Lab 04 Notes

Fernando Miguez

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Lab 4: Leaf area index, light geometries and canopy light interception

In this lab we will first explore different approaches to modeling LAI. First we can model LAI simply as a function of GDD. For example, (Clifton-Brown et al. 2000) modeled Miscanthus LAI as a function of Thermal Units using 10°C as base temperature and a slope of 0.0102.

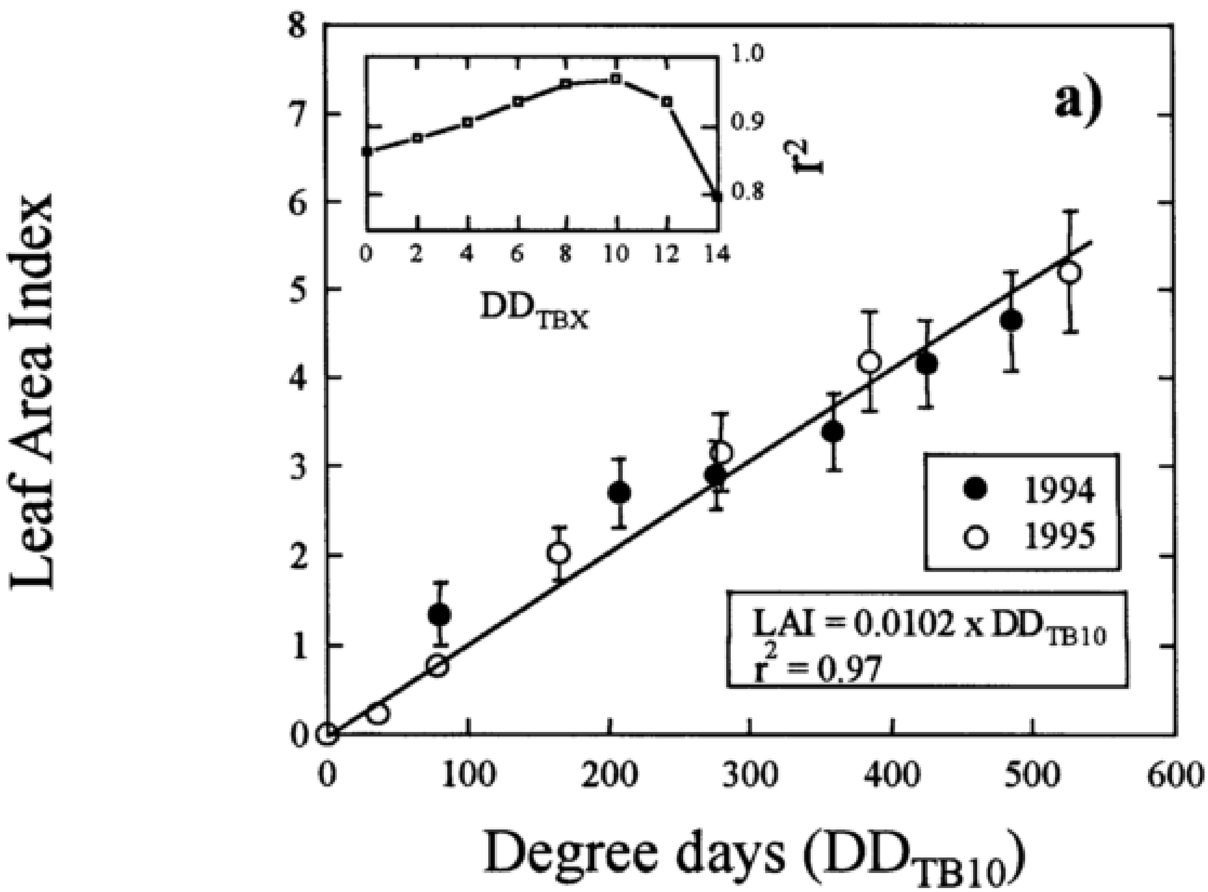


Figure 1: Leaf Area Index modeled as a function of Thermal Units accumulation

To reproduce the code in this lab you would need to install the `BioCro` R package.

```r
install.packages("BioCro", repos="http://R-Forge.R-project.org")
library(BioCro)
```

Implementation of the simple simulation of LAI based on Thermal Time.

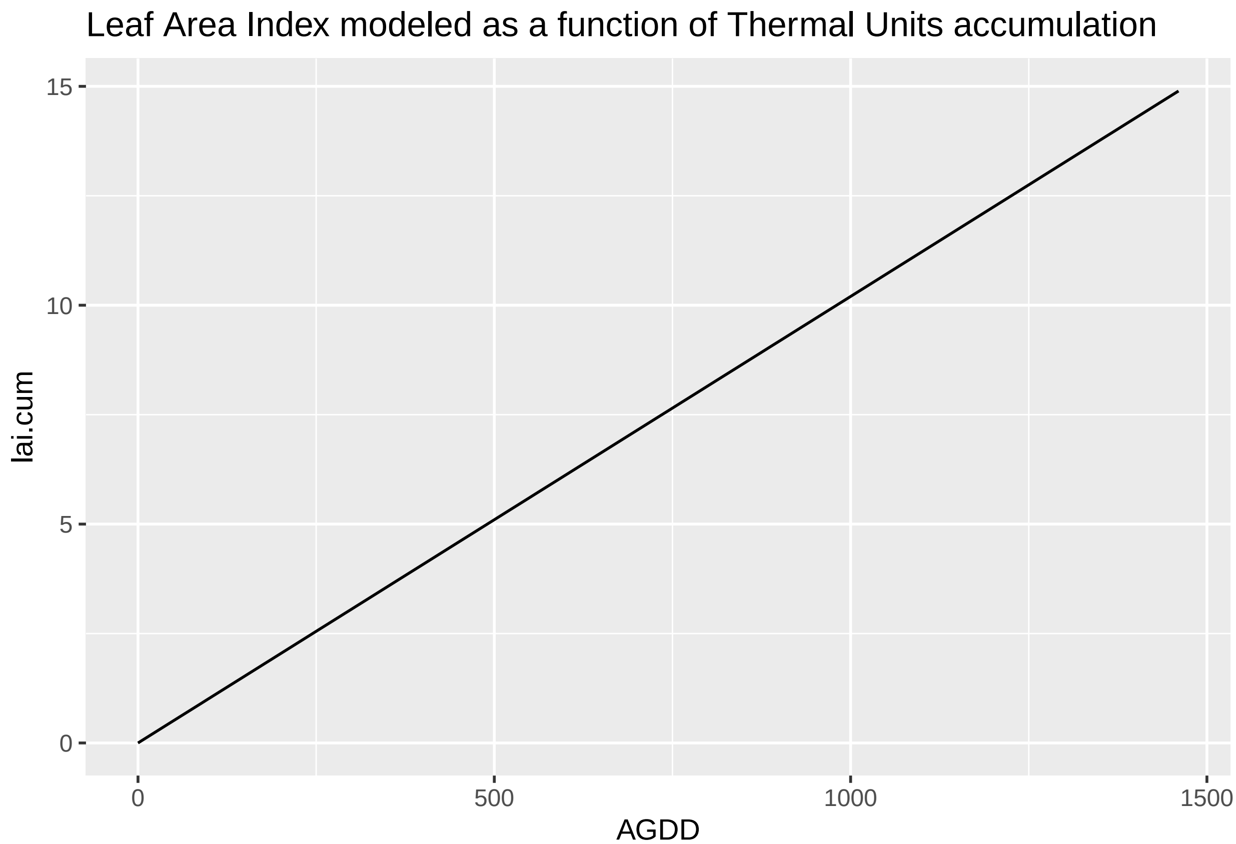
```r
data(cmiWet)
names(cmiWet) <- c("year","day","solar","avgTemp")
## Solar MJ/m2
## Temp in F
cmi05 <- cmiWet[cmiWet$year == 2005,]

args(RUEmod)
```

```
## function (Rad, T.a, doy.s = 91, doy.f = 227, lai.c = 0.0102,
##     rue = 2.4, T.b = 10, k = 0.68)
## NULL
```

```r
res <- RUEmod(cmi05$Rad, cmi05$avgTemp)

names(res)
```

```
## [1] "doy"     "lai.cum" "AG.cum"  "AGDD"    "Int.e"
```

```r
res.d <- as.data.frame(res[-3])

ggplot(data = res.d, aes(x = AGDD, y = lai.cum)) + geom_line() +
  ggtitle("Leaf Area Index modeled as a function of Thermal Units accumulation")
```



The implementation of the model results in Fig **??**. We did not limit our estimate of LAI so probably our prediction is too high (14.9).

A different approach to modeling leaf area index is based on modeling individual leaves as in (Lizaso, Batchelor, and Westgate 2003). The model describes three processes of the life cycle of leaves articulated in a thermal time framework: expansion, longevity and senescence. Figure 2 illustrates the idealized progression of leaf expansion and senescence.

These type of relationships are normally modeled using a logistic function. In this case:

$$LA_i = \frac{Ae_i}{1 + \exp(-ke_i(t - te_i))}$$

where

Ae_i is the final surface area of the ith leaf (cm^2), te_i the thermal time when the leaf reaches 50% of its final area (growing degrees after emergence, base temperature 8° C), and ke$_i$ is a unitless parameter controlling the slope of the curve.

In this model Ae_i is described using the following relationship

$$Ae_i = Ae_x \exp(A_1((LN_i - LN_x)/(LN_x - 1))^2 + A_2((LN_i - LN_x)/(LN_x - 1))^3)$$

Ae_x and LN_x are the area (cm^2) and node of the largest leaf blade, and LN_i the nodal

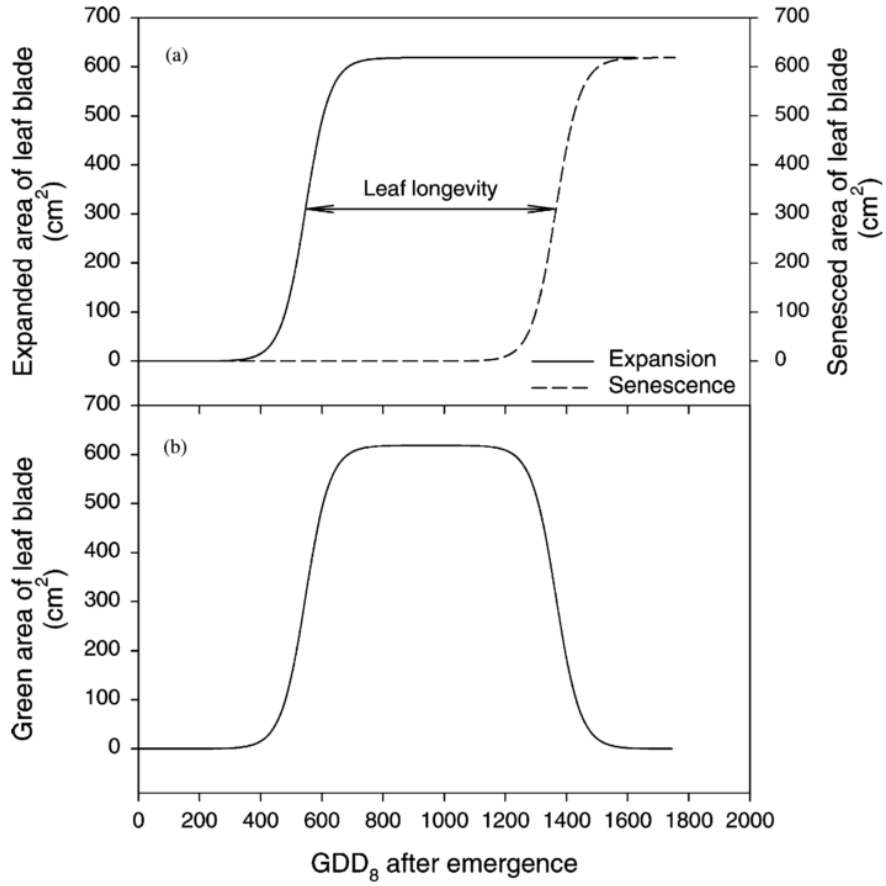


Fig. 1. General framework of the leaf area model. Curves exemplify the dynamics of the 13th leaf. (a) The three main components—leaf expansion, leaf longevity and leaf senescence. (b) Green leaf area of an individual leaf.

Figure 2: Leaf Area Index modeled as individual leaves from Lizaso

position of the ith leaf blade. A_1 and A_2 are shape parameters; A_1 controls the width of the curve and A_2 controls the degree of skewness.

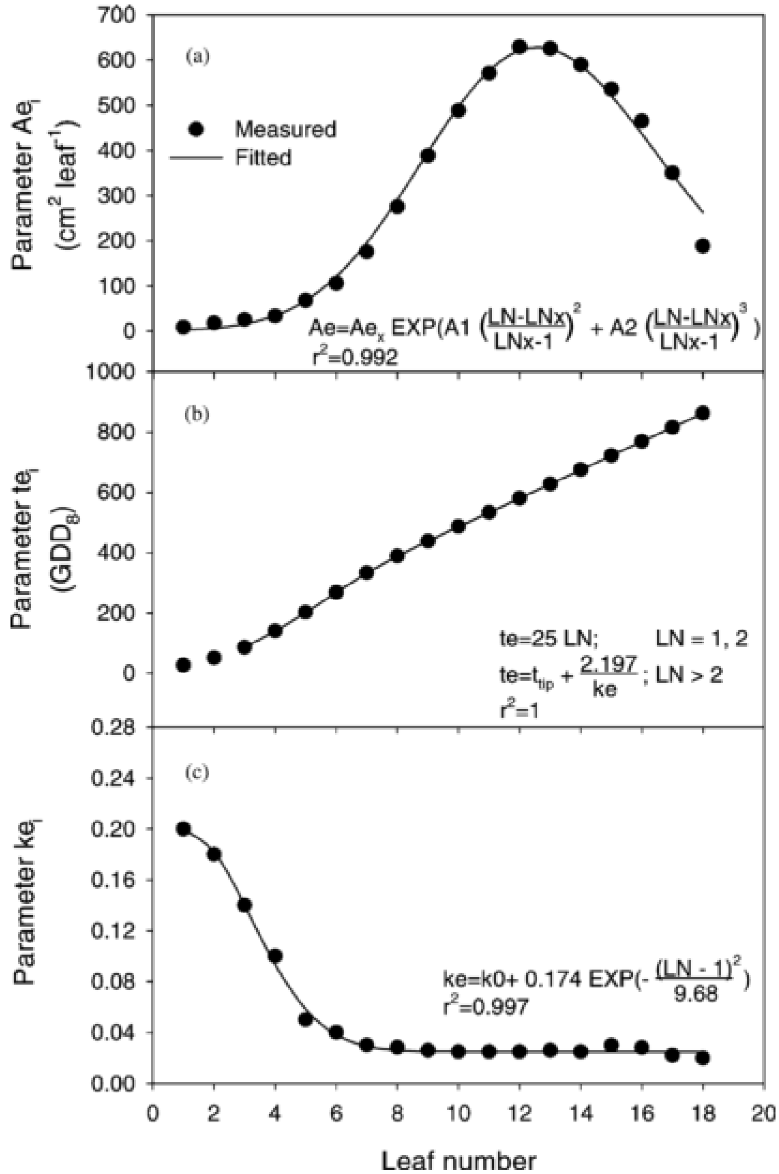


Fig. 2. Seasonal change in the three parameters required to simulate leaf expansion, and fitted relationships used to predict them: (a) Ae$_i$, maximum area of each leaf (cm^2 per leaf); (b) te$_i$, thermal time after emergence when 50% of the leaf is expanded (GDD$_8$); (c) ke$_i$, parameter controlling the slope of the expansion process.

Figure 3: Leaf relationships for an individual leaf area index model

Testing the model as implemented in R in the package BioCro.

```
data(weather04)
laiP <- laiParms(lai.method = "ind-leaf-Lizaso")
res <- MaizeGro(weather04, plant.day = 110, emerge.day = 120,
                harvest.day=300, laiControl = laiP)
```

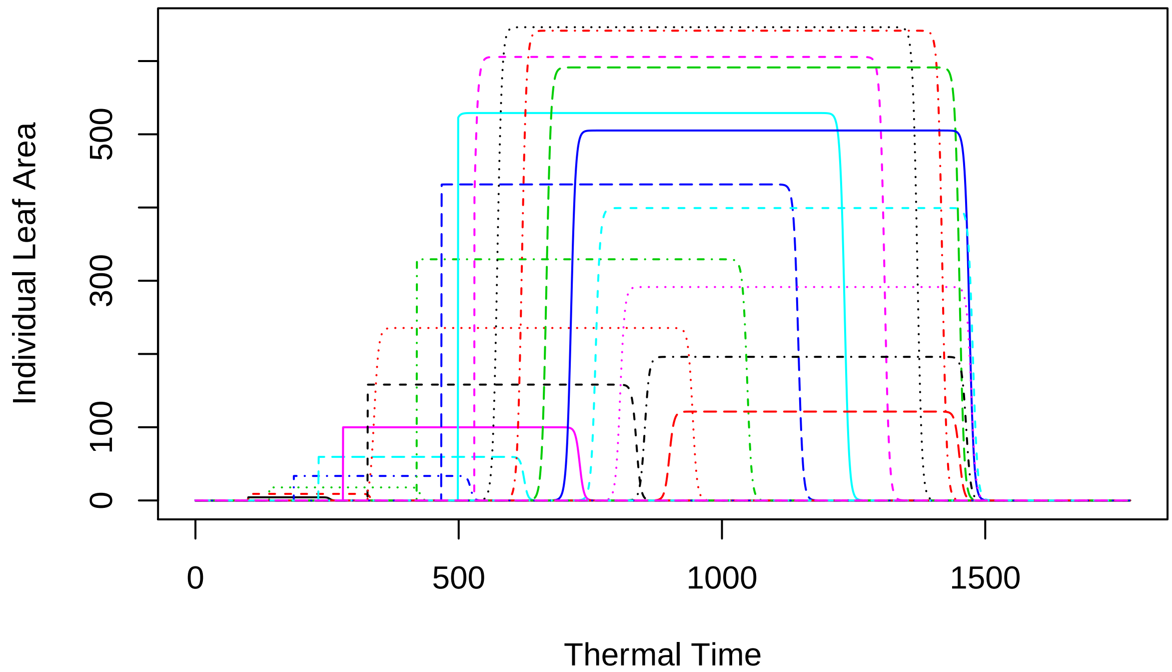


Figure 4: Individual leaf area model as implemented in R

```
plot(res, plot.kind="LAI")
```

Canopy Architecture

There are also models for canopy architecture which have considered, in detail, the complex geometries of leaf arrangements, the height of the crop, sun inclination, time of the day, etc.

There are many steps in the description of the light environment in a canopy. The topic is also described at length in either (Thornley and Johnson 2000) and (Campbell and Norman 1998). Since there are many steps and it is hard to cover all of them in detail I will only describe a few of them.

Steps in WIMOVAC

1. Separate solar radiation into direct and diffuse
2. Calculate a canopy extinction coefficient based on sun declination and canopy architecture. Diffuse radiation has a different extinction coefficient.
3. Calculate the amount of sunlit and shaded leaf area for different layers of the canopy
4. Calculate leaf-level photosynthesis and transpiration for each layer taking into account the changes in relative humidity, wind, light, etc.

Another motivation for canopy light interception is to understand the theory behind the derivation of a leaf area index from light interception data as it is done in the LI-80 Accupar ceptometer.

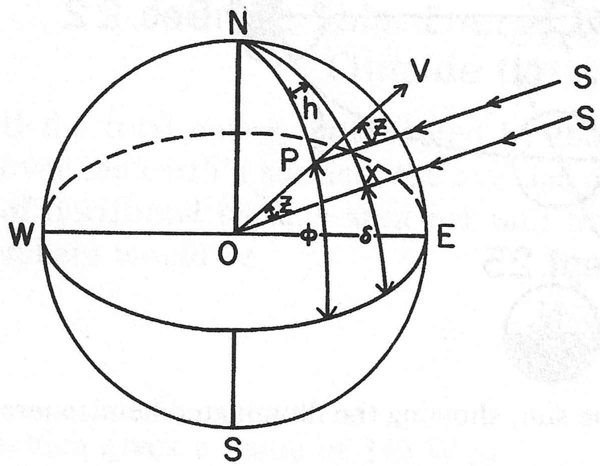


Figure 6.2. Geometrical relations of solar zenith angle z to latitude ϕ, solar declination δ, and hour angle h. The observer is at P.

Figure 5: Earth geometry

Solar radiation beam angles

Some equations:

$$\delta = -23.5 \times \cos(\frac{360(doy + 10)}{365} dtr)$$

where dtr is degrees to radians $\pi/180$.

δ is the angle which the line of the sun's rays at the earth makes with the equatorial plane and it is called the declination of the sun. For the Northern Hemisphere, maximum declination varies from -23.5° at the winter solstice on December 22 to +23.5° at the summer solstice on June 22 (Fig. 6).

From the time of the day and the day of the year the zenith (θ) angle can be derived

$$\cos(\theta) = \sin(\Omega) \cdot \sin(\delta) + \cos(\Omega) \cdot \cos(15 \cdot (t - t_{sn}))$$

From here our additional assumption is that a certain proportion of light actually reaches the surface, this is termed atmospheric transmissivity (α).

$$I_{dir} = irr \times (\alpha^{PP_o / \cos(\theta)}) \times \cos(\theta)$$

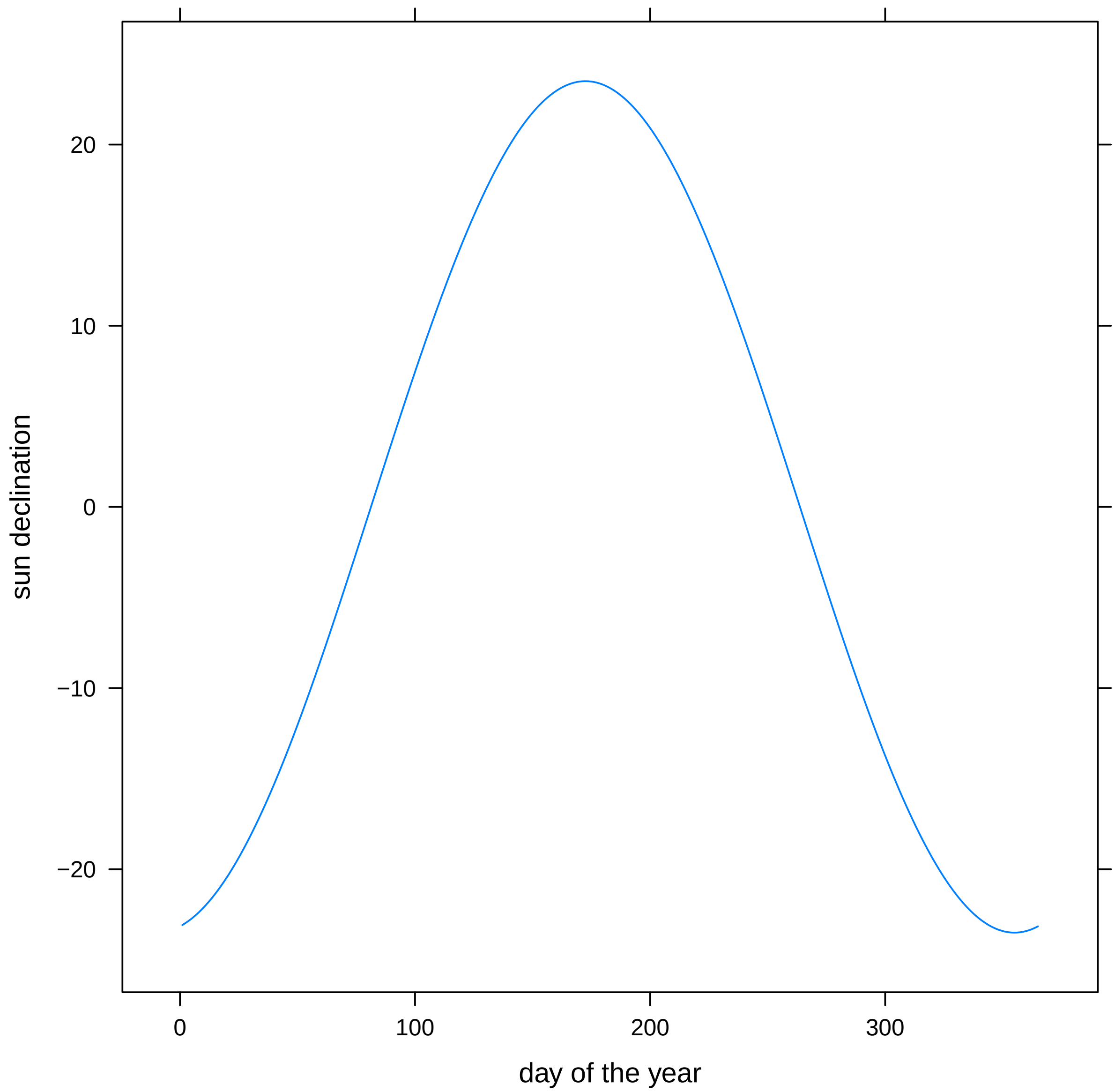


Figure 6: Sun declination

$$I_{diff} = 0.5 \times irr \times (1 - \alpha^{PP_o/\cos(\theta)}) \times \cos(\theta)$$

where

$$irr = \text{Solar irradiance}$$
$$PP_o = 10,000/atm.P$$
$$atm.P = atmosphericpressure$$

Extinction coefficient

The extinction coefficient can be modeled as a function of the zenith angle and canopy architecture. In this case the canopy architecture is described by one parameter χ which is the ratio of average projected area of canopy elements on horizontal and vertical surfaces.

$$\chi = \frac{\text{horizontal projection}}{\text{vertical projection}}$$

For a perfect vertical canopy $\chi = 0$ and for a perfect horizontal canopy χ approaches infinity.

$$k = \frac{\cos(\theta)\sqrt{\chi^2 + \tan\theta^2}}{\chi + 1.744(\chi + 1.183)^{-0.733}}$$

Code

```
## Calculating the sun declination
del <- function(doy){
  dtr <- pi / 180
  ans <- -23.5 * cos((360 * ( doy + 10) /365) * dtr)
  ans
}

xx <- 1:365
yy <- del(xx)

qplot(xx, yy, geom="line",
       xlab = 'day of the year',
       ylab = 'sun declination')
```

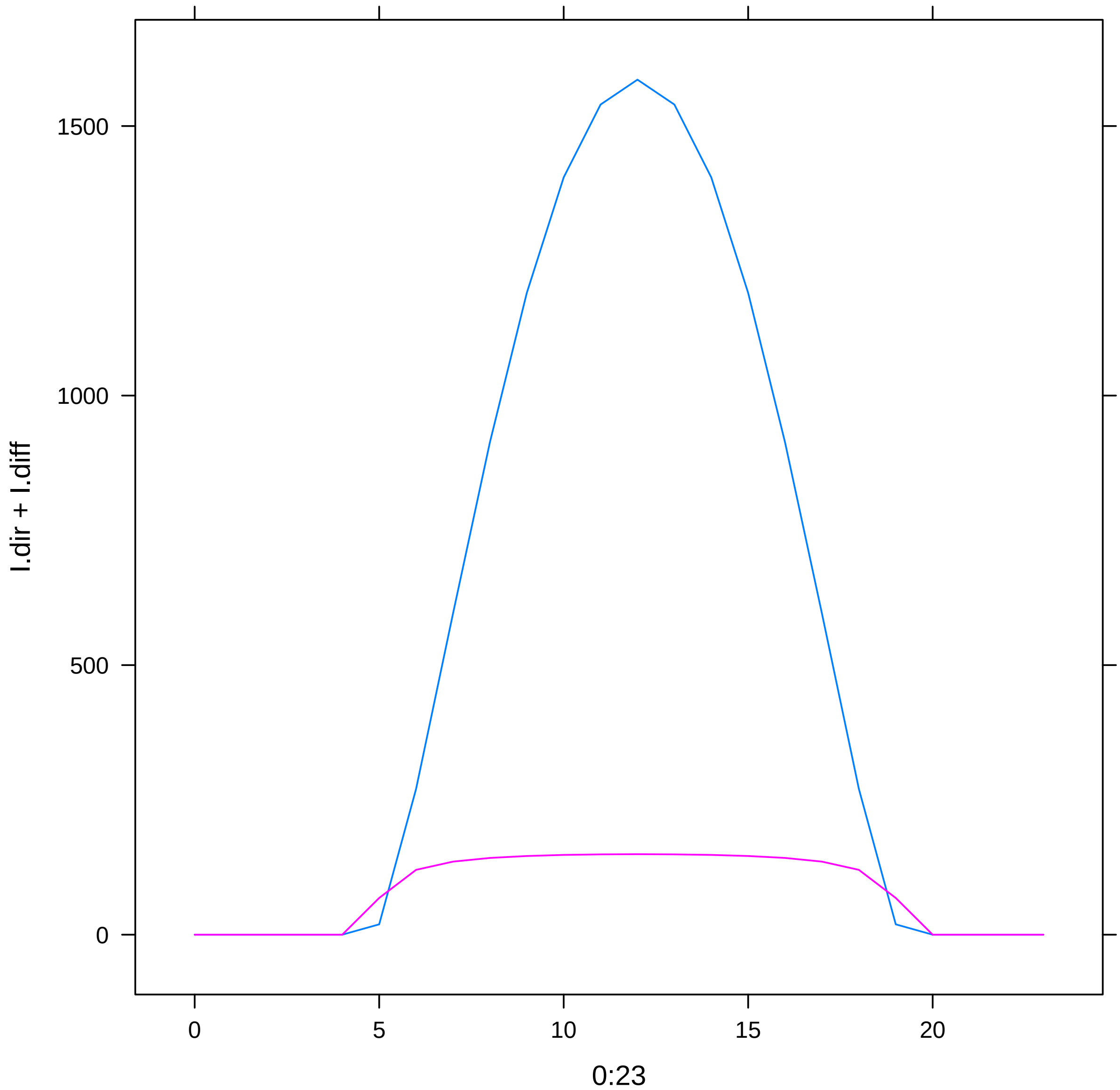
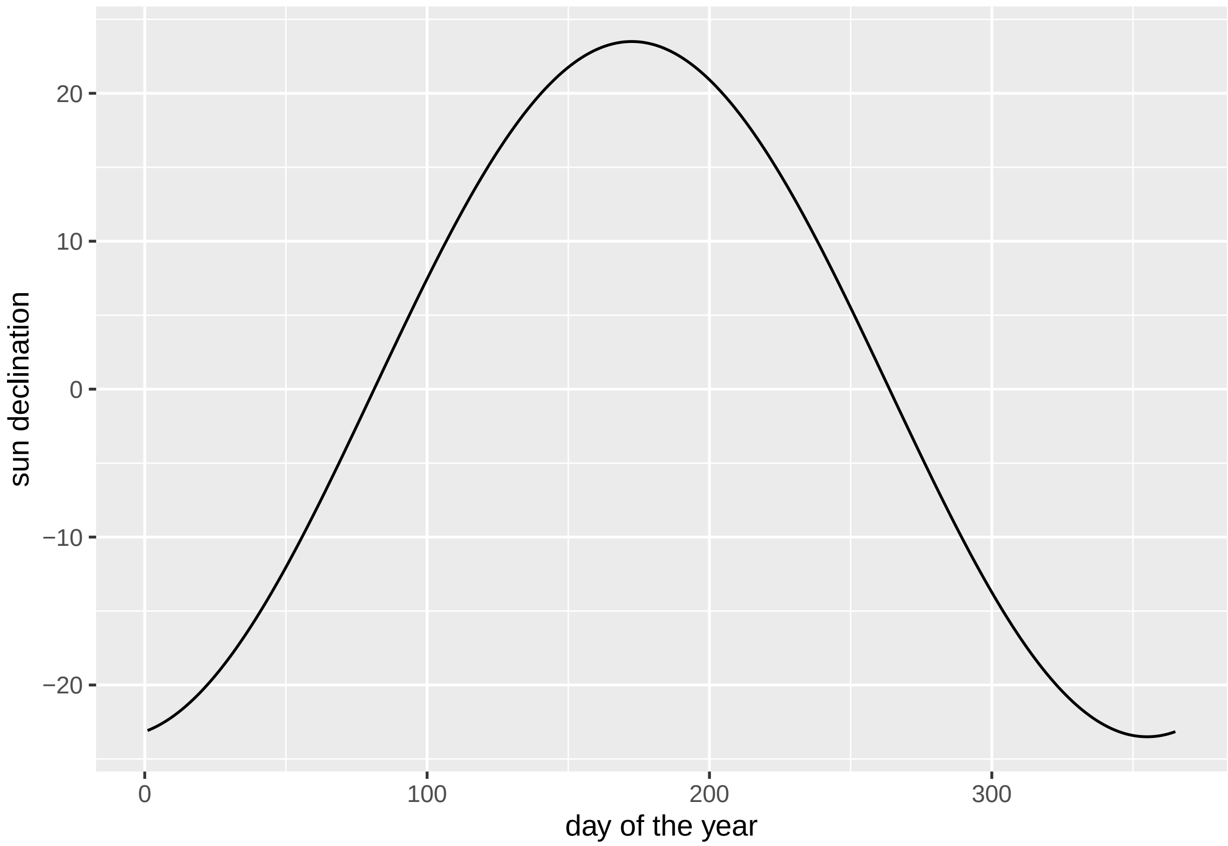


Figure 7: Simulated direct and diffuse light

TABLE 15.1. Values of the leaf angle distribution parameter x for various crop canopies (from Campbell and van Evert, 1994)

Crop	x	**Crop**	x
Ryegrass	0.67–2.47	Cucumber	2.17
Maize	0.76–2.52	Tobacco	1.29–2.22
Rye	0.8–1.27	Potato	1.70–2.47
Wheat	0.96	Horse Bean	1.81–2.17
Barley	1.20	Sunflower	1.81–4.1
Timothy	1.13	White clover	2.47–3.26
Sorghum	1.43	Strawberry	3.03
Lucerne	1.54	Soybean	0.81
Hybrid swede	1.29–1.81	Maize	1.37
Sugar beet	1.46–1.88	J. artichoke	2.16
Rape	1.92–2.13		

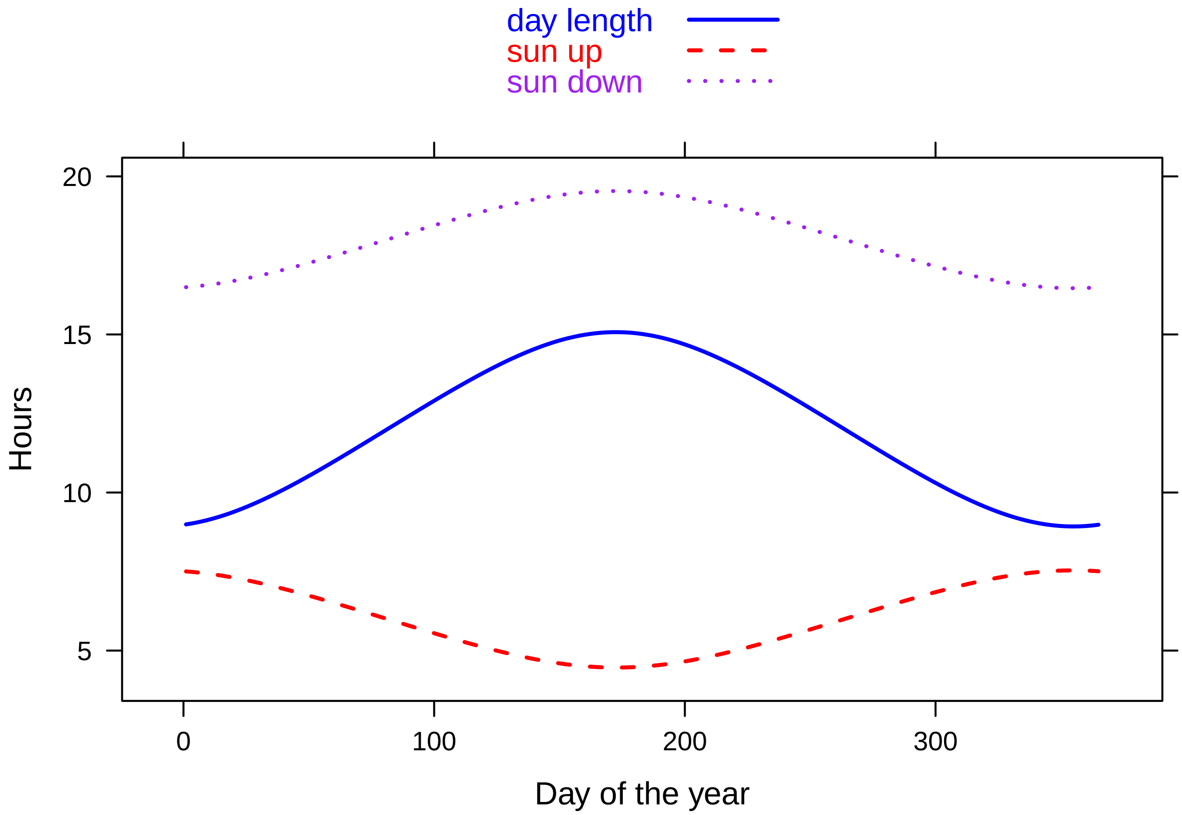
Figure 8: χ values for different crops




```
## Function to calculate the cosine of the zenith angle
coszang <- function(lat, doy, t.d, t.sn = 12){

  dtr <- pi/180
  omega <- lat * dtr
  deltR <- del(doy) * dtr
  t.f <- (15 * (t.d - t.sn)) * dtr

  ss <- sin(deltR) * sin(omega)
  cc <- cos(deltR) * cos(omega)

  coszenithangle <- ss + cc * cos(t.f)
  coszenithangle <- ifelse(coszenithangle < 1e-10, 1e-10, coszenithangle)

  coszenithangle
}


lat <- 42
dtr <- pi / 180

k <- 1
```

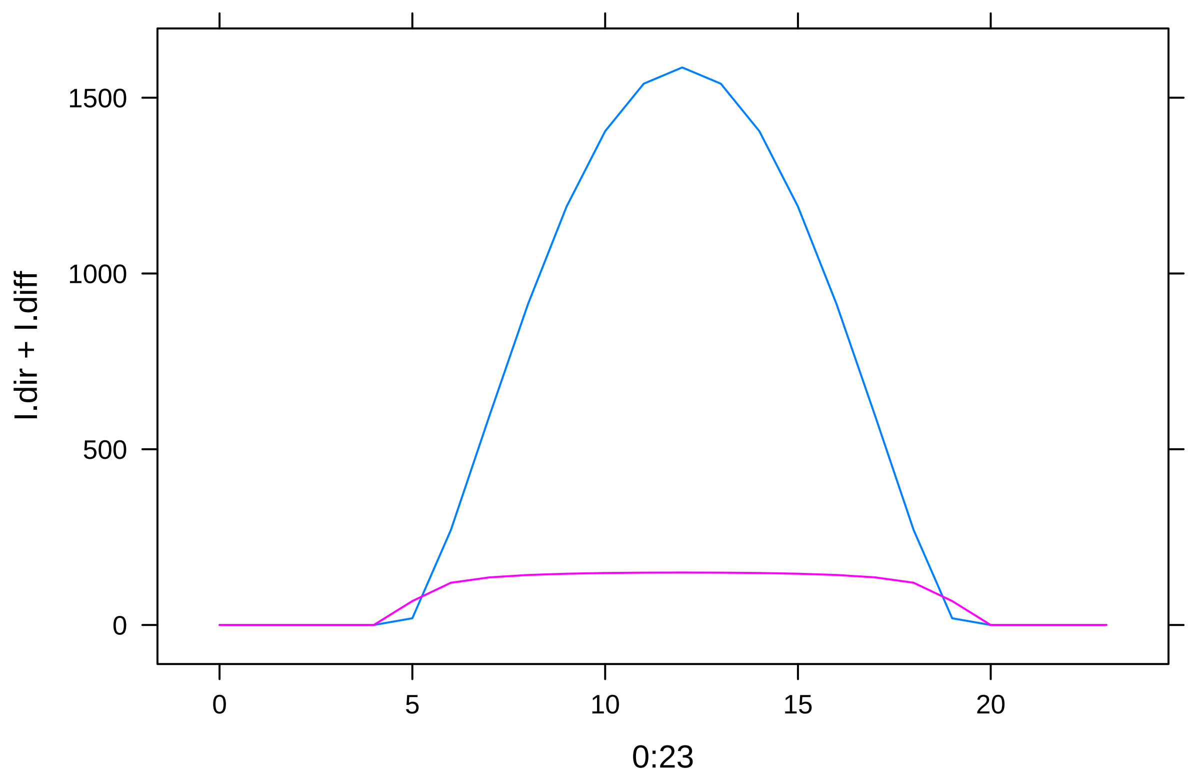
```r
doyhr.angle <- expand.grid(hour = 0:23, doy = 1:365, cos.th = NA)

for(i in 1:365){

  for(j in 0:23){

   doyhr.angle[k,3] <- coszang(lat = lat, doy = i, t.d = j)

   k <- k + 1
 }
}

## Include plot


## We can also calculate daylength, sunup and sundown
daylen <- function(lat, doy){

  dtr <- pi/180

  omega <- lat * dtr

  deltR <- del(doy) * dtr

  coshour <- -tan(omega) * tan(deltR)
  coshourdeg <- (1/dtr) * coshour
  daylength <- 2 * (1/dtr) * acos(coshour) / 15
  sunup <- 12 - daylength / 2
  sundown <- 12 + daylength / 2

  list(daylength = daylength, sunup = sunup, sundown = sundown)

}

## Testing the daylength function

xx <- 1:365
yy <- daylen(lat = 42, xx)

xyplot(yy$daylength + yy$sunup + yy$sundown ~ xx,
       type = 'l',
       lty = 1:3,
       lwd = 2,
       col = c('blue', 'red', 'purple'),
```

```
       xlab = 'Day of the year',
       ylab = 'Hours',
       key=list(text = list(c('day length', 'sun up', 'sun down')),
         lty = 1:3, col = c('blue', 'red', 'purple'),
         lines = TRUE, lwd = 2))
```



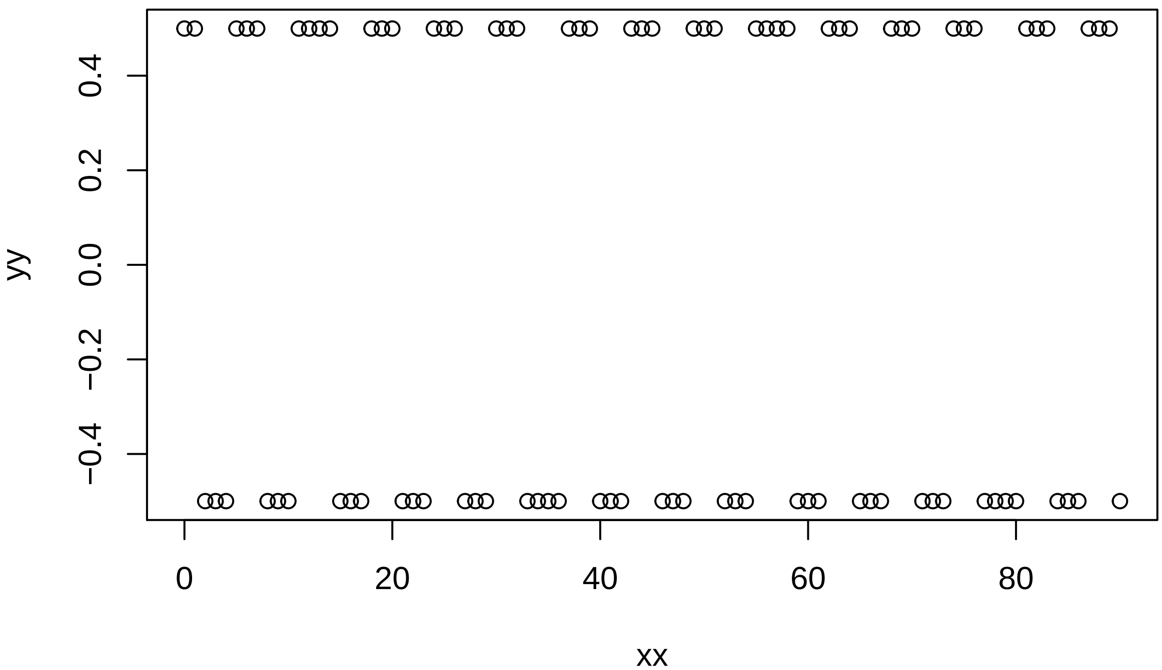

```
## Proportion of direct and diffuse radiation

light <- function(irr=NULL, lat, doy, t.d, atm.P = 1e5){

  dtr <- pi / 180

  PP.o <- 1e5/atm.P
  Solar_Constant <- 2650
  if(missing(irr)) irr <- Solar_Constant
  alpha <- 0.85

  coszenithangle <- coszang(lat = lat, doy = doy, t.d = t.d)

  I.dir <- irr * (alpha^((PP.o)/coszenithangle)) * coszenithangle

  I.diff <- 0.5 * irr * (1 - alpha ^ ((PP.o)/coszenithangle)) * coszenithangle
```

```
  list(I.dir = I.dir, I.diff = I.diff)

}


doyhr.light <- expand.grid(hour = 0:23, I.dir = NA, I.diff = NA)

for(j in 0:23){
  tmp <- light(irr = 2000, lat = lat, doy = 190, t.d = j)
  doyhr.light[j + 1,2] <- tmp$I.dir
  doyhr.light[j + 1,3] <- tmp$I.diff
}

## the previous function is vectorized, even if you don't notice
## So there is no need for a loop

ans <- light(irr = 2000, lat = lat, doy = 190, t.d = 0:23)

xyplot(I.dir + I.diff ~ 0:23 ,
       data = ans, type = 'l')
```




```
## Illustrating the ellipsoidal distribution

extcoef <- function(psi, chi) {
```

```r
 ## Enter psi in degrees
 # psi <- psi * (pi/180)
  num <- sqrt(chi^2 + tan(psi)^2) * cos(psi)
  den <- chi + 1.774*(chi + 1.182)^-0.733

  ans <- num/den

  ans

}

xx <- seq(0, 90)
yy <- extcoef(xx, 1)

plot(xx, yy)
```



Campbell, Gaylon, and John Norman. 1998. *An Introduction to Environmental Biophysics*. New York, NY: Springer.

Clifton-Brown, John, B. Neilson, Iris Lewandowski, and Michael B. Jones. 2000. "The Modelled Productivity of Miscanthus X Giganteus (Greef et Deu) in Ireland." *Industrial Crops and Products* 12: 97–109.

Lizaso, J, W. D. Batchelor, and M E Westgate. 2003. "A leaf area model to simulate cultivar-specific expansion and senescence of maize leaves." *Field Crops Research* 80 (1): 1–17. doi:10.1016/S0378-4290(02)00151-X.

Thornley, J, and I Johnson. 2000. *Plant and Crop Modelling. a Mathematical Approach to*

Plant and Crop Physiology. The Blackburn Press.