《Discrete Mathematics》Midterm Exam (2021-2022)

Questions	1	2	3	4	5	6	7	8	9	Total
Mark										

1. Multiple choice questions.

[15 marks, 3 marks for each]

1) Let p, q and r be propositions where

p is "I work at night"

q is "I work at Burger King"

r is "I walk to work"

The sentence "When I work at night and I work at Burger King, I don't walk to work" can be written using propositions and logical connectives as ____.

A. $(p \wedge q) \rightarrow \neg r$ B. $(p \vee q) \rightarrow p$

C. $(p \vee q) \rightarrow \neg r$ D. $(p \wedge q) \vee \neg r$

2) Which of the following is a tautology (永真式)? ____.

A. $(p \rightarrow q) \leftrightarrow (\neg p \rightarrow \neg q)$ B. $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

C. $(p \wedge q) \rightarrow (p \vee q)$ D. $(p \wedge q \wedge r) \leftrightarrow (p \wedge r \wedge q)$

3) Which of the following is the power set (幂集) of some set? ____.

A. $\{\{\boldsymbol{a}\}, \varnothing\}$ B. $\{\{\boldsymbol{a}\},\{\boldsymbol{b}\},\{\boldsymbol{a},\boldsymbol{b}\}\}$

C. $\{\{\boldsymbol{a}\},\{\boldsymbol{b}\},\varnothing\}$ D. $\{\{\boldsymbol{a}\},\{\boldsymbol{b}\},\{\boldsymbol{a},\boldsymbol{b}\},\varnothing\}$

4) $x^4 + 12x^2 \log(x) + x$ is ____.

A. $O(x^4)$. B. $\Omega(x^5)$.

C. $\Theta(x^4)$ D. $O(x^3)$.

5) Which of the following functions is a bijection (双射函数) from R to R ? ____.

A. $f(x) = -3x + 4$

B. $f(x) = -x^2 + 1$.

C. $f(x) = (x + 1) / (x + 5)$

D. $f(x) = 3x^3 + 1$

2. Fill in the blanks.

[15 marks, 3 marks for each]

1) Convert 1001011101_2 to base 16 : _____.

2) Compute 7^{11} mod 10 : _____.

3) The greatest common divisor (最大公约数) of 5454 and 2700 is gcd(5454, 2700)= _____.

4) The composition (复合函数) of the function f with g is $f \circ g$ = _____, where $g(x) = 0.5x + 1, f(x) = 3x + 4$.

5) Given three sets A, B and C as follows, the filled-in part of the following Venn diagram is _____.

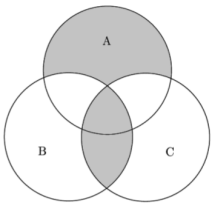

Figure 1 Sets

3. Show that the argument with the following premises (前提) and conclusion (结论) is valid by using rules of inference.

Premises: $\neg(p \wedge q) \vee r, \quad r \rightarrow s, \quad \neg s, \quad p$

Conclusion: $\neg q$

[10 marks]

4. Show that $p \rightarrow q \rightarrow r$ and $(p \wedge q) \rightarrow r$ are logically equivalent (逻辑等值).

[10 marks]

5. Let A, B and C be three sets. Show that

$$A - (B \cap C) = (A - B) \cup (A - C).$$

[10 marks]

6. Use the bubble sort (冒泡排序) algorithm to sort 10, 2, 8, 1, 6, 4, 3, please show the lists obtained at each step.

[10 marks]

7. Use the Euclidean algorithm (欧几里得算法) to find the gcd of 39 and 57, please also write the gcd as a linear combination of 39 and 57.

[10 marks]

8. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using
   a) a proof by contraposition.
   b) a proof by contradiction.

[10 marks]

9. Use the Chinese remainder theorem (中国剩余定理) to find the smallest non-negative integer satisfying the three congruence.

$$x \equiv \begin{cases} -3 \mod 19 \\ -3 \mod 20 \\ -3 \mod 21 \end{cases}$$

[10 marks]