《Discrete Mathematics》Final Exam (2021-2022)

Questions	1	2	3	4	5	6	7	8	9	10	11	12	Total
Mark													

1. Show that $p \to (q \wedge r)$ and $(p \to q) \wedge (p \to r)$ are logically equivalent.
请证明 $p \to (q \wedge r)$ 和 $(p \to q) \wedge (p \to r)$ 两个命题公式等值。

[6 marks]

2. Show that the argument with the following premises and conclusion is valid by using rules of inference.
请构造如下推理的证明。
Premises (前提): $\neg(p \wedge \neg q)$, $\neg q \vee r$, $\neg r$
Conclusion (结论): $\neg p$

[6 marks]

3. An undirected graph G has 15 edges, where there are 4 nodes with degree 5 and all the other nodes have degree at most 2, what is the least number of nodes in this graph G ?
已知无向图 G 有 15 条边, 4 个 5 度顶点, 其余顶点的度数均小于等于 2, 请问无向图 G 至少包含多少个顶点?

[6 marks]

4. Which of the graphs in Figures 1, 2, and 3 have an Euler circuit, an Euler path but not an Euler circuit, or neither? If have, please give one of such circuit or path.
图 1、2、3 中是否存在欧拉回路？或没有欧拉回路但存在欧拉通路？或两者都不存在？请说明理由，如存在请给出各图中的一条欧拉回路或欧拉通路。

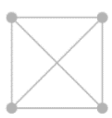

Figure 1 G_1

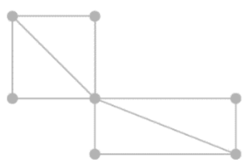

Figure 2 G_2

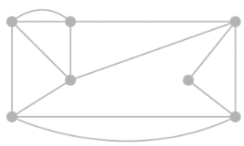

Figure 3 G_3

[6 marks]

5. How many people must you have to guarantee that at least 9 of them will have birthdays in the same day of the week. For example, the birthdays of 9 people are all on Monday.

   请问最少需要多少人，才能保证其中至少 9 人在一周的同一天过生日？比如有 9 人的生日那天都是星期一。

   [6 marks]

6. Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $ad = bc$. Show that R is an equivalence relation.

   对于一个以正整数构成的有序对为元素的集合，令 R 为该集合上的二元关系，并且 $((a, b), (c, d)) \in R$ 当且仅当 $ad = bc$ 成立. 请证明 R 是一个等价关系。

   [10 marks]

7. Use the Chinese remainder theorem to find the smallest non-negative integer x such that

$$x \equiv 2 \quad (mod \;\; 5)$$
$$x \equiv 3 \quad (mod \;\; 7)$$
$$x \equiv 10 \quad (mod \;\; 11)$$

   请使用中国剩余定理求满足上述同余方程组的最小正整数。

   [10 marks]

8. Please use the principle of mathematical induction to prove that for all $n \geq 1$, the sum of the squares of the first $2n$ positive integers is given by

$$1^2 + 2^2 + 3^2 + \cdots + (2n)^2 = \frac{n(2n+1)(4n+1)}{3}.$$

   请使用数学归纳法证明对于任意正整数n，前$2n$ 个正整数的平方和满足上式。

   [10 marks]

9. Find the solution to the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2}$, the initial conditions are $a_0 = 4$ and $a_1 = 17$.

   给出递推关系$a_n = 7a_{n-1} - 10a_{n-2}$ 的解（显式公式），初始项为 $a_0 = 4, a_1 = 17$.

   [10 marks]

10. Use the inclusion-exclusion principle to determine the number of integers that are not divisible by 5, nor by 7, nor by 9 among the first 1000 positive integers.
使用容斥原理求 1000 以内的正整数中，能够同时不被 5、7 和 9 整除的整数的个数。

[10 marks]

11. Given the following directed graph D,
    1) How many paths (including circuits) of length 2 and 3, respectively?
    2) How many circuits of length 2 and 3, respectively?

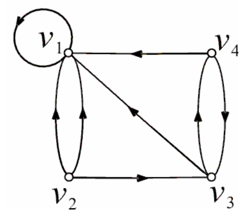

Figure 4 Directed graph D

在上面有向图 D 中，
1) 长度为 2, 3 的通路（包含回路）各有多少条？
2) 长度为 2, 3 的回路各有多少条？

[10 marks]

12. For a finite-state automata $M = (S, I, f, s_0, F)$ given by the following state table, the starting state is s_0, and the set F of acceptance states is $\{s_2\}$.
    1) Please draw the state diagram of M.
    2) Find the language accepted by this finite-state automata M.

State	***f***		
	Input		
	x	y	z
s_0	s_1	s_3	s_4
s_1	s_4	s_2	s_4
s_2	s_4	s_4	s_4
s_3	s_4	s_4	s_2
s_4	s_4	s_4	s_4

对于上面状态表格确定的有限自动机 M，起始状态为s_0，终止状态为$\{s_2\}$.
1) 请画出 M 的状态转移图；
2) 请写出可以被有限自动机 M 识别的语言。

[10 marks]