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CS520 INTRO TO AI

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Division of Labor:

Suyu Huang: Implemented DFS, Astar Manhatton, Astar Euclidean and Genetic Algorithm in Java. Run genetic algorithm to collect and analysis data. Build java GUI to visualize maze and path.

Cong Deng: Implement Astar Manhattan and Astar Euclidean by using C++, reconstruct code on python 3.7, collect and analysis data about Astar algorithm. Use Matlab to generate figures through data.

Bowei Feng: Implement DFS, BFS and Genetic Algorithms, visualize by using matplotlib and python 3.7, and run genetic algorithms to collect and analysis data about DFS and BFS.

Maze Runner

Part 1: Path Planning

1. We test different sizes and try to find how large the map could be within one minute and a reasonable and fixed value of dim. However, the first solution we are looking for varies from device to device. The result is shown below.

Table 1. Average time of each algorithm

Algorithm	DFS	BFS	A* with ED	A* with MD
Ave. Time (s)	0.218024	59.459048	3.03401	2.99085

where $p = 0.1$, the value of dim is 2800 * 2800 and ED means Euclidean Distance and MD means Manhattan Distance (same below).

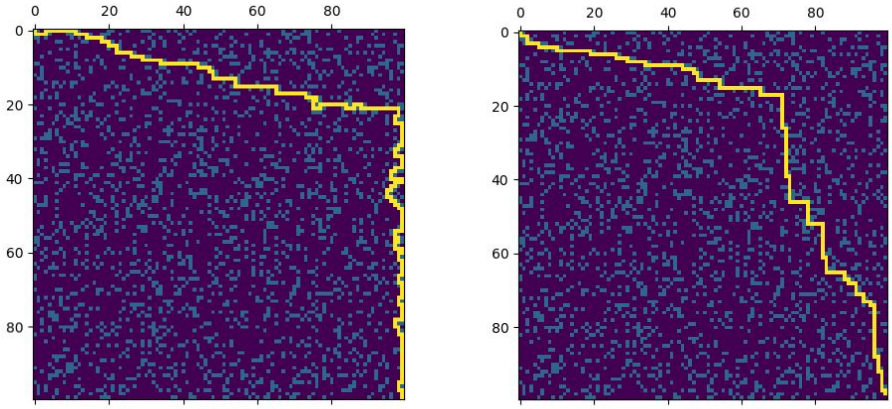
Table 2. Average time of each algorithm

Algorithm	DFS	BFS	A* with ED	A* with MD
Ave. Time (s)	0.000568	0.066590	0.022253	0.007836

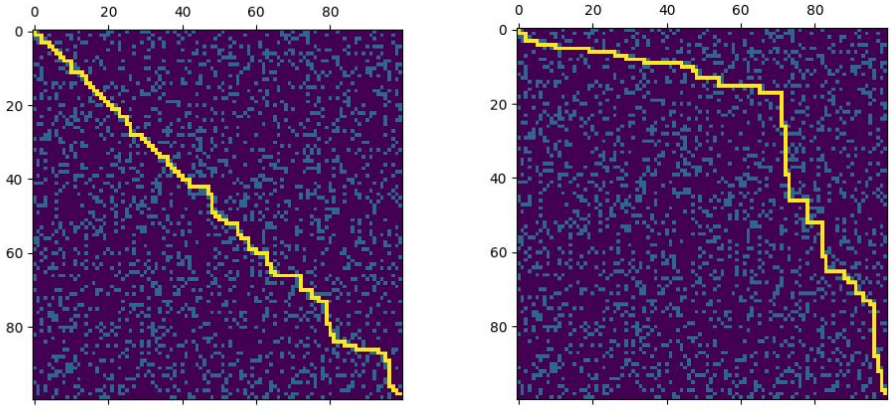
where $p = 0.1$ and the value of dim is 100 * 100.

It is shown that BFS is the slowest algorithm. In order to run code more efficiently, we choose 100*100 as the value of dim.

2.



(a) DFS (b) DFS



(a) A* with ED (b) A* with MD

Figure 1. The paths returned from each algorithm

3. As the result of Problem 1, we choose 100*100 as the value of dim. For each *p* from 0.1 to 0.9, we generate 10,000 random maps to estimate the probability that a map has a complete path from start to goal. The result is shown as below.

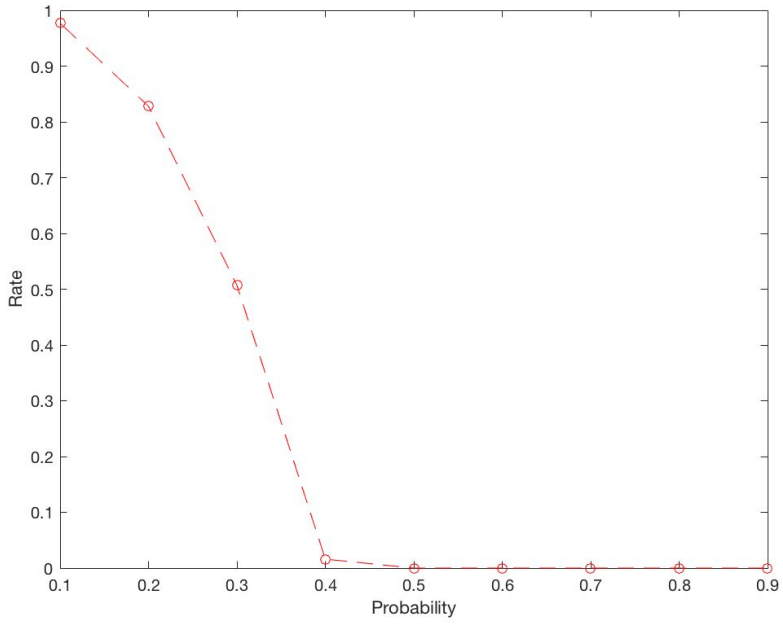


Figure 2. The success rate of random maps for each *p*

We can conclude that $p_0 = 0.2$ and when $p < p_0$, there is usually a clear path. Then we run four algorithms on 10,000 random maps where $p_0 = 0.2$ and the value of dim is 100*100 to estimate the average time for each algorithm to find a clear path. The result is shown as below.

Table 3. Average time for each algorithm to find a clear path

Algorithm	DFS	BFS	A* with ED	A* with MD
Ave. Time (s)	0.003579	0.052069	0.190667	0.028100

We can conclude that DFS is most useful here since we just need to know there is a clear path but don't care weather the path is the shortest, DFS could respond as soon as possible.

4. Since the DFS does not apply the shortest path, we only need to compare A* with EM, A* with MD and BFS. The result is shown as below.

Table 4. The length of shortest path of each algorithm

	P	0.1	0.15	0.2	0.25
A* with ED	Ave. Steps	200.0	200.1	200.4	203.6
A* with MD	Ave. Steps	200.0	200.1	200.4	203.6
BFS	Ave. Steps	200.7	200.8	201.4	204.4

From the data, BFS and A* are both good at finding the shortest path.

5.

Table 5. The average steps of each algorithm

	P	0.1	0.15	0.2	0.25
A* with ED	Ave. Steps	200.0	200.1	200.4	203.6
A* with MD	Ave. Steps	200.0	200.1	200.4	203.6

DFS	Ave. Steps	221.7	238.3	262.0	292.3

From the data, A* could find shorter path than DFS, and with the p increasing, the average of steps of A* is stable but DFS's steps are gradually increase.

6.For the range of *p* values from 1.0 to 2.0, we estimate the average number of nodes expanded in total for a random map by A* with ED and BA* with MD. The result is shown below.

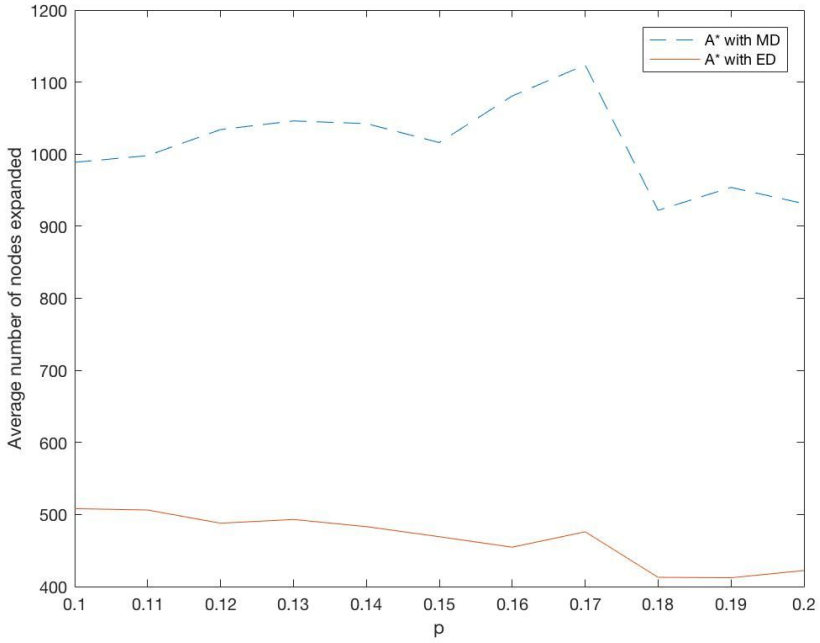


Figure 3. The average number of nodes expanded for A* with MD and ED

7. For the range of *p* values from 1.0 to 2.0, we estimate the average number of nodes expanded in total for a random map by DFS and BFS. The result is shown below.

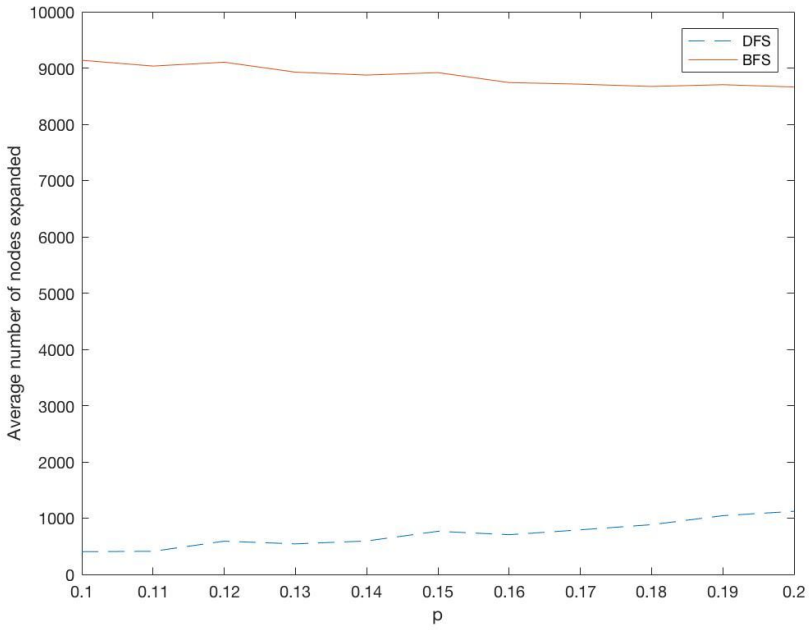


Figure 4. The average number of nodes expanded for DFS and BFS

It's easy to conclude that DFS typically expands fewer nodes. Theorily, that is because BFS algorithm needs to expand every node to find the shortest path while the DFS algorithm just need to find one possible path then quit.

To compare with A* with ED and A* with MD, the result is shown below.

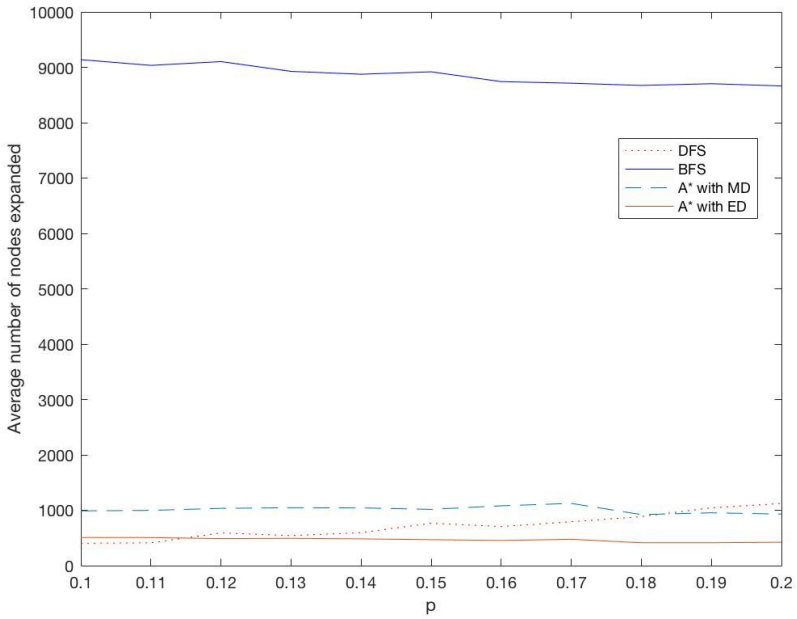


Figure 5. The average number of nodes expanded for each algorithm

We can conclude that BFS still expands the most nodes and A* with ED expands fewer nodes comparing with other three algorithms.

Part 2: Building Hard Mazes

8. We pick Genetic Algorithms to solve this problem since there seems no hardest maze and we could just find a harder maze, and different rules to judge maze direct to different termination.

We use a two-dimensional n*n array to store the maze, n means the length of row and col, and each element has a value of 0 or 1, 0 means it is a path and 1 means it is a wall.

For a maze, each time we generate the path, and the length of the shortest path is a good evaluation standard. When we randomly generate some maze and we could evaluate the maze by the length of the shortest path which makes we could easily select suitable mazes from random mazes, so we apply genetic algorithms to this problem.

9. We set a specific number of iterations as termination condition, for example, when we iterates 300 times, genetic algorithm stops re-producing offsprings.

Based on the data (e.g shortest path) from every generation during iterations, we thought that the evaluation of generations tends to being stable after a particular number of iteration. This particular number is related to the dimension of maze, to the population of generation, and to how many elements of the maze mutated during GA.

Thus, the shortcomings : if the number of iterations we set is lower than the number of iterations that makes the evaluation of generation vary in a short range relatively, given the dimensions of maze, the population of each generation and the number of mutations, we can not be able to find the relative stable state of new mazes whose evaluation value only vary in a short range.Also, if we set the number of termination too high, GA cannot be able to generate better offsprings after a particular number of iterations.

The advantage of this termination condition is obvious. It's straightforward. We could change the number of iterations to find the number that makes the evaluation of generation vary in a short range relatively.

10.

(a) DFS:

We do GA by using 100*100 mazes, 100 populations of each generation and 20 mutation for each mazes after crossover and 500 iteration.For each generation, we calculate the average of

evaluation, the median of evaluation, the maximum of evaluation and the minimum of evaluation, all of these can be founded in attached file.

a.i): Length of Solution path returned

For the first 5 iterations:

```
================= THIS IS THE 0 ITERATION
  Total 100 of mazes
  Average: 218.96 Median: 217.0 MAX: 249.0 MIN: 203.0
================= THIS IS THE 1 ITERATION
  Total 100 of mazes
  Average: 220.28 Median: 219.0 MAX: 241.0 MIN: 203.0
================= THIS IS THE 2 ITERATION
  Total 100 of mazes
  Average: 220.28 Median: 219.0 MAX: 249.0 MIN: 203.0
================= THIS IS THE 3 ITERATION
  Total 100 of mazes
  Average: 221.24 Median: 219.0 MAX: 265.0 MIN: 201.0
================= THIS IS THE 4 ITERATION
  Total 100 of mazes
  Average: 222.0 Median: 221.0 MAX: 265.0 MIN: 203.0
```

For the last 5 iterations:

```
================= THIS IS THE 495 ITERATION
  Total 100 of mazes
  Average: 333.54 Median: 331.0 MAX: 573.0 MIN: 251.0
================= THIS IS THE 496 ITERATION
  Total 100 of mazes
  Average: 335.64 Median: 331.0 MAX: 573.0 MIN: 253.0
================= THIS IS THE 497 ITERATION
  Total 100 of mazes
  Average: 333.82 Median: 327.0 MAX: 573.0 MIN: 251.0
================= THIS IS THE 498 ITERATION
  Total 100 of mazes
  Average: 332.62 Median: 327.0 MAX: 573.0 MIN: 251.0
================= THIS IS THE 499 ITERATION
  Total 100 of mazes
  Average: 329.38 Median: 327.0 MAX: 573.0 MIN: 251.0
```

Based on the data from 500 iterations, even though sometimes steps of path on average decreased, after 500 iterations, the average, median, MAX and MIN has increased.Compare the data from 500th iteration to the data from 1st iteration, the average steps of path has increased by 50% approximately, which shows that the direction of evolution is towards making mazes which have more steps to complete.

Besides, GA cannot be able to push new generation of mazes being “harder” after iterating 160 times approximately because after that, average steps of path is constrained in the range between 320 to 340 basically.

a.ii) Nodes expanded

```
================= THIS IS THE 0 ITERATION
  Total 100 of mazes
  Average: 558.5 Median: 523.0 MAX: 1327.0 MIN: 464.0
================= THIS IS THE 1 ITERATION
  Total 100 of mazes
  Average: 589.93 Median: 525.0 MAX: 1409.0 MIN: 471.0
================= THIS IS THE 2 ITERATION
  Total 100 of mazes
  Average: 569.63 Median: 525.0 MAX: 1271.0 MIN: 464.0
================= THIS IS THE 3 ITERATION
  Total 100 of mazes
  Average: 606.14 Median: 527.0 MAX: 1409.0 MIN: 476.0
================= THIS IS THE 4 ITERATION
  Total 100 of mazes
  Average: 616.2 Median: 527.0 MAX: 1511.0 MIN: 456.0
```

```
================= THIS IS THE 145 ITERATION
  Total 100 of mazes
  Average: 2058.52 Median: 1556.0 MAX: 8987.0 MIN: 661.0
================= THIS IS THE 146 ITERATION
  Total 100 of mazes
  Average: 2050.28 Median: 1516.0 MAX: 8955.0 MIN: 779.0
================= THIS IS THE 147 ITERATION
  Total 100 of mazes
  Average: 2140.0 Median: 1618.0 MAX: 8217.0 MIN: 710.0
================= THIS IS THE 148 ITERATION
  Total 100 of mazes
  Average: 2128.96 Median: 1519.0 MAX: 8212.0 MIN: 692.0
================= THIS IS THE 149 ITERATION
  Total 100 of mazes
  Average: 2180.23 Median: 1691.0 MAX: 8330.0 MIN: 659.0
```

```
================= THIS IS THE 496 ITERATION
  Total 100 of mazes
  Average: 999.16 Median: 968.0 MAX: 1432.0 MIN: 656.0
================= THIS IS THE 497 ITERATION
  Total 100 of mazes
  Average: 994.96 Median: 965.0 MAX: 1432.0 MIN: 656.0
================= THIS IS THE 498 ITERATION
  Total 100 of mazes
  Average: 981.91 Median: 964.0 MAX: 1432.0 MIN: 656.0
================= THIS IS THE 499 ITERATION
  Total 100 of mazes
  Average: 1002.43 Median: 974.0 MAX: 1432.0 MIN: 750.0
```

The total number of nodes expanded gradually increases to a very high point(at 147th iteration), then starts to decrease.Finally,it seems to stabilized at 1000 expanded nodes on average.

a.iii)

The Maximum Size of fringe increase to a very high point (at about 150 iteraitons),then starts to decrease.Finally, it seems to stabilized at 600 on average.But it still increased by 17% compared to origin.

(b) BFS:

We do GA by using 100*100 mazes, 100 populations of each generation and 5% of number of elements in maze mutation for each mazes after crossover and 100 iteration. All results can be founded in attached file.

b.i)The length of path first decreases to a lowest point, then gradually increases longer than first a few iterations, and finally stabilizes.

b.ii)The total number of nodes expanded gradually decreases and finally stabilizes.

b.iii)The maximum size of fringe during runtime gradually decreases and finally stabilizes.

For the length of path, in the first few iterations, the number of wall in the maze is rare, so the length decreases at first, with the iterations, the number of maze with longer path increases, and the length of path gradually increases, finally the maze stabilizes and the length of path stabilizes. But for number of nodes expanded and maximum size of fringe, they are at the highest point, with wall increasing, they gradually decrease, and finally the maze stabilizes and they stabilize.

So, by using BFS, we would better choose length of path to evaluate weather the maze is hard, and in our search algorithm, it is difficult to reach hardest since mutation will balance the number of wall and path, if we want to get the hardest, we should do more operations based on the times of iteration.

(c)A* Manhattan

We do GA by using 100*100 mazes, 100 populations of each generation and 20 mutation for each mazes after crossover and 500 iteration.For each generation, we calculate the average of evaluation, the median of evaluation, the maximum of evaluation and the minimum of evaluation, all of these can be founded in attached file.

c.i)Length of Solution path

```
================= THIS IS THE 0 ITERATION
  Total 100 of mazes
  Average: 199.0 Median: 199.0 MAX: 199.0 MIN: 199.0
================= THIS IS THE 1 ITERATION
  Total 100 of mazes
  Average: 199.0 Median: 199.0 MAX: 199.0 MIN: 199.0
================= THIS IS THE 2 ITERATION
  Total 100 of mazes
  Average: 199.0 Median: 199.0 MAX: 199.0 MIN: 199.0
================= THIS IS THE 3 ITERATION
  Total 100 of mazes
  Average: 199.0 Median: 199.0 MAX: 199.0 MIN: 199.0
================= THIS IS THE 4 ITERATION
  Total 100 of mazes
  Average: 199.02 Median: 199.0 MAX: 201.0 MIN: 199.0
================= THIS IS THE 5 ITERATION
  Total 100 of mazes
  Average: 199.02 Median: 199.0 MAX: 201.0 MIN: 199.0
```

```
================= THIS IS THE 495 ITERATION
  Total 100 of mazes
  Average: 323.66 Median: 319.0 MAX: 421.0 MIN: 247.0
================= THIS IS THE 496 ITERATION
  Total 100 of mazes
  Average: 315.58 Median: 305.0 MAX: 421.0 MIN: 247.0
================= THIS IS THE 497 ITERATION
  Total 100 of mazes
  Average: 316.96 Median: 307.0 MAX: 421.0 MIN: 247.0
================= THIS IS THE 498 ITERATION
  Total 100 of mazes
  Average: 315.56 Median: 305.0 MAX: 429.0 MIN: 247.0
================= THIS IS THE 499 ITERATION
  Total 100 of mazes
  Average: 318.18 Median: 313.0 MAX: 429.0 MIN: 247.0
```

Based on the data from 500 iterations, even though sometimes steps of path on average decreased, after 500 iterations, the average, median, MAX and MIN has increased.Compare the data from 300th iteration to the data from 1st iteration, the average steps of path has increased by 60% approximately, which shows that the direction of evolution is towards making mazes which have more steps to complete.

Besides, GA cannot be able to push new generation of mazes being “harder” after iterating 300 times approximately because after that, average steps of path is constrained in the range between 300 to 330 basically.

c.ii)Nodes expanded

```
================= THIS IS THE 0 ITERATION
  Total 100 of mazes
  Average: 14101.16 Median: 14112.0 MAX: 15113.0 MIN: 12745.0
================= THIS IS THE 1 ITERATION
  Total 100 of mazes
  Average: 14007.69 Median: 13965.0 MAX: 15132.0 MIN: 12654.0
================= THIS IS THE 2 ITERATION
  Total 100 of mazes
  Average: 13899.62 Median: 13897.0 MAX: 15089.0 MIN: 12689.0
================= THIS IS THE 3 ITERATION
  Total 100 of mazes
  Average: 13729.99 Median: 13772.0 MAX: 14935.0 MIN: 12527.0
================= THIS IS THE 4 ITERATION
  Total 100 of mazes
  Average: 13650.47 Median: 13690.0 MAX: 14880.0 MIN: 12470.0
================= THIS IS THE 5 ITERATION
  Total 100 of mazes
  Average: 13581.84 Median: 13574.0 MAX: 14760.0 MIN: 12369.0
```

```
================= THIS IS THE 495 ITERATION
  Total 100 of mazes
  Average: 766.96 Median: 724.0 MAX: 1132.0 MIN: 525.0
================= THIS IS THE 496 ITERATION
  Total 100 of mazes
  Average: 771.91 Median: 724.0 MAX: 1132.0 MIN: 551.0
================= THIS IS THE 497 ITERATION
  Total 100 of mazes
  Average: 784.34 Median: 741.0 MAX: 1132.0 MIN: 523.0
================= THIS IS THE 498 ITERATION
  Total 100 of mazes
  Average: 789.8 Median: 713.0 MAX: 1486.0 MIN: 523.0
================= THIS IS THE 499 ITERATION
  Total 100 of mazes
  Average: 774.86 Median: 706.0 MAX: 1544.0 MIN: 544.0
```

Based on the data from 500 iterations, after 500 iterations, the average, median, MAX and MIN has decreased. Compare the data from 500th iteration to the data from 1st iteration, the average steps of path has decreased by 90% approximately, which shows that the direction of evolution is towards making mazes which have lese nodes to expanded.

Besides, GA cannot be able to push new generation of mazes being “harder” after iterating 430 times approximately because after that, average steps of path is constrained in the range between 780 to 800 basically.

c.iii)Max nodes in Fringe

```
================= THIS IS THE 0 ITERATION
  Total 100 of mazes
  Average: 285.42 Median: 285.0 MAX: 317.0 MIN: 229.0
================= THIS IS THE 1 ITERATION
  Total 100 of mazes
  Average: 287.17 Median: 286.0 MAX: 318.0 MIN: 238.0
================= THIS IS THE 2 ITERATION
  Total 100 of mazes
  Average: 288.63 Median: 288.0 MAX: 324.0 MIN: 238.0
================= THIS IS THE 3 ITERATION
  Total 100 of mazes
  Average: 292.55 Median: 291.0 MAX: 336.0 MIN: 234.0
================= THIS IS THE 4 ITERATION
  Total 100 of mazes
  Average: 294.42 Median: 294.0 MAX: 335.0 MIN: 249.0
================= THIS IS THE 5 ITERATION
  Total 100 of mazes
  Average: 297.01 Median: 297.0 MAX: 336.0 MIN: 244.0
```

```
================= THIS IS THE 56 ITERATION
  Total 100 of mazes
  Average: 423.83 Median: 429.0 MAX: 518.0 MIN: 314.0
================= THIS IS THE 57 ITERATION
  Total 100 of mazes
  Average: 424.61 Median: 423.0 MAX: 528.0 MIN: 328.0
================= THIS IS THE 58 ITERATION
  Total 100 of mazes
  Average: 424.79 Median: 426.0 MAX: 527.0 MIN: 333.0
================= THIS IS THE 59 ITERATION
  Total 100 of mazes
  Average: 425.0 Median: 425.0 MAX: 511.0 MIN: 325.0
================= THIS IS THE 60 ITERATION
  Total 100 of mazes
  Average: 432.88 Median: 437.0 MAX: 535.0 MIN: 325.0
```

```
================= THIS IS THE 495 ITERATION
  Total 100 of mazes
  Average: 16.24 Median: 15.0 MAX: 28.0 MIN: 11.0
================= THIS IS THE 496 ITERATION
  Total 100 of mazes
  Average: 16.51 Median: 15.0 MAX: 27.0 MIN: 11.0
================= THIS IS THE 497 ITERATION
  Total 100 of mazes
  Average: 16.33 Median: 15.0 MAX: 28.0 MIN: 11.0
================= THIS IS THE 498 ITERATION
  Total 100 of mazes
  Average: 16.44 Median: 16.0 MAX: 28.0 MIN: 11.0
================= THIS IS THE 499 ITERATION
  Total 100 of mazes
  Average: 16.25 Median: 15.0 MAX: 26.0 MIN: 11.0
```

The number of max nodes in list gradually increases to a very high point(at 60 iteration), then starts to decrease. Finally,it seems to stabilized at 20 on average.

d) A* Euclidean

We do GA by using 100*100 mazes, 100 populations of each generation and 20 mutation for each mazes after crossover and 300 iteration.For each generation, we calculate the average of evaluation, the median of evaluation, the maximum of evaluation and the minimum of evaluation, all of these can be founded in attached file.

d.i) Length of Solution path

```
================= THIS IS THE 0 ITERATION
  Total 100 of mazes
  Average: 199.0 Median: 199.0 MAX: 199.0 MIN: 199.0
================= THIS IS THE 1 ITERATION
  Total 100 of mazes
  Average: 199.0 Median: 199.0 MAX: 199.0 MIN: 199.0
================= THIS IS THE 2 ITERATION
  Total 100 of mazes
  Average: 199.0 Median: 199.0 MAX: 199.0 MIN: 199.0
================= THIS IS THE 3 ITERATION
  Total 100 of mazes
  Average: 199.0 Median: 199.0 MAX: 199.0 MIN: 199.0
================= THIS IS THE 4 ITERATION
  Total 100 of mazes
  Average: 199.0 Median: 199.0 MAX: 199.0 MIN: 199.0
================= THIS IS THE 5 ITERATION
  Total 100 of mazes
  Average: 199.0 Median: 199.0 MAX: 199.0 MIN: 199.0


================= THIS IS THE 295 ITERATION
  Total 100 of mazes
  Average: 311.9 Median: 301.0 MAX: 479.0 MIN: 243.0
================= THIS IS THE 296 ITERATION
  Total 100 of mazes
  Average: 299.64 Median: 297.0 MAX: 407.0 MIN: 243.0
================= THIS IS THE 297 ITERATION
  Total 100 of mazes
  Average: 299.72 Median: 293.0 MAX: 479.0 MIN: 243.0
================= THIS IS THE 298 ITERATION
  Total 100 of mazes
  Average: 305.3 Median: 297.0 MAX: 479.0 MIN: 243.0
================= THIS IS THE 299 ITERATION
  Total 100 of mazes
  Average: 305.18 Median: 299.0 MAX: 479.0 MIN: 243.0
```

Based on the data from 300 iterations, after 300 iterations, the average, median, MAX and MIN has decreased. Compare the data from 500th iteration to the data from 1st iteration, the average steps of path has increased by 53% approximately, which shows that the direction of evolution is towards making mazes which have more steps to complete.

Besides, GA cannot be able to push new generation of mazes being “harder” after iterating 241 times approximately because after that, average steps of path is constrained in the range between 290 to 310 basically.

d.ii)Nodes expanded

```
================= THIS IS THE 0 ITERATION
  Total 100 of mazes
  Average: 15783.81 Median: 15780.0 MAX: 16209.0 MIN: 15442.0
================= THIS IS THE 1 ITERATION
  Total 100 of mazes
  Average: 15710.41 Median: 15710.0 MAX: 16077.0 MIN: 15330.0
================= THIS IS THE 2 ITERATION
  Total 100 of mazes
  Average: 15636.99 Median: 15643.0 MAX: 15984.0 MIN: 15278.0
================= THIS IS THE 3 ITERATION
  Total 100 of mazes
  Average: 15566.63 Median: 15563.0 MAX: 15890.0 MIN: 15263.0
================= THIS IS THE 4 ITERATION
  Total 100 of mazes
  Average: 15494.19 Median: 15495.0 MAX: 15792.0 MIN: 15087.0
```

```
================= THIS IS THE 295 ITERATION
  Total 100 of mazes
  Average: 1537.75 Median: 1460.0 MAX: 2701.0 MIN: 1000.0
================= THIS IS THE 296 ITERATION
  Total 100 of mazes
  Average: 1515.81 Median: 1440.0 MAX: 2701.0 MIN: 1042.0
================= THIS IS THE 297 ITERATION
  Total 100 of mazes
  Average: 1461.6 Median: 1405.0 MAX: 2209.0 MIN: 951.0
================= THIS IS THE 298 ITERATION
  Total 100 of mazes
  Average: 1473.98 Median: 1429.0 MAX: 2678.0 MIN: 942.0
================= THIS IS THE 299 ITERATION
  Total 100 of mazes
  Average: 1464.21 Median: 1409.0 MAX: 2682.0 MIN: 919.0
```

Based on the data from 300 iterations, after 300 iterations, the average, median, MAX and MIN has decreased. Compare the data from 300th iteration to the data from 1st iteration, the average steps of path has decreased by 90% approximately, which shows that the direction of evolution is towards making mazes which have lese nodes to expanded.

d.iii)Max nodes in Fringe

```
================= THIS IS THE 0 ITERATION
  Total 100 of mazes
  Average: 177.1 Median: 177.0 MAX: 189.0 MIN: 168.0
================= THIS IS THE 1 ITERATION
  Total 100 of mazes
  Average: 177.24 Median: 177.0 MAX: 190.0 MIN: 168.0
================= THIS IS THE 2 ITERATION
  Total 100 of mazes
  Average: 176.64 Median: 176.0 MAX: 190.0 MIN: 163.0
================= THIS IS THE 3 ITERATION
  Total 100 of mazes
  Average: 176.83 Median: 177.0 MAX: 191.0 MIN: 164.0
================= THIS IS THE 4 ITERATION
  Total 100 of mazes
  Average: 175.9 Median: 176.0 MAX: 188.0 MIN: 163.0


================= THIS IS THE 295 ITERATION
  Total 100 of mazes
  Average: 21.72 Median: 21.0 MAX: 41.0 MIN: 14.0
================= THIS IS THE 296 ITERATION
  Total 100 of mazes
  Average: 22.42 Median: 22.0 MAX: 39.0 MIN: 14.0
================= THIS IS THE 297 ITERATION
  Total 100 of mazes
  Average: 22.0 Median: 22.0 MAX: 39.0 MIN: 15.0
================= THIS IS THE 298 ITERATION
  Total 100 of mazes
  Average: 21.91 Median: 22.0 MAX: 39.0 MIN: 16.0
================= THIS IS THE 299 ITERATION
  Total 100 of mazes
  Average: 21.66 Median: 21.0 MAX: 39.0 MIN: 15.0
```

Based on the data from 300 iterations, after 300 iterations, the average, median, MAX and MIN has decreased. Compare the data from 300th iteration to the data from 1st iteration, the average steps of path has decreased by 88% approximately, which shows that the direction of evolution is towards making mazes which have lese nodes to expanded.