion, the angle of internal friction, and the vertical and horizontal initial stresses acting on the shape and depth of the plastic zones. These authors stated that the plastic zone around the tunnel provides a relevant theoretical basis for defining and designing the support. In that respect, an excessive plastic zone would significantly affect the stability and functionality of a tunnel. Reducing the extension of the plastic zone around tunnels is, therefore, of great importance in engineering tunnel design projects.

Using parametric three-dimensional numerical analyses, Chortis and Kavvadas [7, 8] investigated the effect of building a transverse tunnel that intersected deep twin tunnels perpendicularly, focusing the study on the axial forces and the circumferential and longitudinal bending moments acting on the primary support of the intersection regions, respectively. According to the authors, the potential interaction between deep twin tunnels lined with shotcrete must be taken into account, especially when the distance between them is less than or equal to twice their diameter.

According to Fortsakis [3], in a realistic construction context, twin tunnels are excavated and supported with a delay, so that the second tunnel is usually built after the first one has advanced enough to maintain a longitudinal separation distance between the faces. The advance of the subsequent tunnel mobilizes the redistribution of stresses and deformations in the zone between the tunnels, resulting in additional loading of the preceding tunnel.

As for transverse tunnels, these are generally built far enough behind the advanced face of the main tunnel to ensure that their excavation has virtually no effect during the construction of the junction tunnel [4]. The interaction at the intersection, between the main tunnel and the transverse tunnel, significantly modifies the stress state of the primary support and that of the surrounding rock mass in these areas, compared to that of the singular tunnel, making three-dimensional finite element analyses essential for developing a realistic and safe design for tunnel junctions [1].

During the construction of the transverse tunnel, the surrounding rock mass is subjected to a redistribution of stresses, causing an additional load on the main tunnel, precisely in the intersection zone. If these additional loads exceed the load capacity of the primary support of the main tunnel, a potentially unstable region can develop, leading to failure, especially in adverse geotechnical conditions [4].

While the simulation of tunnel convergence in single tunnels has been widely investigated and reported in published literature, few works have addressed the computational evaluation of deformation in twin tunnels. Less attention has been dedicated to assessing the mutual mechanical interaction induced by the excavation of the transverse gallery connecting the twin tunnels.

In this context, the main contributions of this paper may be summarized at both the material and tunnel analysis levels. At the material level, the constitutive state equations of the rock mass are formulated within the framework of coupled plasticity-viscoplasticity, which is relevant for clayey rocks. Such a framework allows capturing the irreversible instantaneous tunnel response (plasticity) as well as the delayed irreversible response (viscoplasticity). As regards the mechanical behavior of concrete material defining the lining, which is classically modeled through linear elastic relationships, the present analysis considers an aging viscoelastic rheological model relying upon the Bažant and Prasannan Solidification theory [10, 11]. At the structure analysis level, the simulation of deformation in the highly interacting material system components (namely, rock mass and lining), resulting from the excavation process of twin tunnels and transverse gallery, is handled using finite element simulations performed in a three-dimensional setting. From the computational viewpoint, the excavation process and lining placement are simulated by means of the activation/deactivation technique. The constitutive models formulated for the rock mass and lining constituent as well as the related numerical integration schemes are implemented into the same procedure UPF/USERMAT

customization tool [12] of ANSYS standard software. The three-dimensional finite element analysis developed in this paper is specifically devised for addressing the three-dimensional interaction induced by the construction process, twin tunnels proximity, and the presence of the transverse gallery.

## 2. Fundamental assumptions

The basic assumptions of the constitutive and computational modeling, as well as related limitations, are summarized as follows:

- (a) Only the configuration of deep tunnels shall be considered in the subsequent analysis, thus neglecting deformations caused by surface loads and settlements arising from the excavation process.
- (b) Although material heterogeneity and behavior anisotropy are inherent features of soils and rocks, the rock mass is modeled throughout the paper as a homogeneous and isotropic continuous medium. At the scale adopted for tunnel modeling (macroscopic scale), this assumption means in particular that the possible micro-heterogeneities, such as isotropic distributions of joints or cracks present at the finer scale, are accounted for in the homogenized behavior by means of a preliminary homogenization process (e.g., [13, 14, 15, 16, 17]). Clearly enough, the framework of continuum modeling adopted in the paper would reveal questionable when the rock mass is cut by a few macroscale fracture joints.
- (c) The rock mass is phenomenologically modeled using an elastoplastic-viscoplastic rheological law to capture instantaneous and long-term responses. This approach disregards the aspect connected temperature gradients, water flow, and poromechanics coupling.
- (d) Despite the complexity of the stress distribution prevailing in the rock mass before the process of tunnel excavation, which is mainly affected by the geological history, the present study assumes a geostatic initial stress reflected by vertical and horizontal stresses.
- (e) Twin tunnels are often designed considering a time gap between excavation fronts. However, the finite element simulations assume synchronous excavation steps to ensure symmetry conditions.
- (f) The simulation excavation processes are carried out assuming a constant tunnel advancement rate (i.e., constant excavation speed), together with a constant thickness of concrete lining.
- (g) Effects of temperature and humidity that may affect the viscoelastic behavior of lining concrete are disregarded.
- (h) Perfect bonding is assumed at the interface between concrete lining and the rock mass.
- (i) The framework of infinitesimal strain analysis, together with quasi-static evolutions, is adopted in the paper. In particular, dynamic excitations and related inertial forces, such as those induced, for instance, by earthquakes or explosions, shall not be considered in the numerical analysis.

## 3. Constitutive Model of the Rock Material

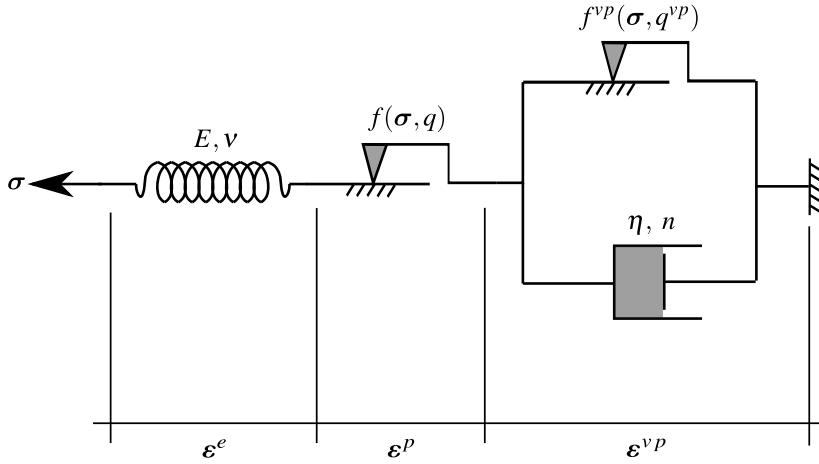
Time-dependent phenomena associated with the delayed behavior of the constitutive material are key aspects of deformation in tunnel structures excavated in deep clayey rocks (see for instance [18, 19] or [20], to cite a few). In most computational analyses developed for tunnel engineering design, this issue is generally addressed by means of viscoplastic constitutive behavior. While such constitutive models could relevantly model the transient and long-term deformation, they seem however inadequate to capture the influence of short-term events (tunnelling and support placement phases) on the final stability of the structure. In particular, an analysis of tunnel deformation based on a viscoplastic model would suggest that the ultimate support pressure at tunnel structure equilibrium mainly depends on the closure rate at the moment when the contact between lining and rock mass is achieved (e.g., [19]), thus disregarding the irreversible effects rising in the initial construction phases. Indeed, during the primary stages of tunnel excavation, the surrounding rock mass is subjected to severe loading conditions and high strain rates, which may lead to yielding associated with high instantaneous irreversible strains near the tunnel wall, and can therefore affect the long-term equilibrium of the structure. It is thus of fundamental concern to formulate a constitutive model that incorporates both instantaneous and delayed irreversible components of the rock material. For this purpose, the present analysis considers a constitutive model that includes both instantaneous plasticity to describe shorth-term material yielding and viscoplasticity to represent delayed behavior. The formulation of the coupled plasticity-viscoplastic rheological model is based on that originally proposed in [19] and [18]. Previous studies have implemented this plastic-viscoplastic model for computational analysis of deformation in single tunnels (e.g., [21, 22, 20, 23]. For the sake of brevity, only

the main features of this constitutive model shall be summarized below. Detailed description of the model, including application and validation in the context of single tunnel structures may be found in [24]. Finite element implementation of this model in the USERMAT procedure of ANSYS software is also described in [23].

The elastoplastic-viscoplastic model is formulated based on a serial association of the elastoplastic and viscoplastic constitutive models. The local strain rate  $\dot{\epsilon}$  is split into three contributions  $\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^p + \dot{\epsilon}^{vp}$ , so that the constitutive relationships relating the Cauchy stress rate  $\dot{\sigma}$  and strain rate components can be written as:

$$\dot{\sigma} = \mathbf{D} : \dot{\epsilon}^e = \mathbf{D} : (\dot{\epsilon} - \dot{\epsilon}^p - \dot{\epsilon}^{vp}). \quad (1)$$

In the above relationship,  $\dot{\epsilon}^e$ ,  $\dot{\epsilon}^p$  and  $\dot{\epsilon}^{vp}$ , represent respectively the elastic, plastic and viscoplastic strain rate, and  $\mathbf{D}$  denote the fourth-order isotropic elastic linear constitutive tensor. Tensor  $\mathbf{D}$  is defined by the rock mass elastic Young modulus  $E$  and Poisson ratio  $\nu$ . The one-dimensional representation of the constitutive behavior is shown in Fig. 1. In



**Figure 1:** Rheological representation of the elastoplastic-viscoplastic model.

this model is used a Drucker-Prager plastic flow surface given by

$$f(\sigma, q) = f(I_1, J_2, q) = \beta_1 I_1 + \beta_2 \sqrt{J_2} - q(\alpha), \quad (2)$$

which  $I_1$  is the first invariant of the stress tensor,  $J_2$  the second invariant of the deviator tensor and  $\beta_1$ ,  $\beta_2$  and  $q(\alpha)$  are strength parameters related to the friction angle  $\phi$  and cohesion  $c(\alpha)$ , respectively. In the present model Drucker-Prager surface been inner of the Mohr-Coulomb surface [25], that is,

$$\beta_1 = \frac{(k-1)}{3}, \quad \beta_2 = \frac{(2k+1)}{\sqrt{3}}, \quad q(\alpha) = 2\sqrt{k} c(\alpha), \quad (3)$$

where  $k = (1 + \sin \phi)/(1 - \sin \phi)$ . The internal variable  $\alpha$  is the equivalent plastic strain  $\bar{\epsilon}^p$  used to simulate strain hardening/softening phenomena. However, for this study, we adopt perfect plasticity, meaning that  $c$  is a constant. For the viscoplasticity surface  $f^{vp}$  the same surface is employed, but with  $\phi^{vp}$  in  $\beta_1$  and  $\beta_2$ , and  $q^{vp} = 2\sqrt{k^{vp}} - c^{vp}$  where  $k^{vp} = (1 + \sin \phi^{vp})/(1 - \sin \phi^{vp})$  and  $c^{vp}$  is a constant, i.e., perfect viscoplasticity. The plastic flow rule is given by:

$$\dot{\epsilon}^p = \begin{cases} \lambda \frac{\partial g}{\partial \sigma} & \text{for } f > 0 \\ \mathbf{0}, & \text{for } f \leq 0 \end{cases}, \quad (4)$$

where  $\lambda$  is the plasticity multiplier and  $g$  is a potential flow function analogous to  $f$  used to simulate the volume dilatation during the evolution of plastic deformations. However, for this analysis, was used associated plasticity, i.e.,  $g = f$ . The plastic multiplier is obtained through the consistency condition  $\dot{f} = 0$ . Numerical details of this implementation can be found in [24]. For viscoplastic flow rule we have,

$$\dot{\epsilon}^{vp} = \lambda^{vp} \frac{\partial f^{vp}}{\partial \sigma} \quad (5)$$

In contrast to the plastic multiplier, the viscoplastic multiplier  $\lambda^{vp}$  is independent of a consistency like condition. As a result, its expression is explicit. Based on the framework of generalized Perzyna's overstress theory [26], its expression may be derived as follows:

$$\lambda^{vp} = \frac{\Phi(\sigma, q^{vp})}{\eta} \quad \text{and} \quad \Phi = \left\langle \frac{f^{vp}(\sigma, q^{vp})}{f_0} \right\rangle^n, \quad (6)$$

where  $\Phi$  is the overstress function,  $\eta$  is the dynamic viscosity constant,  $n$  is the dimensionless parameter that gives the form of the power law,  $f_0$  a parameter conveniently adopted and  $\langle *\rangle$  is the McCauley function which is 0 when  $* < 0$ , i.e. viscoplastic flow will only occur when the overstress function is positive.

In this coupled model, when  $\phi = \phi^{vp}$ , cohesion entirely controls the evolution of local mechanical fields. Specifically, when  $c \rightarrow \infty$  and  $c^{vp} \rightarrow \infty$ , the system achieves a purely elastic solution. The solution becomes purely elastoviscoplastic with  $c \rightarrow \infty$ , while a pure elastoplastic solution emerges with  $c^{vp} \rightarrow \infty$ . In the coupled analysis, condition  $c^{vp} < c$  is adopted, allowing the viscoplastic domain to occur without plasticity. However, in the presence of plasticity, viscous effects become inevitable. Fig. 2 illustrates these domains in principal stress space.

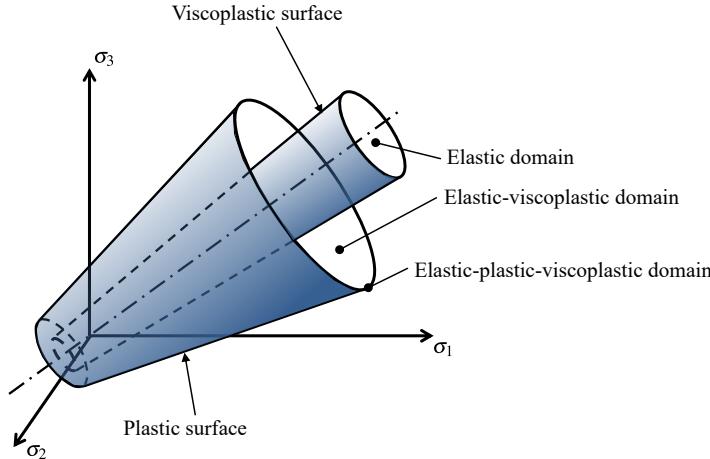


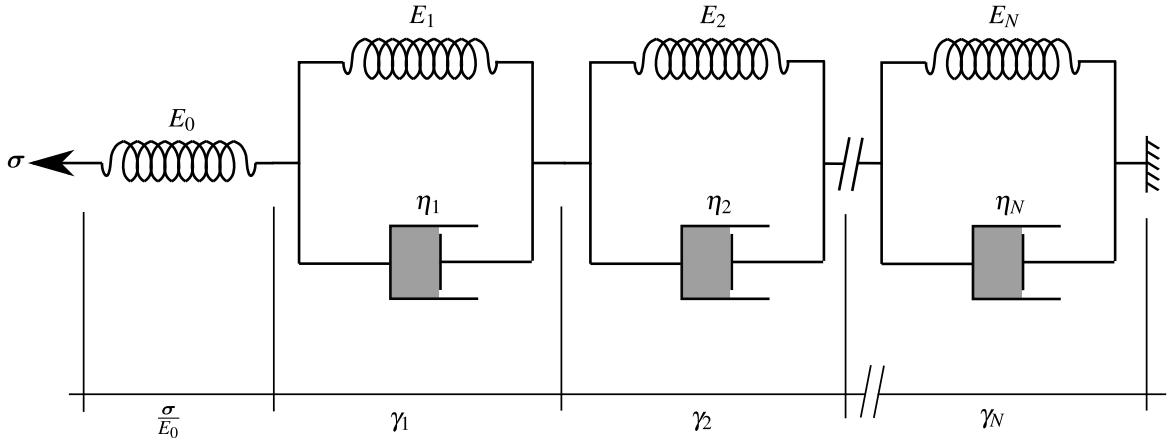
Figure 2: Elastoplastic-viscoplastic domains.

#### 4. Constitutive Model of the Lining

Shrinkage and creep phenomena represent fundamental components of concrete deformation processes that are expected to naturally affect the instantaneous as well as the transient and long-term behavior of structures involving such material. However, most of the tunnel design analyses consider the concrete involved in lining systems as a linear elastic material. From a phenomenological point of view, creep of concrete refers to the time-dependent deformation induced by sustained loading, whereas shrinkage deformation refers to the volume decrease caused by drying. As far as deformation in tunnel structures is concerned, creep and shrinkage have an important effect on the performance of the concrete lining and consequently on its contribution to controlling the long-term convergence of the tunnel. To account for such constitutive features, the concrete creep deformation is addressed by means of an aging viscoelastic rheological model relying on Bažant and Prasannan Solidification Theory [10, 11]. The viscoelastic model is described by a Generalized Kelvin-chain as depicted in Fig. 3. The mechanical parameters that define such a rheological model are the springs stiffness and dash-pots viscosity. The model parameters are calibrated based on the CEB-FIP MC90 standard specifications formulation reported in [27]. One may refer to [28, 29] for detailed description of the calibration procedure. As regards the concrete deformation associated with shrinkage, the isotropic formulation proposed in CEB-FIP MC90 standard [27] is adopted in the present modeling and subsequent computational analyses. Full details regarding model definition and related finite element implementation may be found in [30] and [29].

Accordingly, the constitutive equations for concrete lining relating the stress and strain rate can be expressed in the framework of infinitesimal strain analysis as:

$$\dot{\sigma} = D : \dot{\epsilon}^e = D : \dot{\epsilon} - D : \dot{\epsilon}^{sh} - D^* : \dot{\epsilon}^{cr} \quad (7)$$



**Figure 3:** Generalized Kelvin model for uniaxial concrete viscoelasticity.

In the above relationship,  $\dot{\epsilon}^{sh}$  and  $\dot{\epsilon}^{cr}$  are respectively the shrinkage and creep strain rates. The fourth-order tensors  $\mathbf{D}$  and  $\mathbf{D}^*$  refer to the isotropic elastic linear constitutive tensor and modified constitutive tensor that incorporate the aging viscoelastic properties of the concrete, respectively.

For the numerical implementation purposes, relationship (7) may conveniently be written in incremental form:

$$\Delta\sigma = \mathbf{D} : \Delta\epsilon - \mathbf{D} : \Delta\epsilon^{sh} - \mathbf{D}^* : \Delta\epsilon^{cr} \quad (8)$$

As mentioned above, isotropic formulation is considered for shrinkage, so that increment of shrinkage strain reads:

$$\Delta\epsilon^{sh} = \Delta\epsilon_{sh}(t_s)\mathbf{1} \quad (9)$$

where  $t_s$  represents the concrete curing time, and  $\Delta\epsilon_{sh}$  is the variation in magnitude of the concrete deformation associated with shrinkage (the dependency  $\Delta\epsilon_{sh}$  of on current time is omitted). The latter expression is determined based on CEB-FIP MC90 standard specifications [27].

Regarding the increment of creep strain  $\Delta\epsilon^{cr}$ , its value is computed making use of the incremental algorithm developed by Bažant and Prasannan [10, 11], together with a model calibration that incorporates CEB-FIP MC90 standard formulation [27]. More precisely, the three-dimensional ageing viscoelastic behavior of isotropic concrete is defined by the Generalized kelvin model for the relaxation modulus under uniaxial stress, whereas the Poisson ratio is assumed to be time independent within the time interval of analysis. The procedure for the identification of model parameters is achieved by comparing the creep functions provided in references [10, 11] and [27], leading to the following equivalence:

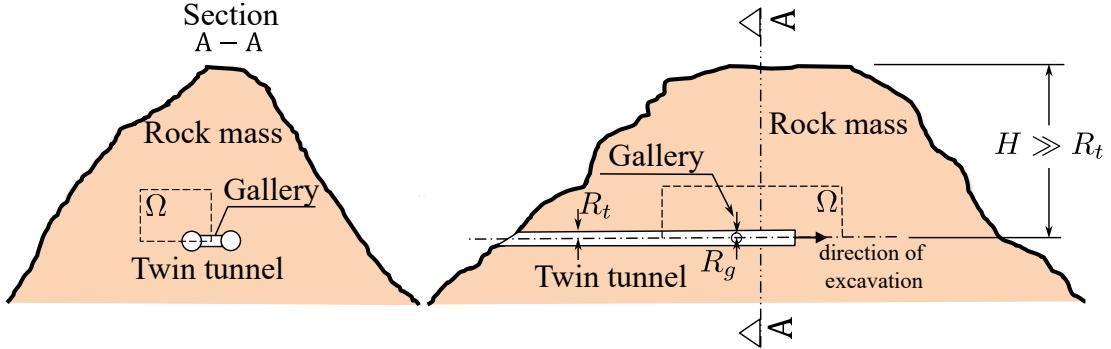
$$E_0 = E_c(t_0), \gamma(t - t_0) = \beta_c(t - t_0), \frac{1}{v(t)} = \frac{\phi_0(t_0)}{E_{ci}} \text{ and } \frac{1}{\eta(t)} \rightarrow 0 \quad (10)$$

in which  $t$  refers to the current time value and  $t_0$  to the concrete age at the instant of load application (time interval  $t - t_0$  is generally referred to as loading time or loading age). In the Generalized kelvin model introduced by Bažant and Prasannan [10, 11],  $E_0$  is the instantaneous elasticity modulus of the concrete formed aggregates and cement paste

particles,  $\gamma(t - t_0) = \sum_{i=1}^N \gamma_i$  is the microviscoelastic deformation of the volume fraction  $v(t)$  of solidified concrete and  $\eta(t)$  is the apparent macroscopic viscosity. In the CEB-FIP MC90 formulation [27],  $E_c(t_0)$  stands for the tangent elastic modulus of concrete at the instant of the loading application  $t_0$ ,  $\beta_c(t - t_0)$  is a coefficient that depends on the loading age  $t - t_0$ ,  $\phi_0(t_0)$  is a coefficient defining the delayed strain when loaded at age  $t_0$  of the concrete, and  $E_{ci}$  represents the tangent elasticity modulus of the concrete at the age of 28 day.

## 5. Spatial and time discretization of the domain

The geometry model of analyzed domain  $\Omega$  is schematically displayed in Fig. 4. It consists of a system of deep twin tunnels connected with a transverse gallery. The radius of the circular longitudinal tunnels is denoted by  $R_t$ , whereas that of the circular connecting gallery is denoted by  $R_g \leq R_t$ . The underground structure is excavated in a homogeneous rock mass at great depth  $H \gg R_t$ .



**Figure 4:** Schematic representation of the twin tunnels geometry problem.

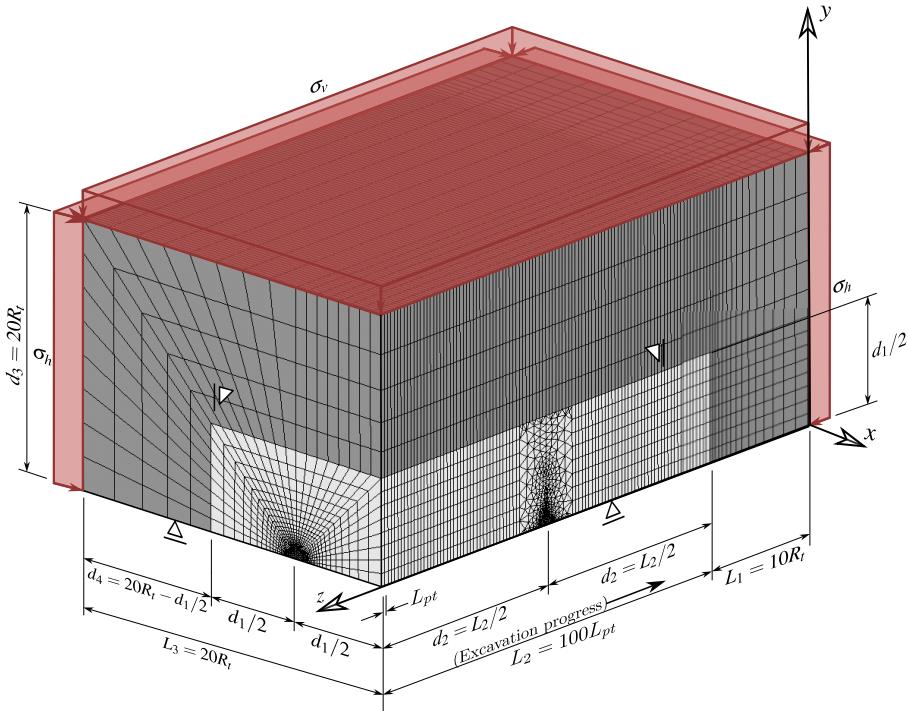
Within the analyzed material domain, the initial stress state prevailing in the rock mass prior to the tunnel excavation process is defined by constant vertical and horizontal geostatic stress  $\sigma_v$  and  $\sigma_h$ , taking the following form:

$$\sigma_0 = -\sigma_v e_y \otimes e_y - \sigma_h (1 - e_y \otimes e_y) \quad (11)$$

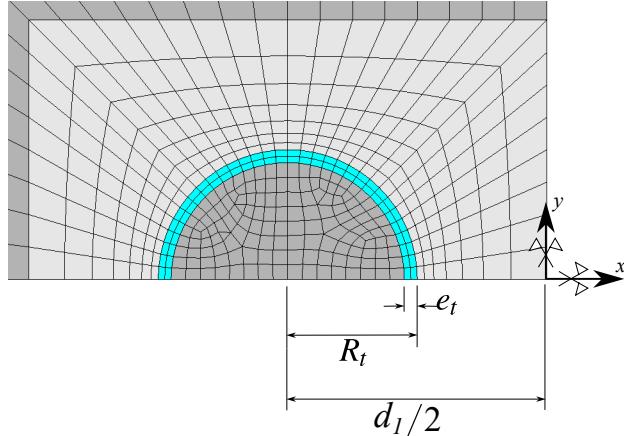
where  $e_y$  is the upward unit vector parallel to vertical direction. The initial horizontal stress is generally related to the vertical stress by means of the horizontal thrust coefficient  $\sigma_h = k_0 \sigma_v$ . Starting from the initial configuration of the material system  $\Omega$ , the processes of excavation (advancing face) and lining placement are simulated by means of the “activation/deactivation” technique [31, 32, 33, 29].

The geometry material domain  $\Omega$  considered for the finite element simulations, including tunnelling and deformation analysis, is defined by a parallelepiped volume of dimensions  $(L_1 + L_2) \times L_3 \times d_3$  (Fig. 5). Owing to the symmetry of the problem, only the material portion  $\{x \leq 0, y \geq 0\}$  is considered for F.E discretization and analysis. Referring to the notations of Fig. 5,  $d_1$  is the distance between the axes of longitudinal tunnels,  $L_2$  represents the total length along longitudinal direction  $e_z$  of the cylindrical volume to be excavated that is considered in the numerical simulation,  $d_3$  is the thickness along vertical direction  $e_y$  of material domain  $\Omega$ ,  $L_1$  stands for the length of unexcavated region after total excavation process,  $L_3$  is the total length along transversal direction  $e_x$  of discretized material domain,  $d_2$  characterizes the location of the circular transverse axis gallery that intersects the longitudinal tunnel at  $z = L_1 + d_2$ . The length of the excavation step adopted will be denoted by  $L_{pt}$ . The finite element model including geometrical discretization and boundary conditions is illustrated in Fig. 5. The mesh used in the simulations consists of 119740, 182470 or 221104 total elements (hexahedra and tetrahedra), depending on the value of spacing between longitudinal tunnels. To increase the accuracy of the model predictions in the intersection zone, the region surrounding the transverse gallery (including part of the longitudinal tunnel) is discretized by means 10-node quadratic tetrahedral elements, whereas 8-node trilinear hexahedral elements are used for the remaining part of the structure. Furthermore, a refined meshing is used for discretizing the zones surrounding the longitudinal and transverse gallery. These zones whose mechanical state is significantly affected by the tunnelling process are indicated by light gray color in Fig. 5.

Figures 6 to 10 display some details regarding the geometry and F.E discretization of the structure. Fig. 6 presents some details of the longitudinal tunnel cross-section in a  $xy$  plane, together with the layer of concrete lining (in sky blue color), parameter  $e_l$  being the thickness of the lining. Installation of the lining (shotcrete or precast concrete) is simulated in the F.E modeling by progressive activation of the corresponding elements, which consists in assigning to these elements the concrete mechanical properties.



**Figure 5:** Mesh, dimensions and boundary conditions of the 3D twin tunnel domain.



**Figure 6:** Detail 1 - Mesh in longitudinal tunnel cross-section with spacing  $d_1 = 4R_t$ .

An important issue investigated in this work is the influence of the spacing  $d_1$  between twin tunnels on their convergence. Fig. 7 and Fig. 8 illustrate the spatial discretization of the gallery region as well as of the connection with the longitudinal tunnel. Three values shall be considered for the spacing  $d_1$  in the numerical simulations, namely  $d_1 = 16R_t$ ,  $8R_t$  and  $4R_t$ . The layer of concrete lining of thickness  $e_g$  installed along the gallery wall is indicated by red color in the figures. Without introducing additional modeling restriction and for the sake of simplicity, the value of the gallery radius is fixed to  $R_g = 2/3R_t$ . The same lining system (same concrete material and layer thickness) is considered for both longitudinal tunnels and gallery. As regards the discretization of the region surrounding the gallery, parameters  $d_5$  and  $d_1$  define the size in a  $yz$  plane of the transition region involving the tetrahedral finite elements. Fig. 9 provides a view of the transition region and tunnel/gallery intersection zone.

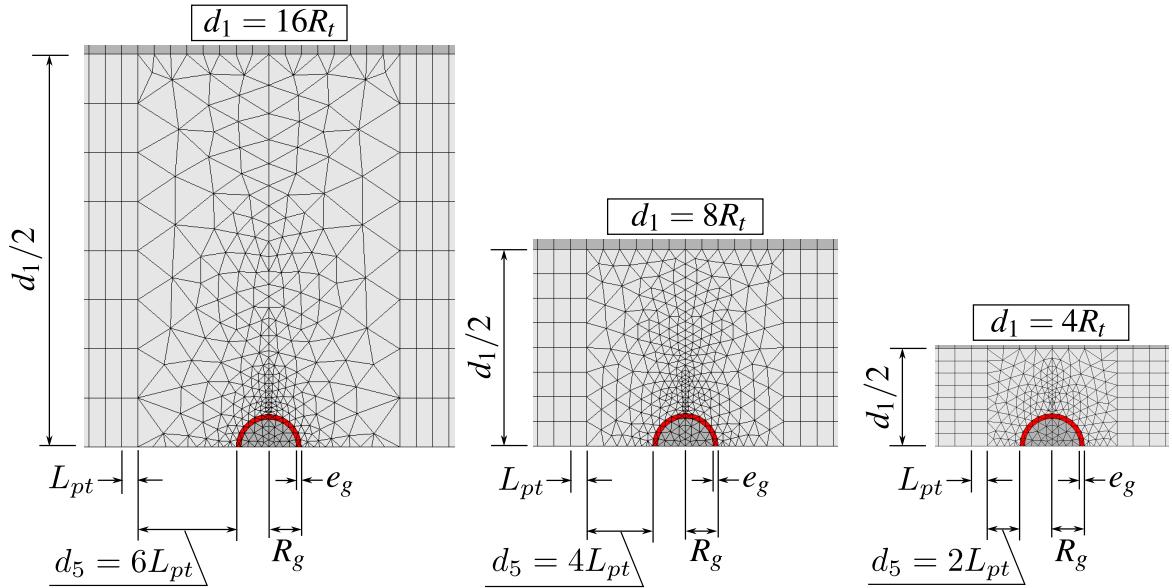


Figure 7: Geometry and F.E mesh of gallery cross-section for configurations  $d_1 = 16R_t$ ,  $d_1 = 8R_t$  and  $d_1 = 4R_t$ .

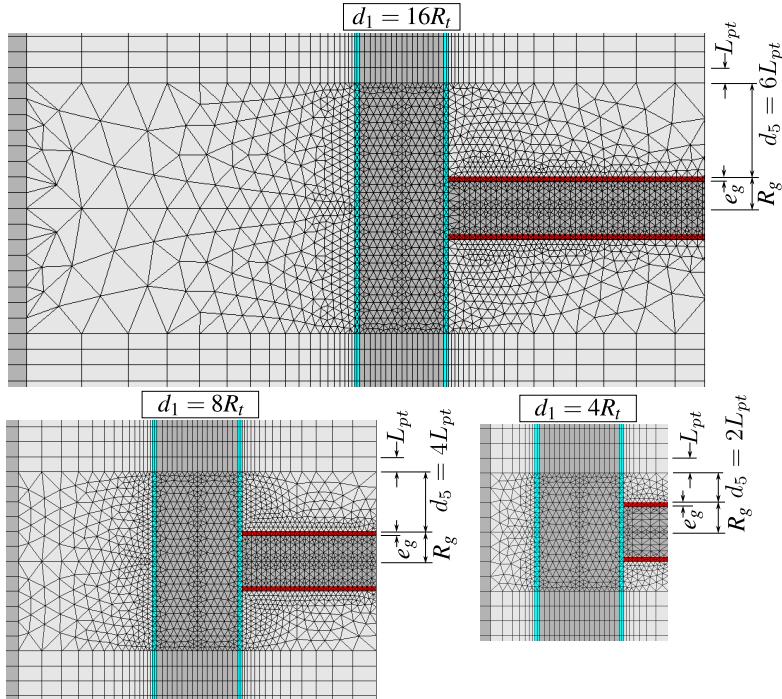
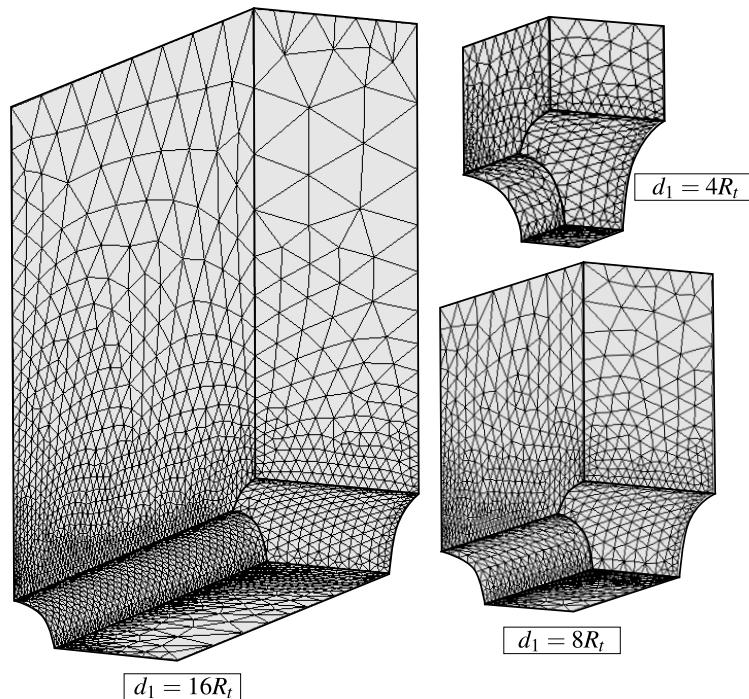
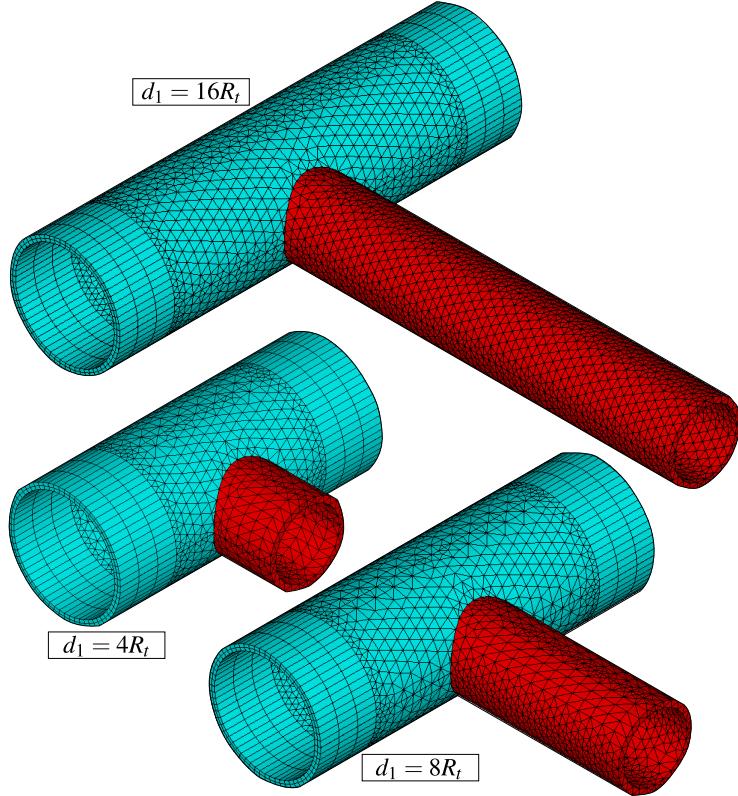


Figure 8: Views of longitudinal tunnel and gallery in the symmetry plane  $y = 0$  for configurations  $d_1 = 16R_t$ ,  $d_1 = 8R_t$  and  $d_1 = 4R_t$ .



**Figure 9:** View of the transition and tunnel/gallery intersection zones for configurations  $d_1 = 16R_t$ ,  $d_1 = 8R_t$  and  $d_1 = 4R_t$ .

Finally, Fig. 10 presents the F.E mesh used for the layer of concrete lining in both the longitudinal layer (in sky blue color) and the gallery (in red color) for the three configurations  $d_1 = 16R_t$ ,  $d_1 = 8R_t$  and  $d_1 = 4R_t$ , with specific details on the junction region of the gallery and the longitudinal tunnel. For the illustration purposes, symmetry with respect to plane  $y = 0$  has been used to complete the geometry representation of each configuration. It is emphasized that the tetrahedral elements used for the discretization of the region surrounding the transverse gallery exactly fits excavation steps (elements removal or deactivation).



**Figure 10:** Isometric view of the lining at the intersection for  $d_1 = 16R_t$ ,  $d_1 = 8R_t$  and  $d_1 = 4R_t$  - expansion of symmetry in the  $xz$  plane.

As mentioned previously, the tunnelling process, including the excavation steps and lining installation, is simulated resorting to the activation-deactivation method. Each excavation step is modeled by deactivation of the corresponding elements (the elements stiffness is reduced by a factor  $1E8$ ), whereas installation of elements of lining at a distance  $d_{0t}$  from the excavation face (unlined length) is achieved through activation of the corresponding elements by assigning them concrete properties. The F.E solution of the time-dependent problem is performed for each excavation step associated with time interval  $t_p = L_p/V_p$ , where  $L_{pt}$  represents the length of the excavation step and  $V_{pt}$  is the speed of the excavation face. Fig. 11 schematically displays the consecutive phases of excavation process. In this Figure,  $n_p$  is the total number of excavation steps and  $n_{pig}$  represents the number of longitudinal tunnel excavation steps prior to gallery excavation. After achievement of the  $n_{pig}$  excavation steps, the excavation of the gallery is initiated starting from the longitudinal tunnel wall. Referring to the notation of Fig. 11,  $L_{pg}$  is the considered step length for the gallery excavation,  $V_{pg}$  is the speed of the gallery excavation, and  $d_{0g}$  is the unlined length of the gallery. Each gallery excavation step is associated with time interval  $t_{pg} = V_{pg}/L_{pg}$ . After the gallery excavation is completed, we proceed to further excavation steps of the longitudinal tunnel.

For the sake of clearness, the main parameters defining the geometry domain as well as and excavation process and lining installation are summarized in Table 1.

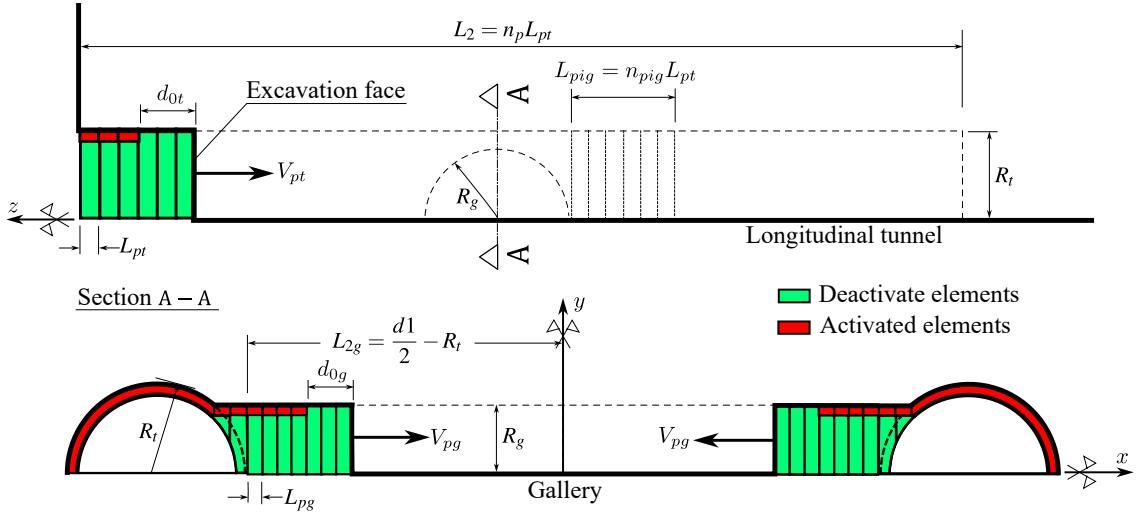


Figure 11: Schematic representation of the excavation process.

**Table 1**

Parameters related to the geometry of the domain, excavation and installation of the lining.

PARAMETERS	SYMBOL	UNIT	VALUES
Longitudinal tunnels			
Radius of the longitudinal tunnel	$R_t$	m	$R_t$
Thickness of the lining	$e_t$	m	$0.1 R_t$
Step length of the excavation process	$L_{pt}$	m	$1/3 R_t$
Unlined length	$d_{0t}$	m	$2 L_{pt}$
Speed of the excavation face	$V_{pt}$	m/day	12.5
Excavation step time	$t_p$	day	$L_{pt}/V_{pt}$
Gallery			
Radius of the gallery	$R_g$	m	$2/3 R_t$
Thickness of the concrete lining	$e_g$	m	$0.1 R_t$
Step length of the excavation process <sup>1</sup>	$L_{pg}$	m	$1/3 R_g$
Unlined length	$d_{0g}$	m	$2 L_{pg}$
Speed of the excavation face	$V_{pg}$	m/day	12.5
Number of steps that starts gallery excavation	$n_{pig}$	un	15
Rest of domain			
Distance between longitudinal tunnel axes	$d_1$	m	$4R_t$ $8R_t$ $16R_t$
Length of the unexcavated region	$L_1$	m	$10R_t$
Total excavated length	$L_2$	m	$100L_{pt}$
Domain height	$L_3$	m	$20R_t$

<sup>1</sup>Value of  $L_{pg}$  is slightly different for the last excavation step to match the gallery length.

During the tunnel construction phases, the time increment used for the time-dependent analysis is automatically managed by the ANSYS solver. The latter makes use of a semi-implicit scheme for the viscoplasticity solution, together with an automatic time stepping algorithm [34] in which the time step is defined as a fraction of time  $t_p$  for the phases of longitudinal tunnel excavation and as a fraction of  $t_{pg}$  for the phases of transverse gallery excavation. Furthermore, distinct time steps are considered for the time-dependent analysis during tunnelling process and post-excavation stage. After complete tunnel construction phases, the analysis is carried out for a period of about 3000 days to assess the time evolving deformation as well as long-term viscous effects on the final equilibrium of the tunnel structure. At that respect and in anticipation of the numerical results of the subsequent sections, the characteristic viscoplastic relaxation time [35] is equal to  $\bar{\tau} = \eta f_0/E$ , which is close to 30 days for model data of Table 2.

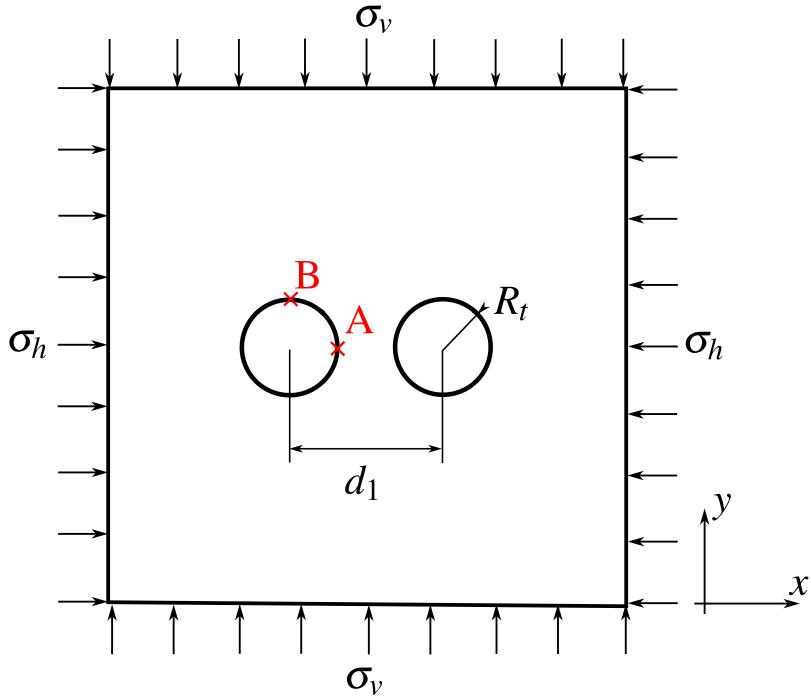
## 6. Preliminary numerical simulations and computational model verification

This section is aimed at applying the computational modeling to simulate deformation and stress in two academic twin tunnels configurations. The numerical results provided in these illustrative applications may be viewed as preliminary verifications of the F.E formulation. The first application refers to unlined twin tunnels excavated in an elastic rock mass, whereas the second application addresses the situation of unlined twin tunnels excavated in an elastoplastic medium.

### 6.1. Unlined twin tunnels in elastic medium

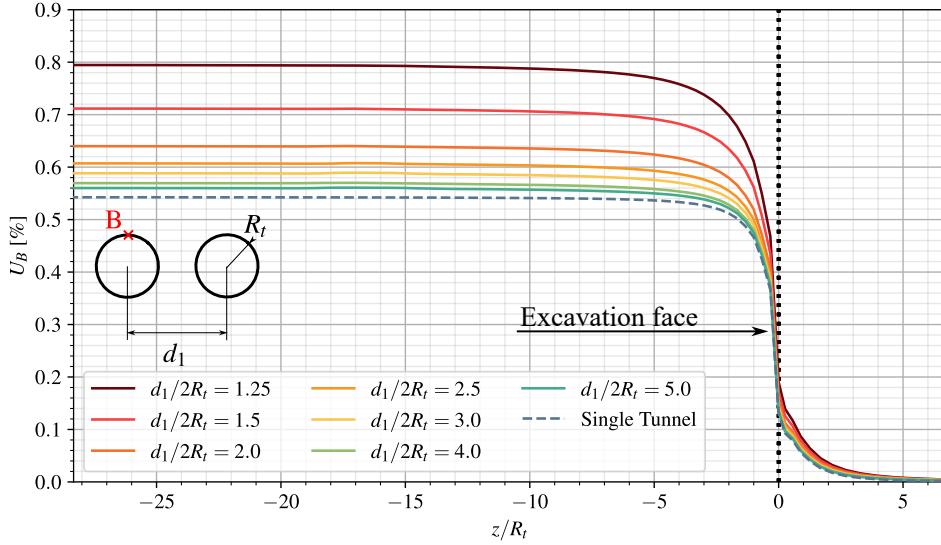
In the context of plane strain conditions, Guo et al. [6] addressed the configuration of deep twin tunnels excavated in a homogeneous elastic medium in which prevails a hydrostatic initial stress distribution. The authors formulated approximate analytical solutions for the stress distribution establishing far behind the face, which are induced in the rock mass by the excavation of two parallel circular tunnels. The model geometry of the twin circular tunnels as well as the loading associated with initial hydrostatic stress ( $\sigma_h = \sigma_v$ ) are displayed in Fig. 12.

Simulation of the problem has been addressed by means of the 3D finite element model and the numerical results obtained for the stress distribution far behind the faces of the twin tunnel shall be compared to the analytical stress solution derived by Guo et al. [6] in the framework of plane strain conditions. The simulations have been performed taking advantage of symmetry with respect to the midplane between twin tunnels and considering the following model data: tunnel radius  $R_t = 4$  m, rock Young modulus  $E = 500$  MPa and Poisson ratio  $\nu = 0.23$ , isotropic initial stresses of  $\sigma_v = \sigma_h = 2.2$  MPa.



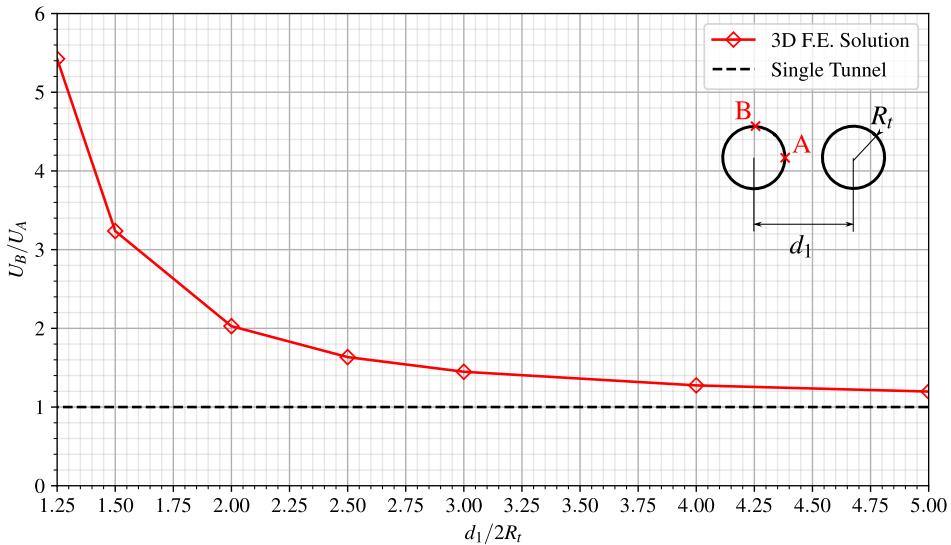
**Figure 12:** Geometry model and loading mode of the twin circular tunnels studied in Guo et al. [6].

Denoting by  $u_y$  the displacement component following the  $y$ -axis, Fig. 13 displays the convergence curves  $U_B = -u_y(B)/R_t$  that characterize the inward movement at the tunnel roof  $B(x = 0, y = R_t, z)$  as a function of normalized longitudinal distance to the facing. Several values of normalized distances between the twin tunnels axes  $d_1/2R_t$  have been investigated, and the configuration of single tunnel may be viewed as the limiting case  $d_1/2R_t \gg 1$ . It is recalled that in the latter configuration, the convergence far from the tunnel face that is obtained from an elastic analysis reads  $U = \sigma_v(1 + \nu)/E$ . As expected, this figure indicates that the closer the longitudinal tunnels, the greater the convergence at the roof.



**Figure 13:** Convergence profiles in the crown (point B).

The tunnel deformation anisotropy induced by the twin tunnels proximity is illustrated in Fig. 14, which plots the ratio  $U_B/U_A = u_y(B)/u_x(A)$  between the vertical displacement  $u_y$  at the roof B and the horizontal displacement  $u_x$  at the side wall A( $x = R_t$ ,  $y = 0$ ,  $z$ ). The results refer to a tunnel section located far behind the facing at normalized distance  $z/R_t = -25$ . They emphasize the significative tunnel ovalization induced by the proximity of twin tunnel as the distance  $d_1/2R_t$  decreases.



**Figure 14:** Illustration of the tunnel wall deformation anisotropy induced by twin tunnels proximity.

The stress distribution prevailing far from the facing that were obtained from the 3D numerical simulations are compared in Fig. 15 to the stress solutions derived analytically and numerically in Guo et al. [6]. In this figure, the tangential stress concentration factor  $\sigma_{yy}/\sigma_v$  computed at the side wall A is plotted for several values of the normalized

twin tunnels distance. The results of the theoretical solution to a plate containing two circular holes of equal size presented in Ling et al. [37] are also reported in Fig. 15. It is observed that the results of the 3D finite element simulations correspond to a tunnel section located at normalized distance  $z/R_t = -25$  from the facing, which is considered sufficient for the plane strain conditions to establish. Interestingly, the tangential stress concentration obtained for a deep single tunnel under plane strain condition simply reads  $\sigma_{yy}/\sigma_v = 2$ . Although the overall agreement observed between the different predictions, it appears from the comparison that the approximate analytical stress solution provided in [6] slightly overestimates the tangential stress computed at point A as the value of distance  $d_1/2R_t$  increases.

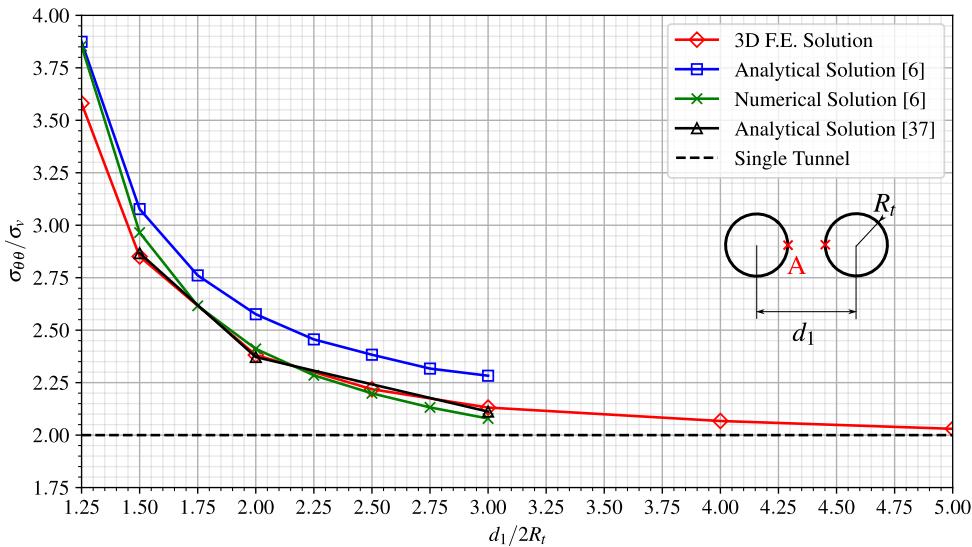


Figure 15: Tangential stress concentration factor at the side wall A versus twin tunnels distance  $d_1/2R_t$ .

Finally, Fig. 16 displays the distribution of tangential (orthoradial) stress  $\sigma_{\theta\theta}$  around the tunnel boundary  $\{r = R_t, 0 \leq \theta \leq \pi\}$  considering  $d_1/2R_t = 1.5$ . The predictions of stress component  $\sigma_{\theta\theta}$  obtained from the 3D finite element simulations far behind the facing are shown together with the strain plane solutions derived analytically in [6], emphasizing the ability of the computational model to accurately capture the effect of tunnels proximity on stress distribution.

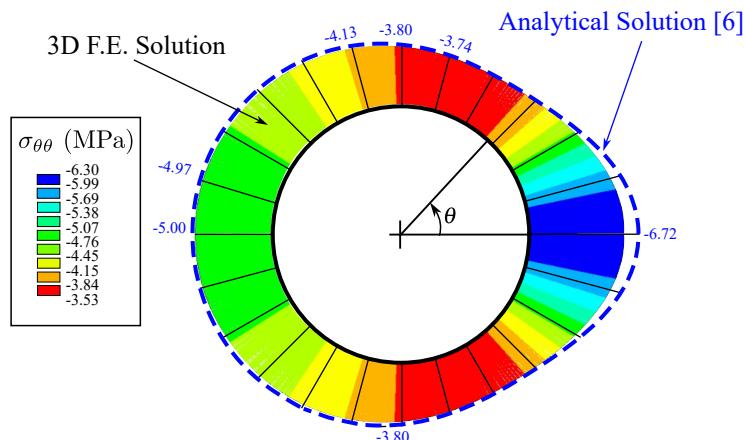


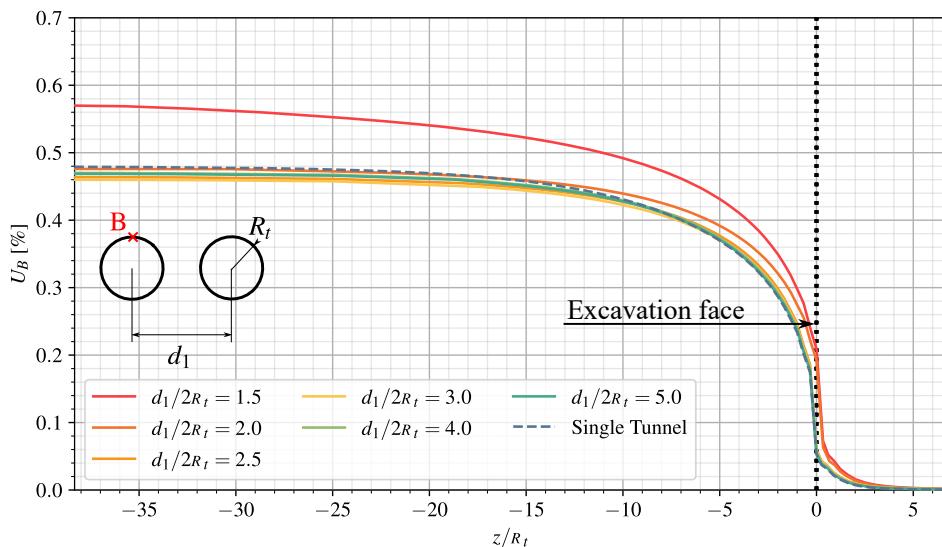
Figure 16: Distribution of tangential stress  $\sigma_{\theta\theta}$  around the tunnel wall prevailing far behind the facing (twin tunnels distance  $d_1/2R_t = 1.5$ ).

Keeping in mind it addresses only an academic configuration, the results provided in this section may be viewed as a first preliminary verification of the accuracy of the computational model formulated for the mechanical interaction in deep twin tunnels.

## 6.2. Unlined twin tunnels in elastoplastic medium

In the context of plane strain conditions, Ma et al. [9] developed an approximate analytical solution for the stresses and the plastic zone boundary around deep twin circular tunnels excavated in a homogeneous elastoplastic medium. The geometric model and initial stress state are the same as those shown in Fig. 12. However, this model does not assume necessarily equal initial stresses, and the initial stress along the tunnel axis is assumed as the middle stress, i.e.,  $\sigma_{zz} = (\sigma_v + \sigma_h)/2$ . For the constitutive model, the authors considered perfectly plastic Mohr-Coulomb criterion with associated plastic flow rule.

Numerical results of the convergence and the stress distribution localized in a section far from the excavation face, obtained from the 3D finite element model, are compared with the analytical solution for the plane strain derived by [9]. The simulations also utilized symmetry concerning the midplane between the twin tunnels. Denoting by  $u_y$  the displacement component along the y-axis, Fig. 17 displays the convergence curves  $U_B = -u_y(B)/R_t$ , that characterize the inward movement at the tunnel roof  $B(x = 0, y = R_t, z)$  as a function of normalized longitudinal distance to the facing, using the following data:



**Figure 17:** Convergence profiles in the crown (Point B).

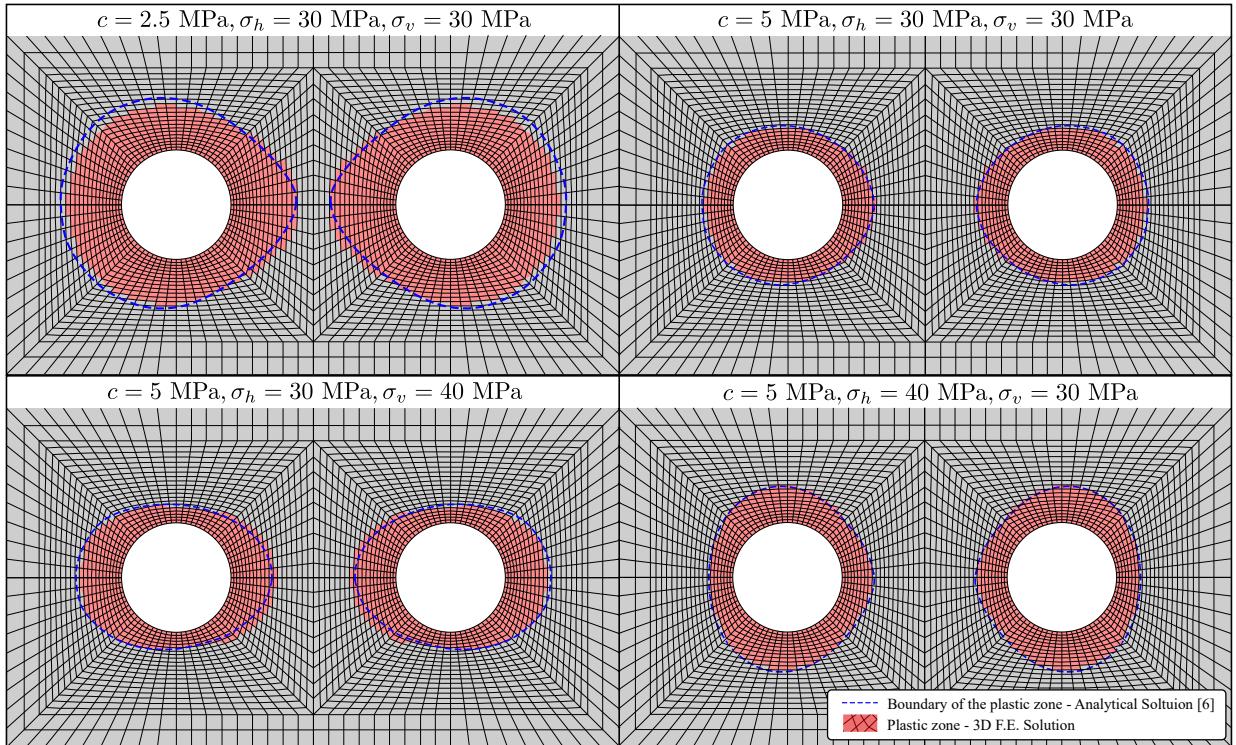
continue from here

The authors consider symmetry in the midplane between twin tunnels and assume that the plastic zones around each tunnel are isolated from each other and completely surround each tunnel. Their analytical solution is verified with plane strain numerical solutions obtained using the finite difference program FLAC3D.

Denoting by  $u_y$

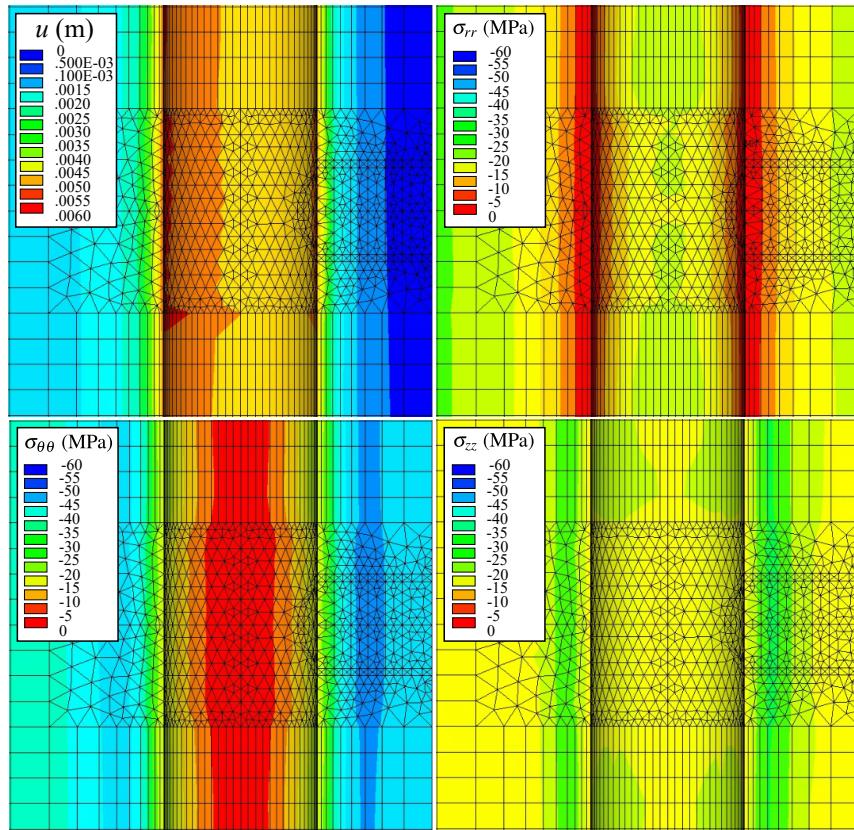
As a result, the authors obtained the contour of the plastic zone for several initial conditions and constitutive parameters, i.e., cohesion  $c$  and friction angle  $\phi$ .

Fig. 18 shows the comparison between the 3D F.E. Solution (from a far behind the excavation face) and the analytical solution for plastic zone boundary provided in [9]. For these analysis,  $R_t = 1$  m,  $d_1 = 5$  m, Young's modulus  $E = 20$  GPa, Poisson's ratio  $\nu = 0.3$  and, friction angle  $\phi = 30^\circ$ . The boundary of the plastic zone agrees very well with the elements that have reached the plasticity criterion.



**Figure 18:** Comparisons between the numerical and analytical solution for the plastic zones considering  $d_1 = 5$  and  $\phi = 30^\circ$  for different initial conditions and cohesion values.

Fig. 19 displays the magnitude of displacements  $u$  and stresses  $\sigma_{rr}$ ,  $\sigma_{\theta\theta}$  and  $\sigma_{zz}$  at the element level for the case with  $c = 5$  MPa,  $d_1 = 5$  m,  $\sigma_h = \sigma_v = 30$  MPa. We adopted a case without excavating the gallery to assess the quality of the mesh. The smoothness observed in the solution between the elements indicates satisfactory discretization.



**Figure 19:** Element solution with  $c = 5$  MPa,  $d_1 = 5$  m,  $\sigma_v = \sigma_h = 30$  MPa at the junction of the longitudinal tunnel with the gallery from the viewpoint of the symmetry plane  $y = 0$

## 7. Numerical Results and Discussion

To develop the parametric analyses, we employed the constitutive parameters of the clay rock mass in the Paris basin (specifically in Aisne), as detailed in Piepi [22] and Rousset [18]. These parameters, shown in Table 2, were derived qualitatively from various axisymmetric triaxial compression tests, including cyclic and creep tests conducted under undrained conditions. Previous research by Giraud [20] indicates that for Aisne clay rocks, characterized by low porosity (typically less than 20%), hydromechanical coupling has minimal significance. The delayed effects primarily stem from material viscosity, with a low proportion attributable to pore pressure redistribution (hydraulic diffusion). The material exhibits high density (ranging from 2.01 to 2.57) and a low average water content (between 3 to 11%). Another characteristic is that irreversible deformations are observed in cyclic tests even at very small values of axial deformation (less than 0.3%). Furthermore, for confinement values exceeding 10 MPa (approximately the in situ confinement), the maximum deviation remains practically constant, suggesting a Tresca-type failure criterion.

During creep tests on this material, it was observed that creep deformations are of comparable magnitude to deformations observed during instantaneous tests, and there exists a deviatoric stress threshold beyond which creep phenomena initiate. Moreover, it was found that the influence of confining pressure on creep phenomena can be disregarded. Comparing both behaviors—instantaneous and delayed—reveals that short-term cohesion exceeds long-term cohesion, with the ratio between these two cohesion values ranging between 1.2 and 2.

For the lining, we employed typical values for ordinary reinforced concrete.

These parameters are shown in Table 2. In these analyses the radius is  $R_t = 1$  m and excavation speed is 12.5 m/day.

**Table 2**

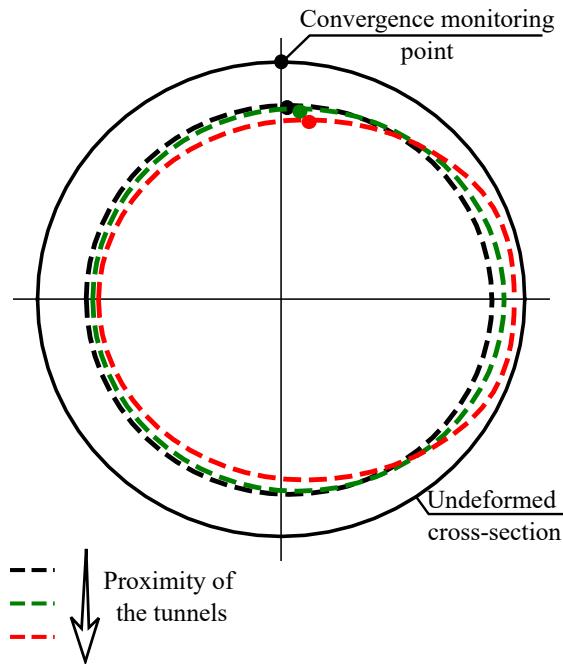
Constitutive parameters used in the parametric analysis

PARAMETERS	SYMBOL	UNIT	VALUES
Constitutive model of rock mass			
Initial stress state (isotropic)	$\sigma_x, \sigma_y, \sigma_z$	MPa	9
Young's modulus	$E$	MPa	1500
Poisson's ratio	$\nu$	adm	0.498
Plastic cohesion	$c$	MPa	$4\sqrt{3}/2$
Plastic friction angle	$\phi$	°	0
Viscoplastic cohesion	$c_{vp}$	MPa	$2\sqrt{3}/2$
Viscoplastic friction angle	$\phi_{vp}$	°	$\phi$
Power law parameter	$n$	adm	1
Reference parameter	$f_0$	MPa	1
Viscosity coefficient	$\eta$	day	40000
Constitutive model of lining			
Compressive strength	$f_{ck}$	MPa	20
Young's modulus at 28 days	$E_{c_{28}}$	MPa	30303
Poisson's ratio	$\nu$	adm	0.2
Coefficient which depends on the type of cement	$s$	adm	0.2
Relative humidity of ambient environment	$RH$	%	70
Fictitious thickness <sup>2</sup> (longitudinal tunnel)	$h_f$	cm	0.2111
Fictitious thickness (transverse gallery)	$h_{fg}$	cm	0.2176
Drying time of the concrete	$t_s$	days	7
Coefficient in shrinkage which depends on the type of cement	$\beta_{sc}$	adm	8
Temperature	$T$	°C	20°
Age of concrete at loading	$t_0$	days	1

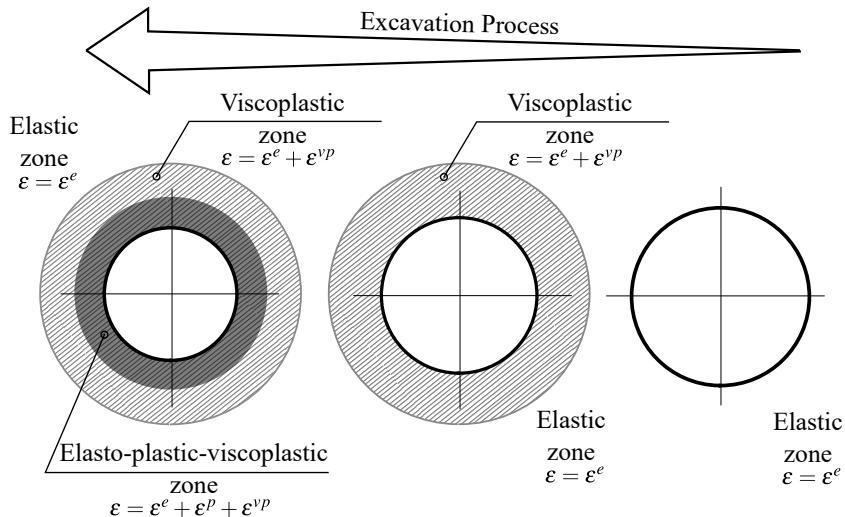
<sup>2</sup> $h_f = [R_t^2 - (R_t - e)^2]/(R_t - e)$  for a circular tunnel cross-section with internal perimeter in contact with atmosphere

In presenting the results,  $U_{eq}$  denotes the equilibrium convergence value at the convergence profile outside the region of influence of the excavation face and the gallery. When the gallery is present, the highest convergence value,  $U_{peak}$  is highlighted at the gallery position. In addition, it is necessary to highlight some important points:

- **Observation 1:** All the results presented in the following analyses pertain to the point located at the top of the tunnel section (crown), and we will monitor its convergence throughout the excavation process. Fig. 20 presents this point. Likewise, we will only analyze the convergence of the point located at the crown of the gallery.
- **Observation 2:** Under material isotropy and initial stress state, the symmetry of the tunnel wall is preserved throughout the excavation process. Thus, the deformed tunnel wall remains circular. On the other hand, one of the effects of the mutual interaction induced by the proximity of the tunnels is the loss of symmetry of the deformed tunnel wall, as illustrated in Fig. 20. In this context, the point chosen to follow the convergences (on the crown) is not representative of the entire deformation of the tunnel wall.
- **Observation 3:** Referring to the material properties shown in Table 2, the value adopted for plastic cohesion ( $c$ ) is higher than the value for viscoplastic cohesion ( $c_{vp}$ ):  $c > c_{vp}$ . This implies that in the regime of irreversible deformations, the viscoplasticity of the material will be activated first. Throughout the excavation process, viscoplastic deformations will appear without plasticization of the massif. The generic configurations of the deformation zones of the massif throughout the excavation process are illustrated in Fig. 21.



**Figure 20:** Monitoring point and ovalization effect



**Figure 21:** Configurations for the zones with irreversible deformations in the rock mass

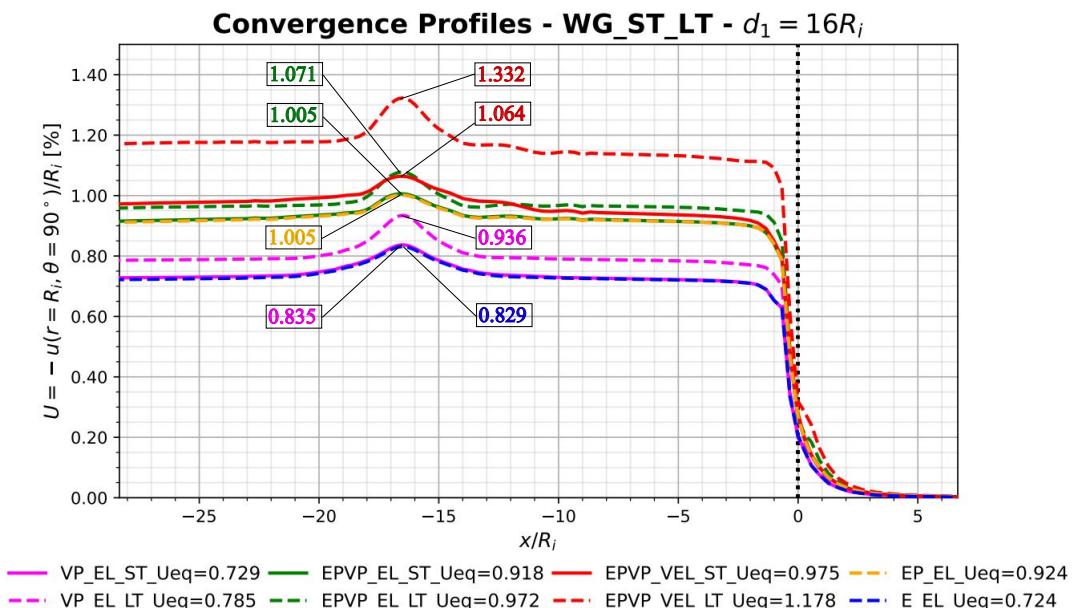
Table 3 presents the abbreviation to read the title and legend of the results.

**Table 3**

Abbreviation to the title and legend of the results

DESCRIPTION	ABBREVIATION
Elastic rock mass	E
Elastoplastic rock mass	EP
Elastoviscoplastic rock mass	VP
Elasto-Plastic-Viscoplastic rock mass	EPVP
Not lining	NL
Elastic lining	EL
Viscoelastic lining	VEL
Long-term	LT
Final excavation (Short-term)	ST
With Gallery	WG
Not Gallery	NG

Figs. 22, 23, and 24 show the convergence profiles of the twin tunnels with gallery (WG) for all the constitutive models of the rock mass (E - blue, EP - yellow, VP - magenta, EPVP - red) and the lining (EL and VEL) in the short-term (solid lines) and the long-term (dashed lines), for  $d_1 = 16R_t$ ,  $8R_t$  and  $4R_t$  respectively.

**Figure 22:** Convergence Profiles - with gallery (WG), short-term (ST) and long-term (LT) for  $d_1 = 16R_t$

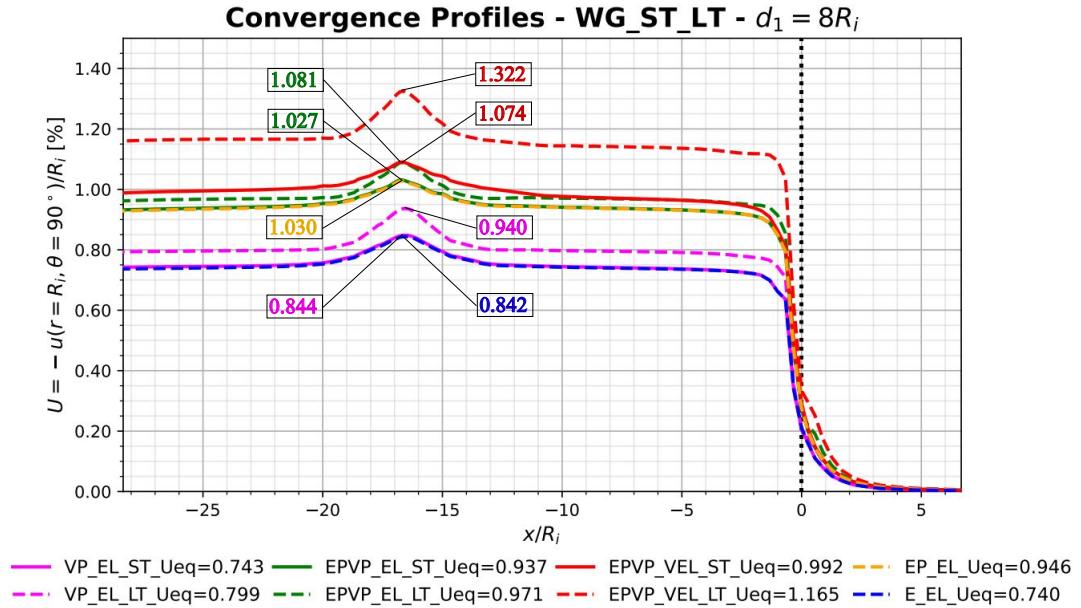


Figure 23: Convergence Profiles - with gallery (WG), short-term (ST) and long-term (LT) for  $d_1 = 8R_i$

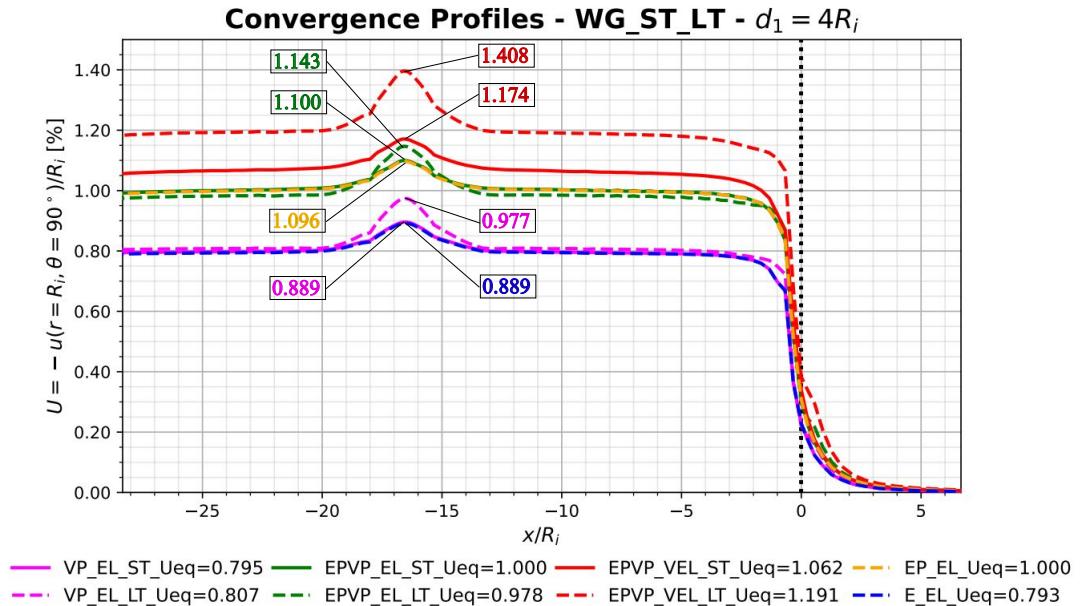


Figure 24: Convergence Profiles - with gallery (WG), short-term (ST) and long-term (LT) for  $d_1 = 4R_i$

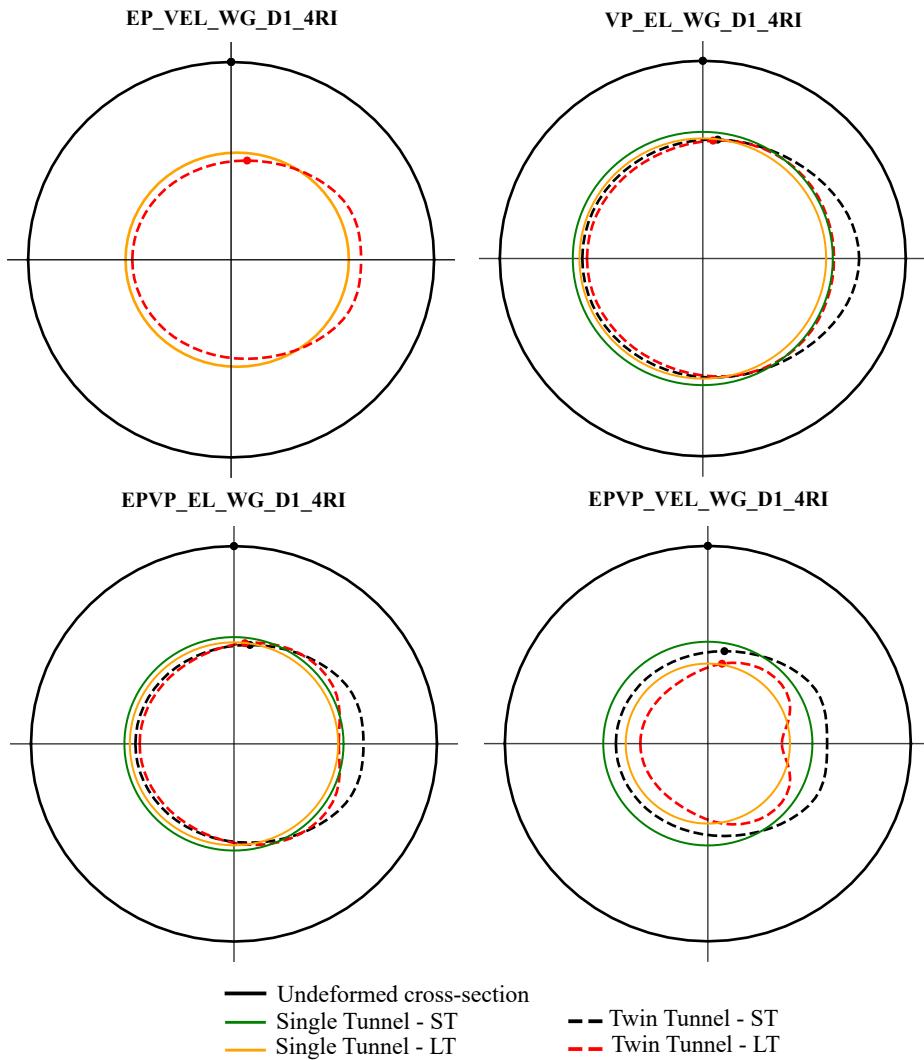
In all  $d_1$  distances, the convergence profiles of the E-EL model (blue dashed line) and the VP-EL (magenta solid line) in the short-term (ST) are equivalent, probably due to the high excavation speed. The high speed of the excavation and installation of the lining limits the time for the viscous effects to manifest themselves also taking into account the restriction imposed by the stiffness of the lining.

In the short-term (ST), the EPVP-EL model (green solid line) is equivalent to the EP-EL (yellow dashed line) because, although plasticization around the section has already occurred due to excavation, the viscous effects have not yet evolved considerably due to the short time variation between the start of excavation and the end of the excavation process. After the long-term convergence occurs (green dashed line) are a difference. However, when the rheological effect of the lining is present, the profile continues to evolve considerably over the long-term, for example, EPVP-VEL (red solid and dashed line).

It's worth noting that the stiffness of the elastic lining significantly impedes the evolution of convergence due to viscous effects, particularly evident in the VP-EL model with  $d_1 = 4R_t$  (magenta solid and dashed line). In this scenario, the interaction between nearby twin tunnels causes a substantial rise in the value of  $U_{eq}$  in the short-term (ST). However, the profile in the long-term (LT) practically remains unchanged, staying close to the short-term due to the limitation imposed by the stiffness of the lining.

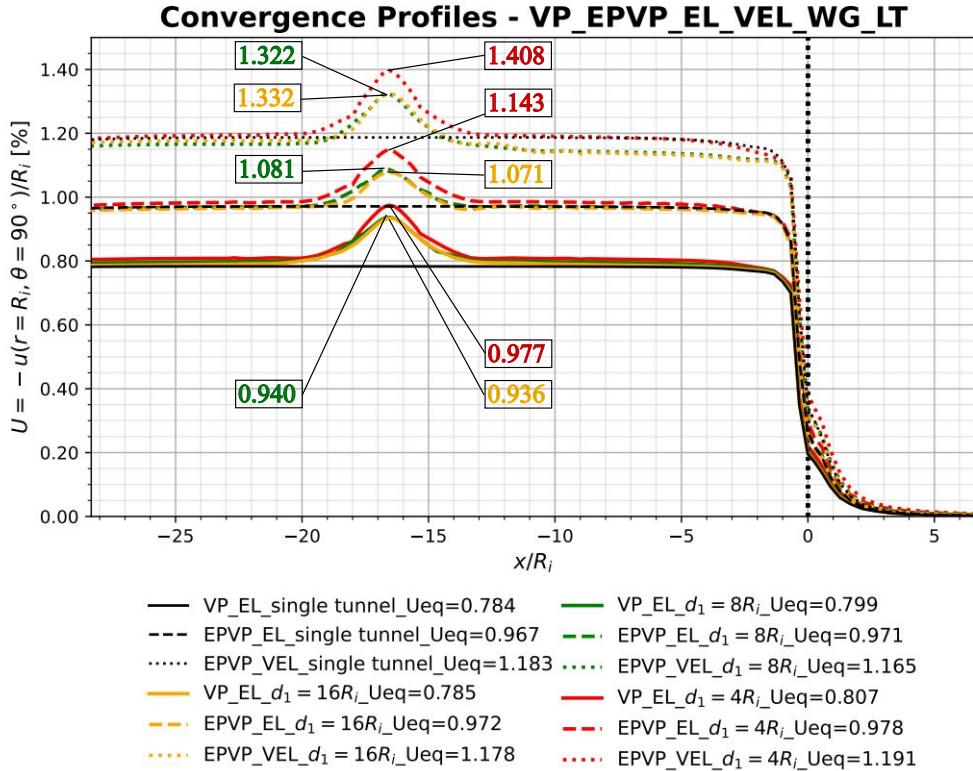
Another noteworthy aspect is that the EPVP-VEL model with  $d_1 = 16R_t$  (red dashed line) experiences a reduction in  $U_{eq}$  convergence after 15 excavation steps ( $n_{pig}$ ) following the gallery. This phenomenon is due to the evolving viscous effects of the already-excavated longitudinal tunnel during the gallery excavation. This effect becomes more pronounced with  $d_1 = 16R_t$ . When the gallery is smaller ( $d_1 = 8R_t$  and  $4R_t$ ), the time elapsed is shorter, and this effect is less pronounced.

Note another aspect: the EPVP-EL-LT (green solid line) model converges slightly lower than the EVP-EL-ST (green dashed line) model with  $d_1 = 4R_t$ . The ovalization effect is responsible for the crown's convergence decreasing over time. However, another point in the section experiences an increase in convergence. Fig. 25 illustrates this effect away from the gallery region with a single tunnel reference.



**Figure 25:** Ovalization effect for  $d_1 = 4R_i$  scaled 50x

Fig. 26 compares the convergence profiles of the viscoplastic rock mass (VP) with elastic lining (EL) models (solid lines) with the elasto-plastic-viscoplastic rock mass (EPVP) with elastic (EL) and viscoelastic lining (VEL) models in the long-term (LT) (dashed lines and dot lines, respectively). As a reference, it also shows the results for a single tunnel (black lines).

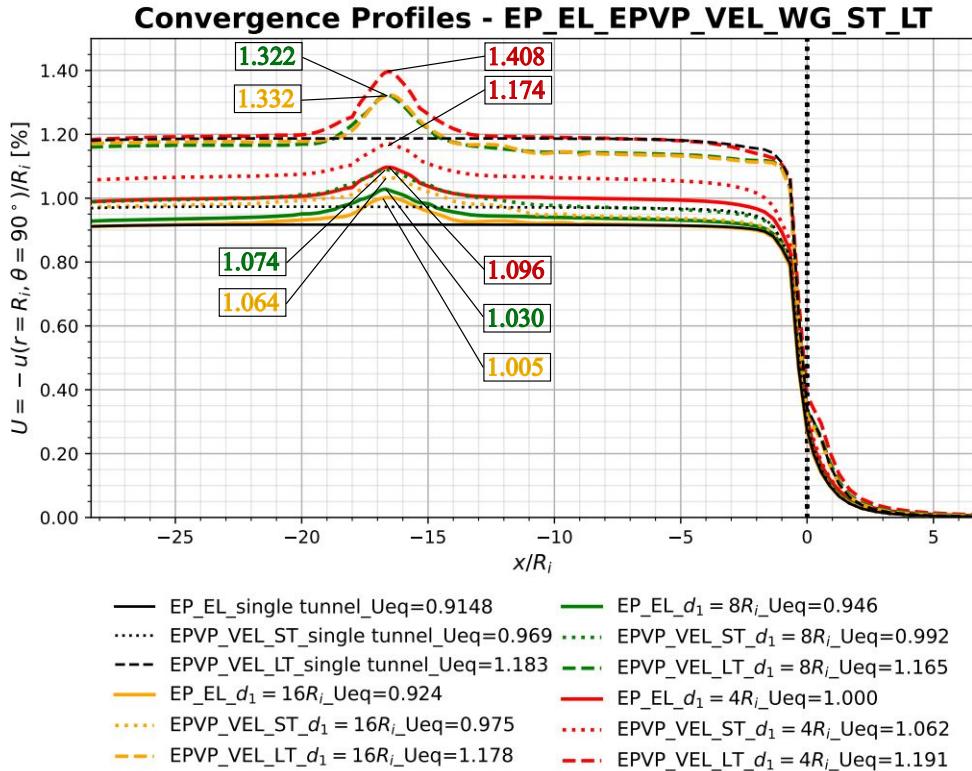


**Figure 26:** Convergence Profiles - viscoplastic rock mass (VP) with elastic lining (EL) versus elasto-plastic-viscoplastic rock mass (EPVP) with elastic (EL) and viscoelastic lining (VEL) in long-term (LT)

Figure 26 shows a slight increase in the peak value of convergence  $U_{peak}$  for the EPVP-VEL model when comparing  $d_1 = 16R_t$  (dotted yellow line) and  $d_1 = 8R_t$  (dotted green line). In the case of  $d_1 = 8R_t$ , the proximity of the tunnel compensates the convergence difference due the gallery excavation elapsed time between  $d_1 = 8R_t$  and  $16R_t$ . However, when  $d_1 = 4R_t$  (dotted red line) the effect of the gallery is more pronounced due to the interaction between the proximity of the twin tunnels and the viscous effect.

Moreover, one can observe a more pronounced effect in the section before the gallery, specifically at the plateau of the convergence profile, in the EPVP-VEL model for  $d_1 = 8R_t$  and  $d_1 = 16R_t$  (green and yellow dotted lines). This effect is due to the viscoelastic behavior of the lining and elapsed time to excavate the gallery. Unlike the highly rigid elastic lining, the viscoelastic lining allows the convergence to evolve during the excavation of the gallery. Consequently, the values of convergences before the gallery tend to be higher than after the gallery.

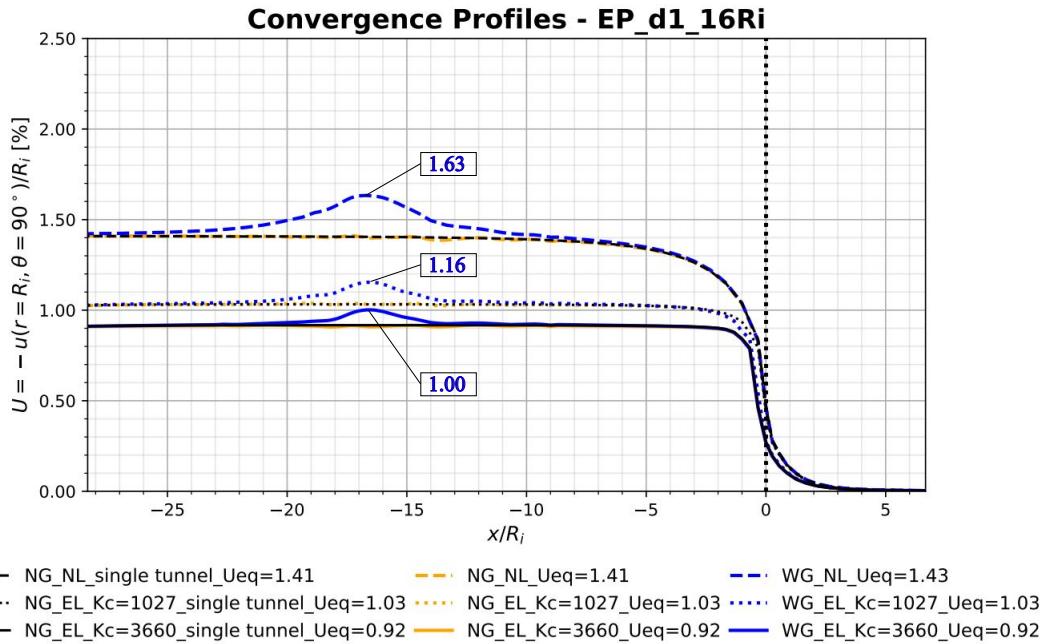
Fig. 27 compares the convergence profiles of the elastoplastic rock mass (EP) and elastic lining (EL) models (solid lines) with the elasto-plastic-viscoplastic rock mass (EPVP) and viscoelastic lining (VEL) models in the short-term (ST) (dot lines) and long-term (LT) (dashed lines). As a reference, it also shows the results for a single tunnel (black lines).



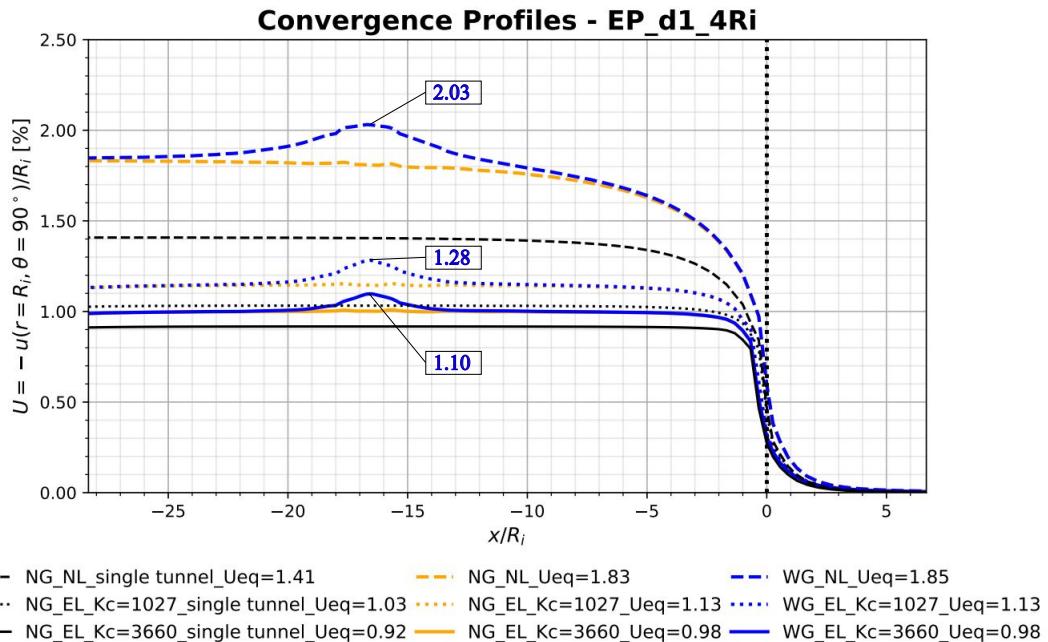
**Figure 27:** Convergence Profiles - elastoplastic rock mass (EP) with elastic lining (EL) versus elasto-plastic-viscoplastic rock mass (EPVP) with viscoelastic lining (VEL) in short-term (ST) and long-term (LT)

This Figure shows the crucial effect of the viscoelastic lining to the convergence profile of the tunnels. In the short term (ST), the elasto-plastic-viscoplastic rock mass (EPVP) with viscoelastic lining (VEL) (dotted lines) shows superior convergences compared to the elastoplastic (EP) model with elastic lining (EL) (solid lines). Because the young age of the viscoelastic lining (VEL) has a lower modulus of elasticity, resulting in lower stiffness. Therefore, compared to the elastic lining (EL), the lower initial value of the modulus of elasticity contributes more to the development of convergence. In the long term (LT), even though the viscoelastic lining (VEL) (dashed lines) has a higher stiffness due to a aging of lining, the viscous effects over time result in a significantly more discrepant convergence profile compared to the elastoplastic model (EP) with elastic lining (EL) (solid lines). There is a noticeable increase in the magnitude of  $U_{peak}$  between the short term and the long term at the gallery position, highlighting the influence of the viscoelastic lining.

To study the effect of the lining, Fig. 28 and 29 show the elastoplastic rock mass (EP) under various conditions: without lining (NL), with a moderately stiff elastic lining ( $K_c = 1027$  MPa), and with a highly stiff lining ( $K_c = 3660$  MPa) with (WG) and without gallary (NG).



**Figure 28:** Convergence Profiles - elastoplastic rock mass (EP) without lining (NL) with a highly stiff elastic lining ( $K_c = 3660$  MPa) and a moderately stiff elastic lining ( $K_c = 1027$  MPa), without (NG) and with gallery (WG) for  $d_1 = 16R_t$



**Figure 29:** Convergence Profiles - elastoplastic rock mass (EP) without lining (NL) with a highly stiff elastic lining ( $K_c = 3660$  MPa) and a moderately stiff elastic lining ( $K_c = 1027$  MPa), without (NG) and with gallery (WG) for  $d_1 = 4R_t$

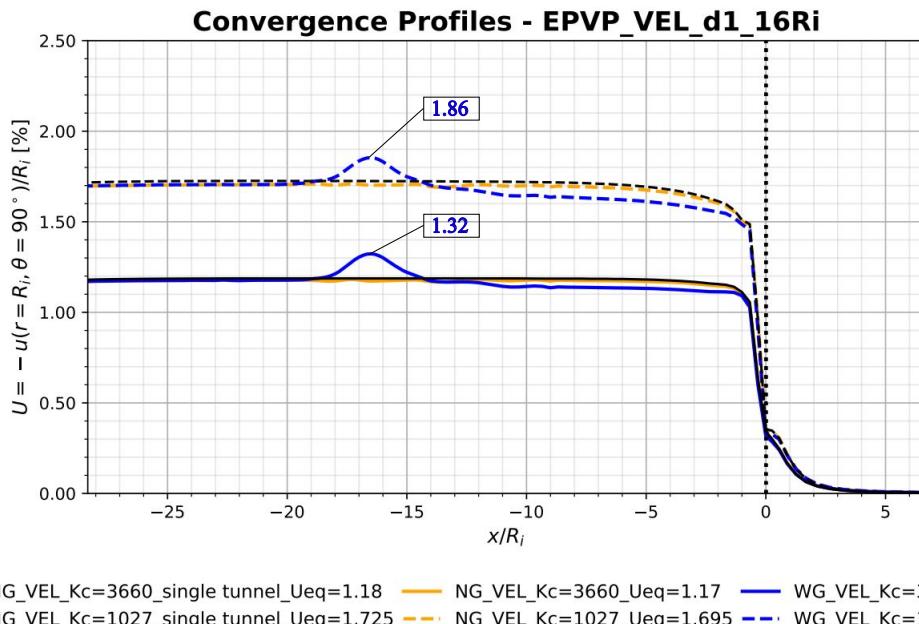
For the single tunnel, a high stiffness lining (black solid line) decreases convergence by approximately 35% compared to the unlined model (black dashed line). Conversely, a moderately stiff lining (black dotted line) increases convergence by 12% compared to the rigid lining.

When  $d_1 = 16R_t$  between the twin tunnels (blue and yellow lines), the results of  $U_{eq}$  are similar to the isolated tunnel (black line). However, with a distance reduced to  $d_1 = 4R_t$ , the interaction between the tunnels becomes significant. A smaller  $d_1$ , the high stiffness lining (solid yellow and blue lines) can restrict convergence by up to 46% of the unlined (dashed yellow and blue lines) convergence. A moderate stiffness lining (dotted lines) leads to an increase of up to 16% in convergence compared to the high stiffness lining (solid lines).

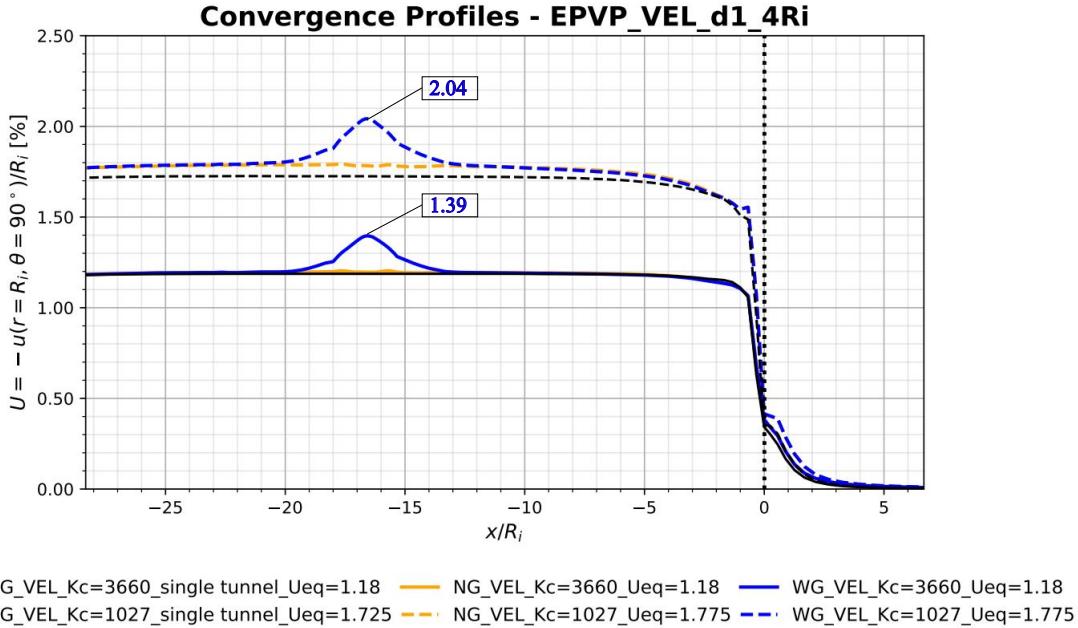
When comparing results between twin lined tunnels with spacings of  $16R_t$  and  $4R_t$ , differences of 6% with high stiffness lining (solid yellow and blue lines), 10% with moderate stiffness lining (dotted yellow and blue lines), and 30% without lining (dashed yellow and blue lines) are observed. These results show the direct impact of lining stiffness and the distance between twin tunnels on  $U_{eq}$  convergence.

When analyzing the convergence  $U_{peak}$  at the point where the gallery meets the longitudinal tunnel, there is an increase of 16% when using a moderate stiffness elastic lining (dotted blue line) compared to a high stiffness lining (solid blue line). However, when analyzing the difference between the  $U_{eq}$  and  $U_{peak}$ , there is a difference of up to 12% for the high stiffness elastic lining (solid blue line to  $4R_t$  and  $16R_t$ ) and up to 13% for the moderate stiffness elastic lining (dotted blue line to  $4R_t$  and  $16R_t$ ) for  $d_1 = 4R_t$ .

Applying the same type of previously analysis, we examine the elasto-plastic-viscoplastic model for the rock mass (EPVP), considering the presence of a viscoelastic lining (VEL) (Figs. 30 and 31). This analysis aims to comprehend the influence of the lining's stiffness, particularly when the twin tunnels are nearby. The results reveal that, once again, the convergence profile under these conditions, with a distance of  $16R_t$  between the tunnels, closely resembles that of an isolated tunnel. Differences are on the order of 1.8% for a moderately stiff lining and 0.8% for a highly stiff lining.



**Figure 30:** Convergence Profiles - elasto-plastic-viscoplastic rock mass (EPVP) with a highly stiff viscoelastic lining ( $K_c = 3660$  MPa) and a moderately stiff viscoelastic lining ( $K_c = 1027$  MPa), without (NG) and with gallery (WG) for  $d_1 = 16R_t$



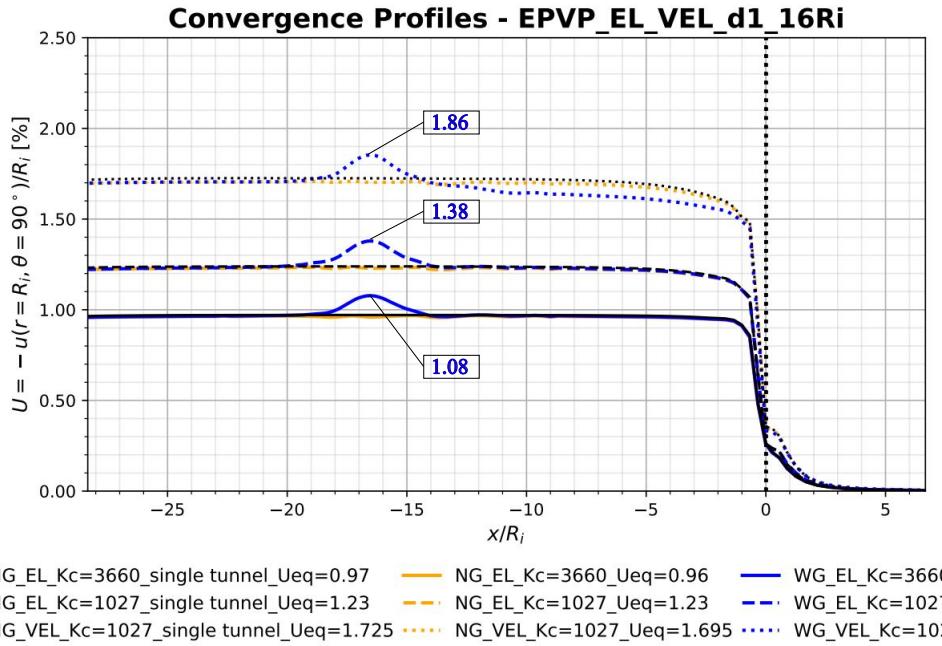
**Figure 31:** Convergence Profiles - elasto-plastic-viscoplastic rock mass (EPVP) with a highly stiff viscoelastic lining ( $K_c = 3660$  MPa) and a moderately stiff viscoelastic lining ( $K_c = 1027$  MPa), without (NG) and with gallery (WG) for  $d_1 = 4R_t$ ,

In the case of  $d_1 = 4R_t$ , when a high stiffness lining is applied (blue and yellow solid lines), there is practically no difference compared to the isolated tunnel (black solid line). This occurs because the high rigidity of the lining blocks convergence in the interaction between the tunnels. However, when using a moderately stiff lining (blue and yellow dashed lines), there is a difference of approximately 3% in the convergence  $U_{eq}$  compared to the single tunnel (black dashed line).

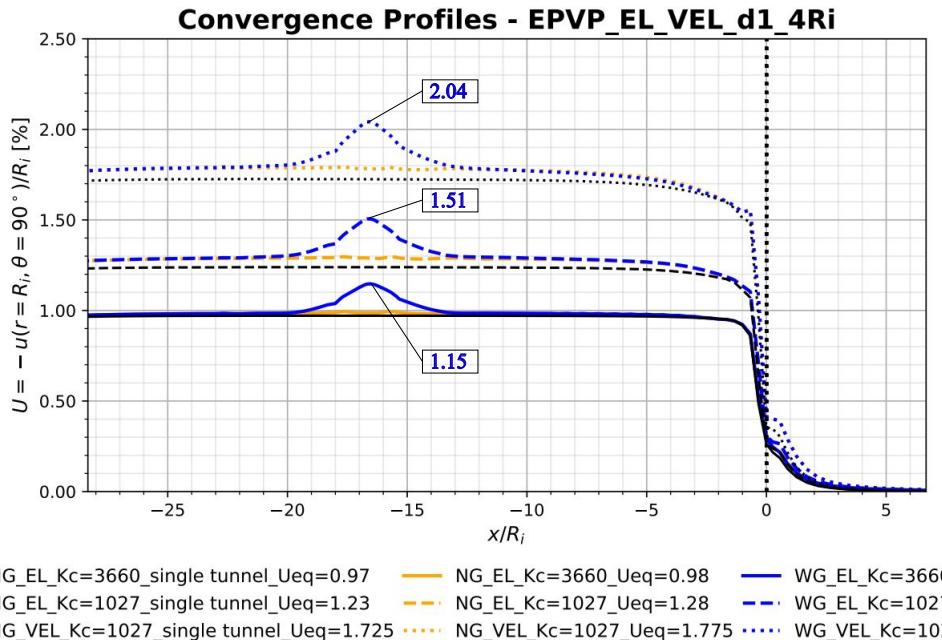
When comparing the results for  $d_1 = 16R_t$  and  $4R_t$ , considering each lining separately, there is a difference of 0.8% when there is a high stiffness lining (solid yellow and blue lines to  $16R_t$  and  $4R_t$ ) and 4.8% for a moderate stiffness lining (dashed yellow and blue lines to  $16R_t$  and  $4R_t$ ). Thus, once again, the importance of the stiffness of the lining when associated with the distance between the twin tunnels.

When analyzing the convergence  $U_{peak}$  at the point where the gallery meets the longitudinal tunnel, there is an increase of 47% for  $d_1 = 4R_t$  when using an moderate stiffness lining (dashed blue line) compared to a high stiffness lining (solid blue line). However, when analyzing the difference between the  $U_{eq}$  and  $U_{peak}$ , there is a difference of up to 18% for the high stiffness elastic lining (solid blue line) and up to 15% for the moderate stiffness lining (dashed blue line) for  $d_1 = 4R_t$ .

The following results (Figs. 32 and 33) compare the elastic (EL) and viscoelastic (VEL) lining with high and moderate stiffness, considering the elastoplastic-viscoplastic (EPVP) model for the rock mass.



**Figure 32:** Convergence Profiles - elasto-plastic-viscoplastic rock mass (EPVP) without lining (NL) with a highly stiff ( $K_c = 3660$  MPa) and a moderately stiff ( $K_c = 1027$  MPa) elastic (EL) and viscoelastic (VEL) lining, without (NG) and with gallery (WG) for  $d_1 = 16R_i$



**Figure 33:** Convergence Profiles - elasto-plastic-viscoplastic rock mass (EPVP) without lining (NL) with a highly stiff ( $K_c = 3660$  MPa) and a moderately stiff ( $K_c = 1027$  MPa) elastic (EL) and viscoelastic (VEL) lining, without (NG) and with gallery (WG) for  $d_1 = 4R_i$

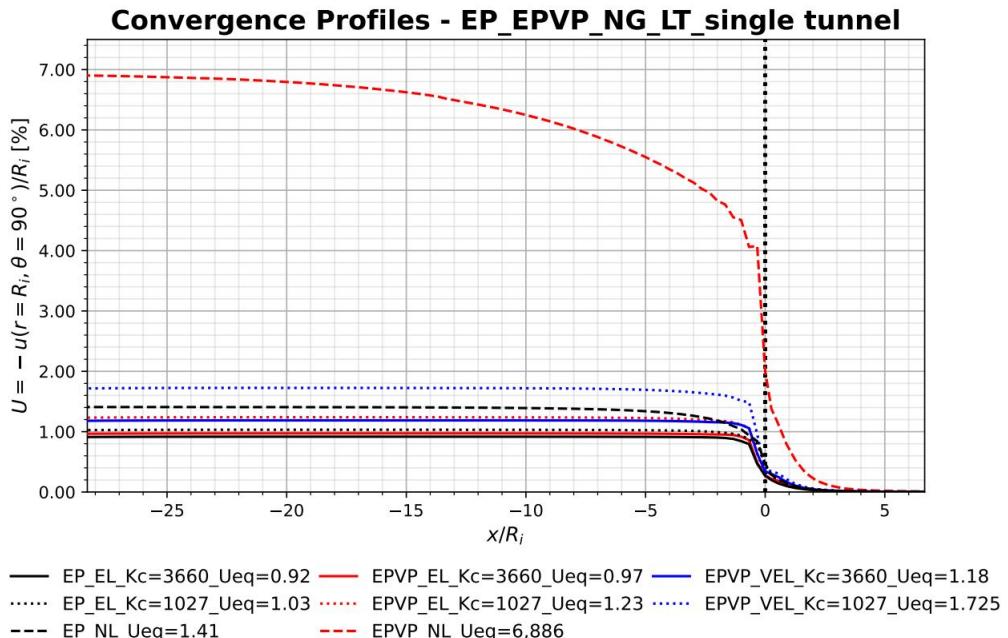
For  $d_1 = 16R_t$ , the significant difference in  $U_{eq}$  compared to the isolated tunnel (dotted black line) occurs when considering the moderate stiffness viscoelastic lining (dotted blue line), with approximately 1.8%. In contrast, for  $d_1 = 4R_t$ , the differences are 4% and 3% for the elastic (dashed lines) and viscoelastic (dotted lines) linings with moderate stiffness, respectively.

When adopting the high stiffness elastic lining as a reference (solid lines), for  $d_1 = 16R_t$ , the differences increase to 27% and 78% when comparing the elastic (dashed lines) and viscoelastic (dotted lines) lining with moderate stiffness, respectively, and to 31% and 81% in the case of  $d_1 = 4R_t$ .

When analyzing the convergence at the peak  $U_{peak}$ , which corresponds to the point where the gallery meets the longitudinal tunnel, and using the value of the high stiffness elastic lining (solid lines) as the reference, an increase of up to 31% and 77% is observed for  $d_1 = 4R_t$  when using the elastic (dashed lines) and viscoelastic (dotted lines) linings of moderate stiffness, respectively.

However, when analyzing the difference between the  $U_{eq}$  and  $U_{peak}$ , there is a difference of up to 18% for the moderate stiffness elastic lining (dashed blue line) and up to 15% for the moderate stiffness viscoelastic lining (dotted blue line) for  $d_1 = 4R_t$ .

Finally, Fig. 34 a comparison between the lining elastic and viscoelastic, considering high and moderate stiffness, and between the elastoplastic and elastoplastic-viscoplastic models for the rock mass. We adopt the condition of an isolated tunnel, starting from the reference value of the elastoplastic rock mass with high stiffness elastic lining.



**Figure 34:** Convergence Profiles – single tunnel with elastoplastic (EP) and elasto-plastic-viscoplastic rock mass (EPVP), without lining (NL) with a highly stiff ( $K_c = 3660$  MPa) and a moderately stiff ( $K_c = 1027$  MPa) elastic (EL) and viscoelastic (VEL) lining

Taking as a reference the elastoplastic model with a highly stiffness elastic lining (solid black line) we observed a difference of 12% and 53.5%, respectively, in the cases of the elastoplastic rock mass with an elastic lining of moderate stiffness (dotted black) and without a lining (dashed black line). We also identified a difference of 5.5%, 33.5%, and 748% when using the elastoplastic-viscoplastic model for the rock mass with an elastic lining of high stiffness (solid red line), moderate stiffness (dotted red line), and no lining (dashed red line). In addition, we found a difference of 28% and 87.5% when comparing the elastoplastic-viscoplastic model for the rock mass and the viscoelastic model for the high stiffness (solid blue line) and moderate stiffness lining (dotted blue line), respectively.

## 8. Conclusions

The fundamental role of the stiffness of the concrete lining in the convergence profile of twin tunnels is understood from the analyses. Depending on the value of this stiffness, it is possible to condition the restriction of viscous effects that tend to manifest over time after the completion of the excavation process.

Additionally, the effect of the interaction between longitudinal tunnels is notable when considering proximity, with significant influence from a distance of 4 radii. However, in many cases, this effect may be subtle or almost imperceptible due to the presence of a highly rigid lining.

In models considering the viscosity of the rock mass, the time factor plays a significant role in convergence. In this scenario, when excavating the gallery with  $d_1 = 16R_t$ , the portion of the tunnel already excavated remains subject to viscous effects for a more extended period compared to other distances. In any case, when  $d_1 = 4R_t$ , the proximity interaction between twin tunnels, along with viscous effects over time, results in a higher value compared to cases where  $d_1 = 16R_t$  and  $8R_t$ .

Another important observation related to EPVP-EL and EPVP-VEL models concerns the possible ovalization of the section over time, concerning the analyzed reference point on the section perimeter (in the crown). Instead of following the logic of following the closing direction of the section, it may undergo some negative displacement in the long term. The result, for the same observation point, is a convergence value that in the short term may indicate section closure but in the long term may indicate the opposite.

Another crucial observation regarding EPVP-EL and EPVP-VEL models pertains to the potential ovalization of the section over time, particularly concerning the analyzed reference point on the section perimeter (in the crown). Instead of conforming to the expected logic of closing in the section's direction, it might experience negative displacement in the long term. Consequently, for the same observation point, the convergence value may initially suggest section closure in the short-term but indicate the opposite in the long-term.

Concerning the EPVP-VEL model, specifically with distances of 16 and 8 radii between twin tunnels, we observe that Ueq in the section ahead of the gallery region is slightly smaller than Ueq in the section preceding the gallery. This variation in the convergence profile results from the shorter exposure time to viscous effects in the portion excavated later than the transverse gallery. Therefore, during the gallery excavation process, the portion of the longitudinal tunnel already excavated experiences viscous effects until completing the gallery excavation.

However, concerning the existence of a transverse gallery and adopting the constitutive parameters while considering the presence of lining, its influence is highly localized, spanning approximately four radii on each side from its axis. Consequently, there is no significant impact on the remaining convergence profile of the tunnels, except for the model with viscoelastic lining, where lower convergences occur after the gallery.

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