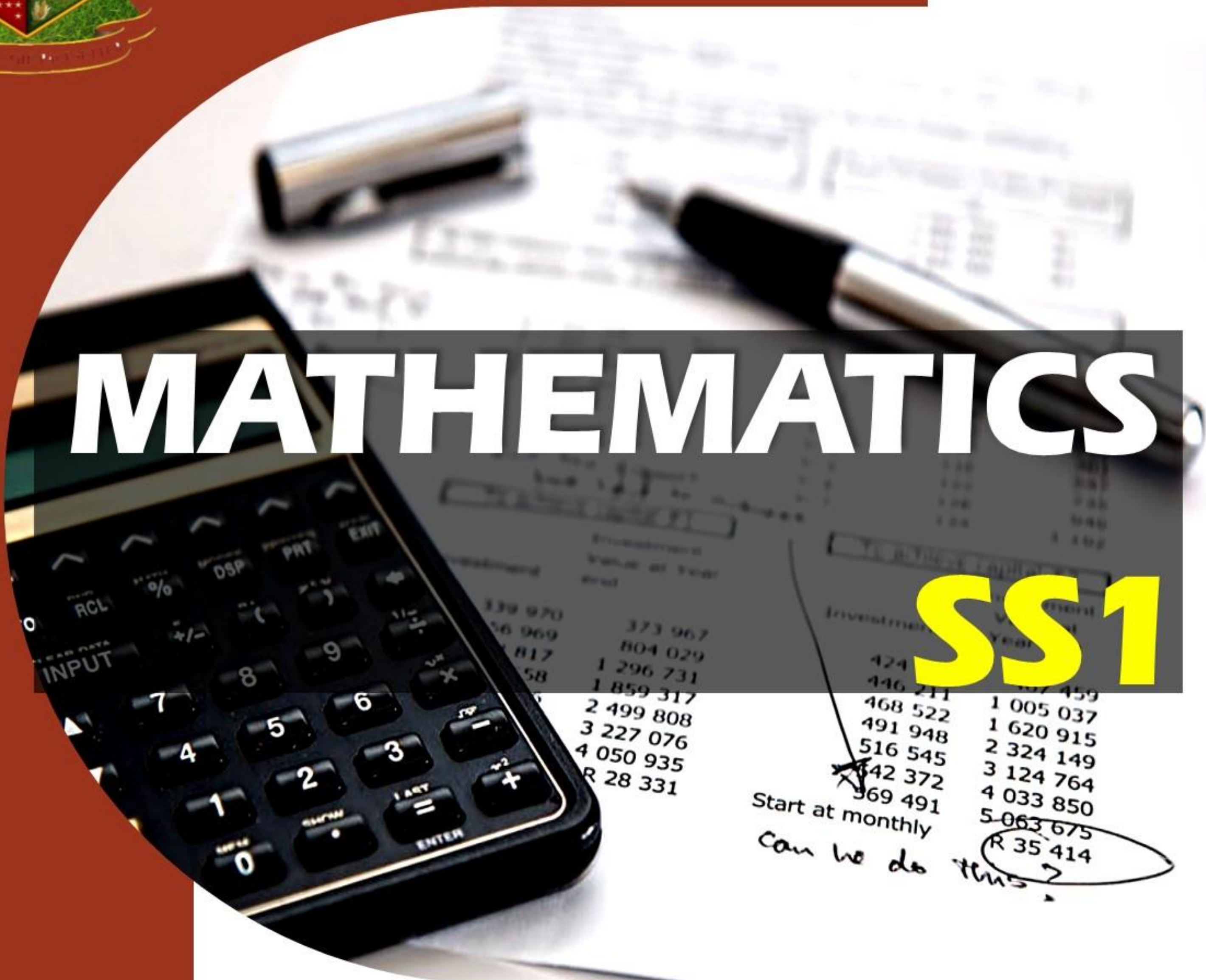
OYO STATE LECTURE NOTES



MATHEMATICS

SS1

COMPILED BY: Mr. J.A. Farayibi

REVIEWED BY: Mr. Y.A. Fasasi (Dean, School of Science, Oke Ogun Polytechnic, Saki)

EDITED BY: Mr. O.I. Olawale

DESIGNED BY: BROWT -ESSCON

FIRST TERM

WEEK 1: NUMBER BASE SYSTEM

Conversion can be done from other bases to base 10

For instance

i. Denary or decimal number which is base 10 and the highest digit is 9 i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
ii. Binary system: base 2 which are 0 and 1.
iii. Octal System: base 8 which has digits as 0, 1, 2, 3, 4, 5, 6 and 7
iv. Base 12 (duo decimal) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A and B
v. Base 16 (Hexadecimal System) the digit used are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A(10), B(11), C(12), D(13), E(14), and F(15)

A. Conversion from other bases to base ten

Example 1

a. Convert 5603_{eight} to base 10
b. Convert $3B9_{twelve}$ to base 10

Solution:

a. Convert 5603_{eight} to base 10

$= 5 \times 8^3 + 6 \times 8^2 + 0 \times 8^1 + 3 \times 8^0$

$= 5 \times 512 + 6 \times 64 + 0 + 3 \times 1$

$= 2560 + 384 + 0 + 3$

Therefore, $5603_{eight} = 2947_{ten}$

b. Convert $3B9_{twelve}$ to base 10

$= 3 \times 12^2 + B \times 12^1 + 9 \times 12^0$

$= 3 \times 144 + 11 \times 12 + 9 \times 1$

$= 432 + 132 + 9$

Therefore, $3B9_{twelve} = 573_{ten}$

Example 2

Convert 1011.101_{two} to base 10

Solution:

1011.101

$= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$

$= 8 + 0 + 2 + 1 + ½ + 0 + 0.125$

Therefore, $1011.101_{two} = 11.625_{ten}$

B. Converting Numbers in Base ten to other bases

Example

Express 208_{ten} in
i. Binary
ii. Hexadecimal

Solution:

i. 208_{ten} in binary

2	**208**	**Remainder**
2	104	0 ↑
2	52	0
2	26	0
2	13	0
2	6	1
2	3	0
2	1	1
2	0	1

Therefore, $208_{ten} = 11010000_{two}$

ii. 208_{ten} in hexadecimal

16	**208**	**Remainder**
16	13	0 ↑
	0	D

Therefore, $208_{ten} = D0_{sixteen}$

WEEK 2: NUMBER BASE

Addition and subtraction in other bases

Addition and subtraction in other bases are carried out exactly the same way as base ten numbers. For example to add or subtract in base two:

Hints

1+0 = 1	1 – 0 = 1
0+1 = 1	1 – 1 = 0
1 + 1 = 1 0	10 – 1 = 1

Examples: Evaluate the following

a. $56_{eight} + 243_{eight}$
b. $1001_{two} - 111_{two}$

Solution:

a. 56 + 243 321_{eight}	Procedure: 1st Column 6 + 3 = 9(i.e. 1 eight and 1) write 1 carry 1 forward. 2nd Column: 5 + 4 + 1= 10(i.e. 1 eight and 2) , write 2 carry 1 forward 3rd Column: 2 + 1=3 (carry forward = 3)
b. 1001 - 111 010	1st Column: 1 – 1 = 0 2nd Column: 0 – 1 is not possible, we borrow 1 from next column to give 2 i.e. 10. Then 10 – 1 = 1. 3rd Column: 1-1 = 0.

Multiplication and Division in other bases

Multiplication and Division in other bases are carried out exactly the same way as base ten numbers

Hints:

a. To multiply in base 2
   i. 0 x 0 = 0
   ii. 0 x 1 = 0
   iii. 1 x 1 = 1
b. To multiply in base 8
   i. 2 x 4 = 8 but 8 = 1eight and 0
      Thus 2 x 4 = 10
   ii. 3 x 4 = 12, But 8 = 1eight and 4
   Thus 3 x 4 = 14

Example

Calculate the following:

a. $321_{five} \times 21_{five}$
b. $110.11_{two} \times 11.1_{two}$

Solution

a. $321_{five} \times 21_{five}$

```
      321
     x 21
      321
1142
12241five
```

Hence $321_{five} \times 21_{five} = 12241_{five}$

b. $110.11_{two} \times 11.1_{two}$

First, ignore the binary point and multiply 1011_{two} by 111_{two} as follows:

```
      11011
     x  111
      11011
+11011
 11011
10111101two
```

Now insert the binary point (b.p) by counting the number of digits before binary point in the question 10111.101

Thus: $110.11_{two} \times 11.1_{two} = 10111.101_{two}$

APPLICATION TO COMPUTER PROGRAMMING

Computers usually store information as a series of bits. A bit is the smallest unit information stored in a computer. A group of 8bits is called a byte, each byte corresponding to one character. Binary system contains two numbers 0 for OFF and 1 for ON. Therefore, a bit is either a 0 or a 1. The amount of data that can be stored on a disk is measured in kilobytes (i.e.One thousand bytes is 1024 kilobytes (KB) or 1048576 bytes .2^{20} is 1 megabyte (MB).

WEEK 3: MODULAR ARITHMETIC

Concept of modular arithmetic:

Modular arithmetic (sometimes called clock arithmetic) is a system of arithmetic in which numbers wrap around after they reach a certain value. It can also be regarded to as arithmetic that involves integers.The plural of modulus is moduli.

Example 1:

Simplify the following

a. 76 (Mod 5)

b. 340 (mod 7)

Solution

a. $76 \div 5 = 15$ Remainder 1

OR $76 = 15 \times 5 + 1$

Therefore, $76 = 1 \pmod 5$

b. 340 (mod 7)

$340 \div 7 = 48$ Remainder 4

OR $340 = 4 \pmod 7$

In general, two integers **a and b** are said to be contingent modulo **m,** written as **$a \equiv b$** (mod m).

Operation in Modular Arithmetic

Addition and Subtraction in modular arithmetic :

Addition in modulo 5

+	**0**	**1**	**2**	**3**	**4**
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2

4	4	0	1	2	3

Also, Subtraction in modulo 5

-	**0**	**1**	**2**	**3**	**4**
0	0	4	3	2	1
1	1	0	4	3	2
2	2	1	0	4	3
3	3	2	1	0	4
4	4	3	2	1	0

Example 2:

a. Evaluate 36+20 (mod 6)

Solution:

36 + 20 = 56

56 = 9 x 6 + 2 = **2 (mod 6)**

b. Work out the simplest positive form of –3(mod 8)

Solution:

–3 = -1 x 8 + 5 = **5 (mod 8)**

c. Evaluate the following:

i. 12 – 3 (mod 5)

ii. 11 – 24 (mod 7)

Solution:

i. $12 - 3 \pmod 5$

$12 - 3 = 9$

$9 = 5 + 4 =$ **4 (mod 5)**

ii. $11 - 24 \pmod 7$

$11 - 24 = -13$

$= -2 \times 7 + 1 =$ **1 (mod 7)**

Multiplication

Multiplication can be interpreted as repeated addition. The table below shows multiplication in modulo 5

X	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Example 3:

Calculate the following in (mod 5)

a. 3×2 b. 28×12

Solution:

a. $3 \times 2 = 6$

$6 = 1 \times 5 + 1$

= 1 (mod 5)

b. 28 x 12= 336

336 = 67 x 5 + 1

= 1 (mod 5)

Division

Since division is the inverse of multiplication, to solve division problem, for example :

if 2 ÷ 4 =x (mod 5), then

2/4 = x

4x = 2, therefore x = 2

Multiplication inverse

The multiplication inverse of a number x is a number y such that xy = 1 e.g. multiplication inverse of 7 is 1/7 because 7 x 1/7 = 1. We can apply the above to find the inverse of residue in modular arithmetic as shown in the following example.

Example 1:

Find the inverse of:

a. 5(mod 7) b. 3 (mod 4)

Solution

a. 5(mod 7)

The inverse of 5 in (mod 7) is the number that when multiplied by 5 gives 1 thus 5 x 3 = 15 = **1 (mod 7)**

b. 3 (mod 4)

The inverse of 3 in (mod 4) is the number that when multiplied by 3 gives 1

Thus 3 x 3 = 9 therefore, = **1 (mod 4)**

This means the inverse of 3 in modulo 4 is 3

Example 2:

Simplify the following inmod 5

a. 3 ÷ 4 b. 3 ÷ 3

Solution:

a. 3 ÷ 4 (mod 5)

Let 3 ÷ 4 = x

Then 4x = 3 ………*

Multiply both sides by a number that will make the coefficient of x unityi.e 1

Try 4 x 1 =4 = 4 (mod 5)

4 x 2 = 8 = 3 (mod 5)

4 x 3 = 12 =2 (mod 5)

4 x 4 = 16 = 1 (mod 5)

Multiply both sides of * by 4 i.e. the multiplicative inverse of 4 in modulo 5

(4 x 4)x = 3 x 4

1x = 12 = 2 x 5 + 2

x = 2 (mod 5)

b. 3 ÷ 3

Let 3 ÷ 3 =x

then 3x = 3 (mod 5)

To make the coefficient of x unity, multiply both sides by 2

(3 x 2)x = 3 x 2

1x = 6, therefore, **x = 1 (mod 5)**

Solving Simple Equations in Modular Arithmetic.

Example :

Given that

x + 5 = 2 (mod 7), find x

Solution:

x + 5 = 2 (mod 7)

Look for a number in arithmetic (mod 7) that can be added to 5 tomake it 0 . Thus, add the additive inverse of 5 to both sides which is 2, to both sides. We have,

x + 5 + 2 = 2 + 2 (mod 7)

x + 0 = 4 (mod 7)

x = 4 (mod 7)

Check

4 + 5 = 9 = 2 (mod 7)

Application of Modular Arithmetic

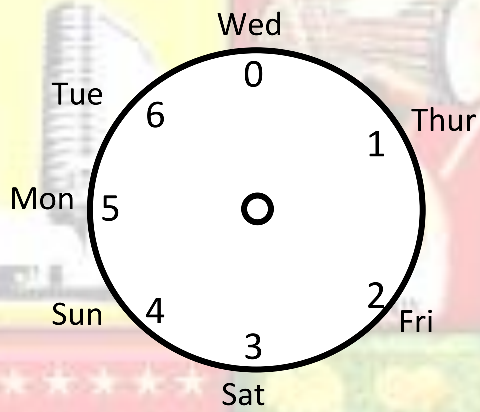
Example:

In one year, New Year Day is on Wednesday. On what day of the week would the New Year day be in the following year if it is:

(a) Common Year? (b) A leap year?

Solution:

Since there are 7 days in a week, so we work in arithmetic (mod 7). We take Wednesday as 0 as shown in the number cycle below:



a. Common year = 365 days = 52 weeks +1 day = 52 x 7 + 1 =1 (mod 7)

   i.e. 1 day after Wednesday is Thursday

   Therefore, the day of the week is Thursday.

b. Leap year, 366 days = 52 weeks + 2 days = 52 x 7 + 2 = 2 (mod 7)

   i.e. 2 days after Wednesday is Friday

   Therefore,the day of the week is Friday.

WEEK 4: STANDARD FORM AND INDICES

Objectives: By the end of the lesson students should be able to:

i. Write numbers in standard form

ii. Relate indices to standard form

iii. State the laws of indices and deduce their meaning

iv. Solve problems using the laws of indices

v. Solve problems on simple indices equation

STANDARD FORM

The standard form is generally expressed in term of $\mathbf{a \times 10^n}$.

Where a is a number between 1 and 10 and n is a positive or negative integer.

Example 1:

a. Express 7090000 in standard form

b. Express 0.00007 in standard form

Solution:

a. $7090000 = 7.09 \times 1000000 = 7.09 \times 10^6$

b. $0.00007 = 7.0 \times (\frac{1}{100000}) = 7 \times 10^{-5}$

Example 2:

Change these standard forms to ordinary numbers

a. 6.3×10^5

b. 4.08×10^{-5}

Solution:

a. 6.3×10^5

$= \left(\frac{63}{10}\right) \times 100000$

= 630000

b. 4.08×10^{-5}

$= \frac{408}{100} \times \frac{1}{100000}$

$= \frac{408}{10000000})$

= 0.0000408

INDICES

Laws of Indices

An index tells us how many times the base number has been multiplied by itself.

e.g. $3^5 = 3 \times 3 \times 3 \times 3 \times 3$

In general , we have,

a^m = axaxax…. (Where a = base and m = power).

Law 1:

$a^m \times a^n = a^{m+n}$ (addition law)

i.e. multiplication involving same base results in addition of powers of the common base.

Example

i. $a^5 \times a^7$

$= a^{5+7}$

$= a^{12}$

ii. $8^5 \times 8^7$

$= 8^{5+7}$

$= 8^{12}$

Law 2 :$a^m \div a^n = a^m / a^n = a^{m-}$(Division law)

i.e division involving same base results in subtraction of powers of the common base.

When dividing in indices we are subtracting

Example:

i. $a^8 \div a^5$

$= a^{8-5}$

$=a^3$

ii. $10^9 \div 10^7$

$= 10^{9-7}$

$= 10^2$

law 3 :

$a^0 = 1$(Zero index)

Example:

i. $a^8 - a^8$

$=a^{8-8}$

$=a^0$

$= 1$ (since $a^0 =1$)

law 4:

$a^{-m} = 1 \div a^m = 1/a^m$ (Negative index)

Example

a. $a^{-5} = 1/a^5$

b. $c^{-8} = 1/c^8$

General Examples :

Simplify the following :

a. $6a^{-4}$

b. $3x^3 y^{-4} z^{-2}$

c. $27^{x-1} \times 27^{1-x}$

d. $48e^{-5} \div 12e^{-10}$

Solution:

a. 6a-4

$= 6 \times 1/a^4$

$= 6/a^4$

b. $3x^3 y^{-4} z^{-2}$

$= 3x^3 \times 1/y^4 \times 1/z^2$

$= 3x^3/y^4z^2$

c. $27^{x-1} \times 27^{1-x}$

$= 3^{3(x-1)} x 3^{3(1-x)}$

$= 3^{3x-3+3-3x}$

$= 3^0 \quad = 1$

d. $48e^{-5} \div 12e^{-10}$

$= 48e^{-5}/12e^{-10}$

$= \frac{48}{12} \times (e^{-5}/ e^{-10})$

$= 4e^{-5-(-10)}$

$= 4e^{-5+10}$

$= 4e^5$

Law5 :Raising an index to another power

In general ,$(a^m)^n = a^{mn}$

Example :

Simplify the following :

a. $[(3g)^2]^{-2}$ b. $(-2x^6)^4$

Solution :

a. $[(3g)^2]^{-2}$

$= [(3^4g)^4]^{-!}$

$= 1/(3^4g^4)$

$= 1/81g^4$

b. $(-2x^6)^4$

$= (-2^{4} \text{ x } ^{24})$

$= 16 \text{ x } ^{24}$

OR

$(-2x^6)^4$

$= -2x^6 \text{ x } -2x^6 \text{ x } -2x^6 \text{ x } -2x^6$

$= 16x^{6+6+6+6}$

$= 16x^{24}$

WEEK 5: FRACTIONAL INDICES SQUARE ROOTS

Law 6 (Root Law)

$a^{½} = \sqrt{a}$

i.e. $25^{1/2} = \sqrt{25}$

Similarly in algebra $a^{½} \times a^{½} = a^{½+½} = \sqrt{a} \times \sqrt{a} = a$

Cube roots and higher roots using the same method as above

$a^{1/3} \times a^{1/3} \times a^{1/3}$

$= a^{1/3+1/3+1/3}$

$= a^1 \quad = a$

But

$\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a^1$

Therefore, $\sqrt[3]{a} = a^{1/3}$

Also $\sqrt[4]{a} = a^{1/4}$

Note also that:,

$\sqrt[m]{a} = a^{1/m}$

When the numerator of the fractional index is not 1 for instance

a. $a^{2/3} = a^{2 \times 1/3}$

$=(a^2)^{1/3}$

$=(a^{1/3})^2$

$=(\sqrt[3]{a})^2$ or $\sqrt[3]{a^2}$

In general,

$a^{m/n} = (\sqrt[n]{a})^m$

Example:

Evaluate: a. $16^{1/2}$ b. $\sqrt{25b^4}$

Solution:

a. $16^{1/2} = (\sqrt{16})^1 = (4)^1 = 4$

b. $\sqrt{25b^4} = \sqrt{25} \times \sqrt{b^4} = (5^2)^{1/2} \times b^{1/2\times4} = 5b^2$

Simple exponential equation rules

Examples :

1. Solve these equations

   a. $x^{1/3} = 5$ b. $3x^{-1/2} = -27$ c. $8^x = 0.125$

Solution:

a. $x^{1/3} = 5$

$x^{1/3 \times 3} = 5^3$ (Take the cube of both sides),

$x^1 = 5^3$

$x = 125$

b. $3x^{-1/2} = -27$

$x^{-1/2} = -9$ (dividing both sides by 3)

$(x^{-1/2})^2 = (-9)^2$ (Squaring both sides)

$x^{-1} = 81$

since $a^{-m} = 1/a^m$,then

$\frac{1}{x} = 81$

$X = \frac{1}{81}$

c. $8^x = 0.125$

$8^x = \frac{125}{1000}$

$8^x = \frac{1}{8}$

$8^x = 8^{-1}$

$x = -1$

Example 2

If $9^{2x+1} = 81^{x-2}/3^{x}$, find x.

Solution

$9^{2x+1} = 81^{x-2}/3^{x}$

$3^{2(2x+1)} = 3^{4(x-2)-x}$

$3^{4x+2} = 3^{4x-8-x}$

$3^{4x+2} = 3^{3x-8}$

Equate the base

$4x + 2 = 3x - 8$

$4x - 3x = -8 - 2$

$x = -10$

WEEK 5 LOGARITHM

Behavioral objectives: By the end of the lesson students should be able to:

i. Show logarithms as the inverse of indices
ii. Define logarithm
iii. Relate indices to standard form
iv. Find the logarithm and antilogarithms of numbers greater than one from four figure tables
v. Use logarithm and antilogarithms tables in calculation
vi. Apply logarithm to solve real life problems

Relationship between indices and logarithms

Recall in index notation

8 = 23

Similarly if $81 = 3^4$ then

$Log_3 81 = 4$

In general

If $y = a^x$, then $x = log_a y$

Remarks :,

i. If $1 = 10^0$ then $Log_{10}1 = 0$
ii. If $10 = 10^1$ then $log_{10}10 = 1$

Logarithm written to base 10 is called common logarithm and can be written as $log_{10}10 = log10$ and,$log_{10}100 = log100$.

Definition :

Logarithm simply means the inverse operation of indices.

Using tables to find the logarithms of a number :

If a number is expressed as a power of 10, then the index is called the logarithm of the number e.g. $69 = 10^{1.8388}$

Example

Use logarithm table to find the log of the following

a) 8.4 b) 840

Solution

a) 8.4

Express 8.4 in form of $ax10^n$i.e 8.4×10^0 in standard form.

From thelogarithm table ,check 84 under 0. It corresponds to 0.9243

$8.4 = 10^{0.9243}$

:. Log 8.4 = 0.9243

b) 840

Express in form of a x 10^n

840 = 8.4 x 10^2 in standard form

From logarithm table, 840 = $10^{2+0.9243}$

:. log 840 = 2.9243

Note :

The logarithm of a number contain two parts i.e. the characteristics and the mantissa e.g.

Log 840 = 2.9243(2 is the characteristics and .9243 is the mantissa).

Reading of logarithm and antilogarithm (antilog) of a number means to find the number whose logarithm is given. Therefore antilogarithm is the reverse of logarithm.

Illustration:

$\log_{10}1 = 0$, then antilog of 0 = $10^0 = 1$

Also

$\log_{10}10 = 1$, then antilog of 1 = $10^1 = 10$ etc

Examples :

Use the antilogarithm tables to find the following :

a) The antilog of 0.6894
b) The number whose logarithm is 4.56

Solution :

a. Antilog 0.6894

The mantissa .6894 in the antilog table = 4887 + 4 = 4891. The characteristic is 0. This means there must be 1 digit before the decimal point.

:. antilog of 0.6894 = 4.891

b. Antilog 4.56

The mantissa .56 in the antilog table = 3631. The characteristic is 4, then 4+1 = 5. This means there must be 5 digits before the decimal point.

:. antilog of 4.56 = 36310.

WEEK 7:

Use of logarithm table in solving problems.

Multiplication :

To find the product of number ;

i. Find the logarithm of each number
ii. Add the logarithm together then
iii. Find the antilogarithm of the result

Example:

a). Use logarithm tables to evaluate 9.456 x 867.4

Solution :

Number	Log
9.456	0.9757
867.4	+2.9382
	3.9139

8202 ← Antilog of 3.9139

Division :

b). Use logarithm table to evaluate $\frac{786.8}{12.54}$

Solution :

Number	Log
786.8	2.8959
12.54	-1.0983
	1.7876

Antilog of 1.7976 = 62.75

Power :

Examples :

Evaluate the following using logarithm tables :

(a) $(89.67)^4$ b) $(7.0698)^3$

Solution:

a) $(89.67)^4$

Number	Log
89.67	1.9526
89.67^4	1.9526
	x 4
64620000	7.8104

Roots:

Examples :

Evaluate the following using logarithm tables :

a) $\sqrt{79.84}$ b) $\sqrt[4]{894.5}$

Solution

a) $\sqrt{79.84}$

Number	Log
79.48	1.9002
$79.48^{1/4}$	1.9002 ÷ 2
	0.9501

Antilog of 0.9501 = 8.915

b) $\sqrt[4]{894.5}$

Number	Log
894.5	2.9516
$894.5^{1/4}$	2.9516 ÷ 4
	0.7379

Antilog of 0.7379 = 5.469

Example 3:

Use algorithm table to evaluate

$$\frac{(8.3552)^3 X \sqrt[5]{893.4}}{\sqrt[4]{(7.245 X 25.34)^2}}$$

Solution

$$\frac{(8.3552)^3 X \sqrt[5]{893.4}}{\sqrt[4]{(7.245 X 25.34)^2}} = \frac{(8.3552)^3 X (893.4)^{\frac{1}{5}}}{(7.245 X 24.35)^{\frac{1}{4} X 2}}$$

$$= \frac{(8.3552)^3 X (893.4)^{\frac{1}{5}}}{(7.245 X 24.35)^{\frac{1}{2}}}$$

Number	Log	
8.352^3	0.9218 x 3 = 2.7654	
$(893.4)^{1/5}$	2.9511 ÷ 5 = 0.5902	
Numerator	3.3556	3.3556
7.245	0.8600	
X 25.34	+1.4038	-
	2.2638	
$(7.245 \text{ x } 25.34)^{1/2}$	2.2638 ÷ 2 = 1.1319	1.1319
		2.2237

Antilog of 2.2237 = 167.4

Exercise

Use logarithm and antilogarithm tables to evaluate the following:

1. $4.927^2 \times 45.1$
2. $\dfrac{(18.6)^3 X 8.09}{\sqrt[5]{85000}}$

WEEK 8: SETS

Behavioral Objective : By the end of this lesson, students should be able to define and identify various types of sets.

Definition 1 :

A set can be defined as a group or a collection of well defined objects or numbers. E.g. A = {Set of fruits} i.e. A = {Mango, Guava etc} .

Set is usually denoted by a capital letters i.e. A, B, C, D etc.

Definition2: Elements of a set

The elements of a set are the objects or numbers that belong to the set. Elements of a set are also called members of a set.

Example:

If M = {1, 3, 5, 7}, then n(M) = 4

and 3 is a member of Mis written as 3 Є M

A set can be described by:

i. Listing all its elements or members within the brackets { } e.g.
   F = { mango, grape, orange, guava}
ii. In words
   e.g. M = {Multiple of 3 from 3 to 18}
   C = {Consonants}
iii. Algebraically
   E.g. B = {x: -10 < x ≤ 3, x is an integer} this means B is the set of numbers x such that x is a whole number greater than -10 but less than or equal to 3. A set specified in this way is known as a set builder notation. Hence, B = {-9,-8,…..3}.

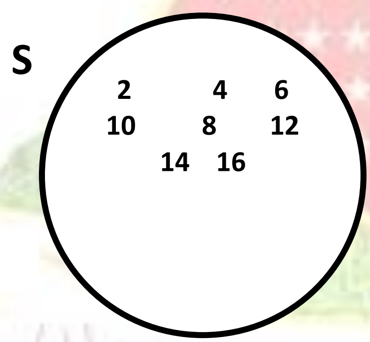
Types of Sets

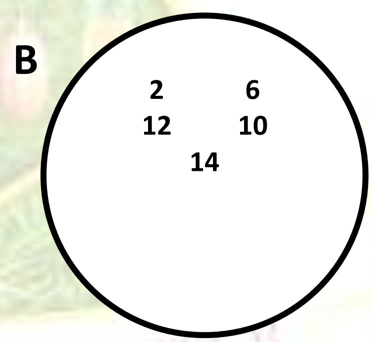
1. Finite Set:
   A finite set is a set in which all its members can be listed.
   E.g. If A = {factors of 6}, then A = {1,2,3,6}
2. Infinite Set:
   An infinite set is a set in which all its members cannot be listed

E.g. If B = {Odd numbers greater than 5}, then B = {7, 9, 11, 13, 15…}

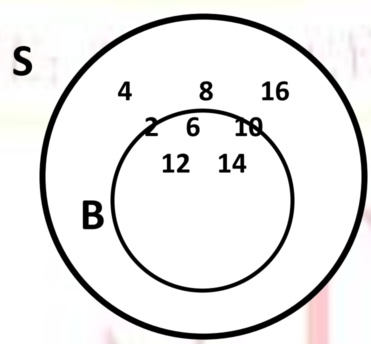
3. Empty Set (Or Null Set)
   A Set without any members or elements, is called a null set or an empty set.
   It is usually represented by { } or ϕ
   E.g. M = {Months with two letters} :. M = ϕor M={ }

4. Subsets:
   Set A is a subset of set B if and only if every member of A is a member of B.
   It is denoted by C. e.g. B c S reads as B is a subset of S
   S ɔB reads asS is a superscript of B
   Consider the set below
   S = {2, 4, 6, 8, 12, 14, 16}
   B = {2, 6, 10, 12, 14}
   B C S
   S Ɔ B
   i.e.





B C S



5. Power Set and the Number of Subsets
   The power set of a set is a list of all its possible subset. It is denoted asP(A)
   If A = {1, 2, 3, ...} the power set of A is 2^4
   Thus, if A = {1, 3}, the power of set A = 2^2 = 4 and the list of all possible subsets are ;

{1}, {3}, {1,3}, {}

Example

A set contains 7 numbers; find the number of subsets that can be obtained from it

Solutions:

Number of possible subsets $N = 2^n$

Where n = 7, then

$N = 2^7 = 128$

The number of possible subsets of the sets is 128.

6. Universal set

   It is denoted by ƹ or U

   The Universal set for any given problem is a set that contains all the members, which can be used for that problem

   E.g. if ƹ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

   A = {2, 3, 5, 6, 7}

   A is a subset of ƹ i.e. A⊂ U

7. Set Builder Notation

   A set specified by using an algebraic expression such as A = {x : x ≤ 8}.

   Also

   ifA = {x : $x^2 - 4x = 5$}, then it means , A is the set of x such that $x^2 - 4x = 5$.

   We can solve this expression by factorization:

   $x^2 - 4x = 5$

   $x^2 - 4x - 5 = 0$

   $(x + 1)(x - 5) = 0$

   x+1 =0 0r x -5) =0

   X = -1 or x = 5

   :. X=-1 or 5

   Hence x = { -1, 5 }

8. The number of elements in a set is the total number of elements in a particular or given set , say A, and it is denoted by n(A). It reads numbers of elements in set A or cardinality of set A.

   Consider the set P = {2, 3, 5, 7, 11} . :. n(P) =5

   .Also, ifM = {Datsun, Toyota, Mazda, Nisan} . :. n(P)=4

9. Equivalent sets

   Two sets are said to be equivalent if they have equal number of elements

   Let P = {a, b, c} and

   Q = {mango, apple, guava}

   Since each set contains three elements, P is equivalent to Q i.e.

   $P \equiv Q$ since n(P) = n(Q)

   Example

   Determine whether the following are equal or not

   - C = {4, 6, 2, 8}
     E = {2, 4, 6, 2, 8, 4}
   - M = {x : x is factors of 12}
     N = {1, 3, 2, 12, 6, 4}

   Solution

   a) C = {4, 6, 2, 8} and E = {2, 4, 6, 2, 8, 4}.
      In E = {2, 4, 6, 2, 8, 4} 2 and 4 are repeated, remember that any element that is repeated is counted once only . Now,
      N (C) =4 and n(E) = 4 .
      Hence, n (C) = n(E) $\longrightarrow$ $C \equiv E$

   b) M = {x : x is factors of 12}
      : . M = {1, 2, 3, 4, 6, 12} and N = {1, 3, 2, 12, 6, 4}
      Both sets have the same number of elementsi.e n(M)=n(N)=6.
      Since n(M) =n(N) $\longrightarrow$ $M \equiv N$.

WEEK 9

SET OPERATIONS :Union and Intersection Of Sets .

1. **Union of sets (U)**

The union of sets A and B is the set of all members that belong to A or B or to both A and B.It is denoted by U.

The Union of sets A and B can be written as A

Illustrations :

If A = {a, b, c, d} and B = {b, d, e, f}then,

A U B = {a, b, c, d, e, f}

2. **Intersection of Sets**

The intersection of sets A and B is the set of members that are common to both A and B. The Intersection of A and B is usually written as A∩ B when '∩' is the symbol used to denote intersection.

Illustrations :

If A = {a, b, c, d} and B = {a, c, e}, then the elements common to A and B are "a and c" .

Hence, A ∩ B = {a, c}

If A = {2, 5, 8, 11, 14, 17, 20}

B = {2, 3, 5, 7, 9, 11}

C = {5, 11, 17} and

D = {1, 2, 3, 4}

Using Venn diagram find (a) A U C (b) A ∩ B (c) C U D

(d) Present the answers on Venn Diagram

Solution:

A = {2, 5, 8, 11, 14, 17, 20}

B = {2, 3, 5, 7, 9, 11}

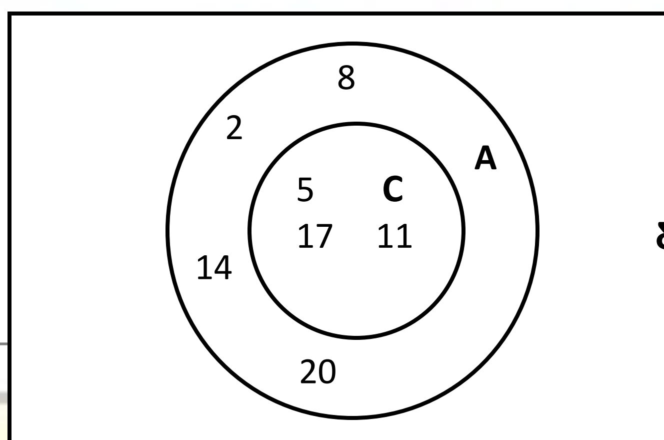
C = {5, 11, 17} and

D = {1, 2, 3, 4}

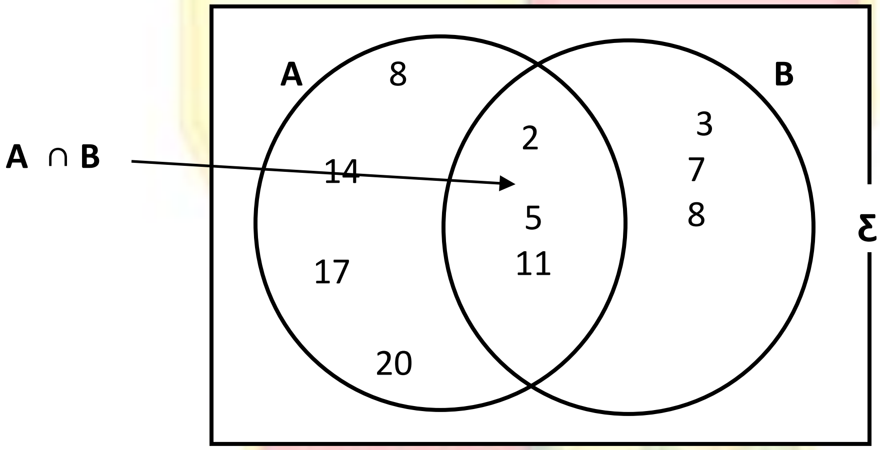
(a) A U C = {2, 5, 8, 11, 14, 17, 20}.

Here , A and C have same elements in common. Hence, the Venn diagram becomes

A U C

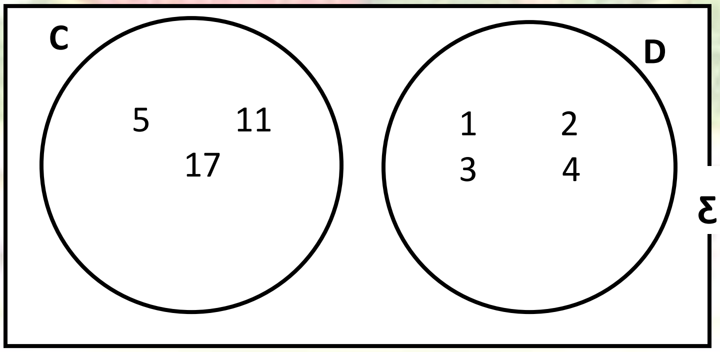


(b) A ∩ B= {2, 5, 11}



(c) C U D= {1, 2, 3, 4, 5, 11, 17}

Hence, the two sets C and D are disjoint because they have no elements in common.



3. Difference of Sets

The difference of two sets of B and D denoted by B – D (i.e. B minus D or B difference D) is the set of members which only belongs to B but do not belong to D e.g Let B = { 2, 3, 5, 7} and D ={1, 2, 4, 8 }

:. B – D ={ 3, 5, 7}

4. Disjoint Sets

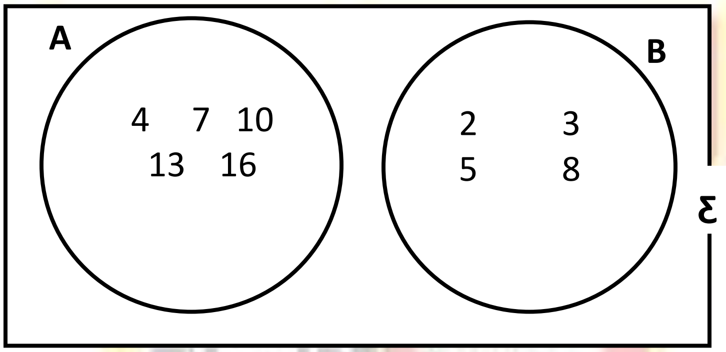
Two sets A and B are said to be disjoint, if they have no common members i.e. if no member of A is in B and vice versa. In other words, two sets A and B are said to be disjoint if and only if A ∩ B = ϕ

For example

(a) If A = {4, 7, 11, 13, 16} and B = {2, 3, 5, 8} then A and B are disjoint because they have no members in common. $\therefore A \cap B = \phi$

This is shown in the Venn diagram below :

A∩B = ϕ



(b) If Q = {Odd numbers} and M = {multiples of 4} then,

Q = { 1, 3, 5, 7, 9 …}

M = {4, 8, 12, 16, 20, 24 …}

Notice that multiple of 4 are even numbers, since sets Q and M cannot have common elements. $\therefore$ **Q∩M =ϕ.**

5. Complement of Sets

The members in the universal set ε that are not in set A are called the complement of set A and is denoted by **A’** or **A^c** .

Illustration:

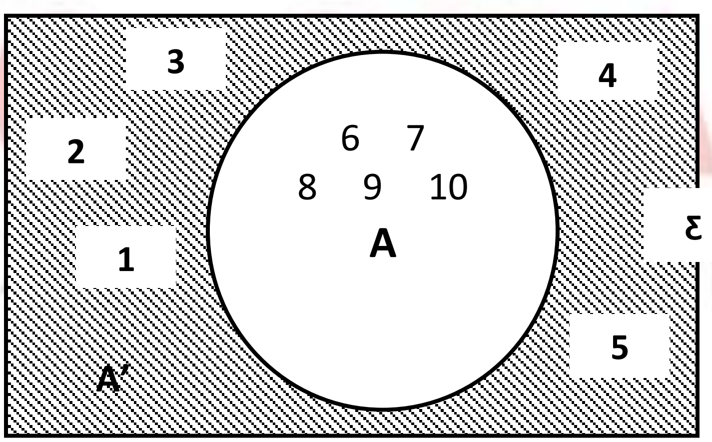
If ε = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and

A = {6, 7, 8, 9, 10}

Then

A’ = {1, 2, 3, 4, 5}

The Venn diagram of the above relationship is shown below. The shaded portion represents A’



Example

If ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} ,

A = {2, 3, 5, 7} ,B = {2, 4, 5, 7} and

D = {1, 2, 5, 7} , find

(a) A' (b) (A ∩ B)' (c) (A U D)'

Solution

(a) The set of members in ξ which are not in A is given by A'

: . A' = {1, 4, 6, 8, 9, 10}

(b) (A ∩ B)'

A ∩ B = {2, 5, 7}

: . (A∩B)' = {1, 3, 4, 6, 8, 9, 10}

(c) (A U D)'

A U D = {1, 2, 3, 5, 7}

: . (A U D)' = {4, 6, 8, 9, 10}

WEEK 10:

VENN DIAGRAM AND APPLICATION UP TO 3 SET PROBLEMS

Definition :

Venn diagram is a diagrammatical way of solving problems on set. It consists of circles and rectangular box.

Example1 :

In a class of 30 students 25 like football and 15 like wrestling, every pupil like at least one of these sports. How many students like both sports?

Solution :

Let F represent football, and W represent wrestling

:.n (F) = 25

n(W) =15

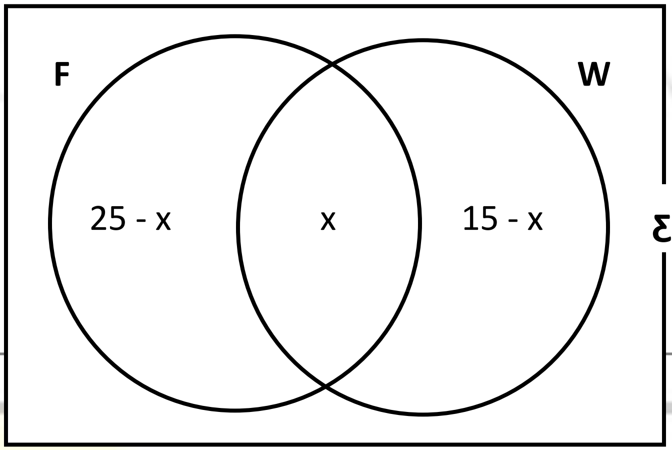
n (ξ) = 30,

Let the number of students who like both sports be x.

:. n(F∩W) = x

Also, number of students who like Football only = 25 – x

i.e. n(F∩W)' = 25 – x



and number of students who like wrestling only = 15 – x

: . n(F∩W) = 15 – x

n(Ꜫ) = 30

Then the total number of students in the class is

30 = (25 – x) + x + (15 – x)

30 = 25 – x + x + 15 – x

30 = 40 – x

x = 40-30

X = 10

Therefore, 10 students like both sports.

Solving problems involving three sets

Example2 :

2000 people were asked which make of cars would they like to have. 300 saidPeugeot(P) only, 500 said Toyota (T) only, 450 said Datsun (D) only , 200 said Peugeot and Toyota, 180 said Peugeot and Datsun and 250 said Toyota and Datsun.If 420 said none of these cars,

(a). draw a Venn diagram to illustrate this information and hence find the number of people who like the three types of cars.

(b). How many people like at least two of these cars?

(c). How many people like both Toyota and Datsun?

Solution :

(a).

Let the number who like the three types of cars be xn(P∩T∩D) = x

Those who like Peugeot only = 300n(P∩T'∩D') = 300

Toyota onlyn(P'∩T∩D') = 500

Datsun onlyn(P'∩T'∩D) = 450

Peugeot and Toyotan(P∩T∩D') = 200 – x

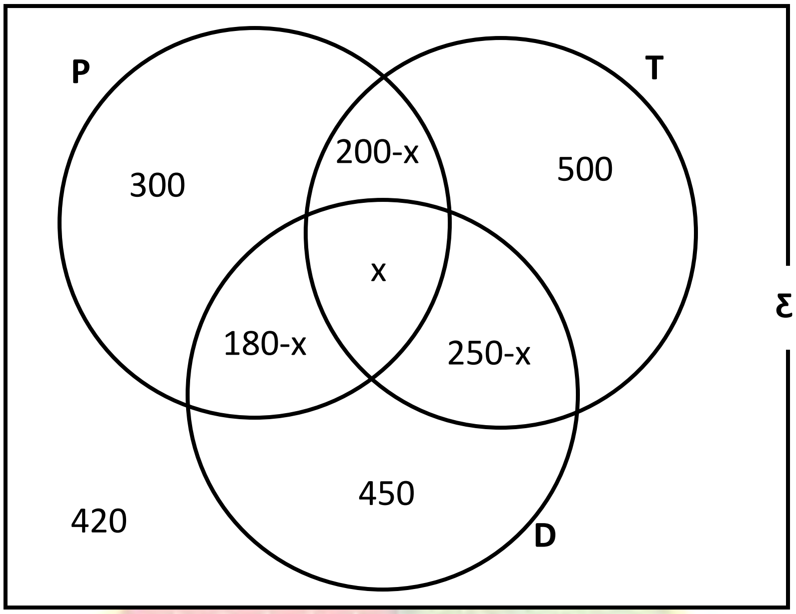
Toyota and Datsunn(P'∩T∩D) = 250 – x

Peugeot and Datsunn(P∩T'∩D) = 180 – x

Those who like none of these carsn(P'∩T'∩D') or n(P∩T∩D)' = 420

n (Ꜫ) = 2000

Now, these information are shown in the Venn diagram below;



Since n (ε) = 2000, we have

300 + 500 + 450 + 200 – x + 180 –x + 250 – x + x + 420 = 2000

2300 – 2x = 2000

Collecting like terms, we have

-2x = 2000 – 2300

-2x = -300

Dividing both sides by -300, we have

x = $\frac{-300}{-2}$

x = 150

Hence, the number of people who like the three types of cars is 150

b) Those who like at least two of the cars

= 200 – x + x + 250 – x + 180 – x

= 630 – 2x

Substituting for x = 150 from (a) above, we have

= 630 – (2 x 150)

= 630 – 300

= 330

: . those who likes at least two types of car is 330

b) From the Venn diagram,
those who like both Toyota and Datsun

$= 250 - x$

$= 250 - 150$

$= 100$

CLASSWORK

In a certain class, 22 students take one or more of Chemistry (C), Economics (E) and Government (G). 12 take Economics (E), 8 take Government (G) and 7 take Chemistry (C). Nobody takes Economics and Chemistry and 4 students take Economics and Government.

a (i). Using set notation and letter indicated above write down the two statements in the last sentences

(ii). Draw a Venn diagram to illustrate the information

b. How many students take:

(i). Both Chemistry and Government

(ii). Government only

SECOND TERM

1. SIMPLE LINEAR EQUATION

Behavioral Objectives

By the end of the lesson student should be able to:

i. Identify and solve linear equations
ii. Solve linear equations with brackets and fractions
iii. Substitute values into a given formula
iv. Change the subject of a given formula

Illustrations :

An equation is simply a statement showing that two algebraic expressions are equal in value.

A. Equation with brackets

Examples :

1. Solve the equation $3(4c - 7) - 4(4c - 1) = 0$

Solution

$3(4c - 7) - 4(4c - 1) = 0$
Remove the brackets, we have
$12c - 21 - 16c + 4 = 0$
$-4c - 17 = 0$
Add 17 to both sides, we have
$-4c = 17$
Divide both sides by -4,we have
$c = \frac{-17}{4}$
$c = -4\frac{1}{4}$

B. Equations with Fractions

Examples

1. Find the value of x in the equation
$\frac{3x}{4} - 1\frac{2}{3} = \frac{2x}{3}$

Solution

$$\frac{3x}{4} - 1\frac{2}{3} = \frac{2x}{3}$$

To clear the fraction , multiply through by the L.C.M of 4 and 3i.e 12

$\frac{3x}{4} \times 12 - 1\frac{2}{3} \times 12 = \frac{2x}{3} \times 12$

$9x - 20 = 8x$
Collecting like terms, we have

9x – 8x = 20
x = 20
: . the value of x is 20

Example 2
Solve the equation

$$\frac{2(6x-1)}{5} = \frac{3\,(3x+2)}{4} - 2$$

Solution:

$$\frac{2(6x-1)}{5} = \frac{3\,(3x+2)}{4} - 2$$

To clear the fraction , multiply through by the l.c.m of 5 and 4 i.e 20

$$\frac{2(6x-1)}{5}\text{x}20 = \frac{3\,(3x+2)}{4}\ \text{x}20 \quad - 2\text{x}20$$

2(6x -1) x 4 = 3(3x+2) x 5 – 40
8(6x –1) = 15(3x + 2) – 40
Clear brackets, we have
48x – 8 = 45x + 30 – 40
Collect like terms, we have
48x – 45x = 30 – 40 + 8
3x = – 2
Dividing both sides by 3, we have
x = $-\frac{2}{3}$

C. Change of subject of formulae (Transposition)

To transpose a formula, it means to rearrange it so that a different letter becomes the subject

Examples

1. Make x the subject of the formula
   a = b(1–x)

Solution:
a = b(1–x)
By opening the bracket, we have
a = b - bx
To make x the subject , we have
a - b = –bx
-bx = a - b

Divide both side by –b

$\frac{-bx}{-b} = \frac{a}{-b} - \frac{b}{-b}$

$X = \frac{a}{-b} \quad - \quad \frac{b}{-b}$

$X \quad = \quad \frac{a}{-b} \quad + \quad \frac{1}{1}$

$x \quad = \frac{-a+b}{b}$ since l,c.m =b

$\therefore x = \frac{b-a}{b}$

2. Make x the subject of the formula

$b = \frac{1}{2}\sqrt{a^2 - x^2}$

Solution

$b = \frac{1}{2}\sqrt{a^2 - x^2}$

To clearthe fraction, multiply both sides by 2, we have

$2 \times b = 2 \times \frac{1}{2}\sqrt{a^2 - x^2}$

: $2b = a^2 \sqrt{x^2}$

Squaring both sides, we have

$(2b)^2 = (a^2 - x^2)^{1/2 \times 2}$

$4b^2 = a^2 - x^2$

Rearrange to give term in x on the one side

$x^2 = a^2 - 4b^2$

Take square root of both sides, we have

$x = \pm\sqrt{a^2 - 4b^2}$

3. Make x the subject of the formula

$p = y^2 + \frac{1}{2x^3}$

Solution:

Eliminate the fraction by multiplying each term by $2x^3$

$2x^3 p = (2x^3 \times y^2) + 2x^3 \times \frac{1}{2x^3}$

$2x^3 p = 2x^3 \times y^2 + 1$

To make x the subject, we have

$2x^3 p - 2x^3 \times y^2 = 1$

Factorizing, we have

$x^3(2p - 2y^2) = 1$

Dividing both sides by $2p - 2y^2$, we have

$$x^3 = \frac{1}{2p - 2y^2}$$

Taking cube root of both sides, we have

$$x = \sqrt[3]{\frac{1}{2(p - y^2)}}$$

$$x = \frac{1}{\sqrt[3]{2(p - y^2)}}$$

D. Substitution in formulae

Example

Calculate the value of x in the equation

$q = \frac{m + n}{y - x}$ if q = 4, m = –5, n = 11 and y = –8.5

Substituting

q = 4, m = –5, n = 11 and y = –8.5 into the given equation, we have

$$4 = \frac{-5 + 11}{-8.5 - x}$$

Eliminate the fraction by cross multiplication

4(–8.5 – x) = –5 + 11

Remove the bracket, we have

–34 –4x = 6

Collect like terms, we have

–4x = 6 + 34

–4x = 40

Divide both sides by -4, we have

$$x = -\frac{40}{4}$$

: . x = –10

CLASSWORK

Solve the following equations

1. 4a – (3–a) = 17
2. 8n – (5n + 13) = 7
3. $\frac{x}{2} = \frac{x}{3} + \frac{1}{2}$

4. $\frac{2d-7}{6}-\frac{d+5}{3}=0$
5. Make x the subject of the following equations
   a. $ax + bx = c$
   b. $a(b–x) = cx$
   c. $a = \frac{2b-3x}{3b+2x}$
6. If $y = 2x^2 – 5x – 3$, find the value of y when
   (a) $x = –1$ (b) $x = 0$ (c) $x = 1$

2. VARIATIONS

Behavioral Objectives

By the end of the lesson, students should be able to solve problems involving direct, inverse, joint and partial variations.

Definition :

Variation is the relationship that exits between two or more quantities in which a change in one quantity leads to change in the others. Variations can be classified into direct, inverse, joint and partial variations.

A. Direct Variation

Y varies directly as x is written as $y \alpha x$,

$y\alpha x$ also means y is directly proportional to x, meaning $y=kx$

Where k is a constant of proportionality or simply a constant.

Example1:

If $y \alpha \sqrt{h}$ and h = 16 when y =9 , Find

(a) The relationship between y and h

(b) The value of y when h = 36

(c) The value of h when y = 27

Solution

(a) If $y \alpha \sqrt{h}$

Then $y = k\sqrt{h}$ (where k is a constant)

Substitute h = 16and y = 9 into the relation, we have

$9 = k\sqrt{16}$

$9 = 4k$

$k = \frac{9}{4}$

The required relation is $y = \frac{9}{4}\sqrt{h}$

(b) When h = 36, then the above relation is y = $\pm\frac{9}{4}\sqrt{36}$

y = ±2.25 x 6

y = ±13.5

(c) When y = 27, then the relation becomes

27 = $\pm\frac{9}{4}\sqrt{h}$

Cross multiply by 4 to clear the fraction, we have

27 x 4 = 9$\sqrt{h}$

Divide both sides by 9, we have

27 x 4/ 9 = $\sqrt{h}$

12 = $\sqrt{h}$

Square both sides, we have

$(\sqrt{h})^2 = 12^2$

:. h = 144

B. Inverse Variations

y varies inversely as x is written as y α $\frac{1}{x}$

:. y = $\frac{k}{x}$(where k is constant)

Example :

Ifp is inversely proportional to $\sqrt[3]{q}$,

Find a. the constant of proportionality when p=8 and q = 27

b. the equation connecting p and q

c. the value of q when p = 3

Solution :

a. p α $\frac{1}{\sqrt[3]{q}}$ then

p = $\frac{k}{\sqrt[3]{q}}$ where k is constant

When p = 8, q = 27 then

8 = $\frac{k}{\sqrt[3]{27}}$

$8 = \frac{k}{3}$

Cross multiply, we have

k = 8 x 3

:. k = 24

b. The required equation of the variation is

$p = \frac{24}{\sqrt[3]{q}}$

c. To find the value of q when P = 3,recall that the equation connecting

d. p and q is

$p = \frac{24}{\sqrt[3]{q}}$

When p=3 then, we have

$3 = \frac{24}{\sqrt[3]{q}}$

Cube both sides, we have

$3^3 = \frac{24^3}{q^{1/3}\ )3}$

$3^3 = \frac{24^3}{q}$

Cross multiply, we have

3^3 q = 24^3

Divide both sides by 3^3 ,we have

q = $24^3/3^3$

$q = \frac{24X24X24}{3X3X3}$

q = 8 x 8 x 8

q = 8^3

: .q = 512

C. Joint Variation

In joint variation, three or more variables are involved for example, if A varies directly as F and inversely as M then

$A \alpha \frac{F}{M}$ or $A = \frac{KF}{M}$, Where K is a constant.

Therefore, joint variation is a combination of direct and inverse variation.

Note that

(a) If p α q, then q α p

(b) If p α 1/q then q α 1/p

(c) If p α q^2 then q α $p^{1/2}$etc

Example

P varies directly as Q and inversely as the square of R and Q = 40 When R = 25 and P = 2/5

(a) Find the formula connecting the variables

(b) Find R when Q = 8 and P = 2

Solution

P α Q and P α $1/R^2$

→ P α Q/R^2

: . $P = KQ/R^2$ Where K is a constant

(a). To get the formula connecting the variables :

$P = KQ/R^2$

When Q = 40, R = 25and P = 2/5 , we have

$2/5 = 40K/25^2$

Cross multiply to clear the fraction, we have

$5 \times 40K = 2 \times 25^2$

$200K = 2x\ 625$

Divide both sides by 200, we have

$\frac{200K}{200} = \frac{2x625}{200}$

$K = 2 \times 625/200$

$K = \frac{25}{4}$

: . The required formula is**$P = 25Q/4R^2$**

(b) To find the value of R when Q = 8 and P = 2 using the formula

$P = 25Q/4R^2$, we have

$2 = 25 \times 8/4R^2$

$2 = 25x2/R^2$

Cross multiply byR^2, we have

$2R^2 = 25 \times 2$

Divide both sides by 2, we have

$R^2 = 25 \times 2/2$

$R^2 = 25$

Taking square root of both sides, we have

$R = \sqrt{25}$

$R = 5$

Partial Variation

When a variation is expressed as a sum of two or more parts it is called partial variation.

Illustration:

If y varies directly as x and partly varies inversely as P^2, then we write

$y \alpha x$ and $y \alpha 1/p^2$ or

$y = ax + \frac{b}{p2}$(where a and b are constants of proportionality).

Example:

X is partly constant and partlyvaries as y. When y=5, x = 7 and when y = 8, x = 10. Find the relationship between y and x. Hence, find the value of x when y = 15.

Solution :

$x \alpha k + y \longrightarrow x = K + c y$ (where k and c are constants)

To find the constants in the formula, we substitute

Y = 5 and x = 7, we have

7 = k + 5 c ………..(i)

and also y =8 when x = 10. We have

10 = k + 8 c…………(ii)

To eliminate one of the constants , we subtract equation (i) from equation (ii) i.e

_ 10 = k + 8 c…………(ii)

7 = k + 5 c ………..(i)

3 = 0 + 3c

$3 = 3c$

Divide both sides by 3, we have

$\frac{3}{3} = \frac{3c}{3}$

$1 = c$

$\therefore c = 1$

Put c = 1 into either (i) or (ii), we have

$10 = k + 8c$............(ii)

$10 = k + 8 \times 1$

$10 = k + 8$

Collect like terms, we have

$10 - 8 = k$

$2 = k$

$\therefore k = 2$

The relationship between y and x in the formula **x = K + c y** is

x = 2 + 1 y or

x = 2 + y

To find the value of x when y = 15, we use the formula x = 2 + y and we have

$X = 2 + 15$

$\therefore X = 17$