Capitulo_4_Uso_de_la_Regresion_Multiple

Econometría para la Gestión (ECO_EPG) - FEN UAH

Tabla de contenidos

1. 1. Material descargable

Descargar PDF de contenidos teóricos

2. Configuración inicial en R

2.1. Carga de librerías

```
library(openxlsx)
library(MASS)      # funciones adicionales para modelos lineales
library(corrplot)  # correlaciones gráficas
library(lmtest)    # pruebas como Durbin-Watson, Breusch-Pagan
library(ggplot2)   # gráficos avanzados
```

2.2. Definir ruta de trabajo

En tu proyecto utilizaremos la ruta:

```
ruta_datos <- "C:/Users/manue/Desktop/lab-econometria/labs_epg/data_epg"

# Verificamos que la carpeta exista y revisamos algunos archivos
list.files(ruta_datos)
```

```
 [1] "annos_mantenimiento.xlsx" "auto_peso_consumo.xlsx"
 [3] "costos.xlsx"              "data_PCA_Decathlon.csv"
 [5] "data_PCA_ExpertWine.csv"  "Ejemplo1.xlsx"
 [7] "Ejemplo2.xlsx"            "millaje.txt"
 [9] "orange.csv"               "tabla_ejemplo_R.xlsx"
```

> **Tip**
>
> Si copias este laboratorio a otro computador, solo deberás **cambiar la ruta** de `ruta_datos` para que apunte a la nueva carpeta donde estén `millaje.txt` y otros archivos de datos.

3. Parte 1: Regresión múltiple con inversión publicitaria (tv, radio, periodico)

En esta primera parte trabajaremos con un ejemplo clásico de marketing:

- **tv**: gasto en publicidad en TV (en miles de dólares).
- **radio**: gasto en publicidad en radio.
- **periodico**: gasto en publicidad en periódicos.
- **ventas**: ventas del producto (en miles de unidades).

La idea es entender **cómo se relacionan las ventas con los distintos medios publicitarios**, usando regresión múltiple.

3.1. Crear el conjunto de datos

El script genera los vectores directamente en R y luego los combina en un `data.frame`:

```r
tv <- c(230.1, 44.5, 17.2, 151.5, 180.8, 8.7, 57.5, 120.2, 8.6, 199.8, 66.1, 214.7,
        23.8, 97.5, 204.1, 195.4, 67.8, 281.4, 69.2, 147.3, 218.4, 237.4, 13.2,
        228.3, 62.3, 262.9, 142.9, 240.1, 248.8, 70.6, 292.9, 112.9, 97.2, 265.6,
        95.7, 290.7, 266.9, 74.7, 43.1, 228.0, 202.5, 177.0, 293.6, 206.9, 25.1,
        175.1, 89.7, 239.9, 227.2, 66.9, 199.8, 100.4, 216.4, 182.6, 262.7, 198.9,
        7.3, 136.2, 210.8, 210.7, 53.5, 261.3, 239.3, 102.7, 131.1, 69.0, 31.5,
        139.3, 237.4, 216.8, 199.1, 109.8, 26.8, 129.4, 213.4, 16.9, 27.5, 120.5,
        5.4, 116.0, 76.4, 239.8, 75.3, 68.4, 213.5, 193.2, 76.3, 110.7, 88.3,
        109.8, 134.3, 28.6, 217.7, 250.9, 107.4, 163.3, 197.6, 184.9, 289.7,
        135.2, 222.4, 296.4, 280.2, 187.9, 238.2, 137.9, 25.0, 90.4, 13.1, 255.4,
        225.8, 241.7, 175.7, 209.6, 78.2, 75.1, 139.2, 76.4, 125.7, 19.4, 141.3,
        18.8, 224.0, 123.1, 229.5, 87.2, 7.8, 80.2, 220.3, 59.6, 0.7, 265.2,
        8.4, 219.8, 36.9, 48.3, 25.6, 273.7, 43.0, 184.9, 73.4, 193.7, 220.5,
        104.6, 96.2, 140.3, 240.1, 243.2, 38.0, 44.7, 280.7, 121.0, 197.6, 171.3,
```

```r
         187.8, 4.1, 93.9, 149.8, 11.7, 131.7, 172.5, 85.7, 188.4, 163.5, 117.2,
         234.5, 17.9, 206.8, 215.4, 284.3, 50.0, 164.5, 19.6, 168.4, 222.4, 276.9,
         248.4, 170.2, 276.7, 165.6, 156.6, 218.5, 56.2, 287.6, 253.8, 205.0,
         139.5, 191.1, 286.0, 18.7, 39.5, 75.5, 17.2, 166.8, 149.7, 38.2, 94.2,
         177.0, 283.6, 232.1)

radio <- c(37.8, 39.3, 45.9, 41.3, 10.8, 48.9, 32.8, 19.6, 2.1, 2.6, 5.8, 24.0,
           35.1, 7.6, 32.9, 47.7, 36.6, 39.6, 20.5, 23.9, 27.7, 5.1, 15.9, 16.9,
           12.6, 3.5, 29.3, 16.7, 27.1, 16.0, 28.3, 17.4, 1.5, 20.0, 1.4, 4.1,
           43.8, 49.4, 26.7, 37.7, 22.3, 33.4, 27.7, 8.4, 25.7, 22.5, 9.9, 41.5,
           15.8, 11.7, 3.1, 9.6, 41.7, 46.2, 28.8, 49.4, 28.1, 19.2, 49.6, 29.5,
           2.0, 42.7, 15.5, 29.6, 42.8, 9.3, 24.6, 14.5, 27.5, 43.9, 30.6, 14.3,
           33.0, 5.7, 24.6, 43.7, 1.6, 28.5, 29.9, 7.7, 26.7, 4.1, 20.3, 44.5,
           43.0, 18.4, 27.5, 40.6, 25.5, 47.8, 4.9, 1.5, 33.5, 36.5, 14.0, 31.6,
           3.5, 21.0, 42.3, 41.7, 4.3, 36.3, 10.1, 17.2, 34.3, 46.4, 11.0, 0.3,
           0.4, 26.9, 8.2, 38.0, 15.4, 20.6, 46.8, 35.0, 14.3, 0.8, 36.9, 16.0,
           26.8, 21.7, 2.4, 34.6, 32.3, 11.8, 38.9, 0.0, 49.0, 12.0, 39.6, 2.9,
           27.2, 33.5, 38.6, 47.0, 39.0, 28.9, 25.9, 43.9, 17.0, 35.4, 33.2, 5.7,
           14.8, 1.9, 7.3, 49.0, 40.3, 25.8, 13.9, 8.4, 23.3, 39.7, 21.1, 11.6,
           43.5, 1.3, 36.9, 18.4, 18.1, 35.8, 18.1, 36.8, 14.7, 3.4, 37.6, 5.2,
           23.6, 10.6, 11.6, 20.9, 20.1, 7.1, 3.4, 48.9, 30.2, 7.8, 2.3, 10.0,
           2.6, 5.4, 5.7, 43.0, 21.3, 45.1, 2.1, 28.7, 13.9, 12.1, 41.1, 10.8,
           4.1, 42.0, 35.6, 3.7, 4.9, 9.3, 42.0, 8.6)

periodico <- c(69.2, 45.1, 69.3, 58.5, 58.4, 75.0, 23.5, 11.6, 1.0, 21.2, 24.2,
               4.0, 65.9, 7.2, 46.0, 52.9, 114.0, 55.8, 18.3, 19.1, 53.4, 23.5,
               49.6, 26.2, 18.3, 19.5, 12.6, 22.9, 22.9, 40.8, 43.2, 38.6, 30.0,
               0.3, 7.4, 8.5, 5.0, 45.7, 35.1, 32.0, 31.6, 38.7, 1.8, 26.4, 43.3,
               31.5, 35.7, 18.5, 49.9, 36.8, 34.6, 3.6, 39.6, 58.7, 15.9, 60.0,
               41.4, 16.6, 37.7, 9.3, 21.4, 54.7, 27.3, 8.4, 28.9, 0.9, 2.2, 10.2,
               11.0, 27.2, 38.7, 31.7, 19.3, 31.3, 13.1, 89.4, 20.7, 14.2, 9.4,
               23.1, 22.3, 36.9, 32.5, 35.6, 33.8, 65.7, 16.0, 63.2, 73.4, 51.4,
               9.3, 33.0, 59.0, 72.3, 10.9, 52.9, 5.9, 22.0, 51.2, 45.9, 49.8,
               100.9, 21.4, 17.9, 5.3, 59.0, 29.7, 23.2, 25.6, 5.5, 56.5, 23.2,
               2.4, 10.7, 34.5, 52.7, 25.6, 14.8, 79.2, 22.3, 46.2, 50.4, 15.6,
               12.4, 74.2, 25.9, 50.6, 9.2, 3.2, 43.1, 8.7, 43.0, 2.1, 45.1, 65.6,
               8.5, 9.3, 59.7, 20.5, 1.7, 12.9, 75.6, 37.9, 34.4, 38.9, 9.0, 8.7,
               44.3, 11.9, 20.6, 37.0, 48.7, 14.2, 37.7, 9.5, 5.7, 50.5, 24.3,
               45.2, 34.6, 30.7, 49.3, 25.6, 7.4, 5.4, 84.8, 21.6, 19.4, 57.6,
               6.4, 18.4, 47.4, 17.0, 12.8, 13.1, 41.8, 20.3, 35.2, 23.7, 17.6,
               8.3, 27.4, 29.7, 71.8, 30.0, 19.6, 26.6, 18.2, 3.7, 23.4, 5.8, 6.0,
               31.6, 3.6, 6.0, 13.8, 8.1, 6.4, 66.2, 8.7)
```

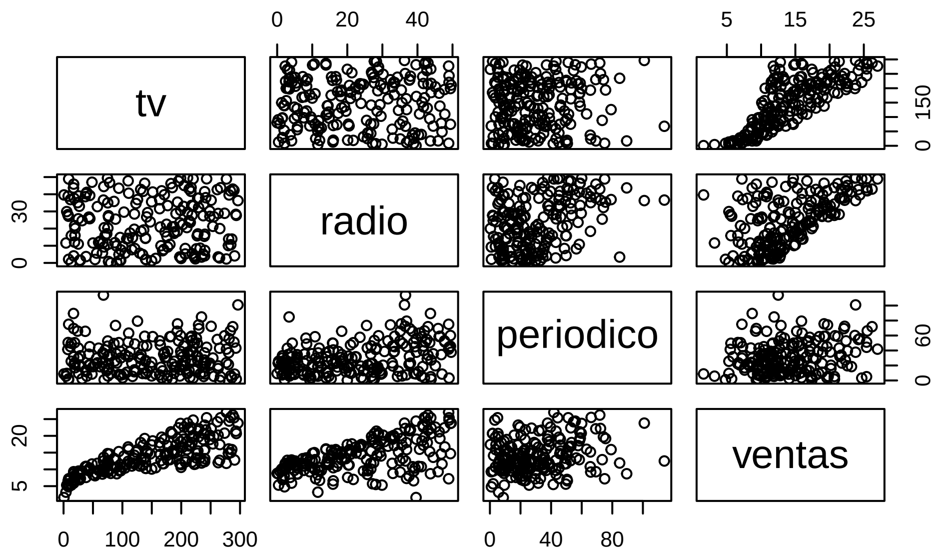
```r
ventas <- c(22.1, 10.4, 9.3, 18.5, 12.9, 7.2, 11.8, 13.2, 4.8, 10.6, 8.6, 17.4,
            9.2, 9.7, 19.0, 22.4, 12.5, 24.4, 11.3, 14.6, 18.0, 12.5, 5.6, 15.5,
            9.7, 12.0, 15.0, 15.9, 18.9, 10.5, 21.4, 11.9, 9.6, 17.4, 9.5, 12.8,
            25.4, 14.7, 10.1, 21.5, 16.6, 17.1, 20.7, 12.9, 8.5, 14.9, 10.6, 23.2,
            14.8, 9.7, 11.4, 10.7, 22.6, 21.2, 20.2, 23.7, 5.5, 13.2, 23.8, 18.4,
            8.1, 24.2, 15.7, 14.0, 18.0, 9.3, 9.5, 13.4, 18.9, 22.3, 18.3, 12.4,
            8.8, 11.0, 17.0, 8.7, 6.9, 14.2, 5.3, 11.0, 11.8, 12.3, 11.3, 13.6,
            21.7, 15.2, 12.0, 16.0, 12.9, 16.7, 11.2, 7.3, 19.4, 22.2, 11.5, 16.9,
            11.7, 15.5, 25.4, 17.2, 11.7, 23.8, 14.8, 14.7, 20.7, 19.2, 7.2, 8.7,
            5.3, 19.8, 13.4, 21.8, 14.1, 15.9, 14.6, 12.6, 12.2, 9.4, 15.9, 6.6,
            15.5, 7.0, 11.6, 15.2, 19.7, 10.6, 6.6, 8.8, 24.7, 9.7, 1.6, 12.7,
            5.7, 19.6, 10.8, 11.6, 9.5, 20.8, 9.6, 20.7, 10.9, 19.2, 20.1, 10.4,
            11.4, 10.3, 13.2, 25.4, 10.9, 10.1, 16.1, 11.6, 16.6, 19.0, 15.6,
            3.2, 15.3, 10.1, 7.3, 12.9, 14.4, 13.3, 14.9, 18.0, 11.9, 11.9, 8.0,
            12.2, 17.1, 15.0, 8.4, 14.5, 7.6, 11.7, 11.5, 27.0, 20.2, 11.7, 11.8,
            12.6, 10.5, 12.2, 8.7, 26.2, 17.6, 22.6, 10.3, 17.3, 15.9, 6.7, 10.8,
            9.9, 5.9, 19.6, 17.3, 7.6, 9.7, 12.8, 25.5, 13.4)

datos <- data.frame(tv, radio, periodico, ventas)
head(datos)
```

```
     tv radio periodico ventas
1 230.1  37.8      69.2   22.1
2  44.5  39.3      45.1   10.4
3  17.2  45.9      69.3    9.3
4 151.5  41.3      58.5   18.5
5 180.8  10.8      58.4   12.9
6   8.7  48.9      75.0    7.2
```

3.2. Correlaciones y multicolinealidad

Primero, miramos las correlaciones entre variables:

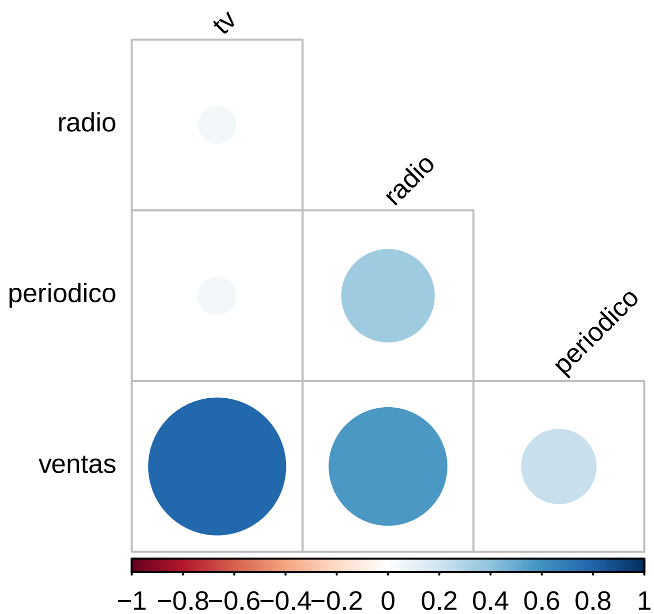
```r
pairs(datos)
```




```r
r <- cor(datos)
r
```

```
                   tv      radio  periodico    ventas
tv         1.00000000 0.05480866 0.05664787 0.7822244
radio      0.05480866 1.00000000 0.35410375 0.5762226
periodico  0.05664787 0.35410375 1.00000000 0.2282990
ventas     0.78222442 0.57622257 0.22829903 1.0000000
```

```r
corrplot(r, method="circle", type="lower", diag=FALSE,
         tl.col="black", tl.cex=0.8, tl.srt=45)
```



ℹ Nota

Interpretación:

- La columna **ventas** te muestra cómo se relaciona la variable respuesta con cada medio.
- Si dos regresores (por ejemplo, `tv` y `radio`) tienen **correlación muy alta**, podría haber multicolinealidad.
- El `corrplot` ayuda a ver estas relaciones de forma más clara que solo con la matriz numérica.

3.3. Modelo con las tres variables

Ajustamos el modelo completo:

$$\text{ventas} = \beta_0 + \beta_1 \text{tv} + \beta_2 \text{radio} + \beta_3 \text{periodico} + \varepsilon$$

```
modelo_full <- lm(ventas ~ tv + radio + periodico, data = datos)
summary(modelo_full)
```

```
Call:
lm(formula = ventas ~ tv + radio + periodico, data = datos)

Residuals:
    Min      1Q  Median      3Q     Max 
-8.8277 -0.8908  0.2418  1.1893  2.8292 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.938889   0.311908   9.422   <2e-16 ***
tv           0.045765   0.001395  32.809   <2e-16 ***
radio        0.188530   0.008611  21.893   <2e-16 ***
periodico   -0.001037   0.005871  -0.177     0.86    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.686 on 196 degrees of freedom
Multiple R-squared:  0.8972,	Adjusted R-squared:  0.8956 
F-statistic: 570.3 on 3 and 196 DF,  p-value: < 2.2e-16
```

> **Nota**
>
> Mira especialmente:
>
> - El **p-value** de cada coeficiente → te indica si esa variable es significativa.
> - El **p-value** de la prueba F → si el modelo completo explica significativamente a `ventas`.
> - El **R^2** y **R^2 ajustado** → qué porcentaje de la variación se explica por los regresores.

3.4. Modelo sin variable no significativa

Si el coeficiente de `periodico` no es significativo, podemos intentar un modelo más parsimonioso:

```r
modelo <- lm(ventas ~ tv + radio, data = datos)
summary(modelo)
```

```
Call:
lm(formula = ventas ~ tv + radio, data = datos)

Residuals:
    Min      1Q  Median      3Q     Max 
-8.7977 -0.8752  0.2422  1.1708  2.8328 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.92110    0.29449   9.919   <2e-16 ***
tv           0.04575    0.00139  32.909   <2e-16 ***
radio        0.18799    0.00804  23.382   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.681 on 197 degrees of freedom
Multiple R-squared:  0.8972,	Adjusted R-squared:  0.8962 
F-statistic: 859.6 on 2 and 197 DF,  p-value: < 2.2e-16
```

Tip

Eliminar variables no significativas:

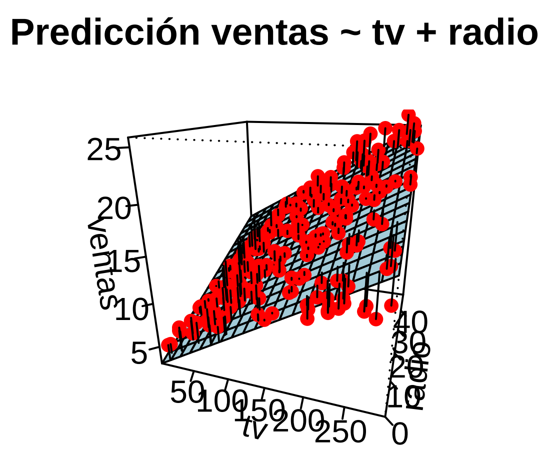
- Simplifica la interpretación.
- Puede mejorar la capacidad predictiva fuera de muestra.
- Siempre es recomendable comparar modelos (por ejemplo, con ANOVA o criterios de información).

3.5. Superficie de regresión en 3D

Como ahora el modelo solo depende de **tv** y **radio**, podemos visualizar la "superficie de regresión" y cómo se ubican los datos alrededor de ella.

```r
rango_tv <- range(datos$tv)
nuevos_valores_tv <- seq(from = rango_tv[1], to = rango_tv[2], length.out = 20)

rango_radio <- range(datos$radio)
nuevos_valores_radio <- seq(from = rango_radio[1], to = rango_radio[2],
```

```
                            length.out = 20)

predicciones <- outer(
  X = nuevos_valores_tv,
  Y = nuevos_valores_radio,
  FUN = function(tv, radio) {
    predict(object = modelo, newdata = data.frame(tv, radio))
  }
)

superficie <- persp(
  x = nuevos_valores_tv,
  y = nuevos_valores_radio,
  z = predicciones,
  theta = 18, phi = 20,
  col = "lightblue", shade = 0.1,
  xlab = "tv", ylab = "radio", zlab = "ventas",
  ticktype = "detailed",
  main = "Predicción ventas ~ tv + radio"
)

observaciones <- trans3d(datos$tv, datos$radio, datos$ventas, superficie)
error <- trans3d(datos$tv, datos$radio, fitted(modelo), superficie)

points(observaciones, col = "red", pch = 16)
segments(observaciones$x, observaciones$y, error$x, error$y)
```



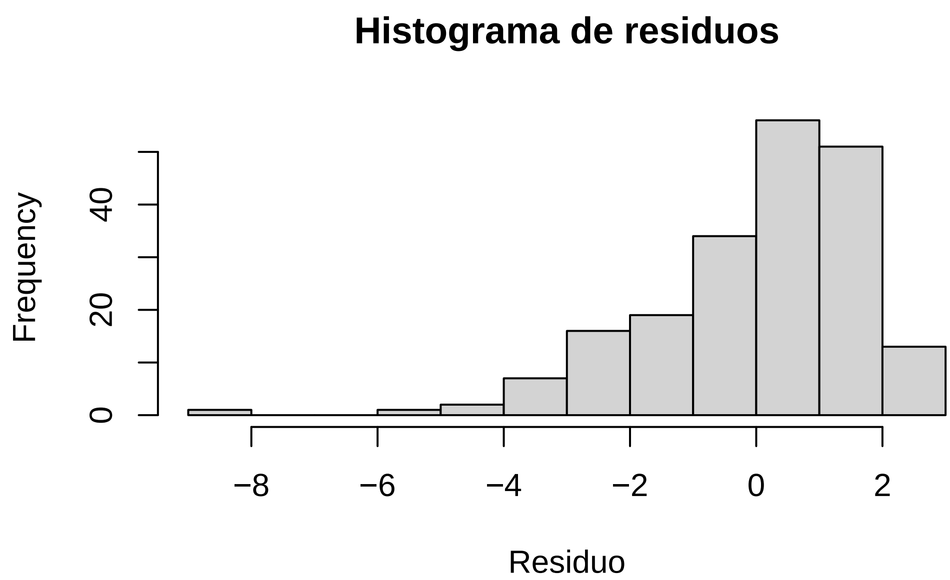
> ℹ Nota
>
> - Los puntos rojos son las **observaciones reales**.
> - Las líneas verticales muestran la **distancia** entre la superficie de predicción y los datos → son los errores del modelo.
> - Si las líneas son pequeñas, el ajuste es bueno.

3.6. Análisis de residuos del modelo reducido

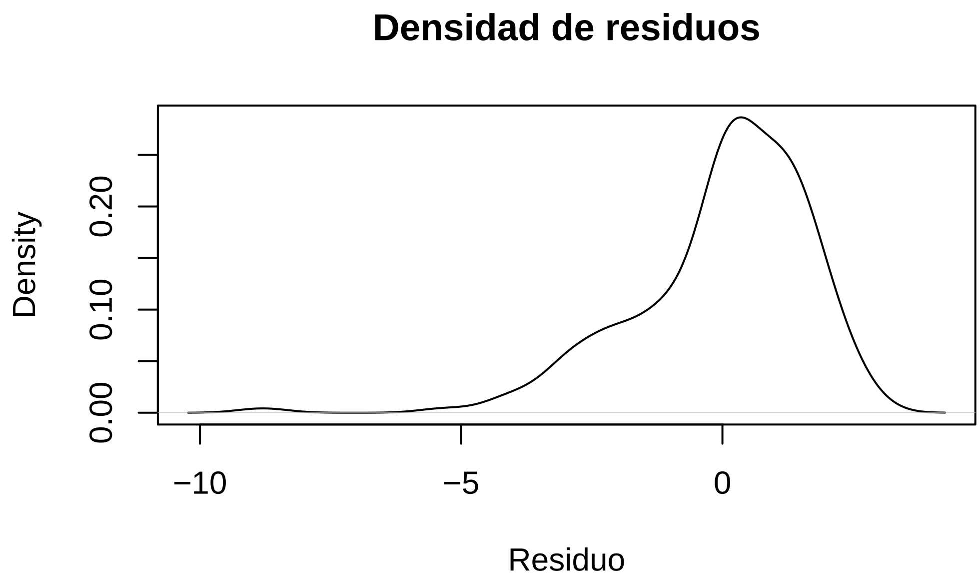
```r
shapiro.test(modelo$residuals)
```

```
	Shapiro-Wilk normality test

data:  modelo$residuals
W = 0.91804, p-value = 4.19e-09
```

```r
hist(modelo$residuals, main="Histograma de residuos", xlab="Residuo")
```




```r
plot(density(modelo$residuals), main="Densidad de residuos", xlab="Residuo")
```



Pruebas de autocorrelación y heterocedasticidad:

```
dwtest(modelo, alternative ="two.sided", iterations = 1000)
```

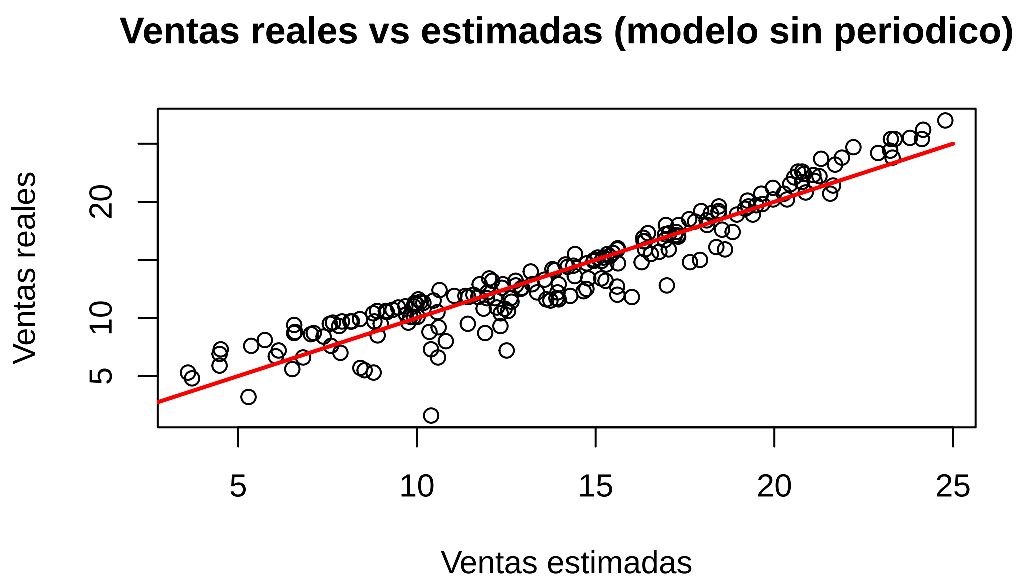
```

	Durbin-Watson test

data:  modelo
DW = 2.0808, p-value = 0.5656
alternative hypothesis: true autocorrelation is not 0
```

```
bptest(modelo)
```

```

	studentized Breusch-Pagan test

data:  modelo
BP = 4.8093, df = 2, p-value = 0.0903
```

Gráfico de valores reales vs estimados:

```r
plot(modelo$fitted.values, datos$ventas,
     xlab = "Ventas estimadas", ylab = "Ventas reales",
     main = "Ventas reales vs estimadas (modelo sin periodico)")
lines(c(0, 25), c(0, 25), col = "red", lwd = 2)
```



Nota

- Si los puntos se alinean alrededor de la diagonal roja → el modelo predice razonablemente bien.
- Desviaciones sistemáticas o patrones curvos indicarían que falta estructura (no linealidad, interacciones, etc.).

3.7. Incorporar una interacción tv * radio

Ahora probamos un modelo donde el efecto de la TV **depende del nivel de radio** (y viceversa).

```r
tv_radio <- tv * radio
```

```r
modelo_interaccion <- lm(ventas ~ tv + radio + tv:radio, data = datos)
summary(modelo_interaccion)
```

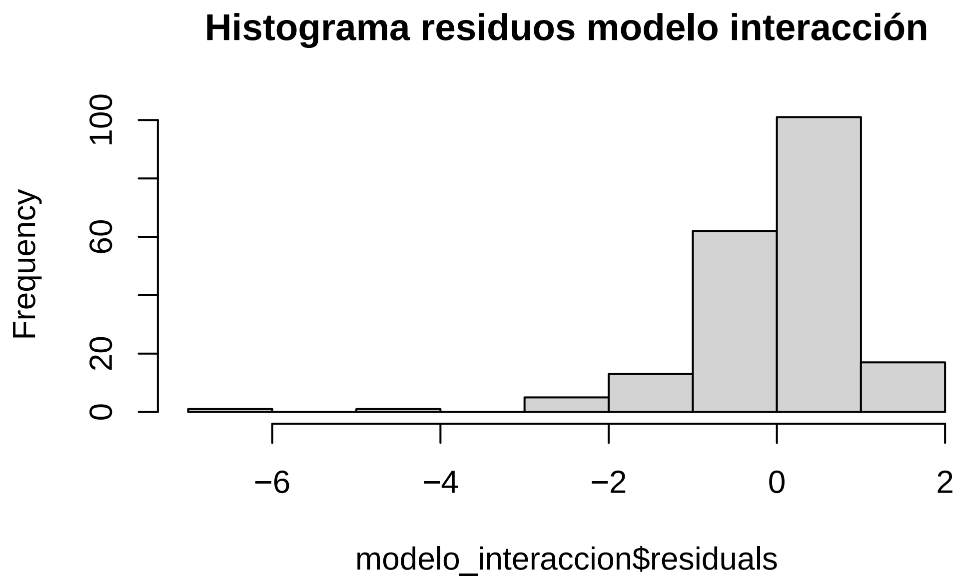
```

Call:
lm(formula = ventas ~ tv + radio + tv:radio, data = datos)

Residuals:
    Min      1Q  Median      3Q     Max 
-6.3366 -0.4028  0.1831  0.5948  1.5246 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 6.750e+00  2.479e-01  27.233   <2e-16 ***
tv          1.910e-02  1.504e-03  12.699   <2e-16 ***
radio       2.886e-02  8.905e-03   3.241   0.0014 ** 
tv:radio    1.086e-03  5.242e-05  20.727   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9435 on 196 degrees of freedom
Multiple R-squared:  0.9678,    Adjusted R-squared:  0.9673 
F-statistic:  1963 on 3 and 196 DF,  p-value: < 2.2e-16
```

Analizamos los residuos del nuevo modelo:

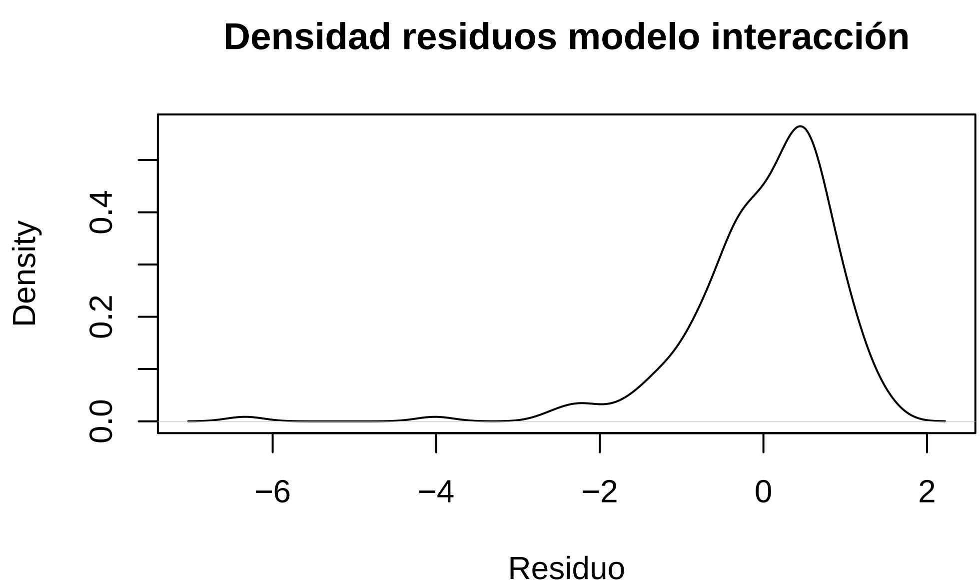
```r
shapiro.test(modelo_interaccion$residuals)
```

```

    Shapiro-Wilk normality test

data:  modelo_interaccion$residuals
W = 0.8469, p-value = 3.047e-13
```

```r
hist(modelo_interaccion$residuals, main="Histograma residuos modelo interacción")
```




```r
plot(density(modelo_interaccion$residuals),
     main="Densidad residuos modelo interacción", xlab="Residuo")
```



```r
dwtest(modelo_interaccion, alternative ="two.sided", iterations = 1000)
```

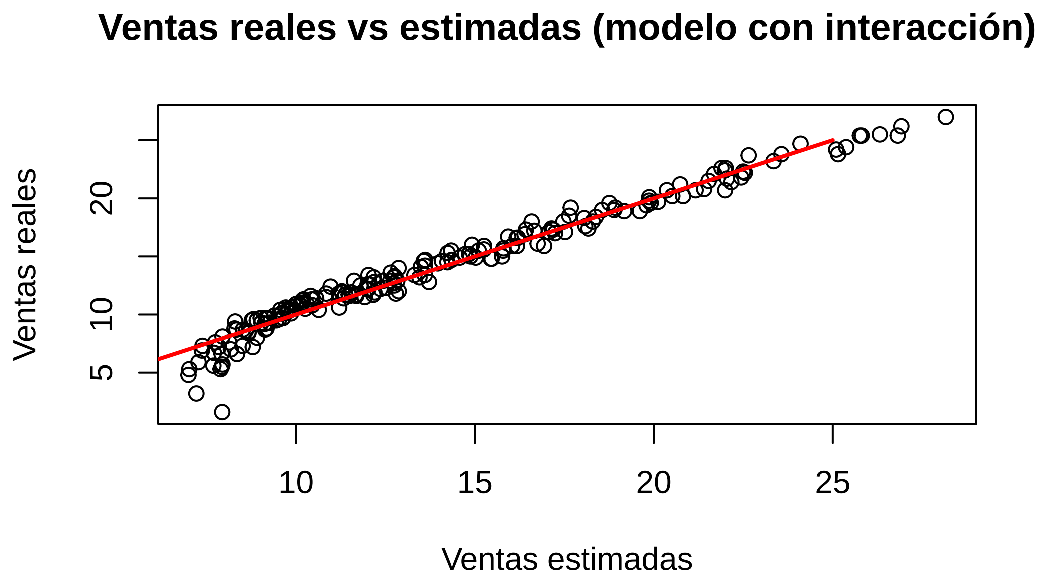
```

	Durbin-Watson test

data:  modelo_interaccion
DW = 2.2236, p-value = 0.1103
alternative hypothesis: true autocorrelation is not 0
```

```r
bptest(modelo_interaccion)
```

```

	studentized Breusch-Pagan test

data:  modelo_interaccion
BP = 14.324, df = 3, p-value = 0.002495
```

Ventas reales vs estimadas con interacción:

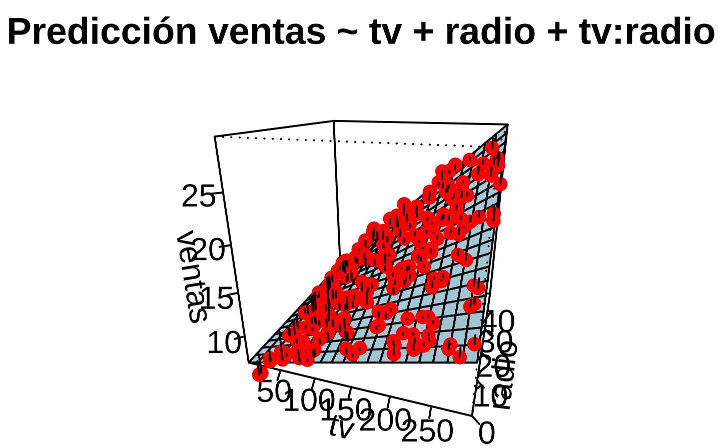
```r
plot(modelo_interaccion$fitted.values, datos$ventas,
     xlab = "Ventas estimadas",
     ylab = "Ventas reales",
     main = "Ventas reales vs estimadas (modelo con interacción)")
lines(c(0, 25), c(0, 25), col = "red", lwd = 2)
```



3.8. Superficie del modelo con interacción

```
rango_tv <- range(datos$tv)
nuevos_valores_tv <- seq(from = rango_tv[1], to = rango_tv[2], length.out = 20)

rango_radio <- range(datos$radio)
nuevos_valores_radio <- seq(from = rango_radio[1], to = rango_radio[2], length.out = 20)

predicciones <- outer(
  X = nuevos_valores_tv,
  Y = nuevos_valores_radio,
  FUN = function(tv, radio) {
    predict(object = modelo_interaccion,
            newdata = data.frame(tv, radio))
  }
)

superficie <- persp(
  x = nuevos_valores_tv,
  y = nuevos_valores_radio,
  z = predicciones,
```

```r
  theta = 18, phi = 20,
  col = "lightblue", shade = 0.1,
  xlab = "tv", ylab = "radio", zlab = "ventas",
  ticktype = "detailed",
  main = "Predicción ventas ~ tv + radio + tv:radio"
)

observaciones <- trans3d(datos$tv, datos$radio, datos$ventas, superficie)
error <- trans3d(datos$tv, datos$radio, fitted(modelo_interaccion), superficie)

points(observaciones, col = "red", pch = 16)
segments(observaciones$x, observaciones$y, error$x, error$y)
```



3.9. Comparación de modelos con ANOVA

Comparamos el modelo sin interacción y el modelo con interacción:

```r
anova(modelo, modelo_interaccion)
```

```
Analysis of Variance Table
```

```
Model 1: ventas ~ tv + radio
Model 2: ventas ~ tv + radio + tv:radio
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    197 556.91
2    196 174.48  1    382.43 429.59 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

> **Nota**
>
> - Si la prueba ANOVA da un **p-value pequeño**, la interacción aporta información estadísticamente significativa.
>
> - Además de la significancia, es importante revisar residuos y lógica económica del modelo.

3.10. Modelo con interacción y término cuadrático en tv

Probamos un modelo más flexible:

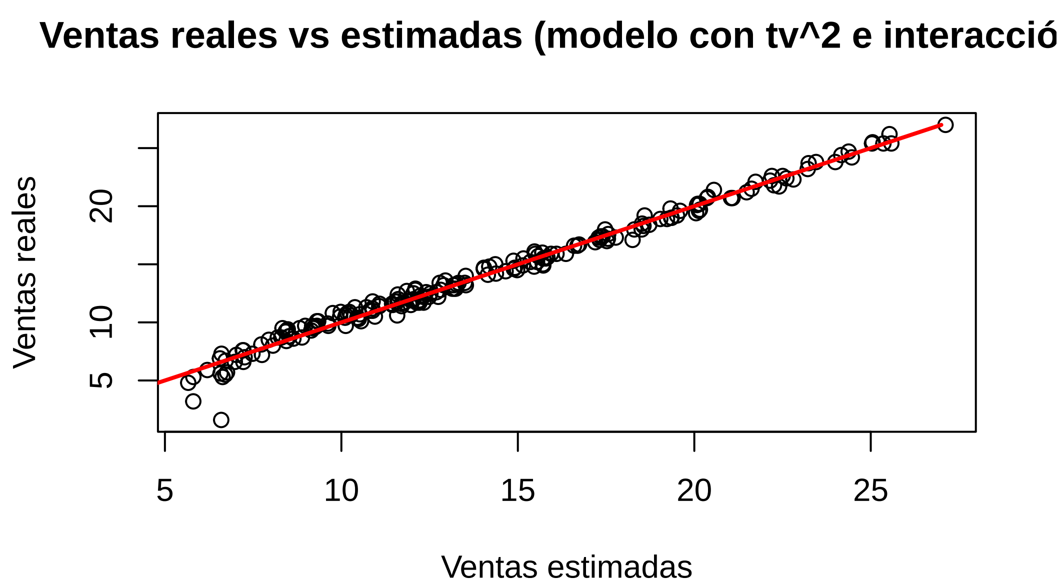
$$\text{ventas} = \beta_0 + \beta_1 \text{tv} + \beta_2 \text{radio} + \beta_3 \text{tv}^2 + \beta_4 (\text{tv} \cdot \text{radio}) + \varepsilon$$

```r
modelo_interaccion_1 <- lm(ventas ~ tv + radio + I(tv^2) + tv:radio, data = datos)
summary(modelo_interaccion_1)
```

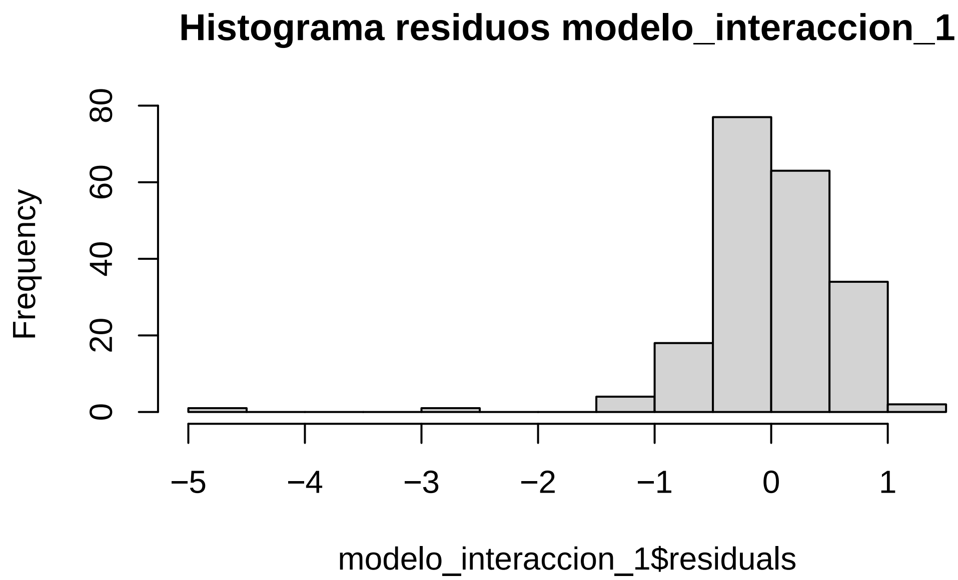
```
Call:
lm(formula = ventas ~ tv + radio + I(tv^2) + tv:radio, data = datos)

Residuals:
    Min      1Q  Median      3Q     Max
-4.9949 -0.2969 -0.0066  0.3798  1.1686

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  5.137e+00  1.927e-01  26.663  < 2e-16 ***
tv           5.092e-02  2.232e-03  22.810  < 2e-16 ***
radio        3.516e-02  5.901e-03   5.959 1.17e-08 ***
I(tv^2)     -1.097e-04  6.893e-06 -15.920  < 2e-16 ***
tv:radio     1.077e-03  3.466e-05  31.061  < 2e-16 ***
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6238 on 195 degrees of freedom
Multiple R-squared:  0.986, Adjusted R-squared:  0.9857
F-statistic:  3432 on 4 and 195 DF,  p-value: < 2.2e-16
```

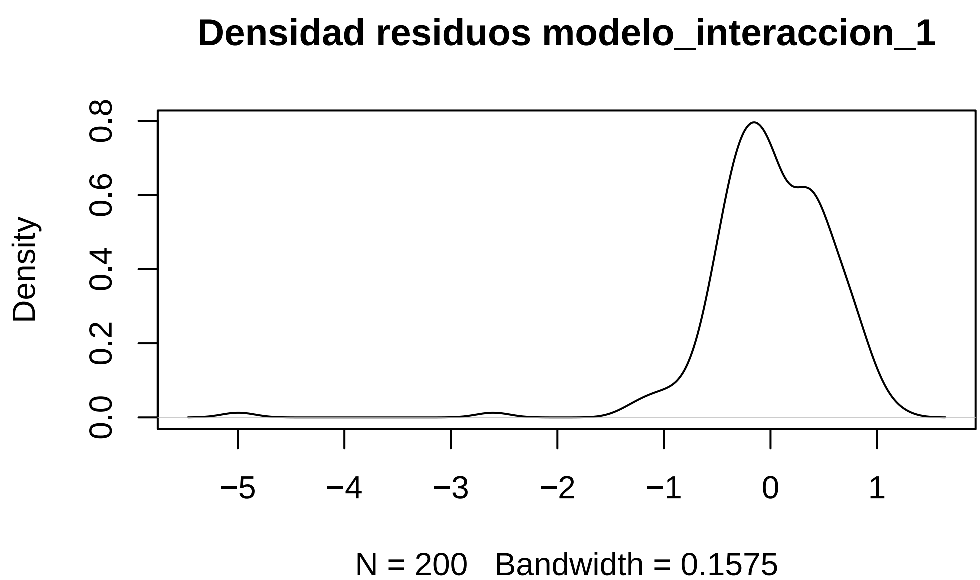
```
plot(modelo_interaccion_1$fitted.values, datos$ventas,
     xlab = "Ventas estimadas", ylab = "Ventas reales",
     main = "Ventas reales vs estimadas (modelo con tv^2 e interacción)")
lines(c(0, 27), c(0, 27), col = "red", lwd = 2)
```




```
hist(modelo_interaccion_1$residuals, main="Histograma residuos modelo_interaccion_1")
```




```
plot(density(modelo_interaccion_1$residuals),
     main="Densidad residuos modelo_interaccion_1")
```



```r
shapiro.test(modelo_interaccion_1$residuals)
```

```

	Shapiro-Wilk normality test

data:  modelo_interaccion_1$residuals
W = 0.80888, p-value = 6.359e-15
```

```r
dwtest(modelo_interaccion_1, alternative ="two.sided", iterations = 1000)
```

```

	Durbin-Watson test

data:  modelo_interaccion_1
DW = 2.204, p-value = 0.1432
alternative hypothesis: true autocorrelation is not 0
```

```r
bptest(modelo_interaccion_1)
```

```

	studentized Breusch-Pagan test

data:  modelo_interaccion_1
BP = 19.986, df = 4, p-value = 0.0005027
```

4. Parte 2: Regresión polinomial y transformaciones (ejemplo de millaje)

En esta parte trabajamos con el archivo `millaje.txt`, que contiene información de autos:

- `mpg`: millas por galón (consumo).
- `hp`: horsepower (potencia del motor).
- `vol`: alguna medida de volumen/cilindrada del motor.

Queremos modelar el **consumo de combustible** (`mpg`) en función de la potencia (`hp`) y otras características, usando polinomios y transformaciones.

4.1. Cargar los datos de millaje

```
archivo_millaje <- file.path(ruta_datos, "millaje.txt")

millaje <- read.table(file = archivo_millaje, header = TRUE)
head(millaje)
```

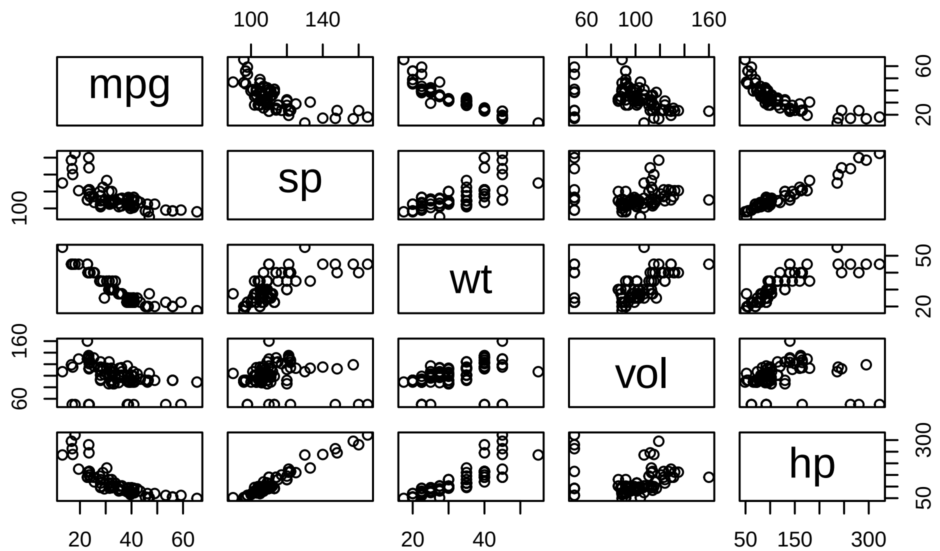
```
   mpg  sp   wt vol hp
1 65.4  96 17.5  89 49
2 56.0  97 20.0  92 55
3 55.9  97 20.0  92 55
4 49.0 105 20.0  92 70
5 46.5  96 20.0  92 53
6 46.2 105 20.0  89 70
```

4.2. Correlaciones y gráficos

```
r_auto <- cor(millaje)
r_auto
```

```
           mpg          sp         wt         vol          hp
mpg  1.0000000 -0.68844623 -0.9050849 -0.36861368 -0.78985635
sp  -0.6884462  1.00000000  0.6785339 -0.04306242  0.96654517
wt  -0.9050849  0.67853388  1.0000000  0.38495423  0.83222021
vol -0.3686137 -0.04306242  0.3849542  1.00000000  0.07647905
hp  -0.7898564  0.96654517  0.8322202  0.07647905  1.00000000
```

```
pairs(millaje)
```




```r
corrplot(r_auto, method="circle", type="lower", diag=FALSE,
         tl.col="black", tl.cex=0.8, tl.srt=45)
```

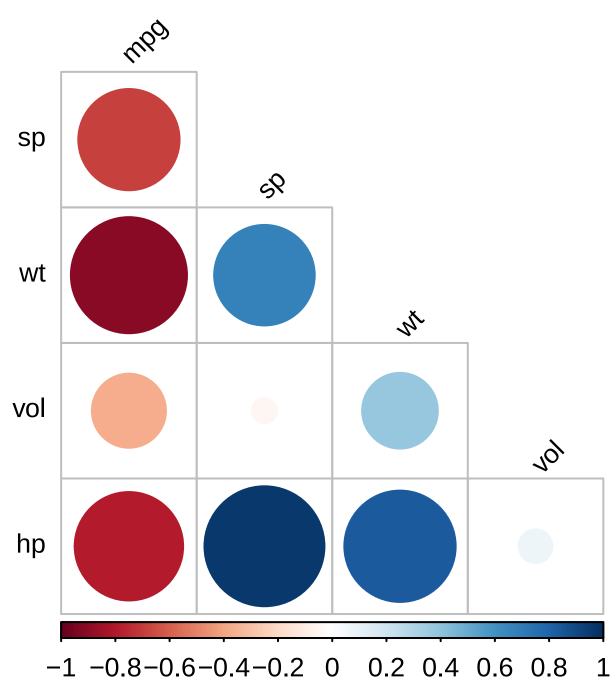
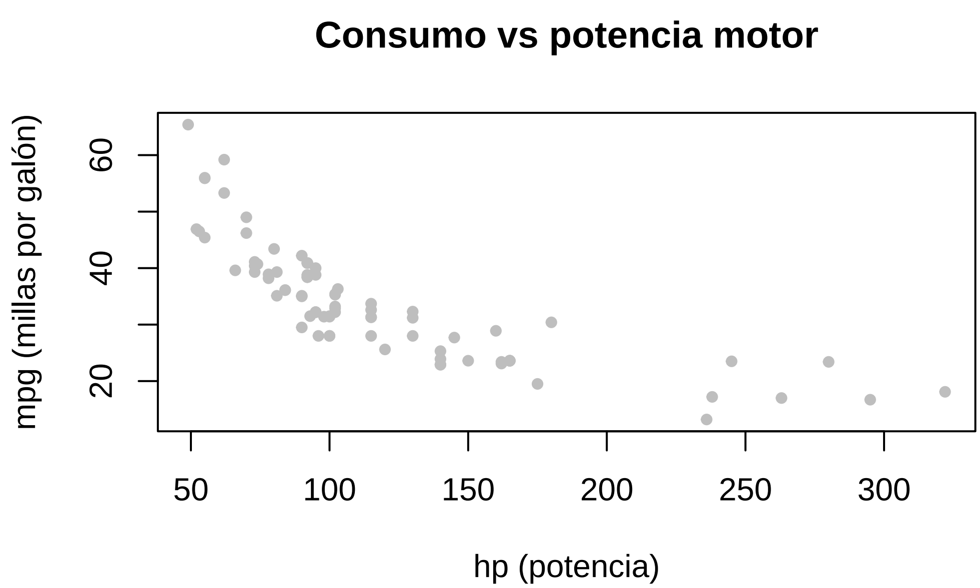


Gráfico simple de `mpg` vs `hp`:

```
plot(
  x = millaje$hp,
  y = millaje$mpg,
  main = "Consumo vs potencia motor",
  xlab = "hp (potencia)",
  ylab = "mpg (millas por galón)",
  pch = 20,
  col = "grey"
)
```



4.3. Modelo lineal simple en hp y vol

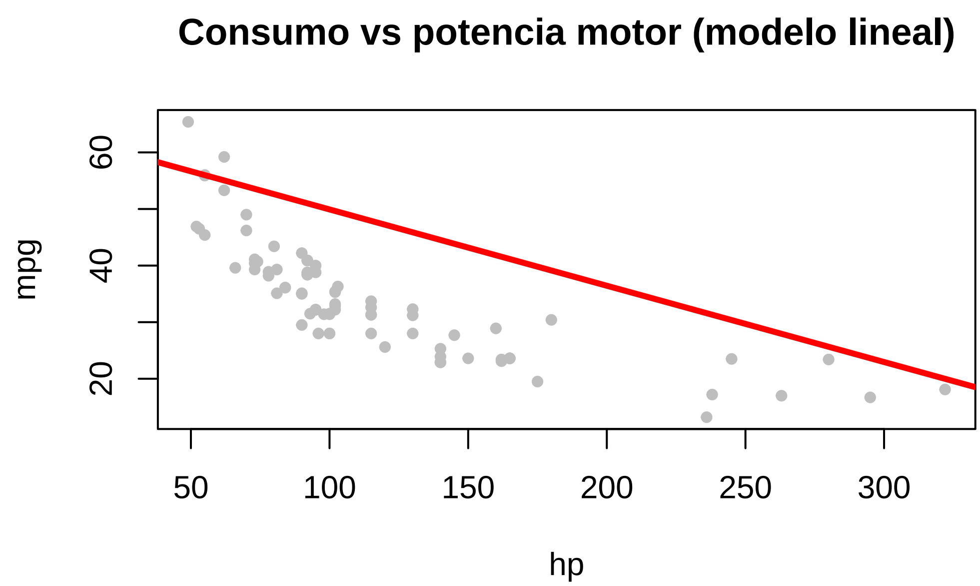
```
modelo_lineal <- lm(mpg ~ hp + vol, data = millaje)
summary(modelo_lineal)
```

```
Call:
lm(formula = mpg ~ hp + vol, data = millaje)
```

```
Residuals:
    Min      1Q  Median      3Q     Max 
-10.556  -3.411  -0.687   2.736  21.058 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 63.40255    2.91066  21.783  < 2e-16 ***
hp          -0.13485    0.01052 -12.818  < 2e-16 ***
vol         -0.13993    0.02698  -5.187 1.61e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.366 on 79 degrees of freedom
Multiple R-squared:  0.7194,	Adjusted R-squared:  0.7123 
F-statistic: 101.3 on 2 and 79 DF,  p-value: < 2.2e-16
```

Visualizamos la recta de regresión en función de `hp` (manteniendo fijo `vol` en el promedio, de manera implícita):

```r
plot(
  x = millaje$hp,
  y = millaje$mpg,
  main = "Consumo vs potencia motor (modelo lineal)",
  xlab = "hp",
  ylab = "mpg",
  pch  = 20,
  col  = "grey"
)
abline(modelo_lineal, lwd = 3, col = "red")
```



> **Nota**
>
> Este gráfico es más ilustrativo que riguroso (porque el modelo usa también `vol`), pero sirve para visualizar la tendencia lineal negativa: a mayor `hp`, menor `mpg`.

4.4. Modelo polinomial cuadrático

Ahora permitimos una relación **no lineal** entre `hp` y `mpg`:

```
modelo_pol2 <- lm(mpg ~ vol + hp + I(hp^2), data = millaje)
summary(modelo_pol2)
```

```
Call:
lm(formula = mpg ~ vol + hp + I(hp^2), data = millaje)

Residuals:
    Min      1Q  Median      3Q     Max 
-8.4677 -2.9686 -0.6293  2.3102 15.0791 

Coefficients:
```

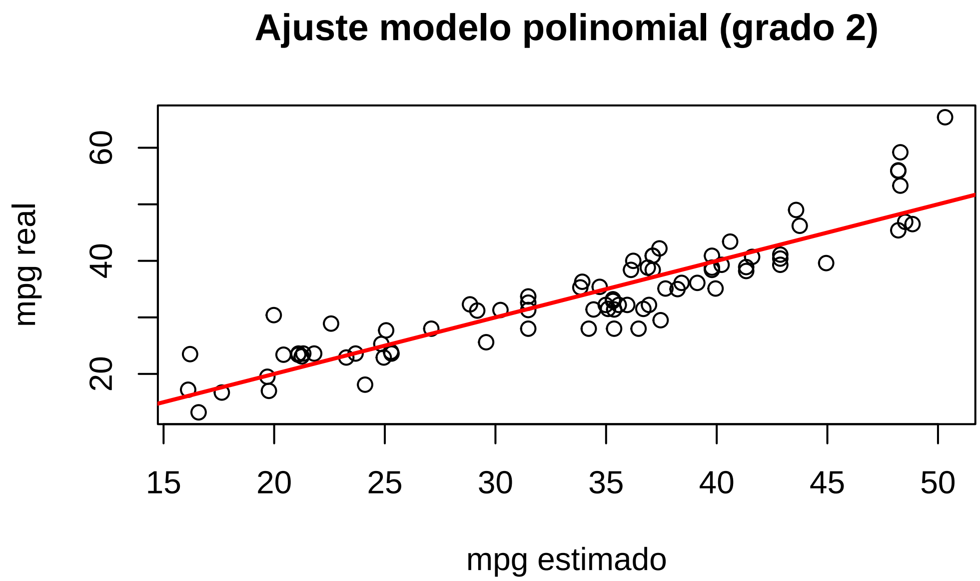
```
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 73.3557314  2.8205235  26.008  < 2e-16 ***
vol         -0.0546235  0.0255711  -2.136   0.0358 *
hp          -0.4115233  0.0436316  -9.432 1.57e-14 ***
I(hp^2)      0.0008294  0.0001283   6.466 8.01e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.357 on 78 degrees of freedom
Multiple R-squared:  0.8173,    Adjusted R-squared:  0.8103
F-statistic: 116.3 on 3 and 78 DF,  p-value: < 2.2e-16
```

```r
modelo_cuadratico <- lm(mpg ~ poly(hp, 2), data = millaje)
summary(modelo_cuadratico)
```

```
Call:
lm(formula = mpg ~ poly(hp, 2), data = millaje)

Residuals:
    Min      1Q  Median      3Q     Max
-8.2059 -3.3067 -0.4611  2.4724 14.3716

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)   33.7817     0.4919  68.674  < 2e-16 ***
poly(hp, 2)1 -71.1198     4.4545 -15.966  < 2e-16 ***
poly(hp, 2)2  38.4953     4.4545   8.642 4.87e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.454 on 79 degrees of freedom
Multiple R-squared:  0.8067,    Adjusted R-squared:  0.8018
F-statistic: 164.8 on 2 and 79 DF,  p-value: < 2.2e-16
```

Comparación gráfico predicho vs real:

```r
plot(modelo_pol2$fitted.values, millaje$mpg,
     xlab = "mpg estimado", ylab = "mpg real",
     main = "Ajuste modelo polinomial (grado 2)")
lines(c(10, 60), c(10, 60), col = "red", lwd = 2)
```



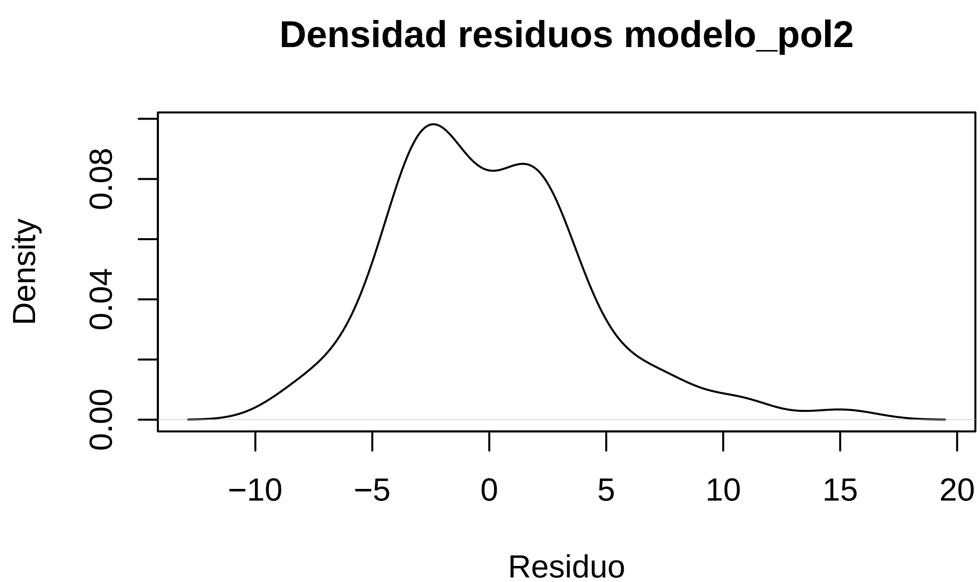
4.5. Análisis de residuos del modelo polinomial

```
shapiro.test(modelo_pol2$residuals)
```

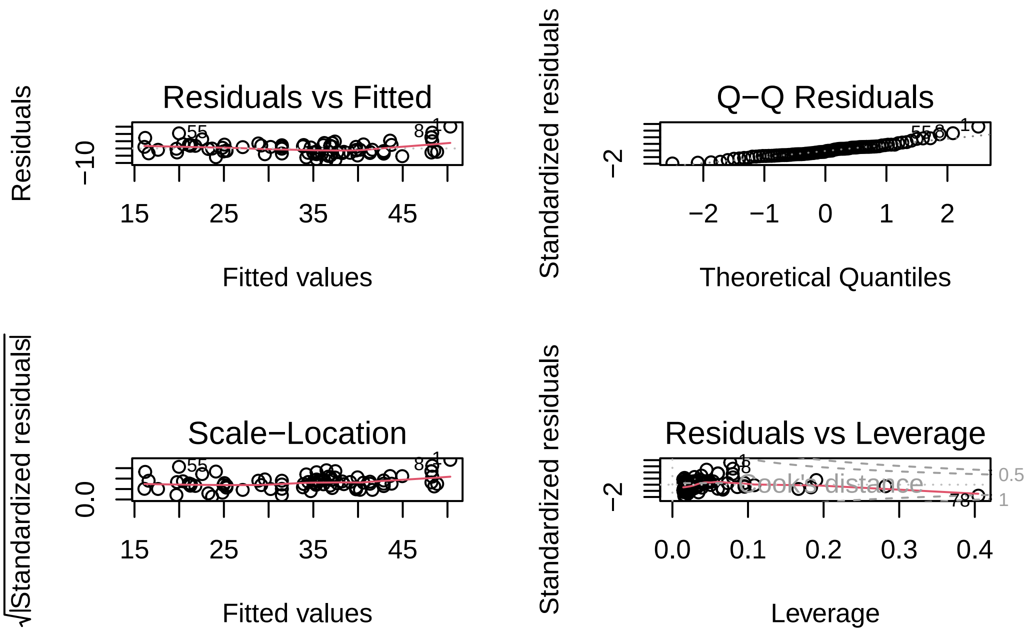
```
	Shapiro-Wilk normality test

data:  modelo_pol2$residuals
W = 0.95926, p-value = 0.0107
```

```
hist(modelo_pol2$residuals, main="Histograma residuos modelo_pol2")
```


```
plot(density(modelo_pol2$residuals),
     main="Densidad residuos modelo_pol2", xlab="Residuo")
```



```r
par(mfrow = c(2, 2))
plot(modelo_pol2)
```




```r
par(mfrow = c(1, 1))
```

Comparación formal entre el modelo lineal y el polinomial:

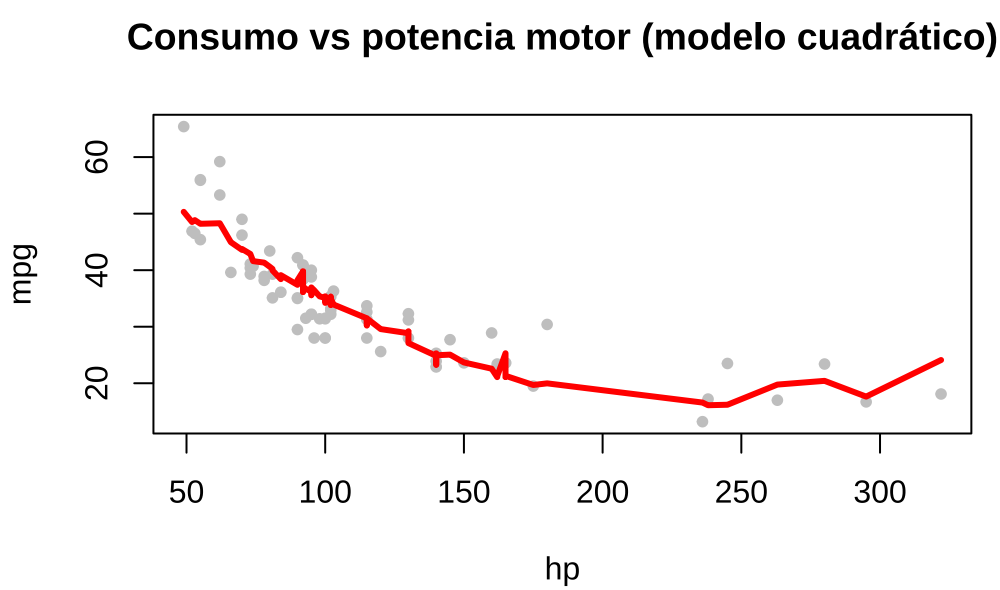
```r
anova(modelo_lineal, modelo_pol2)
```

```
Analysis of Variance Table

Model 1: mpg ~ hp + vol
Model 2: mpg ~ vol + hp + I(hp^2)
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1     79 2274.8
2     78 1480.9  1    793.86 41.813 8.009e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Si el p-valor es pequeño, el término cuadrático mejora significativamente el modelo.

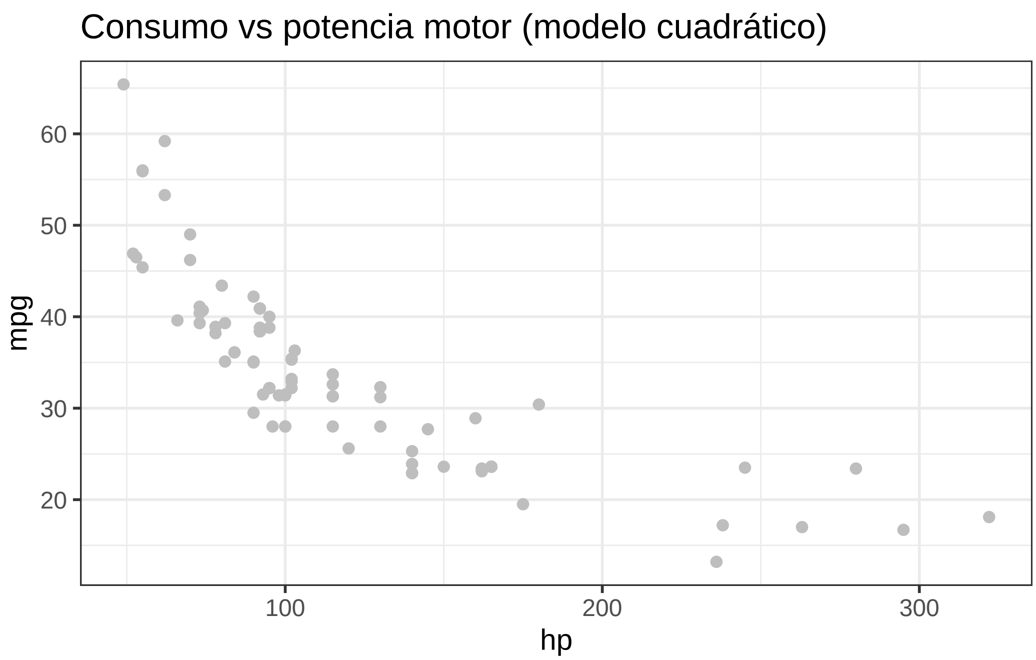
4.6. Curva predicha del modelo polinomial

```
plot(
  x = millaje$hp,
  y = millaje$mpg,
  main = "Consumo vs potencia motor (modelo cuadrático)",
  xlab = "hp",
  ylab = "mpg",
  pch  = 20,
  col  = "grey"
)

puntos_interpolados <- seq(from = min(millaje$hp), to = max(millaje$hp), by = 1)

prediccion <- predict(
  object = modelo_pol2,
  newdata = data.frame(hp = millaje$hp, vol = millaje$vol)
)

lines(sort(millaje$hp), prediccion[order(millaje$hp)],
      col = "red", lwd = 3)
```



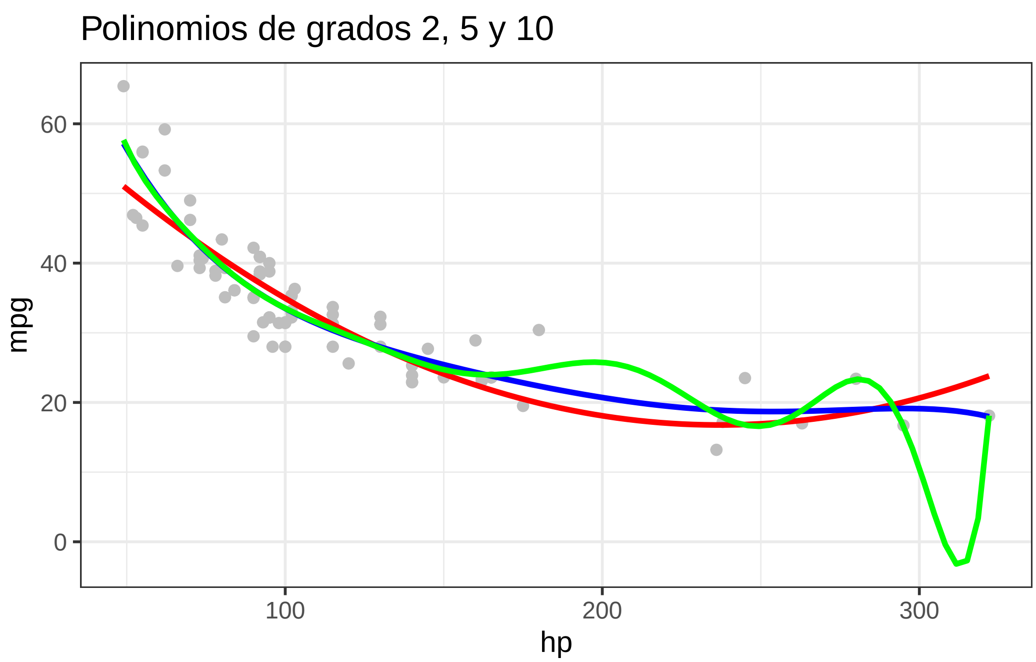
4.7. Visualización con ggplot2

```
ggplot(millaje, aes(x = hp, y = mpg)) +
  geom_point(colour = "grey") +
  stat_smooth(method = "lm", formula = y ~ hp + I(hp^2)) +
  labs(title = "Consumo vs potencia motor (modelo cuadrático)") +
  theme_bw()
```



4.8. Polinomios de grados más altos

```
ggplot(millaje, aes(x = hp, y = mpg)) +
  geom_point(colour = "grey") +
  stat_smooth(method = "lm", formula = y ~ poly(x, 2),  colour = "red",   se = FALSE) +
  stat_smooth(method = "lm", formula = y ~ poly(x, 5),  colour = "blue",  se = FALSE) +
  stat_smooth(method = "lm", formula = y ~ poly(x, 10), colour = "green", se = FALSE) +
  labs(title = "Polinomios de grados 2, 5 y 10") +
  theme_bw()
```



ℹ Nota

Observa cómo los polinomios de grados más altos se ajustan fuertemente a los datos, pero pueden **sobreajustar** (overfitting) y producir curvas muy oscilantes poco realistas.

4.9. Modelos polinomiales y comparación

```
modelo_5 <- lm(mpg ~ poly(hp, 5), data = millaje)
summary(modelo_5)
```

```
Call:
lm(formula = mpg ~ poly(hp, 5), data = millaje)

Residuals:
    Min      1Q  Median      3Q     Max
-7.9505 -2.5323 -0.4598  3.2027 10.9823

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept)   33.7817     0.4503  75.018  < 2e-16 ***
poly(hp, 5)1 -71.1198     4.0778 -17.441  < 2e-16 ***
poly(hp, 5)2  38.4953     4.0778   9.440 1.92e-14 ***
poly(hp, 5)3 -15.3033     4.0778  -3.753  0.00034 ***
poly(hp, 5)4   7.5552     4.0778   1.853  0.06780 .
poly(hp, 5)5  -3.5388     4.0778  -0.868  0.38822
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.078 on 76 degrees of freedom
Multiple R-squared:  0.8441,	Adjusted R-squared:  0.8339
F-statistic: 82.31 on 5 and 76 DF,  p-value: < 2.2e-16
```

```r
modelo_5_correjido <- lm(mpg ~ vol + hp + I(hp^2) + I(hp^3), data = millaje)
summary(modelo_5_correjido)
```

```
Call:
lm(formula = mpg ~ vol + hp + I(hp^2) + I(hp^3), data = millaje)

Residuals:
    Min      1Q  Median      3Q     Max
-7.6503 -2.6022 -0.3181  2.6926 11.4477

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  9.236e+01  5.361e+00  17.229  < 2e-16 ***
vol         -6.226e-02  2.345e-02  -2.655 0.009634 **
hp          -8.414e-01  1.135e-01  -7.410 1.38e-10 ***
I(hp^2)      3.765e-03  7.355e-04   5.119 2.20e-06 ***
I(hp^3)     -5.782e-06  1.430e-06  -4.044 0.000124 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.983 on 77 degrees of freedom
Multiple R-squared:  0.8493,	Adjusted R-squared:  0.8415
F-statistic: 108.5 on 4 and 77 DF,  p-value: < 2.2e-16
```

```r
anova(modelo_cuadratico, modelo_5_correjido)
```

```
Analysis of Variance Table
```

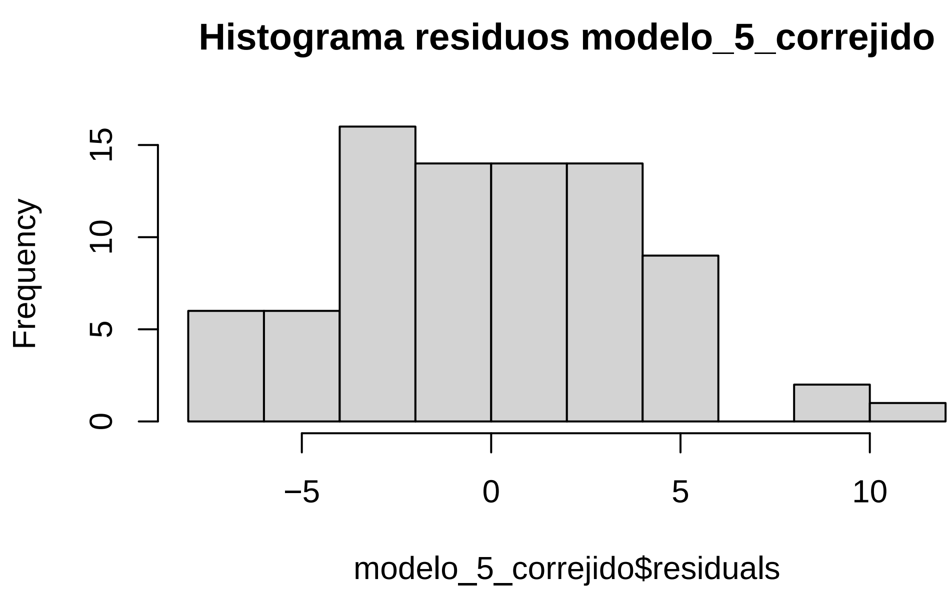
```
Model 1: mpg ~ poly(hp, 2)
Model 2: mpg ~ vol + hp + I(hp^2) + I(hp^3)
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1     79 1567.5
2     77 1221.5  2    346.02 10.906 6.758e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Análisis de residuos:

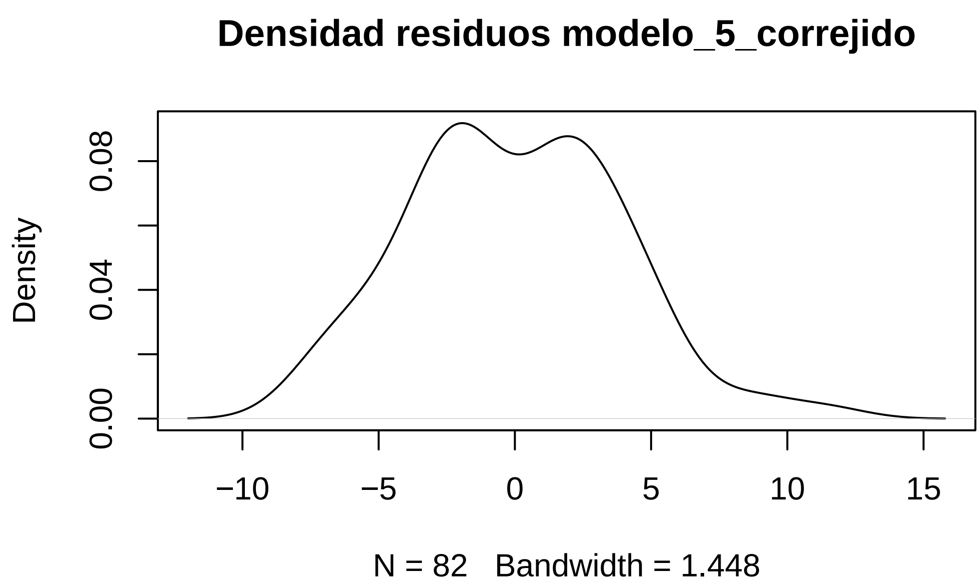
```r
shapiro.test(modelo_5_correjido$residuals)
```

```

    Shapiro-Wilk normality test

data:  modelo_5_correjido$residuals
W = 0.98456, p-value = 0.4319
```

```r
hist(modelo_5_correjido$residuals, main = "Histograma residuos modelo_5_correjido")
```

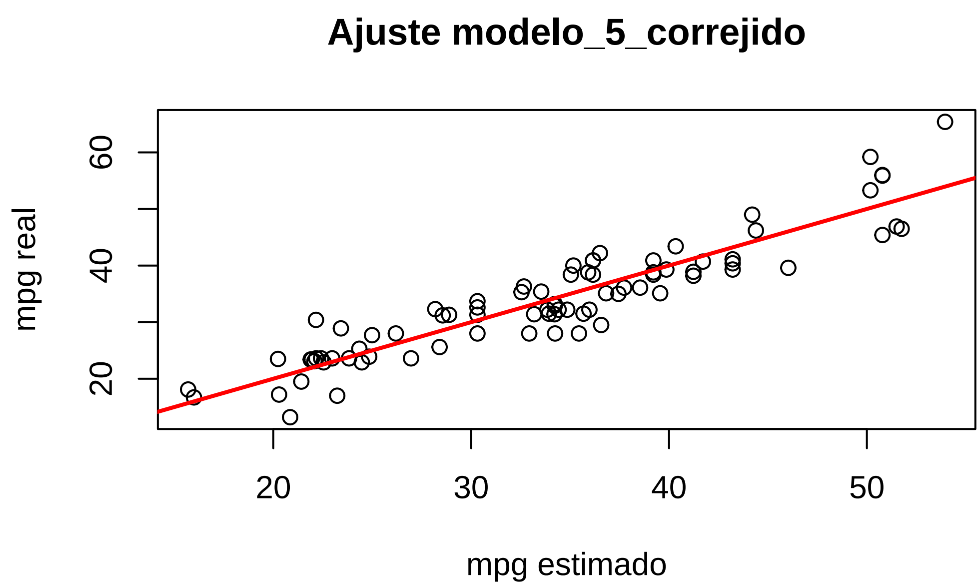


```r
plot(density(modelo_5_correjido$residuals),
     main = "Densidad residuos modelo_5_correjido")
```




```r
plot(modelo_5_correjido$fitted.values, millaje$mpg,
     xlab = "mpg estimado", ylab = "mpg real",
     main = "Ajuste modelo_5_correjido")
lines(c(10, 60), c(10, 60), col = "red", lwd = 2)
```

Ajuste modelo_5_correjido



4.10. Transformaciones de la variable respuesta

Buscamos mejorar la normalidad de los residuos y la homocedasticidad usando transformaciones de `mpg`:

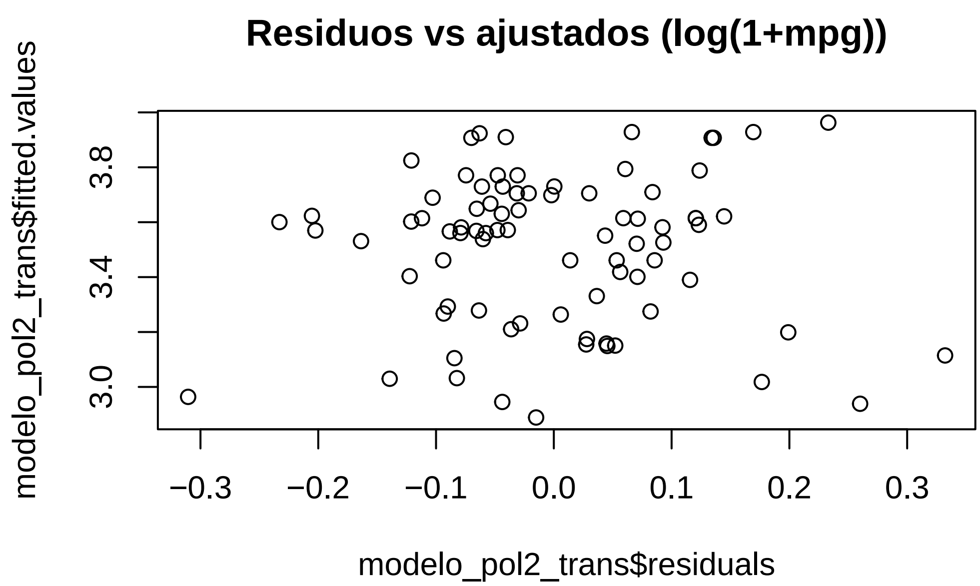
4.10.1. Transformación logarítmica

```
modelo_pol2_trans <- lm(log(1 + mpg) ~ vol + hp + I(hp^2), data = millaje)
summary(modelo_pol2_trans)
```

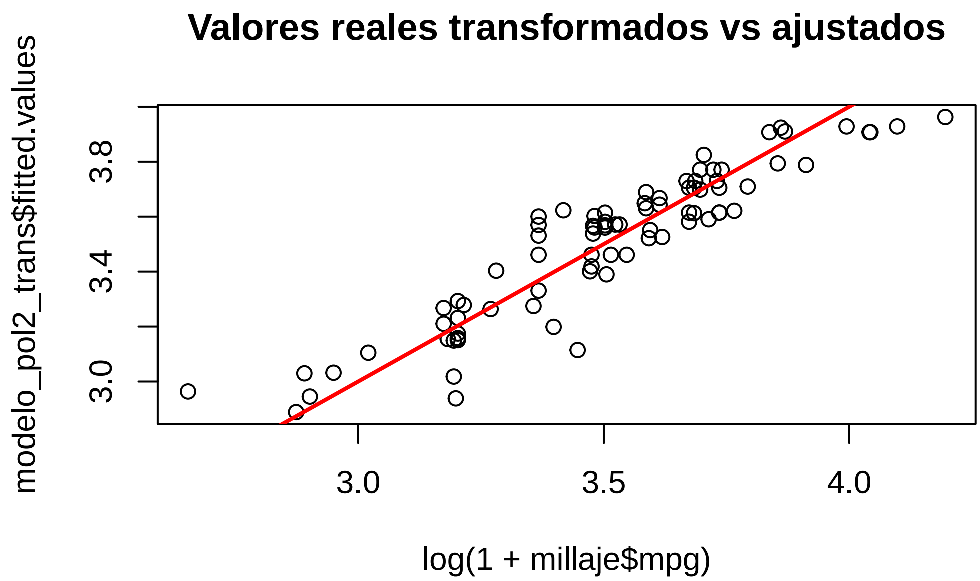
```
Call:
lm(formula = log(1 + mpg) ~ vol + hp + I(hp^2), data = millaje)

Residuals:
     Min       1Q   Median       3Q      Max
-0.31049 -0.06894 -0.02497  0.07082  0.33219

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept)  4.581e+00  7.376e-02  62.104  < 2e-16 ***
vol         -1.846e-03  6.687e-04  -2.761  0.00718 **
hp          -1.011e-02  1.141e-03  -8.858 2.03e-13 ***
I(hp^2)      1.734e-05  3.354e-06   5.171 1.75e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1139 on 78 degrees of freedom
Multiple R-squared:  0.8557,    Adjusted R-squared:  0.8501
F-statistic: 154.1 on 3 and 78 DF,  p-value: < 2.2e-16
```

```r
plot(modelo_pol2_trans$residuals, modelo_pol2_trans$fitted.values,
     main = "Residuos vs ajustados (log(1+mpg))")
```



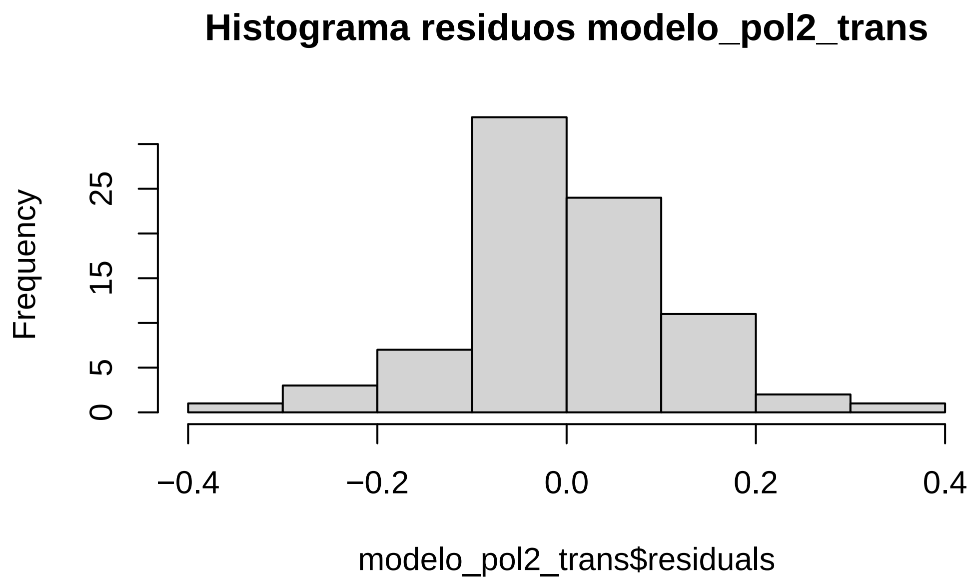

```r
plot(log(1 + millaje$mpg), modelo_pol2_trans$fitted.values,
     main = "Valores reales transformados vs ajustados")
lines(c(2, 5), c(2, 5), col = "red", lwd = 2)
```



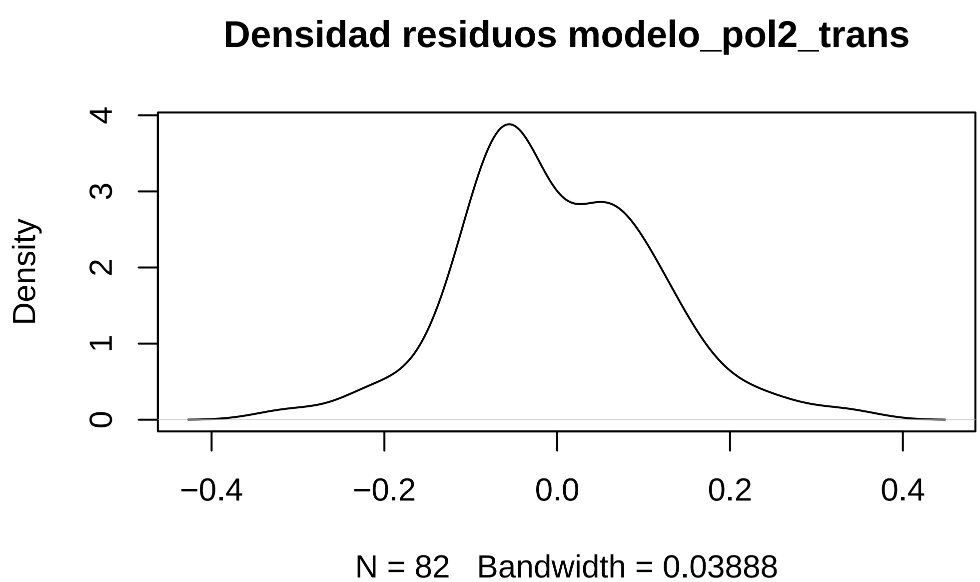

```
shapiro.test(modelo_pol2_trans$residuals)
```

```
	Shapiro-Wilk normality test

data:  modelo_pol2_trans$residuals
W = 0.98398, p-value = 0.4003
```

```
hist(modelo_pol2_trans$residuals, main = "Histograma residuos modelo_pol2_trans")
```




```r
plot(density(modelo_pol2_trans$residuals),
     main = "Densidad residuos modelo_pol2_trans")
```



4.10.2. Transformación raíz cuadrada

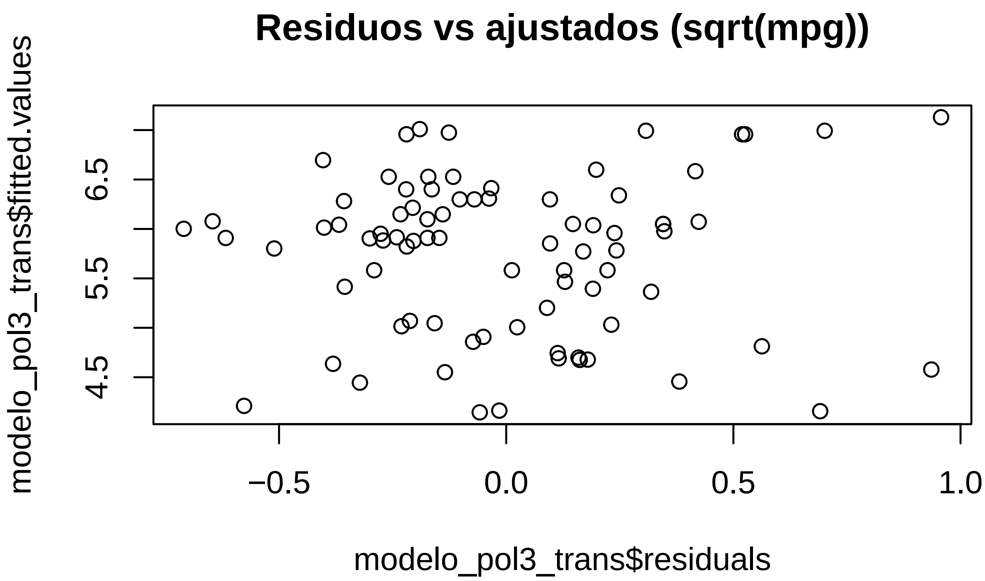
```r
modelo_pol3_trans <- lm(sqrt(mpg) ~ vol + hp + I(hp^2), data = millaje)
summary(modelo_pol3_trans)
```

```

Call:
lm(formula = sqrt(mpg) ~ vol + hp + I(hp^2), data = millaje)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.70979 -0.22004 -0.05431  0.19640  0.95714 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  9.031e+00  2.232e-01  40.459  < 2e-16 ***
vol         -5.083e-03  2.024e-03  -2.512   0.0141 *  
hp          -3.256e-02  3.453e-03  -9.429 1.59e-14 ***
I(hp^2)      6.117e-05  1.015e-05   6.027 5.21e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3448 on 78 degrees of freedom
Multiple R-squared:  0.8442,	Adjusted R-squared:  0.8383 
F-statistic: 140.9 on 3 and 78 DF,  p-value: < 2.2e-16
```

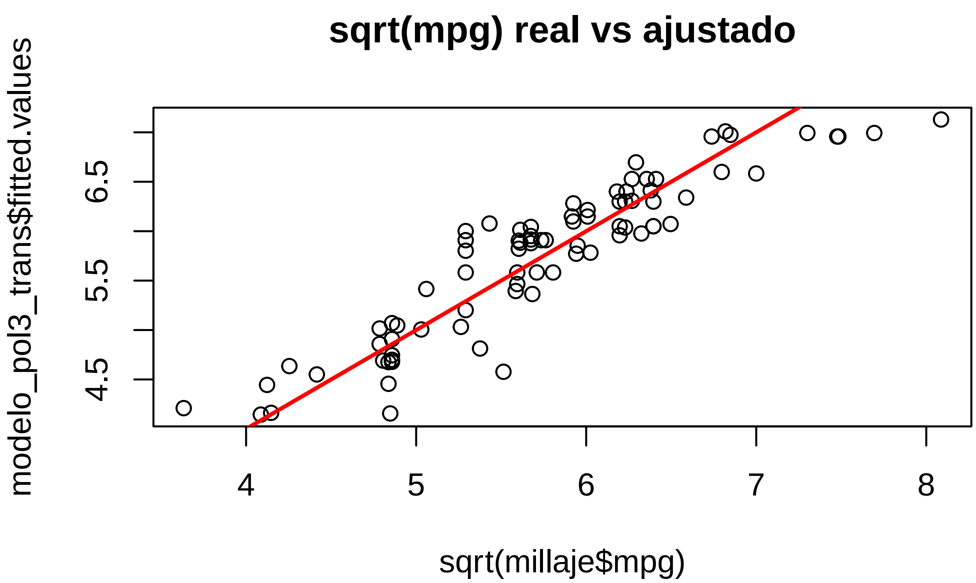
```r
shapiro.test(modelo_pol3_trans$residuals)
```

```

	Shapiro-Wilk normality test

data:  modelo_pol3_trans$residuals
W = 0.97571, p-value = 0.1217
```

```r
plot(modelo_pol3_trans$residuals, modelo_pol3_trans$fitted.values,
     main = "Residuos vs ajustados (sqrt(mpg))")
```




```
plot(sqrt(millaje$mpg), modelo_pol3_trans$fitted.values,
     main = "sqrt(mpg) real vs ajustado")
lines(c(4, 8), c(4, 8), col = "red", lwd = 2)
```

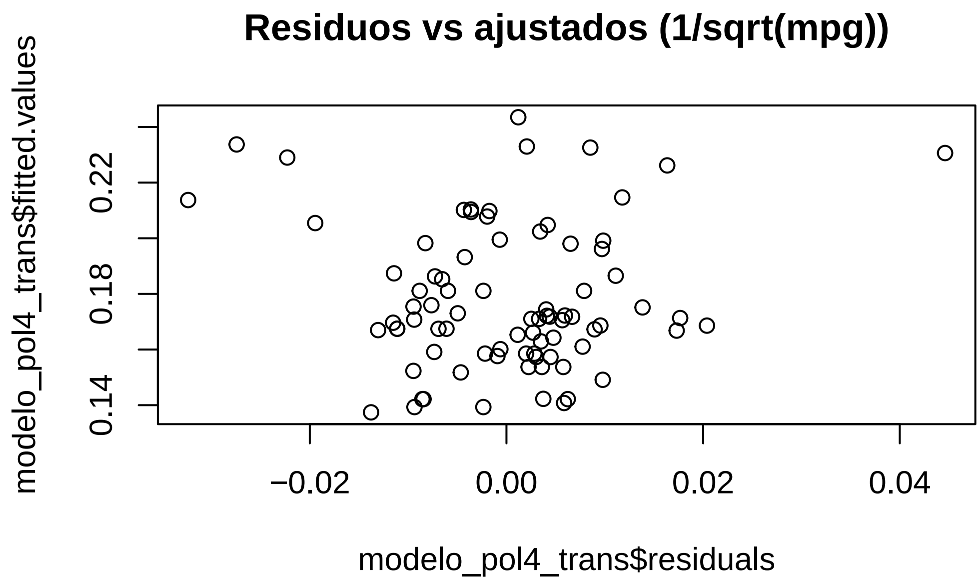


4.10.3. Transformación 1/sqrt(mpg)

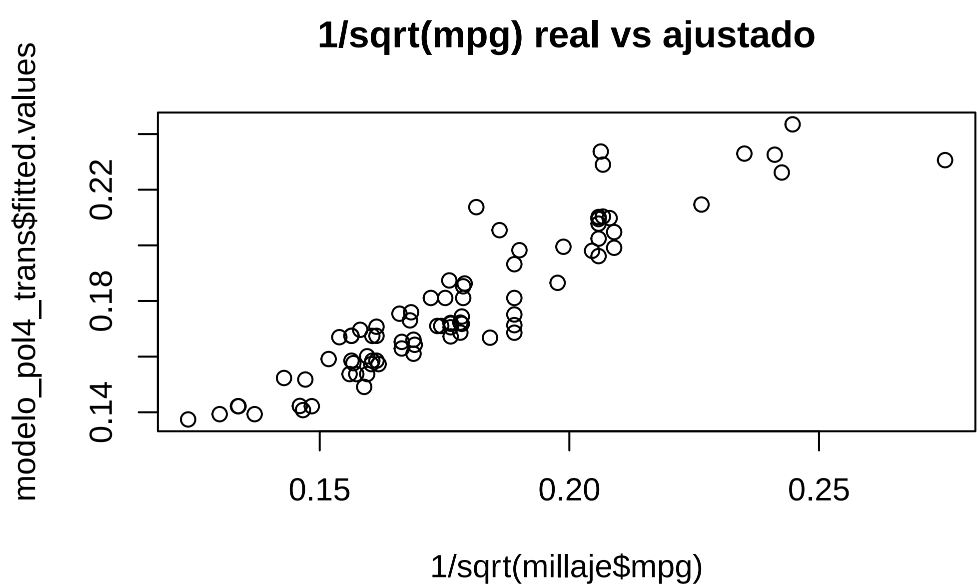
```
modelo_pol4_trans <- lm(1/sqrt(mpg) ~ vol + hp + I(hp^2), data = millaje)
summary(modelo_pol4_trans)
```

```
Call:
lm(formula = 1/sqrt(mpg) ~ vol + hp + I(hp^2), data = millaje)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.032371 -0.007178  0.001601  0.005842  0.044607 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  8.354e-02  7.154e-03  11.677  < 2e-16 ***
vol          1.823e-04  6.486e-05   2.811 0.006249 ** 
hp           8.284e-04  1.107e-04   7.485 9.28e-11 ***
I(hp^2)     -1.219e-06  3.253e-07  -3.747 0.000341 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01105 on 78 degrees of freedom
Multiple R-squared:  0.8501,	Adjusted R-squared:  0.8444 
F-statistic: 147.5 on 3 and 78 DF,  p-value: < 2.2e-16
```

```
plot(modelo_pol4_trans$residuals, modelo_pol4_trans$fitted.values,
     main = "Residuos vs ajustados (1/sqrt(mpg))")
```




```
plot(1/sqrt(millaje$mpg), modelo_pol4_trans$fitted.values,
     main = "1/sqrt(mpg) real vs ajustado")
```



```r
shapiro.test(modelo_pol4_trans$residuals)
```

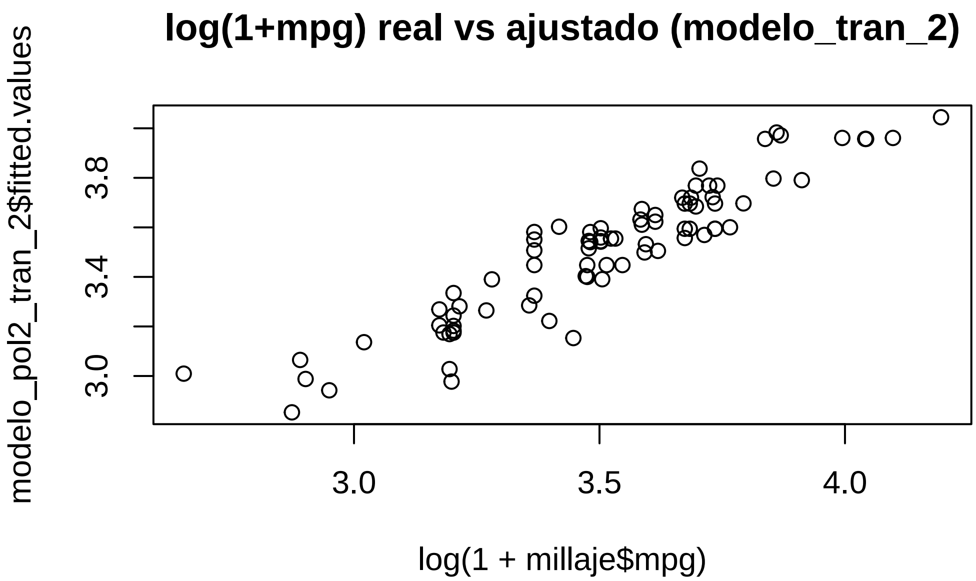
```

	Shapiro-Wilk normality test

data:  modelo_pol4_trans$residuals
W = 0.94789, p-value = 0.002249
```

4.10.4. Transformaciones más complejas

```r
modelo_pol2_tran_2 <- lm(log(1 + mpg) ~ vol + hp + log(1 + hp) + I(hp^2),
                         data = millaje)
summary(modelo_pol2_tran_2)
```

```

Call:
lm(formula = log(1 + mpg) ~ vol + hp + log(1 + hp) + I(hp^2),
    data = millaje)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.35624 -0.06687 -0.00430  0.07828  0.29355 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  7.340e+00  1.236e+00   5.937 7.87e-08 ***
vol         -2.076e-03  6.602e-04  -3.144  0.00237 ** 
hp           2.231e-03  5.630e-03   0.396  0.69297    
log(1 + hp) -8.215e-01  3.675e-01  -2.236  0.02827 *  
I(hp^2)     -2.564e-06  9.486e-06  -0.270  0.78768    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1111 on 77 degrees of freedom
Multiple R-squared:  0.8645,	Adjusted R-squared:  0.8574 
F-statistic: 122.8 on 4 and 77 DF,  p-value: < 2.2e-16
```

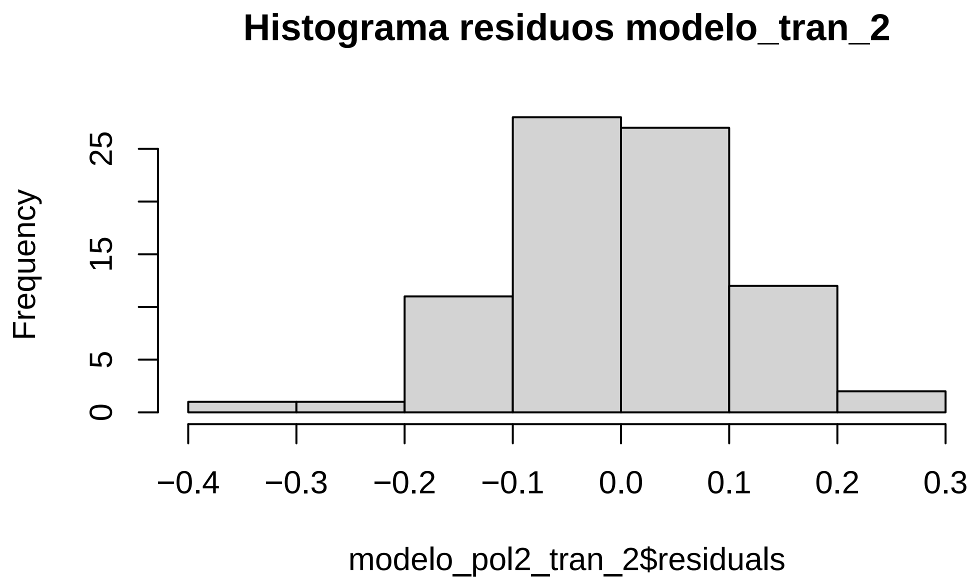
```r
plot(log(1 + millaje$mpg), modelo_pol2_tran_2$fitted.values,
     main = "log(1+mpg) real vs ajustado (modelo_tran_2)")
```



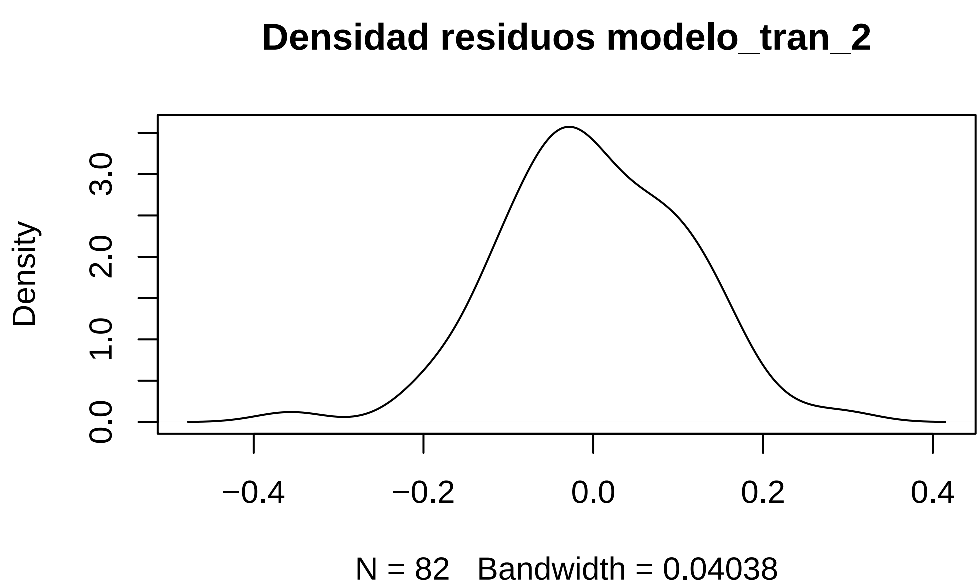

```
shapiro.test(modelo_pol2_tran_2$residuals)
```

```
	Shapiro-Wilk normality test

data:  modelo_pol2_tran_2$residuals
W = 0.98972, p-value = 0.7645
```

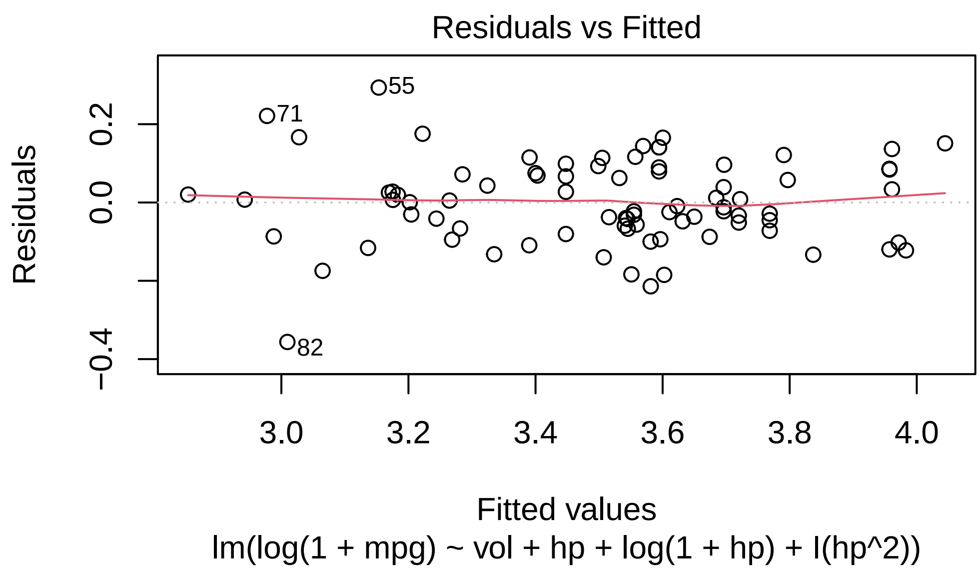
```
hist(modelo_pol2_tran_2$residuals, main = "Histograma residuos modelo_tran_2")
```

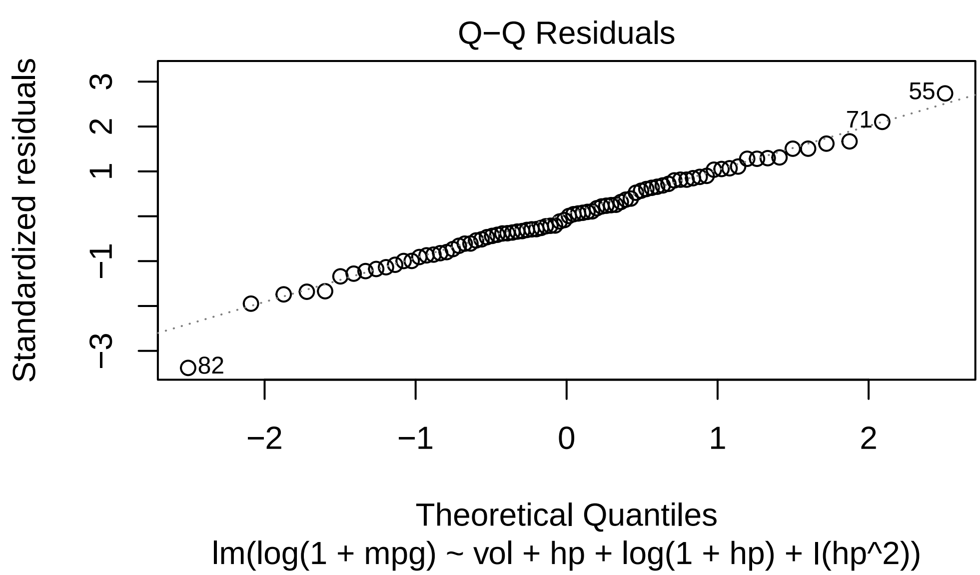


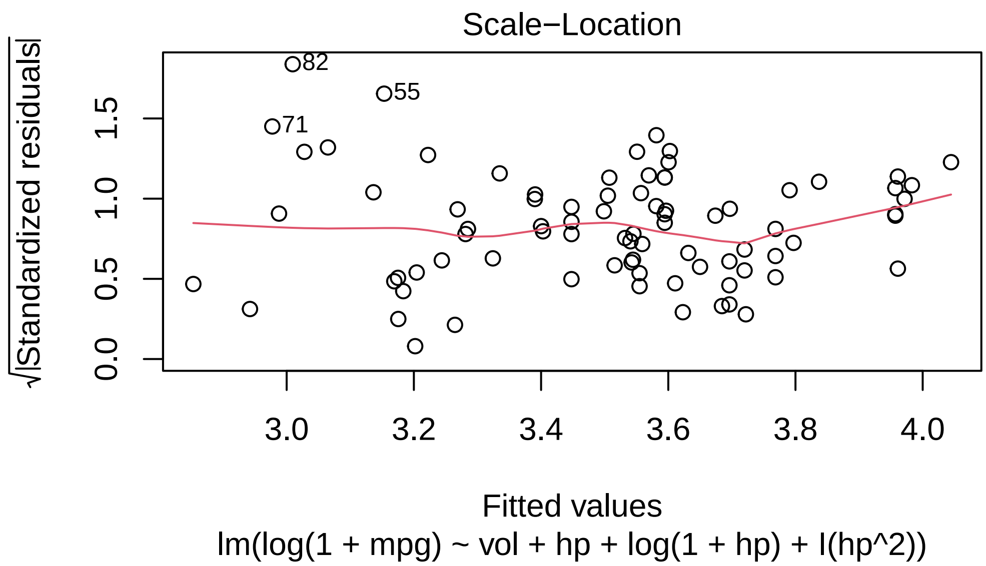

```
plot(density(modelo_pol2_tran_2$residuals),
     main = "Densidad residuos modelo_tran_2")
```

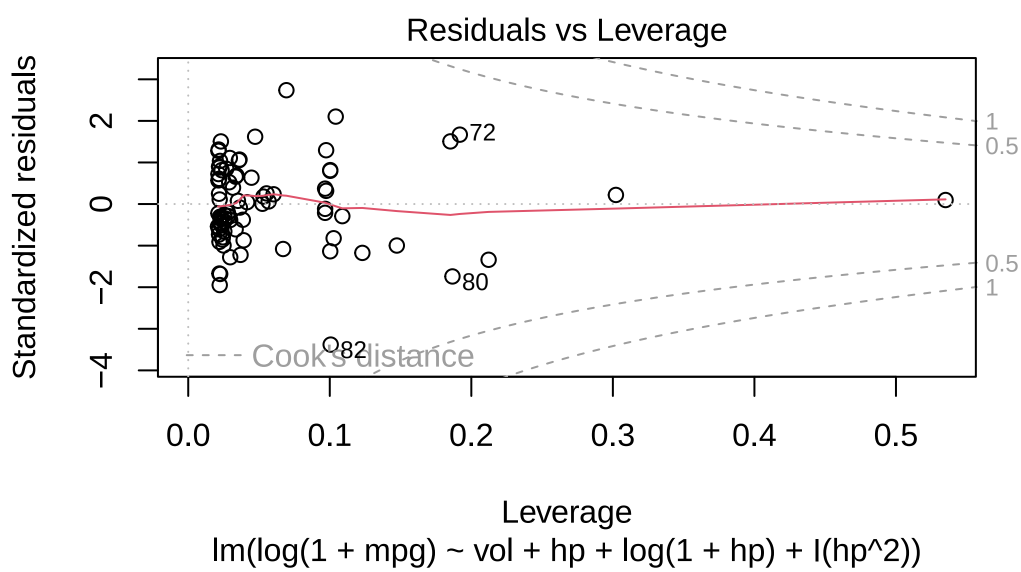


```r
plot(modelo_pol2_tran_2)
```









Otro modelo más flexible:

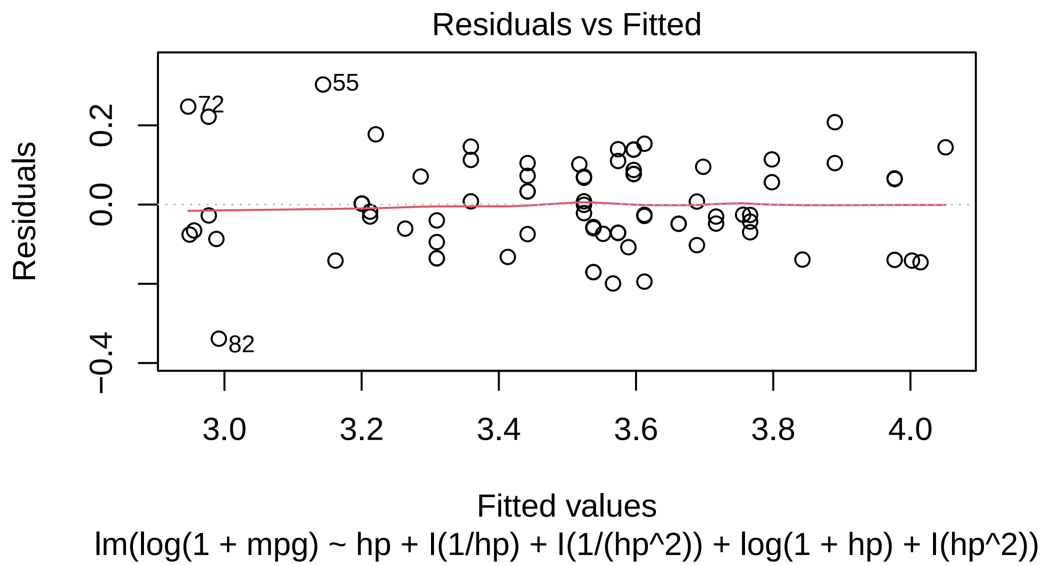
```r
modelo_pol2_tran_3 <- lm(log(1 + mpg) ~ hp + I(1/hp) + I(1/(hp^2)) +
                           log(1 + hp) + I(hp^2),
                         data = millaje)
summary(modelo_pol2_tran_3)
```

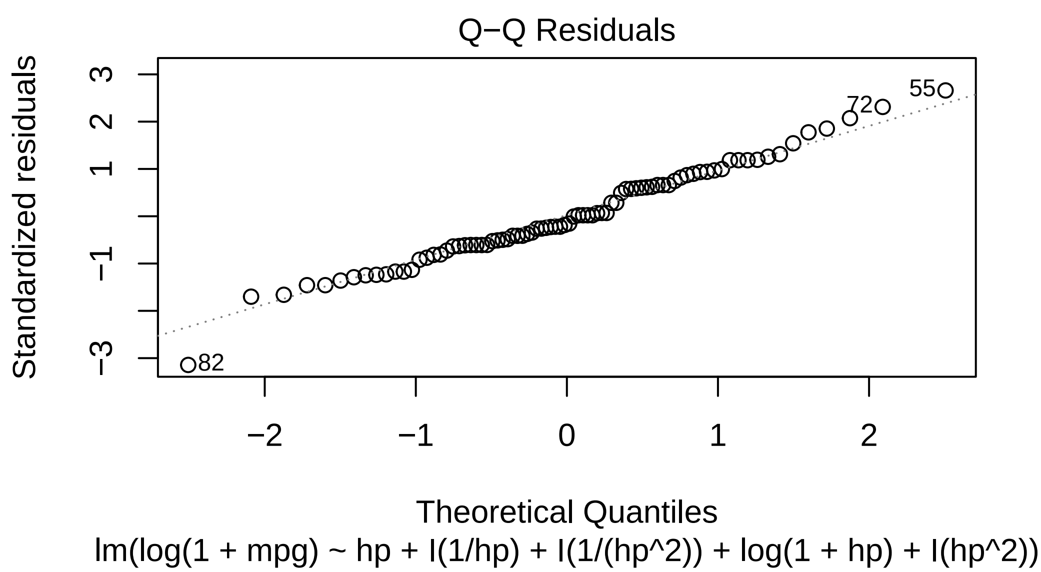
```
Call:
lm(formula = log(1 + mpg) ~ hp + I(1/hp) + I(1/(hp^2)) + log(1 +
    hp) + I(hp^2), data = millaje)

Residuals:
     Min       1Q   Median       3Q      Max
-0.33860 -0.07127 -0.01969  0.07723  0.30307

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.034e+01  7.517e+01  -0.271    0.787
hp          -3.355e-02  8.029e-02  -0.418    0.677
I(1/hp)      4.093e+02  1.224e+03   0.334    0.739
I(1/(hp^2)) -5.165e+03  1.705e+04  -0.303    0.763
log(1 + hp)  5.049e+00  1.558e+01   0.324    0.747
I(hp^2)      3.600e-05  7.142e-05   0.504    0.616

Residual standard error: 0.1187 on 76 degrees of freedom
Multiple R-squared:  0.8475,	Adjusted R-squared:  0.8374
F-statistic: 84.45 on 5 and 76 DF,  p-value: < 2.2e-16
```

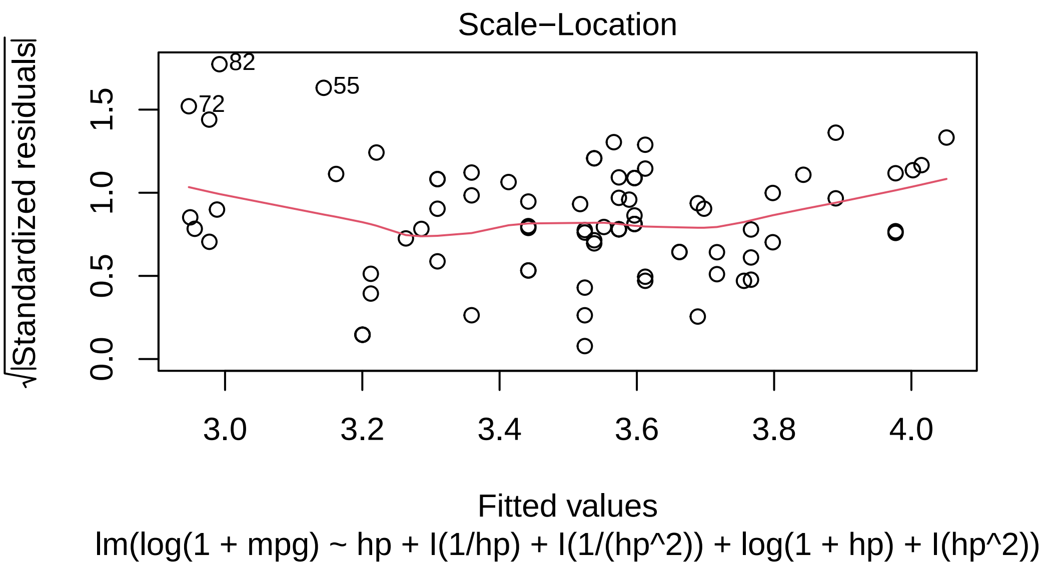
```r
shapiro.test(modelo_pol2_tran_3$residuals)
```

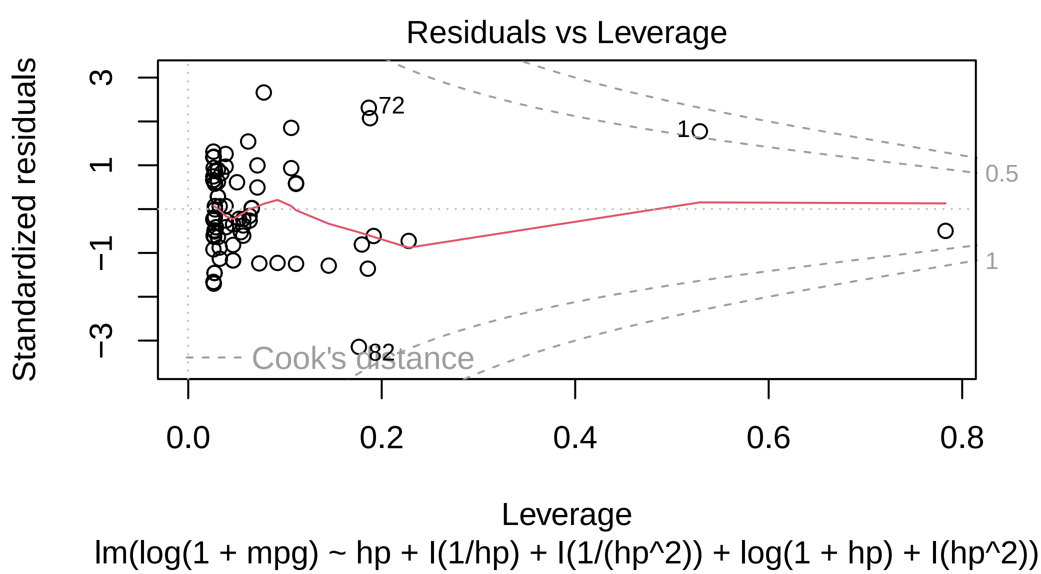
```
	Shapiro-Wilk normality test

data:  modelo_pol2_tran_3$residuals
W = 0.98744, p-value = 0.6104
```

```r
plot(modelo_pol2_tran_3)
```

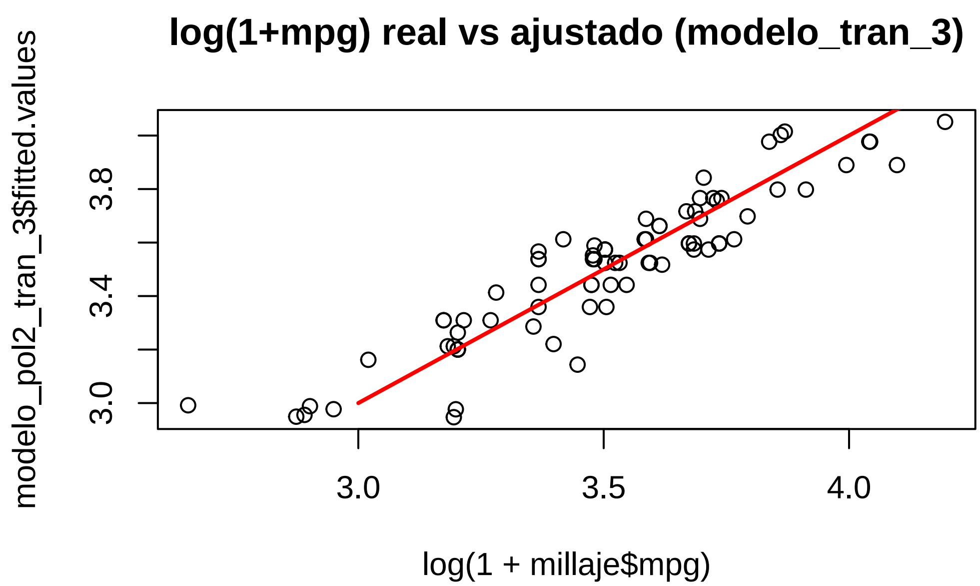

Residuals vs Fitted
Residuals
0.2
0.0
−0.4
72
55
82
3.0
3.2
3.4
3.6
3.8
4.0
Fitted values
lm(log(1 + mpg) ~ hp + I(1/hp) + I(1/(hp^2)) + log(1 + hp) + I(hp^2))


Q−Q Residuals
Standardized residuals
3
2
1
−1
−3
72
55
82
−2
−1
0
1
2
Theoretical Quantiles
lm(log(1 + mpg) ~ hp + I(1/hp) + I(1/(hp^2)) + log(1 + hp) + I(hp^2))






```
plot(log(1 + millaje$mpg), modelo_pol2_tran_3$fitted.values,
     main = "log(1+mpg) real vs ajustado (modelo_tran_3)")
lines(c(3, 4.5), c(3, 4.5), col = "red", lwd = 2)
```



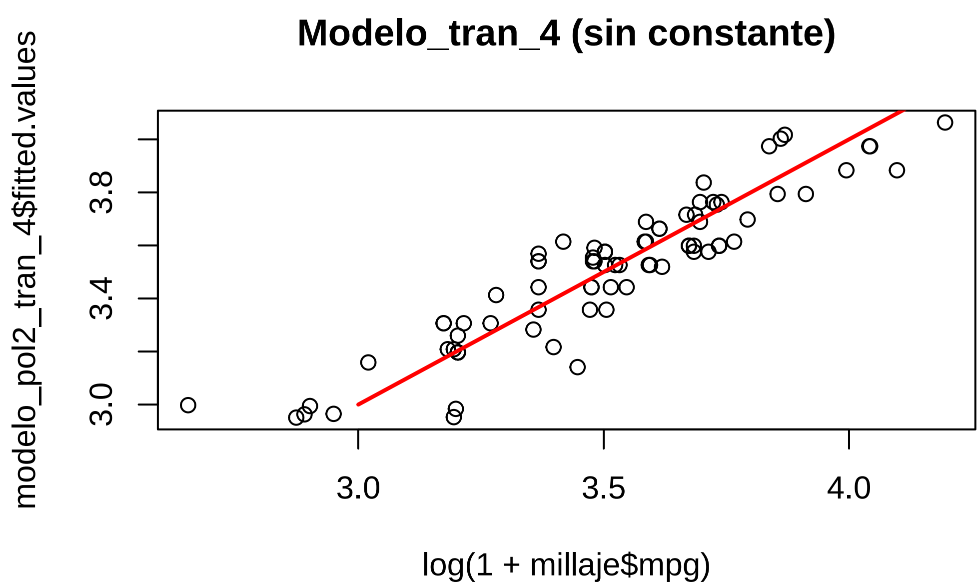
4.10.5. Modelos sin constante y selección

```
modelo_pol2_tran_4 <- lm(log(1 + mpg) ~ hp + I(1/hp) + I(1/(hp^2)) +
                           log(1 + hp) + I(hp^2) - 1,
                         data = millaje)
summary(modelo_pol2_tran_4)
```

```
Call:
lm(formula = log(1 + mpg) ~ hp + I(1/hp) + I(1/(hp^2)) + log(1 +
    hp) + I(hp^2) - 1, data = millaje)

Residuals:
     Min       1Q   Median       3Q      Max
-0.34452 -0.07355 -0.01464  0.07509  0.30562

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
hp          -1.202e-02  1.098e-02  -1.094   0.2772
I(1/hp)      7.934e+01  1.074e+02   0.738   0.4625
I(1/(hp^2)) -6.299e+02  3.147e+03  -0.200   0.8419
```

```
log(1 + hp)  8.319e-01  3.658e-01   2.274   0.0258 *
I(hp^2)      1.725e-05  1.732e-05   0.996   0.3223
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1179 on 77 degrees of freedom
Multiple R-squared:  0.9989,    Adjusted R-squared:  0.9989
F-statistic: 1.459e+04 on 5 and 77 DF,  p-value: < 2.2e-16
```

```
plot(log(1 + millaje$mpg), modelo_pol2_tran_4$fitted.values,
     main = "Modelo_tran_4 (sin constante)")
lines(c(3, 4.5), c(3, 4.5), col = "red", lwd = 2)
```




```
modelo_pol2_tran_5 <- lm(log(1 + mpg) ~ hp + I(1/hp) + log(1 + hp) + I(hp^2) - 1,
                         data = millaje)
summary(modelo_pol2_tran_5)
```

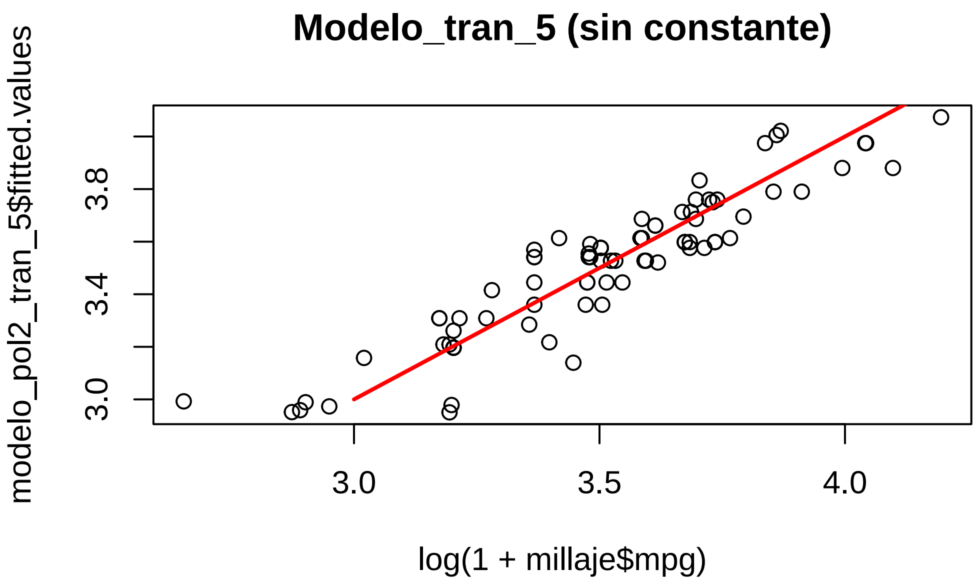
```
Call:
lm(formula = log(1 + mpg) ~ hp + I(1/hp) + log(1 + hp) + I(hp^2) -
    1, data = millaje)
```

```
Residuals:
     Min       1Q   Median       3Q      Max 
-0.33926 -0.07379 -0.01700  0.07535  0.30724 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
hp          -1.410e-02  3.554e-03  -3.967 0.000161 ***
I(1/hp)      5.795e+01  1.116e+01   5.194  1.6e-06 ***
log(1 + hp)  9.030e-01  8.687e-02  10.394  < 2e-16 ***
I(hp^2)      2.041e-05  7.119e-06   2.867 0.005328 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1172 on 78 degrees of freedom
Multiple R-squared:  0.9989,    Adjusted R-squared:  0.9989 
F-statistic: 1.846e+04 on 4 and 78 DF,  p-value: < 2.2e-16
```

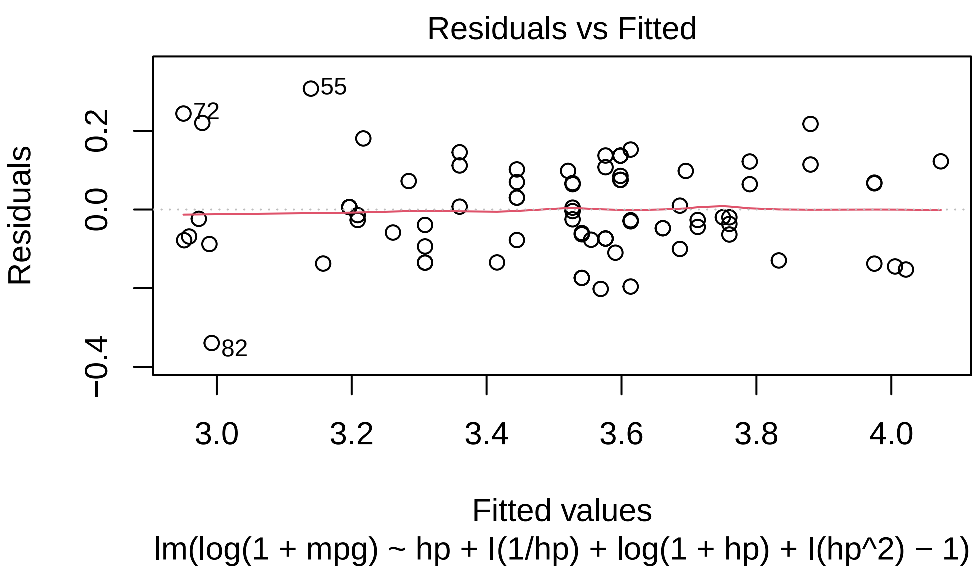
```r
shapiro.test(modelo_pol2_tran_5$residuals)
```

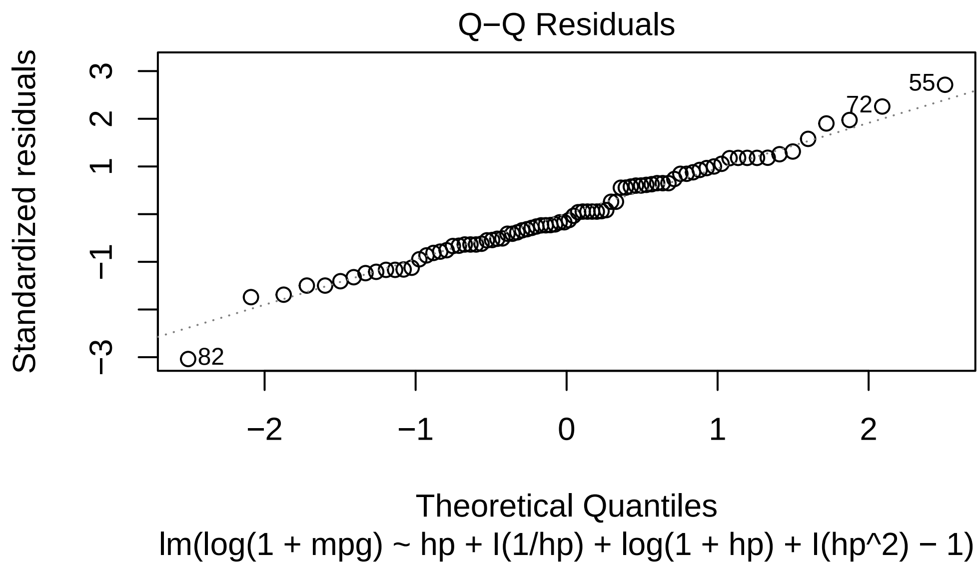
```
    Shapiro-Wilk normality test

data:  modelo_pol2_tran_5$residuals
W = 0.98884, p-value = 0.7058
```

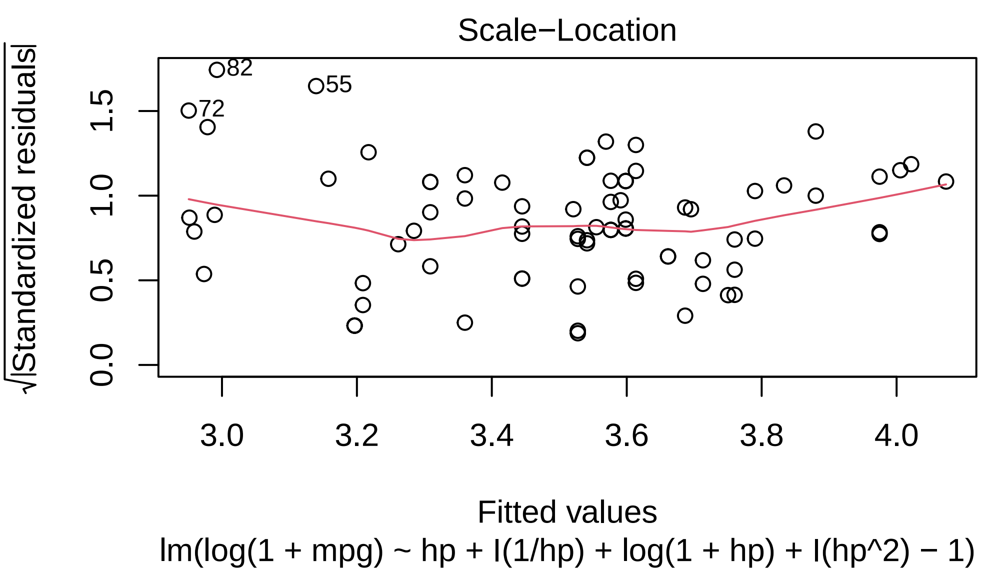
```r
plot(log(1 + millaje$mpg), modelo_pol2_tran_5$fitted.values,
     main = "Modelo_tran_5 (sin constante)")
lines(c(3, 4.5), c(3, 4.5), col = "red", lwd = 2)
```

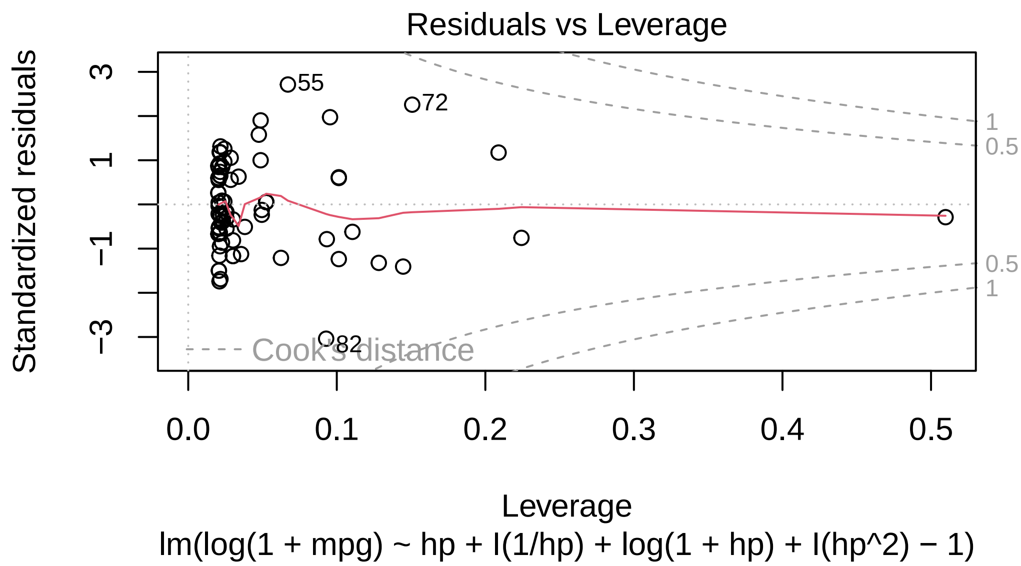



```r
plot(modelo_pol2_tran_5)
```




Q−Q Residuals
Standardized residuals
Theoretical Quantiles
lm(log(1 + mpg) ~ hp + I(1/hp) + log(1 + hp) + I(hp^2) − 1)
55
72
82


Scale−Location
√|Standardized residuals|
Fitted values
lm(log(1 + mpg) ~ hp + I(1/hp) + log(1 + hp) + I(hp^2) − 1)
82
55
72



4.10.6. Un modelo candidato "bueno"

El script sugiere como uno de los mejores:

```
modelo_pol2_tran_6 <- lm(log(1 + mpg) ~ log(1 + hp) - 1, data = millaje)
summary(modelo_pol2_tran_6)
```

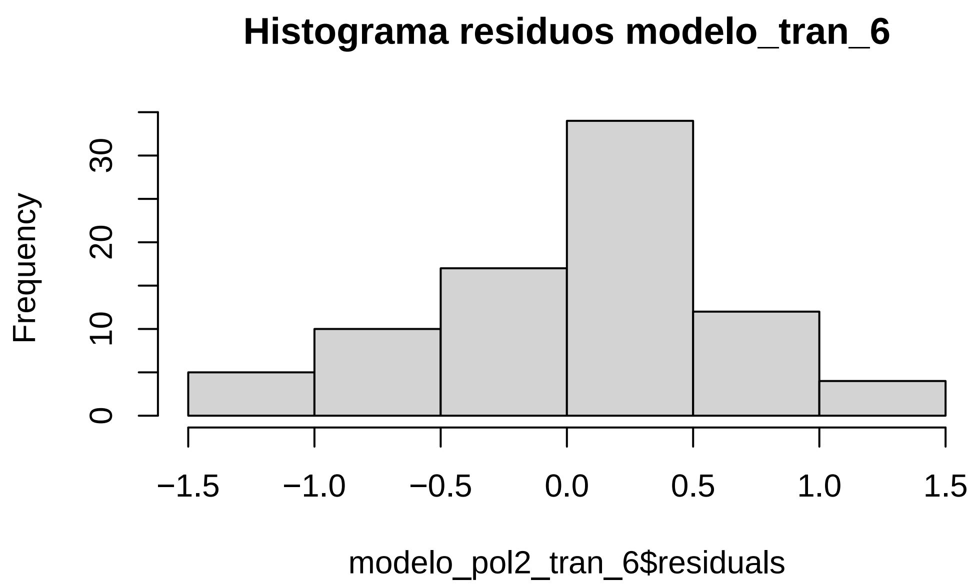
```
Call:
lm(formula = log(1 + mpg) ~ log(1 + hp) - 1, data = millaje)

Residuals:
    Min      1Q  Median      3Q     Max
-1.3860 -0.3480  0.1168  0.3870  1.3059

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
log(1 + hp)  0.73870    0.01383   53.42   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5883 on 81 degrees of freedom
```

```
Multiple R-squared:  0.9724,    Adjusted R-squared:  0.9721 
F-statistic:  2854 on 1 and 81 DF,  p-value: < 2.2e-16
```

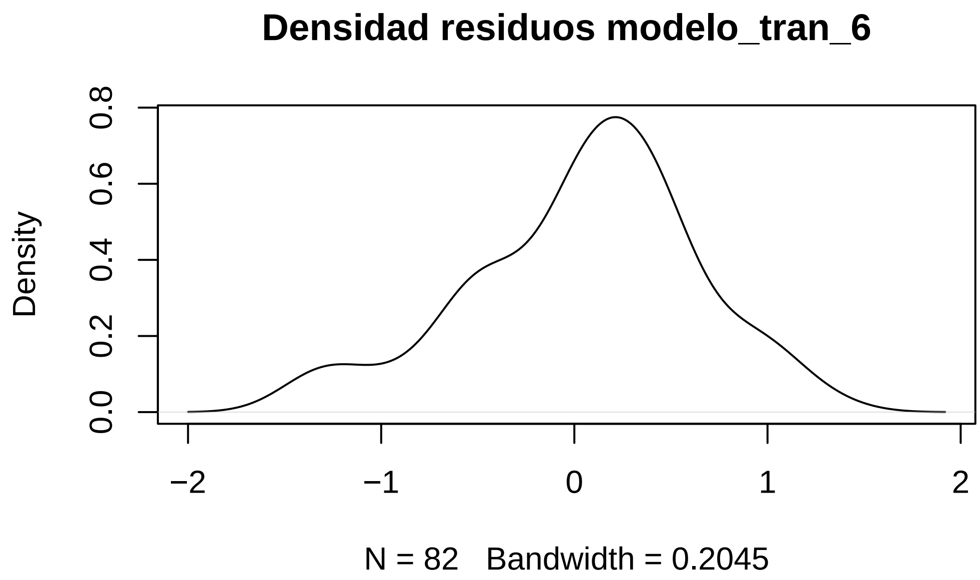
```r
shapiro.test(modelo_pol2_tran_6$residuals)
```

```
    Shapiro-Wilk normality test

data:  modelo_pol2_tran_6$residuals
W = 0.97148, p-value = 0.06443
```

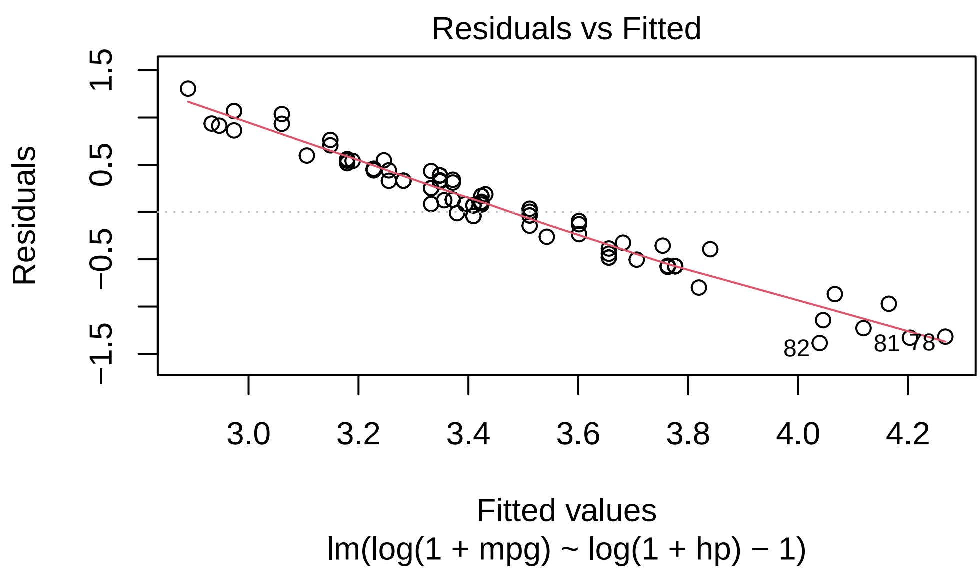
```r
hist(modelo_pol2_tran_6$residuals,
     main = "Histograma residuos modelo_tran_6")
```

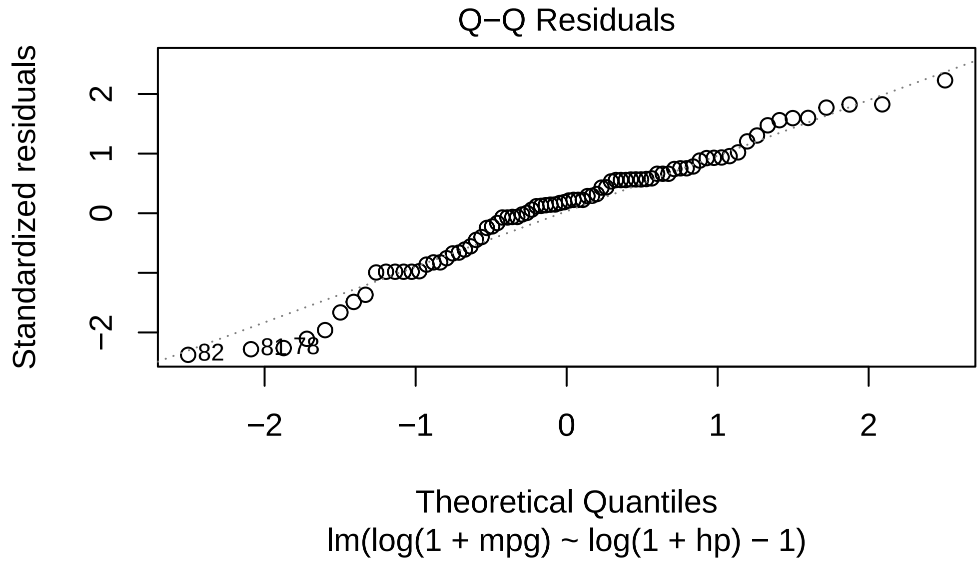


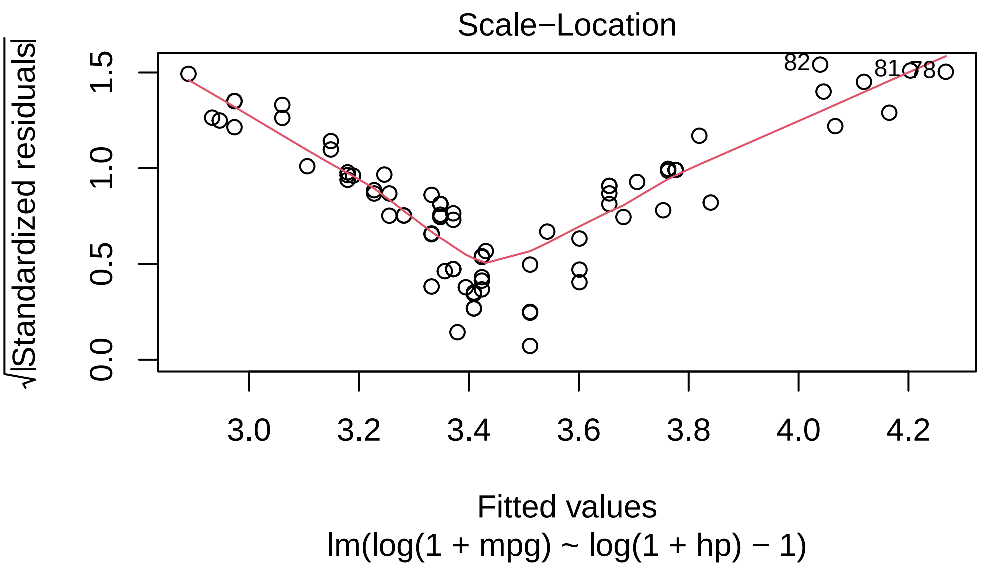

```r
plot(density(modelo_pol2_tran_6$residuals),
     main = "Densidad residuos modelo_tran_6")
```

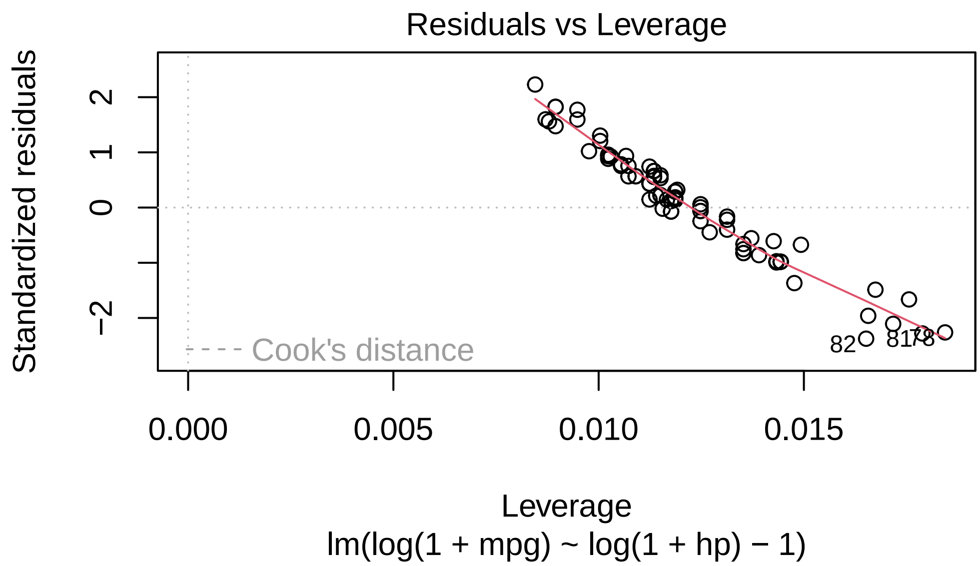



```r
plot(modelo_pol2_tran_6)
```




Q−Q Residuals
Standardized residuals
Theoretical Quantiles
lm(log(1 + mpg) ~ log(1 + hp) − 1)
82
81
78
−2
−1
0
1
2


Scale−Location
√|Standardized residuals|
Fitted values
lm(log(1 + mpg) ~ log(1 + hp) − 1)
82
81
78
0.0
0.5
1.0
1.5
3.0
3.2
3.4
3.6
3.8
4.0
4.2




```r
modelo_pol2_tran_7 <- lm(log(1 + mpg) ~ vol + log(1 + hp) - 1, data = millaje)
summary(modelo_pol2_tran_7)
```

```
Call:
lm(formula = log(1 + mpg) ~ vol + log(1 + hp) - 1, data = millaje)

Residuals:
    Min      1Q  Median      3Q     Max
-1.3617 -0.3668  0.1028  0.4405  1.2853

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
vol         0.003047   0.002943   1.035    0.304
log(1 + hp) 0.674633   0.063404  10.640   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.588 on 80 degrees of freedom
Multiple R-squared:  0.9728,    Adjusted R-squared:  0.9721
F-statistic:  1429 on 2 and 80 DF,  p-value: < 2.2e-16
```

> **Nota**
>
> En la práctica, al elegir entre varios modelos transformados, debes considerar:
>
> - Supuestos sobre los **residuos** (normalidad, homocedasticidad).
> - Interpretabilidad económica de los coeficientes.
> - Capacidad predictiva (idealmente evaluada fuera de muestra).
> - Parsimonia: preferir el modelo más simple que explique bien los datos.

5. Cierre del laboratorio

En este laboratorio trabajaste con:

- Regresión múltiple con **eliminación de variables irrelevantes**.
- Inclusión de **interacciones** y términos **cuadráticos**.
- Visualizaciones 3D de superficies de regresión.
- Modelos polinomiales de distintos grados.
- Comparación de modelos vía **ANOVA**.
- Uso de **transformaciones** (log, raíz, recíprocos) para mejorar los supuestos.

Todo esto forma parte del "arsenal" que usarás en cursos posteriores de econometría y en aplicaciones reales para ajustar modelos más realistas y robustos