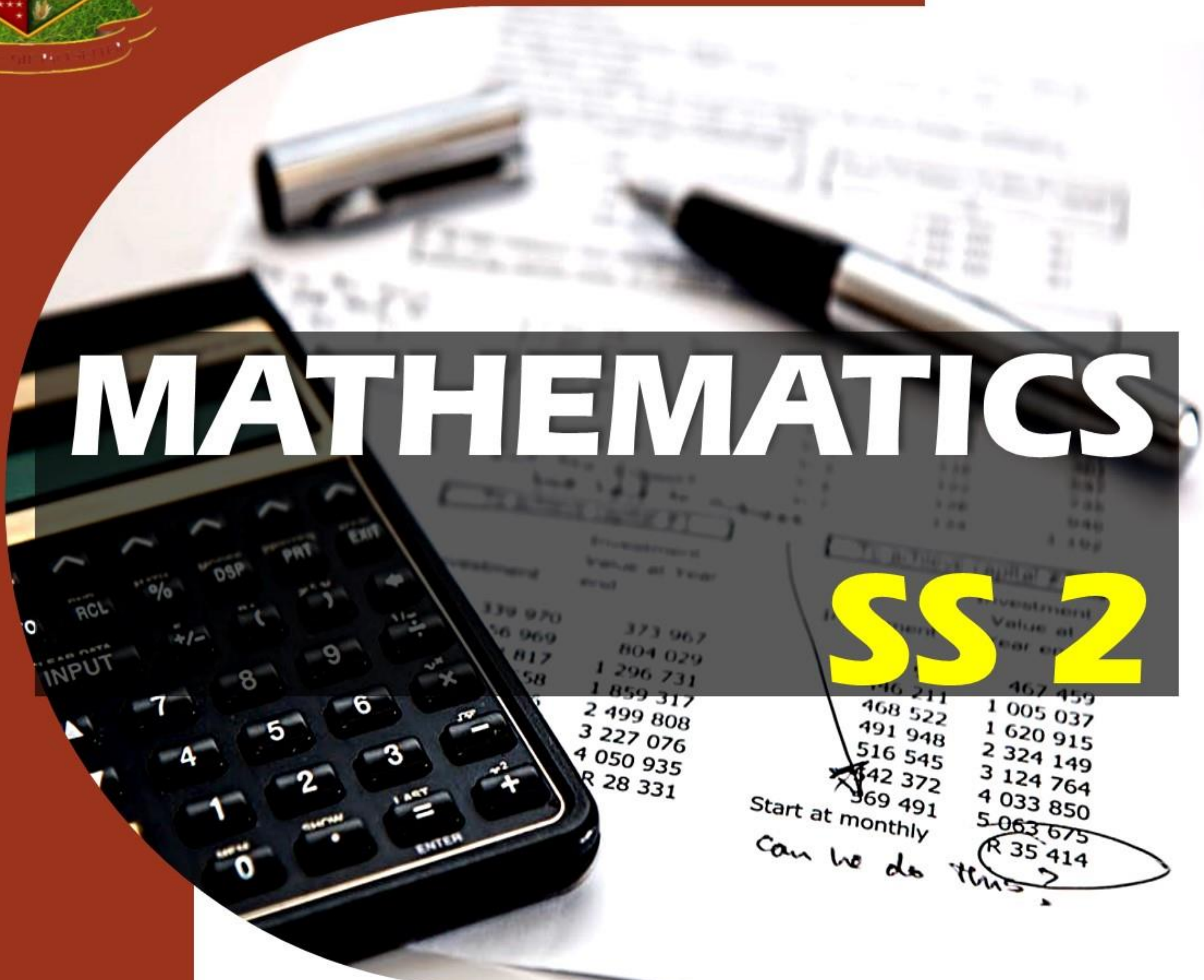
OYO STATE LECTURE NOTES



MATHEMATICS

SS 2

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SS 2

1^{ST} TERM WORK

LOGARITHM

The logarithm of base 10 numeral consists of two parts:

- Characteristics
- Mantissa

A. Logarithm of number greater than 1.

Example: Use the table of logarithms to evaluate the following ; a. $\frac{13.95 \times 364}{4.32}$ b. $\sqrt{\frac{6.147 \times 29.3^2}{437}}$

SOLUTION: a.

a.

No	Log
13.95	1.1446
364	2.5611+
	3.7057
4.32	0.6355-
1176	3.0702
	= 1176 Ans

b.

No	Log	Working
6.147	3.7887	3.7887
29.3^2	1.4669 ×2	2.9338+
		6.7225
437		2.6405-
		4.0820÷2
109.9		2.0410
		= 109.9 Ans

B. Logarithms of n umber less than 1

Example 1: write each of the following in standard form (a) 0.765 (b)0.074

Solution: (a) $\frac{0.7654}{1000} = \frac{765}{1000}$ (b) $\frac{0.074}{1000} = \frac{74}{1000}$

(a)	(b)
765×10^{-3}	74×10^{-3}
$7.65\times10^{2}\times10^{-3}$	$7.4\times10^{1}\times10^{-3}$
7.65×10^{-1}Ans	7.4×10^{-2} Ans

Example 2: Find the logarithms of the following

(a) 0.364 (b) 0.01395

Solution:

(a) $0.364 = 3.64 \times 10^{-1}$ in standard form

$= 10^{0.5611} \times 10^{-1}$

$= 10^{0.5611 + (-1)}$ law of indices

$= 10^{\bar{1}.5611}$ log of 0.364 = $\bar{1}.5611$

(b) $0.01395 = 1.395 \times 10^{-2}$ in standard form

$= 10^{0.1446} \times 10^{-2}$

$= 10^{0.1446 + (-2)}$

$= 10^{\bar{2}.1446}$

$= \bar{2}.1446$

Example 3: Simplify the following (a) $\begin{array}{r} \bar{5}.1 \\ +\,\bar{3}.4 \end{array}$ (b) $\begin{array}{r} 4.9 \\ -\,6.3 \end{array}$ (c) $\bar{5}.6 \div 4$

Solution: (a) $\begin{array}{r} \bar{5}.1 \\ +\,\bar{3}.4 \\ \hline \bar{8}.5 \end{array}$ (b) $\begin{array}{r} 4.9 \\ -\,6.3 \\ \hline \bar{2}.6 \end{array}$ (c)

$\bar{5}.6 \div 4$

$= (\bar{5} + 0.6) \div 4$

$= (\bar{8} + 3.6) \div 4$

$= \bar{2} + 0.9$

$= \bar{2}.9$

Example 4: Evaluate $(0.3684)^3$

Solution:

	No	Log
	0.3684	$\bar{1}.5663$
	$(0.3684)^3$	$\bar{1}.5663 = \bar{1} + 0.5663 \times 3$
4999×10^2		$\bar{2}.6989 \leftarrow \bar{3} + 16989$
$(0.3684)^3$	0.04999	

APPROXIMATION

An approximation is an estimate of a number or an amount that is almost correct but not exact.

A. Rounding off numbers

Rounding off is a way of approximating numbers when rounding off, all numbers 0,1,2,3,4 are rounded down to 0 while numbers 5,6,7,8,9 are rounded up to 1.E.g. 4.681 ≅ 4.7 to 1 decimal place.

Also 4.681 ≅ 4.68 to 2 decimal places.

B. Significant Figures.

In any number, the first significant figure is the first digit which is not a zero (0)e.g$\underline{1}$ 648, $\underline{9}$ 63 , $\underline{3}$ 9684 first significant figure is underlined.

Example1: Round 16418.39 to

(a) 3 significant figures
(b) 2 significant figures

Solution: (a) 16418.39 = 164.00 to 3 significant figure.

16418.39 = 16.000 to 2 significant figure.

Example 2: Simplify $8.4\times10^3-7.5\times10^2+3\times10$ giving your answer in standard form

Solution: $8.4\times10^3-7.5\times10^2+ 3\times10$

= 8400-750 +30

= 8400+30 – 750

= 8430 – 750

= 7680

= 7.680×10^3

C. PERC ENTAGE ERROR

Every measurement, no matter how carefully carried out, is an approximation. The error involved can not exceed ± 0.05 it is called the maximum absolute error. Relative error helps to precisely and satisfactorily judge the degree of accuracy of any measurement it is called absolute error.

Relative error = maximum absolute error ∕ Actual value

Percentage error = $\frac{Error}{Actual\ Measurement}\ X\ \frac{100}{1}$

Example 1: The length of a pencil is 16.5cm, correct to the nearest cm. Find the percentage error to one significant figure.

Solution: The length given is 16.45cm and 16.54cm

: the maximum absolute error is 0.05cm

The relative error = 0.05/16.5 = 0.003

And the percentage error = 0.05/16.5 × 100/1 = 0.30%

Sequence and Series

Sequence simply means number of patterns consist of list of numbers that follow a rule. They are called number of sequences. E.g Find the nth term of the following sequence.

(a) 7,12, 17, 22 (b) 5,9,13,17

Solutiuon:

Rule

(a) $U_1 = 7 = 5+2$

$U_2 = 7+5 = (5\times2) + 2 + 12$

$U_3 = 7 + 10 = (5\times3) +2 = 17$

$U_4 = 7+ 15 = (5\times4) + 2 = 22$

$U_n = (5\times n) +2 = 5n + 2$

$\therefore U_n = 5n + 2$

(b) $T_1 = 5=1+4$
$T_2 = 5+ 4 = 1 + (4\times2) = 9$
$T_3 = 5+8 = 1 + (4\times3) = 13$
$T_4 = 5+ 12 = 1+ (4\times4) = 17$
$T_n = 1+ 4\times n = 1+ 4n.$

$\therefore T_n = 1 + 4n$

Series

A series is the sum of the terms of a sequence, this means that when we add the terms in any given sequence, it makes up a series e.g.

Finite series are:

(a) 4+2+0 – 2 – 4 + ………………….+ 32-34
(b) 1+2+3+4+……..+98+99+100

Infinite series are:

(a) 4+8+12+16 +------------

(b) $1 + \frac{1}{2} + {}^{1}/_{4} + {}^{1}/_{8} +$ ----------

ARITHMETIC PROGRESSION (AP)

The terms are said to form an arithmetic progression (AP). The constant term is called the common difference d. From the sequence 1st term is a and nth term is U_n. The nth term of an AP is :

$U_n = a + (n-1)d$

Where:

a = First term

n = number of terms

d = common difference

U_n = nth term

Example 1: The 16th term of an Ap is 93 and its common difference is 6. Find the first term of the AP and hence, evaluate the 30 th term of the AP.

Solution: $U_n = a + (n-1)\, d$

$U_{16} = 93$, n = 16, a=?, d=6

$93 = a + (16-1)6$

$93 = a + 15 \times 6$

$93 = a + 90$

$a = 93 - 90$

$a = 3$

∴the first term of the AP is 3.

Hence,

To evaluate the 30th term of the AP:

$U_n = a + (n-1)d$; a = 3, d= 6 and n = 30

$U_{30} = 3 + (30-1)(6)$

$= 3 + (29)(6)$

= 3 + 174

= 177

Therefore, the 30th term of the AP is 177

Sum of terms of an AP

The sum of n term of an AP is given by

$S_n = \frac{n}{2}[2a + (n-1)d]$ - Equation (i)

OR

$S_n = \frac{n}{2}(a + l)$ - Equation (ii)

When:

S = The sum of terms

n = The number of tems.

l = The last term

a = the first term

d = The common difference.

Example: Find the sum of the first 12 positive even integers.

Solution: Positive integers are 2,4,6,8,-----

First term a= 2

Common difference d = 8-6 =2

nth = 12

Since we don't know the value of the last term (l) then, we use equation (i) to solve the problem.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{12}{2}[2(2) + (12-1)2]$

= 6(4 + 22)

= 6(26)

= 156

∴The sum of the first 12 positive even integers is 156.

To check: The first 12 positive even integers are: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 22 and 24.

The sum becomes: 2+4+ 6+ 8+ 10+ 12+ 14+ 16+ 18+ 20+ 22 + 24 = 156

Example: the sum of the first n terms of an AP is 252. If the first term is -16 and the last term is 72. Find the number of term in the series.

Solution:

Since the last term (l) is known then, we apply equation (ii).

Formula $S_n = \frac{n}{2}(a+l)$

Where $S_n = 252$, a=-16, l=72 n=?

252 = n/2(-16+72)

252×2 =(56)n

252×2 =n (56)

n= 252×2/56

n=9

∴Number of terms in the series is 9

GEOMETRIC PROGRESSION (GP)

A geometric progression GP is a sequence which two consecutive terms differ by a multiple. The multiple by which the terms differ is known as the common ratio denoted by r. Note that the common ratio r is obtained by dividing the succeeding term by the preceding term.

$U_n = ar^{n-1}$

Example: The 5^{th} term of a GP is 48 and the 8^{th} term is 384. Find the first term and the common ratio.

Solution:

$U_n = ar^{n-1}$

$U_5 = 48 = ar^{5-1} = ar^4 = 48$ ……………….. (i)

$U_8 = 384 = ar^7 = 384$ ………………………..(ii)

Divide equation 2 by equation 1, we have

$$\frac{ar^7}{ar^4} = \frac{384}{48}$$

→ r3 = 8

$r = \sqrt[3]{8}$

∴ r = 2

Substrate for r in equation (i), we have

$ar^4=48$

$a(2)^4=48$

16a=48

a=48/16

a=3.

The first term of GP is 3 and the common ratio is 2.

The sum of an nth term of a GP is given by

$S_n = \frac{a\,(1-r^n)}{(1-r)}$ when r < 1 - Equation (3)

OR

$S_n = \frac{a\,(r^n-1)}{(r-1)}$ when r > 1 - Equation (4)

Example: The third term of a GP is 360 and the sixth term is 1215. Find the (a) common ratio (b) first term (c) sum of the first four terms.

(a) $U_n = ar^{n-1}$ or $T_n = ar^{n-1}$
$U_3 = ar^{3-1} = 360$
$ar^2 = 360$-------------- (1)
Also
$U_6 = ar^{6-1} = 1215$
$ar^5 = 1215$ ------------- (2)
Divide equation 2 by 1

$$\frac{ar^5}{ar^2} = \frac{1215}{360}$$

$$\rightarrow r^3 = \frac{27}{8}$$

$$r = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$$

∴ The common ratio is $\frac{3}{2}$ or $1\frac{1}{2}$

(b) To find the first term from equation 1;

$ar^2=360$

Since r= 3/2 then,

$a(3/2)^2=360$

$a=360\times(2/3)^2 =360\ (4/9)$

a=160

∴The first term is 160

(c) To find the sum of the first four term ;

$$S_n = \frac{a(r^n - 1)}{(r - 1)}, r > 1$$

But

a=160, r=3/2 and a=4

Now

$$S_4 = \frac{160\left[(\frac{3}{2})^4 - 1\right]}{(\frac{3}{2} - 1)}$$

$$= \frac{160\ (\frac{81}{16} - 1)}{(\frac{3}{2} - 1)}$$

$$= \frac{160\ (\frac{65}{16})}{(\frac{1}{2})}$$

$$= 160 \text{ X } \frac{65}{16} \text{ X } \frac{2}{1}$$

$$= \frac{20800}{16}$$

$= 1300$

∴ S_4 = 1300

: the sum of the first four term of GP is 1300

The sum of a G.P in infinity

The infinite sequence has an incoding term n →∞ which means that n is so large that it is limitless or without a boundary.The sum to infinity of a GP is

$S_\infty = \frac{a}{1-r}$ for /r/ < 1.

Example: Find the sum of the infinity progression

3,1, 1/3, 1/9, 1/27,------------------

Solution: Given a=3, $r = \frac{1}{3} \div 1 = \frac{1}{3}$ or $r = \frac{1}{9} \div \frac{1}{3} = \frac{1}{9} \text{ X } \frac{3}{1} = \frac{1}{3} \therefore r = \frac{1}{3}$

Now,

$S_\infty = \frac{a}{1-r}$

$= \frac{3}{1-\frac{1}{3}} = \frac{3}{2/3}$

$= 3 \div \frac{2}{3}$

$= 3 \times \frac{3}{2}$

$= \frac{9}{2} \ or \ 4\frac{1}{2}$

QUADRATIC EQUATION

Example 1: what should be added to x^2-3x to make it a perfect square.

Step I : find half of coefficient of x i.e -3/2

Step II: square this result i.e $(-3/2)^2=9/4$

Step III: Add this square to the original expression to obtain a perfect square i.e $x^2-3x+9/4$.

This can be verified as follows

$(x-3/2)^2 =(x-3/2)(x-3/2)$

$X^2-3/2x - 3/2x + 9/4$

$X^2 -3x +9/4$.

Example 2: solve the equation $x^2 +3x - 1 = 0$

Solution: The expression $x^2 +3x -1$ does not factorise

Step I: Add 1 to both sides

Step II: Add the term that will make x^2+3x a perfect square to both side. Which is $(\frac{3}{2})^2$

i.e $x^2 + 3x + (3/2)^2= 1+(3/2)^2$

$x^2 + 3x + (3/2)^2$

$= 1+9/4$, $(x+3/2)^2=13/4$.

StepIII: Take the square root of both sides

$X + \frac{3}{2} = \pm\sqrt{\frac{13}{4}}$

$X = -\frac{3}{2} \pm \sqrt{\frac{13}{4}}$

$$\therefore \; x = -\frac{3}{2} + \sqrt{\frac{13}{4}} \;\; or \; -\frac{3}{2} - \sqrt{\frac{13}{4}}$$

FACTORIZATION

Example :

Solve for the values of x in the equation $2x^2 - 5x - 3 = 0$

Solution

2x2 – 5x – 3 = 0

2x2 – 6x + x – 3 = 0

(2x2 – 6x) + (x-3) = 0

2x (x - 3) + 1 (x – 3) = 0

(2x + 1) (x – 3) = 0

2x + 1 = 0 or x – 3 = 0

2x = -1 of x = 3

X = -1 or x = 3

∴ x = -1 or 3

Completing the square

Solve $2x^2-5x-3$

Solution : $2x^2-5x-3+3=0+3$

2x-5x=3, divide the coefficient of x^2 which is 2.

i.e $x^2-5x/2-3/2$,

$x^2-5x/2+(5/4)^2=3/2+(5/4)^2$

$(x-5/4)^2=3/2+25/16$

$(x-5/4)^2=49/16$, x-5/4= ± ∫49/16, x=5/4±7/4

X=-2/4=-1/2 or x=12/4 =3

Quadratic Formula

The Quadratic formular is

X = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solve $2x^2-5x-3=0$

a=2, b=-5, c=-3,, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Solution:

X = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Given that

$2x^2 – 5x – 3 = 0$

Compare with the general form of the quadratic equation i.e.

$ax^2 + bx + c = 0$

now,

a = 2, b = -5 and c = -3

x = $\frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$

$= \frac{5 \pm \sqrt{25 - 24}}{4} = \frac{5 \pm \sqrt{49}}{4} = \frac{5 \pm 7}{4} = \frac{5+7}{4}$ or $\frac{5-7}{4}$

$= \frac{12}{4}$ or $\frac{-2}{4}$

$= 3$ or $-\frac{1}{2}$

$\therefore x = -\frac{1}{2}$ or 3

GRAPH

(a) Draw the graph of the equation y+2=x(x+1), for values of x in the range $-3 \leq x \leq 3$.

(b) Also draw the graph of y=2x on the same axis the graph above ,from your graphs read off the solution to the following equations:

(i) $x^2+x-2=0$ (ii) $x^2-x-2=0$ (iii) $x^2+x=5$ (iv) $3-x-x^2=0$.

Solution:

a. Given that y + 2 = x (x + 1)

$y + 2 = x^2 + x$

$\therefore y = x^2 + x - 2$

The following procedure should be followed in other to draw the graph :

(i) Construct a table of values (ii)find convenient scales on the x and y axis (iii) Plot your point (iv) Join the points by a smooth curve.

Table of values of $y = x^2 + x - 2$

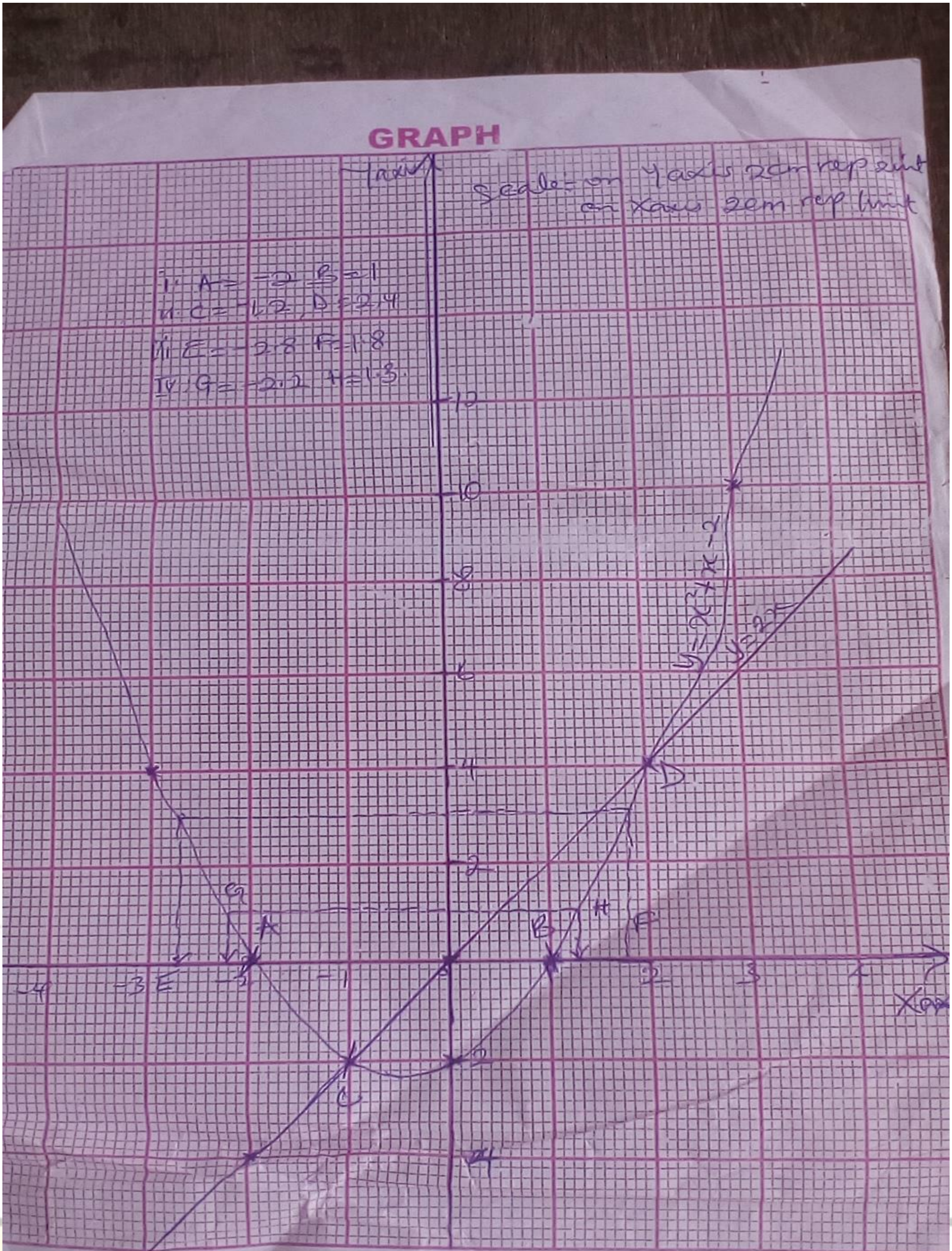
X	-3	-2	-1	0	1	2	3
X^2	9	4	1	0	1	4	9
+x	-3	-2	-1	0	1	2	3
-2	-2	-2	-2	-2	-2	-2	-2
Y	4	0	-2	-2	0	4	10

Use a scale of 2cm to represent unit 0n x-axis and 2cm to 2units on y-axis.

Y=+2x

X	-3	-2	-1	0	1	2	3

2x	-6	-4	-2	0	2	4	6


GRAPH
Y axis
Scale:- on Y axis 2cm rep 1 unit
on X axis 2cm rep 1 unit
i. A = -2 B = 1
ii. C = -1.2, D = 2.4
iii. E = -2.8 F = 1.8
IV. G = -2.2 H = 1.3
12
10
8
6
4
2
-2
-4
-4
-3
E
-2
-1
2
3
4
X axis
A
B
C
D
F
G
H
y = 2x
y = x² + x - 2

1. $X^2+x-2=0$

Solutions are given by points A and B in the graph x=-2, 1.

1. $X^2-x-2=0$
2. This is not the given quadratic curve but when $X^2+x-2=2x$, then $X^2-x-2=0$. Hence, solution are the points C,D of intersection between the curve y= X^2+x-2 and y= 2x, x=-1, 2.
3. $X^2+x=5$. This can be written as , $X^2+x-2=0$, the solutions are therefore the points of intersection of the curve y= X^2+x-2 and liney=3, whichare the points E and F : x=-2.8, 1.8
4. $3-x-x^2=0$, The solutions are therefore the points of intersection of thecurve y= X^2+x-2 and the limit y=1 which are the points G and h

GRADIENTS OF A CURVE

Linear graph gives a straight line graph from any given straight line equation which is in the general from mx+c=y or ax+by+c=0.

Example: Draw the graph of equation 4x+2y=5. Solution : Choose any three values of x and find their corresponding values in y…

4x+2y=5

2y=5-4x

Y=5-4x/2

X	0	1	2
Y	2 ½ ½	-1 ½	

Example2: find the point of intersection of the lines y=3x+2 and y=2x+5.

Solution : y = 3x+2----------(i)

Y= 2x+5----------(ii)

At the point of intersection

3x+2=2x+5

3x-2x=5-2

X=3

Substitute 3 for x in equation (i)

Y=3(3)+2=11, therefore 3, 11 is the point of intersection.

Gradient of a straight line :

The gradient of straight line is defined the ratio change in y/change in x, note that : change in y/change means $\frac{y_1 - y_2}{x_2 - x_1}$

Examples: find the gradient of the line passing through (2, -3) and (12, 17)

Example 3:
Find the gradient and the angle of slope of the line passing through (1,3) and (-4,2)
Solution:
The gradient : $m=y_2-y_1/x_2-x_1$
$X_1=1$, $y=3$ and $x_2=-4$, $y_2=2$
M= 2-3/-4-1 = -1/-5 =1/5
M=1/5 which gives the required gradient
letϴ be the angle of slope
M= tan ϴ
1/5= tan ϴ
tan ϴ =0.2
$\theta=\tan^{-1} 0.2$
$\theta= 11.31^0$
Equation of a straight line
(i) Equation of a line with gradient m and y intercept c is given as y=mx+c
(ii) Equation of a line passing through the point (x,y) with gradient.
Gradient m and which passes through the point (x,y) is given as m = $\frac{y - y_1}{x - x_1}$

Example: Find the equation of the line with gradient 2 and which passes through the point (-3,2)
Solution:
The equation of the line with a known gradient which passes through a given point is
$\frac{y - y_1}{x - x_1} = m$
m=2 , $(x_1, Y_1) = (-3,2)$
$x_1 = -3$ & $y_1 = 2$

Now,
$$\frac{y - 2}{x - (-3)} = 2$$
$$\frac{y - 2}{x + 3} = \frac{2}{1}$$
$$y - 2 = 2(x + 3)$$
y – 2 = 2x + b
y = 2x + 6 + 2
∴ y = 2x – 8 which is the required equation of the line.

Example 2: find the equation of the line passing through points A(3,1) and B(2,-3)
Solution:
Recall that $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

Here $(x_1,y_1)=(3,1)$and$(x_2,y_2)=(2,-3)$
$x_1 = 3$, $y_1 = 1$, $x_2 = 2$ and $y_2 = -3$

$$\frac{y-1}{x-3} = \frac{-3-1}{2-3}$$

$$\frac{y-1}{x-3} = \frac{-4}{-1}$$

$$\frac{y-1}{x-3} = 4$$

y-1 = 4 (x-3)
y-1 = 4x-12
y = 4x – 12 + 1
∴ y = 4x – 11 is the required equation of the line

LINEAR INEGQUALITIES
A. Linear inequalities in one variable
Inequalities are symbols or signs used consuming elements in arithmetic operations instead of quality signs

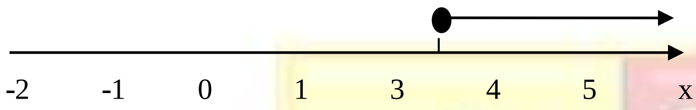
>means as greater than
≥ means as greater than or equal to
< means as less than
≤ means as less than or equal to

Example: solve 4x +6<2
Solution:
4x +6<2
4x+6-6<2-6
4x<-4
4x/4<-4/4
x<-1

B. Number line.
Example: solve the following inequalities and represent their solution on the line graph
1/5(x+4) ≥ 1/3 (x+1)
Solution:
L.C.M = 3by 5 =15
15×1/5 (x+4) ≥1/3 × 15 (x-1)
3(x+4) ≥ 5 (x+1)
3x +12 ≥ 5x+5
3x-5x ≥ 5-12
-2x ≥-7
Multiply through by -1
2x ≥7
2x≥7

$x \geq \frac{7}{2}$

$\therefore x \geq 3\frac{1}{2}$



C. Linear Inequalities in two variables.

Example: Determine the solution set for the inequality $3x-8y \geq 12$

Solution:

Solve for y,

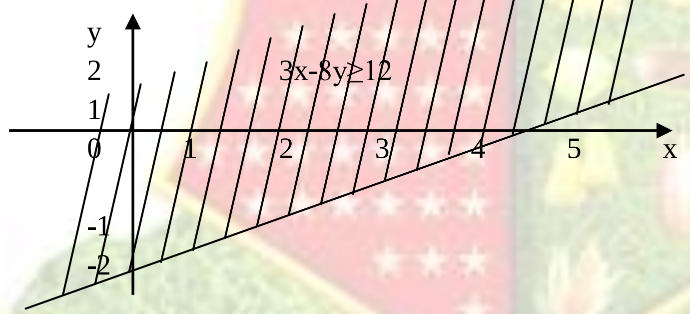
$-8y \geq -3x+ 12$

$Y = \geq 3/8x - 3/2$

$y \geq 3/8x - 3/2$

when $x = 0$, $y = -3/2$

when $y=0$, $x = 4$



D. Solutions of simultaneous linear inequalities

Example: solve graphically the simultaneous inequalities. $4x+3y <12$, $y \geq 0$, $x>0$ for integral values of x and y

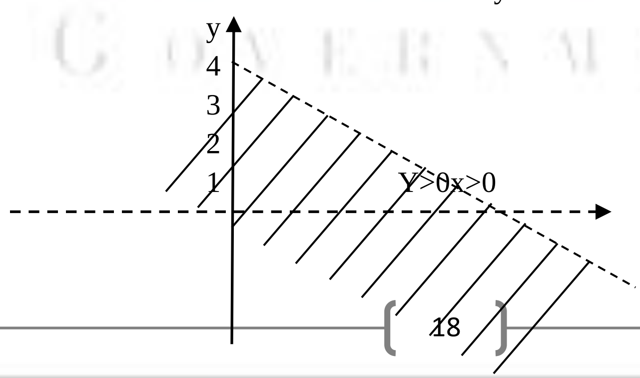
Solution: from the diagram $4x +3y = 12$

Broken y=0 solid

X= broken

$4x+3y=12$

When $y=0$	when $x=0$
$4x+0 =12$	$4x+3y=12$
$4x=12$	$0+3y=12$
$X=12/4$	$3y=12$
$X=3$	$y=12/3=4$



-2 -1 0 1 2 3 x
-1
-2

ALGEBRAIC FRACTIONS

Simplification of algebraic fractions

Example 1: simplify $\frac{x^2+2x}{x^2-4}$

Solution:

$$\frac{x^2+2x}{x^2-4} = \frac{x\,(x+2)}{(x+2)(x-2)}$$

$$= \frac{x}{x-2}$$

Example2 : simplify $\frac{2}{3x+3} + \frac{1}{2x+4}$

Solution:

$$\frac{2}{3x+3} + \frac{1}{2x+4}\quad ; L.C.M = (3x+3)(2x+4).$$

$$= \frac{2(2x+4)+3x+3}{(3x+3)(2x+4)}$$

Example3 : simplify xy/3x-6y × 4x-8y/x^2y

Solution:

Xy/3(x-2y) × 4(x-2y)/x(xy)

= 4/3x

EQUATION INVOLVING ALGE BRAIC FRACTION

Example: solve the equation $\frac{t}{4} + \frac{2t}{3} = \frac{55}{12}$

Solution:

$$\frac{t}{4} + \frac{2t}{2} = \frac{55}{12}$$

To clear the fraction, multiply through by 12

$12(\frac{t}{4}) + 12\left(\frac{2t}{3}\right) = 12\left(\frac{55}{12}\right)$

3t + 4(2t) = 55

3t + 8t = 55

11t = 55

Divide both sides by 11, we have

t = $\frac{55}{11}$

∴ t = 5

Example: if $\frac{a}{b} = \frac{3}{4}$ $, evaluate\ 2a - \frac{b}{2a} + b$

Solution:

$2a-b/2a+b$ if $a/b = 3/4$

Divide the numerator and denominator of

$2a-b/2a+b$ by b

$2a-b/2a+b = 2(a/b)-1/2(a/b) +1$

$2(3/2)-1/2(3/4)+1 = 3/2 -1/3/2+1 = 3-2/2/3+2/2$

$3-2/2 \times 2/3+2 = 1/5$

Simultaneous linear equation involving Fractions

Example :Solve the following simultaneous equation

(a) $x/2+y/4= 1,\ x/4-y/4+1=0$

Solution

$x/2+y/4 =1$ ………………………..(i)

$x/4 - y/4 +1 = 0$ …………………(ii)

multiply by 4 in equation 1and 2

$4(x/2+y/4=1)$

$4(x/2)+4 (y/4) =4$

$2x+y=4$ ………………………………..(iii)

$4(x/4-y/4+1=0)$

$4(x/4)-4(y/4) + 4=0$

$x-y +4 = 0$ ………………………..(iv)

from equation 3

$2x+y =4$

$Y=4-2x$……………………….(v)

Substitute equation (v) into equation (iv), we have

$x-(4-2x) +4 =0$

$x-4+2x+4=0$

$3x=0$

$X=0$

Substitute for x in equation (iii)

$2(0) +y =4$

$Y=4.$

$\therefore$ $x = 0$ and $y = 4$

Undefined value of a fraction

Example: find the value of x for which the fraction $x+1/x-2$ is undefined.

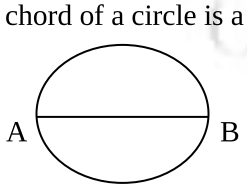
Solution: $x+1/x-2$ is undefined when $x-2=0$

If $x-2 =0$ then $x=2$

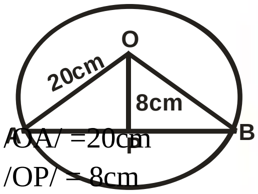
The fraction is undefined when $x=2$

CHORD PROPERTY

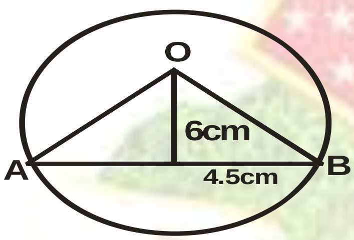
A chord of a circle is a line segment whose two end point lie on the circle.



Angles subtended by chord at the centre and at the circumference of a circle.
Example: calculate the length of a chord of a circle of radius 20cm, if the chord is 8cm from the centre of the circle.
Solution:

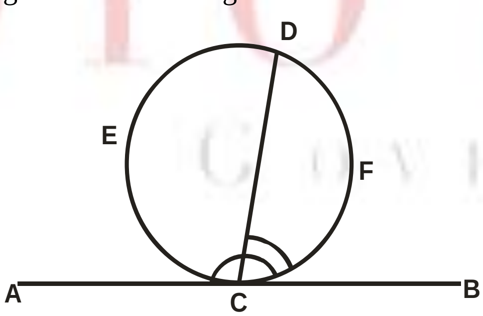


/OA/ = 20cm
/OP/ = 8cm
ΛAPO
$/AP/^2 = /OA/^2 - /OP/^2$
$20^2 - 8^2$
400-64
=336
$/AP/ = \sqrt{336} = 18.3$ cm
Length of a chord AB = 2×18.3 = 36.6cm
Perpendicular Bisector of chords
Example: find the radius of the circle. If a chord 9cm long is 6cm from the centre of the circle
Solution:



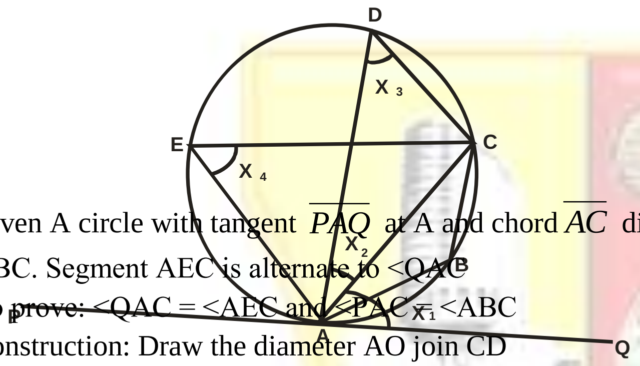
Since OC bisect AB
/AB/ = /CB/ = 4.5cm
$/AD/^2 = /OC/^2 + /AC/^2$
$6^2 + 4.5^2$
36+ 20.25cm
56.25 cm
/AO/ = 7.5cm

Angle in Alternate Segments



Theorem: the alternate segment.

An angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.



Given A circle with tangent $\overline{PAQ}$ at A and chord $\overline{AC}$ dividing the circle into two segments AEC and ABC. Segment AEC is alternate to <QAC

To prove: <QAC = <AEC and <PAC = <ABC

Construction: Draw the diameter AO join CD

Proof from the lettering:

$X_1 + x_2 = 90^0$ ……………………………….(i) (DA AQ)

Also $<ACD = 90^0$ (angle in a semicircle in ΛACD)

$X_2 + x_3 <ACD = 180^0$ (sum of angles in a triangle)

$X_2 + x_3 + 90^0 = 180^0$

$X_2 + x_3 = 90^0$ ……………………………….(ii)

Subtracting x_2 from equation 1and 2

$x_1 = x_3 = x_4$

: <QAC = < AEC

Also, B is a point in the minor segment

< PAC + <CAQ = 180^0(as on a straight line)

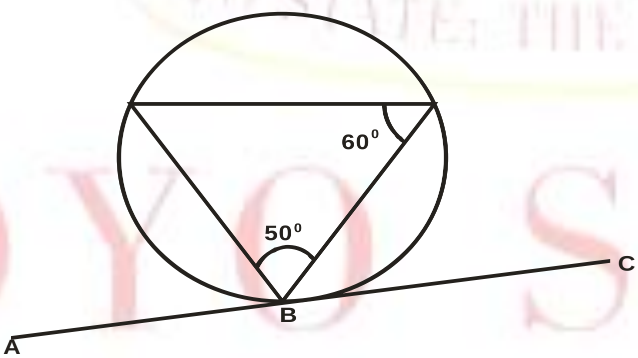
<PAC + $x_1 = 180^0$

<PAC = $180^0 - x_1$

= $180^0 - x_4$(proved $x_1 = x_4$)

= <ABC (Opposite angles of a cyclic quadrilateral)

Example: In the diagram below ABC is a tangent to circle BDE calculate <DBC



Solution:

In Δ BDE

ΔBED =180^0 $(60+50)^0$(sum of <S in a Δ)

$180^0 - 110^0$

= 70^0 (Angles in alternative segments)

<DBE = 70^0(Alternate <s in segment)

CIRCLE THEOREM

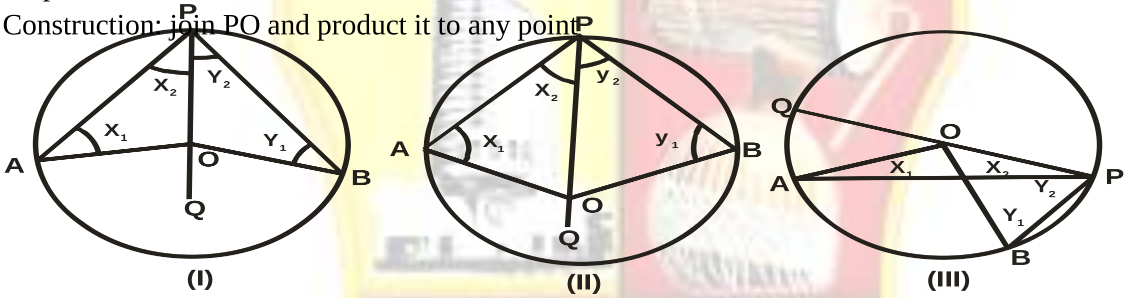
A theorem is a statement that has been proved. It starts with certain assumptions that will help established the result and ends with a conclusion.

Theorem : The angle which an arc of a circle subtends at the centreof the circle is twice the angle which it subtends at any point on the remaining part of the circumference.

From a given circle APB with centre O

To prove AOB = 2×APB

Construction: join PO and product it to any point



Q

Proof: /QA/= /OB/ (radii)

$X_1 = x_2$ (base angles of isosceles∆)

AOQ = $x_1 + x_2$ (exterior angle of ∆ AOB)

AOQ = $2x_2$ ($x_1=x_2$)

Similary BOQ = $2y_2$

In diagram (i) AOB

(ii) reflex AOB

$= 2x_2 + 2y_2$

$=2 (x_2+ y_2)$

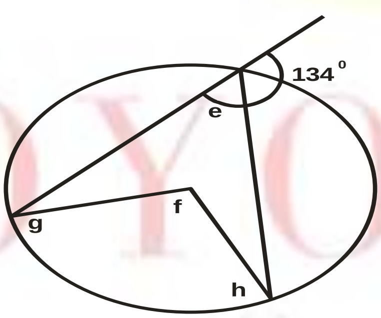
2× APB

AOB = BOQ – AOQ = $2x_2 – 2y_2$

$2 (x_2-y_2)$

2× APB

Example: Find the lettered angles in the diagram below



Solution: e+ $134^0=180^0$ (<on a straight line)

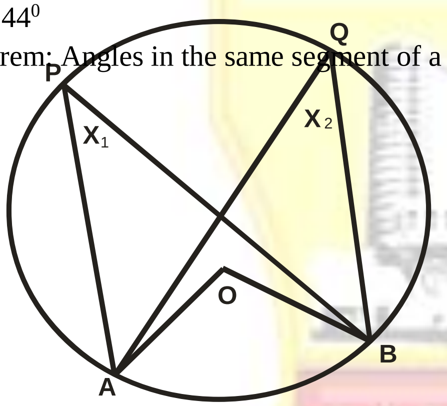
e=180^0- 134 = 46^0

f=2×e (< at the centre = 2× < at the O^{cc})

f=2×46^0=92^0

g=h (base <s of issoc)

$f+g+h = 180^0$(sum of <s in a triangle)
$92^0 + g + g = 180^0$
$2g = 180 – 92$
$2g = 88^0$
$g = 88/2 = 44^0$
$g = h = 44^0$
Theorem: Angles in the same segment of a circle and equal.

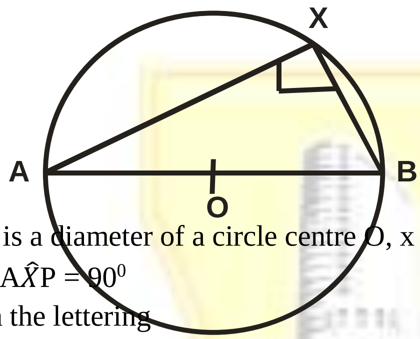


Given: Pand Q are any point on the major arc of a circle APQB
To prove: $A\hat{P}B = A\hat{Q}B$
Construction: Join A and B to O, the centre of the circle
Proof: with the lettering in the diagram
$A\hat{O}B = 2x$ (<at centre twice the < at O^{cc})
$A\hat{O}B = 2x_2$ (same reason)
$X_1 = x_2$
$A\hat{P}B = A\hat{Q}B$
Since P and Q are any points on the major arc all angles in the major segment are equal to each other.
Example: in the figure below, $\overline{PQ}$ is a diameter of circle PMQN, centre O, if $P\hat{Q}M$ is 63^0 find $Q\hat{N}M$

Solution:
In Δ QPM or angle QPM
$P\hat{M}Q = 90^0$ (<s in a semi circle)
QPM = $180^0 - 90 - 63^0$ (sum of < in triangle)

QNM = 27^0($Q\hat{P}M = 27^0$, in same segment as $Q\hat{P}M$

Theorem: The angles in a semi circle is a right angle.



Given: $\overline{AB}$ is a diameter of a circle centre O, x is any point on the circumference of the circle

To prove: $A\hat{X}P = 90^0$

Proof: with the lettering

$A\hat{O}B$ = 2 AXB (angle at centre twice the angle at O^{ce})

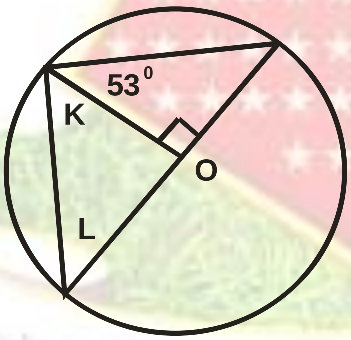
$A\hat{O}B = 180^0$(straight line)

$2A\hat{X}B = 180^0$

$A\hat{X}B$= 180/2

= 90^0

Example: Find the lettered angles in the following diagram.



Solution:

53^0+k = 90^0(Angle in a semi circle)

K=90^0- 53^0

= 37^0

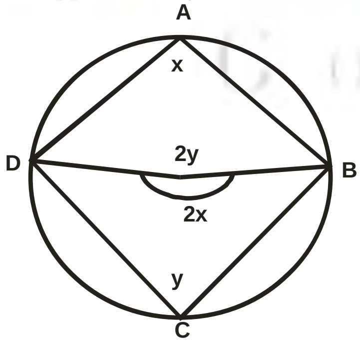
K+ L + 90^0=180^0 (sum of <s in a triangle)

37^0+ L = 90^0= 180^0

L=180-127

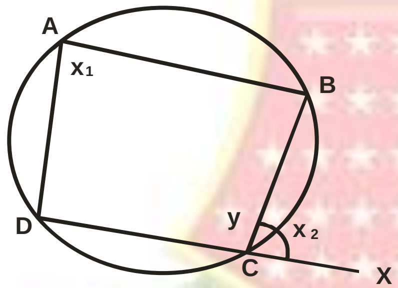
L=53^0

Theorem: The opposite angles of a cyclic quadrilateral are supplementary or angles in opposite segment are supplementary.

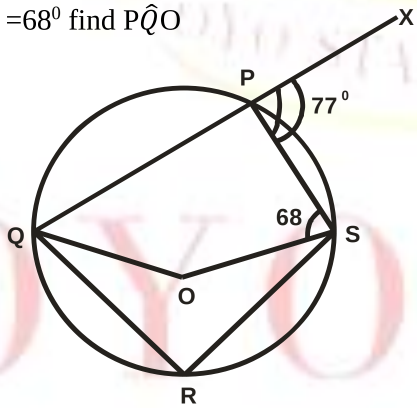


Given: a cyclic quadrilateral ABCD
To prove: BAD + BCD = 180^0
Construction: Join B and D to the centre O of circle ABCD
Proof: with the lettering of the figure above
BOD = 2y (angle at centre twice < at O^{ce})
Reflex $B\hat{O}D$ = 2x (same reason)
$2x + 2y = 360^0$(angles at a point)
$X + y = 180^0$

$B\hat{A}D + B\hat{C}D = 180^0$
It follows from the above theorem that " The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle"



From the figure above
$X_1 + y = 180$ (opp <s of cyclic quad)
$X_1 = x_2$ $(180^0 – y)$
$B\hat{C}X = B\hat{A}D$
Example: in the figure, PQRS are point on a circle centre O. OP is produced to X if XPS = 77^0 and PSO $=68^0$ find $P\hat{Q}O$



Solution:

$Q\hat{R}S = 77^0$(ext angle of cyclic Quad.)
: $Q\hat{O}S = 2\times 77^0$ (< at a centre twice < at O^{ce})\
$= 154^0$
$Q\hat{P}S = 180^0 – 77^0$(straight angle)

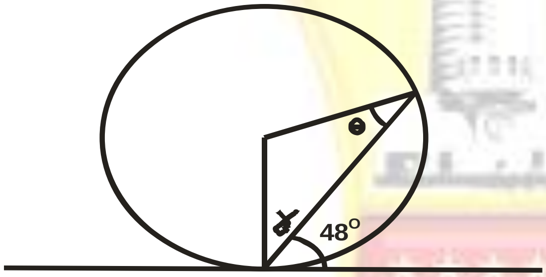
$= 103^0$

In quad. PQOS,

$P\hat{Q}O = 360^0 - 154^0 - 103 - 68^0$ (angle sum of quad.)

$P\hat{Q}O = 35^0$

Example: calculate the size of angle α and Q in each part of the following O is the centre of each circle.



Solution:

$\alpha + 48^0 = 90^0$ (tangent to circle)

$\alpha = 90^0 - 48^0$

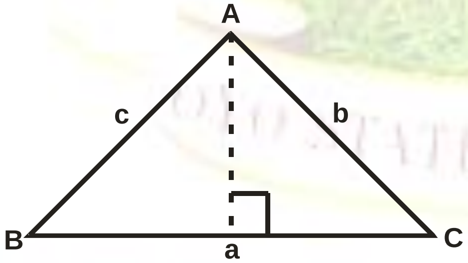
$\alpha = 42^0$

$\alpha = Q = 42^0$(base angles of isosceles. Triangle)

TRIGONOMETRIC.

Sine rule: in any triangle, the angles are denoted by the upper case letters such as A,B and C while the side opposite these angles are denoted by the letters a,b and c

Given: Any acute angle ΔABC as shown



Required to prove

a/sineA = b/sinB=c/sinC

construction: draw a perpendicular line from A to BC and call it h

Proof: using the triangle above figure

SinB = h/c and SinC =h/b

h=C SinB and h =bSinC

h=C SinB =b SinC

CSinB = b SinC

Dividing both side by SinB and SinC

c/SinC = b/SinB

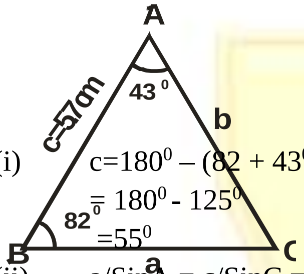
Similarly, by drawing a perpendicular line from B to AC,

a/SinA = c/SinC

a/SinA= b/SinB= c/SinC

Example:

In a triangle ABC, $A=43^0$, $B=82^0$ and C= 5.7cm .find (i) c (ii) a (iii) b



(i) $c=180^0 - (82 + 43^0)$ sum of angles in a triangle)

$= 180^0 - 125^0$

$=55^0$

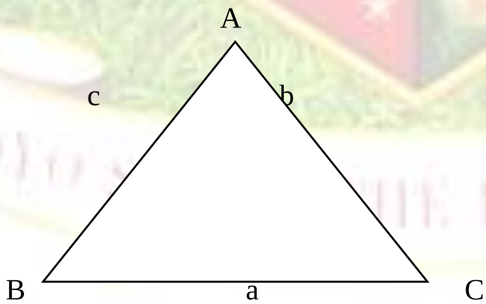
(ii) a/SinA = c/SinC = $a/\text{Sin}43^0 = 5.7/\text{Sin}55^0$

$a=5.7\times \text{Sin } 43^0/\text{Sin } 55^0$

No	Log
5.7	0.7559
$\text{Sin}43^0$	$\bar{1}.8338$ +
	0.5897
$\text{Sin } 55^0$	$\bar{1}.9134$ –
4745	0.6763
4.745Ans	

Cosine rule

In any given triangle with the usual notation A, B and C as shown below



$a^2=b^2 + c^2 - 2bc \text{ Cos } A$

$b^2= a^2+c^2 - 2ac \text{ Cos } B$

$c^2= a^2 + b^2 - 2ab \text{ Cos } c$

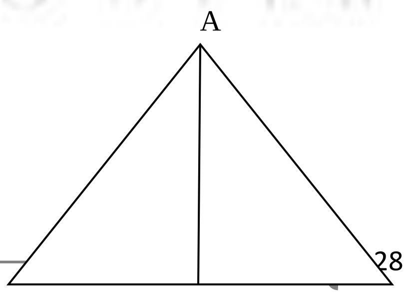
To find the sides of the triangle.

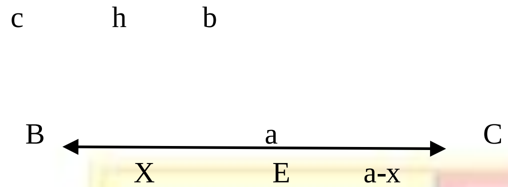
$\text{Cos } A = b^2 + c^2 - a/ 2bc$

$\text{Cos } B= a^2+ c^2- b^2/2ac$

$\text{Cos } C= a^2 + b^2 - c^2/2ab$

Given: an acute angled ABC





Required to prove: $a^2=b^2 + c^2 - 2\,bc\text{Cos }A$

Construction: Draw $\overline{AE}\ /\overline{BC}$ denote

/AE/ by h, /BE/ By X and /CE/ by a-x

Proof: $b^2 = (a-x)^2+h^2$ (pythagoras theorem)

$= a^2- 2ax + x^2 + h^2$

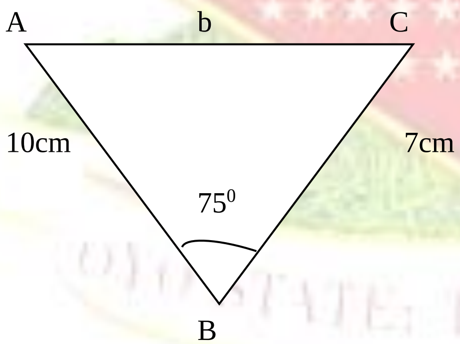
$=a^2 - 2ax + c^2$(since in triangle ACE, $C^2 =x^2+h^2$)

$a^2 = b^2 + c^2 - 2bc \text{ Cos } A$ (in triangle ABE, CosB= x/c)

$b^2=a^2+c^2 - 2ac \text{ Cos } B$

$c^2=a^2 + b^2- 2ab \text{ Cos } C.$

Example: in a triangle ABC, a = 7cm c=10cm , $B=75^0$ find (i) b (ii) A (iii) C



$b^2 = a^2+ c^2 - 2ac \text{ Cos } B$

$b^2= 7^2+ 10^2 - 2\times 7\times 10 \text{ Cos } 75^0$

$b^2 = 49+100-140^0\text{Cos } 75^0$

$b^2 = 149-140 \times 0.2588$

$b^2 = 149-36.232$

$b^2 = 112.768$

$b=\sqrt{112.768} = 10.6192$

$\therefore b= 10.62\text{cm}$

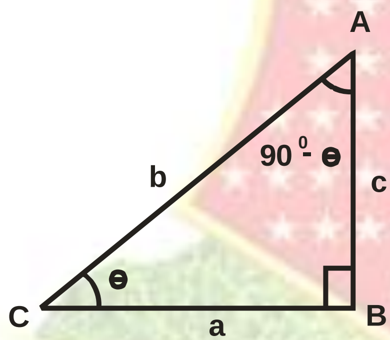
(iii) SinA/a = SinB/b
SinA/7cm = Sin 75^0/10.6195
=7×0.9659/10.6192
$=\frac{6.7615}{10.6192}=$ 0.6367
A= $\sin^{-1}$ (0.6367) = 39.5462^0
A= 39.55^0 (2dp)

(iv) C= $180^0 - (75^0 + 39.5462^0)$ (sum of angles in a triangle)
180^0-114.5462
=65.4538
: C = 65.45^0 (2d.p)

TRIGONOMETRIC RATIO

Complementary Angles

Two angles are said to be complementary when they add up to 90^0



Sin θ = $c/_b$ $\rightarrow Cos\ (90^0 - \theta)$

Tan θ = $c/_a$ $\rightarrow Cot\ (90^0 - \theta)$

Cot θ = $a/_c$ $\rightarrow tan\ (90^0 - \theta)$

Example : Solve the following equation

(a) Sin θ = Cos 50^0
(b) Cos x = Sin (x + 55^0)

Solution

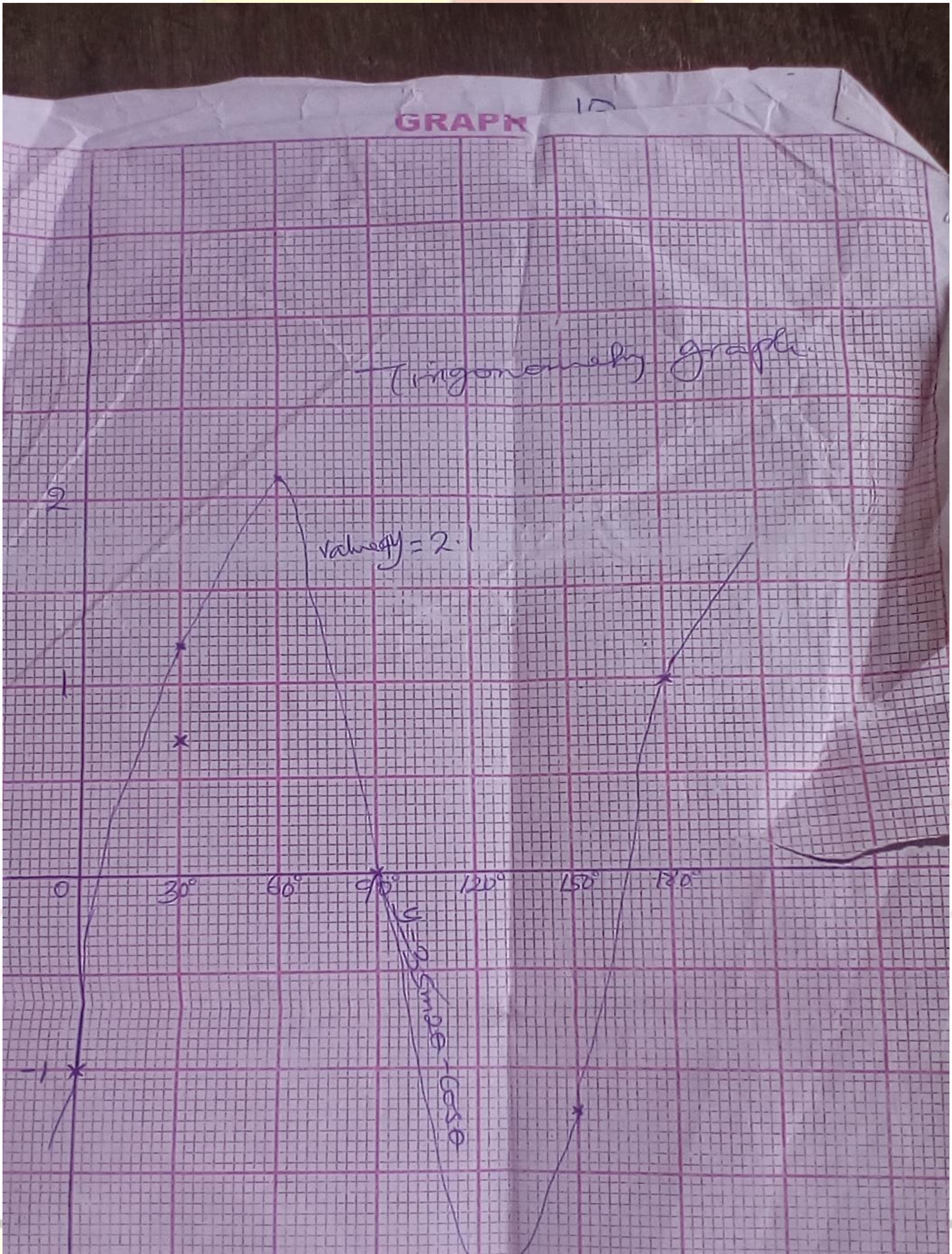
Sin θ = Cos 50^0

Sin θ = Cos (90^0 – θ)

Cos (90^0 – θ) = Cos 50^0

$90^0 - \theta = 50^0$

Θ = 40^0


GRAPH
Trigonometry graph
value of y = 2·1
2
1
-1
0
30°
60°
90°
120°
150°
180°
y = 2Sin2θ - Cosθ

Trigonometric Graph

(a) Copy and complete the following table of value of $y = 3\sin 2\theta - \cos\theta$

Θ	0^0	30^0	60^0	90^0	120^0	150^0	180^0
Y	-1.0			0			1.0

(b) Using a scale of 2cm to 30^0 on the Θ axis and 4cm to 1 unit on the y axis, draw the graph of $y = 3\sin 2\Theta - \cos\Theta$ for $0^0 \leq \Theta \leq 180^0$

(c) Use your graph to find the

(i) Solution fo the equation $3\sin 2\Theta - \cos\Theta = 0$, Correct to the nearest degree.

(ii) Maximum value of y, correct to 1 d.p

Solution

$Y = 3\sin 2\Theta - \cos\Theta$

Θ	0^0	30^0	60^0	90^0	120^0	150^0	180^0
$3\sin 2\Theta$	0	2.598	2.598	0	-2.598	-2.598	0
$-\cos\Theta$	-1	-0.866	-0.500	0	0.500	0.500	1
Y	-1.0	1.732	2.098	0	-2.098	-2.098	1.0

(b) $\cos x = \sin(x + 55^0)$

$\cos x = \sin(90^0 - x)$

$\sin(90^0 - x) = \sin(x + 55^0)$

$90^0 - x = x + 55^0$

$90^0 - 55^0 = x + x$

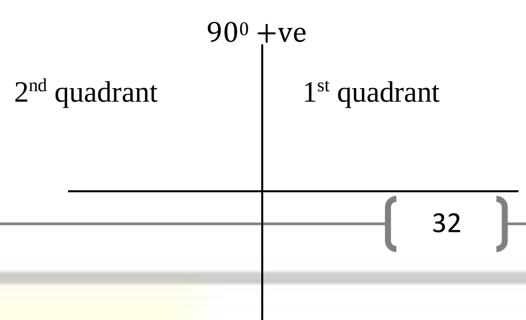
$35 = 2x$

$x = \frac{35}{2}$

$x = 17.5^0$

General Angles

Angles which range from Θ^0 to 360^0 could be divided into four main quadrants as shown below



$90^0 \leq \Theta \leq 180^0$ $0^0 \leq \Theta \leq 90^0$

180 –ve +ve 0^0, 360^0

3^{rd} quadrant 4^{th} quadrant

$180^0 \leq \Theta \leq 270^0$ $270^0 \leq \Theta \leq 360^0$

-ve 270^0

First quadrant ($0^0 \leq \theta \leq 90^0$)

$\text{Sin}\theta = \frac{O}{H} = \frac{+ve}{+ve} = +\text{ve}$

$\text{Cos}\theta = \frac{A}{H} = \frac{+ve}{+ve} = +\text{ve}$

$\tan\theta = \frac{O}{A} = \frac{+ve}{+ve} = +\text{ve}$

$\text{Sec}\theta = \frac{H}{A} = \frac{+ve}{+ve} = +\text{ve}$

Second quadrant ($90^0 \leq \theta \leq 180^0$)

$\text{Sin}(180^0 - \theta) = \frac{O}{H} = \frac{+ve}{+ve} = +\text{ve}$

$\cos(180^0 - \theta) = \frac{A}{H} = \frac{-ve}{+ve} = \text{-ve}$

$\tan(180^0 - \theta) = \frac{O}{A} = \frac{+ve}{-ve} = \text{-ve}$

$\sec(180^0 - \theta) = \frac{H}{A} = \frac{+ve}{-ve} = \text{-ve}$

$\text{cosec}(180^0 - \theta) = \frac{H}{O} = \frac{+ve}{+ve} = +\text{ve}$

Third quadrant ($180^0 \leq \theta \leq 270^0$)

$\text{Sin}(\theta - 180^0) = \frac{-ve}{+ve} = \text{-ve}$

$\text{Cos}(\theta - 180^0) = \frac{+ve}{+ve} = +\text{ve}$

$\tan(\theta - 180^0) = \frac{-ve}{-ve} = +\text{ve}$

$\cot(\theta - 180^0) = \frac{-ve}{-ve} = +\text{ve}$

$\sec(\theta - 180^0) = \frac{+ve}{-ve} = \text{-ve}$

Fourth quadrant ($270^0 \leq \theta \leq 360^0$)

$\text{Sin}(360^0 - \theta) = \frac{-ve}{+ve} = \text{-ve}$

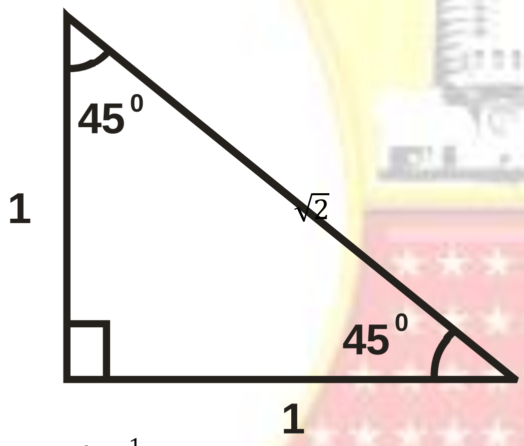
$\text{Cos}(360^0 - \theta) = \frac{+ve}{+ve} = +\text{ve}$

$\tan(360^0 - \theta) = \frac{-ve}{+ve} = \text{-ve}$

$\text{Cot}(360^0 - \theta) = \frac{-ve}{+ve} = \text{-ve}$

$\text{Sec}(360^0 - \theta) = \frac{+ve}{+ve} = +\text{ve}$

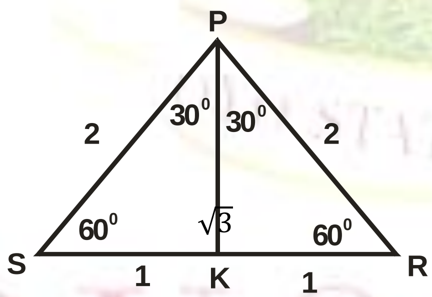
Trigonometric Ratios of special Angles



Sin $45^0 = \frac{1}{\sqrt{2}}$

Cos $45^0 = \frac{1}{\sqrt{2}}$

tan $45^0 = \frac{1}{1} = 1$



Sin $60^0 = \frac{\sqrt{3}}{2}$

Cos $60^0 = \frac{1}{2}$

tan $60^0 = \frac{\sqrt{3}}{1} = \sqrt{3}$

cot $60^0 = \frac{1}{\sqrt{3}}$

Sin $30^0 = \frac{1}{2}$

Cos $30^0 = \frac{\sqrt{3}}{2}$

tan $30^0 = \frac{1}{\sqrt{3}}$

Cot $30^0 = \frac{\sqrt{3}}{1} = \sqrt{3}$

Example: Evaluate $\frac{Cos3\theta - 2Cos4\theta}{Sin3\theta + 2sin4\theta}$

Where θ = 150^0

$= \frac{Cos\,450 - 2Cos\,600}{Sin\,450 + 2Sin600}$

$= \frac{0 - 2(-^1/_2)}{1 - \frac{2\sqrt{3}}{2}}$

$= \frac{1}{1 - \sqrt{3}}$

$= \frac{1}{1 - \sqrt{3}} \; X \; \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$

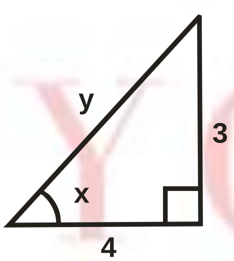
$= \frac{1 + \sqrt{3}}{1 - 3}$

$= \frac{1 + \sqrt{3}}{-2}$

∴ = - $\frac{1 + \sqrt{3}}{2}$

Example 2: If x is an acute angle and tanx= $^3/_4$, evaluate, $\frac{Cosx - Sinx}{Cosx + Sin\,x}$

Solution



Use Pythagoras theorem

$y^2 = 4^2 + 3^2$

$y^2 = 16 + 9$

$y^2 = 25$

y = $\sqrt{25}$

y = 5

cos x = $^4/_5$, Sin x = $^3/_5$

$\therefore \frac{Cosx - Sinx}{Cosx + Sinx}$

$= \frac{^4/_5 - ^3/_5}{^4/_5 + ^3/_5}$

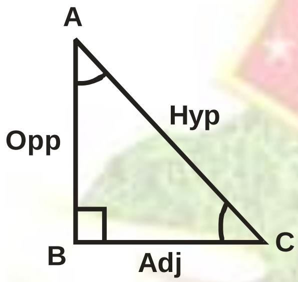
$= \left(\frac{4-3}{5}\right) \div \left(\frac{4+3}{5}\right)$

$= ^1/_5 \div ^7/_5$

$= ^1/_7$

BEARING AND DISTANCE

Trigonometric Ratios



SOHCAHTOA

SOH ≡Sinɵ = Opp/Hyp = AB/AC

CAH ≡Cosɵ = Adj/Hyp = BC/AC

TOA ≡Tanɵ = Opp/Adj = AB/BC

Reciprocal of trigonometric

1/Sinɵ= 1/opp/Hyp = cosecant ɵ = Cosec θ

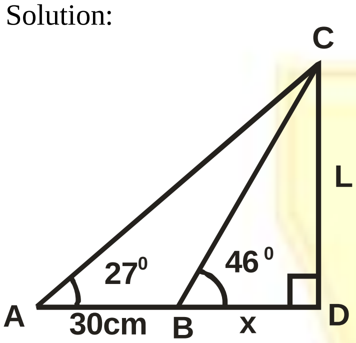
1/Cosɵ= 1/Adj/Hyp = secant ɵ = Sec θ

1/tanɵ = 1/Opp/Adj = cotagentɵ = Cot θ

Angle of Elevation and Depression

Example: A building has a fence round it. Two points A and B 30m apart, are chosen outside the fence on a straight line with the foot of the building on a horizontal plane. The angles of elevation of the top of the building from A and B are 27^0 and 46^0 respectively. Find (i) the length of the building (ii)The distance of the building from point B

Solution:



/CD/ represent the building from triangle BCD

$CD/BD = \tan 46^0$

$CD= BD \tan 46^0$--------------------(i)

From triangle ACO

$CD/AD = \tan 27^0$
$CD = Ad \tan 27^0$ ……………………………………..(ii)

From (i) & (ii)

$BD \tan 46^0= AD \tan 27^0$

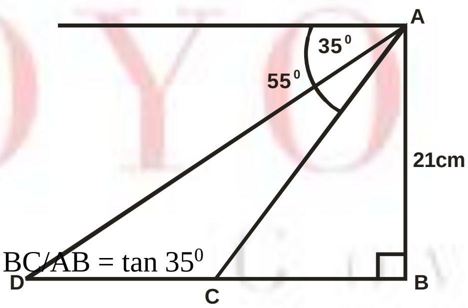
$(30+BD) \tan 27^0$

$/BD/ = 30 \tan 27^0/\tan 46^0 \tan 27^0 = 29.1m$

From (1) $CD = 29.1 \tan 46$

$= 30.1m$

Example2: Two boat on the sea are in line with on observe standing on top of a cliff. Their angles of depression are 35^0 and 55^0 if the height of the observer is 21m above sea level. Find the distance between the two boats



$BC/AB = \tan 35^0$

$BC = 21 \tan 35 = 14.7m$

From AB D, angle $DAB = 55^0$

$BD/AB = \tan 55^0$

$BD = 21 \tan 55^0 = 30m$

$/CD/ = /BD/ - /BC/$

$= (30-14.7)\ m = 15.3\ m$

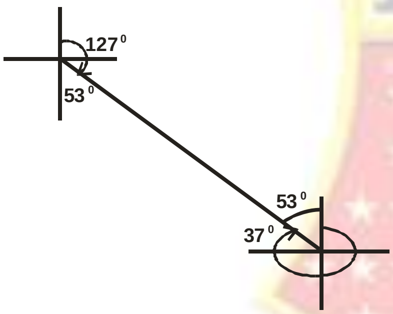
The distance between the two boats is 15.3m

Bearing and Distance

Bearing is the most convenient method of locating area of town or city.

Example: If the bearing of P from Q is 127^0 find the bearing of Q from P

Solution



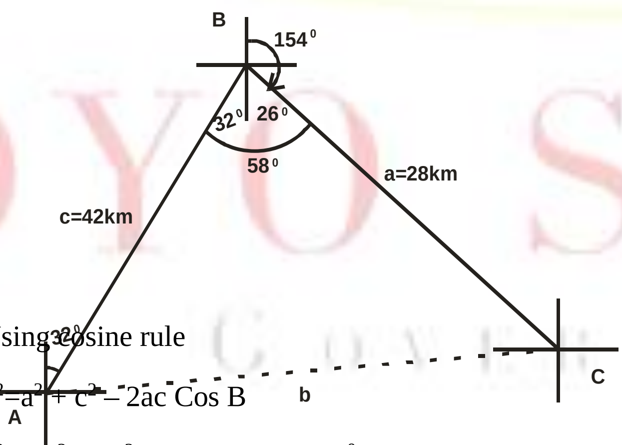
$Q = 180 – 127$

$=53^0$

Bearing of Q from P = $360 - 53 = 307^0$ OR $= 270^0 + 37^0 = 307^0$

Example 2: A man prospecting for oil leaves his base camps and drives 42 km on a bearing of 032. He then drives 28km on a bearing of 154^0. How far is he then from his base camp and what is his bearing from it?

Solution



Using cosine rule

$b^2 = a^2 + c^2 – 2ac \text{ Cos } B$

$b^2 = 28^2 + 42^2 - 2×28×42 \text{ Cos } 58^0$

$b^2 = 784 + 1764 - 2352 \times 0.5299$

$= 2548 - 1246.3701$

$b^2 = 1301.6299$

$b = \sqrt{1301.6299}$

$= 36.0781$

$\therefore\ b \cong 36.1$ km (to 1d.p)

The bearing of C from A is $32 + \theta$

Using sine rule

$$\frac{a}{SinA} = \frac{b}{SinB}$$

$$\frac{28}{Sin\theta} = \frac{36.1}{Sin58^0}$$

$36.1 Sin\theta = 28 Sin58^0$

$Sin\theta = \frac{28Sin58^0}{36.1} = \frac{23.7453}{36.1}$

$Sin\theta = 0.6578$

$\Theta = Sin^{-1}(0.6578)$

$\therefore \theta = 41.1297^0 \cong 41.1^0$ (to 1 d.p)

The bearing of C from A = 32+41.1

= $73.1^{0.}$

A. MEASURES OF CENTAL TENDENCY

Measures of central tendency also known as measures of location are measure that gives us those data that are found among sets of data.

1. Mean: is the most common measure of central tendency. It is the sum of all the items in a set of data divided by the number of items involved.

$$\bar{x} = \frac{x_1 + x_2 + ---x_n}{n}$$

In general, $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$ *"For ungrouped data"*

$\sum_{2}^{n} i \big/ n$

Example 1: Find the mean of 10 students whose mathematics scores in a test are: 15,13,16,12,26,14,22,23,23,27

$$\bar{X} = \frac{\sum x}{n}$$

$$= \frac{15+13+16+12+26+14+22+23+23+27}{10}$$

$$\frac{191}{10} = 19.1$$

Example 2: The mark distribution of an English language test in which the mean mark is 3. Find the value of y

Marks	1	2	3	4	5
Frequency(f)	y	3	y+3	3	4-y

Solution:

$$\bar{x} = \frac{y \times 1 + 3 \times 2 + (y+3)3 + 4 \times 3 + (4-y)5}{y+3+y+3+3+4-y}$$

$$\frac{y+6+3y+9+12+20-5y}{y+3+y+3+3+4-y} = 3$$

$$\frac{47-y}{y+13} = 3$$

$$\therefore 47-y = 3(y+13)$$

$$47-y = 3y+39$$

$$-y-3y = 39-47$$

$$-4y = -8$$

$$4y = 8$$

$$y = 8/4$$

$$y = 2$$

Hint: $\bar{x} = \frac{\sum fx}{\sum f}$ "for grouped data"

Attentively,

Mark (x)	f	fx
1	y	y
2	3	6
3	y+3	3y+9

4	3	12
5	4-y	20-5y
$\sum$	13+y	47 – y

If the mean = 3 then $\bar{x}$ = 31.0

$$\frac{\sum fx}{\sum f} = 3$$

From the table above,

$\sum$f = 13 + y and $\sum$fx = 47 – y

Now,

$$\frac{47 - y}{13 + y} = \frac{3}{1}$$

47-y = 3(13 + y)

47 – y = 39 + 3y

Collect the like terms

-y-3y = 39 – 47

-4y = -8

Divide both side by -4

Y = $\frac{-8}{-4}$

$\therefore$ y = 2

2. Median: when the number of items odd the item that lies in the middle after the set of item (data) have been arranged in order of magnitude either in ascending or descending order is called median.

$$\left.\begin{array}{l} (N+1/2) + h \text{ (ie) N odd} \\ (N/2) \text{ th} + (N+1/2) \text{ th (ie) N=even)} \end{array}\right\} \text{For “ungrouped data”}$$

Example 1: Find the median of the following numbers 3,9,7,5,2,13,10

Solution

Rearrange

2,3,5,7,9,10,13

Formula = (N+1/2) th

=(7+1/2) th

(8/2)th

=4th Items from the rearranged data

∴The median item is 7

Example 2: Represent the scores of 40 student in a mathematic test. Find the median score.

Score(x)	1	2	3	4	5
Frequency(f)	15	12	8	3	2

Solution:

Score	f	cf	position
1	15	15	1-15th
2	12	27	16th-27th
3	8	35	28th- 35th
4	3	38	36th-38th
5	2	40	39th-40th
	∑f=40		

N=40 (even)

(N/2)th + (N/2+1) th score

40/2th + (40/2+1)th/2

20+21/2

By observing through the positional column, we see that both the 20th and 21st term is 2

The median is 2+2/2 = 4/2=2.

B. Measurement of Dispension

Disperse means to spread, scatter or vary measure of spread is a statistical measure that showns the extent to which numerical data cluster around a measure of location.

1. Mean deviation is the algebraic sum of the absolute deviation of each observation from the mean divided by the number of observed values.

Remarks:

(i) M.D $= \frac{\Sigma / x_i - \bar{x} /}{n}$ "for ungrouped data"

(ii) M.D $= \frac{\Sigma f / x_i - \bar{x} /}{\Sigma f}$ "*For grouped data*"

Example 1: Find the mean deviation of the set of number 4,8,10,6,2,6

Solution

$\bar{x}$= 4+8+10+6+2+6/6 = 36/6 = 6

∴$\bar{x}$= 6

Mean deviation: /4-6/+/8-6/+/10-6/+/6-6/+/2-6/+/6-6 /

$= \frac{2+2\ 4+0+4+0}{6} = \frac{12}{6} = 2$

∴ M.D = 2

Example 2: Find the mean deviation of the following distribution

X	3	4	5	6	7	8	9
F	1	1	3	1	1	2	1

Solution:

X	f	Fx	x-x̄	/x-x/	f/x-x/
3	1	3	-3	3	3
4	1	4	-2	2	2
5	3	15	-1	1	3
6	1	6	0	0	0
7	1	7	1	1	1
8	2	16	2	2	4
9	1	9	3	3	3
∑	10	60			16

Since the given data is a grouped data then

M.D = $\frac{\sum f/x_i - \bar{x}/}{\sum f}$

Where:

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{60}{10} = 6 \quad \therefore \ \bar{x} = 6$$

Now,

From the table above, we have

∑f/xi - $\bar{x}$/ = 16 and ∑f = 10

Hence,

M.D = $\frac{16}{10}$

$\therefore$ M.D = 1.6

VARIANCE

Variance is the mean of the squared deviation from the mean.

Formula for calculating variance is $S^2 = \frac{\sum d^2}{n}$ where $d = x - \bar{x}$

Or for a frequency distribution

$S^2 = \frac{\sum f d^2}{\sum f}$

Where $d = x_i - \bar{x}$

F = frequency

similarly, for a grouped frequency distribution

Similarly,

$S^2 = \frac{\sum f d^2}{\sum f} - (\frac{\sum f d}{\sum f})^2$

STANDARD DEVIATION

Standard deviation is the square root of the variance. i.e.

S.D = $\sqrt{S^2} = \sqrt{\frac{\sum/x_i - \bar{x}/^2}{n-1}}$ *"for ungrouped data"*

S.D = $\sqrt{S^2} = \sqrt{\frac{\sum f/x_i - \bar{x}/^2}{\sum f}} = \sqrt{\frac{\sum f d^2}{\sum f} - (\frac{\sum f d}{\sum f})^2}$ *"for grouped data"*

Example 1:

Calculate, correct to the nearest whole under the variance and standard deviation of the distribution.

Size of shoes	40	41	42	43	44	45
No of athletes	5	10	6	4	7	3

Solution

x	f	fx	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f(x_i - \bar{x})^2$
40	5	200	-2.2	4.84	24.2
41	10	410	-1.2	1.44	14.4

42	6	252	-0.2	0.04	0.24
43	4	172	0.8	0.64	2.56
44	7	308	1.8	3.24	22.68
45	3	135	2.8	7.84	23.52
$\sum$	35	1477			87.6

SD = S = $\sqrt{\frac{\sum f/x_i - \bar{x}/^2}{\sum f}}$

Where: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{1477}{35} = 42.2$

$\therefore \bar{x} = 42.2$

Now,

From the table above, we have

$\sum f = 35$ and

$\sum f(xi - \bar{x})^2 = 87.6$

Hence,

(i) Variance is:

$S^2 = \frac{\sum f/x_i - \bar{x}/^2}{\sum f}$

$S^2 = \frac{87.6}{35}$

$\therefore S^2 = 2.50$

(ii) Standard deviation:

$S = \sqrt{Variance}\ or \sqrt{\frac{\sum f/x_i - \bar{x}/^2}{\sum f}}$

$= \sqrt{2.50}$

$= 1.58$

$\therefore$ The Standard deviation is 1.58

The variance and standard deviation of grouped data using the assured mean.

Example: The following are the marks in percentage of forty candidates in an examination.

27, 17, 16, 38, 13, 12, 48, 37, 12, 15, 36, 46, 28, 64, 24, 27, 26, 36, 15, 52, 38, 39, 47, 36, 15, 57, 39, 29, 42, 28, 6 5, 45, 44, 39, 58, 23, 36, 28, 82.

(a) Draw up a frequency distribution table using equal intervals of 5-14, 15-24, --------,75-84
(b) Using an assume mean of 39.5% find (i) the mean and (ii) the standard deviation of the distribution

Solution

Class intervals	f	x	d=x-A	fd	fd^2
5-15	5	9.5	-30	-150	4500
15- 24	7	19.5	-20	-140	2800
25- 34	7	29.5	-10	-70	700
35 – 44	12	39.5	0	0	0
45 – 54	5	49.5	10	50	500
55-64	3	59.5	20	60	1200
65 – 74	0	69.5	30	0	0
75 – 84	1	79.5	40	40	1600
	40			-210	11300

i) Mean:

$\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$

Where:

A = Assumed mean value = 39.5

d = X - A

$\bar{x} = 39.5 + \frac{(-210)}{40}$

$= 39.5 - \frac{210}{40}$

= 39.5 – 5.25

= 34.25%

∴ The mean of the distribution is 34.25%.

(iii) Standard deviation

$$= \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$$

$$= \sqrt{\frac{11300}{40} - \left(\frac{-210}{40}\right)^2}$$

$$= \sqrt{282.5 - (5.25)^2}$$

$$= \sqrt{282.5 - 27.5625}$$

$$= \sqrt{254.9375}$$

$= 15.967$

$\therefore$ S$\cong$ 15.97%

Measures of central tendency (grouped data)

Mean median and mode

Example: A set of data was given in a frequency table as following

Group	1-20	21-40	41-60	61-81	81-100	101- 120	
Frequency	9	13	12	11	13	27	

Determine the mean, median and mode of the distribution above.

Solution

Group	Freq (f)	mid point (x)	fx	cf	
1-20	9	10.5	94.5	9	
21- 40	13	30.5	396.5	22	
41-60	12	50.5	606.0	34	
61-80	11	70.5	775.5	45	→Median Row or Class
81-100	13	90.5	1176.5	58	
101-120	27	110.5	2983.5	85	
$\sum$	85		6032.5		

$\bar{x} = \frac{\sum fx}{\sum f} = \frac{6032.5}{85} = 70.9706$

$\therefore \bar{x} \cong 70.97$

Median = Lm + $\frac{\left(\frac{N}{2} - Cf_b\right)c}{fme}$

Where :

Lm = lower limit of median class

Efb = cumulative frequency before the median class

fme = Frequency with or corresponding to the median class

C = Class interval

and N = ∑f or total frequency.

Remark: To locate the median class we use $\left(\frac{N}{2}\right) th\ items.$

Now,

$\left(\frac{N}{2}\right) th = \left(\frac{85}{2}\right) th = 42.5 th\ item\ from\ the\ table, we\ have\ 61 - 80$

∴The median class is 61 – 80.

The class boundary or limt of the median class is 60.5 – 80.5

∴ The lower limit of the median class is 60.5 i.e. Lm = 60.5

Hence,

Median = Lm + $\frac{\left(\frac{N}{2} - Cf_b\right)c}{fme}$

Where:

Lm = 60.5, $\frac{N}{2}$ = 42.5, Cf_b = 34, C = 20 and Fme = 11

Median = 60.5 + $\frac{(42.5-34)\ X\ 20}{11}$

= 60.5 + $\frac{(8.5)(20)}{11}$

= 60.5 + $\frac{170}{11}$ = 60.5 + 15.4545

= 75.9545

∴ Median≅ 75.95 (to 2 d.p)

(ii) Mode = Lm + $\left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) C$

Where:

Lc = Lower class boundary f the modal class

Δ_1 = fm – fa

fm = frequency of the modal class

fa= Frequency above the modal class

$\Delta 2 = \text{fm} - \text{fb}$

Fb = Frequency below the modal class.

C = Class size

Solution:

Mode = Lc + $\left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) C$

To locate the modal class:

The class with the highest frequency is known as the modal class.

∴ The modal class = 101 – 120 because it has the highest frequency of 27.

Also,

The corresponding class boundary fo the modal class is 100.5 - 120.5

∴The lower class boundary of the modal class is 100.51 i.eLc = 100.5

Now,

Lc = 100.5, Δ_1 = fm – fa = 27 – 13 = 14 ∴ $\Delta_1 = 14$, $\Delta_1 = \text{fm} - \text{fb} = 27 - 0 = 27$ ∴ $\Delta_2 = 27$ and c = 20

Hence,

$$\text{Mode} = 100.5 + \left(\frac{14}{14 + 27}\right) X\ 20$$

$$= 100.5 + \left(\frac{14}{41}\right) X\ 20$$

$$= 100.5 + (0.3415)(20)$$

$$= 100.5 + 6.8293$$

$= 107.3293$

$\therefore$ Mode $\cong 107.33$ (to 2 d.p)

DATA PRESENTATION

1. Bar Chart and Histogram

The table below shows the mass in kg of 50bags in a particular village

Mass(kg)	41	42	43	44	45	46
Frequency	6	10	5	8	13	8

Use the table to draw bar chart and histogram.

Solution :

Mass (kg)	Frequency	class boundaries
41	6	40.5-41.5
42	10	41.5-42.5
43	5	42.5-43.5
44	8	43.5-44.5
45	13	44.5-45.5
46	8	45.5-46.5

DIAGRAMS/CHARTS

BAR CHART& HISTOGRAM& FREQUENCY POLYGON

2. PIE CHART:

Example:

The following table represents the scholarship awards made by some philanthropists over the years

YEAR	NO OF SCORLARSHIPS
2001	100
2002	215
2003	185
2004	125
2005	165
2006	201

Draw the pie chart to represent the scholarship given for the 6years

SOLUTION: Sum up all the item i.e 100+215+185+125+165+210= 1000

YEAR	NO OF SCORLARSHIPS	proportion	corresponding degree
2001	100	100/1000	$100/1000 \text{x} 360^{0}=^{0}\ 36^{0}$
2002	215	215/1000	$215/1000 \text{x} 360^{0}=77.4^{0}$
2003	185	185/1000	$185/1000 \text{x} 360^{0}=66.6^{0}$
2004	125	125/1000	$125/1000 \text{x} 360^{0}=45^{0}$
2005	165	165/1000	$165/1000 \text{x} 360^{0}=59.4^{0}$
2006	201	210/1000	$210/1000 \text{x} 360^{0}=72.6^{0}$

***** pie chart

CUMMMULATIVE FREQUENCY CURVE (Ogive)

This curve can be described as measure of portion .

Quartile:

The quartile divide the distribution into four equal parts. Those quartiles achieve this division they are : 1^{st} quartile denoted as Q_1(lower quartile), 2^{ND} quartile denoted as Q_2 (median), 3^{rd} quartile denoted as Q_3 (upper quartile).

If n is the total frequency then: lower quartile $Q_1 = ¼(n+1)$th value, median quartile $Q_2 = 1/2(n+1)$th value, upper quartile $Q_1 = 3/4(n+1)$th value.

Interquartile range = $Q_3 – Q_1$

Semi interquartile range = Quartile Deviation = $\frac{Q3 - Q1}{2}$ or ½ ($Q_3 - Q_1$)

range=highest value – lowest value

percentile : The percentile divides the distribution into hundred equal parts

Percentile = p/100(n+1)th value e.g.

P_{25} = 25^{th}= 25/100(n+1)th value

P_{50}= 50^{th} =50/100(n+1)th value

P_{75} = 75th = 75/100(n+1)th value

Example: The table below shows the marks of 20 students in physics test

MARKS	2-6	7-11	12-16	17-21	22-26
Frequency	2	5	7	4	2

a. Draw the cumulative frequency curve b. find the interquartile range c. find the pass mark if 60% of the students passed.

SOLUTION:

MARK	FREQUENCY	CF	CLASS BOUNDARY
2-6	2	2	1.5-6.5
7-11	5	7	6.5-11.5
12-16	7	14	11.5-16.5
17-21	4	18	16.5-21.5
22-26	2	20	21.5-26.5

*** the curve

b. Intercept range= Q3-Q1, Q3=3/4X20=15, Q1= 1/4X20=5, Q3-Q1= 15-5=10
c. 60% passed means 40% failed
d. 40% of the students scored lower marks= 40/100x20= 8.

PARTITION VALUES:

These are measures which divide a distribution into variation segments. They are :

Quantiles----------it divides a distribution into 4 equal parts

Quintiles----------it divides a distribution into 5 equal parts

Deciles ----------it divides a distribution into 10 equal parts

percentile----------it divides a distribution into 100 equal parts

Range--------------- Highest value – Lowest value

GROUP DATA

Remarks:

i. Median = Lm + $\frac{\left(\frac{N}{2} - cf_b\right)c}{fme}$

ii. First Quartile = Q_1 = Lm + $\frac{\left(\frac{N}{4} - cf_b\right)c}{fq_1}$

iii. Second Quartile = Q_2 = Median

iv. Third Quartile = Q_3 = Lm + $\frac{\left(\frac{N}{2} - cf_b\right)c}{fq_3}$

SIMPLE SERIES

Obtain the 1st and the 3rd quartiles of the distribution below:

X	f	cf	Class boundaries
0-4	2	(2)	-0.5 - 4.5
5-9	6	8-----------Q1	4.5 - 9.5
10-14	8	16	9.5 - 14.5
15-19	5	21 --------Q3	14.5 - 19.5
20-24	4	25	19.5 - 24.5

1ST QUARTILE IN GROUP DATA

1st Quartile = Q_1 = Lm + $\frac{\left(\frac{N}{4} - cf_b\right)c}{fq_1}$

To locate Q1 class, we use $\left(\frac{N}{4}\right) th \;\; items$

$$\left(\frac{N}{4}\right) th = \left(\frac{25}{4}\right) th = 6.25^{th} \; from \; the \; table,$$

$\therefore \; Q_1 \; class \; is \; 5 - 9$

Now,

Lm = 4.5, $\frac{N}{4} = 6.25,$

$Cf_b = 2$, $fq_1 = 6$ and $c = 5$

Hence,

$Q1 = 4.5 + \frac{(6.25-2)X5}{6}$

$= 4.5 + \frac{(4.25)(5)}{6}$

$= 4.5 + \frac{21.25}{6}$

$= 4.5 + 3.5417$

$= 8.0417$

$\therefore Q_1 \cong 8.042$ (to 3 d.p)

3rd quartile3N/4=3(25/4)=18.75

LQ3=14+15/2=14.5

CfbQ3= 16, Fq3=5,C=5

$Q3=14.5+(18.75-16/5)^5$

14.5+2.75=17.25

Quartile

Lower quartile= $lq1+ (n/5-cfbq1)^c/fq1$

Upper quartile= $lq4+ (4n/5-cfbq4)^c/fq4$

Similarly for Decile:

1st decile = $Ld1+(n/10-cfbd1)^c/fd1$

9th decile= $lq9+ (9n/10-cfbd9)^c/fd9$

Decile range=9th decile -1st decile

80th percentile =80% of the distribution

30th percentile =30% of the distribution

50th percentile =50% of the distribution

75th percentile =3rd quartile Q3 of the distribution

25th percentile =1st quartile Q1 of the distribution

80^{th} percentile= LM+ $(80n\text{-}cfbm/fm)^{c}$

Interpercentile range= pq0-p10.

Range P90 – P10 = Lm + $\left(\frac{80N - Cfbm}{fm}\right) c$

PROBABILITIY

Probability is the study of chances. These probabilities are found by studying the symmetry of problem.When probability is one, it means that the event must surely happen. The probability can be less than but cannot be more than one. When probability is zero, it means that the event can never happen. Probability lies between $0 \leq p \leq 1$.

Mutually exclusive events: two events Are Mutually exclusive if the occurrence of either of them takes place differently and both can not occur.The word *or *is very common in this case ofx and y are Mutually exclusive events their probabilities are expressed as P(xuy)=p (x)+p(y) – p(xny) = 0

Example1: What is the probability of having an odd no in a single toss of a fair die?

Soln : s= {1,2,3,4,5,6} A die has 6 faces

Odd no = {1,3,5}

Pr (odd)=n(0)/n(s)

=3/6=1/2

E. A man kept 6 blacks , 5 brown and 7 purple shirts in a drawer. What is the probability of his picking a purple shirt with eyes closed?

Solution : black=6, brown = 5, purple =7, total =18, so pr (purple) = 7/18.

ADDITION OF probability

It is probability that involved either or as the symmetry of the problem. Lets consider some examples

F. Find the probability that a no is chosen at random from the integers between 10 and 20 inclusive is either a prime or a multiple of 5.
Solution : the universal set U= [10,11,12,13,14…..20] and n (U)=11

The subset A of primes = [11,13,17,19], so n(A)=4, the subset B of multiple of 5 is= (10,15,20], so n (B) =3

The favourable subset C= AuB. Since any element in c will either be A OR B OR both and hence n (c)=n(A)+n(B). pr{either a prime or multiple of 5}= 4/11= 3/11= 4+3/11=7/11.

NB: n[AuB]=n[A]+n[B] only because AnB=ɵ as there is no no between 10 and 20 which is a prime and a multiple of 5.

G. Two dice are thrown in a single tossed. What are the probabilities that they will show : i. sun of 8 appear? ii. Sum of 5 appear? Iii. Sum of 7 or 8 appear? Iv. Not obtaining 9? V. at least 10?
Soln:

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

i. Sum of 8= [NB crosses line of 8]= n [sum of 8]=5, simple space = 6x6 =36
ii. Pr(sum of 8)=5/36
Ii. Sum of 5 (crosses the line of 50= [sum of 5 appear]=4, pr(sum of of 5 appear)= 4/36
iii. Sum of 7 appear= 6, pr (sum of 7 appear)= 6/36=1/6
Pr(sum of 7 or 8 appear), pr(7)+pr(8= 6/36+5/36=6=5/36=11/36)
iv. Pr(sum of 9 appear)=4/36, pr(not obtaining 9)=1-4/36= 36-4/36=32/36=8/9
v. Probability of at least the sum of two dices is 10 means any score from 10 and above=10 to 12….(sum at least 10)=6, pr (at least 10)= 6/36=1/6

Example 5:

The probability that two hunters P and Q hit their target are 2/3 and ¾ respectively.

a. The hunters aimat a target together
b. If the target is hit, what ids the probability that: i. only P hit it. Ii. Only one of them hit it . iii. Both hunters hit the target.

SOLN:

Pr(p hitting the target)=2/3

Pr9p missing the target)=1-2/3=1/3

Pr(q hitting the target)=3/4

Pr(q missing the target)= 1-3/4=1/4

a. Pr [only p hit it]=1/4x1/4=1/16
b. Pr [only p hit it)=2/3x1/4=1/12
Pr[only one of them hit it]= pr[PQ^1] or [QP^1]
P^1=MISSING TARGET
c. PR(only one of them hit it)= 2/3x1/4+3/4x1/3=2/12+3/12=5/12
Pr[both hit the target]=2/3x3/4=6/12=1/2