Homework 1

Problem 1.1

2.1 Order the following functions by growth rate: N, $\sqrt{N}$, $N^{1.5}$, N^2, $N \log N$, $N \log \log N$, $N \log^2 N$, $N \log(N^2)$, $2/N$, 2^N, $2^{N/2}$, 37, $N^2 \log N$, N^3. Indicate which functions grow at the same rate.

Problem 1.2

2.7 For each of the following six program fragments:

a. Give an analysis of the running time (Big-Oh will do).

b. Implement the code in Java, and give the running time for several values of N.

c. Compare your analysis with the actual running times.

(1)
```
sum = 0;
for( i = 0; i < n; i++ )
    sum++;
```

(2)
```
sum = 0;
for( i = 0; i < n; i++ )
    for( j = 0; j < n; j++ )
        sum++;
```

(3)
```
sum = 0;
for( i = 0; i < n; i++ )
    for( j = 0; j < n * n; j++ )
        sum++;
```

(4)
```
sum = 0;
for( i = 0; i < n; i++ )
    for( j = 0; j < i; j++ )
        sum++;
```

---

(5)
```
sum = 0;
for( i = 0; i < n; i++ )
    for( j = 0; j < i * i; j++ )
        for( k = 0; k < j; k++ )
            sum++;
```

(6)
```
sum = 0;
for( i = 1; i < n; i++ )
    for( j = 1; j < i * i; j++ )
       if( j % i == 0 )
           for( k = 0; k < j; k++ )
               sum++;
```

Problem 1.3

2.11 An algorithm takes 0.5 ms for input size 100. How long will it take for input size 500 if the running time is the following (assume low-order terms are negligible):

a. linear
b. $O(N \log N)$
c. quadratic
d. cubic

Problem 1.4

2.15 Give an efficient algorithm to determine if there exists an integer i such that $A_i = i$ in an array of integers $A_1 < A_2 < A_3 < \cdots < A_N$. What is the running time of your algorithm?

Problem 1.5

2.25 Programs A and B are analyzed and found to have worst-case running times no greater than $150N \log_2 N$ and N^2, respectively. Answer the following questions, if possible:

a. Which program has the better guarantee on the running time, for large values of N ($N > 10{,}000$)?

b. Which program has the better guarantee on the running time, for small values of N ($N < 100$)?

c. Which program will run faster *on average* for $N = 1{,}000$?

d. Is it possible that program B will run faster than program A on *all* possible inputs?