What determines catchment water balance partitioning?

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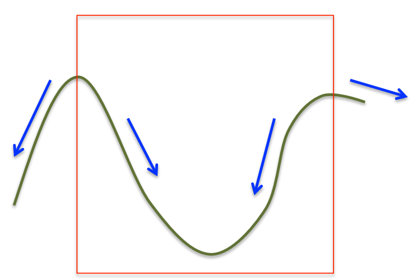
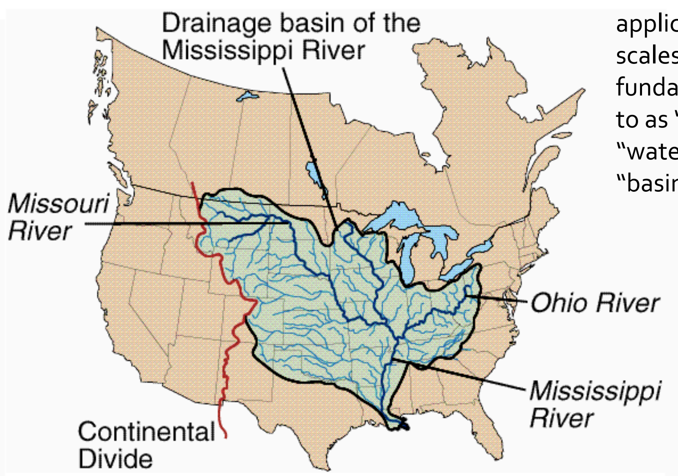
Catchment: a basic unit of hydrology

- *"The area of land from which any water flowing through a particular point in a stream channel must have originated"*
- This definition relies on water not flowing upward.
- The boundaries are strategically placed at local maxima (ridges, hilltops) so that they effectively define a **control volumes for water fluxes.**



Catchment: a basic unit of hydrology



The definition can be applicable over a range of scales… so you'll hear this fundamental unit referred to as "catchments," "watersheds," and "basins."

Catchment: a basic unit of hydrology

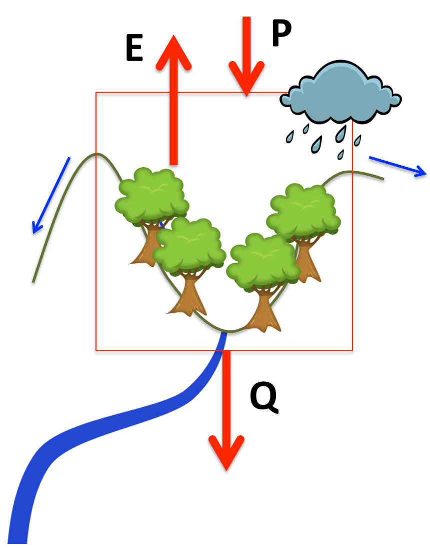
- Inside this **control volume**, all flows converge toward the lowest point – a single outlet represented by a river or a stream or a lake.

Storage   Evaporation + Transpiration
Precipitation   Streamflow

$$\frac{dS}{dt} = P - E - Q$$

- If we assume long term, steady state conditions, then *dS/dt* → *o*. This gives us *P* = *E* + *Q*, and

$$1 = \frac{Q}{P} + \frac{E}{P}$$



Rivers do not exist in isolation!

It shapes the landscape through which it flows, but its flow regime is also controlled by how its surrounding landscape "filters" incoming water (i.e., precipitation).

Water balance partitioning

$$1 = \frac{Q}{P} + \frac{E}{P}$$

The central question (v.1): how does the input P get partitioned between the two dominant modes of outputs Q and E?

This is roughly divided between water that plants use ("green water," E), and water we humans use ("blue water," Q).

So why do we care about water balance partitioning?

- Information from streamflow are used operationally to improve water security and reduce hazard risks.
  - Managing **dams** – for hydropower production, erosion and sediment transport, aquatic habitats.
  - Assessing **flood and drought risks**
  - Allocating water from **aquifers**
  - Managing **irrigation districts**

- Indicators of climate and land use change
  - Preserving river or wetland **habitat**

- In the context of this summer institute, streamflow can also tell us about **landscape evolution.**

Water balance partitioning

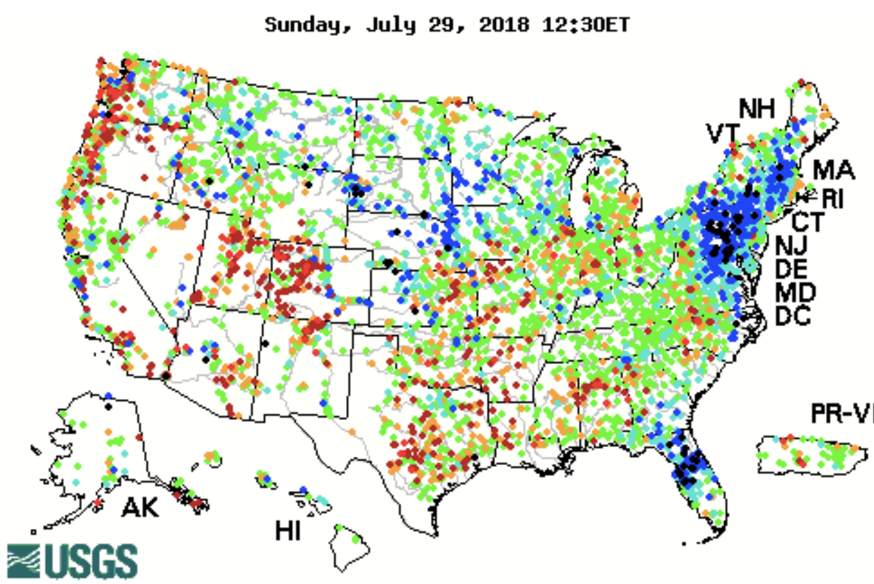
$$1 = \frac{Q}{P} + \frac{E}{P}$$

Let's take a closer look at how we measure these fluxes.

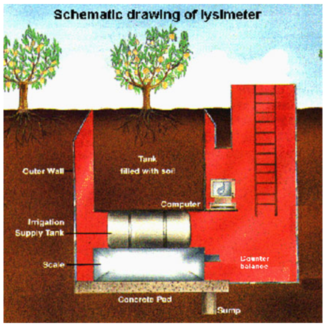
Streamflow measurements

- Available over a long time
  - U. S. Geological Service (USGS) started its first stream gage on 1889 on the Rio Grande in New Mexico
- Relatively straightforward to obtain
  - Need stage-discharge relationship (height to volumetric flow)
  - Extrapolate using continuously measured stage

https://waterdata.usgs.gov/nwis/rt



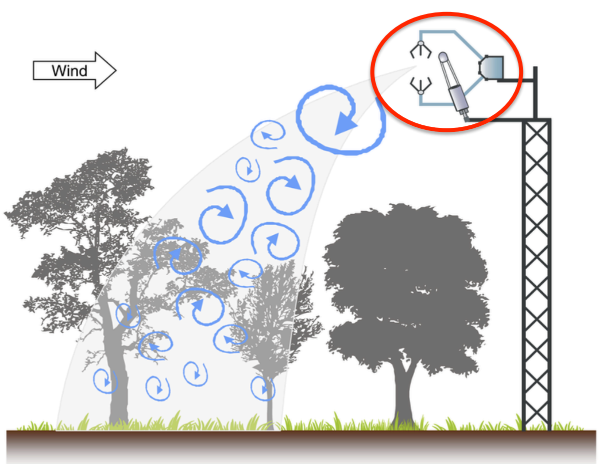
Evapotranspiration measurements

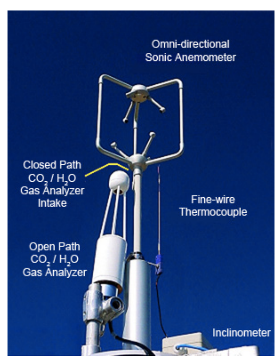


Direct measurements are only possible at relatively small scales. For example, these large weighing lysimeters measure mass changes – they are huge!! – but tiny compared to the whole landscape

Evapotranspiration measurements

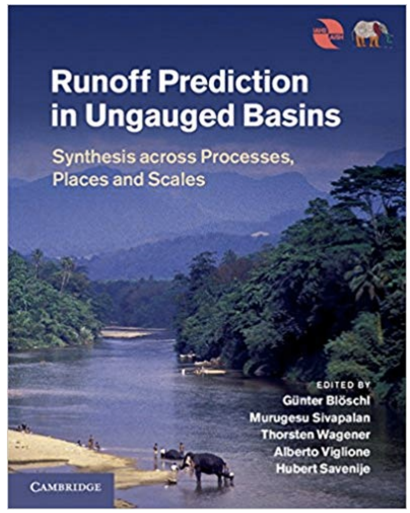
Flux towers relies on principles of turbulent mass transport – which relies on assumptions. Their footprints is also relatively small compared to a catchment.



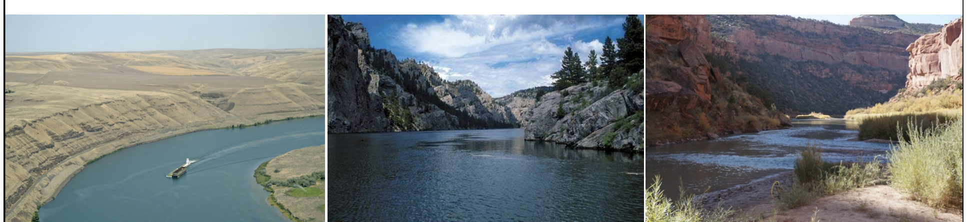


Our direct knowledge is still incomplete

- Limited by scale of measurement
  - The total E from a catchment is much larger than what we can directly measure!
  - Satellite sensing of E has many associated uncertainties.
- Limited by spatial coverage of stream gages
  - This is especially the case in rural sites and developing countries.



The central question (v.2): in the absence of direct measurements, can we predict variations in Q and E across different types of catchments?



INVITED COMMENTARY

HYDROLOGICAL PROCESSES
Hydrol. Process. **17**, 3163–3170 (2003)
Published online in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/hyp.5155

Prediction in ungauged basins: a grand challenge for theoretical hydrology

In this session, we attempt to answer:
"What determines catchment water balance partitioning?"

Part I : I talk
Introduction to water balance partitioning
Empirical observations • process-based modeling

Part II : You sleuth
Investigating controls for water balance partitioning

Part III : You talk
Presentation of findings

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Let's start by considering that transpiration dominates the global water cycle

$$1 = \frac{Q}{P} + \frac{E}{P}$$

LETTER

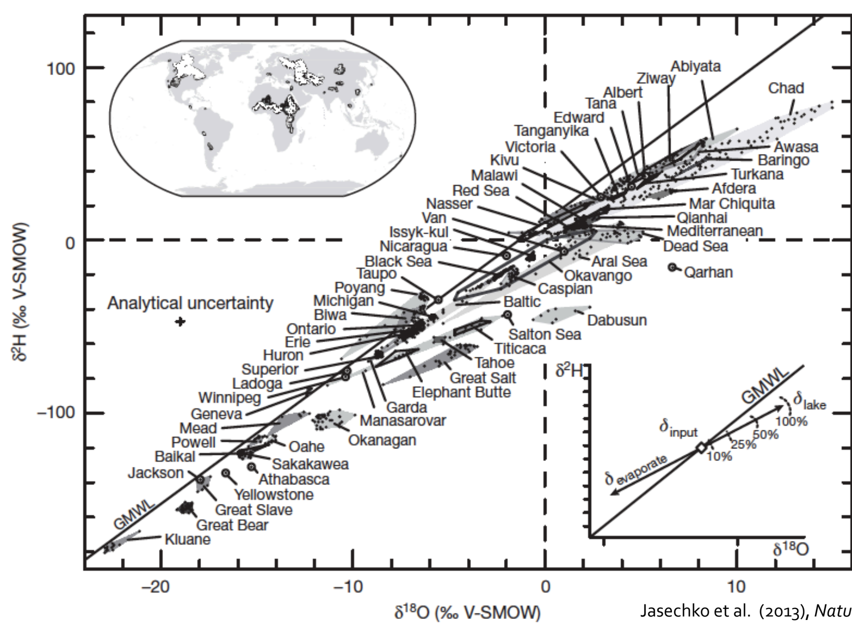
doi:10.1038/nature11983

Terrestrial water fluxes dominated by transpiration

Scott Jasechko[1], Zachary D. Sharp[1], John J. Gibson[2,3], S. Jean Birks[2,4], Yi Yi[2,3] & Peter J. Fawcett[1]

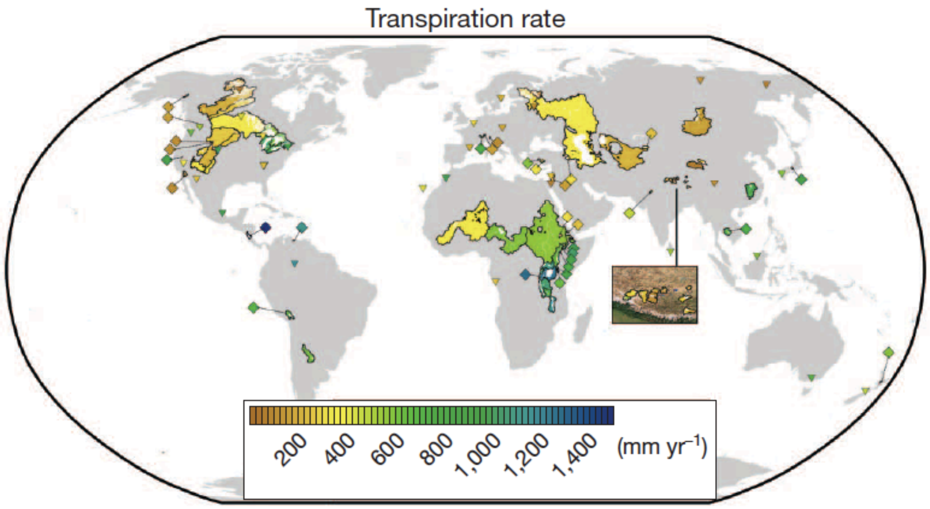
- Transpiration consist of 80-90% of terrestrial evaporation E
- 56% of terrestrial precipitation… i.e., E/P ~ 0.56
- Approach: mass and isotope balance of global lake catchments

Varying isotopic signature of lake catchments



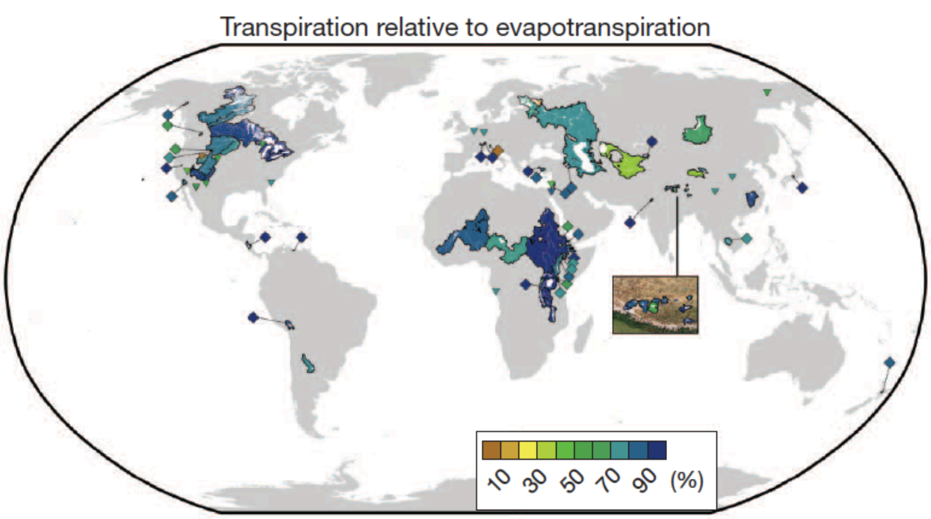
Jasechko et al. (2013), *Nature.*

Variations of transpiration across climates

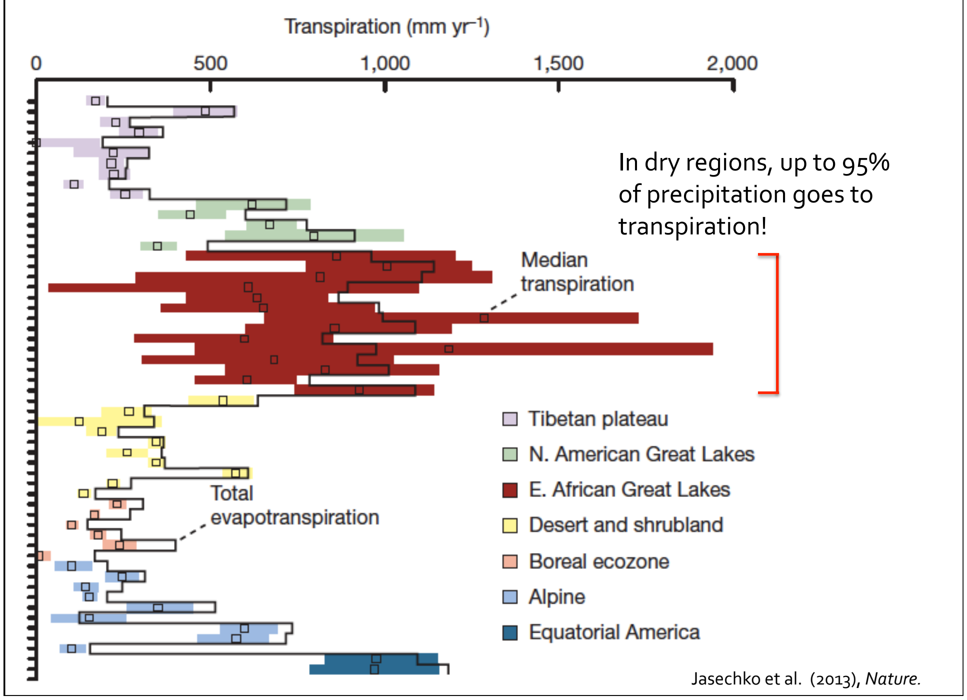


Jasechko et al. (2013), *Nature.*

And as % of total E



Jasechko et al. (2013), *Nature.*

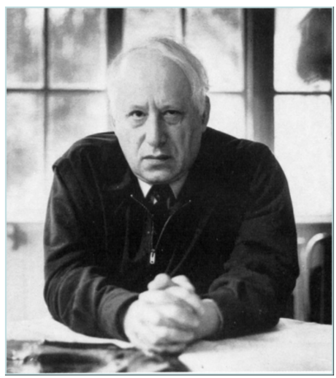


The relative dominance of transpiration is *climate-mediated*

- If it is hot and dry, most precipitation will evaporate. There also probably will not be a lot of plants around.

- If it is cold and humid, most precipitation will not evaporate. Doesn't matter how many plants you have around.

Controlling for the hydroclimate via Budyko's curve



Mikhail Ivanovich Budyko

In Budyko's framework, water scarcity is represented by

Dryness index = PET / P

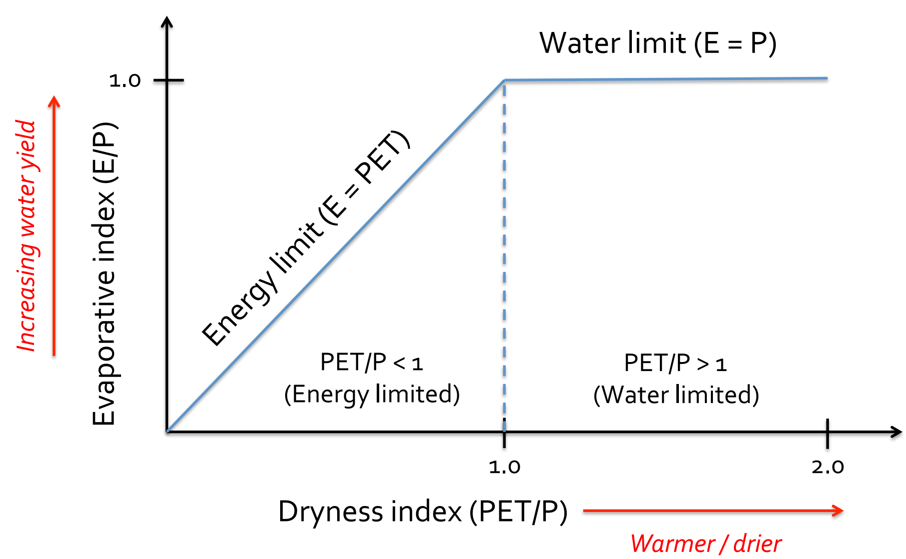
P : precipitation (water supply)

PET : potential evapotranspiration (energetic water demand) – based on temperature, radiation, humidity

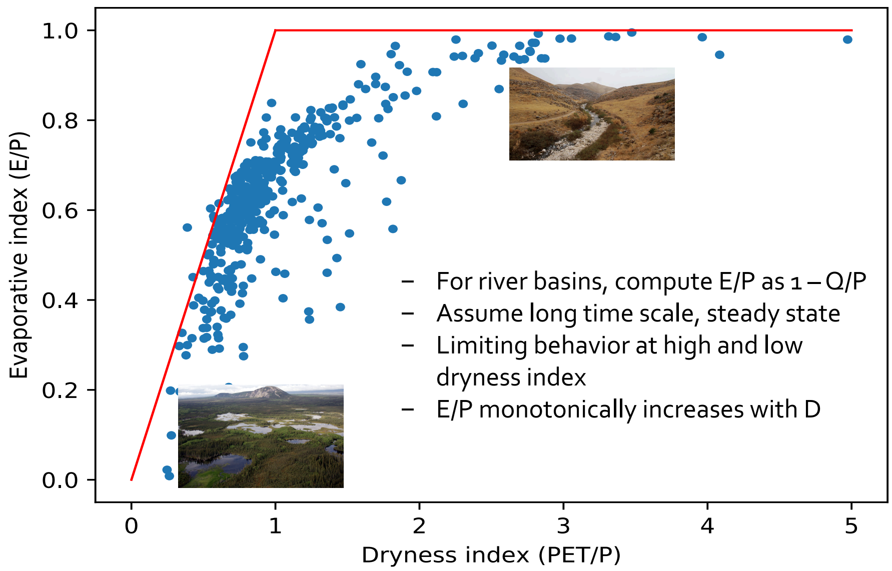
D > 1 → more energy than water

D < 1 → more water than energy

Budyko's curve explained



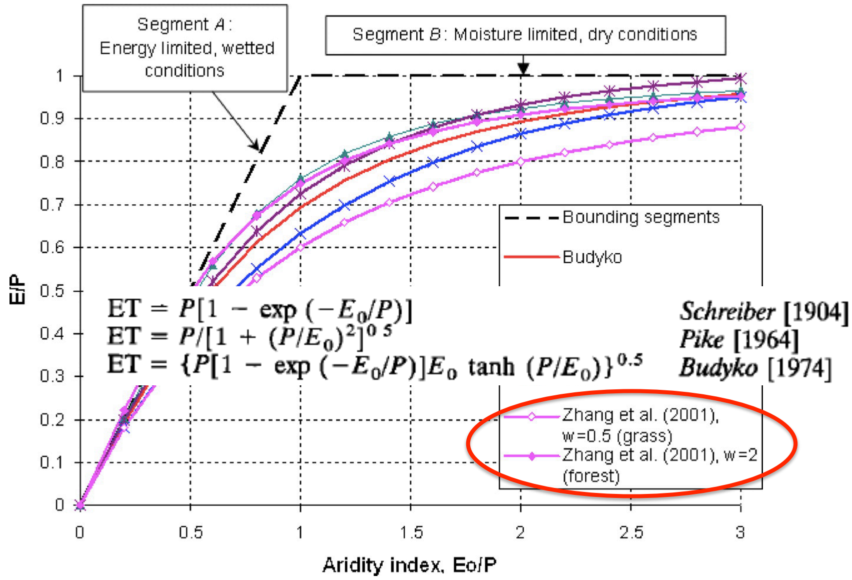
USGS streamflow on Budyko's curve



"What determines catchment water balance partitioning?"

✓ **Climate** (Increase DI → Increase in E/P)

Empirical functions have been proposed to describe Budyko's curve



Budyko's curve with different vegetation covers

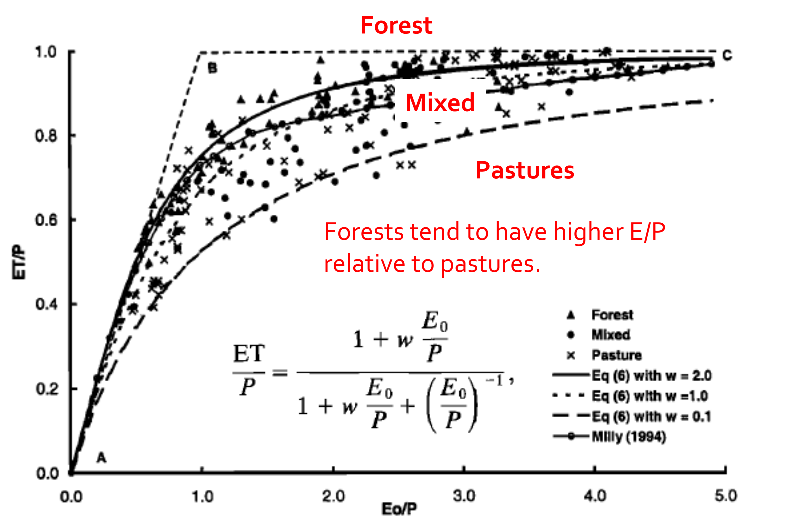


Figure 5. Comparison of equation (6) with measurements for catchments with different vegetation covers. Also shown is curve of *Milly* [1994].

Zhang et al. (2001), *Water Resources Research.*

"What determines catchment water balance partitioning?"

- ✓ **Climate** (Increase DI → Increase in E/P)
- ✓ **Vegetation** (Forest E/P > Pasture E/P)

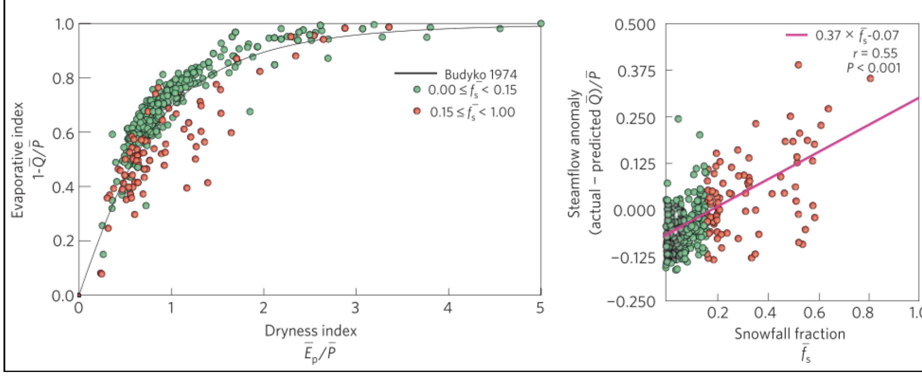
Another empirical observation based on snow cover

nature climate change LETTERS
PUBLISHED ONLINE: 18 MAY 2014 | DOI: 10.1038/NCLIMATE2246

Snow fraction is defined as the proportion of precipitation that fall on days with average T<1°C.

A precipitation shift from snow towards rain leads to a decrease in streamflow

W. R. Berghuijs[1,2*], R. A. Woods[2] and M. Hrachowitz[1]



"What determines catchment water balance partitioning?"

✓ **Climate** (Increase DI → Increase in E/P)
✓ **Vegetation** (Forest E/P > Pasture E/P)
✓ **Snow** (increase snow fraction → decrease E/P)

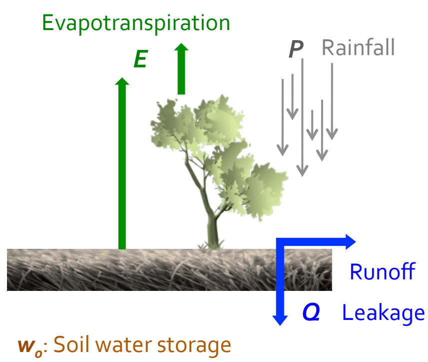
Question: if we think of the original Budyko's curve as a "null hypothesis" of E/P regulated by climate, why do you think factors like vegetation and snow fraction can influence deviations from this curve?

We now turn to a ***process-based, modeling approach*** *to get at a better understanding of water balance partitioning.*

Stochastic soil moisture modeling

$S = w_0 \bullet s$

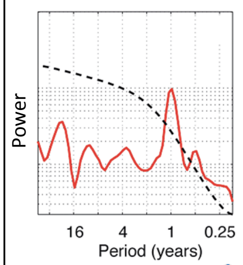
$$w_0 \frac{ds}{dt} = P - E - Q$$



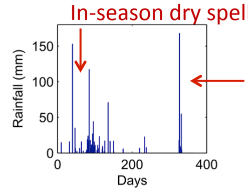
Key Premises

1. Rainfall at the daily scale is a random variable (RV)
2. Variables within the soil water balance are therefore RVs:
   - Soil moisture (s)
   - Evapotranspiration (E)
   - Leakage & Runoff (Q)
3. Randomness can be described by simple rainfall statistics.

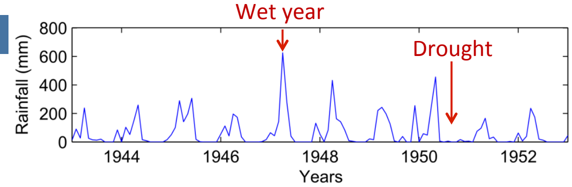
1. Rainfall is a random variable



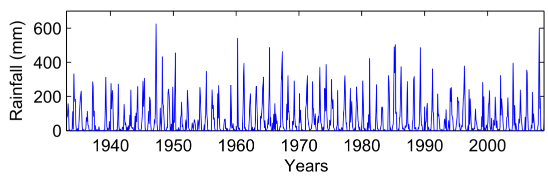
Daily



Seasonal



Interannual



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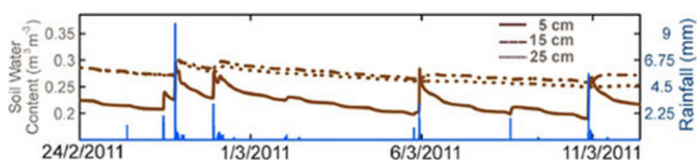
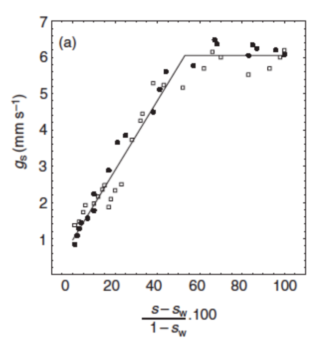
2. Randomness within the soil-plant-atmosphere continuum (SPAC)

Soil moisture responds to rainfall pulses



Plants respond to soil moisture availability

E increases linearly with s:
E(s) = PET • s



3. Capturing randomness via rainfall statistics

WATER RESOURCES RESEARCH, VOL. 29, NO. 11, PAGES 3755–3758, NOVEMBER 1993

An Analytic Solution of the Stochastic Storage Problem Applicable to Soil Water

WATER RESOURCES RESEARCH, VOL. 37, NO. 3, PAGES 457–463, MARCH 2001

A minimalist probabilistic description of root zone soil water

P. C. D. Milly
U.S. Geological Survey, Princeton, New Jersey
Geophysical Fluid Dynamics Laboratory, NOAA, Princeton, New Jersey

Chris Milly (USGS)

Ignacio Rodriguez-Iturbe (Texas A&M)

Amilcare Porporato (Princeton)

THE ROYAL SOCIETY

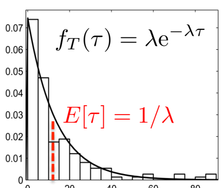
Probabilistic modelling of water balance at a point: the role of climate, soil and vegetation

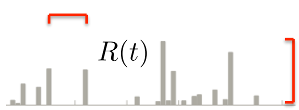
By I. Rodriguez-Iturbe[1], A. Porporato[2], L. Ridolfi[2], V. Isham[3] and D. R. Cox[4]

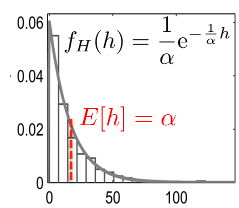
3. Capturing randomness via rainfall statistics

τ : waiting times (days)

h : rainfall intensity (mm)



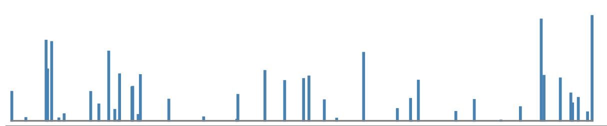




Daily rainfall as a marked Poisson (stochastic) process:

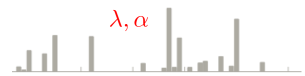
1. Waiting times are exponentially distributed
2. Event sizes (intensity) are exponentially distributed

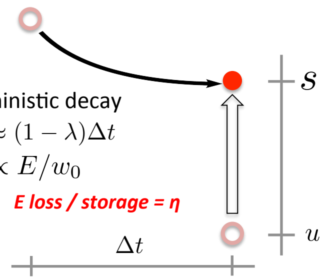
Simulated rain series:



3. Capturing randomness via rainfall statistics

From rainfall stochasticity to a probabilistic description of soil moisture dynamics





Deterministic decay
$\text{prob} \approx (1-\lambda)\Delta t$
$\Delta s \propto E/w_0$
E loss / storage = η

Stochastic jump
$\text{prob} \approx \lambda\Delta t$
$\Delta s \propto w_0/\alpha$
Storage / mean rainfall depth = γ

$$p(s,t+\Delta t)ds = (1-\lambda\Delta t)p(s+\Delta s,t)d(s+\Delta s)$$ → Deterministic decay

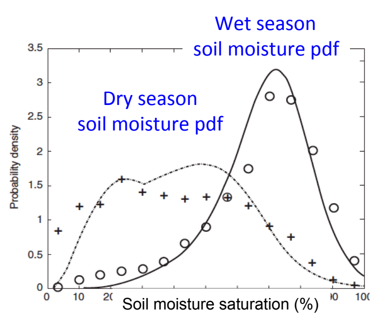
$$+\lambda\Delta t\int_0^s p(u+\Delta u,t)f(s-u;u)d(u+\Delta u)ds$$ → Stochastic jumps

Rearrange, $\Delta t \to 0$ $$\frac{\partial}{\partial t}p(s,t) = \frac{\partial}{\partial s}\left[\rho(s)p(s,t)\right] - \lambda p(s,t) + \lambda\gamma\int_0^s e^{-\gamma(s-u)p(u,t)}du$$

Rodriguez-Iturbe et al. (1999), *Proc. Royal Society A.*

3. Capturing randomness via rainfall statistics

Result : steady state soil moisture PDFs



$$p(s) = \frac{\gamma^{\lambda/\eta}}{\Gamma(\lambda/\eta,0,\gamma)}s^{\lambda/\eta-1}e^{-\gamma s}$$

λ: Mean rainfall frequency
γ: Soil water storage / mean rainfall depth
η: Losses via E / soil water storage

Long term mean soil moisture

$$\langle s\rangle = \frac{\lambda}{\eta\gamma} - \frac{\gamma^{\lambda/\eta-1}e^{-\gamma}}{\Gamma(\lambda/\eta,0,\gamma)}$$

Porporato et al. (2004) *The American Naturalist.*

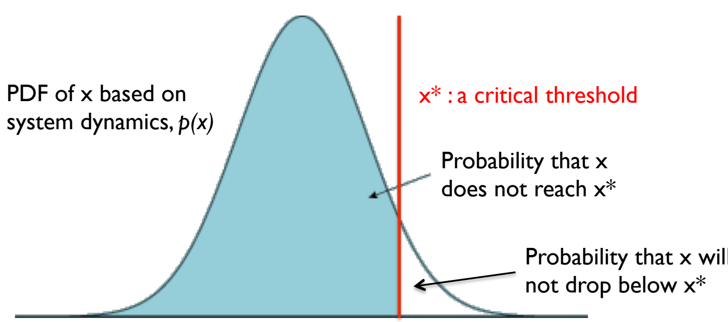
This is a key result, but barriers exit to adoption

1. Probabilities are notoriously difficult to communicate
2. The predictions need to be related to management outcomes

A way forward? Risk analysis

The concept of risk can be quantitatively defined:

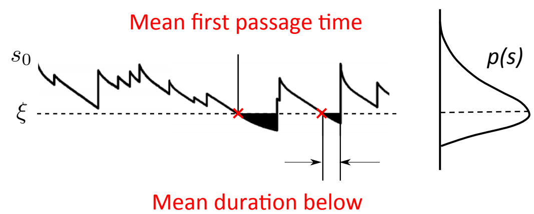
1. Non-exceedance probability of a desirable threshold
2. Exceedance probability of a undesirable threshold



Risk analysis

Risk analysis based on (1) a steady state PDF and (2) a critical threshold can also lead to nuanced quantification of:

1. Frequency of crossing (how often?)
2. Mean duration below/above threshold (how long?)
3. Mean first passage time to threshold (how urgent?)



What are some relevant physical thresholds in your field?

Let's get back to Budyko

Dryness index $D = \frac{PET}{\langle P \rangle} = \frac{PET}{\lambda\alpha} = \frac{\eta\gamma}{\lambda}$

λ: Mean rainfall frequency
γ: Soil water storage / mean rainfall depth
η: Losses via E / soil water storage

Evaporative index $\frac{\langle E \rangle}{\langle P \rangle} = \frac{PET\langle s \rangle}{\langle P \rangle} = D\langle s \rangle$

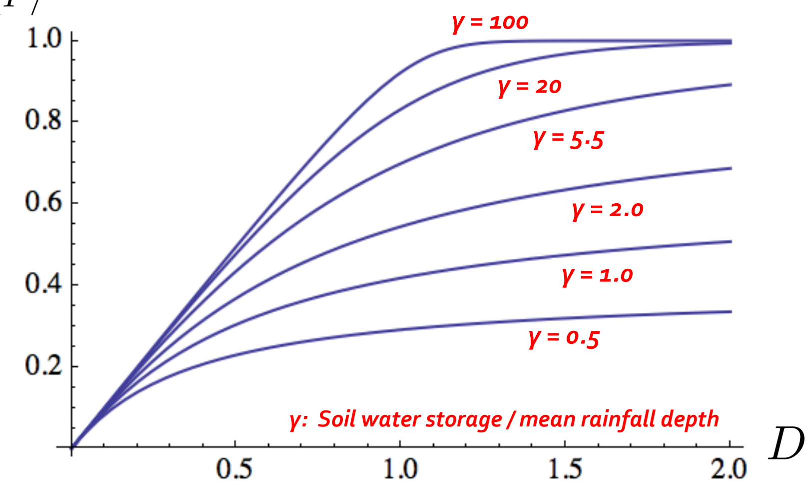
$$\langle s \rangle = \frac{\lambda}{\eta\gamma} - \frac{\gamma^{\lambda/\eta-1}e^{-\gamma}}{\Gamma(\lambda/\eta, 0, \gamma)} \longrightarrow \langle s \rangle = \frac{1}{D} - \frac{\gamma^{\gamma/D-1}e^{-\gamma}}{\Gamma(\gamma/D, 0, \gamma)}$$

We can now plot Budyko's curve as a function of D and γ.

Porporato et al. (2004) *The American Naturalist*.

Now we recast long-term mean soil moisture within Budyko's framework

$$\frac{\langle E \rangle}{\langle P \rangle} = D\langle s \rangle$$



Porporato et al. (2004) *The American Naturalist.* Feng et al. (2012), *Water Resources Research.*

"What determines catchment water balance partitioning?"

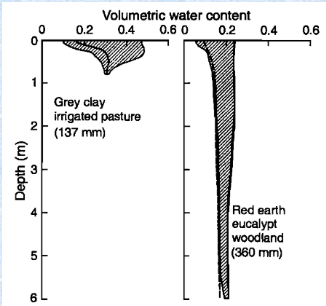
✓ **Climate** (Increase DI → Increase in E/P)
✓ **Vegetation** (Forest E/P > Pasture E/P)
✓ **Snow** (increase snow fraction → decrease E/P)

✓ **Effective soil water storage** (increase γ → increase E/P)

Question: Effective soil water storage depends on porosity and effective rooting depth. How do you think they relate to vegetation type?

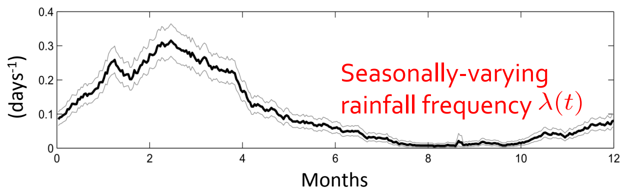
"What determines catchment water balance partitioning?"

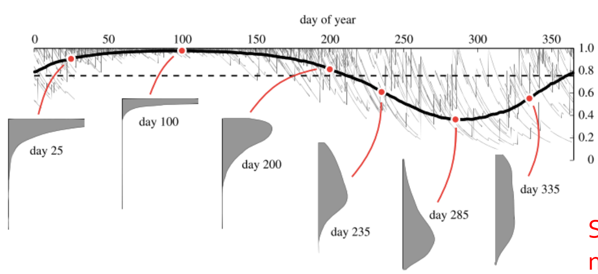
- ✓ **Climate** (Increase DI → Increase in E/P)
- ✓ ~~**Vegetation** (Forest E/P > Pasture E/P)~~
- ✓ **Snow** (increase snow fraction → decrease E/P)
- ✓ **Effective soil water storage** (increase γ → increase E/P)



Zhang et al. (2001), *Water Resources Research.*

Modification of soil water balance due to climate seasonality





Feng et al. (2015), *Proceedings of Royal Society A.*

Modification of soil water balance due to climate seasonality

Master equation
$$\frac{\partial}{\partial t}p(s,t) = \frac{\partial}{\partial s}\left[\rho(s)p(s,t)\right] - \lambda p(s,t) + \lambda\gamma \int_0^s e^{-\gamma(s-u)p(u,t)}du$$

$$\langle s(t)\rangle = \int_0^1 up(u,t)du$$

λ: Mean rainfall frequency
γ: Soil water storage / mean rainfall depth
η: Losses via E / soil water storage

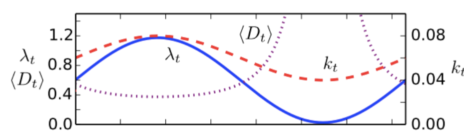
<Rainfall> <Evapotranspiration> <Leakage & runoff>

Mean soil moisture changes
$$\frac{d\langle s(t)\rangle}{dt} = \frac{\lambda(t)}{\gamma} - \int_0^1 \frac{E}{w_0}p(u,t)du - \frac{\lambda(t)}{\gamma}\int_0^1 e^{-\gamma(1-s)}p(u,t)du$$

Linearization of *E*:
E = PET x
$$\frac{d\langle s(t)\rangle}{dt} = \frac{\lambda(t)}{\gamma} - \eta(t)\langle s(t)\rangle - \frac{\lambda(t)}{\gamma}\langle e^{-\gamma(1-s)}\rangle$$

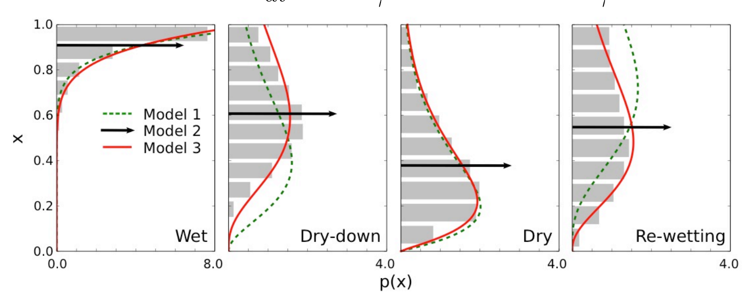
Feng et al. (2015), *Proc. Royal Society A.*

Seasonal evolution of mean soil moisture



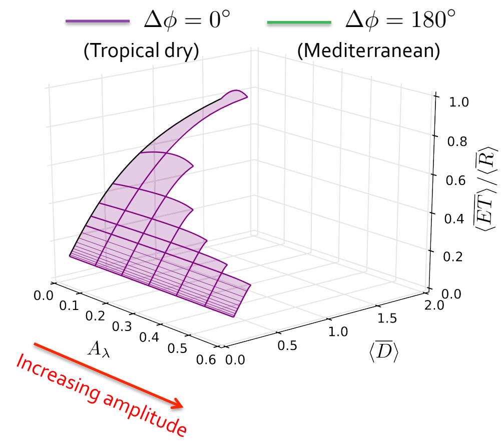
Seasonal rainfall frequency: $\lambda(t) = \mu_\lambda + A_\lambda \cdot \sin(wt + \phi_\lambda)$
Seasonal PET / storage: $\eta(t) = \mu_\eta + A_\eta \cdot \sin(wt + \phi_\eta)$

$$\frac{d\langle s(t)\rangle}{dt} = \frac{\lambda(t)}{\gamma} - \eta(t)\langle s(t)\rangle - \frac{\lambda(t)}{\gamma}\langle e^{-\gamma(1-s)}\rangle$$



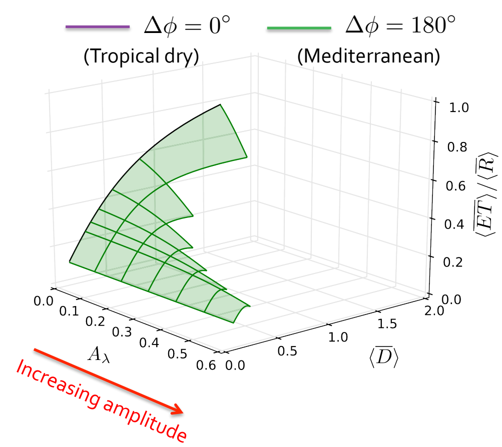
Feng et al. (2015), *Proc. Royal Society A.*

Rainfall seasonality (amplitude & phase) decreases *E/P*



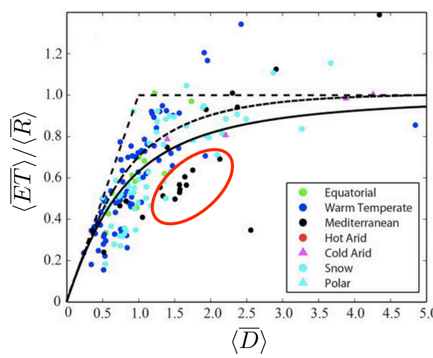
Feng et al. (2015), *Proc. Royal Society A*.

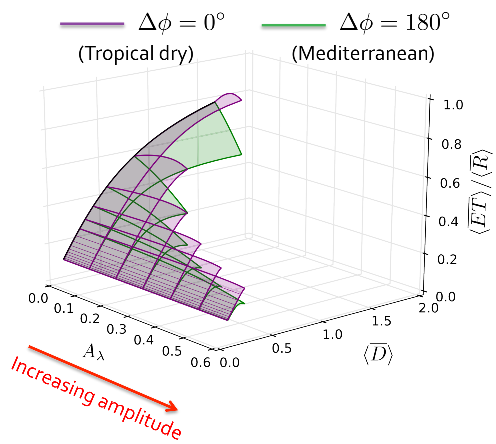
Rainfall seasonality (amplitude & phase) decreases *E/P*



Feng et al. (2015), *Proc. Royal Society A*.

Rainfall seasonality (amplitude & phase) decreases *E/P*





Williams et al. (2012), *Water Resources Research*.

Feng et al. (2015), *Proc. Royal Society A*.

"What determines catchment water balance partitioning?"

✓ **Climate** (Increase DI → Increase in E/P)

✓ ~~**Vegetation** (Forest E/P > Pasture E/P)~~

✓ **Snow** (increase snow fraction → decrease E/P)

✓ **Effective soil water storage** (increase γ → increase E/P)

✓ **Climate seasonality** (increase amplitude, decrease PET and P synchronicity → decrease E/P)

Question: How do you think this relates to snow fraction?

"What determines catchment water balance partitioning?"

✓ **Climate** (Increase DI → Increase in E/P)
✓ ~~**Vegetation** (Forest E/P > Pasture E/P)~~
✓ ~~**Snow** (increase snow fraction → decrease E/P)~~

✓ **Effective soil water storage** (increase γ → increase E/P)
✓ **Climate seasonality** (increase amplitude, decrease PET and P synchronicity → decrease E/P)

What have we learned?

- **Process-based models** have enormous explanatory power.
- Long term E/P (and Q/P) is controlled by **climate**, **soil water storage** (expressed by vegetation type?), and **seasonal distributions** of water supply (P) and demand (PET) (expressed by snow fraction?).
- Long term catchment water balance partitioning could be related to the **"efficiency"** with which water is delivered back into the atmosphere via E.
- Loss in "efficiency" – leading to decrease in E/P – could manifest via **boundary conditions** (i.e., atmospheric water supply do not occur when there is high demand), or **within the landscape** (i.e., low soil water storage leading to nonlinear losses to Q when rainfall exceeds storage).

Your turn

- As part of a group, investigate a hypothesis about a determinant of catchment water balance partitioning
- You'll be asked to present at the end of the session (aim for 5 minutes each) – to tell everyone about:
  1. Your hypothesis
  2. Your approach
  3. Your findings & remaining questions
- Some starting topics:
  1. Topography – mountainous vs. flat terrain
  2. Soil / substrate type – sandy vs. rocky vs. loamy
  3. Land use type – agricultural (irrigation!) vs. residential vs. urban
  4. Groundwater – connectivity to regional groundwater reservoirs
  5. Spatial organization – laterally interconnected vs. vertical flow
  6. Temporal scale – climate change, vegetation dynamics (wildfires!)

Your turn

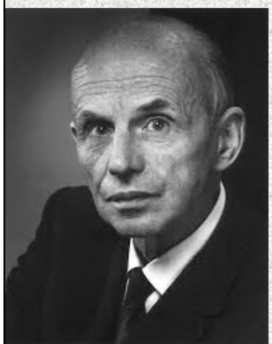
- Some possible starting approaches:

  1. In-depth review & synthesis – pick a few interesting / seminal papers on that topic and relate your conclusions based on critical readings

  2. Data analysis – MOPEX data are available on GitHub to be imported to your favorite computing environment: https://github.com/feng-ecohydro/SIESD_2018

  3. Your own model?

CLORPT for Catchments?



Hans Jenny's state equation (1941):

Soil = *f* (Cl, O, R, P, T…)

5-factors of
soil formation

Climate
Organism
Relief
Parent material
Time

Can we come up with a general framework that similarly captures the co-evolution of geomorphology and ecohydrology?