

# **Capítulo\_7\_Análisis\_Factorial**

Econometría para la Gestión (ECO\_EPG) - FEN UAH

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## 1. 1. Material descargable

Descargar PDF de contenidos teóricos

Descargar PDF de contenidos teóricos

## 2. Recordatorio teórico: PCA y análisis factorial

A muy grandes rasgos, el **Análisis de Componentes Principales (PCA)** busca:

- Reducir un conjunto de  $p$  variables originales ( $X_1, \dots, X_p$ ) a un número menor de **componentes** ( $Z_1, \dots, Z_z$ ), con  $z < p$ .
- Cada componente es una **combinación lineal** de las variables originales que captura la máxima varianza posible.
- Es un método de **aprendizaje no supervisado**: no hay variable respuesta, sólo interesa la estructura interna de las  $X$ .

Pasos habituales:

1. **Estandarizar** las variables si están en escalas diferentes (media 0, desviación estándar 1).
2. Calcular la **matriz de correlación R**.
3. Obtener **valores propios** ( $\lambda_i$ ) y **vectores propios** (direcciones principales).
4. Decidir cuántas componentes retener (porcentaje de varianza explicada, gráfico de sedimentación, criterio de Kaiser, etc.).

5. Interpretar las componentes mirando las **cargas** (correlación variable–componente).
6. Analizar gráficos de **individuos** y **variables** para comprender la estructura de los datos.

Además, se suele evaluar:

- **Test de Bartlett:** contrasta si la matriz de correlación es suficientemente distinta de la identidad (si es esférica, PCA no tiene mucho sentido).
  - **KMO:** mide la adecuación muestral (valores altos indican que es apropiado usar análisis factorial/PCA).
- 

### 3. Configuración inicial en R

#### 3.1. Carga de librerías

```
#install.packages(c(
  # "FactoMineR",           # PCA y métodos multivariados
  # "psych",                # Análisis psicométrico (KMO, etc.)
  # "corrplot",              # Gráficos de correlación
  # "PerformanceAnalytics" # Estadísticos financieros y correlaciones
))

library(FactoMineR)      # PCA y funciones de análisis factorial
library(dplyr)            # Manipulación de datos
library(psych)             # KMO, análisis psicométrico
library(corrplot)          # Matriz de correlaciones graficadas
library(PerformanceAnalytics) # chart.Correlation
```

#### 3.2. Ruta de trabajo

Usaremos la ruta estándar de tus labs:

```
ruta_datos <- "C:/Users/manue/Desktop/lab-econometria/labs_epg/data_epg"

list.files(ruta_datos)
```

```
[1] "annos_mantenimiento.xlsx" "auto_peso_consumo.xlsx"
[3] "costos.xlsx"                 "data_PCA_Decathlon.csv"
[5] "data_PCA_ExpertWine.csv"    "Ejemplo1.xlsx"
[7] "Ejemplo2.xlsx"               "millaje.txt"
[9] "orange.csv"                  "tabla_ejemplo_R.xlsx"
```

---

## 4. Parte 1: PCA con datos de Decathlon

En el primer ejercicio analizaremos resultados de atletas en un **decathlon**.

- Cada fila: un atleta.
- Columnas 1 a 10: variables cuantitativas de desempeño (distancias, tiempos, puntajes).
- Otras columnas: información complementaria (competición, ranking, etc.).

### 4.1. Importar datos de Decathlon

En el script original se usa un archivo CSV con ; como separador:

```
archivo_decathlon <- file.path(ruta_datos, "data_PCA_Decathlon.csv")

decathlon <- read.table(
  archivo_decathlon,
  header      = TRUE,
  sep         = ";",
  dec         = ".",
  row.names   = 1,
  check.names = FALSE
)

summary(decathlon)
```

100m	Long jump	Shot put	High jump	400m
Min. :10.44	Min. :6.61	Min. :12.68	Min. :1.850	Min. :46.81
1st Qu.:10.85	1st Qu.:7.03	1st Qu.:13.88	1st Qu.:1.920	1st Qu.:48.93
Median :10.98	Median :7.30	Median :14.57	Median :1.950	Median :49.40
Mean   :11.00	Mean   :7.26	Mean   :14.48	Mean   :1.977	Mean   :49.62

3rd Qu.:11.14	3rd Qu.:7.48	3rd Qu.:14.97	3rd Qu.:2.040	3rd Qu.:50.30
Max. :11.64	Max. :7.96	Max. :16.36	Max. :2.150	Max. :53.20
110m H	Discus	Pole vault	Javeline	
Min. :13.97	Min. :37.92	Min. :4.200	Min. :50.31	
1st Qu.:14.21	1st Qu.:41.90	1st Qu.:4.500	1st Qu.:55.27	
Median :14.48	Median :44.41	Median :4.800	Median :58.36	
Mean :14.61	Mean :44.33	Mean :4.762	Mean :58.32	
3rd Qu.:14.98	3rd Qu.:46.07	3rd Qu.:4.920	3rd Qu.:60.89	
Max. :15.67	Max. :51.65	Max. :5.400	Max. :70.52	
1500m	Rank	Points	Competition	
Min. :262.1	Min. : 1.00	Min. :7313	Length:41	
1st Qu.:271.0	1st Qu.: 6.00	1st Qu.:7802	Class :character	
Median :278.1	Median :11.00	Median :8021	Mode :character	
Mean :279.0	Mean :12.12	Mean :8005		
3rd Qu.:285.1	3rd Qu.:18.00	3rd Qu.:8122		
Max. :317.0	Max. :28.00	Max. :8893		

### 💡 Tip

Observa:

- Número de **individuos** (atletas).
- Número de **variables cuantitativas activas** (primeras 10 columnas).
- Variables adicionales (competición, ranking, etc.) que luego usaremos como **suplementarias**.

## 4.2. Estandarización manual (opcional)

El PCA de FactoMineR::PCA ya estandariza por defecto las variables cuantitativas, pero el script muestra cómo hacerlo explícitamente:

```
decathlonnorm <- decathlon[, 1:10] %>%
  mutate_all(~ scale(.) %>% as.vector)

head(decathlonnorm)
```

	100m	Long jump	Shot put	High jump	400m	110m H
Sebrle	-0.5628739	1.8331131	2.283919585	1.60955474	-1.0892025	-1.17818270
Clay	-2.1216730	2.2123779	0.913271989	0.93502243	-0.3696226	-1.00861538

Karpov	-1.8935560	1.7382969	1.762345721	1.27228858	-2.4329962	-1.34775002
Macey	-0.4107959	0.6637134	1.519753226	1.94682089	-0.5603546	-0.09719103
Warners	-1.4373221	1.5170591	0.003550134	-0.07677604	-1.4273183	-1.26296636
Zsivoczky	-0.3347570	-0.3792648	1.010308987	1.60955474	-0.1875602	0.72944966
	Discus	Pole vault	Javeline		1500m	
Sebrle	1.3009450	0.8545364	2.52825135	0.08439142		
Clay	1.7124500	0.4948240	2.36043901	0.25486670		
Karpov	2.1683620	-0.5843134	-0.57524110	-0.07837391		
Macey	1.1884472	-1.3037383	0.02971203	-1.16547502		
Warners	-0.1763283	0.4948240	-0.60631746	-0.08351387		
Zsivoczky	0.3832000	-0.2246009	1.06351893	-0.81253124		

### 4.3. Matriz de correlación

Antes de hacer PCA, miramos las correlaciones entre variables:

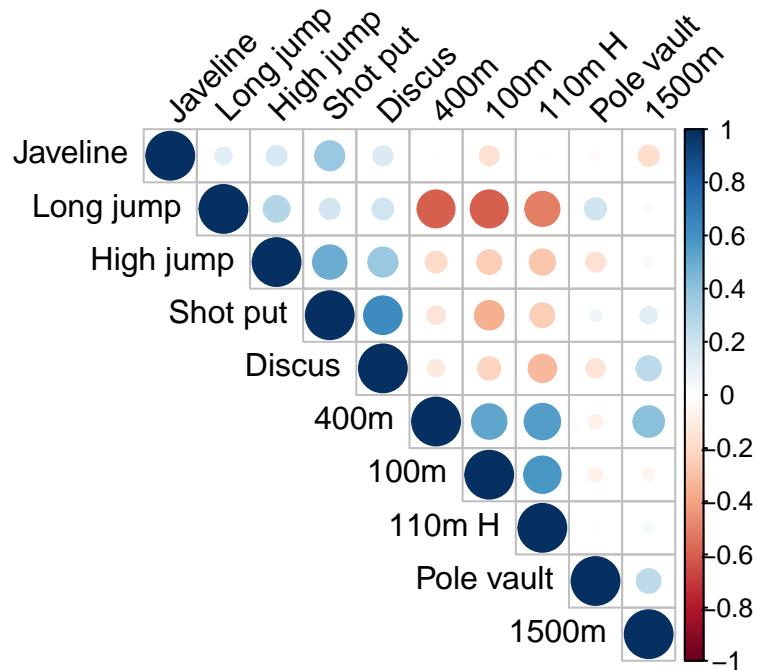
```
cor.mat <- round(cor(decathlon[, 1:10]), 2)
cor.mat
```

	100m	Long jump	Shot put	High jump	400m	110m H	Discus	Pole vault	
100m	1.00	-0.60	-0.36	-0.25	0.52	0.58	-0.22	-0.08	
Long jump	-0.60	1.00	0.18	0.29	-0.60	-0.51	0.19	0.20	
Shot put	-0.36	0.18	1.00	0.49	-0.14	-0.25	0.62	0.06	
High jump	-0.25	0.29	0.49	1.00	-0.19	-0.28	0.37	-0.16	
400m	0.52	-0.60	-0.14	-0.19	1.00	0.55	-0.12	-0.08	
110m H	0.58	-0.51	-0.25	-0.28	0.55	1.00	-0.33	0.00	
Discus	-0.22	0.19	0.62	0.37	-0.12	-0.33	1.00	-0.15	
Pole vault	-0.08	0.20	0.06	-0.16	-0.08	0.00	-0.15	1.00	
Javeline	-0.16	0.12	0.37	0.17	0.00	0.01	0.16	-0.03	
1500m	-0.06	-0.03	0.12	-0.04	0.41	0.04	0.26	0.25	
		Javeline	1500m						
100m		-0.16	-0.06						
Long jump		0.12	-0.03						
Shot put		0.37	0.12						
High jump		0.17	-0.04						
400m		0.00	0.41						
110m H		0.01	0.04						
Discus		0.16	0.26						
Pole vault		-0.03	0.25						
Javeline		1.00	-0.18						
1500m		-0.18	1.00						

```

corrplot(
  cor.mat,
  type = "upper",
  order = "hclust",
  tl.col = "black",
  tl.srt = 45
)

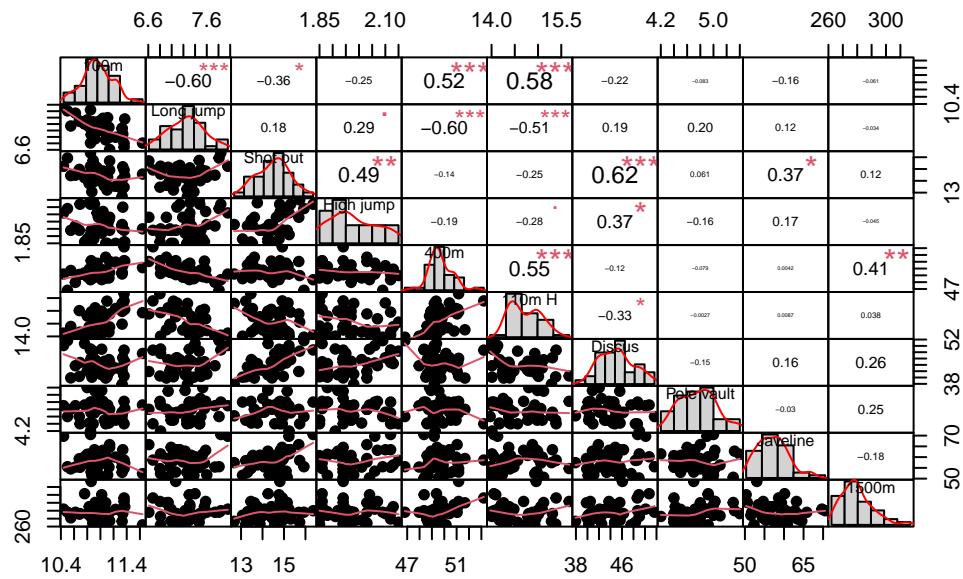
```



```

chart.Correlation(decathlon[, 1:10],
                  histogram = TRUE,
                  pch       = 19)

```



### Nota

- Correlaciones altas (en valor absoluto) entre variables sugieren **redundancia de información**, lo que hace atractivo aplicar PCA.
- **corrplot** agrupa visualmente las variables con patrones de correlación similares.

#### 4.4. PCA con variables activas (primeras 10 columnas)

```
res <- PCA(decathlon[, 1:10], ncp = 5)
summary.PCA(res)
```

Call:  
 PCA(X = decathlon[, 1:10], ncp = 5)

	Eigenvalues	Dim.1	Dim.2	Dim.3	Dim.4	Dim.5	Dim.6	Dim.7
Variance		3.272	1.737	1.405	1.057	0.685	0.599	0.451

% of var.	32.719	17.371	14.049	10.569	6.848	5.993	4.512
Cumulative % of var.	32.719	50.090	64.140	74.708	81.556	87.548	92.061
	Dim.8	Dim.9	Dim.10				
Variance	0.397	0.215	0.182				
% of var.	3.969	2.148	1.822				
Cumulative % of var.	96.030	98.178	100.000				

#### Individuals (the 10 first)

	Dist	Dim.1	ctr	cos2	Dim.2	ctr	cos2	Dim.3	
Sebrle	4.843	4.038	12.158	0.695	1.366	2.619	0.080	-0.290	
Clay	4.647	3.919	11.451	0.711	0.837	0.984	0.032	0.231	
Karpov	5.006	4.620	15.911	0.852	0.040	0.002	0.000	-0.042	
Macey	3.434	2.233	3.719	0.423	1.042	1.524	0.092	-1.864	
Warners	2.979	2.168	3.505	0.530	-1.803	4.565	0.366	0.851	
Zsivoczky	2.566	0.925	0.638	0.130	1.169	1.918	0.207	-1.477	
Hernu	1.824	0.889	0.589	0.238	-0.618	0.537	0.115	-0.898	
Nool	3.098	0.295	0.065	0.009	-1.546	3.354	0.249	1.355	
Bernard	2.827	1.906	2.709	0.455	-0.086	0.010	0.001	-0.757	
Schwarzl	1.971	0.081	0.005	0.002	-1.353	2.572	0.472	0.822	
	ctr	cos2							
Sebrle	0.146	0.004							
Clay	0.093	0.002							
Karpov	0.003	0.000							
Macey	6.034	0.295							
Warners	1.257	0.082							
Zsivoczky	3.790	0.332							
Hernu	1.401	0.242							
Nool	3.189	0.191							
Bernard	0.995	0.072							
Schwarzl	1.174	0.174							

#### Variables

	Dim.1	ctr	cos2	Dim.2	ctr	cos2	Dim.3	ctr	cos2
100m	-0.775	18.344	0.600	0.187	2.016	0.035	-0.184	2.420	0.034
Long jump	0.742	16.822	0.550	-0.345	6.869	0.119	0.182	2.363	0.033
Shot put	0.623	11.844	0.388	0.598	20.607	0.358	-0.023	0.039	0.001
High jump	0.572	9.998	0.327	0.350	7.064	0.123	-0.260	4.794	0.067
400m	-0.680	14.116	0.462	0.569	18.666	0.324	0.131	1.230	0.017
110m H	-0.746	17.020	0.557	0.229	3.013	0.052	-0.093	0.611	0.009
Discus	0.552	9.328	0.305	0.606	21.162	0.368	0.043	0.131	0.002
Pole vault	0.050	0.077	0.003	-0.180	1.873	0.033	0.692	34.061	0.479
Javeline	0.277	2.347	0.077	0.317	5.784	0.100	-0.390	10.807	0.152
1500m	-0.058	0.103	0.003	0.474	12.946	0.225	0.782	43.543	0.612

```
100m      |
Long jump |
Shot put   |
High jump  |
400m      |
110m H    |
Discus    |
Pole vault|
Javeline   |
1500m     |
```

```
print(res)
```

```
**Results for the Principal Component Analysis (PCA)**
The analysis was performed on 41 individuals, described by 10 variables
*The results are available in the following objects:
```

	name	description
1	"\$eig"	"eigenvalues"
2	"\$var"	"results for the variables"
3	"\$var\$coord"	"coord. for the variables"
4	"\$var\$cor"	"correlations variables - dimensions"
5	"\$var\$cos2"	"cos2 for the variables"
6	"\$var\$contrib"	"contributions of the variables"
7	"\$ind"	"results for the individuals"
8	"\$ind\$coord"	"coord. for the individuals"
9	"\$ind\$cos2"	"cos2 for the individuals"
10	"\$ind\$contrib"	"contributions of the individuals"
11	"\$call"	"summary statistics"
12	"\$call\$centre"	"mean of the variables"
13	"\$call\$ecart.type"	"standard error of the variables"
14	"\$call\$row.w"	"weights for the individuals"
15	"\$call\$col.w"	"weights for the variables"

Aquí usamos sólo las **variables activas** (sin incluir todavía variables suplementarias).

#### 4.4.1. Adecuación del análisis factorial: Bartlett y KMO

```
bartlett.test(decathlon[, 1:10])
```

```
Bartlett test of homogeneity of variances

data: decathlon[, 1:10]
Bartlett's K-squared = 1268.8, df = 9, p-value < 2.2e-16
```

```
KMO(decathlon[, 1:10])
```

```
Kaiser-Meyer-Olkin factor adequacy
Call: KMO(r = decathlon[, 1:10])
Overall MSA = 0.6
MSA for each item =
      100m Long jump Shot put High jump        400m      110m H     Discuss
          0.69      0.72     0.57      0.70      0.61      0.83      0.54
Pole vault    Javeline   1500m
          0.24      0.44     0.29
```

#### i Nota

- Un p-value pequeño en **Bartlett** indica que la matriz de correlaciones **no es esférica**, por lo que PCA tiene sentido.
- **KMO** cercano a 1 indica buena adecuación; valores muy bajos desaconsejan el análisis factorial.

## 4.5. PCA con variables suplementarias

Supongamos que las columnas 11 y 12 son **cuantitativas suplementarias** (no influyen en la construcción de las componentes) y la 13 es una **cualitativa suplementaria** (por ejemplo, tipo de competición).

```
res <- PCA(
  decathlon,
  ncp      = 5,
  quanti.sup = 11:12,
  quali.sup = 13
)
```

```
summary(res, nbelements = Inf)
```

Call:

```
PCA(X = decathlon, ncp = 5, quanti.sup = 11:12, quali.sup = 13)
```

#### Eigenvalues

	Dim.1	Dim.2	Dim.3	Dim.4	Dim.5	Dim.6	Dim.7
Variance	3.272	1.737	1.405	1.057	0.685	0.599	0.451
% of var.	32.719	17.371	14.049	10.569	6.848	5.993	4.512
Cumulative % of var.	32.719	50.090	64.140	74.708	81.556	87.548	92.061
	Dim.8	Dim.9	Dim.10				
Variance	0.397	0.215	0.182				
% of var.	3.969	2.148	1.822				
Cumulative % of var.	96.030	98.178	100.000				

#### Individuals

	Dist	Dim.1	ctr	cos2	Dim.2	ctr	cos2	Dim.3
Sebrle	4.843	4.038	12.158	0.695	1.366	2.619	0.080	-0.290
Clay	4.647	3.919	11.451	0.711	0.837	0.984	0.032	0.231
Karpov	5.006	4.620	15.911	0.852	0.040	0.002	0.000	-0.042
Macey	3.434	2.233	3.719	0.423	1.042	1.524	0.092	-1.864
Warners	2.979	2.168	3.505	0.530	-1.803	4.565	0.366	0.851
Zsivoczky	2.566	0.925	0.638	0.130	1.169	1.918	0.207	-1.477
Hernu	1.824	0.889	0.589	0.238	-0.618	0.537	0.115	-0.898
Nool	3.098	0.295	0.065	0.009	-1.546	3.354	0.249	1.355
Bernard	2.827	1.906	2.709	0.455	-0.086	0.010	0.001	-0.757
Schwarzl	1.971	0.081	0.005	0.002	-1.353	2.572	0.472	0.822
Pogorelov	2.383	0.540	0.217	0.051	0.771	0.834	0.105	1.348
Schoenbeck	1.797	0.114	0.010	0.004	-0.040	0.002	0.000	0.740
Barras	2.224	0.002	0.000	0.000	0.360	0.182	0.026	-1.570
Smith	3.536	0.870	0.565	0.061	1.059	1.576	0.090	-1.643
Averyanov	2.521	0.349	0.091	0.019	-1.559	3.411	0.382	0.283
Ojaniemi	2.338	0.380	0.108	0.026	-0.772	0.838	0.109	-0.371
Smirnov	2.021	-0.485	0.175	0.057	-1.061	1.580	0.275	-1.228
Qi	1.764	-0.434	0.141	0.061	-0.326	0.149	0.034	-1.070
Drews	3.423	-0.249	0.046	0.005	-3.082	13.334	0.811	1.055
Parkhomenko	3.486	-1.069	0.853	0.094	2.093	6.152	0.361	-1.000
Terek	3.282	-0.682	0.347	0.043	0.536	0.403	0.027	2.209
Gomez	2.613	-0.290	0.063	0.012	-1.197	2.011	0.210	-1.306

Turi		3.069		-1.542	1.772	0.252		0.427	0.256	0.019		0.514
Lorenzo		3.510		-2.409	4.324	0.471		-1.583	3.518	0.203		-1.502
Karlivans		2.704		-1.994	2.965	0.544		-0.294	0.122	0.012		-0.343
Korkizoglou		3.975		-0.958	0.684	0.058		2.066	5.995	0.270		2.587
Uldal		2.946		-2.562	4.894	0.757		0.245	0.085	0.007		-0.419
Casarsa		4.921		-2.857	6.085	0.337		3.798	20.252	0.596		0.031
SEBRLE		2.369		0.792	0.467	0.112		0.772	0.836	0.106		0.827
CLAY		3.507		1.235	1.137	0.124		0.575	0.464	0.027		2.141
KARPOV		3.396		1.358	1.375	0.160		0.484	0.329	0.020		1.956
BERNARD		2.763		-0.610	0.277	0.049		-0.875	1.074	0.100		0.890
YURKOV		3.018		-0.586	0.256	0.038		2.131	6.376	0.499		-1.225
WARNERS		2.428		0.357	0.095	0.022		-1.685	3.986	0.482		0.767
ZSIVOCZKY		2.563		0.272	0.055	0.011		-1.094	1.680	0.182		-1.283
McMULLEN		2.561		0.588	0.257	0.053		0.231	0.075	0.008		-0.418
MARTINEAU		3.742		-1.995	2.968	0.284		0.561	0.442	0.022		-0.730
HERNU		2.794		-1.546	1.782	0.306		0.488	0.335	0.031		0.841
BARRAS		1.952		-1.342	1.342	0.472		-0.311	0.136	0.025		0.000
NOOL		3.734		-2.345	4.099	0.394		-1.966	5.429	0.277		-1.336
BOURGUIGNON		4.299		-3.979	11.802	0.857		0.200	0.056	0.002		1.326
		ctr		cos2								
Sebrle		0.146		0.004								
Clay		0.093		0.002								
Karpov		0.003		0.000								
Macey		6.034		0.295								
Warners		1.257		0.082								
Zsivoczky		3.790		0.332								
Hernu		1.401		0.242								
Nool		3.189		0.191								
Bernard		0.995		0.072								
Schwarzl		1.174		0.174								
Pogorelov		3.153		0.320								
Schoenbeck		0.952		0.170								
Barras		4.278		0.498								
Smith		4.689		0.216								
Averyanov		0.139		0.013								
Ojaniemi		0.239		0.025								
Smirnov		2.619		0.369								
Qi		1.987		0.368								
Drews		1.932		0.095								
Parkhomenko		1.736		0.082								
Terek		8.472		0.453								
Gomez		2.962		0.250								
Turi		0.459		0.028								

Lorenzo	3.918	0.183	
Karlivans	0.204	0.016	
Korkizoglou	11.615	0.423	
Uldal	0.305	0.020	
Casarsa	0.002	0.000	
SEBRLE	1.187	0.122	
CLAY	7.960	0.373	
KARPOV	6.644	0.332	
BERNARD	1.375	0.104	
YURKOV	2.606	0.165	
WARNERS	1.020	0.100	
ZSIVOCZKY	2.857	0.250	
McMULLEN	0.303	0.027	
MARTINEAU	0.925	0.038	
HERNU	1.227	0.091	
BARRAS	0.000	0.000	
NOOL	3.101	0.128	
BOURGUIGNON	3.055	0.095	

#### Variables

	Dim.1	ctr	cos2	Dim.2	ctr	cos2	Dim.3	ctr
100m	-0.775	18.344	0.600	0.187	2.016	0.035	-0.184	2.420
Long jump	0.742	16.822	0.550	-0.345	6.869	0.119	0.182	2.363
Shot put	0.623	11.844	0.388	0.598	20.607	0.358	-0.023	0.039
High jump	0.572	9.998	0.327	0.350	7.064	0.123	-0.260	4.794
400m	-0.680	14.116	0.462	0.569	18.666	0.324	0.131	1.230
110m H	-0.746	17.020	0.557	0.229	3.013	0.052	-0.093	0.611
Discus	0.552	9.328	0.305	0.606	21.162	0.368	0.043	0.131
Pole vault	0.050	0.077	0.003	-0.180	1.873	0.033	0.692	34.061
Javeline	0.277	2.347	0.077	0.317	5.784	0.100	-0.390	10.807
1500m	-0.058	0.103	0.003	0.474	12.946	0.225	0.782	43.543
			cos2					
100m		0.034						
Long jump		0.033						
Shot put		0.001						
High jump		0.067						
400m		0.017						
110m H		0.009						
Discus		0.002						
Pole vault		0.479						
Javeline		0.152						
1500m		0.612						

```

Supplementary continuous variables
      Dim.1   cos2    Dim.2   cos2    Dim.3   cos2
Rank      | -0.671  0.450 |  0.051  0.003 | -0.058  0.003 |
Points    |  0.956  0.914 | -0.017  0.000 | -0.066  0.004 |

Supplementary categories
      Dist   Dim.1   cos2 v.test    Dim.2   cos2 v.test    Dim.3
Decastar |  0.946 | -0.600  0.403 -1.430 | -0.038  0.002 -0.123 |  0.289
OlympicG |  0.439 |  0.279  0.403  1.430 |  0.017  0.002  0.123 | -0.134
              cos2 v.test
Decastar     0.093  1.050 |
OlympicG     0.093 -1.050 |

```

```
print(res)
```

\*\*Results for the Principal Component Analysis (PCA)\*\*  
The analysis was performed on 41 individuals, described by 13 variables  
\*The results are available in the following objects:

```

name
1  "$eig"
2  "$var"
3  "$var$coord"
4  "$var$cor"
5  "$var$cos2"
6  "$var$contrib"
7  "$ind"
8  "$ind$coord"
9  "$ind$cos2"
10 "$ind$contrib"
11 "$quanti.sup"
12 "$quanti.sup$coord"
13 "$quanti.sup$cor"
14 "$quali.sup"
15 "$quali.sup$coord"
16 "$quali.sup$v.test"
17 "$call"
18 "$call$centre"
19 "$call$ecart.type"
20 "$call$row.w"
21 "$call$col.w"
description

```

```

1 "eigenvalues"
2 "results for the variables"
3 "coord. for the variables"
4 "correlations variables - dimensions"
5 "cos2 for the variables"
6 "contributions of the variables"
7 "results for the individuals"
8 "coord. for the individuals"
9 "cos2 for the individuals"
10 "contributions of the individuals"
11 "results for the supplementary quantitative variables"
12 "coord. for the supplementary quantitative variables"
13 "correlations suppl. quantitative variables - dimensions"
14 "results for the supplementary categorical variables"
15 "coord. for the supplementary categories"
16 "v-test of the supplementary categories"
17 "summary statistics"
18 "mean of the variables"
19 "standard error of the variables"
20 "weights for the individuals"
21 "weights for the variables"

```

#### 4.5.1. Valores propios y varianza explicada

```

eigenvalues <- res$eig
eigenvalues[, 1:3]

```

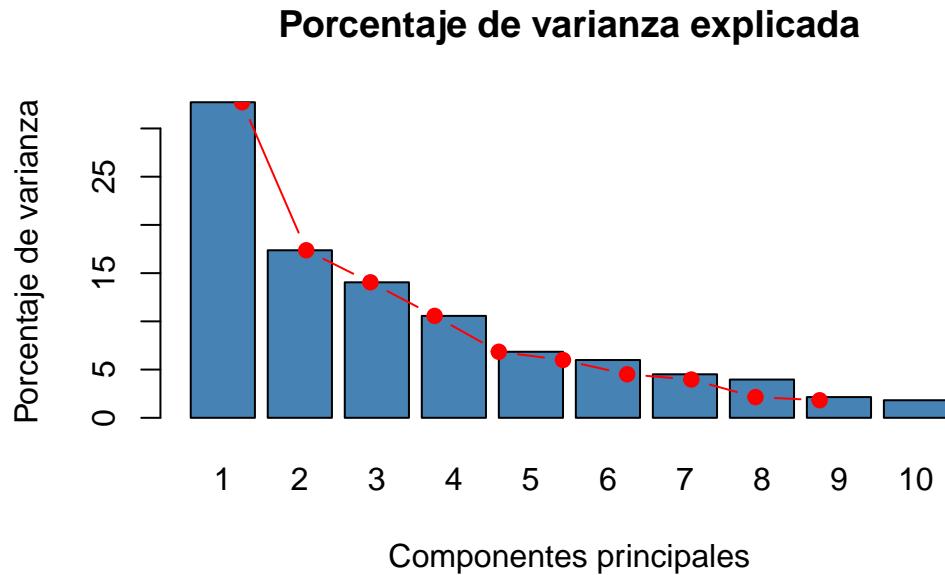
	eigenvalue	percentage of variance	cumulative percentage of variance
comp 1	3.2719055	32.719055	32.71906
comp 2	1.7371310	17.371310	50.09037
comp 3	1.4049167	14.049167	64.13953
comp 4	1.0568504	10.568504	74.70804
comp 5	0.6847735	6.847735	81.55577
comp 6	0.5992687	5.992687	87.54846
comp 7	0.4512353	4.512353	92.06081
comp 8	0.3968766	3.968766	96.02958
comp 9	0.2148149	2.148149	98.17773
comp 10	0.1822275	1.822275	100.00000

```

barplot(
  eigenvalues[, 2],
  names.arg = 1:nrow(eigenvalues),
  main = "Porcentaje de varianza explicada",
  xlab = "Componentes principales",
  ylab = "Porcentaje de varianza",
  col = "steelblue"
)

lines(x = 1:nrow(eigenvalues),
      y = eigenvalues[, 2],
      type = "b",
      pch = 19,
      col = "red")

```



? Tip

- La primera columna son los **valores propios** (`lambda_i`).
- La segunda columna es el **porcentaje de varianza explicada**.
- Fíjate cuántas componentes necesitas para explicar, por ejemplo, el **70 %–80 %** de

la varianza.

#### 4.6. Descripción de las dimensiones

`dimdesc` ayuda a interpretar cada componente, relacionándola con las variables originales:

```
dimdesc(res)
```

\$Dim.1

Link between the variable and the continuous variables (R-square)

	correlation	p.value
Points	0.9561543	2.099191e-22
Long jump	0.7418997	2.849886e-08
Shot put	0.6225026	1.388321e-05
High jump	0.5719453	9.362285e-05
Discus	0.5524665	1.802220e-04
Rank	-0.6705104	1.616348e-06
400m	-0.6796099	1.028175e-06
110m H	-0.7462453	2.136962e-08
100m	-0.7747198	2.778467e-09

\$Dim.2

Link between the variable and the continuous variables (R-square)

	correlation	p.value
Discus	0.6063134	2.650745e-05
Shot put	0.5983033	3.603567e-05
400m	0.5694378	1.020941e-04
1500m	0.4742238	1.734405e-03
High jump	0.3502936	2.475025e-02
Javeline	0.3169891	4.344974e-02
Long jump	-0.3454213	2.696969e-02

\$Dim.3

Link between the variable and the continuous variables (R-square)

	correlation	p.value

```

1500m      0.7821428 1.554450e-09
Pole vault  0.6917567 5.480172e-07
Javeline    -0.3896554 1.179331e-02

```

```
dimdesc(res, axes = c(1, 2, 3, 4, 5))
```

\$Dim.1

Link between the variable and the continuous variables (R-square)

```
=====
correlation      p.value
Points          0.9561543 2.099191e-22
Long jump       0.7418997 2.849886e-08
Shot put         0.6225026 1.388321e-05
High jump        0.5719453 9.362285e-05
Discus          0.5524665 1.802220e-04
Rank             -0.6705104 1.616348e-06
400m            -0.6796099 1.028175e-06
110m H          -0.7462453 2.136962e-08
100m            -0.7747198 2.778467e-09
```

\$Dim.2

Link between the variable and the continuous variables (R-square)

```
=====
correlation      p.value
Discus          0.6063134 2.650745e-05
Shot put         0.5983033 3.603567e-05
400m            0.5694378 1.020941e-04
1500m           0.4742238 1.734405e-03
High jump        0.3502936 2.475025e-02
Javeline         0.3169891 4.344974e-02
Long jump        -0.3454213 2.696969e-02
```

\$Dim.3

Link between the variable and the continuous variables (R-square)

```
=====
correlation      p.value
1500m          0.7821428 1.554450e-09
Pole vault     0.6917567 5.480172e-07
Javeline       -0.3896554 1.179331e-02
```

\$Dim.4

Link between the variable and the continuous variables (R-square)

	correlation	p.value
Javeline	0.7122773	1.761578e-07
Pole vault	0.5515340	1.857748e-04

\$Dim.5

Link between the variable and the continuous variables (R-square)

	correlation	p.value
High jump	0.5554396	0.0001635051
Pole vault	0.3299593	0.0351316637
Rank	-0.3500257	0.0248682140

Link between the variable and the categorical variable (1-way anova)

	R2	p.value
Competition	0.1092271	0.0348183

Link between variable and the categories of the categorical variables

	Estimate	p.value
Competition=Decastar	0.2938609	0.0348183
Competition=OlympicG	-0.2938609	0.0348183

```
dimdesc(res, proba = 0.2)
```

\$Dim.1

Link between the variable and the continuous variables (R-square)

	correlation	p.value
Points	0.9561543	2.099191e-22
Long jump	0.7418997	2.849886e-08
Shot put	0.6225026	1.388321e-05
High jump	0.5719453	9.362285e-05
Discus	0.5524665	1.802220e-04
Javeline	0.2771108	7.942460e-02

Rank	-0.6705104	1.616348e-06
400m	-0.6796099	1.028175e-06
110m H	-0.7462453	2.136962e-08
100m	-0.7747198	2.778467e-09

Link between the variable and the categorical variable (1-way anova)

---

	R2	p.value
Competition	0.05110487	0.1552515

Link between variable and the categories of the categorical variables

---

	Estimate	p.value
Competition=OlympicG	0.4393744	0.1552515
Competition=Decastar	-0.4393744	0.1552515

\$Dim.2

Link between the variable and the continuous variables (R-square)

---

	correlation	p.value
Discus	0.6063134	2.650745e-05
Shot put	0.5983033	3.603567e-05
400m	0.5694378	1.020941e-04
1500m	0.4742238	1.734405e-03
High jump	0.3502936	2.475025e-02
Javeline	0.3169891	4.344974e-02
110m H	0.2287933	1.501925e-01
Long jump	-0.3454213	2.696969e-02

\$Dim.3

Link between the variable and the continuous variables (R-square)

---

	correlation	p.value
1500m	0.7821428	1.554450e-09
Pole vault	0.6917567	5.480172e-07
High jump	-0.2595119	1.013160e-01
Javeline	-0.3896554	1.179331e-02

#### 4.7. Coordenadas, cos2 y contribuciones de las variables

```
# Coordenadas de las variables
res$var$coord
```

	Dim.1	Dim.2	Dim.3	Dim.4	Dim.5
100m	-0.77471983	0.1871420	-0.18440714	-0.03781826	0.30219639
Long jump	0.74189974	-0.3454213	0.18221105	0.10178564	0.03667805
Shot put	0.62250255	0.5983033	-0.02337844	0.19059161	0.11115082
High jump	0.57194530	0.3502936	-0.25951193	-0.13559420	0.55543957
400m	-0.67960994	0.5694378	0.13146970	0.02930198	-0.08769157
110m H	-0.74624532	0.2287933	-0.09263738	0.29083103	0.16432095
Discus	0.55246652	0.6063134	0.04295225	-0.25967143	-0.10482712
Pole vault	0.05034151	-0.1803569	0.69175665	0.55153397	0.32995932
Javeline	0.27711085	0.3169891	-0.38965541	0.71227728	-0.30512892
1500m	-0.05807706	0.4742238	0.78214280	-0.16108904	-0.15356189

```
# Calidad de representación (cos2)
res$var$cos2
```

	Dim.1	Dim.2	Dim.3	Dim.4	Dim.5
100m	0.600190812	0.03502213	0.0340059930	0.0014302206	0.091322660
Long jump	0.550415232	0.11931587	0.0332008675	0.0103603165	0.001345279
Shot put	0.387509426	0.35796686	0.0005465513	0.0363251605	0.012354505
High jump	0.327121422	0.12270561	0.0673464410	0.0183857880	0.308513117
400m	0.461869674	0.32425938	0.0172842817	0.0008586058	0.007689811
110m H	0.556882084	0.05234639	0.0085816841	0.0845826853	0.027001375
Discus	0.305219255	0.36761593	0.0018448960	0.0674292539	0.010988725
Pole vault	0.002534268	0.03252860	0.4785272696	0.3041897208	0.108873151
Javeline	0.076790421	0.10048206	0.1518313365	0.5073389244	0.093103658
1500m	0.003372945	0.22488818	0.6117473613	0.0259496775	0.023581254

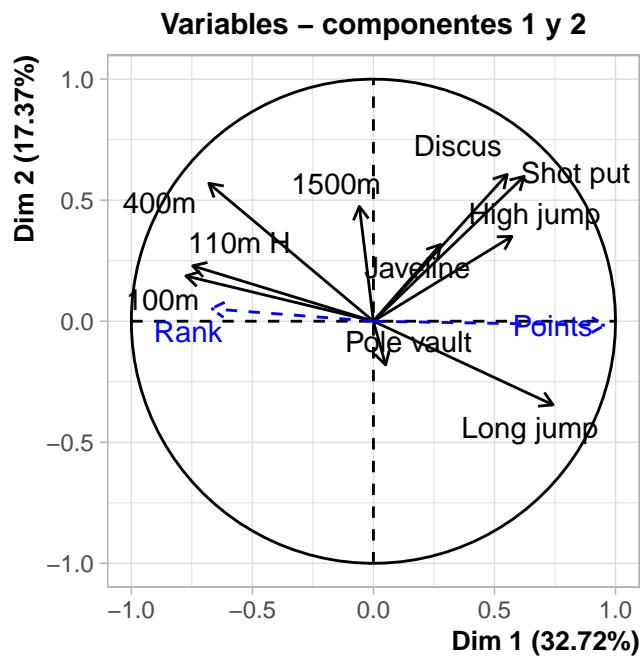
```
# Contribuciones (en %)
res$var$contrib
```

	Dim.1	Dim.2	Dim.3	Dim.4	Dim.5
100m	18.34376957	2.016090	2.42049891	0.13532858	13.336184
Long jump	16.82246707	6.868559	2.36319121	0.98030118	0.196456
Shot put	11.84353954	20.606785	0.03890276	3.43711486	1.804174
High jump	9.99788710	7.063694	4.79362526	1.73967752	45.053306

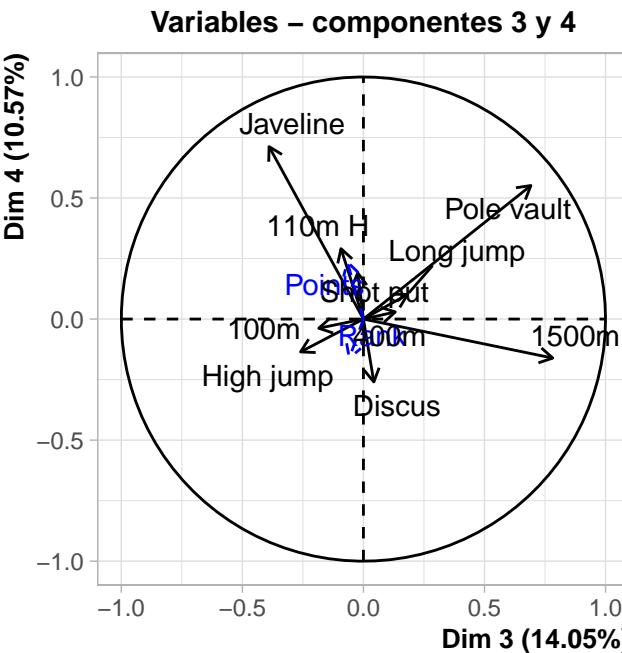
400m	14.11622887	18.666374	1.23027094	0.08124195	1.122971
110m H	17.02011495	3.013382	0.61083225	8.00327927	3.943110
Discus	9.32848615	21.162245	0.13131711	6.38020830	1.604724
Pole vault	0.07745541	1.872547	34.06090024	28.78266727	15.899147
Javeline	2.34696326	5.784369	10.80714169	48.00480246	13.596270
1500m	0.10308808	12.945954	43.54331962	2.45537861	3.443657

#### 4.7.1. Gráfico de variables

```
plot(res, choix = "var", title = "Variables - componentes 1 y 2", axes = 1:2)
```



```
plot(res, choix = "var", title = "Variables - componentes 3 y 4", axes = 3:4)
```



**i** Nota

- Las variables cercanas unas a otras están **positivamente correlacionadas**.
- Variables opuestas en un mismo eje tienen correlación **negativa**.
- La distancia al origen indica la **importancia** en esas componentes ( $\cos^2$  alto).

#### 4.8. Individuos: coordenadas, cos2 y contribuciones

```
# Coordenadas de los individuos
head(res$ind$coord)
```

	Dim.1	Dim.2	Dim.3	Dim.4	Dim.5
Sebrle	4.0384485	1.36582606	-0.2899565	1.9411341	0.3769545
Clay	3.9193652	0.83696136	0.2311753	1.4939721	-1.0376085
Karpov	4.6199873	0.03999523	-0.0415858	-1.3135257	0.1877295
Macey	2.2334606	1.04176620	-1.8643620	-0.7432135	0.9772701
Warners	2.1683964	-1.80320025	0.8510173	-0.2845996	-0.1513946
Zsivoczky	0.9251322	1.16865180	-1.4774803	0.8075947	0.8729726

```
# Calidad de representación  
head(res$ind$cos2)
```

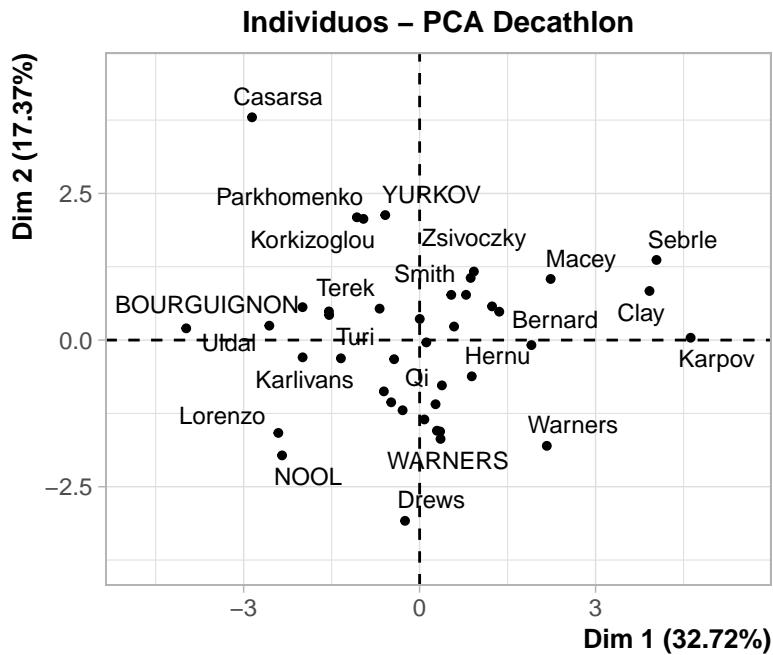
	Dim.1	Dim.2	Dim.3	Dim.4	Dim.5
Sebrle	0.6954102	7.954314e-02	0.0035849052	0.160665653	0.006058846
Clay	0.7112052	3.243204e-02	0.0024742661	0.103335239	0.049846023
Karpov	0.8517553	6.383365e-05	0.0000690118	0.068851025	0.001406366
Macey	0.4230486	9.203950e-02	0.2947774222	0.046844751	0.080995884
Warners	0.5299437	3.664716e-01	0.0816261239	0.009128951	0.002583291
Zsivoczky	0.1299979	2.074432e-01	0.3315677581	0.099063996	0.115752425

```
# Contribuciones  
head(res$ind$contrib)
```

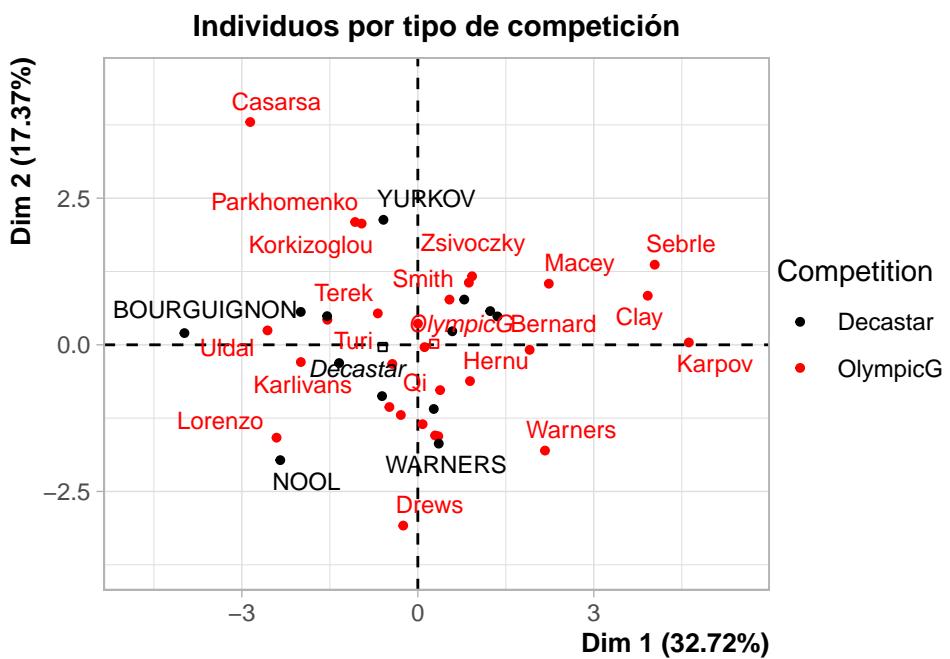
	Dim.1	Dim.2	Dim.3	Dim.4	Dim.5
Sebrle	12.1575058	2.619234357	0.145959136	8.6958838	0.50611259
Clay	11.4510904	0.983545343	0.092778749	5.1509536	3.83474272
Karpov	15.9109806	0.002245949	0.003002311	3.9818030	0.12552619
Macey	3.7185358	1.523786399	6.034288443	1.2747642	3.40171873
Warners	3.5050382	4.565322740	1.257310032	0.1869266	0.08163758
Zsivoczky	0.6380034	1.917581489	3.789736079	1.5051840	2.71437829

#### 4.8.1. Gráfico de individuos

```
# Sin mostrar las variables cualitativas  
plot(res, cex = 0.8, invisible = "quali",  
      title = "Individuos - PCA Decathlon")
```

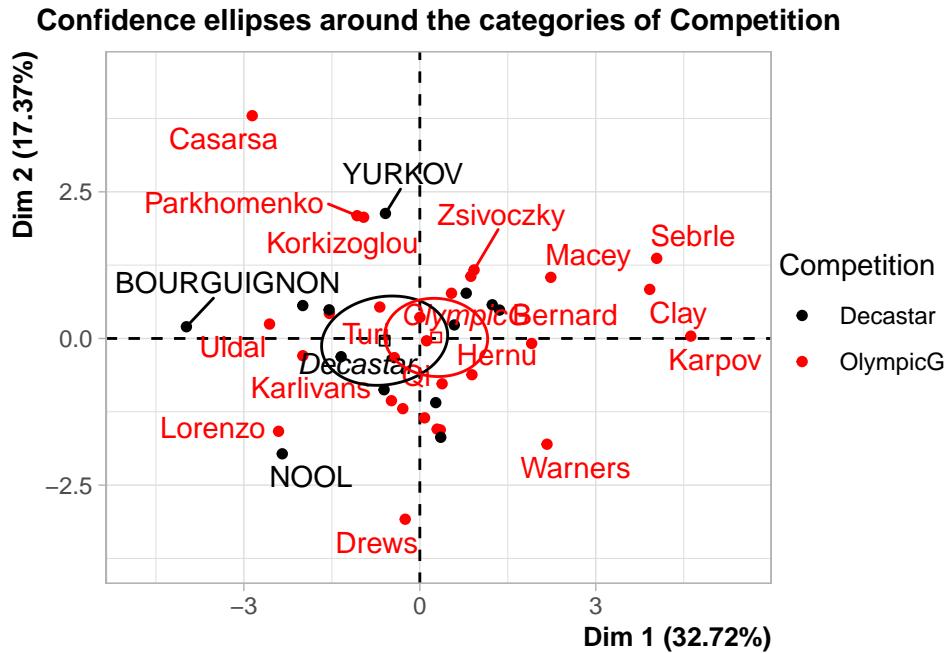


```
# Coloreando por competición (columna 13)
plot(res, cex = 0.8, habillage = 13,
     title = "Individuos por tipo de competición")
```



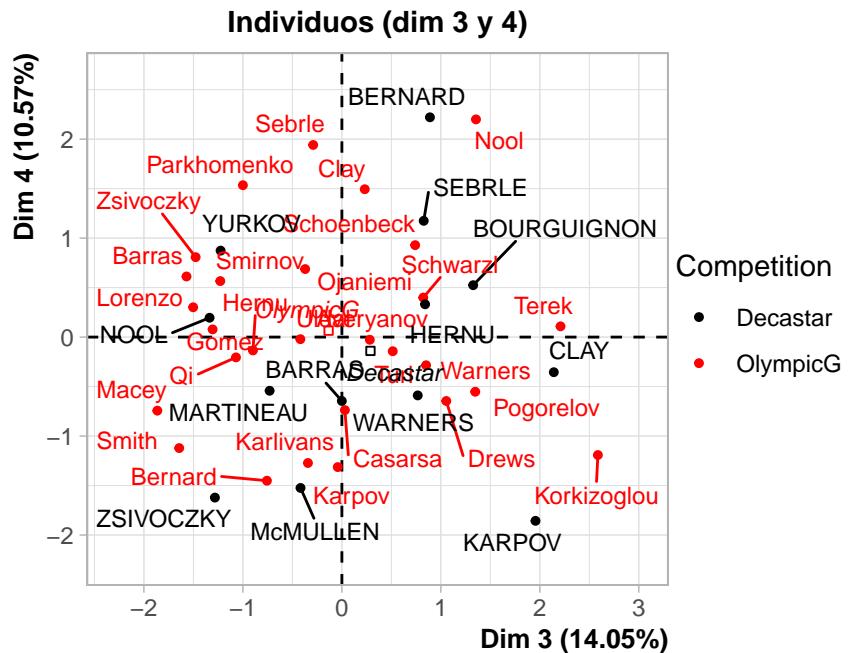
#### 4.8.2. Elipses de confianza por categoría

```
plotellipses(res)
```

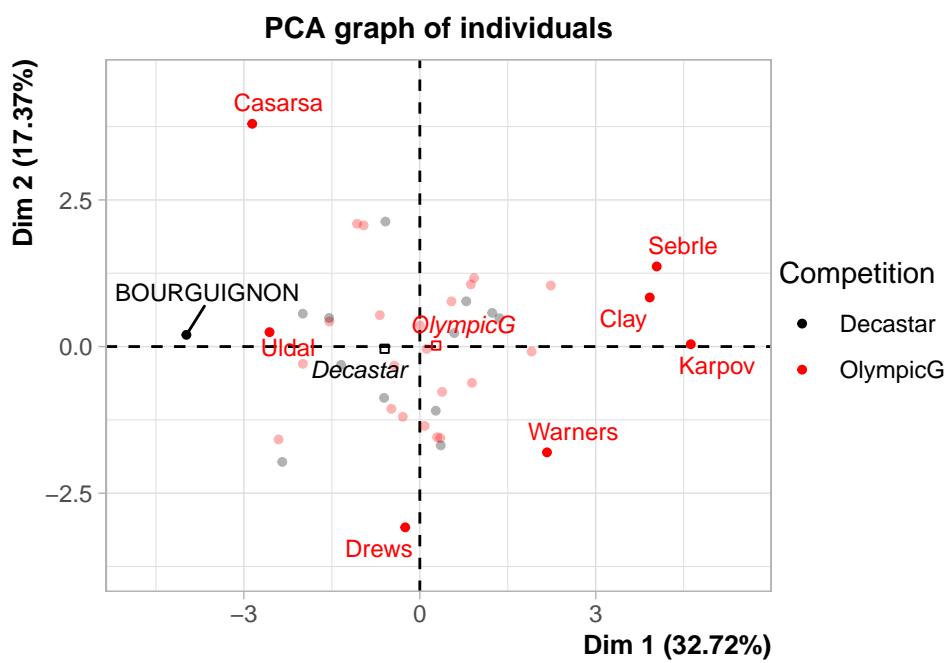


#### 4.8.3. Otras vistas y selección de individuos

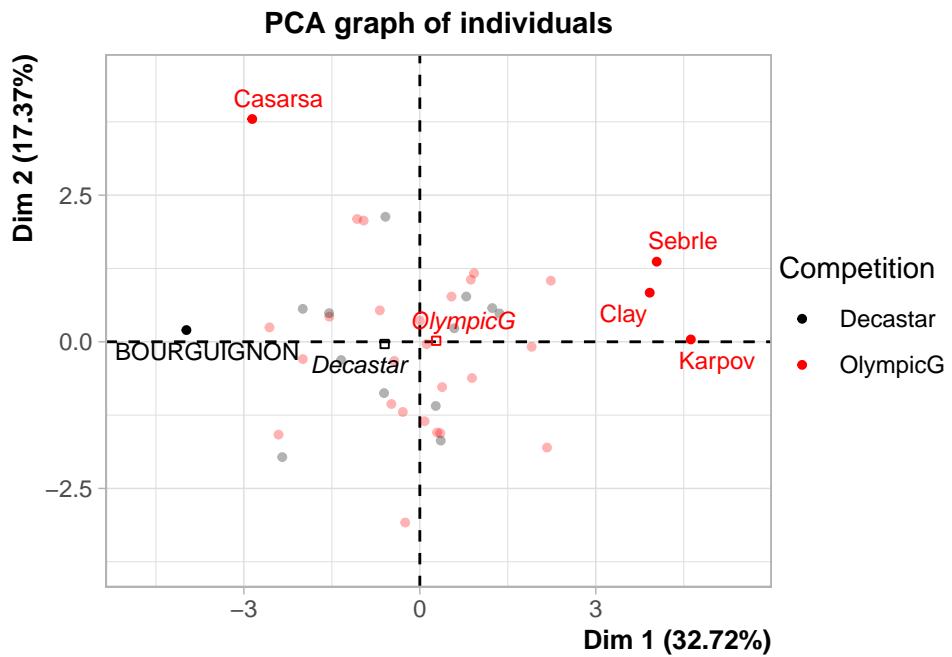
```
# Dimensiones 3 y 4
plot(res, choix = "ind", cex = 0.8, habillage = 13,
      title = "Individuos (dim 3 y 4)", axes = 3:4)
```



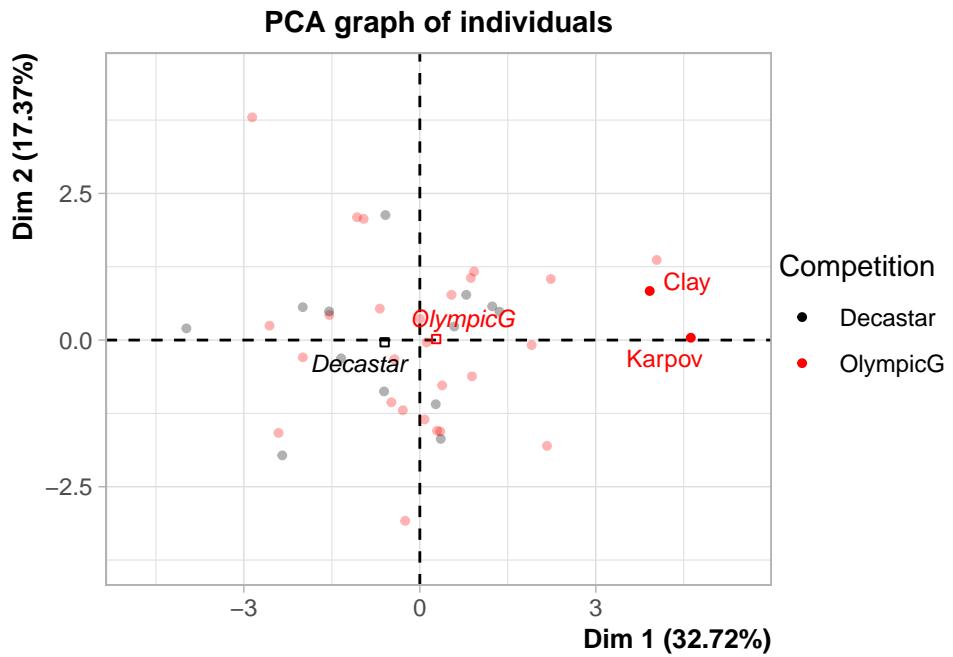
```
# Seleccionar individuos bien representados  
plot(res, cex = 0.8, habillage = 13, select = "cos2 0.7")
```



```
# Seleccionar individuos con mayor contribución  
plot(res, cex = 0.8, habillage = 13, select = "contrib 5")
```

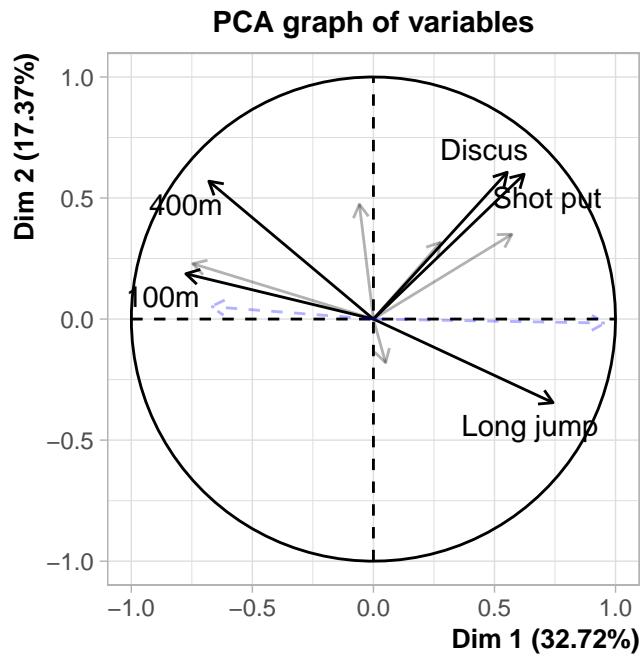


```
# Seleccionar individuos específicos (por nombre)  
plot(res, cex = 0.8, habillage = 13,  
     select = c("Clay", "Karpov"))
```



#### 4.8.4. Selección de variables importantes

```
plot(res, choix = "var", select = "contrib 5")
```

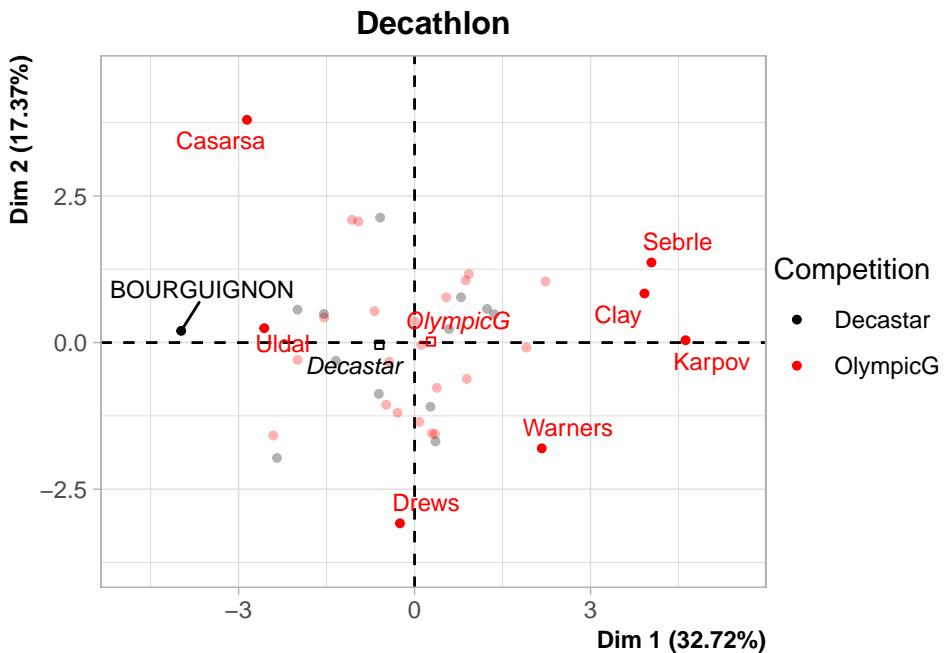


#### 4.8.5. Gráfico con varios argumentos

```

plot(
  res,
  cex      = 0.8,
  habillage = 13,
  select   = "cos2 0.7",
  title    = "Decathlon",
  cex.main = 1.1,
  cex.axis = 0.9,
  shadow   = TRUE,
  auto     = "y"
)

```



## 5. Parte 2: PCA con datos de jugos de naranja (orange)

En el segundo bloque analizamos una base de datos de jugos de naranja:

- Columnas 1 a 14: **variables cuantitativas** (por ejemplo, atributos químicos/sensoriales).
- Columnas 15 y 16: **variables cualitativas supplementarias** (por ejemplo, origen, tipo de marca, etc.).

### 5.1. Importar datos de orange

```
archivo_orange <- file.path(ruta_datos, "orange.csv")

orange <- read.table(
  archivo_orange,
  header      = TRUE,
  sep         = ";",
```

```

dec      = ".",
row.names = 1,
check.names = FALSE
)

```

```
summary(orange)
```

	Odour intensity	Odour typicality	Pulpiness	Intensity of taste
Min.	:2.760	Min. :2.530	Min. :1.660	Min. :3.120
1st Qu.	:2.775	1st Qu.:2.625	1st Qu.:1.722	1st Qu.:3.265
Median	:2.825	Median :2.775	Median :2.625	Median :3.410
Mean	:2.907	Mean :2.762	Mean :2.710	Mean :3.362
3rd Qu.	:3.010	3rd Qu.:2.865	3rd Qu.:3.603	3rd Qu.:3.458
Max.	:3.200	Max. :3.020	Max. :4.000	Max. :3.540
	Acidity	Bitterness	Sweetness	Glucose
Min.	:2.330	Min. :1.760	Min. :2.600	Min. :17.33
1st Qu.	:2.453	1st Qu.:1.998	1st Qu.:2.825	1st Qu.:22.94
Median	:2.800	Median :2.320	Median :3.110	Median :24.48
Mean	:2.802	Mean :2.328	Mean :3.057	Mean :24.76
3rd Qu.	:3.125	3rd Qu.:2.612	3rd Qu.:3.335	3rd Qu.:26.70
Max.	:3.310	Max. :2.970	Max. :3.380	Max. :32.42
	Fructose	Saccharose	Sweetening power	pH
Min.	:20.00	Min. :22.92	Min. : 82.55	Min. :3.590
1st Qu.	:25.40	1st Qu.:37.07	1st Qu.: 90.14	1st Qu.:3.620
Median	:26.50	Median :41.55	Median : 92.79	Median :3.750
Mean	:27.06	Mean :40.06	Mean : 92.80	Mean :3.738
3rd Qu.	:28.95	3rd Qu.:45.39	3rd Qu.: 96.10	3rd Qu.:3.842
Max.	:34.54	Max. :52.12	Max. :102.22	Max. :3.890
	Citric acid	Vitamin C	Way of preserving	Origin
Min.	:0.6700	Min. :27.00	Length:6	Length:6
1st Qu.	:0.6950	1st Qu.:33.67	Class :character	Class :character
Median	:0.7250	Median :36.80	Mode :character	Mode :character
Mean	:0.7667	Mean :36.04		
3rd Qu.	:0.8150	3rd Qu.:38.88		
Max.	:0.9500	Max. :43.44		

## 5.2. PCA con variables activas (1 a 14)

```

res <- PCA(orange[, 1:14], ncp = 3)
summary(res)

```

Call:

PCA(X = orange[, 1:14], ncp = 3)

Eigenvalues

	Dim.1	Dim.2	Dim.3	Dim.4	Dim.5
Variance	8.065	2.583	1.461	1.011	0.880
% of var.	57.607	18.450	10.438	7.220	6.285
Cumulative % of var.	57.607	76.057	86.496	93.715	100.000

Individuals

	Dist	Dim.1	ctr	cos2	Dim.2	ctr	cos2
Pampryl amb.	3.700   -3.090 19.727 0.697   -1.026 6.797 0.077						
Tropicana amb.	3.885   2.358 11.486 0.368   -2.825 51.491 0.529						
Fruvita fr.	3.536   2.688 14.928 0.578   1.385 12.373 0.153						
Joker amb.	4.300   -3.895 31.359 0.821   -0.127 0.104 0.001						
Tropicana fr.	3.800   3.092 19.757 0.662   0.533 1.830 0.020						
Pampryl fr.	3.128   -1.152 2.743 0.136   2.061 27.405 0.434						
	Dim.3	ctr	cos2				
Pampryl amb.	0.022 0.006 0.000						
Tropicana amb.	-1.076 13.205 0.077						
Fruvita fr.	-0.076 0.067 0.000						
Joker amb.	1.184 15.976 0.076						
Tropicana fr.	1.734 34.298 0.208						
Pampryl fr.	-1.788 36.449 0.327						

Variables (the 10 first)

	Dim.1	ctr	cos2	Dim.2	ctr	cos2	Dim.3
Odour intensity	0.352 1.533 0.124						
Odour typicality	0.938 10.898 0.879						
Pulpiness	0.666 5.498 0.443						
Intensity of taste	-0.554 3.800 0.306						
Acidity	-0.878 9.560 0.771						
Bitterness	-0.897 9.978 0.805						
Sweetness	0.958 11.384 0.918						
Glucose	-0.786 7.652 0.617						
Fructose	-0.781 7.554 0.609						
Saccharose	0.891 9.841 0.794						
	ctr	cos2					
Odour intensity	1.936 0.028						
Odour typicality	2.336 0.034						
Pulpiness	0.006 0.000						

```

Intensity of taste 21.785 0.318 |
Acidity           11.316 0.165 |
Bitterness        2.301 0.034 |
Sweetness         0.315 0.005 |
Glucose           5.376 0.079 |
Fructose          5.683 0.083 |
Saccharose         2.925 0.043 |

```

```
print(res)
```

\*\*Results for the Principal Component Analysis (PCA)\*\*

The analysis was performed on 6 individuals, described by 14 variables

\*The results are available in the following objects:

name	description
1 "\$eig"	"eigenvalues"
2 "\$var"	"results for the variables"
3 "\$var\$coord"	"coord. for the variables"
4 "\$var\$cor"	"correlations variables - dimensions"
5 "\$var\$cos2"	"cos2 for the variables"
6 "\$var\$contrib"	"contributions of the variables"
7 "\$ind"	"results for the individuals"
8 "\$ind\$coord"	"coord. for the individuals"
9 "\$ind\$cos2"	"cos2 for the individuals"
10 "\$ind\$contrib"	"contributions of the individuals"
11 "\$call"	"summary statistics"
12 "\$call\$centre"	"mean of the variables"
13 "\$call\$ecart.type"	"standard error of the variables"
14 "\$call\$row.w"	"weights for the individuals"
15 "\$call\$col.w"	"weights for the variables"

### 5.3. PCA con variables suplementarias

Consideramos ahora que las columnas 15 y 16 son cualitativas suplementarias (por ejemplo, **origen** del jugo).

```
res2 <- PCA(orange, quali.sup = 15:16)
summary(res2, nbelements = Inf)
```

Call:

PCA(X = orange, quali.sup = 15:16)

#### Eigenvalues

	Dim.1	Dim.2	Dim.3	Dim.4	Dim.5
Variance	8.065	2.583	1.461	1.011	0.880
% of var.	57.607	18.450	10.438	7.220	6.285
Cumulative % of var.	57.607	76.057	86.496	93.715	100.000

#### Individuals

	Dist	Dim.1	ctr	cos2	Dim.2	ctr	cos2	Dim.3	ctr	cos2
Pampryl amb.	3.700   -3.090 19.727 0.697   -1.026 6.797 0.077									
Tropicana amb.	3.885   2.358 11.486 0.368   -2.825 51.491 0.529									
Fruvita fr.	3.536   2.688 14.928 0.578   1.385 12.373 0.153									
Joker amb.	4.300   -3.895 31.359 0.821   -0.127 0.104 0.001									
Tropicana fr.	3.800   3.092 19.757 0.662   0.533 1.830 0.020									
Pampryl fr.	3.128   -1.152 2.743 0.136   2.061 27.405 0.434									
		Dim.3	ctr	cos2						
Pampryl amb.	0.022 0.006 0.000									
Tropicana amb.	-1.076 13.205 0.077									
Fruvita fr.	-0.076 0.067 0.000									
Joker amb.	1.184 15.976 0.076									
Tropicana fr.	1.734 34.298 0.208									
Pampryl fr.	-1.788 36.449 0.327									

#### Variables

	Dim.1	ctr	cos2	Dim.2	ctr	cos2	Dim.3
Odour intensity	0.352 1.533 0.124						
Odour typicality	0.938 10.898 0.879						
Pulpiness	0.666 5.498 0.443						
Intensity of taste	-0.554 3.800 0.306						
Acidity	-0.878 9.560 0.771						
Bitterness	-0.897 9.978 0.805						
Sweetness	0.958 11.384 0.918						
Glucose	-0.786 7.652 0.617						
Fructose	-0.781 7.554 0.609						
Saccharose	0.891 9.841 0.794						
Sweetening power	0.194 0.467 0.038						
pH	0.964 11.525 0.930						
Citric acid	-0.906 10.173 0.820						
Vitamin C	-0.105 0.136 0.011						
		ctr	cos2				

Odour intensity	1.936	0.028	
Odour typicality	2.336	0.034	
Pulpiness	0.006	0.000	
Intensity of taste	21.785	0.318	
Acidity	11.316	0.165	
Bitterness	2.301	0.034	
Sweetness	0.315	0.005	
Glucose	5.376	0.079	
Fructose	5.683	0.083	
Saccharose	2.925	0.043	
Sweetening power	0.778	0.011	
pH	0.776	0.011	
Citric acid	10.368	0.152	
Vitamin C	34.099	0.498	

#### Supplementary categories

	Dist	Dim.1	cos2	v.test	Dim.2	cos2	v.test	
Ambient	2.040	-1.543	0.572	-1.215	-1.326	0.423	-1.845	
Fresh	2.040	1.543	0.572	1.215	1.326	0.423	1.845	
Florida	2.741	2.712	0.979	2.136	-0.303	0.012	-0.421	
Other	2.741	-2.712	0.979	-2.136	0.303	0.012	0.421	
	Dim.3	cos2	v.test					
Ambient	0.043	0.000	0.080					
Fresh	-0.043	0.000	-0.080					
Florida	0.194	0.005	0.359					
Other	-0.194	0.005	-0.359					

#### 5.4. Valores propios y varianza explicada

```
eigenvalues <- res$eig
eigenvalues[, 1:3]
```

	eigenvalue	percentage of variance	cumulative percentage of variance
comp 1	8.0649505	57.606789	57.60679
comp 2	2.5830341	18.450244	76.05703
comp 3	1.4613887	10.438490	86.49552
comp 4	1.0107601	7.219715	93.71524
comp 5	0.8798667	6.284762	100.00000

## 5.5. Test de Bartlett y KMO para orange

```
bartlett.test(orange[, 1:14])
```

```
Bartlett test of homogeneity of variances

data: orange[, 1:14]
Bartlett's K-squared = 210.49, df = 13, p-value < 2.2e-16
```

```
KMO(orange[, 1:14])
```

```
Error in solve.default(r) :
  sistema es computacionalmente singular: número de condición recíproco = 1.37813e-18

Kaiser-Meyer-Olkin factor adequacy
Call: KMO(r = orange[, 1:14])
Overall MSA = 0.5
MSA for each item =
      Odour intensity   Odour typicality       Pulpiness Intensity of taste
          0.5                  0.5                 0.5                  0.5
      Acidity           Bitterness       Sweetness        Glucose
          0.5                  0.5                 0.5                  0.5
      Fructose         Saccharose   Sweetening power        pH
          0.5                  0.5                 0.5                  0.5
      Citric acid       Vitamin C
          0.5                  0.5
```

## 5.6. Descripción de las dimensiones

```
dimdesc(res)
```

```
$Dim.1
```

```
Link between the variable and the continuous variables (R-square)
=====
```

```
correlation     p.value
```

```

pH          0.9641128 0.001908728
Sweetness    0.9581672 0.002588371
Odour typicality 0.9375068 0.005736073
Saccharose    0.8908878 0.017208699
Acidity       -0.8780844 0.021389085
Bitterness     -0.8970717 0.015346133
Citric acid   -0.9057713 0.012900241

```

\$Dim.2

Link between the variable and the continuous variables (R-square)

```

=====
correlation p.value
Sweetening power 0.9147625 0.0105885

```

\$Dim.3

```
dimdesc(res, axes = c(1, 2))
```

\$Dim.1

Link between the variable and the continuous variables (R-square)

```

=====
correlation p.value
pH          0.9641128 0.001908728
Sweetness    0.9581672 0.002588371
Odour typicality 0.9375068 0.005736073
Saccharose    0.8908878 0.017208699
Acidity       -0.8780844 0.021389085
Bitterness     -0.8970717 0.015346133
Citric acid   -0.9057713 0.012900241

```

\$Dim.2

Link between the variable and the continuous variables (R-square)

```

=====
correlation p.value
Sweetening power 0.9147625 0.0105885

```

```
dimdesc(res, proba = 0.2)
```

\$Dim.1

```
Link between the variable and the continuous variables (R-square)
```

	correlation	p.value
pH	0.9641128	0.001908728
Sweetness	0.9581672	0.002588371
Odour typicality	0.9375068	0.005736073
Saccharose	0.8908878	0.017208699
Pulpiness	0.6659126	0.148777121
Fructose	-0.7805085	0.066977593
Glucose	-0.7856011	0.064022682
Acidity	-0.8780844	0.021389085
Bitterness	-0.8970717	0.015346133
Citric acid	-0.9057713	0.012900241

```
$Dim.2
```

```
Link between the variable and the continuous variables (R-square)
```

	correlation	p.value
Sweetening power	0.9147625	0.01058850
Pulpiness	0.7422675	0.09107899

```
$Dim.3
```

```
Link between the variable and the continuous variables (R-square)
```

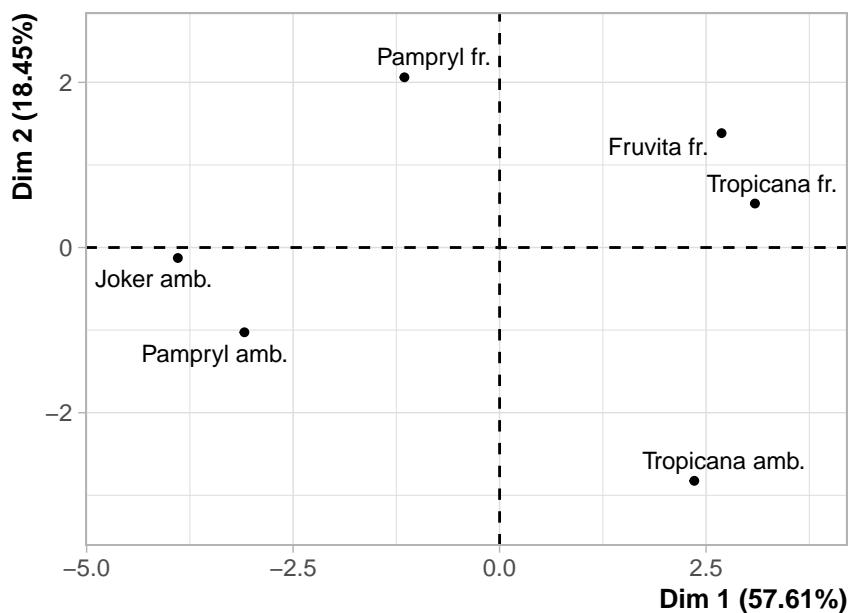
	correlation	p.value
Vitamin C	0.7059171	0.1170103

## 5.7. Gráficos de individuos y variables

### 5.7.1. Individuos

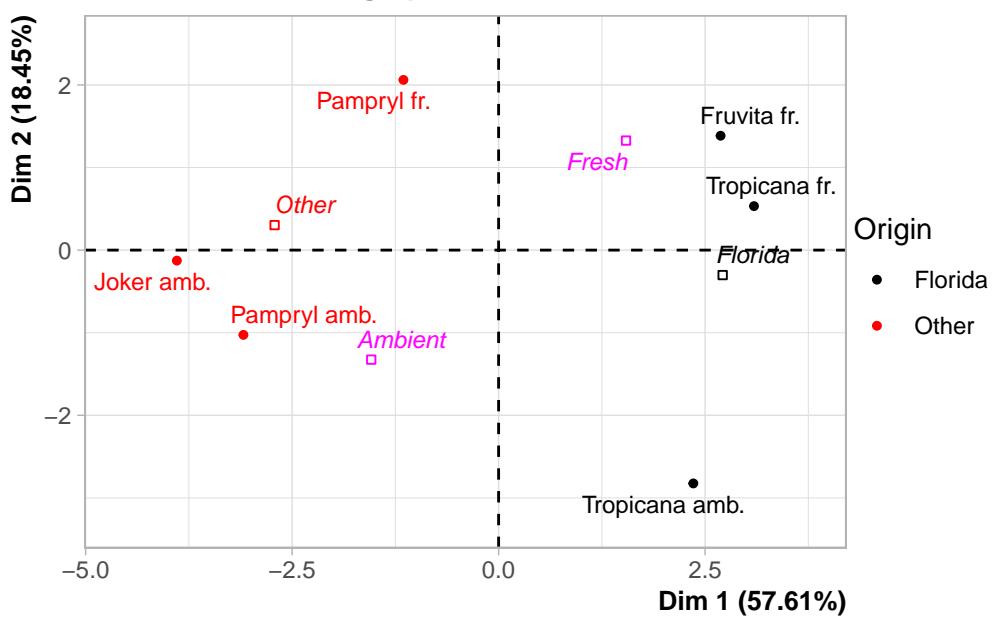
```
# Individuos sin variables cualitativas
plot(res, cex = 0.8, invisible = "quali",
      title = "Individuos - PCA Orange")
```

**Individuos – PCA Orange**



```
# Coloreados según variable de origen (si existe en el dataset)
plot(res2, cex = 0.8, habillage = "Origin")
```

**PCA graph of individuals**



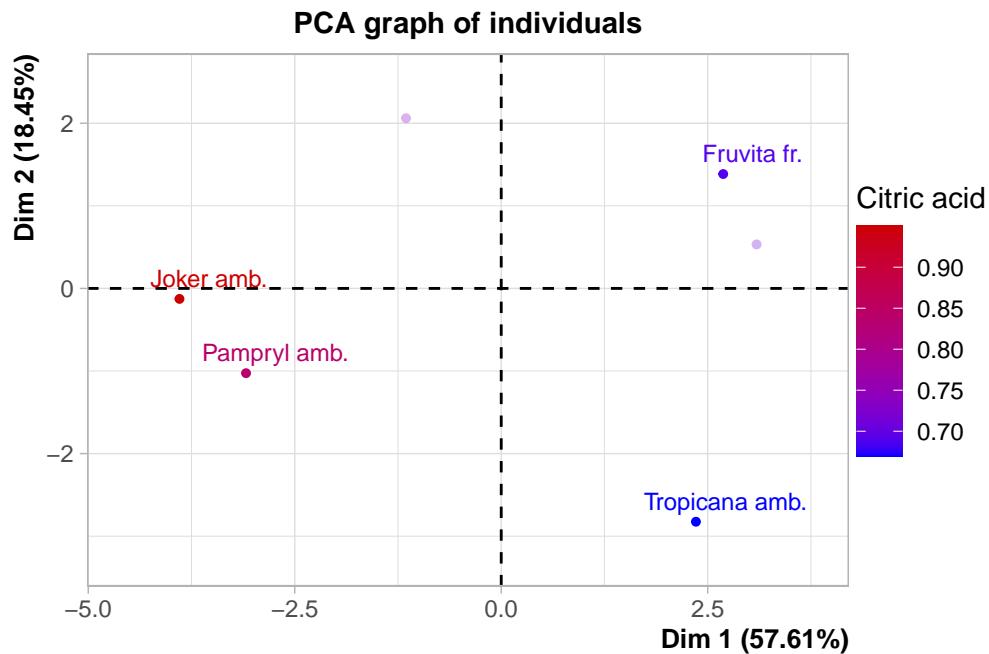
### 5.7.2. Elipses de confianza

```
plotellipses(res)
```

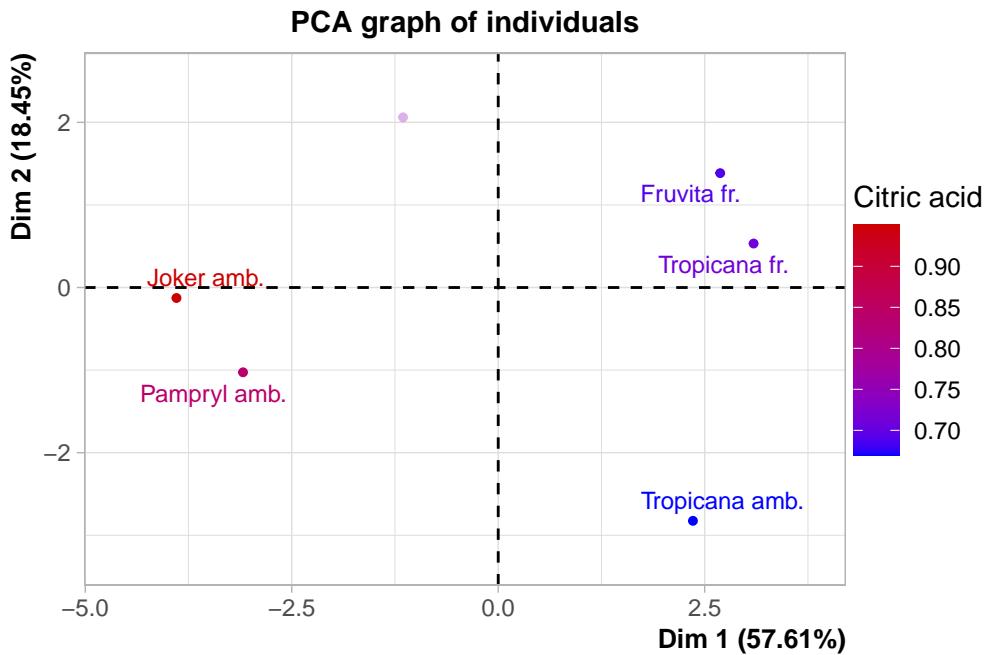
NULL

### 5.7.3. Selección de individuos y variables

```
# Individuos bien representados  
plot(res, cex = 0.8, habillage = 13, select = "cos2 0.7")
```

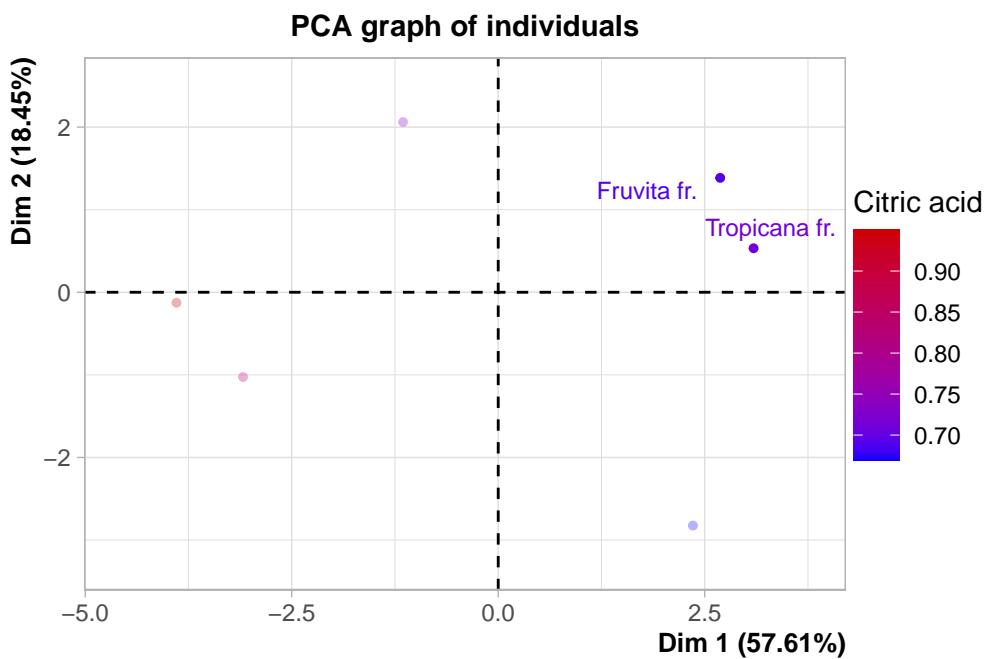


```
# Individuos con mayor contribución  
plot(res, cex = 0.8, habillage = 13, select = "contrib 5")
```

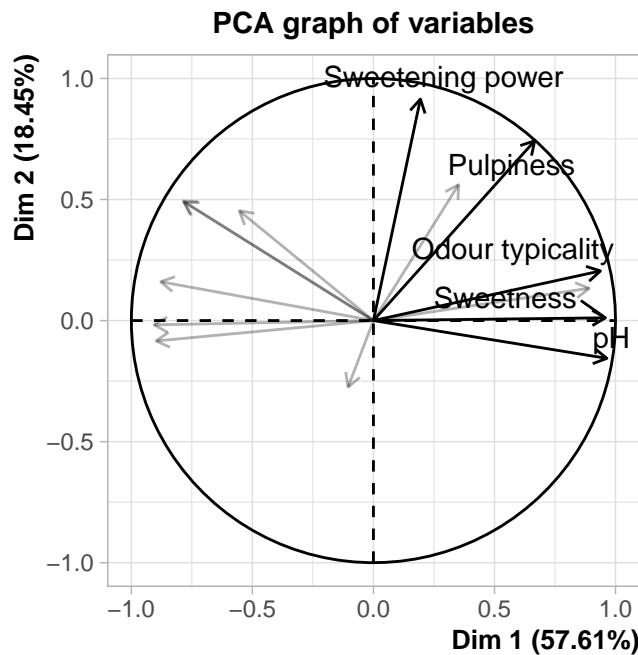


```
# Individuos específicos (ejemplo: marcas específicas)
plot(res, cex = 0.8, habillage = 13,
     select = c("Fruvita fr.", "Tropicana fr."))

```

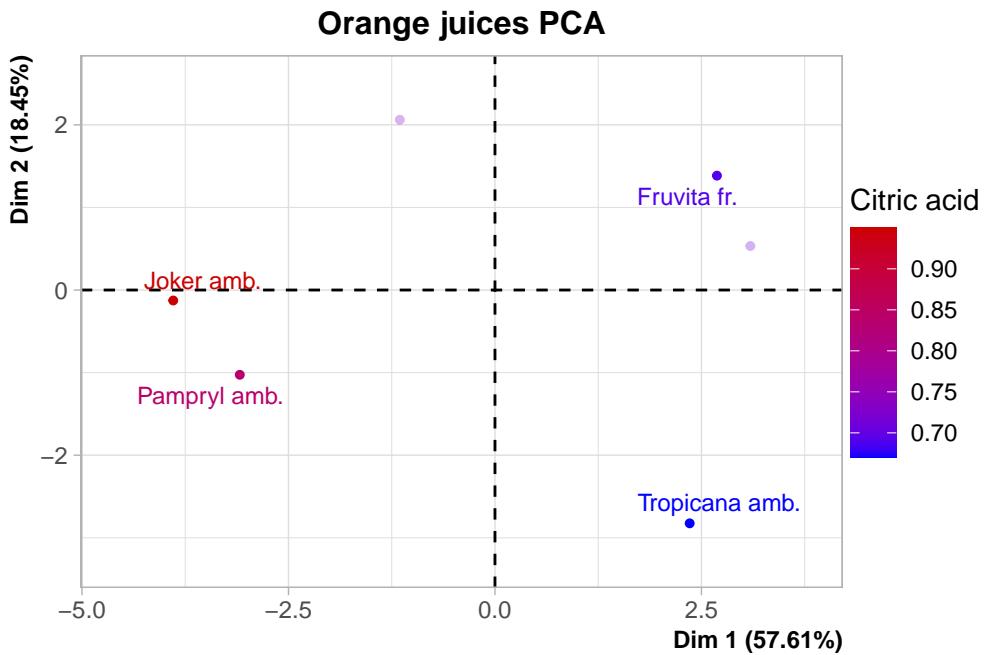


```
# Variables que más contribuyen
plot(res, choix = "var", select = "contrib 5")
```



#### 5.7.4. Gráfico combinado (orange)

```
plot(
  res,
  cex      = 0.8,
  habillage = 13,
  select   = "cos2 0.7",
  title    = "Orange juices PCA",
  cex.main = 1.1,
  cex.axis = 0.9,
  shadow   = TRUE,
  auto     = "y"
)
```



## 6. Cierre del laboratorio

En este laboratorio:

- Aplicaste **PCA** a dos conjuntos de datos reales (Decathlon y jugos de naranja).
- Revisaste la idoneidad del análisis mediante **Bartlett** y **KMO**.
- Interpretaste **valores propios**, varianza explicada y elegiste cuántas componentes retener.
- Analizaste:
  - Cargas de las variables (coordenadas y cos2).
  - Contribuciones de variables e individuos.

- Gráficos de individuos por grupos, con **elipses de confianza**.
- Variables e individuos “más importantes” (por contribución o calidad de representación).

Estos elementos son fundamentales para:

- Reducir dimensionalidad en bases con muchas variables.
- Explorar estructuras latentes antes de aplicar otros métodos (clustering, regresión, etc.).
- Construir indicadores sintéticos a partir de múltiples variables observadas.