

Instructions to execute programs:

All inputs are taken in standard input.

Input must be in the following format:

All algorithms take weighted – directed or weighted undirected graphs.

All the algorithms can be made directed (or undirected) by just modifying one line, the one that assigns the weight of {vertex-1 to vertex-2} to {vertex-2 to vertex-1} as well.

1. Kruskal's Algorithm for Minimum Spanning Tree:

It is a greedy algorithm to find the minimum spanning tree. This algorithm needs weighted directed or undirected graph. Complexity not affected by the directed or undirected nature of the graph. If un-weighted input is to be given, all weights can be given as 1.

Time Complexity of the algorithm is $O(|E| \log |V|)$

Reading input: $O(E)$

Sorting $O(E \log E)$

For all E till count $(V-1) : \Omega(E)$.

$O(\log V)$ for number of sets.

$O(1)$ - adding new edge

$O(1)$ -

$O(1)$

For sorting the edges we use a sort of $O(|E| \log(E))$ complexity. So final complexity is $O(|E| \log |V|)$.

Where E is the number of edges and V is the number of vertices.

Time taken

2. Dijkstra's Single Source Shortest Paths:

It needs weighted directed or undirected graph.

It is a greedy algorithm.

Complexity: $O(|V|^2)$

For all vertices $\theta(v)$

For all vertices (connected, ie degree of vertex) $\theta(v)$

Find min $O(1)$

Update output $O(1)$

Formula: $\text{output}[\text{vertex } a] = \min(\text{output}[\text{vertex } a], (\text{output}[b] + \text{inputcost}[a][b]))$

Base case $\text{output}[\text{vertex } 0] = 0$;

This is because all the pairs of vertices are visited once (two for loops).

If I were to use a priority queue then the complexity would be $\log |V|$.

3. Floyd Warshall's All Pairs Shortest Paths:

This algorithm uses dynamic programming approach.
Needs a weighted directed or undirected graph.

Complexity: $O(n^3)$

Reading $O(n^2)$.

base case calculation : $O(n^2)$

filling dynamic array $O(n^3)$

Dynamic arrays: $S[i][j][k] = \text{min cost from vertex } i \text{ to vertex } j \text{ including } k \text{ number of vertices.}$

Formula: $S[i][j][k] = \min(S[i][j][k-1], S[i][k][k-1] + S[k][j][k-1])$

Result: $S[i][j][n] = \text{min path from } i \text{ to } j \text{ including all the vertices.}$

4. Transitive Closure:

This is a dynamic programming algorithm.

Finds if there is a path from a vertex to any other vertex in the graph, directly or indirectly.

Complexity: $O(n^3)$

Dynamic arrays: $S[i][j][k] = \text{min cost from vertex } i \text{ to vertex } j \text{ including } k \text{ number of vertices.}$

Time table: (all values in Milliseconds)

Time taken	Kruskals (directed)	Kruskal (undirected)	Dijkstra's	Floyd Warshal	Transitivity Closure
NodeCon <10 v= 40	2.2	3.0	14.8	15.5	6.4
NodeCon < 10 v= 60	3.1	3.9	16.0	16.3	8.7
NodeCon <10 v= 100	6.0	7.8	42.9	41.2	11
NodeCon >n/2 v= 40	2.2	2.8	15.3	11.5	4.8
NodeCon >n/2 v= 60	7.0	3.1	75.1	65.4	34.2
NodeCon >n/2 v= 100	9.0	6.2	70.2	78.1	44.0
NodeCon random v= 40	1.0	4.0	15.1	15.0	5.2

NodeCon random v= 60	2.5	4.0	15.2	16.5	8.4
NodeCon random v= 100	5.0	8.1	42	43.2	12.4

Conclusion:

All the above algorithms were understood, implemented and tested successfully.
Greedy and Dynamic programming concepts were used.