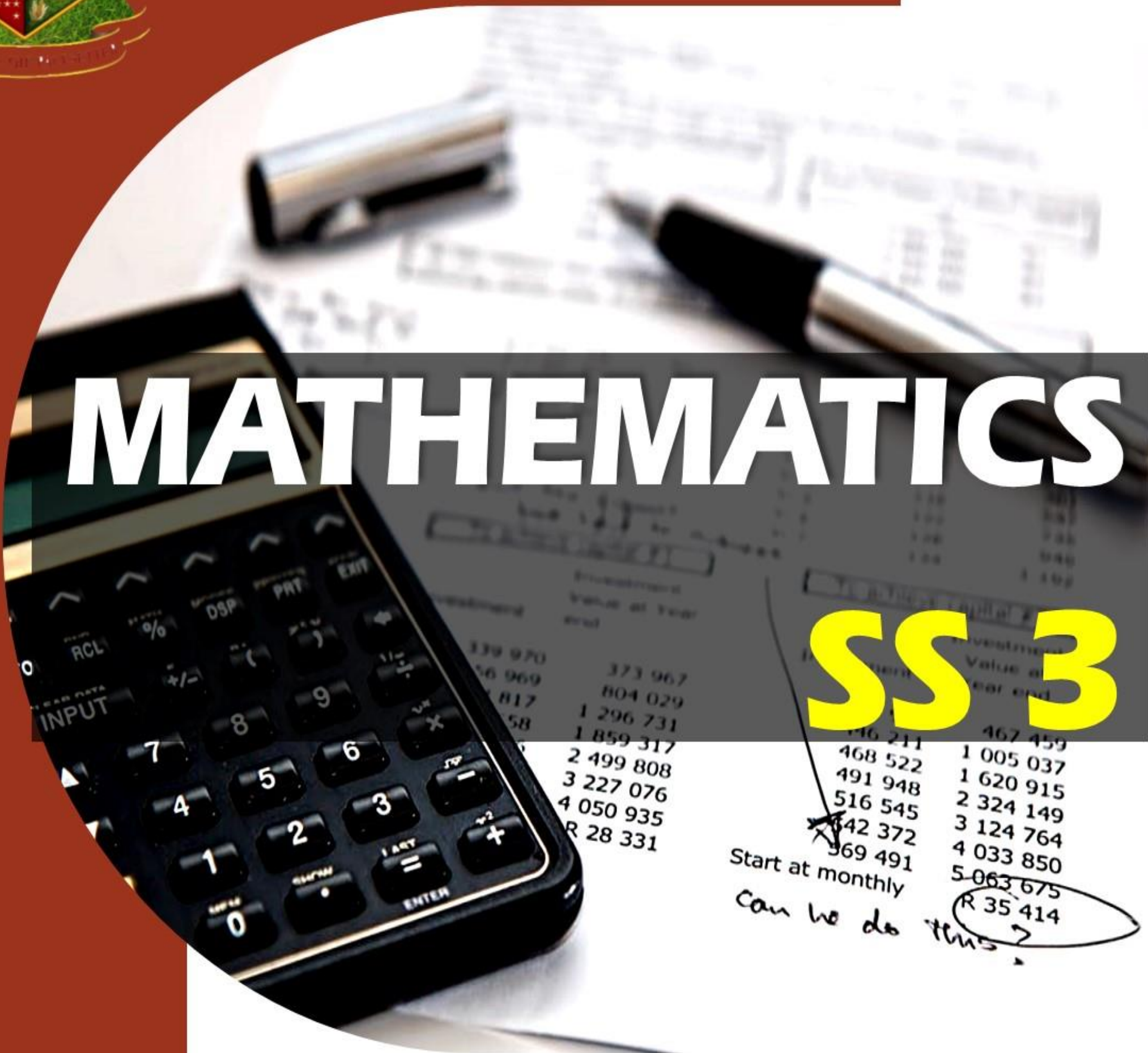
OYO STATE LECTURE NOTES

MATHEMATICS

SS 3



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AJUMOSE LECTURE NOTES

MATHEMATICS

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BASIC OPERATIONS IN SURDS

ADDITION AND SUBTRACTION OF SURDS

Define Surd:

Similar or like surds can only be added together or subtracted from each other.

N.B.Like surds are surds that have the same number under the square roots after reducing them to the surd in basic forms.

Rules or formula of surd

i.e. (i) $\sqrt{m} \; X \sqrt{m} \; = \text{m}$

(ii) $a\sqrt{n} + b\sqrt{n} = (a+b)\sqrt{n}$

(iii) $a\sqrt{n} - b\sqrt{n} = (a-b)\sqrt{n}$

(iv) $\sqrt{\frac{n}{m}} = \frac{\sqrt{n}}{\sqrt{m}}$

Examples

Evaluate each of the following

i) $7\sqrt{3} + \sqrt{3}$

ii) $5\sqrt{5} + 4\sqrt{5}$

iii) $4\sqrt{7} - 3\sqrt{7}$

iv) $3\sqrt{8} + 3\sqrt{100} - \frac{2}{3}\sqrt{162}$

v) $\sqrt{18} + \sqrt{71} - \sqrt{288}$

Solution

i) $7\sqrt{3} + \sqrt{3} = 7\sqrt{3} + 1\sqrt{3} = 8\sqrt{3}$

ii) $5\sqrt{5} + 4\sqrt{5} = 9\sqrt{5}$

iii) $4\sqrt{7} - 3\sqrt{7} = 1\sqrt{7} = \sqrt{7}$

iv) $3\sqrt{8} + 3\sqrt{100} - \frac{2}{3}\sqrt{162}$

First change everything to its basic form

⇨ $3\sqrt{4 \times 2} + 3\sqrt{100 \times 2} - \frac{2}{3}\sqrt{162}$

$$3 \times 2\sqrt{2} + 3 \times 10\sqrt{2} - \frac{2}{3} \times 9\sqrt{2}$$

$$6\sqrt{2} + 30\sqrt{2} - 6\sqrt{2}$$

$= 30\sqrt{2}$

v) $\sqrt{18} + \sqrt{71} - \sqrt{288}$

First change everything to its basic form

⇨ $\sqrt{9 \times 2} + \sqrt{36 \times 2} - \sqrt{144 \times 2}$

$$3\sqrt{2} + 6\sqrt{2} - 12\sqrt{2}$$

$= -3\sqrt{2}$

MULTIPLICATION OF SURDS

(i) $m\sqrt{x} \times n\sqrt{y} = m \times n\sqrt{xy} = mn\sqrt{xy}$

(ii) $(a\sqrt{b} + c\sqrt{d})(e\sqrt{f} + g\sqrt{h}) = ae\sqrt{bf} + ag\sqrt{bh} + ce\sqrt{df} + cg\sqrt{dh}$

Example

Evaluate each of the following

i) $6\sqrt{5} \times 3\sqrt{15}$

ii) $(2 + \sqrt{3})^2$

iii) $(2\sqrt{3}+3)(2-\sqrt{3})$

iv) $(4\sqrt{5}-2\sqrt{3})(4\sqrt{5}+2\sqrt{3})$

Solution

i) $6\sqrt{5}\ \times 3\sqrt{15}$

$= 18\sqrt{5\times 15}$

$= 18\sqrt{5\times 5\ x\ 3}$

$= 18\sqrt{25\times 3}$

$= 18\sqrt{25}\ \times\ \sqrt{3}$

$= 18\ \times 5\times\sqrt{3}$

$= 90\sqrt{3}$

ii) $(2+\sqrt{3})^2$

$= (2+\sqrt{3})(2+\sqrt{3})$

$= 4+2\sqrt{3}+2\sqrt{3}+\ \sqrt{9}$

$= 4+4\sqrt{3}+3$

$$= 7+4\sqrt{3}$$

iii) $(2\sqrt{3}+3)(2-\sqrt{3})$

$= 4\sqrt{3}-2\sqrt{9}+6-3\sqrt{3}$

$= 4\sqrt{3}-6+6-\ 3\sqrt{3}$

$= 4\sqrt{3}-3\sqrt{3}$

$= \sqrt{3}$

iv) $(4\sqrt{5}-2\sqrt{3})(4\sqrt{5}+2\sqrt{3})$

$$16\sqrt{25}+8\sqrt{15}-8\sqrt{15}-4\sqrt{9}$$

$= 16\times 5-4\times 3$

$$= 80 - 12$$

$$= 68$$

DIVISION OF SURDS

RATIONALIZATION

If a given surd is a fraction in which the denominator is a radical, the act of making the denominator to be rational is called rationalization.

i) To rationalize a surd of the form $\frac{a}{\sqrt{b}}$ multiply both by $\sqrt{b}$ such as

$$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{a}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

ii) To rationalize a surd of the form $\frac{a}{b\sqrt{c}}$, multiply both by $\sqrt{c}$

$$\frac{a}{b\sqrt{c}} = \frac{a}{b\sqrt{c}} \times \frac{c}{\sqrt{c}} = \frac{a\sqrt{c}}{bc}$$

iii) To rationalize a surd of the form $\frac{a-\sqrt{b}}{c\sqrt{d}}$, multiply both by the conjugate of the denominator i.e. $c - \sqrt{d}$

N.B: If the denominator is $c - \sqrt{d}$ its conjugate is $c + \sqrt{d}$

(i) i.e. $\frac{a-\sqrt{b}}{c+\sqrt{d}} = \frac{a-\sqrt{b}}{c+\sqrt{d}} \times \frac{c-\sqrt{d}}{c-\sqrt{d}} = \frac{(a-\sqrt{b})(c-\sqrt{d})}{(c+\sqrt{d})(c-\sqrt{d})}$

$= \frac{ac-a\sqrt{d}-c\sqrt{b}+\sqrt{db}}{c^2-c\sqrt{d}+c\sqrt{d}-d}$

$= \frac{ac-a\sqrt{d}-c\sqrt{b}+\sqrt{db}}{c^2-a}$

Also,

(ii) $\frac{a\sqrt{c}+b\sqrt{d}}{a\sqrt{c}-b\sqrt{d}}$

$= \frac{a\sqrt{c}+b\sqrt{d}}{a\sqrt{c}-b\sqrt{d}} \; X \; \frac{a\sqrt{c}+b\sqrt{d}}{a\sqrt{c}+b\sqrt{d}}$

$= \frac{a^2c+ab\sqrt{cd}+ab\sqrt{cd}+b^2d}{a^2+ab\sqrt{cd}-ab\sqrt{cd}-b^2d}$

$$= \frac{a^2c + b^2d \quad 2ab\sqrt{cd}}{a^2 - b^2d}$$

$$= \frac{(a^2c + b^2d)}{(a^2c - b^2d)} + \frac{2ab}{a^2c - b^2d}\sqrt{cd}$$

In form of A + B$\sqrt{m}$

Where,

$$A = \left(\frac{(a^2c + b^2d)}{(a^2c - b^2d)}\right)$$

$$B = \left(\frac{2ab}{a^2c - b^2d}\right)$$

M = cd

Examples

Evaluate each of the following

(1)

i) $\frac{4}{\sqrt{3}}$

ii) $\frac{2}{\sqrt{2}}$

iii) $\frac{5\sqrt{5}}{3\sqrt{2}}$

iv) $\frac{1}{\sqrt{5}}$

v) $\frac{\sqrt{3}-1}{\sqrt{3}+4}$

vi) $\frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}}$

Solution

i) $\frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

ii) $\frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$

iii) $\frac{5\sqrt{5}}{3\sqrt{2}} = \frac{5\sqrt{5}}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$$= \frac{5\sqrt{10}}{3 \times 2}$$

$$= \frac{5\sqrt{3}}{6}$$

iv) $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{5}}$

v) $\frac{\sqrt{3}-1}{\sqrt{3}+4} = \frac{\sqrt{3}-1}{\sqrt{3}+4} \times \frac{\sqrt{3}-4}{\sqrt{3}-4}$

$= \frac{(\sqrt{3}-1)(\sqrt{3}-4)}{(\sqrt{3}+4)(\sqrt{3}-4)}$

$= \frac{3-4\sqrt{3}-\sqrt{3}+4}{3-4\sqrt{3}+4\sqrt{3}-15}$

$= \frac{7-5\sqrt{3}}{-13}$

$= \frac{5\sqrt{3}-7}{13}$

vi) $\frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}} = \frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}} \times \frac{2\sqrt{3}+3\sqrt{2}}{2\sqrt{3}+3\sqrt{2}}$

$= \frac{(2\sqrt{3}+3\sqrt{2})(2\sqrt{3}+3\sqrt{2})}{(2\sqrt{3}-3\sqrt{2})(2\sqrt{3}+3\sqrt{2})}$

$= \frac{4\times3+6\sqrt{6}+6\sqrt{6}+9\times2}{4\sqrt{9}+6\sqrt{6}-6\sqrt{6}-9\sqrt{4}}$

$= \frac{12+12\sqrt{6}+18}{4\times3-9\times2}$

$= \frac{30+12\sqrt{6}}{12-18}$

$= \frac{30+12\sqrt{6}}{-6}$

$= \frac{6(5+2\sqrt{6})}{-6}$

$= -(5 + 2\sqrt{6})$

(2) Express $\frac{3\sqrt{5}-\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$ in the form a + b $\sqrt{c}$ where a, b and c are rational

Solution

$$\frac{3\sqrt{5}-\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} = \frac{3\sqrt{5}-\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} \times \frac{2\sqrt{5}-3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$$

$= \frac{(3\sqrt{5}-\sqrt{2})(2\sqrt{5}-3\sqrt{2})}{(2\sqrt{5}+3\sqrt{2})(2\sqrt{5}-3\sqrt{2})}$

$= \frac{6\times5-9\sqrt{10}-2\sqrt{10}+3\times2}{4\times5-6\sqrt{10}+6\sqrt{10}-9\times2}$

$= \frac{30-11\sqrt{10}+6}{20-18}$

$= \frac{36-11\sqrt{10}}{2}$

$= \frac{36}{2} - \frac{11\sqrt{10}}{2}$

$= 18 - \frac{11}{2}\sqrt{10}$

Where a = 18, b = 11/2 and c = 10

Exercise

i) Without using tables evaluate $\frac{3\sqrt{7}+5}{3\sqrt{7}-5} + \frac{3\sqrt{5}-5}{3\sqrt{7}+5}$

ii) Express $\frac{2\sqrt{5}+5\sqrt{2}}{3\sqrt{5}-7\sqrt{2}}$ in the form $q + r\sqrt{2}$ where q, r and s are rational

iii) If $(3+4\sqrt{3})(2-a\sqrt{3}) = -18+2\sqrt{3}$. Find the value of 'a'

iv) Given that $\sqrt{2} = 1.4142$. find the value of (i) $\frac{4}{\sqrt{2}-1}$ (ii) $\frac{3\sqrt{2}+4}{3+\sqrt{2}}$ to 3.d.p.

MATRICES

A matrix is a rectangular array of nos., itesm, objects or quantities, in row(s) and column(s).

The quantities that are in the rectangular array are called elements or entries.

The horizontal lines of elements in a matrix are called rows while the vertical lines of elements are called columns.

For instance

$$1st\ Column$$
$$\downarrow$$

$$\begin{array}{ll} Row_1 \rightarrow \\ Row_2 \rightarrow \end{array} \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix} \begin{array}{l} 1st\ row \\ 2nd\ row \end{array}$$

DIMENSION OF A MATRIX

The dimension of a matrix is the size of the matrix which normally expressed as m x n, while m is the number of row and n is the number of columns.

Thus, the matrix $\begin{pmatrix} 7 & -2 & 3 \\ 1 & 8 & -6 \end{pmatrix}$ has a matrix of dimension 2 x 3 i.e. 2 rows and 3 columns.

While the matrix $\begin{pmatrix} 10 & -11 \\ 3 & 4 \\ -5 & 6 \end{pmatrix}$ has 3 x 2 i.e. 3 rows and 2 columns.

A matrix which has the same number of rows and columns is called a square matrix e.g.

$\begin{pmatrix} 1 & 5 \\ 2 & 3 \end{pmatrix}$ and $\begin{pmatrix} 3 & 5 & -1 \\ 6 & 8 & 2 \\ 1 & 4 & 7 \end{pmatrix}$ are square matrix.

N.B Matrices is a plural of matrix.

The element a_{ij} of a matrix A is denoted as the element in the ith row and jth column

consider the matrix.

A = $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

i.e.

a_{11} is the element in the 1st row, 1st column

a_{12} is the element in the 1st row, 2nd column

a_{13} is the element in the 1st row, 3rd column

a_{21} is the element in the 2nd row, 1st column

a_{221} is the element in the 2nd row, 2nd column

a_{23} is the element in the 2nd row, 3rd column

a_{31} is the element in the 3rdrow, 1st column

a_{32} is the element in the 3rdrow, 2nd column

a_{33} is the element in the 3rdrow, 3rd column

Example

Given that A = $\begin{pmatrix} 1 & 3 & 4 \\ 7 & 8 & -4 \\ 1 & -7 & 8 \end{pmatrix}$

Find the following designated entries of the matrix

i) a_{11}

ii) a_{23}

iii) a_{32}

iv) a_{13}

v) a_{33}

Solution

i) Element $a_{11} = 1$

ii) Element $a_{23} = -4$

iii) Element $a_{32} = -7$

iv) Element $a_{13} = 4$

v) Element $a_{33} = 18$

Equality of matrices

Two matrices of the same dimension are said to be equal if their corresponding entries are

Let

A = $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and B = $\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

Then A = B if and only if $a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$

Example:

Find the value of P + Q given that

$$\begin{pmatrix} p & 3 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} 7 & 3 \\ 5 & q \end{pmatrix}$$

Solution: -

Since the matrices are equal. Hence their corresponding entries are also equal.

=> p = 7 and q = - 4

$\Leftarrow p + q = 7 + (-4) = 7 - 4 = 3$

BASIC OPERATION OF MATRICES

Addition and subtraction of matrices

A = $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and B = $\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

Then

A + B = $\begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{pmatrix}$ and

A - B = $\begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{pmatrix}$ and

Examples:-

Given that P = $\begin{pmatrix} 3 & 8 \\ 2 & -1 \end{pmatrix}$ and Q = $\begin{pmatrix} 7 & 1 \\ -3 & 4 \end{pmatrix}$ and R = $\begin{pmatrix} -5 & 2 \\ 1 & 3 \end{pmatrix}$

Find:

(i) P + Q

(ii) P + Q – R

(iii) P – Q – R

Solution

(i) P + Q = $\begin{pmatrix} 3 & 8 \\ 2 & -1 \end{pmatrix} + \begin{pmatrix} 7 & 1 \\ -3 & 4 \end{pmatrix}$

= $\begin{pmatrix} 10 & 9 \\ -1 & 3 \end{pmatrix}$

(i) P + Q – R = $\begin{pmatrix} 3 & 8 \\ 2 & -1 \end{pmatrix} + \begin{pmatrix} 7 & 1 \\ -3 & 4 \end{pmatrix} - \begin{pmatrix} -5 & 2 \\ 1 & 3 \end{pmatrix}$

= $\begin{pmatrix} 10 & 9 \\ -1 & 3 \end{pmatrix} - \begin{pmatrix} -5 & 2 \\ 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} 15 & 7 \\ -2 & 0 \end{pmatrix}$

(ii) P – Q – R $\begin{pmatrix} 3 & 8 \\ 2 & -1 \end{pmatrix} - \begin{pmatrix} 7 & 1 \\ -3 & 4 \end{pmatrix} - \begin{pmatrix} -5 & 2 \\ 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} -4 & 7 \\ 5 & -5 \end{pmatrix} \begin{pmatrix} -5 & 2 \\ 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} 1 & 5 \\ 4 & -8 \end{pmatrix}$

Scalar multiplication of matrix

Let K be a scalar quantity and A = $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ the scalar product of the scalar K and the

Matrix A is denoted KA and is defined as:

KA = K $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{pmatrix}$

Examples

Let A = $\begin{pmatrix} 9 & 5 \\ 3 & 2 \end{pmatrix}$ and B = $\begin{pmatrix} 4 & -3 \\ 1 & 7 \end{pmatrix}$

Find (i) – 5B (ii) 3A + 10B (iii) 2A – 3B

Solution

(i) – 5B = $-5 \begin{pmatrix} 4 & -3 \\ 1 & 7 \end{pmatrix} = \begin{pmatrix} -20 & 15 \\ -5 & -35 \end{pmatrix}$

(ii) 3A + 10B = $3 \begin{pmatrix} 9 & 5 \\ 3 & 2 \end{pmatrix} + 10 \begin{pmatrix} 4 & -3 \\ 1 & 7 \end{pmatrix}$

$= \begin{pmatrix} 27 & 15 \\ 9 & -6 \end{pmatrix} + \begin{pmatrix} 40 & -30 \\ 10 & 70 \end{pmatrix}$

3A + 10B = $\begin{pmatrix} 67 & -15 \\ 19 & 64 \end{pmatrix}$

(iii) 2A – 3B = $2 \begin{pmatrix} 9 & 5 \\ 3 & 2 \end{pmatrix} - 3 \begin{pmatrix} 4 & -3 \\ 1 & 7 \end{pmatrix}$

$= \begin{pmatrix} 18 & 10 \\ 6 & -4 \end{pmatrix} - \begin{pmatrix} 12 & -9 \\ 3 & 21 \end{pmatrix}$

$= \begin{pmatrix} 6 & 19 \\ 3 & -25 \end{pmatrix}$

UNITY / IDENTITY MATRIX

An identity matrix/unit matrix is a diagonal or scalar matrix in which all the entries on the principal diagonal are unity and all other entries are zero. An identity matrix is usually denoted by the letter I. sometimes by E

For 2 by 2 I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and

For 3 by 3 I = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Multiplication of two matrices

For the multiplication of two matrices to be possible, the two matrices must be conformable. Two matrices are said to be conformable if the number of columns in the 1st matrix equals the number of rows in the 2nd matrix.

For instance matrices $\begin{pmatrix} 2 & 3 & 4 \\ -5 & 8 & 1 \end{pmatrix}$ and $\begin{pmatrix} 4 & 3 \\ 5 & -2 \\ 1 & 8 \end{pmatrix}$ are conformable

While matrices $\begin{pmatrix} 2 & 3 \\ -5 & 8 \\ 1 & 2 \end{pmatrix}$ and $\begin{pmatrix} 4 & 3 \\ 5 & -2 \\ 1 & 8 \end{pmatrix}$ are not conformable

Multiplication of two matrices

Let A = $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and B = $\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

Let the product of A and B be C $\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$ where

AB = $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

C = $\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$

$C_{11} = a_{11} b_{11} + a_{12} b_{21}$

$C_{12} = a_{11} b_{12} + a_{12} b_{22}$

$C_{21} = a_{21}\, b_{11} + a_{22}\, b_{21}$

$C_{22} = a_{21}\, b_{12} + a_{22}\, b_{22}$

Examples

1) Given that A = $\begin{pmatrix} 2 & 3 \\ -4 & 5 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & 2 \\ 6 & -7 \end{pmatrix}$ find (i) AB (ii) BA

2) Given that A = $\begin{pmatrix} 2 & 5 & 6 \\ 3 & 7 & -8 \\ 1 & 4 & 2 \end{pmatrix}$ and B = $\begin{pmatrix} 7 & 3 & 1 \\ 4 & 2 & 6 \\ -5 & 1 & 8 \end{pmatrix}$ Find AB

Solution

1) Given that A = $\begin{pmatrix} 2 & 3 \\ -4 & 5 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & 2 \\ 6 & -7 \end{pmatrix}$ find (i) AB (ii) BA

(i) AB = $\begin{pmatrix} 2 & 3 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 6 & -7 \end{pmatrix}$

$= \begin{pmatrix} 2+18 & 4-21 \\ -4+30 & -8-35 \end{pmatrix}$

$= \begin{pmatrix} 20 & -17 \\ 26 & -43 \end{pmatrix}$

(ii) BA = $\begin{pmatrix} 1 & 2 \\ 6 & -7 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -4 & 5 \end{pmatrix}$

$= \begin{pmatrix} 2-8 & 3-10 \\ 12+28 & 18-35 \end{pmatrix}$

$= \begin{pmatrix} -6 & 13 \\ 40 & -17 \end{pmatrix}$

Note that AB $\neq$ BA

2) Given that A = $\begin{pmatrix} 2 & 5 & 6 \\ 3 & 7 & -8 \\ 1 & 4 & 2 \end{pmatrix}$ and B = $\begin{pmatrix} 7 & 3 & 1 \\ 4 & 2 & 6 \\ -5 & 1 & 8 \end{pmatrix}$ Find AB

$= \begin{pmatrix} 2 & 5 & 6 \\ 3 & 7 & -8 \\ 1 & 4 & 2 \end{pmatrix}\begin{pmatrix} 7 & 3 & 1 \\ 4 & 2 & 6 \\ -5 & 1 & 8 \end{pmatrix}$

$= \begin{pmatrix} 14+20-30 & 6+10+6 & 2+30+48 \\ 21+28+40 & 9+14-8 & 3+42-64 \\ 7+16-10 & 3+8+2 & 1+24+16 \end{pmatrix}$

$= \begin{pmatrix} 4 & 22 & 80 \\ 89 & 15 & -19 \\ 13 & 13 & 41 \end{pmatrix}$

Transpose of a matrix

Consider the matrix

A= $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and

The matrix A^t = $\begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix}$

The matrix A^t is called the transpose of the matrix A and it is obtained by interchanging the columns and the rows of the given Matrix A.

Examples:

1) Given that A = $\begin{pmatrix} 4 & 5 \\ 3 & 6 \\ -1 & 7 \end{pmatrix}$ and B = $\begin{pmatrix} 5 & -6 & 1 \\ 2 & 3 & 4 \end{pmatrix}$ Find (i) A^t (ii) B^t (iii) B^tA^t

2) Given that A = $\begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix}$ evaluate $A^2 - 4A + 8I$

3) Two matrices A and B are defined as A = $\begin{pmatrix} 5 & 2 \\ -3 & -1 \end{pmatrix}$ B = $\begin{pmatrix} x \\ y \end{pmatrix}$

   If $\measuredangle$ is a scalar such that AB = $\measuredangle$B. show that

   (i) $(5 - \measuredangle)n + 2y = 0$

   (ii) $3x + (\measuredangle + 1)y = 0$

   Hence or otherwise show that $\measuredangle^2 - 4x + 1 = 0$

Solution

1) A = $\begin{pmatrix} 4 & 5 \\ 3 & 6 \\ -1 & 7 \end{pmatrix}$ and B = $\begin{pmatrix} 5 & -6 & 1 \\ 2 & 3 & 4 \end{pmatrix}$

   (i) A^t = $\begin{pmatrix} 4 & 3 & 1 \\ 5 & 6 & 7 \end{pmatrix}$

   (ii) B^t = $\begin{pmatrix} 5 & 2 \\ -6 & 3 \\ 1 & 4 \end{pmatrix}$ \

   (iii) B^tA^t = $\begin{pmatrix} 5 & 2 \\ -6 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 & 1 \\ 5 & 6 & 7 \end{pmatrix}$

$$= \begin{pmatrix} 20+10 & 15+12 & -5+14 \\ -24+15 & -18+6 & 6+21 \\ 4+20 & 3+24 & -1+28 \end{pmatrix}$$

Solution to No. 2

A = $\begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix}$

A^2 = AA = $\begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 20 \\ -5 & 0 \end{pmatrix}$

= $\begin{pmatrix} 5 & 20 \\ -5 & 0 \end{pmatrix}$

4A = $4\begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 12 & 16 \\ -4 & 8 \end{pmatrix}$

8I = $8\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$

∴ $A^2 - 4A + 8I$

$\begin{pmatrix} 5 & 20 \\ -5 & 0 \end{pmatrix} - \begin{pmatrix} 12 & 16 \\ -4 & 8 \end{pmatrix} + \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$

= $\begin{pmatrix} 1 & 4 \\ 9 & 0 \end{pmatrix}$

3(i) AB = $\curlywedge B$

= $\begin{pmatrix} 5 & 2 \\ -3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \curlywedge\begin{pmatrix} x \\ y \end{pmatrix}$

= $\begin{pmatrix} 5x+2y \\ -3n-y \end{pmatrix} = \begin{pmatrix} \curlywedge x \\ \curlywedge y \end{pmatrix}$

=> $5x + 2y = \curlywedge x$ (i)

$-3n - y = \curlywedge y$ (ii)

i) $5x - \curlywedge x + 2y = 0$

$(5 - \curlywedge)x + 2y = 0$ proved

ii) $-3x - y - \curlywedge y = 0$

Multiply through by (-1)

$3x + y + \curlywedge y = 0$

$3x + (\curlywedge + 1)y = 0$ proved

From eq (i) $x = \frac{-2y}{5-\lambda}$

Subst. in eq. (ii)

$$3\left(\frac{-2y}{5-\lambda}\right) + (\lambda + 1)\, y = 0$$

Divide through by 'y'

$$\frac{-6}{5-\lambda} + \lambda + 1 = 0$$

$$\frac{6}{\lambda - 5} + \frac{\lambda + 1}{1} = 0$$

$$6 + (\lambda + 1)(\lambda - 5) = 0$$

$$6 + \lambda^2 = 5\lambda + \lambda - 5 = 0$$

$\lambda^2 - 4\lambda + 1 = 0$ proved

EXERCISE

(1) The matrix A and B are given by A = $\begin{pmatrix} a \\ b \end{pmatrix}$ and B = $\begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}$

Where $A \neq 0, b \neq 0$. If BA = KA where K is a scalar,

(i) Show (5-K)a+2b=0

(ii) Find another equation satisfied by 'a' and 'b'

(iii) Show that $K^2 - 6k + 1 = 0$

DETERMINANT

Determinant is a single No. associated with a square matrix.

Let matrix A = $\begin{pmatrix} a_{11} & a_{21} \\ a_{21} & a_{22} \end{pmatrix}$

Let A = ΔA = $\begin{vmatrix} a_{11} & a_{21} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

Examples:

Evaluate each of the following

i) $\begin{vmatrix} 3 & 4 \\ 2 & 7 \end{vmatrix}$

ii) $\begin{vmatrix} -4 & 3 \\ 5 & 6 \end{vmatrix}$

iii) $\begin{vmatrix} 8 & 4 \\ -1 & -2 \end{vmatrix}$

iv) $\begin{vmatrix} 10 & 4 \\ 8 & -2 \end{vmatrix}$

Solution

i) $\begin{vmatrix} 3 & 4 \\ 2 & 7 \end{vmatrix}$

$$= 3 \times 7 - 4 \times 2 = 21 - 8 = 13$$

ii) $\begin{vmatrix} -4 & 3 \\ 5 & 6 \end{vmatrix}$

$$= -4 \times 6 - 3 \times 5 = -24 - 15 = -39$$

iii) $\begin{vmatrix} 8 & 4 \\ -1 & -2 \end{vmatrix}$

$$= 8 \times -2 - (-1 \times 4) = -16 + 4 = -12$$

iv) $\begin{vmatrix} 10 & 4 \\ 8 & -2 \end{vmatrix}$

$$= 10 \times -2 - 8 \times 4 = -20 - 32 = -52$$

DETERMINANT OF THIRD ORDER

Let M_{ij} be the minor of the entry a_{ij} in a given determinant, the number associated with the given determinant which is obtained by multiplying each A_{ij} by $(-1)^{l+j}$ is denoted by cij and is called the Co-factor of the entry a_{ij} in the given determinant.

Thus $c_{ij} = (-1)^{l+j} M_{ij}$ where i is the number of rows and j the number of columns in the given determinants.

i.e. let A = $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$\Delta A = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \end{array}$$

$$\Delta A = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Example

i) Evaluate the determinant

$$\begin{vmatrix} 4 & 2 & 1 \\ 1 & -5 & 7 \\ 3 & 6 & 8 \end{vmatrix}$$

$$= \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 4 & 2 & 1 \\ 1 & -5 & 7 \\ 3 & 6 & 8 \end{vmatrix} \end{array} = 4\begin{vmatrix} -5 & 7 \\ 6 & 8 \end{vmatrix} - 2\begin{vmatrix} 1 & 7 \\ 3 & 8 \end{vmatrix} + 1\begin{vmatrix} 1 & -5 \\ 3 & 6 \end{vmatrix}$$

$= 4\,(-40 - 42) - 2\,(8 - 21) + 1(6 + 15)$

$= 4\,(-82) - 2\,(-13) + 1(21)$

$= -328 + 26 + 21$

= -281

APPLICATION OF DETERMINANT

Cramer's rule

Consider the following system of two equations in two unknowns

$a_1x + b_1y = c_1$ ……………………….(i)

$a_2x + b_2y = c_2$ ……………………….(ii)

It can be expressed in matrix form as shown below

$$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

Let $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ and $\Delta x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ $\Delta y \begin{vmatrix} c_1 & c_1 \\ c_2 & c_2 \end{vmatrix}$

The Cramer's rule states that

$$x = \frac{\Delta x}{\Delta} \text{ and } y = \frac{\Delta y}{\Delta}$$

Example

Solve the simultaneous equation

$2x + 3y = 18$ ………………. (i)

$5x - 2y = 7$ …………………. (ii)

Solution

$$\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18 \\ 7 \end{pmatrix}$$

Let $\Delta = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19$

$\Delta x = \begin{vmatrix} 18 & 3 \\ 7 & -2 \end{vmatrix} = -36 - 21 = -57$

And $\Delta y = \begin{vmatrix} 2 & 18 \\ 5 & 7 \end{vmatrix} = 14 - 90 = -76$

$\Leftarrow x = \frac{\Delta x}{\Delta} = \frac{-57}{-19} = 3$

$$y = \frac{\Delta y}{\Delta} = \frac{-76}{-19} = 4$$

Hence x = 3 and y = 4

ADJOINT AND INVERSE MATRIX

The transpose of the matrix formed by taking the co-factor of a given square matrix is called the adjoint of the given square matrix.

Let A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the co factor of A is given by c = $\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$

The transpose of the co factor of A is called the adjoint of A.

i.e. Adj A = C^T

⇦Adj A = $\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$

An inverse matrix of a given matrix is another matric which its product and the given matrix equals an identity matrix (I).

Given matrix A and let the inverse of A be A^{-1}

⇦ A^{-1} is defined as $\frac{AdjA}{\Delta A}$

i.e. let A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

thereforeAdj A = $\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$ and ΔA = ad – bc

⇦inverse of A = $A^{-1}\frac{1}{ad-bc}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$

Example :Find the inverse of the matrix

M = $\begin{pmatrix} 6 & 3 \\ -4 & 5 \end{pmatrix}$

Solution :Let the inverse of M be M^{-1}

⇦ $M^{-1} = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} \times \frac{1}{ad-bc}$

$= \frac{1}{30+12}\begin{pmatrix} 5 & 4 \\ -3 & 6 \end{pmatrix}$

⇦ $M^{-1} = \frac{1}{42}\begin{pmatrix} 5 & 4 \\ -3 & 6 \end{pmatrix}$

INVERSE METHOD FOR SOLVING SIMULTANEOUS EQUATION

Given the equations

$a_1x + b_1y = c_1$ ……………………….(i)

$a_2x + b_2y = c_2$ ……………………….(ii)

In matrix form

$$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

Let A = $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ and A^{-1} = B C = $\begin{pmatrix} x \\ y \end{pmatrix}$ and D $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

Since $A\Delta^{-1} = 1$ ⇦ AB = 1

⇦ AC = D

Multiply both sides by B

=> ABC = DB ,But AB = 1 ⇦ C = BD

Example :Solve the simultaneous equation using matrix (inverse) method

$$\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18 \\ 7 \end{pmatrix}$$

Multiply both sides by the inverse of $\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$

The inverse of $\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$

$$\frac{1}{2(-2)-(5\times 3)}\begin{pmatrix} -2 & -3 \\ -5 & -2 \end{pmatrix}$$

$$\frac{1}{-4-15}\begin{pmatrix} -2 & -3 \\ -5 & -2 \end{pmatrix}$$

$$\frac{-1}{19}\begin{pmatrix} -2 & -3 \\ -5 & -2 \end{pmatrix}$$

$=>\frac{-1}{19}\begin{pmatrix} -2 & -3 \\ -5 & -2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

$=\frac{-1}{19}\begin{pmatrix} -2 & -3 \\ -5 & -2 \end{pmatrix}\begin{pmatrix} 18 \\ 7 \end{pmatrix}$

⇦ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{-1}{19}\begin{pmatrix} -36 & -21 \\ -90 & +14 \end{pmatrix}$

$$=\frac{-1}{19}\begin{pmatrix} -57 \\ -76 \end{pmatrix}$$

$= \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{-57}{-19} \\ \frac{-76}{-19} \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$$⇦ x = 3 \; and \; y = 4$$

EXERCISE

1) Use Cramer's rule to solve

$2x + 3y = 18$ ………………… (i)

$5x - 2y = 7$ ………………… (ii)

2) Use the matrix method to solve equation (i) above

3) Evaluate the determinant

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & -1 \\ -1 & 1 & 3 \end{vmatrix}$$

4) The matrices P and Q are P = $\begin{pmatrix} 1 & 3 \\ 4 & -1 \end{pmatrix}$, Q = $\begin{pmatrix} -3 & -k \\ 5 & -2 \end{pmatrix}$ when k is a constant

a) Find |PQ|

b) If |PQ| = 144 find the value of K

5) Given that A = $\begin{pmatrix} 9 & 1 \\ 5 & 3 \end{pmatrix}$ B = $\begin{pmatrix} 1 & 5 \\ 7 & 4 \end{pmatrix}$

Find the matrix X such that 3A + 5B – 2X = 0

6) (a) Calculate the determinant of $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$

(b) Use your result in (a) solve

$2x + 3y = 4$ ………… (i)

$4x + 5y = 3$ ………… (ii)

THEORY OF LOGARITHMS

Logarithm can be define as a power in which a base is raised.

Laws of logarithms (Revision)

i) $\log_a x + \log_a y = \log_a (xy)$

ii) $\log_a x - \log_a y = \log_a (\frac{x}{y})$

iii) $\log_a 1 = 0$

iv) $\log_a a = 1$

v) $\log_a x^n = n \log_a x$

vi) $\log_{a^n} x = \frac{1}{n} \log_a x$

vii) $\log_b a = \frac{\log_x a}{\log_x b}$

viii) $\log_b a = \frac{1}{\log_a b}$

Change of base

ix) Given that $\log_b a = C \Leftarrow a = b^c$

x) Given that $\log_a x = \log_a y$ then x = y

xi) If $\log_a x = \log_b x$ then a = b

Examples

1) Simplify $\frac{\log 5^{\frac{1}{4}}}{\log \sqrt{125}}$

Solution

$\frac{\frac{1}{4}\log 5}{\log 125^{1/2}} = \frac{\frac{1}{4}\log 5}{\log 5^{3/2}}$

$\frac{\frac{1}{4}\log 5}{\frac{3}{2}\log 5} = \frac{1}{4} \times \frac{2}{3}$

$= \frac{1}{6}$

2) If log 2 = 0.3010 and log 2^y = 1.8062, find the value of y to the nearest whole number

Solution

$\log_{10} 2^y$ = 1.8062

$2^y = 10^{1.8062}$

$\Leftarrow \log_{10} 2^y = \log_{10} 10^{1.8062}$

$$y \log_{10} 2 = 1.8062 \log_{10} 10$$

$$y = \frac{1.8062 \times 1}{\log_{10} 2}$$

$= \frac{1.8062}{0.3010}$

y = 6

3) Solve the equation

$$\log_3\left(\frac{4x+1}{3x-5}\right) = 2$$

⇦$\frac{4x+1}{3x-5} = 3^2$

$$4x + 1 = 9\,(3x - 5)$$

$$4x + 1 = 27x - 45$$

$$4x - 27x = -45 - 1$$

$$-23x = -46$$

$$x = \frac{-46}{-23}$$

$$x = 2$$

4) Solve the simultaneous equation

$\log_{10} x + \log_{10} y = 4$ ………….. (i)

$\log_{10} x + 2\log_{10} y = 3$ ………….. (ii)

Solution

$\log_{10} x + \log_{10} y = 4$ ………….. (i)

$\frac{(\log_{10} x + 2\log_{10} y = 3)}{-\log_{10} y = 1}$………….. (ii)

$$\log_{10} y = -1$$

⇦ y = 10^{-1} hence y = $\frac{1}{10}$

Substitute $\frac{1}{10}$ for y in eq (i)

=>$\log_{10} x + \log_{10}\frac{1}{10} = 4$

$$\log_{10} x + \log_{10} 10^{-1} = 4$$

$$\log_{10} x - 1\log_{10} 10 = 4$$

$$\log_{10} x - 1 = 4$$

$$\log_{10} x = 4 + 1$$

$$\log_{10} x = 5$$

$$⇦ x = 100{,}000 \textit{ and } y = \frac{1}{10}$$

Another Method to solve question (4)

Now,

Let $\log_{10} x = a$

Let $\log_{10} y = b$

Then ,

a + b = 4 --------- (i)

a + 2b = 3 ---------- (ii)

5) Given that $\log 3 = 0.4771$, $\log 2 = 0.3010$. Find the value of the following without using logarithm table

(i) $\log \sqrt{6}$

(ii) $\log \sqrt[3]{0.3}$

(iii) $\log 15^{1/2}$

Solution

$\log 2 = 0.3010$ and $\log 3 = 0.4771$

(i) $\log \sqrt{6} = \log 6^{1/2}$

$= \frac{1}{2} \log 6$

$= \frac{1}{2} \log 2 \times 3$

$= \frac{1}{2} [\log 2 + \log 3]$

$= \frac{1}{2} [0.3010 + 0.4771]$

$= \frac{1}{2} \times 0.7781$

$= 0.38905$

(ii) $\log \sqrt[3]{0.3} = \log (0.3)^{\frac{1}{3}}$

$= \frac{1}{3}\log\left(\frac{3}{10}\right)$

$= \frac{1}{3}[\log 3 - \log 10]$

$= \frac{1}{3}[0.4771 - 1]$

$= \frac{1}{3} \times 0.5229$

$= 0.1743$

(iii) $\log 15^{1/2}$

$= \frac{1}{2}\log 15$

$= \frac{1}{2}\log(3 \times 5)$

$= \frac{1}{2}\log\left(3 \times \frac{10}{2}\right)$

$= \frac{1}{2}[\log 3 + \log 10 - \log 2]$

$= \frac{1}{2}[0.4771 + 1 - 0.3010]$

$= \frac{1}{2} \times 1.1761$

$= 0.58805$

6) Evaluate $\frac{2^{n+4} - 2.2^n}{2.2^{n+3}}$

Solution

$$\frac{2^n \times 2^4 - 2 \times 2^n}{2 \times 2^n \times 2^n}$$

$$\frac{2^n(16 - 2)}{2^n \times 16} = \frac{14}{16} = \frac{7}{8}$$

7. Solve the equation

$$3\left(2^{2n+3}\right) - 5\left(2^{2n+2}\right) - 156 = 0$$

Leaving your answer in logarithm form

Solution

$$3\,(2^{2n+3}) - 5\,(2^{2n+2}) - 156 = 0$$

Let 2^n be 'y'

$\Leftarrow 3\,(y^2 \times 8) - 5(y \times 4) - 156 = 0$

Divide through by '4'

$$3\,(y^2 \times 2) - 5y - 39 = 0$$

$$6y^2 - 5y - 39 = 0$$

$$6y^2 - 18y + 13y\ 39 = 0$$

$$6y\,(y-3) + 13\,(y-3) = 0$$

$$(6y+13)(y-3) = 0$$

$$y = -\frac{13}{6}\, or\ 3$$

But $2^x = y$

$\Leftarrow 2^x = -\frac{13}{6}or2^x = 3$

For $2^x = -\frac{13}{6}$ x has no solution

$$2^x = 3x = \log_2 3$$

EXERCISE

1) Given that $\log_x 256 = 2$. Find $\log_8\left(\frac{1}{x}\right)$

2) Solve the equations

   i) $\log_2(x^2-2) = \log_2(x^2-1) + 1$

   ii) $4^{x+1} - 9(2^x) = -2$

3) If $x^2 + y^2 = 7xy$ prove that $2\log\left(\frac{x+y}{3}\right) = \log x + \log y$

4) (a) if $9^{2n+1} = \frac{81^{x-2}}{3^x}$. Find x

(b) If $\log_2(2x+1) - \log_{10}(3x-2) = 1$. Find x

FINANCIAL MATHEMATICS

I = Interest

R = Rate

P = Principal

A = Amount

T = time

Interest is the rent paid for the use of money.

There are two basic types of interest which are: Simple Interest and Compound Interest

SIMPLE INTEREST

This is interest on the amount interested or borrowed at a given rate and for a given time. Simple interest is calculated as follows:

Simple Interest I $= \frac{Principal \times Rate \times Time}{100} = \frac{PRT}{100}$

The amount ‘A’ at the end of the period = Principal + Interest

COMPOUND INTEREST

In calculating compound interest, interest on the principal is paid at regular interval which is added to the principal at the end of each interval.

Compound Interest = Final Amount – Original Principal i.e.

C.I = A – P

PROFIT AND LOSS

Profit percent = $\frac{profit}{cost\ price} \times 100\%$

Profit = Selling Price – Cost Price

Loss percentage = $\frac{loss}{cost\ price} \times 100\%$

Loss = Cost Price – Selling Price

Examples

(1) A man borrowed ₦x at 15% per annum simple interest. He paid back ₦49,875 after 5yrs.

Calculate (i) the value of x (ii) the Simple Interest

i) Principal = ₦x, Rate = 15%

Time = 5 years, Amount = ₦49,875

Simple interest = $\frac{PRT}{100}$

Simple Interest = Amount – Principal

= 49,875 – x

$\Rightarrow$ 49875 – x = $\frac{x \times 5 \times 5}{100}$

100(49875 – x) = 75x

49875 – x = 0.75x

49875 = 0.75x + x

49875 = 1.75x

$x = \frac{49875}{1.75}$

$\therefore$ x = ₦28,500

ii) Simple Interest = ₦49,875 – ₦28,500

= ₦21,375

(2) a) Find the Simple Interest on ₦3000 in 3½ years at 4% per annum. Give your answer to the nearest naira.

b) A man bought 250 electric lanterns in the United State of America (USA) for $5000. He sold it in Lagos at ₦1920 each. At the time of his journey back to USA, the exchange rate was ₦96 to the Dollar.

i) If by selling the lanterns at ₦1920 each, he made a profit of 20%. Calculate the exchange rate at the time he bought the lanterns.

ii) How much in Dollar will the sale amount to at the time of his journey back to USA?

SOLUTION

(a) simple Interest = $\frac{PRT}{100}$

Principal = ₦3000

Time = 3½ years

Rate = 4%

Simple Interest = $\frac{3000 \times 7 \times 4}{2}$

= ₦420

(b) (i) Selling Price = 1920 × 250

= ₦480,000

He made a gain of 20% by selling at ₦1920

Let the cost price = x

$\frac{120x}{100}$ = 1920

$x = \frac{100 \times 1920}{120}$ = ₦1600

The cost price of 250 electric lanterns = ₦1600 ×250

= ₦400,000

The exchange rate of 1 dollar = $\frac{400{,}000}{5000}$

= ₦80

(ii) Selling Price = ₦1920 × 250

= ₦480,000

If the exchange rate is ₦96 to $1

The amount I Dollar = ₦ $\left(\frac{480{,}000}{96}\right)$

= \$5000

(3) Two kind of flour are mixed together in the ratio 6:4, the cost of the second flour is ₦2.70 per kg. If the mixture is sold for ₦4.50 per kg, thereby making a profit of 25%, find the cost of the first flour per kg.

SOLUTION

The ratio is 6:4 = 6+4 = 10

Selling price of the mixture is ₦4.50 per kg

i.e. Total Selling Price of the mixture is ₦4.50 × 10 = ₦45

Cost of the second flour is ₦2.70

Total cost of the second flour = ₦2.70 × 4

= ₦10.80

Gain made on selling the mixture is 25% of ₦45

∴ Cost Price of the mixture is

$$\frac{100}{125} \times ₦45 = ₦36.00$$

∴ Cost of the first flour is

$$₦36 - ₦10.80 = ₦25.20$$

Cost of the first flour per kg

$$\frac{₦25.20}{6} = ₦4.20$$

(4) Find the Simple Interest on ₦6240 at $6^2/_3$ % per annum for the period from Dec 28, 2007 to May 22, 2008. Also find the amount.

SOLUTION

We have:

Dec		Jan		Feb		Mar		Apr		May
3	+	31	+	29	+	31	+	30	+	22

= 146 days

$= \frac{146}{365} year = \frac{2}{5} years$

P = ₦6240, T = $^{2}/_{5}$yr, R = $^{20}/_{3}$%

S.I = $\frac{PRT}{100}$

$= 6240 \times \frac{20}{2} \times \frac{2}{5} \times \frac{1}{100}$

= ₦166.40

Amount = Principal + S.I

= ₦6240 + ₦166.40

= ₦6404.40

(5) Calculate the amount and the Compound Interest on ₦12,000 in 3yrs when the rates of interest for successive years are 8%, 10% and 15% respectively.

SOLUTION

Using the formula

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$\therefore A = P\left(1 + \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right)\left(1 + \frac{r_3}{100}\right)$$

$$A = 1200\left(1 + \frac{8}{100}\right)\left(1 + \frac{10}{100}\right)\left(1 + \frac{15}{100}\right)$$

= ₦16,394.40

The Amount = ₦16,394.40

And Compound I = ₦16,394.40 – ₦12,000

C.I = ₦4,394.40

(6) What sum of money will amount to ₦9922.50 in 2 years at 5% per annum Compound Interest?

SOLUTION

A = ₦9922.50 r = 5% n = 2

$\therefore A = P\left(1 + \frac{r}{100}\right)^n$

$9922.50 = P\left(1 + \frac{5}{100}\right)^2$

P = 9922.50 $\times\left(\frac{20}{21}\right)^2$

∴ P = ₦9000

(7) (a) An electrical shop sells a video recorder for ₦3200 cash price. Hire purchase deal is offered to Mr. Bali at an extra 20% cost. He pays a deposit of ₦1200 and ₦x in equal installments, calculate the value of x.

(b) Find the Compound Interest (without use of formula) on ₦285.38 in 2yrs at the rate of 2½ per annum. Give your answer to the nearest kobo.

SOLUTION

(a) 20% extra cost

$= \frac{120}{100} \times ₦3200 = ₦3840$

Amount to balance

= ₦3840 – ₦1200 = ₦2640

6 equal monthly installments

$= \frac{2640}{6} = ₦440$

$\therefore x = ₦440.00$

(b) Interest for the 1st year

= 2½% ×285.38

$= \frac{5}{200} \times 285.38$

= ₦7.13

Amount at the end of the 1st year

= ₦(285.38 + 7.13)

= ₦292.51

Interest at the end of 2nd year

$= ₦\left(\frac{5}{200} \times 292.51\right)$

= ₦7.31

Amount at the end of the 2nd year

= ₦(292.51 + 7.31)

= ₦299.83

Compound Interest = ₦(299.83 – 285.38)

= ₦14.45

EXERCISE

(1) A radio which a dealer bought for ₦6000 and marked to give a profit of 30% was reduced in a sales by 10%. Find:

(i) The final sales price

(ii) Percentage profit

(2) A shopkeeper buys 40kg of fruits for ₦120, he sells 20kg at ₦5 per kg, 10kg at ₦3 per kg, 5kg at ₦2 per kg and the remaining 5kg at 50k per kg. calculate the:

(i) Amount he realizes from the sales

(ii) Total profit

(iii) Percentage profit on his outlay of ₦120

(3) A man saved ₦3000 in a bank P, whose interest rate was x% per annum and ₦2000 in another bank Q whose interest was y% per annum his total interest in one year was ₦640. If he had saved ₦2000 in P and ₦ 3000 in Q for the same period, he would have gained ₦20 as additional interest. Find the value of x *and* y.

(4) A certain brand of car costs ₦250,000 in January 2001. If the value of Naira depreciates annually by 5%, how much will such car cost in January 2010?

(5) A man borrows ₦10,000 at 5% per annum compound interest. He repays 35% of the sum borrowed at the end of the first year and 42 % of the sum borrowed at the end of the second year. How much must he pay at the end of the third year in order to clear the debt?

LONGITUDE AND LATITUDE

THE EARTH

The earth is approximately spherical in shape. The earth rotates about its axis.

Note:

(1) Longitude and Latitudes are imaginary line drawn on the Earth surface. The earth is spherical with an approximate radius of 6400km, denoted by R.

(2) The equator is the intersection of a horizontal plane with the Earth's surface through the centre of the Earth.

(3) Parallel of latitudes are the intersection of other horizontal planes with the Earth's surface resulting in circles that are parallel to the equator but less than the equator in length. The radius of the parallel of latitudes is less than the radius of the Earth hence they are called small circles. The radius of the parallel of latitudes is denoted by $r = R\cos\alpha$ where α is the latitude through two points and R = 6400km.

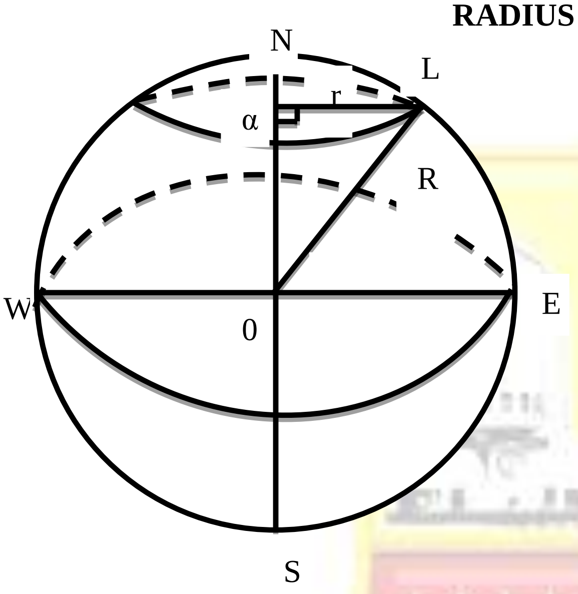
(4) The equator is the greatest amongst the parallel of latitudes with its radius equal to the radius of the Earth. The equator is a great circle.

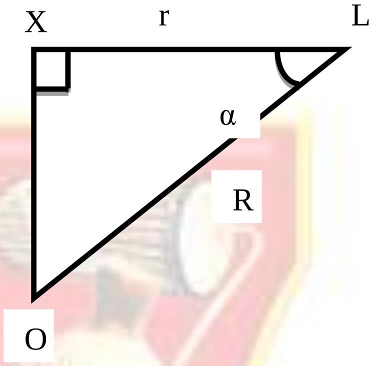
(5) All longitudes are great circle.

(6) Measurements of latitudes are in degrees to the North or South of the equator. Maximum is 90^0N and 90^0S.

(7) Measurements of longitudes are in degrees to the East or West. The maximum longitudes are 180^0E or 180^0W. These two coincide.

RELATIONSHIP BETWEEN RADIUS OF PARALLEL OF LATITUDES AND RADIUS OF THE EARTH





$$\frac{r}{R} = Cos \propto$$

$$\therefore r = Rcos \propto$$

Where R = radius of the Earth and ∝ the latitude at L. the circumference of a parallel of latitude is $2\pi r$ where r = Rcos∝

NB:

(1) Angular difference between 2 points on the same latitude that are in the same direction in East and East or West and West is the positive difference between their longitudes.

(2) Angular different between 2 points on the same latitude that are in different directions i.e. East and West is the sum of their longitude.

(3) Angular difference between 2 points on the same longitude that are in the same hemisphere i.e. North and North or South and South is the positive difference between latitudes.

(4) Angular difference between 2 points on the same longitude that are in different hemisphere i.e. North and South is the sum of their latitudes.

(5) The Earth rotates from West to East through 360^0 in 24hrs.

In 1 hour, the Earth rotates

$$\frac{360}{24} = 15^0$$

$1^0 = \frac{60minutes}{15} = 4\ minutes$ i.e. the Earth rotates 1^0 in 4 mins.

DISTANCE MEASURED ON THE SURFACE OF THE EARTH

Examples

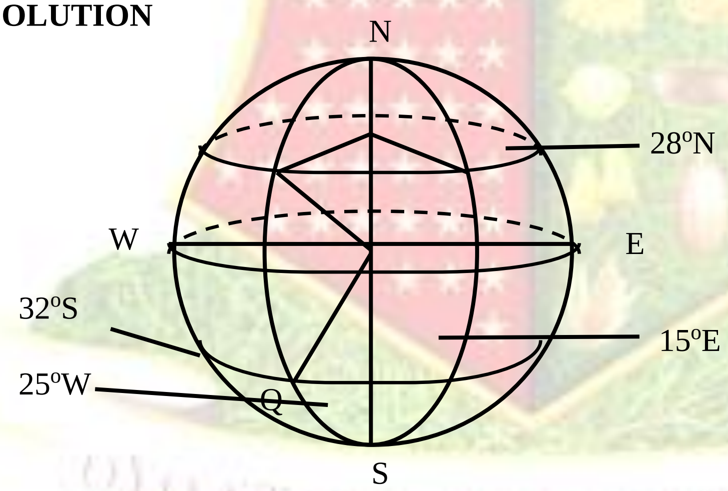
(1) An aircraft moves from a location P (28^0N, 15^0E) to another location T (28^0N, 25^0W) and then to a location Q (32^0S, 25^0W). Calculate:

(i) The radius from P to T

(ii) The distance from P to Q

(iii) The average speed of the aircraft if the journey takes 20 hours. (R = 6400km, π = 3.142)

SOLUTION



(i) Radius of the parallel of latitude 28^0N $= RCos\alpha$

$= 6400\ Cos\ 28$

$= 5650.56$km

(ii) Angular difference between P and T

$= 15^0 + 25^0 = 40^0$

Distance PT $= \frac{40}{360} \times 2 \times \pi \times RCos28$

$= \frac{40}{360} \times 2 \times 3.142 \times 5650.56$

= 3946.35km

(iii) Angular difference between T and Q = 28 + 32 = 60^0

Distance T to Q = $\frac{60}{360} \times 2 \times 3.142 \times 6400$

= 6702.93km

(iv) Total distance covered from P to T to Q = 3945.35 + 6702.93 = 10648.28km

Average speed = $\frac{Distance}{Time}$

= $\frac{10648.28}{20}$

= 532.4km/hr

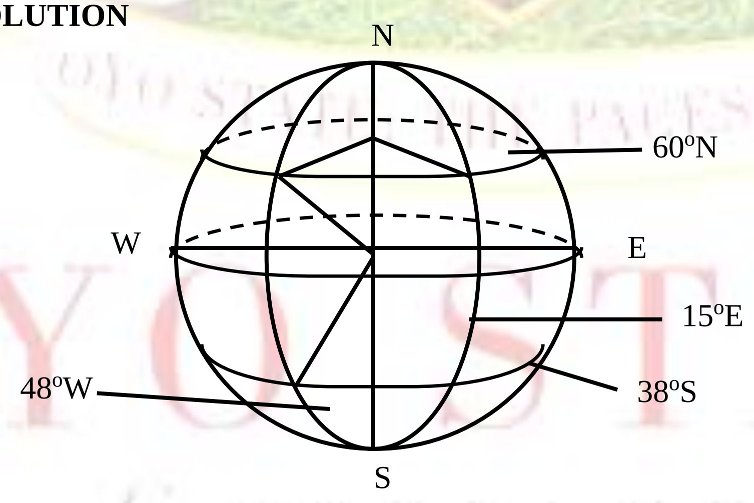
(2) P (60^0N, 15^0E), Q (60^0N, 48^0W) and R (38^0S, 48^0W) are three places on the earth's surface. Calculate to 3 significant figure:

(i) The distance PQ measured along the latitude

(ii) The distance QR measured along the meridian

(iii) The time taken by the aircraft to cover the distance PQ and QR at an average speed of 800km/hr. (π = 3.142, R = 6400km)

SOLUTION



(i) Distance PQ = $\frac{Angular\ diff}{360} \times 2\pi R\cos \propto$

Angular diff between P and Q = 48 + 15 = 63^0

$$\therefore \textit{Distance PQ} = \frac{63}{360} \times 2 \times 3.142 \times 6400cos60$$

= 3519km ≅ 3520km to 3 s.f

(ii) Distance QR = $\frac{Angular\ diff}{360} \times 2\pi R$

Angular diff btw Q and R = 60 + 38 = 98^0

$$\therefore \textit{Distance QR} = \frac{98}{360} \times 2 \times 3.142 \times 6400$$

= 10,948km ≅ $10900km\ to\ 3\ s.f$

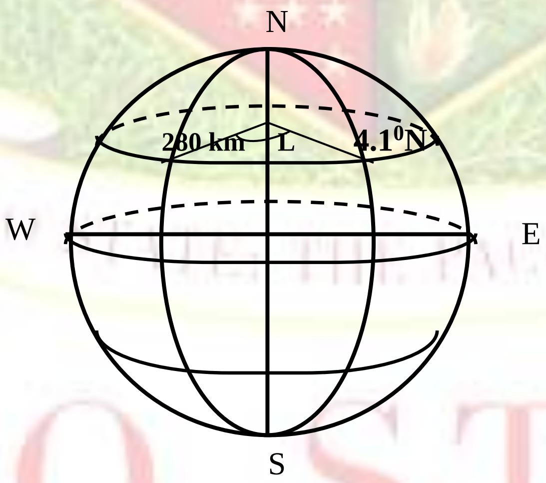
(iii) Total distance travelled

= 3519 + 10,948 = 14,467km

Time taken = $\frac{Distance}{Speed} = \frac{14467}{800}$,= 18.08hrs ≅ 18.1hrs

(3) A ship left the port at Lagos (9.8^0E, 4.1^0N). it sailed 1280km due West and then 90km North to Benue. Find the position of the Benue port.

SOLUTION



Distance between L and C = 1280km

Let the angle diff between L and C be β

$$\therefore 1280 = \frac{\beta}{360} \times 2 \times \frac{22}{7} \times 6400 \cos 4.1$$

$$\therefore \beta = \frac{1280 \times 360 \times 7}{2 \times 22 \times 6400 \cos 4.1}$$

$\beta = 11.48^0$

$\therefore 9.8 + \vartheta^0 W = 11.48$

$\vartheta^0 w = 11.48 - 9.8$

$=1.68^0$W

Distance BC = $\frac{Angular\ diff}{360} \times 2\pi R$

$\therefore Angular\ diff\ btw\ B\ and\ C =$

$\propto - 4.1$

$$\therefore 90 = \frac{\propto - 4.1}{360} \times \frac{2 \times 22}{7} \times 6400$$

$$\propto - 4.1 = \frac{90 \times 360 \times 7}{2 \times 22 \times 6400}$$

$\propto - 4.1 = 0.81$

$\therefore \propto = 4.1 + 0.81$

$\propto = 4.91^0$N

$\therefore$ The position of Benue is (4.91^0N, 1.68^0W)

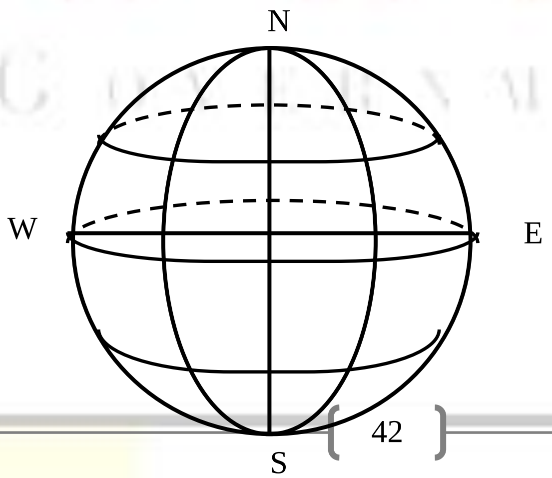
(4) The latitude and longitude of appoint X on the Earth surface are 60^0N and 22^0E and of another point are 60^0N and 38^0W. (R =6400km and $\pi^{22}/_7$). Calculate correct to the nearest 10km:

(i) The radius of the circle of latitudes through X and Y

(ii) The distance between X and Y along the parallel of latitude

(iii) The shortest distance between X and Y

SOLUTION



(i) Radius of latitude through X and Y

Y $= r \cos\propto$

$= 6400 \cos 60$

$= 6400 \times ½$ km

$= 3200$km

(ii) Distance between X and Y along the parallel of latitude =

$$\frac{angular\ diff}{360} \times 2\pi R \cos \propto$$

Angular difference between X and Y along the parallel of latitude

$= 22 + 38 = 60^0$

$\therefore Distance\ XY\ along\ the\ parallel\ of\ latitude$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 6400 cos60$$

$= 3352.40$km

$= 3350$km to the nearest 10km

(iii) The shortest distance between X and Y is the distance measured along the Great circle. Let the angular difference formed on the great circle be β

$$\therefore \beta = 2sin^{-1}\left(\sin\frac{\theta}{2}\cos\propto\right)$$

$$\therefore \beta = 2sin^{-1}\left(\sin\frac{60}{2}\cos 60\right)$$

$$= 2sin^{-1}(\sin 30 \cos 60)$$

$$= 2sin^{-1}\left(\frac{1}{2} \times \frac{1}{2}\right)$$

$= 2sin^{-1}\left(\frac{1}{4}\right)$

$= 2sin^{-1}(0.25)$

$= 2 \times 14.48^0$

$= 28.96^0$

The shortest distance apart between XY = $\frac{\beta}{360} \times 2\pi R$

$= \frac{28.96}{360} \times 2 \times \frac{22}{7} \times 6400km$

= 3236.17km

= 3240km to the nearest 10km.

EXERCISE

(1) A plane flies due East from A (53^0N, 25^0E) to a point B (53^0N, 85^0E) at an average speed of 400km/hr. the plane then flies south from B to a point C 2000km away. Calculate correct to the nearest whole number:

(i) The distance between A and B

(ii) The time the plane takes to reach point B

(iii) The latitude of C

(R = 6400km $\pi = {}^{22}/_7$)

(2) Two towns K and Q are on the parallel of latitude 46^0N. The longitude of town K is 130^0W. a third town P also on latitude 46^0N is on longitude 23^0E. Calculate:

(i) The length of parallel of latitude 46^0N to the nearest 100km

(ii) The distance between K and Q correct to the nearest 100km

(iii) The distance between Q and P measured along the parallel of latitude to the nearest 10 km.

(π = 3.142 R = 6400km)

(3) A moving object takes off from a town P (40^0N, 52^0E) and after flying 1450km due East, it reaches a town Q. it then flies due North to another town S on latitude 60^0N. Calculate:

(i) The radius of line of latitude through P

(ii) The longitude of Q to the nearest degree

(iii) The distance between Q and S along parallel of longitude to 3 s.f

(R= 6400km $\pi = {}^{22}/_{7}$)

(4) A plane flies due East from A (53^0N, 25^oE) to a point B (53^0N, 85^0E) at an average speed of 400km/hr. the plane then flies South from B to a point C 2000km away. Calculate to the nearest whole number:

(i) The distance between A and B

(ii) The time the plane takes to reach point B

(iii) The latitude of C

(R = 6400km π= 3.142)

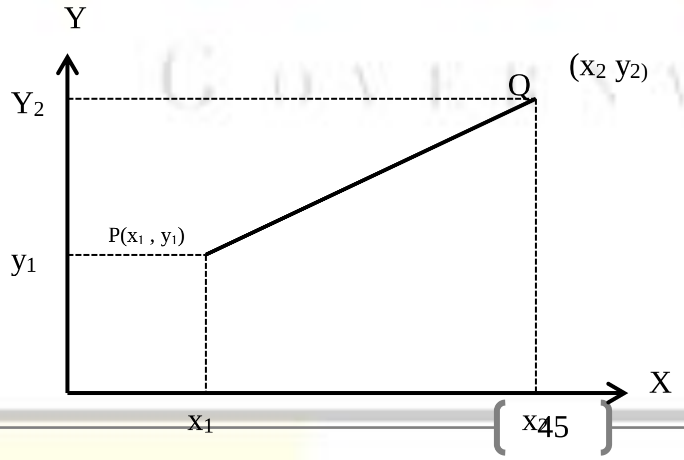
COORDINATES GEOMETRY

A point in a plane can be located from the two perpendicular axes x and y called Cartesian coordinate system.For example, the distance of a point such as P is usually written in brackets as P(x,y) .The x coordinate is written first followed by a comma and then the y coordinate. E.g.IfP(3, 4) that implies x is 3 and y is 4.

LENGTH OF A STRAIGHT LINE JOINING TWO POINTS

Given that P(x, y) and Q(x_2, y_2) are two points on a Cartesian plane. Distance PQ is given as

PQ = $\sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$



∴PQ = $\sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$

MID-POINT OF A STRAIGHT LINE JOINING TWO POINT

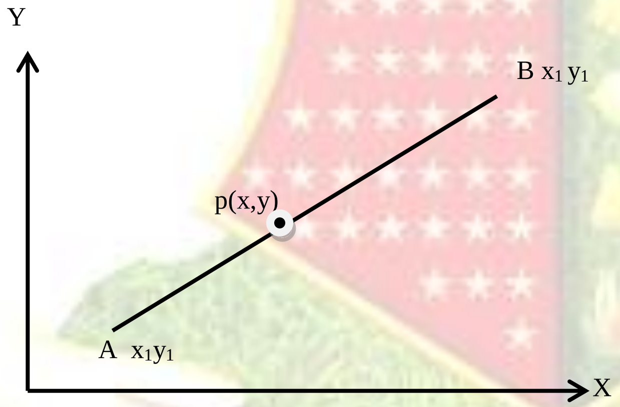
Let P(x, y) be the mid-point of line AB with coordinates A(x_1, y_1) and B(x_2, y_2).

Point P(x, y) can be calculated as follow:

$$x = \frac{x_1+x_2}{2} \text{and} y = \frac{y_1+y_2}{2}$$

∴ The coordinates of the mid-point of A(x_1, y_1) and B (x_2, y_2) are

$$x = \frac{x_1+x_2}{2}\ , y = \frac{y_1+y_2}{2}$$



Examples:

(1) (a) Calculate the length of the line joining P (-3, 4) to Q (5, -2)

(b) Calculate the mid-point of PQ

SOLUTION

(a) (x_1, y_1) = (-3, 4), (x_2, y_2) = (5, -2)

i.e. x_1 = -3, y_1 = 4, x_2 = 5 andy_2= -2

Using PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(5 - -3)^2 + (-2 - 4)^2}$

= $\sqrt{8^2 + (-6)^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100} = 10$

(b) The mid-point of PQ is given by:

$x = \frac{x_1+x_2}{2}$, $y = \frac{y_1+y_2}{2}$

$= \left(\frac{5-3}{2}, \frac{-2+4}{2}\right)$

$= \left(\frac{2}{2}, \frac{2}{2}\right)$

$=(1,1)$

(2) Find the value of $\propto^2 + \beta^2$ if $\propto + \beta = 2$ and the distance between the points $(1,\propto)$ and (P, 1) is 3 units.

SOLUTION

Using distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$3 = \sqrt{(\beta - 1)^2 + (1 - \propto)^2}$

$9 = \beta - 2\beta + 1 + 1 + 2 \propto + \propto^2$

$9 = \beta^2 + \propto^2 - 2 \propto - 2\beta + 2$

$9 - 2 = \beta^2 + \propto^2 - 2\ (\propto + \beta)$

$7 = \beta^2 + \propto^2 - 2(\propto + \beta)$

Since $\propto + \beta = 2$

$7 = \propto^2 + \beta^2 - 2(2)$

$7 = \propto^2 + \beta^2 - 4$

$\therefore\ \propto^2 + \beta^2 = 7 + 4$

$\propto^2 + \beta^2 = 11$

(3) If M(4,q)is the mid point of the line joining l (p, -2) and N(q, p) find the values of P and q

Solution

Midpoint coordinates =$\left(\frac{n1+n2}{2}, \frac{y1+y2}{2}\right)$

i.e (4,q) = $(\frac{p+q}{2}, \frac{-2+p}{2})$

4=$\frac{p+q}{2}$ and q= $\frac{-2+p}{2}$

8= p+q-----------(1)

PERPENDICULARITY AND PARALLELISM

Two lines are parallel if they have the same gradient and two lines are perpendicular if the product of their gradients equal to -1. Gradient of a line segment joining point A(x_1,y_1) and B (x_2,y_2) is given by gradient 'M'= $\frac{y2-y1}{x2-x1}$

Examples:-

(1) Find the gradient of the line joining the points with coordinates (-2,-4) and (3, 11).

Solution

M= $\frac{y2-y1}{x2-x1} = \frac{11-(-4)}{3-(-2)}$

= $\frac{11+4}{3+2}$

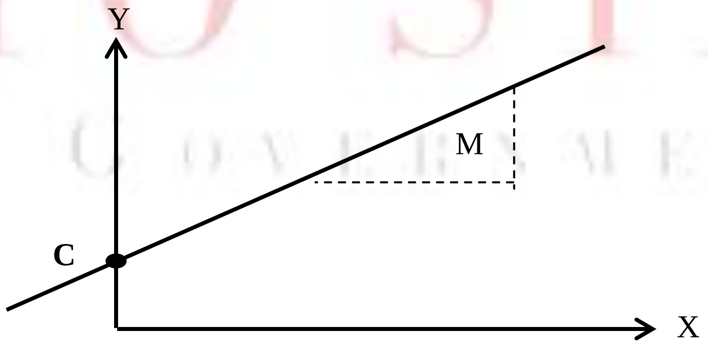
= $\frac{15}{5}$,= 3

:- the gradient is 3

(2) What is the value of 'P' if the gradient of the line joining (-1,p) and (p,4) is $\frac{2}{3}$?

Solution

M= $\frac{y2-y1}{x2-x1}$



'M' is the gradient of the line and 'c' is the intercept on the y axis

The equation of the line is given by y= mn+c. but if the line passes through the orgin (0,0), the equation becomes y=m

Examples :Find the equation of the line of gradient 3 which passes through the point (0,2)

Solution :M= 3 C = 2

;- The equation of the line is y = mn+c, Ic y = 3n+2

1) Find the equation of the straight line which passes through the origin and of gradient -4.

   Solution:M = -4 intercept = origin i.e 'O'

   The equation is y = -4n

2) Find the gradient and the intercept on the y axis for the straight line whose equation is 3y +7n -5 = 0

   Solution :Express it in the form y = mn+c

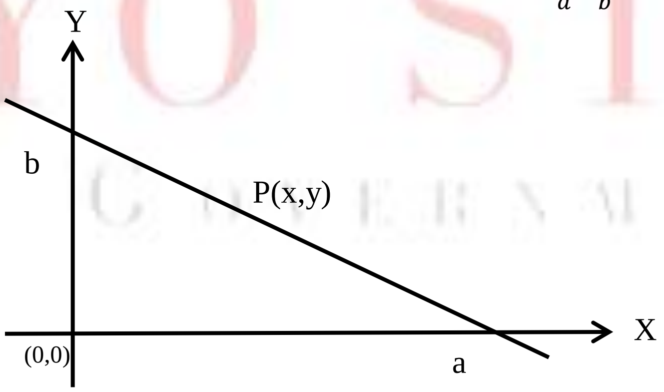
   3y+7n = 5, 3y =-7n+5, y=$^{-7}/_{3}$ n +$^{5}/_{3}$

   Compare with y = mn+c

   :. M =$^{-7}/_{3}$ and C= $^{5}/_{3}$

   Hence the gradient is $^{-7}/_{3}$ and the intercept on the y axis is $^{5}/_{3}$

2) Equation of a line in double intercept form equation of a straight line which has 'a' and 'b' as intercepts at n and y axes respectively is given by $\frac{x}{a}$ +$\frac{y}{b}$ =1



Examples

1) Find the equation of the line whose intercepts on the x and y axes are 2 and -3 respectively

   Solution :a = 2 and b= -3, the equation is given by

   $\frac{x}{a} + \frac{y}{b}$ =1

   $\frac{x}{2}$ -$\frac{y}{3}$ =1, 3x -2y =6, :. The equation is 3x-2y-6=0

2) Given that the equation 4x -3y + 12 =0 find (a) the intercepts on both axes (b) the slope of the line

   Solution: a.Write the equation in the form $\frac{x}{a}$ + $\frac{y}{b}$ =1, 4x -3y =-12,Divide through by 12

$\frac{4x}{-12}$-$\frac{3y}{-12}$ =$\frac{-12}{-12}$

$\frac{x}{-3}$ + $\frac{y}{4}$ =1

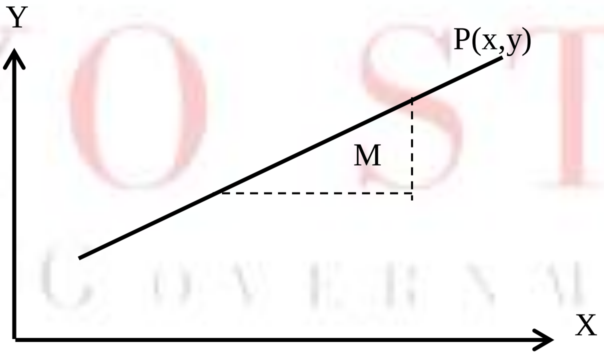
:. X intercept is -3 and Y intercept is 4, a.Write the equation in the form y = mx+c

   3y = 4n +12, Y = $\frac{4}{3}$ x + $\frac{12}{3}$, Y = $\frac{4}{3}$ x + 4

   :. The gradient /slope is $\frac{4}{3}$

3) Equation of A straight line in the gradient and one point form. The equation of a straight line whose gradient is 'm' and passes through the point ( )(, Y) is given by

   $y - y_1 \;=\; m(x - x_1)$



Example

Find the equation of the line of shope 3 which passes through the point (- 2, 5).

Solution :M = 3 (x, y,) = (-2,5) the equation of the line is given by

Y – Y, = M (x – x_1)

- Y – 5 = 3 (x - - 2)

  Y – 5 = 3 (x + 2)

  Y – 5 = 3x + 6

  Y – 3x – 5 – 6 = 0

:- the equation is y – 3n – 11 = 0.

4) Equation of straight line in two points form. Equation of a straight line which passes through points (x_1, y_1) and (x_2, y_2) is given by y –y, = ($\frac{y_2-y_1}{x_2-x_1}$) (n – x_1).

   Because M = $\frac{2-1y}{x_2-x_1}$

Examples.

1) Find the equation of the straight line which passes through the points (- 2,3) and (4, -5).

Solution.:(x_1,y_1) = (- 2, 3) and (x_2,y_2)= (4, -5)

M = $\frac{y2-y}{x_2-x_1} = \frac{-5-3}{4--2} = \frac{-8}{8} = 1$

Equation is $y - y_1 = m\,(x - x_1)$

$$y - 3 = -1\,(x + 2)$$

$$y - 3 = -x - 2$$

$$y + x - 3 + 2 = 0$$

$$y + n - 1 = 0$$

2) Find the equation of the straight line which passes through $(2, 3)$ and parallel to the line

$7x + 3y + 5 = 0$

Solution :Since the two lines are parallel, hence they have the same gradient

i.e.$M_1 = M_2$

$$3y = -7x - 5$$

$Y = -\frac{7}{3}x - \frac{5}{3}$

$\Leftarrow M_1 = -\frac{7}{3}$

hence$M_2 = -\frac{7}{3}$

Equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{7}{3}(x - x_1)$$

$$3y - 9 = -7x + 14$$

$$7x + 3y - 9 - 14 = 0$$

i.e.$7x + 3y - 23 = 0$

3) Find the equation of the straight line which passes through (1,5) and perpendicular to the line $3x + 2y + 5 = 0$

Solution

Since the two lines are perpendicular

$$M_1M_2 = -1$$

$$M_2 = \frac{-1}{m_1}$$

$$3x + 2y + 5 = 0$$

$$2y = -3x - 5$$

$$y = \frac{-3}{2}x - \frac{5}{2}$$

$$M_1 = \frac{-3}{2}$$

$$M_2 = \frac{-1}{-\frac{3}{4}} = \frac{2}{3}$$

Exercise

1) Find the lengths of the lines segments joining the following pairs of points

   (a) $(-1,3)$ and $(4,-2)$, b.$(9,1)$ and $(8,6)$

2) Find the coordinate of the mid points of the line segment joining the following pairs of points

   (a) $(5,3)$ and $(1,5)$

   (b) $(7,-6)$ and $(3,-4)$

3) Find the slopes of the line segment joining the following pairs of points

   (a) $(7,8)$ and $(-2,3)$

   (b) $(-6,3)$ and $(2,5)$

   (c) $(1,-5)$ and $(2,3)$

4) Find the equation of the line through $(-6,4)$ parallel to $5x + 4y = 3$.

5) Find the equation of a straight line which makes intercepts 3 and – 4 on the x and y axes respectively.

6) Find the equation of the line perpendicular to $2y + 3x - 4 = 0$ and passing through the point $(-2,3)$.

Multiplication Laws of Probability

Two or more events are independent if their outcomes don not affect one another. For instance if A,B,C----- are independent events, then the probability of A and B and C happening is p(A) x p(B) x p(C) x --------- is written as Pr(AnBnCn-----) = Pr(A) x Pr(B) x Pr(C) x ------- This is known as multiplication law or rule.

Example:- Two dice, one ;red and the other blue are thrown together. Construct a possibility space diagram to illustrate the outcomes. Find the probability that:

(a) at least one of the dice will show a six.

(b) The sum of the scores is 3 or 5

(c) The sum of the scores is 8 or even double.

Solution

The possibility space diagram of the outcomes is shown below

X 1	1 1,1	2 1,2	3 1,3	4 1,4	5 1,5	6 1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3.4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Outcomes on Red dice.

(a) Number in sample space = n(s) = 36 Let A = event that at least one of the dice showing a six,

Son(A) = 11

P(A) = $\frac{n\ (A)}{n\ (S)} = \frac{11}{36}$

(b) Let C = event that the sum of the schores is 3.

From the table 1 + 2, 2 + 1 = 3

∴ If C = {(1,2)(2,1)}then n© = 2

Let D = event that the sum of the scores is 5

From the table, 1 + 4, 2 + 3, 3 + 2, 4 + 1 all equal to 5

∴ If D = $\{(1,4),(2,3),(3,2),(4,1),\}, the\ n(D) = 4$

Since C and D are mutually exclusive, we have: P(A or B)= P(A) + P (B)

P(AuB) = P(A) + P(B)

= $^{2}/_{36} + {}^{4}/_{36} = {}^{6}/_{36} = {}^{1}/_{6}$

(c) Let E = event that the sum of the scores is 8.

i.e. E = $\{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

∴n (E) = 5

Let F= Event even double

F = $\{(2,2),(4,4),(6,6)\}$

∴n (F) = 3

Notice that {4, 4} is common to both sets

Using P(EuF) = P(E) + P(F) – P(EnF)

$^{5}/_{36} + {}^{3}/_{36} - {}^{1}/_{36}$

$\frac{5+3-1}{36} = {}^{7}/_{36}$

Example 2: A bag contains 9 identical balls, 3 are black, 2 are red and the remaining are whit.

(a) If a ball is picked at random what is the probability that it is not white?

(b) If two balls are picked at random one after the other with replacement fin the probability that:

(i) both of them are white

(ii) One is black and one is white in that order

(iii) at least one of them is red.

Solution:-

(a) P(not white) = P(black or red)

$^{3}/_{9} + ^{2}/_{9} = ^{5}/_{9}$

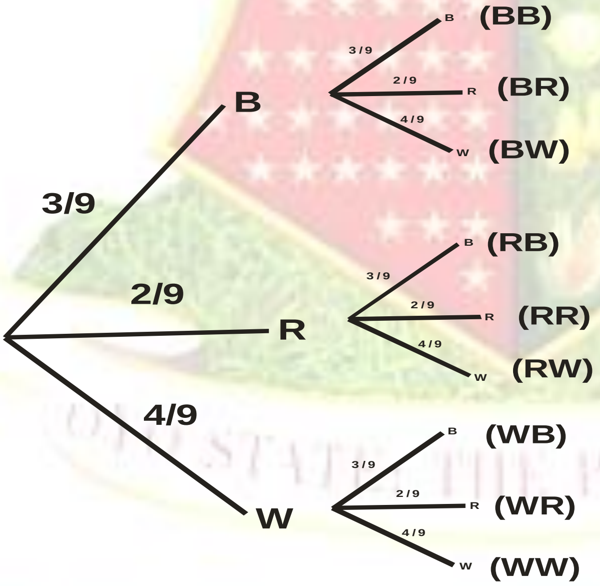
OR

Number of White balls = 9 – 3 – 2 = 4

P(white) = $^{4}/_{9}$

∴P(not white) = $1 - ^{4}/_{9} = ^{5}/_{9}$

(b) The tree diagram is shown below:



(i) P(WW) = $^{4}/_{9}\ X\ ^{4}/_{9} = ^{16}/_{81}$

(ii) P(BW in that order) = $^{3}/_{9}\ X\ ^{4}/_{9} = ^{4}/_{27}$

(iii) There are five ways of choosing at least one red ball i.e. P(at least one Red).

= P(BR) + P(RB) + P(RR) + P(RW) + P(WR)

(Since the outcomes are mutually exclusive)

$= {}^{3}/_{9} \; X \; {}^{2}/_{9} + {}^{2}/_{9} \; X \; {}^{3}/_{9} \; X \; {}^{2}/_{9} + {}^{2}/_{9} \; X \; {}^{4}/_{9} + {}^{4}/_{9} \; X \; {}^{2}/_{9}$

$= {}^{6}/_{81} + {}^{6}/_{81} + {}^{4}/_{81} + {}^{3}/_{81} + {}^{8}/_{81}$

$\frac{6 + 6 + 4 + 8 + 8}{81} = {}^{32}/_{81}$

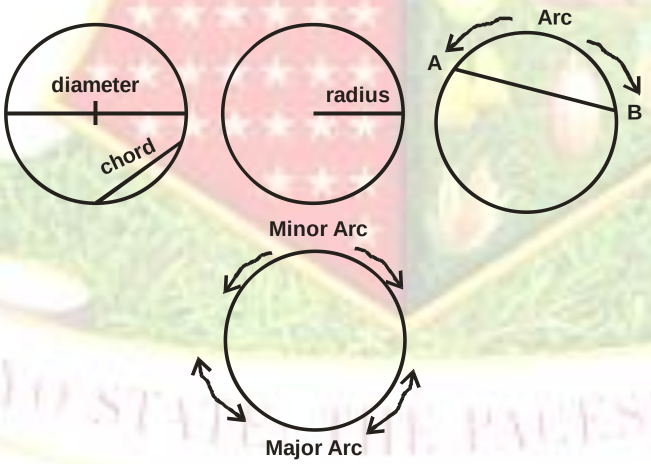
Measurement

A circle is the locus of a point which moves in a plane, so that its distance from a fixed point of that plane is constant. The fixed point is called the centre of circle and the constant distance is the radius.

The boundary line of the circle called its circumference i.e. the distance round the circle.

A chord of circle is a straight line joining two points on the circumference.

A diameter is called a chord which passes through the centre.

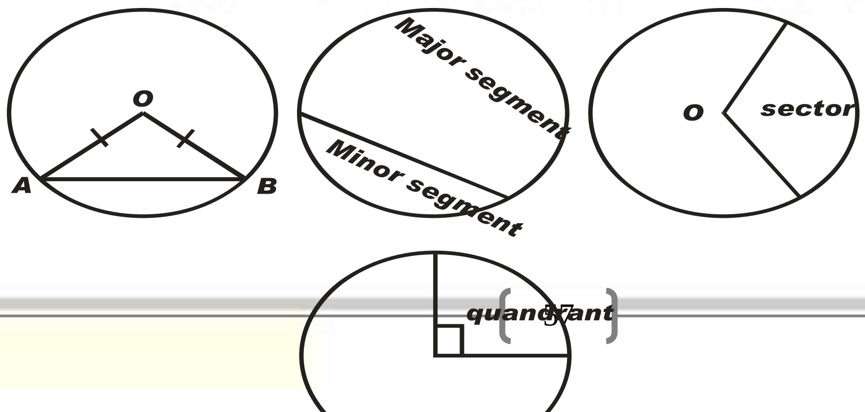


Concentric circles are circles that have the same centre.

An arc of a circle is a part of the circumference.

A major arc is an arc which in length is greater than half of the circumference of the circle.

A minor arc is an arc which in length is less than half of the circumference of the circle



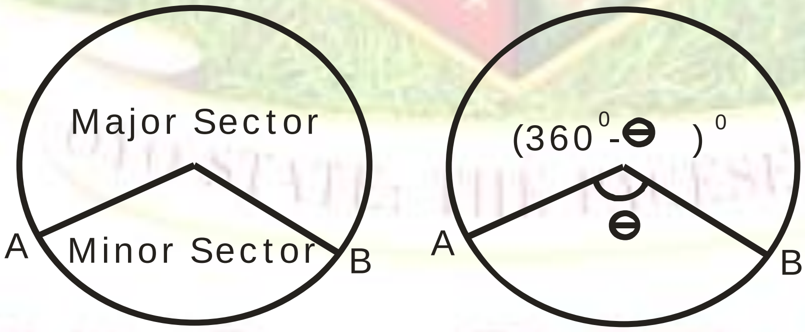
If AB is a chord of a circle with centre O, then the angle AOB is called the angle subtended by the chord AB at the centre of the circle.

A chord divides a circles into two segments. The smaller is called the minor segment while the larger is called the major segment.

A sector of a circle is that arcs bounded by an arc and the two radii passing through the ends of the arc.

The area bounded by two radii at right angle to each other and which they are cut-off is called a quadrant.

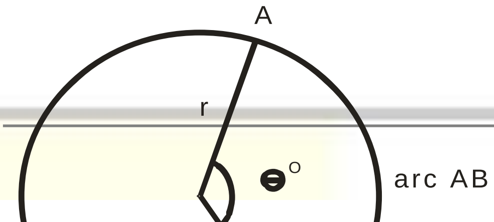
The angle of a sector is the angle between the two radii of the sector when the angle of a minor sector is θ^0, the angle of the corresponding major sector is $(360^0 - \theta)$



We notice that the segment cut-off by a diameter is the semi-circle.

Length of an arc

The length of an arc is proportional to the length of the circumference just as the angle subtended by the arc at the centre is proportional to the angle of the whole circle.



$$\frac{Length\ of\ an\ Arc\ AB}{Circumference\ of\ the\ circle}\ X\ \frac{\theta}{360^0}$$

$$\therefore \frac{Arc\ AB}{2\pi r} = \frac{\theta}{360^0}$$

Length of an arc AB = $\frac{\theta}{360^0}\ X\ 2\pi r$

Example 1. The angle of a sector is 200 and its radius is 15cm, calculate the length of its arc

L = $\frac{\theta}{360}\ X\ 2\pi r$

Where; θ = 200, r = 15cm π= $\frac{22}{7}$

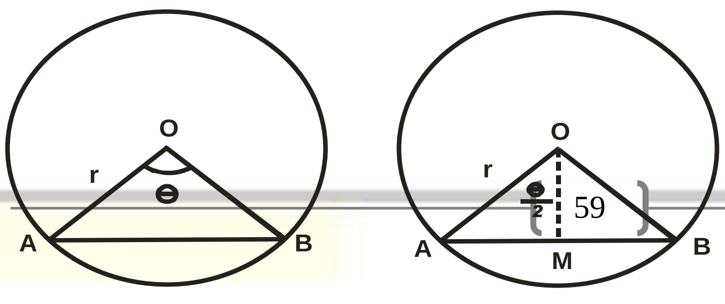
Arch AB = $\frac{20}{360}\ X\ 2\ \frac{22}{7}\ X\ 15$

= $\frac{110}{21}$

= 5.236cm

= 5.2cm

Length of a chord



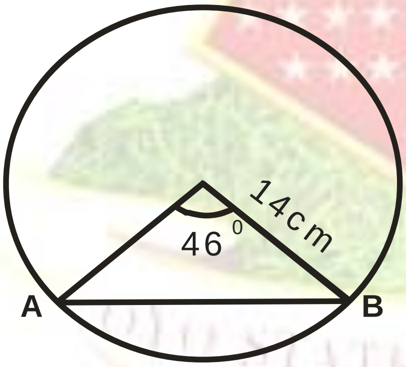
The ΔOAB above is an isosceles triangle M is the mid-point of $\overline{AB}$ *and* A$\hat{O}$M is $\frac{\theta}{2}$.From the right angle ΔA$\hat{O}$M

$$\frac{\overline{AM}}{r} = Sin\ \frac{\theta}{2}$$

$$\overline{AM} = r\sin\frac{\theta}{2}$$

Hence, Chord AB = 2$\overline{AM}$ = 2rsin$\frac{\theta}{2}$

Example 2: Find the length of the chord which sustends an angle of 46^0 at the centre of its circle with radius 14cm.



Length of Chord AB = 2rsin$\frac{\theta}{2}$

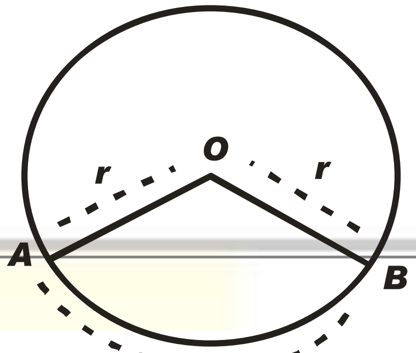
= 2 X 14 X Sin 23^0

= 28 X 0.3907

= 10.9396

≃ 10.9cm

Perimeter of a Sector



Perimeter of a shape is the total distance around the boundary of that shape.
The perimeter of the shaded minor sector AOB = Length OA + length OB + Lent of Minor Arc AB = r + r + $\frac{\theta}{360}$ x $2\pi r$

= $2r + \frac{2\pi r\theta}{360}$

Example 3: Calculate the perimeter of a sector of a circle radius 21cm, where the angle of the sector is 120.

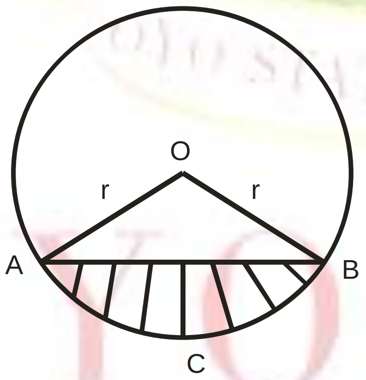
Solution: The required perimeter, P is given by P=2r + $\frac{\theta}{360}$ $X\ 2\pi r$ where r=21cm, $\theta = 120^0$, $\pi = {}^{22}/_{7}$

P = 2 x 21 + $\frac{120}{360}$ x 2 x $\frac{22}{7}$ x 21

= 42 + 44

= 86 cm

Perimeter of a segment



The perimeter P, of the shaded segment ABC is given by

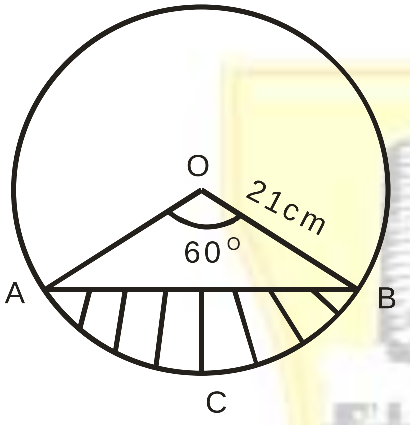
P = Chord AB + Arc ACB

= $2r\mathrm{Sin}{}^{\theta}/_{2} + \frac{2\pi r\theta}{360^0}$

Also, the perimeter of ΔAOB = Lengthe OA + Length OB + Chord AB

= r + r + 2rsin$\frac{\theta}{2}$ = 2r + 2rSin$\frac{\theta}{2}$

Example 4:



In the figure above calculate

(a) The perimeter of the minor sEctor AOB
(b) The perimeter of the triangle AOB
(c) The perimeter of the minor segment ABC

Solution

(a) The perimeter P, ofhte minor sector AOB is given by

P = 2r + $\frac{\theta}{360}\ X\ 2\pi r$

P = 2 x 21 + $\frac{60}{360}\ X\ 2\ X\ \frac{22}{7}\ X\ 21$

= 42 + 22

= 64 cm

(b) The perimeter P, of the triangle AOB is given by:

P = 2r + 2rsin$\frac{\theta}{2}$

= 2 x 21 + 2 x 21 x Sin 30^0

= 42 + 2 x 21 x $^1/_2$

= 42 + 21

= 63 cm

(c) The perimeter P, of the minor Segment ABC is given by

$P = 2r\text{Sin}\frac{\theta}{2} + \frac{\theta}{360}\ X\ 2\pi r$

Where $\theta = 60^0$, r = 21cm

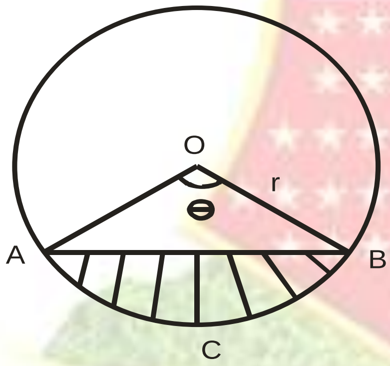
$= 2 \text{ x } 21 \text{ x } \sin 30 + \frac{60}{360} \text{ x } 2 \text{ x } \frac{22}{7} \text{ x } 21$

$= 42 \text{ x } {}^1/_4 + 22$

= 21 + 22

= 43cm

Area of a Segment of a Circle



The minor sector OAB is made up of the isosceles ΔOAB plus the minor segment ABC Area of sector AOBC = $\frac{\theta}{360}\ X\ \pi r^2$

Area of ΔAOB = ${}^1/_2\, r^2 Sin\theta$

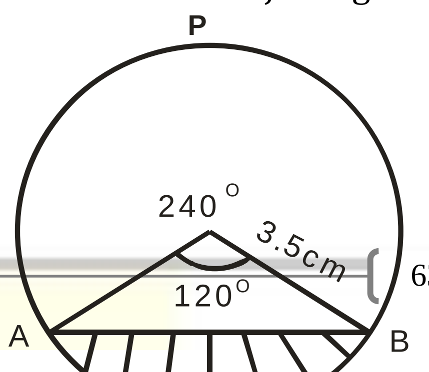
Area of minor segment ACB = Area of sector OACB – Area of ΔAOB

$= \frac{\theta}{360}\ X\ \pi r^2 - {}^1/_2\, r^2 \sin\theta$

Example 5: An arc AB subtends an angle 1200 at the centre of a circle of radius 3.5cm, find;

(a) The area of the minor segment cut-off by the chord AB

(b) The area of the major segment cut-off by the chord AB.



Solution:

$\Theta = 1200\,,\ \pi = \frac{22}{7}\,, r = 3.5\text{cm}$

(a) Area of the minor segment ABC

$= \frac{\theta}{360}\ X\ \pi r^2 - 2r^2 sin\theta$

$= \frac{120}{360}\ X\ \frac{22}{7}\ X\ \frac{7}{2}\ X\ \frac{7}{2} - \frac{1}{2}\ X\ \frac{7}{2}\ X \frac{7}{2}\sin 120^0$

$= \frac{77}{6} - \frac{49}{8}\ X\ 0.866$

$= \frac{97}{6} - \frac{21.217}{4}$

$=$ 12.833 – 5.304

$=$ 7.529cm^2

(b) Area of the major segment APB area of the whole circle – area of minor segment ABC

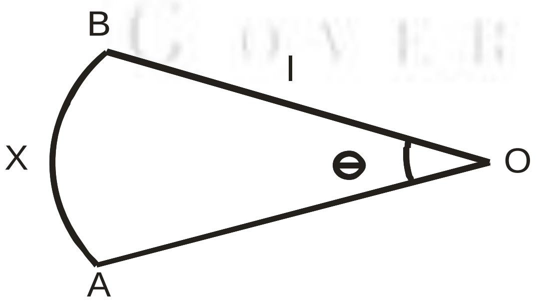
$=$ πr2 – 7.529

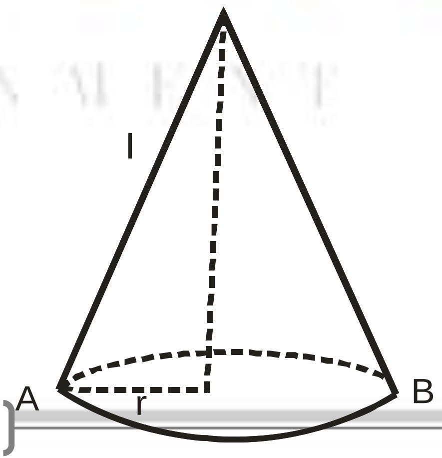
$= {}^{22}/_7\ X\ {}^{7}/_2\ X\ {}^{7}/_2 - 75.29$

$=$ 38.5 – 7.529

$=$ 30.971cm^2

The sector of a circle as a curved surface area of a cone



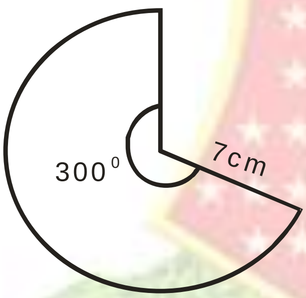


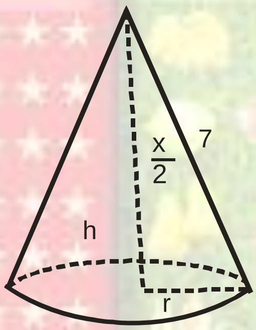
A sector of a circle can be bent to form the curve surface of an open cone

$\therefore$ curved surface area of cone $= \frac{\theta}{360}\ X\ \pi l^2 (area\ of\ the\ sector)$

Example : A 300^0 sector of a circle of radius 7cm is bent to form a cone. Find

(i) The base radius of the cone
(ii) The perpendicular height of the cone
(iii) The vertical angle of the cone.





Let the radius of the base of the cone be rcm and the vertical angle be $\propto$

(i) The circumference of base of cone = length of arc of sector

$2\pi r = \frac{\theta}{360}\ X\ 2\pi l$

$2\pi r = \frac{300}{360}\ X\ 2\pi\ X\ 7$

Divide through by 2π

$r = \frac{5\ X\ 7}{6} = \frac{35}{6} = 5.83cm$

(ii) The perpendicular height

$h = \sqrt{l^2 - r^2}\ \ where\ L = 7cm$

$r = 5.83cm$

$= \sqrt{7^2 - 5.83^2}$

$= \sqrt{49 - 33.9}$

$= \sqrt{15}$

$= 3.873\text{cm}$

(iii) The vertical angle

$\text{Sin}\ {}^{\propto}/_{2} = {}^{r}/_{l} = \frac{5.83}{7} = 0.8328$

$${}^{\propto}/_{2} = Sin^{-1}(0.8328)$$

$= 56.38^0$

$\propto = 56.38^0\ X\ 2$

$= 112.76^0$

Evaluation : The teacher asks the students to solve the question below. A 216^0 sector of a circle of radius 5cm is bent to form a cone. Find.

(i) The base radius of the cone.
(ii) The perpendicular height of the cone
(iii) The Vertical angle of the cone.

Topic: Perimeter and Area of Plane shapes

Objectives: At the end of the lesson, students should be able to list some common plane shapes.

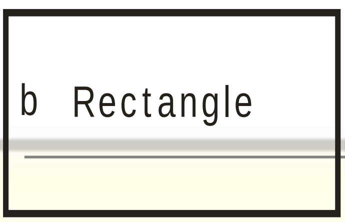
Calculate the perimeter as well as the area of some plane shapes.

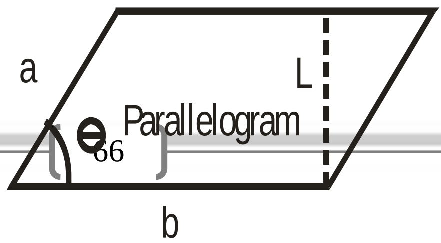
Reference book : J.B Channon New general Mathematics for SS 1 2004 pages 130-131.

Introduction: The teacher introduces the lesson by reminding the student of the properties of some common plane shapes.

PRESENTATION

Perimeter and Area of plane shapes.



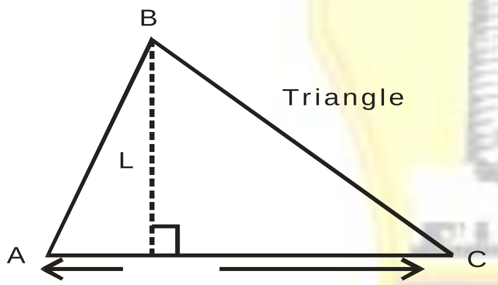


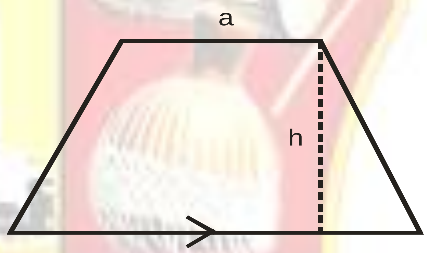
Perimeter = 2(L + B)

Area = L X B

Perimeter = 2(a + b)

Area = b X h = absinθ





Trapezium Area = ½ (a + b)h

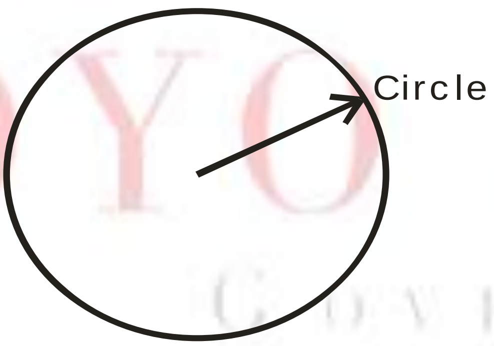
P = a + b + c

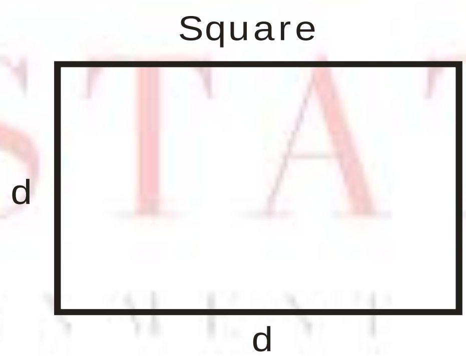
Area = ½ X base X height

½ absinC

½ acSinB

½ bcsinA



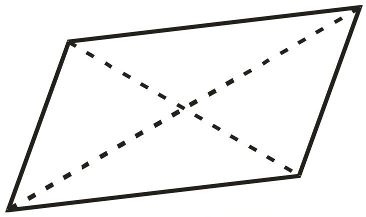


Circumference 2πr

Area = πr^2

P = 4d

Area = d^2



Rhombus

Area = ½ X Product of the diagonal

The common units of area arc cm^2 m^2 km^2. The hectare (ha) is often used for land measure.

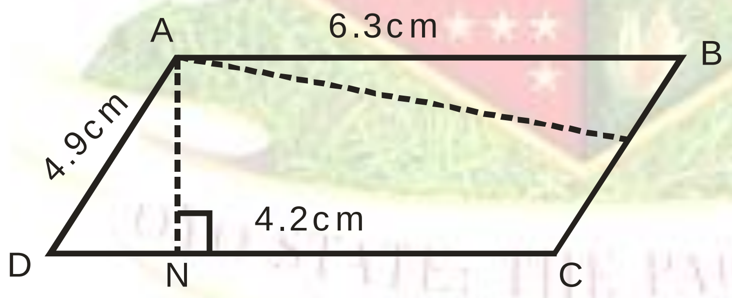
$1ha = 10{,}000m^2$

Example : ABCD is a parallelogram and AM, AN are the perpendicular from A to $\overline{BC}$, CD respectively.

If /AB/ = 6.3cm, /AD/ = 4.9cm and /AN/ = 42cm

Calculate the area of the parallelogram and hence find /AM/

Solution



Area = base X height

= AB = AN

$6.3 \times 4.2 = 26.46cm^2$

Also considering BC as the base Area = BC X AM

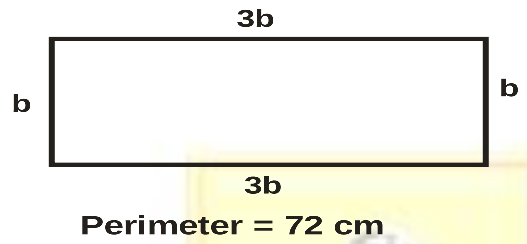
$26.46 = 4.9 \times AM$

$\overline{AM} = 26.46 \div 4.9$

$= 5.4cm$

Example 2: The length of a rectangle is three times its width.

If the perimeter is 72cm, calculate the width of the rectangle.



Let the width of the rectangle be bcm

∴ the height will be 3bcm

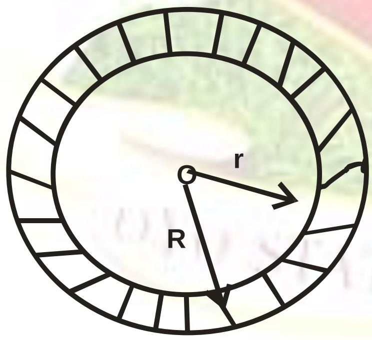
Perimeter $= 3b + b + 3b + b = 8b$

$72 = 8b$

$= 9cm$

Hence, the width of the rectangle is 9cm.

Example 3 : A washer is 4.5cm in diameter with a central hole of diameter 1.5cm. Calculate the surface area of the two sides of the washer.



Solution

Area of the washer = area of the outer circled = Area of the inner circle.

$= \pi r2 - \pi r2$

$= \frac{22}{7} X \frac{9}{4} X \frac{9}{4} - \frac{22}{7} X \frac{3}{4} X \frac{3}{4}$

$= \frac{891}{56} - \frac{99}{56}$

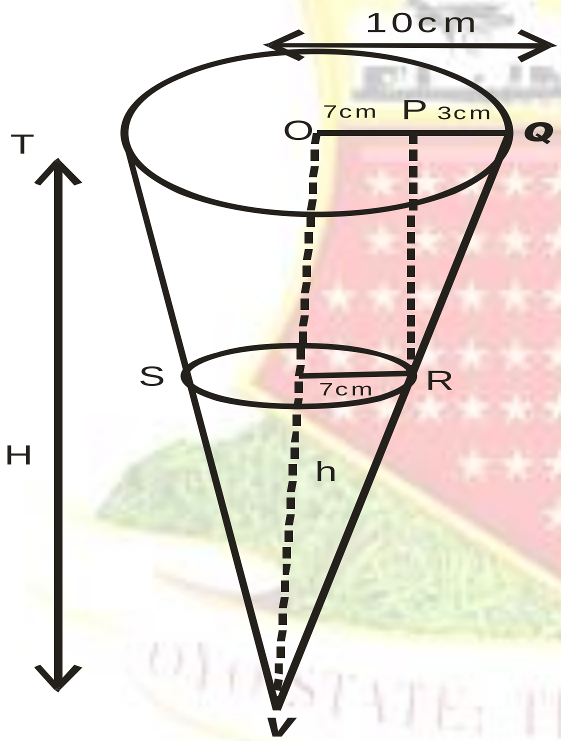
$= \frac{797}{56} = 14.143cm$

Example 4 : A bucket is 20cm in diameter at the top and 14cm in diametr at the bottom and 15cm deep. Calculate:

(a) The capacity of the bucket in litres.
(b) The curved surface area of the bucket.

Solution:

(a) Draw the bucket as shown below. The bucket, TQRS is a frustum of a cone. To carry out the calculation, 16 is necessary to draw a complete cone as shown.



Let the height of the small cone be hcm and the height of the complete cone beHcm. From the diagram

$\frac{h}{7} = \frac{h+15}{10} (Similar\ \Delta S)$

10h = 7h + 105

10h – 7h = 105

3h = 105

H = $^{105}/_{3}$

h = 35

H = 15 + 35

= 50cm

Volume of frustum

= Volume of large cone-volume of small cone.

$= {}^{1}/_{3}\,\pi R^2 H - {}^{1}/_{3}\,X\pi\ X\ 7^2\ X\ 35$

$= {}^{1}/_{3}\ X\ {}^{22}/_{7}\ X\ 3285$

= 3441cm^2

Capacity of the bucket $= \frac{3441}{1000}\ litres$

= 3.44 litresto 3 s.f

(b) In Δ PQR,

S2 = 152 + 32

= 225 + 9

S^2 = 234

S = $\sqrt{234}$

= 15.30cm

Curved surface area = πs(R + r)

= π x 15.30 (10 + 7) cm^2

$= \frac{22}{7}\ \ X\ \ 15.30\ X\ 17$

= 817.46cm^2

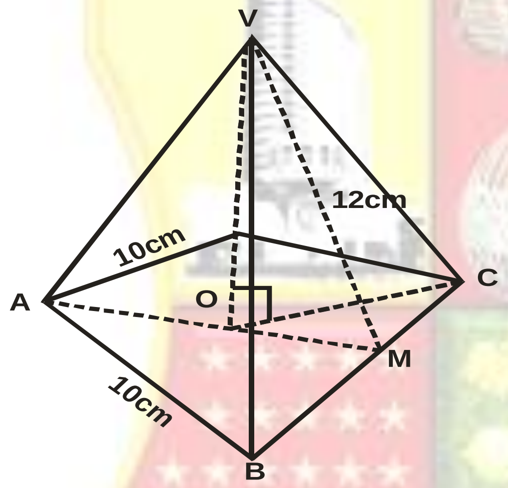
= 817cm^2 to 3 s.f.

Example 5: A pyramid ABCDV with a square base ABCD of side 10cm has triangle faces. With altitudes of 12cm. Calculate giving your answer to 3 significant figures the:

(a) Total surface area
(b) Volume of the pyramid
(c) Angle between the face VBC and the base.

Solution:

(a) Draw the diagram as shown below.



VM is the slanting height of ΔVBC M is the midpoint of BC OM is half of AB.OM = 5cm. VO is the height of the pyramid.

There are four ΔS faces.

Total surface area = Area of square base + 4 X area of a ΔVBC

$= 10 \times 10 + 4(\frac{1}{2} \times 10 \times 12)$

$= 100 + 240\text{cm}^2$

$= 340\text{cm}^2$

(b) In Δ VOM,

$VO^2 = 12^2 - 5^2$

$= 144 - 25$

$= 119$

$VO = \sqrt{119}$

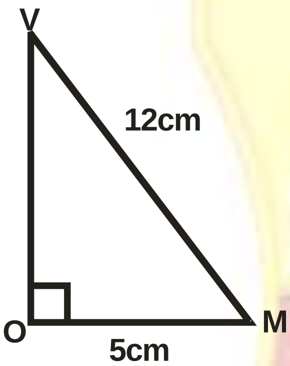
$= 10.91\text{cm}$

Volume = $^{1}/_{3}\ X\ A\ X\ h$

$= \ ^{1}/_{3}\ \ X\ \ 100\ \ X\ 10.91cm^{3}$

$=\ 364.67\text{cm}^3$

$=\ 364\text{cm}^3$ to 3 s.f



(c) The required angle is $V\bar{M}O$

Cos $V\bar{M}O\ =\ ^{5}/_{12} = 0.4167$

$V\bar{M}O\ =\ 65.38^{0}$

$=\ 65.4^{0}$to 3 s.f.

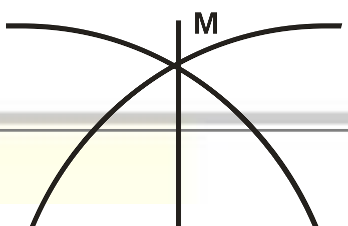
Geometrical constructions and vectors

Rules of constructions

1. Always make a rough sketch before carrying out an accurate construction.
2. Drawing equipment must be clean and in good condition.
3. A construction line must be drawn with a hard and sharp pencil to obtain a consistent thin line. Intersecting arcs must not be too thick.
4. Do not erase your construction lines, i.e. the evidence of the method you use.

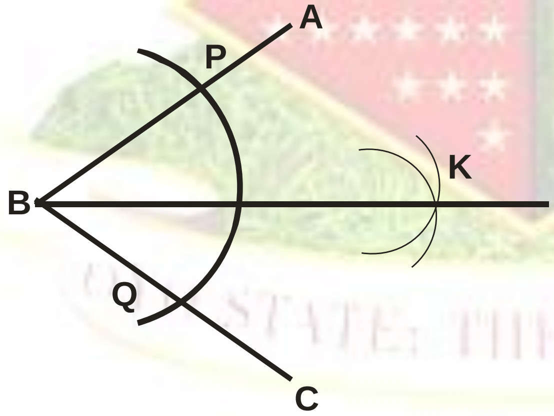
Major Construcitons

(a) Bisection of a straight line segment



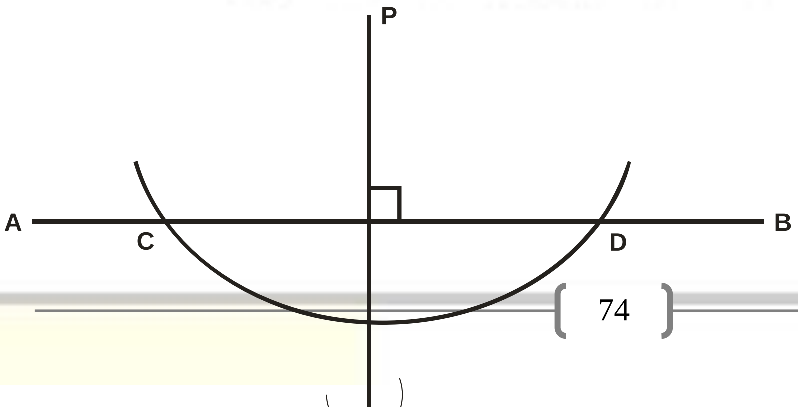
Given line AB with centres A and B and any convenient radius, more than half of the line, make arcs to intersect at M and N. Then join MN. Thus K is the mid-point of AB. Measure $\overline{AB}$ and $\overline{KB}$ are they equal?

(b) Bisection of an angle



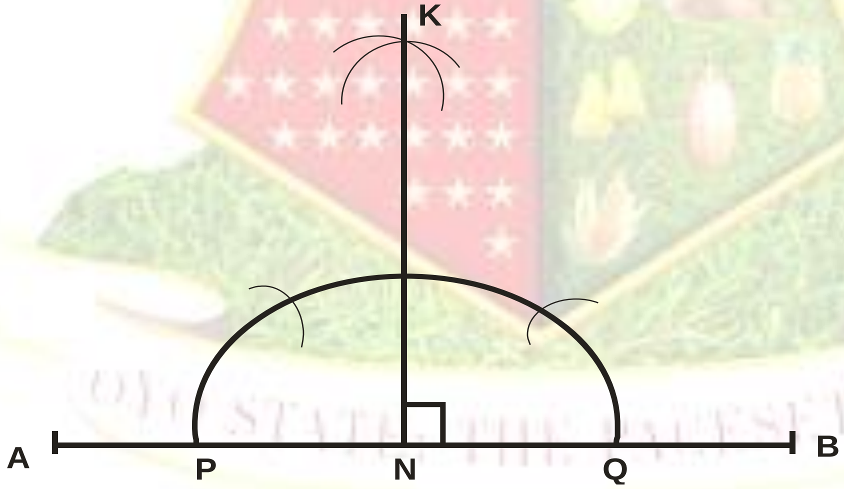
Given angle ABC. With centre B and any radius mark off arcs at P and Q on AB and BC. With centres P and Q and equal radii, mark off arcs to intersect at K. Then from BK which is the angular bisector.

C. To construct a perpendicular line to a given straight line from a given point outside the line.



Given a line AB. With centre P make arc to cut AB at C and D with centres C and D and equal radii mark off arcs to intersect at K. join PK which is perpendicular to line AB.

The construction of Angle 90^0

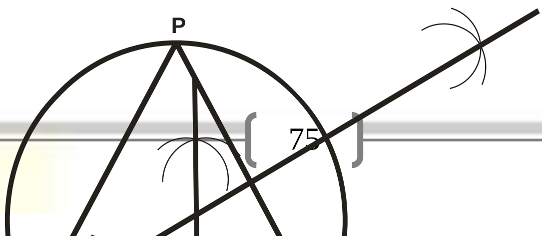


Given any line AB. With centre N on the line make an arc to cut AB at P and Q. With P and Q as centres and equal radii mark off areas to intersect at K. Join KN Angle KNB = 90^0

Application of Loci

(a) Circumscribed circle of a Triangle

Steps: Given an ΔPQR, bisect any two sides and let them meet at O, with centre O and radius OP or OQ, make a circle.



b. Construction of inscribed circle of Triangle.

Steps:

(i) Given ΔPQR bisect any two angles of the triangle and let the bisectors meet at O.

(ii) Through o, construction a perpendicular to any of the sides at k.

(iii) With O as centre and radius OK, make a circle.

Example:

(a) Using a ruler and pair of compasses only, construct.

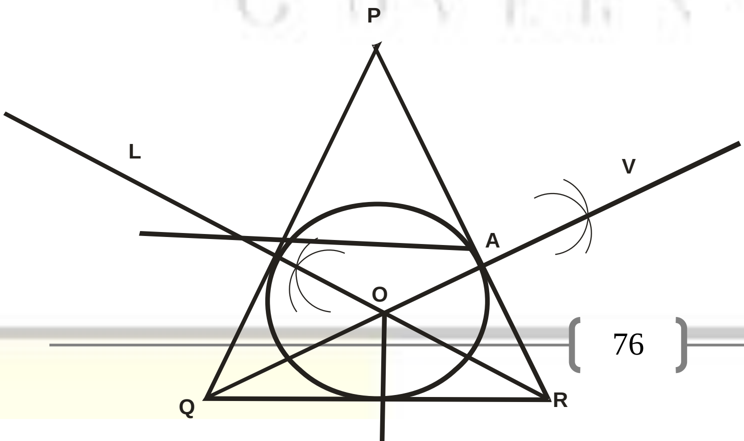
(i) A triangle ABC such that $\overline{AB} = 5cm$ /AC/ = 7.5cm and <CAB = 120^0

(ii) The locus l_1 of points equidistant from A and B

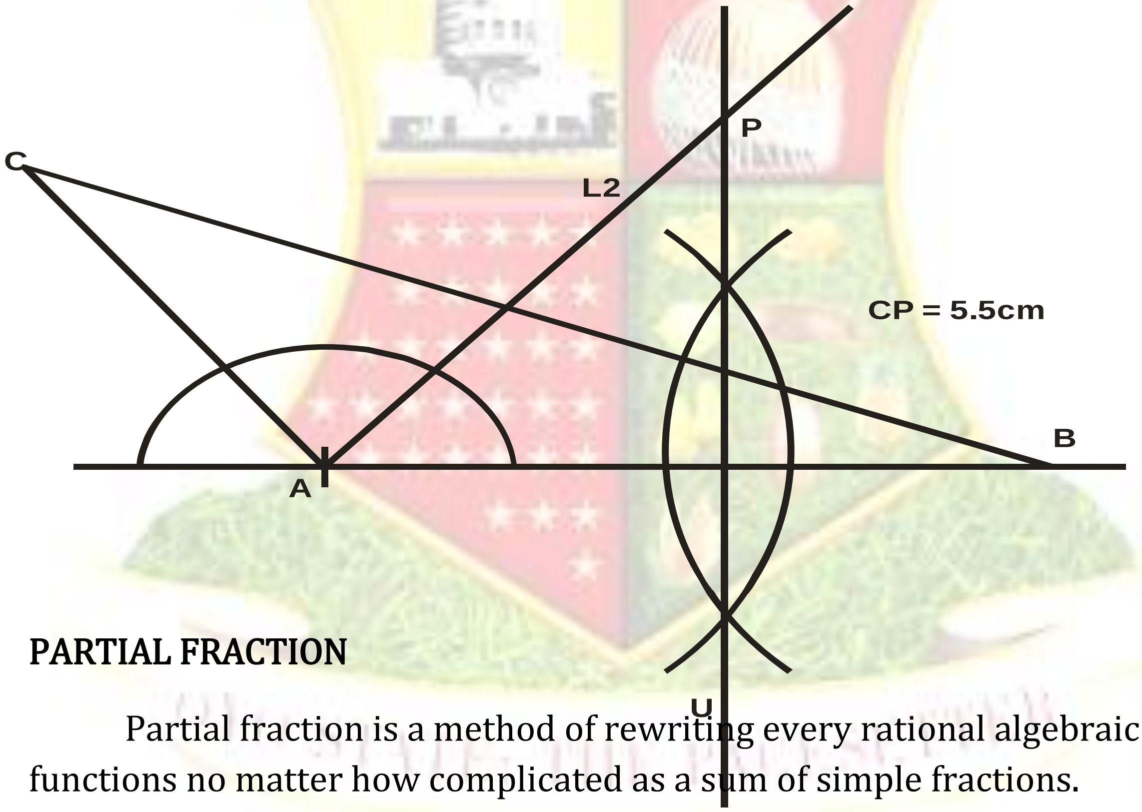
(iii) The locus l2 of points equidistant from AB and AC which passes through triangle ABC

(b) Label the point P where l1 and l2 intersect

(c) Measure $\overline{CP}$



Solution



PARTIAL FRACTION

Partial fraction is a method of rewriting every rational algebraic functions no matter how complicated as a sum of simple fractions.

Rules of solving partial fractions

1. If the degree of the number is greater than or equal to the degree of the denominator, then, carry out a division to obtain a quotient together with a fraction whose numerator is of a lower degree then its denominator.
2. Factorize the denominator into its prime factors, this is important, since the factors obtained determine the shape of the partial fractions.
3. A linear factor of ax + b in the denominator gives a partial fractions of the form $\frac{A}{ax+b}$

4. A repeated linear factor of $(ax + b)^2$ in the denominator gives a partial fraction of the form $\frac{A}{ax+b} + \frac{B}{(ax+b)^2}$
5. A repeated linear factor of (ax – b)2 in the denominator gives a partial fraction of the form: $\frac{A}{ax-b} + \frac{B}{(ax-b)^2} + \frac{C}{(ax-b)^2}$where A,B and C are constants.
6. An irreducible quadratic factor $(ax^2 + bx + c)$ in the denominator gives a partial fraction of the form $\frac{Ax+B}{ax^2+bx+c}$
7. A repeated quadratic factor $(ax + bx + c)^2$ in the denominator gives partial fraction of the form: $\frac{Ax+B}{ax^2+bx+c} + \frac{Cx+D}{(ax^2+bx+c)^2}$

Example 1: Express $\frac{2x+1}{x^2+3x+2}$in partial fraction.

<u>Solution</u>

Factorize the denominator into its prime factors : $x^2 + 3x + 2 = (x + 2)(x + 1)$

Rewrite the fraction

$$\frac{2x+1}{x^2+3x+2} = \frac{2x+1}{(x+2)(x+1)}$$

also,

$$\frac{2x+1}{(x+2)(x+1)} \equiv \frac{A}{x+2} + \frac{B}{x+1} \quad ------equation\ 1$$

Take the L.C.M of the RHS ot gives:

$$\frac{2x+1}{(x+2)(x+1)} \equiv \frac{A(x+1)+B(x+2)}{(x+2)(x+1)}$$

Multiply both side by the denominator (x + 2) (x + 1)

$$\frac{2x+1}{(x+2)(x+1)} X\ (x+2)(x+1) = \frac{A(x+1)+B(x+2)}{(x+2)(x+1)} X\ (x+2)(x+1)$$

2x + 1 = A(x + 1) + B(x + 2)

Choose a value for x that can make one of the expression in the bracket zero;

Let x = -1

2(-1) + 1 = A(-1)(+1) + B(-1 +2)

-2 +1 = A(0) + B(+1)

-1 = B

B = -1

Also let x = -2

2(-2) + 1 = A(-2 + 1) + B(-2 + 2)

-4 + 1 = A(-1) + B(0)

-3 = -A

A = 3

Subtitute A = 3 and B = -1 in equation 1 we have:

$$\frac{2x+1}{x^2+3x+2} = \frac{3}{x+2} - \frac{1}{x+1}$$

Example 2: Express $\frac{35x+17}{(5x+2)^2}$ in partial fraction

Solution:

From Rule 4, rewrite the expression

$$\frac{35x+17}{(5x+2\,)^2} \equiv \frac{A}{5x+5} + \frac{B}{(5x+2)^2} \quad equation\ 1$$

Multiply both sides by the original denominator $(5x + 2)^2$ we have:

35x + 17 = A(35x + 2) + B

35x + 17 = 5Ax + 2A + B

Equate the coefficients fo the power of x we get:

35x = 5Ax

$A = \frac{35}{5}$

$= \ 7$

17 = 2A + B

17 = 2(7) + B

17 = 14 + B

B = 17 – 14

B = 3

Substitute the value of A and B in equation 1 below

$$\frac{35x+17}{(5x+2\,)^2} = \frac{7}{5x+2} + \frac{3}{(5x+2)^2}$$

INTEGRATION BY PATIAL FRACTION

Algebraic fractions can also be expressed interms of partial fractions. This makes integration of such algebraic fractions possible.

Example: Integrate the following by partial fraction.

$$\int \frac{18x+20}{(3x+4)^2}$$

Solution

Let M = $\frac{18x+20}{(3x+4)^2}$

$$\frac{18x+20}{(3x+4)^2} \equiv \frac{A}{3x+4} + \frac{B}{(3x+4)^2} \quad ------- \; equation\ 1$$

Multiply both side by the denominator

$(3x + 4)^2$ we have

$18x + 20 = A(3x + 4) + B$

$18x + 20 = 3AX + 4A + B$

Equating the coefficient of power of x:

$18x = 3Ax$

$A = {}^{18}/_{3}$

$A = 6$

$20 = 4(6) + B$

$20 = 24 + B$

$B = 24 – 20$

$= -4$

Substitute the value of A and B in equation 1

M = $\frac{18x+20}{(3x+4)^2} \equiv \frac{6}{3x+4} - \frac{4}{(3x+4)^2}$

$\therefore \int M dx = \int \frac{18x+20}{(3x+4)^2}\, dx = \int \frac{6}{3x+4} - \frac{4}{(3x+4)}$dx

By Sine rule

$6\int \frac{1}{3x+4}\, dx - \int \frac{1}{(3x+4)^2} = dx$

$2/n/(3x + 4)/$ - $\frac{4}{3(3x+4)} + c$

Calculus

DIFFERENTIATION

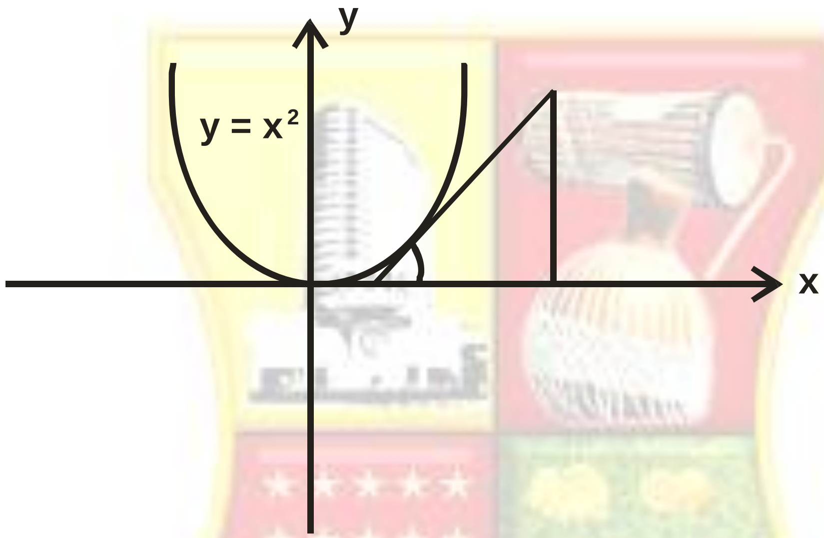
The meaning of $\frac{dy}{dx}$

The process of finding the gradient of slope or tangent of a curve is called differentiation.

The notation: If y is a function of x, the rate of change fo y with respect to x (called the gradient) written as $\frac{dy}{dx}$, is called the derivative or derived function; or slope or first derivative of the function.

First Principle

Example: Find the derivative of $y = x^2$ from the first principle.



Solution

Let $y = x^2$

$y + \Delta y = (x + \Delta x)^2$

$y + \Delta y = x^2 + 2x\Delta x + \Delta x + \Delta x^2$

$\Delta y = (x^2 + 2x\Delta x + \Delta x^2) - x^2$

$\Delta y = x^2 + 2x\Delta x + \Delta x^2 - x^2$

$\Delta y = 2x\Delta x + \Delta x^2$

Divide both sides by Δx

$\frac{\Delta y}{\Delta x} = 2x + \Delta x$

$\text{Lm} \to \left\{\frac{\Delta y}{\Delta x}\right\} \to \frac{dy}{dx}$

$\Delta x \to 0$

$\therefore \frac{dy}{dx} = 2x$

If $y = x^n$, then $\frac{dy}{dx} = nx^{n-1}$

Sum Rule

If $y = u + v + w$ where $U = f(x)$; $v = f(x)$, $w = f(x)$,

then $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx}$

Example: $y = x^3 - 27x^2 + 81x + 19$

Find $\frac{dy}{dx}$

Solution

Given y = x3 – 27x2 + 81x + 19

$\frac{dy}{dx}$ =3x$^{3-1}$ – 27x2x$^{2-1}$ + 81x$^{1-1}$ + 0

$= \ 3x^2 - 5x + 81$

Product Rule

If y = uv, then $\frac{dy}{dx} = u\ \frac{dv}{dx} + v\frac{du}{dx}$

Example: If y = x^2(x^2 -1), find $\frac{dy}{dx}$

Solution:

Let u = x^2, v = x^2 – 1

$\frac{du}{dx} = 2x\ \ and\ \frac{dv}{dx}\ = 2x$

$\therefore \frac{dv}{dx}\ = \ \frac{vdu}{dx}\ \ +\ \ \frac{udv}{dx}$

= (x^2 – 1) (2x) + (x2) (2x)

= 2x^3 – 2x + 2x^3

= 4x^3 – 2x

Quotient Rule

If y = $^u/_v$, $then\ \frac{dy}{dx}\ = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Example: If y = $\frac{x^2+1}{x-1}\ , find\ \frac{dy}{dx}$

Let u = x2 + 1, v = x – 1, y = $^u/_v$

$\frac{du}{dx} = 2x, \qquad \frac{dy}{dx} = 1$

$\frac{dv}{du} = \frac{v\ \frac{du}{dx}\ -\ u\ \frac{dv}{dx}}{v^2}$

$\frac{(x-1)(2x) - (x^2-1)(1)}{(x-1)^2}$

$\frac{2x^2 - 2x - \ x^2 - 1}{(x-1)^2}$

$\frac{x^2 - 2x - 1}{(x-1)^2}$

Chain Rule

If y = un, u = f(x)

$\frac{dy}{du} = nu^{n-1}, \frac{du}{dx} = f^1(x)$

Then, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

= $nu^{n-1} \cdot f^1(x)$

Example : If y = (x2 – 1) 4, find $\frac{dy}{dx}$

Solution:

Let u = $x^2 - 1$, y = u^4

$\frac{du}{dx} = 2x, \qquad \frac{dy}{dx} = 4u^3$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$= 4u^3 \cdot 2x$

$8x(u)^3 \ \ where \ \ u = (x^2 - 1)$

$8x\ (x^2 - 1)^3$

The reciprocal Rule

If y = $\frac{1}{g(x)}$, $where\ g(x) is\ differentiable\ at\ x, then\ \frac{dy}{dx} = \frac{-g^1(x)}{[g(x)]^2}$

Example: let y = $\frac{1}{x^7}$

Solution

Let y = - $\frac{1}{x^7}$

$\therefore \frac{dy}{dx} = \frac{-\,g^1(x)}{[g(x)]^2}$

$= \frac{(-7x^6)}{[g(x^7)]^2}$

$= \frac{7x^6}{x^{14}}$

$= \frac{7}{x^8}$

Example 2:

If y = $\frac{1}{x^3}$, $find\ \frac{dy}{dx}$

Solution,

y = $\frac{1}{x^3} = x^{-3}$

$\frac{dy}{dx} = -3x^{(-3-1)}$

$= -3x^{-4}$

$= \frac{-3}{x^4}$

Implicit Differentiation

Implicit differentiation is finding the derivative of ;a function without solving the equation for y.

Example: Find the slope fo the function: $y^2 + x^2y + x^2 = 1$ at the point (-1, 2)

Solution:

Given: $y^2 + x^2y + x^2 = 1$

Differentiate: y^2, x^3y, x^2 and 1 with respect to x

Using product rule.

$\therefore 2y\frac{dy}{dx} + \left(x^2\frac{dy}{dx} + y.2x\right) + 2x = 0$

$2y\frac{dy}{dx} + x^2\frac{dy}{dx} + 2xy + x^2 = 0$

$\frac{dy}{dx}(2y + x^2) = -x^2 - 2xy$

$\frac{dy}{dx}(2y + x^2) = -(x^2 + 2xy)$

$\therefore \frac{dy}{dx} = \frac{(x^2+2xy)}{2y+x^2}$

At the point (-1, 2), that is, x = -1, y = 2

$$\frac{dy}{dx} = -\frac{(-1)^2 + 2(-1)(2)}{2(2) + (-1)^2}$$

$$= \frac{-(1-4)}{4+1}$$

$$= \frac{-(-3)}{5}$$

$$= \frac{3}{5}$$

Differentiating Trigonometric Functions

1. If y = Sinx, then $\frac{d}{dx}(\sin x) = \cos x$
2. If y = cosx, then $\frac{d}{dx}(\cos x) = -sinx$
3. If y = tanx, then $\frac{d}{dx}(tanx) = sec^2x$
4. If y= cot x, then $\frac{d}{dx})(cotx) = -cosec^2x$
5. If y = secx , then $\frac{d}{dx}(\sec x) = \sec x \tan x$
6. If y = cosec x, then $\frac{d}{dx}(cosec\ x) = -cosec\ x \cot x$

Proof: Given y = tanx, to find $\frac{d}{dx}(tanx)$

Let y = tanx = $\frac{\sin x}{\cos x} \equiv \frac{u}{v}$

Put u = sin x, then $\frac{dv}{dx} = -\sin x$

$\therefore \frac{dy}{dx} = \frac{\frac{vdu}{dx} - \frac{udv}{dx}}{v^2}$ $(Quotient\ rule)$

$= \frac{(cosx)(cosx)-(\sin x)(-sinx)}{cos^2x}$

$= \frac{cos^2x + sin^2x}{cos^2x}$

Since: Cos2x + Sin2x= 1

$\therefore = \frac{1}{cos^2x}$

$= \sec^2 x$

Proof: y = cot x, find $\frac{dy}{dx}$

Let y=cot x = $\frac{1}{tanx} = \frac{1}{tanx} = \frac{\frac{1}{sinx}}{\cos x} = \frac{cosx}{sinx} \equiv \frac{u}{v}$

Let u = cos x, v = sin x

Then $\frac{du}{dx} = -sinx, \frac{dv}{dx} = \cos x$

$\therefore \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{(sinx)(-sinx)-(\cos x)\ (\cos x)}{sin^2x}$

$= \frac{-(sin^2x + cos^2)}{sin^2x}$

$= \frac{1}{sin^2x}$

= -cosec^{2}x

Chain Rule Trigonometric differentiation

The Rules of differentiation and chain rule application apply to trigonometric differentiation.

Example: Differentiation y=sin2x with respect ot x.

Solution:

Given : y = sin2x.

Let u = 2x

Then y = sin u and $\frac{du}{dx}$ = 2

$\therefore \frac{dy}{dx} = \cos u$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

= (cosu) (2)

= 2cosu, where u = 2x

= 2cos2x

Application of Differentiation

Rate of Change

Example; the radius r of a circular disk to increasing at the rate of 0.5cnsec-^{1}. At what rate is the area of the disc increasing when its radius is 6cm?

Solution:

Let Area(A) = πr^2

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$$

= $(2\pi r)\frac{dr}{dt}$

= $2\pi(6)\ (0.5)$

= $2\pi\ (6)$ X ½

= $6\pi cm^2 s^{-1}$

Maxima and Minima

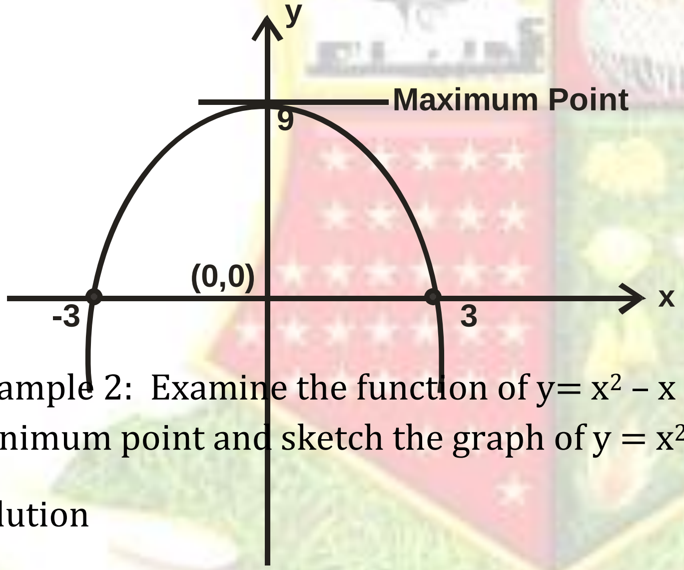
Consider the graph of y = 9 – x^2

When x = 0 , y = 9

When y = 0 , x = ± 3

$\frac{dy}{dx} = -2x, when\ \frac{dy}{dx} = 0, then\ x = 0$

At this point y= 9. The point 9 is maximum point.



Example 2: Examine the function of y= x^2 – x – 2 for maximum or minimum point and sketch the graph of y = x^2 – x – 2

Solution

Let y = x^2 – x – 2

When x = 0 , y = -2

When y = 0 , x^2 – x – 2 = 0

(x – 2) (x + 1) = 0

Either (x – 2) = 0 or (x + 1) = 0

X = 2 or x = -1

This means the graph cuts the x axis at the points x = 2, and x = -1. It passes through the y axis at the point y = -2 for turning point, we need to differentiate the function.

y = x^2 – x – 2

$$\frac{dy}{dx} = 2x - 1$$

At the turning point, $\frac{dy}{dx} = 0$,

2x – 1 = $\frac{dy}{dx} = 0$

Thus, 2x – 1 =0

2x = 1

∴ x = ½

Nature of turning points : Find $\frac{d^2y}{dx^2}$ if it is greater than Zero, it is minimum, if $\frac{d^2y}{dx^2} < 0$ *it is a maximum*

$$\frac{dy}{dx} = 2x - 1 \quad \frac{d^2y}{dx^2} = 2 > 0\ (minimum)$$

When x = ½

Then y = x^2 – x – 2

= (½)2 – (½) -2

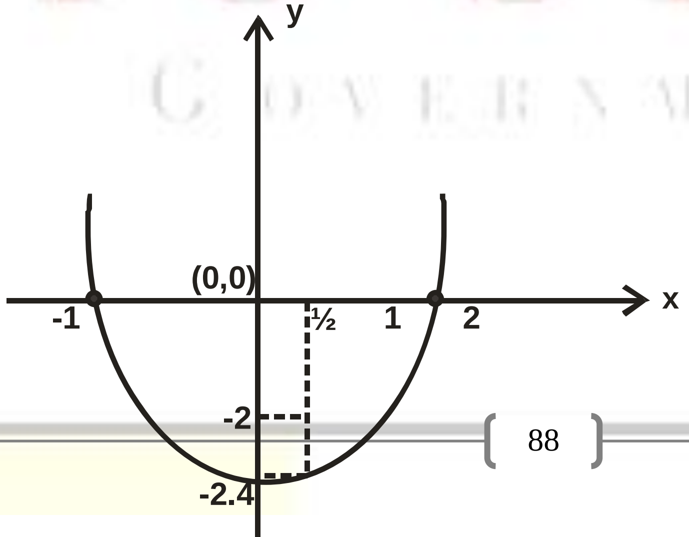
= ¼ - ½ - 2

$= \frac{1-2-8}{4}$

$= \frac{1-10}{4}$

$= \frac{-9}{4}$

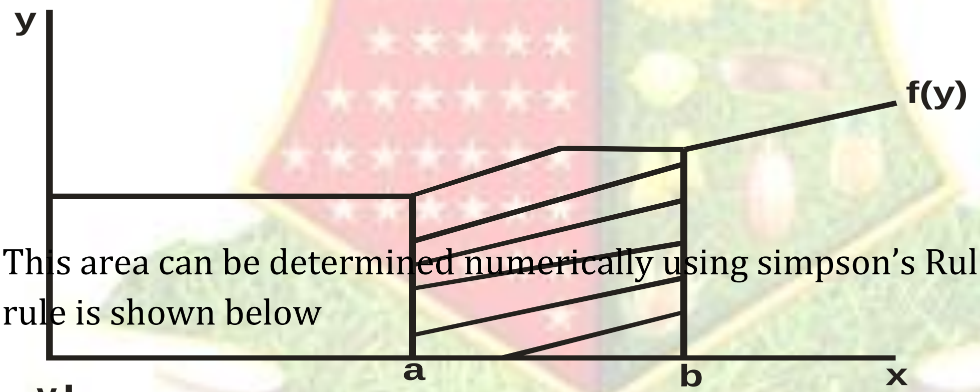
$= -2\frac{1}{4}$



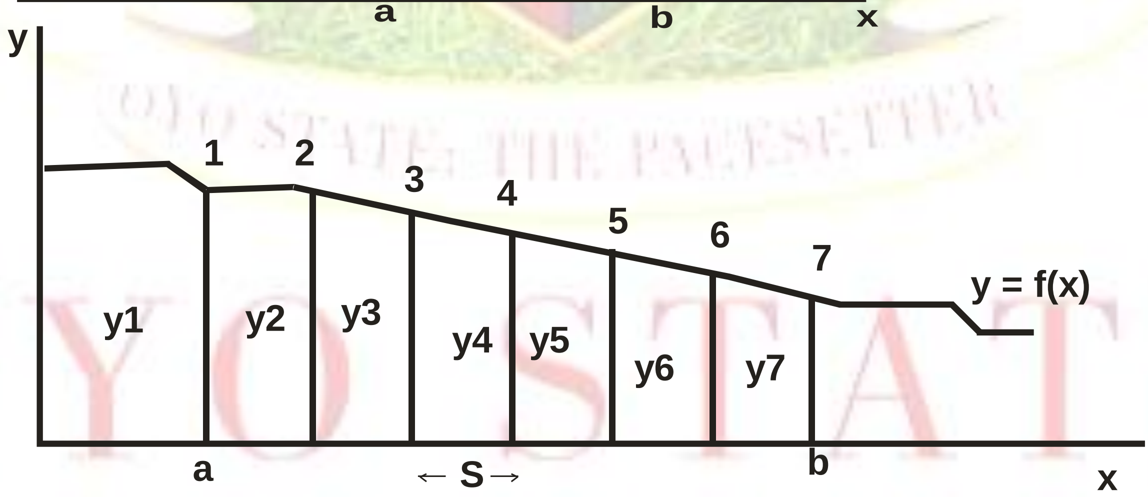
Simpson's Rule and Application of Integral Calculus

There are some integrals that cannot be easily determined by the standard methods, you may have studied so far. A method for determining such integrals is the Simpson's rule.

From definite integrals, the area under the curve ky=f(x) between the points x = a and x = b is given by A = $\int_a^b f(x)dx$



This area can be determined numerically using simpson's Rule. Simpson's rule is shown below



(i) Divide the area into in any even number (n) of equal width strips (width s). Here n= 6

(ii) Number and measure each ordinate y1,y2, y3 --- yn Here are 7 ordinates. The number of ordinates is usually one more than the number of strips

The area A under the curve is given by:

A = $^{S}/_{3}\,[F + L + 4E + 2R]$

Where S = width of each stripe $\frac{b-a}{n}$

F + L = Sum of first and last ordinate

4E = 4 X Sum of the even numbered ordinates.

2R = 2 X Sum of the remaining odd numbered ordinates.

Example: Evaluate $\int_{1.2}^{2} \sqrt{2 + x^3}\ dn\ using\ 8\ interval$

Solution:

Find S = $\frac{b-a}{n}$

$= \frac{2-1.2}{8} = 0.1$

Calculate the values of $\sqrt{2 + x^3}$ at interval of 0.1 from 1.2 to 2.0. this is shown in the table below:

	x	2 + x3	$\sqrt{2 + x^3}$	F + L	E	R
1	1.2	3.728	1.9308	1.9308		
2	1.3	3.197	1.7880		1.7880	
3	1.4	4.744	2.1780			2.1780
4	1.5	5.357	2.3184		2.3184	
5	1.6	6.096	2.4690			2.4690
6	1.7	6.913	2.6293		2.6293	
7	1.8	7.832	2.7986			2.7986
8	1.9	8.859	2.9764		2.9764	
9	2.0	10	2.9764	3.1623		
				5.0931	9.7121	7.4456

From the table,

F + L = 5.0931

4E = 4(9.7121(= 38.8484

2R = 2(7.4456) = 14.8912

Simpson's rule

Area = $^{S}/_{3}\,[(F + L) + 4E + 2R]$

Substituting into the formula

$$\int_0^2 \sqrt{2 + x^2 dx} \simeq \frac{0.1}{3}(5.093 + 38.8484 + 14.8912)$$

$= 1.9611$

Integration

Indefinite Integration is the reverse process of differentiation. The symbol for integration is $\int$. When we integrate a function, we accompany it with a constant of integration.

When the constant of integration is not yet found, we talk about indefinite integration. If $y = kx^n$, then $\int ydx = \int kx^n dx = \frac{kx^{n+1}}{n+1} + C$

Example:

Integrate the following:

(a) $\int 2\sqrt{xdx}$ (b) $\int \left(\frac{x + x^5}{x^3}\right) dx$

Solution

(a) $\int 2\sqrt{xdx} = 2\int \sqrt{xdx}$

$= 2\int x^{½} dx$

$= 2\left(\frac{x^{1/2+1}}{1/2 + 1}\right) + C$

$= \frac{2x^{3/2}}{3/2} + C$

$= 2(2/3\,x)^{3/2} + C$

$= 4/3\, x^{3/2} + C$

(b) $\int \left(\frac{x + x^5}{x^3}\right) dx$

$$\int\left(\frac{x}{x^2} + \frac{x^5}{x^3}\right) dx$$

$$= \int\left(\frac{1}{x^2} + \ x^2\right) dx$$

$$= \frac{-1}{x} + \frac{x^3}{3} + c$$

Example 2:

Evaluate: (a) ∫(5x3 + 3x2 – 6x + 1) dx

(b) $\int[(0.1)x^3 - \ (0.2x^2) + 0.3x]dx$

Solution

∫(5x3 + 3x2 – 6x + 1) dx

$5\int x^2dx + 3\int x^2dx - 6\int xdx + \int 1dx$

$$\frac{5x^4}{4} + \frac{3x^3}{3} - \frac{6x^3}{2} + x + C$$

$$5/_4\, x^4 + \ x^3 - 3x^2 + x + C$$

(b) $\int[(0.1)x^3 - \ (0.2x^2) + 0.3x]dx$

$$\int({}^1/_{10}\, x^2 - {}^2/_{10}\, x^2 - {}^3/_{10}\, x)\ dx$$

$$^1/_{10} \int\ (x^3 - 2x^2 - 3x)dx$$

$$^1/_{10}\left({x^4}/_4 - {2x^2}/_3 - {}^3/_2\, x^3\right) + C$$

Example 2

Evaluate th following integrals

$\int \sin^5 x dx$

Solution:

$\int \sin^5 x dx = \int \sin^4 x\ .\ \int \sin x dx$

From $\sin^2 x + \cos^2 x = 1$

$\text{Sin}^2 x = 1 - \cos^2 x$

$\int \sin^5 x dx$

$= \int \sin^4 x \;.\; \sin x dx$

$= \int (1-\cos^2 x)^2 \sin x dx$

$\therefore \int (1 - 2\cos^2 x + \cos^4 x) \sin x dx$

$= \int (\sin x - 2\cos^2 x \sin x + \cos^4 x \sin x) dx$

$= \int \sin x dx - 2\int \cos^2 x \sin x dx + \int \cos^4 x \sin d x$

$= -\cos x - {}^2/_3 \cos^2 x - {}^1/_5 \cos^5 x + C$

Putting u = cos x, then dn = -sinxdx.

Definite Integrals

The unknown constants of indefinite integrals are removed by the process of definite integrals. The function is evaluated between two limits called the upper and the lower limit e.g. $\int_a^b F(x)dx,$

Integrate F(x)dx and find the area between the upper limit and lower limit.

Example: Integrate.

$$\int_1^2 9x^2\, dx$$

Solution

$= 9\int_1^2 x^2\, dx$

$= \frac{9x^3}{3} + C \;\; \int_1^2$

$= 3x^3 + C\, I_1^2$

$= [3(2)^2 + C] - [3(1)^3 + C]$

$= (24 + C) - (3 + C)$

$= 24 + C - 3 - C$

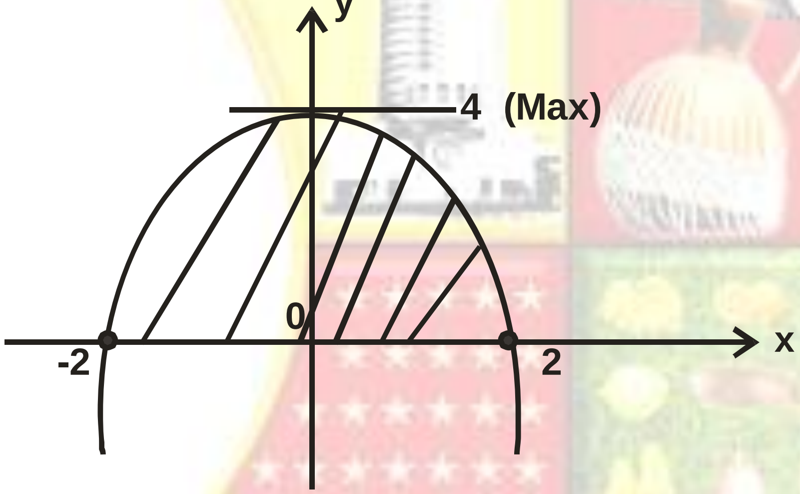
$= 21$

Area using Definite integration

The area under a curve can be evaluated using definite integration this is given by A = $\int_a^b y dx$

Where A is the area, a, b, are the lower and upper limits.

Example: Find the area between the curve y = 4 – 4x^2, the x axis and points x = 2 and x = -2



Given: y = 4 – x2

(i) when y = 0, then x = $\pm$ 2

(ii) When x = 0, then y = 4

(iii) For turning points, ${}^{dy}/_{dx} = 0$

$\rightarrow \frac{dy}{dx} = -2x = 0$

$\therefore$ x = 0

$\frac{d^2y}{dx^2} = -2 < 0$ (*showing maxi point*)

When x = 0, y = 4

Area = $\int_a^b y dx$

$= \int_{-2}^{2} (4 - x^2) dx$

$= [4x - {}^1/_3 x^3 + C] I_{-2}^{2}$

$= {}^{32}/_{3}$