For the tensor pressure of fast ions, the form of $\delta W_k$ is given by

$$\delta W_k = \frac{1}{2}\int d^3x \vec{\xi}_\perp^* \cdot \nabla \cdot \delta \mathbb{P}_h \tag{1}$$

with

$$\delta \mathbb{P}_h = \delta P_\perp \mathbb{I} + \left(\delta P_\parallel - \delta P_\perp\right) \hat{\mathbf{b}}\hat{\mathbf{b}} \tag{2}$$

where the components of pressure tensor $\left\{\delta P_\parallel, \delta P_\perp\right\} = m_h \int d^3v \delta F_h \left\{v_\parallel^2, v_\perp^2/2\right\}$.

Using the identities $\nabla \cdot \left(\vec{A}\vec{B}\right) = \left(\nabla \cdot \vec{A}\right)\vec{B} + \vec{A} \cdot \nabla \vec{B}$ and $\nabla \cdot (\mathbb{T}f) = \nabla f \cdot \mathbb{T} + f\nabla \cdot \mathbb{T}$, one obtains

$$\begin{aligned}
&\nabla \cdot \delta \mathbb{P} \\
&= \nabla \cdot (\delta P_\perp \mathbb{I}) + \nabla \cdot \left[\left(\delta P_\parallel - \delta P_\perp\right)\hat{\mathbf{b}}\hat{\mathbf{b}}\right] \\
&= \mathbb{I} \cdot \nabla P_\perp + \delta P_\perp \nabla \cdot \mathbb{I} + \hat{\mathbf{b}}\hat{\mathbf{b}} \cdot \nabla \left(\delta P_\parallel - \delta P_\perp\right) + \left(\delta P_\parallel - \delta P_\perp\right) \nabla \cdot \left(\hat{\mathbf{b}}\hat{\mathbf{b}}\right) \\
&= \nabla \delta P_\perp + \hat{\mathbf{b}}\hat{\mathbf{b}} \cdot \nabla \left(\delta P_\parallel - \delta P_\perp\right) + \left(\delta P_\parallel - \delta P_\perp\right) \left[\nabla \cdot \hat{\mathbf{b}}\hat{\mathbf{b}} + \hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}\right]
\end{aligned}$$

where $\nabla \cdot \mathbb{I} = 0$. We have

$$\vec{\xi}_\perp^* \cdot \nabla \cdot \delta \mathbb{P}_h = \vec{\xi}_\perp^* \cdot \nabla \delta P_\perp + \vec{\xi}_\perp^* \cdot \vec{\kappa}\left(\delta P_\parallel - \delta P_\perp\right) \tag{3}$$

and

$$\delta W_k = \frac{1}{2}\int d^3x \left[\vec{\xi}_\perp^* \cdot \nabla \delta P_\perp + \vec{\xi}_\perp^* \cdot \vec{\kappa}\left(\delta P_\parallel - \delta P_\perp\right)\right] \tag{4}$$

Since

$$\begin{aligned}
&\int d^3x \left[\vec{\xi}_\perp^* \cdot \nabla \delta P_\perp\right] \\
&= \int d^3x \nabla \cdot \left(\vec{\xi}_\perp^* \delta P_\perp\right) - \int d^3x \delta P_\perp \nabla \cdot \vec{\xi}_\perp^* \\
&= \int d^3x \delta P_\perp 2\vec{\kappa} \cdot \vec{\xi}_\perp^*
\end{aligned}$$

where $\nabla \cdot \vec{\xi}_\perp^* = -2\kappa \cdot \xi_\perp^*$. Therefore

$$\delta W_k = \frac{1}{2}\int d^3x \vec{\xi}_\perp^* \cdot \vec{\kappa}\left(\delta P_\parallel + \delta P_\perp\right) \tag{5}$$

which is half of $\delta W_k = e\int d^3x \int d^3v \left(\frac{i}{\omega}\mathbf{v}_d \cdot \delta\mathbf{E}_\perp\right)^\star g$ used previously. It may result in the critical beta calculated previously was half of the correct one.

For passing ions, we have

$$\delta W_k = m_h \int d^3x d^3v \vec{\xi}_\perp^* \cdot \vec{\kappa} E \delta F_h \tag{6}$$

where $\delta P_\parallel = \int d^3v m_h v_\parallel^2 \delta F_h$, $E = v^2/2 = v_\parallel^2/2$. The above form is consistent with the equation (4) of Graves's paper.

The adiabatic contribution for EPs based on the references given in Graves's paper.

$$\delta F_{hf} = -Ze/m_h \vec{\xi} \cdot \nabla\psi \partial F_h/\partial P_\phi \tag{7}$$

$d\psi_p/d\psi = 1/q$, $2\pi\psi = \int Brdrd\theta$

$\Rightarrow \xi \cdot \nabla\psi_p = \xi_r \frac{d\psi}{dr}\frac{d\psi}{d\psi_p} = rB_0\xi_r/q\left(r\right)$

$P_\phi = Rv_\phi + Ze\psi/m_h \Rightarrow \partial/\partial P_\phi = \Omega_c^{-1}\left[q\left(\bar{r}\right)/\bar{r}\right]\partial/\partial\bar{r}$. Here $\bar{r}$ is a constant of motion.

$$\delta F_{hf} = -\xi_r\left(r/\bar{r}\right)\left(q\left(\bar{r}\right)/q\left(r\right)\right)\partial F_h\left(\bar{r}\right)/\partial\bar{r} \tag{8}$$

To calculate the contribution numerically, the normalized form of $\delta W_{hf}$ is given by

$$\begin{aligned}
\delta\bar{W}_{hf} &= \frac{1}{2}\int \bar{R}^2 x dx d\theta \frac{1}{b\sqrt{b-\frac{\Lambda}{b}}} d\Lambda \bar{E}^{3/2} d\bar{E} \cdot \\
&= \left(\bar{\xi}_\theta^*\bar{\xi}_r\bar{\kappa}_\theta\bar{g}^{\theta\theta} + \bar{\xi}_\theta^*\bar{\xi}_r\bar{\kappa}_r\bar{g}^{rr} + \left|\bar{\xi}_r\right|^2\bar{\kappa}_\theta\bar{g}^{r\theta} + \left|\bar{\xi}_r\right|^2\bar{\kappa}_r\bar{g}^{rr}\right) \cdot \\
&\quad (-1)\left[\frac{\Lambda}{b} + 2\left(1-\frac{\Lambda}{b}\right)\right]\frac{x}{\bar{x}}\frac{q\left(\bar{x}\right)}{q\left(x\right)}\frac{d\bar{F}}{d\bar{x}}
\end{aligned}$$

with

$$\bar{F} = \left(2^{3/2}/C\right)\hat{F}$$

$$\hat{F}\left(x,\bar{\epsilon},\Lambda\right) = \frac{1}{\bar{\epsilon}^{3/2}+\bar{\epsilon}_c^{3/2}} Erfc\left(\frac{\bar{\epsilon}-\bar{\epsilon}_0}{\Delta\bar{\epsilon}}\right)\exp\left[-\left(\frac{x-x_0}{\Delta x}\right)^2\right]\exp\left[-\left(\frac{\Lambda-\Lambda_0}{\Delta\Lambda}\right)^2\right]$$

$$C=\int\sqrt{2}\pi\frac{1}{b\sqrt{1-\frac{\Lambda}{b}}}d\Lambda\bar{\epsilon}^{1/2}d\bar{\epsilon}\frac{1}{\bar{\epsilon}^{3/2}+\bar{\epsilon}_c^{3/2}}Erfc\left(\frac{\bar{\epsilon}-\bar{\epsilon}_0}{\Delta\bar{\epsilon}}\right)\exp\left[-\left(\frac{\Lambda-\Lambda_0}{\Delta\Lambda}\right)^2\right]$$

where $\delta W_{hf}=\pi^2a^2R_0n_0T_h\delta\bar{W}_{hf}$ and $\bar{x}=x-\bar{\rho}_d\cos\theta$.

To calculate the contribution analytically, $\delta F_{hf}$ is expanded around $r$:

$r/\bar{r}=1+\sigma\Delta_b/r\cos\theta$ and ignoring high order terms in $\Delta_b/r\Rightarrow q(\bar{r})/q(r)=1-\sigma\partial\Delta_b/\partial r\cos\theta$

$F(\bar{r})=F(r-\sigma\Delta_b\cos\theta)$ and ignoring quadratic terms in $\Delta_b/r\Rightarrow$

$F(\bar{r})=F(r)-\sigma\Delta_b\cos\theta\partial F/\partial r,\Rightarrow$

$\partial F(\bar{r})/\partial\bar{r}=\partial F(\bar{r})/\partial r(\partial r/\partial\bar{r})\Rightarrow$

$\partial F(\bar{r})/\partial\bar{r}=\partial F/\partial r+(\sigma\Delta_b/r-\sigma\partial\Delta_b/\partial r)\cos\theta\partial F/\partial r-\sigma\Delta_b\cos\theta\partial^2F/\partial r^2$, while in Graves's paper $\partial F(\bar{r})/\partial\bar{r}=\partial F/\partial r-\sigma\Delta_b\cos\theta\partial^2F/\partial r^2$. Then

$$\begin{aligned}\delta F_{hf}&=-\xi_r(r/\bar{r})(q(\bar{r})/q(r))\partial F_h(\bar{r})/\partial\bar{r}\\&=-\xi_r(1+\sigma\Delta_b/r\cos\theta)(1-\sigma\partial\Delta_b/\partial r\cos\theta)\cdot\\&\quad\left(\partial F/\partial r+\sigma\cos\theta\left(\frac{\Delta_b}{r}-\frac{\partial\Delta_b}{\partial r}\right)\frac{\partial F}{\partial r}-\sigma\Delta_b\cos\theta\partial^2F/\partial r^2\right)\end{aligned}$$

Ignoring quadratic terms in $\Delta_b/r$, yields

$$\begin{aligned}\delta\hat{F}_{hf}&=-\xi_0H(r_1-r)\exp(-i\theta)(1+2\sigma\cos\theta[\Delta_b/r-\partial\Delta_b/\partial r])\frac{\partial F}{\partial r}\\&\quad+\xi_0H(r_1-r)\exp(-i\theta)\sigma\Delta_b\cos\theta\partial^2F/\partial r^2\end{aligned}$$

which is reduced into

$$\begin{aligned}\delta\hat{F}_{hf}&=-\xi_0H(r_1-r)\left(\exp(-i\theta)+2\sigma(1+\exp(-i2\theta))\left[\frac{\Delta_b}{r}-\frac{1}{2r}\frac{\partial}{\partial r}r\Delta_b\right]\right)\frac{\partial F}{\partial r}\\&\quad+\xi_0H(r_1-r)\sigma\Delta_b\frac{1}{2}(1+\exp(-i2\theta))\partial^2F/\partial r^2\\&=-\xi_0H(r_1-r)\left(\exp(-i\theta)+\sigma(1+\exp(-i2\theta))\left[\frac{\Delta_b}{r}-\frac{\partial\Delta_b}{\partial r}\right]\right)\frac{\partial F}{\partial r}\\&\quad+\xi_0H(r_1-r)\sigma\Delta_b\frac{1}{2}(1+\exp(-i2\theta))\partial^2F/\partial r^2\end{aligned}$$

Note that there is an additional factor 2 of $\delta F_{hf}$ compared to the equation (3) of Graves's paper. One writes down the $\delta F_h$ as Graves,

$$\delta\hat{F}_h = \delta\hat{F}_{hf} + \delta\hat{F}_{hk}^{(0)} + \delta\hat{F}_{hk}^{(1)} \tag{9}$$

According to WangSJ's paper with $\omega \ll 1$, one obtains

$$\delta\hat{F}_{hk}^{(0)} = \sigma\frac{\Delta_b}{r}\xi_0 H\left(r_1 - r\right) \tag{10}$$

where

$$\delta F_{hk}^{(0)} = \frac{\omega_*}{-\sigma\left|v_\parallel\right|/R}2E\kappa\cdot\xi_\perp\partial_E F$$

$$\omega_* = \frac{q}{r}\frac{1}{\Omega_c}\frac{\partial_r F}{\partial_E F}.$$

Thus

$$\begin{aligned}
\delta W_{hk}^{(0)} + \delta W_{hf}\left(O\left(\Delta_b/r\right)\right) &= m_h\int d^3xd^3vE\kappa\cdot\xi_\perp^*\left(\delta F_{hk}^{(0)} + \delta F_{hf}\left(O\left(\Delta_b/r\right)\right)\right)\\
&= m_h\int d^3xd^3vE\kappa\cdot\xi_\perp^*\left[\left(-\xi_0\right)H\left(r_1-r\right)\cdot\right.\\
&\quad\left(\sigma\left(1+\exp\left(-i2\theta\right)\right)\left[\frac{\Delta_b}{r} - \frac{\partial\Delta_b}{\partial r}\right] - \sigma\frac{\Delta_b}{r}\right)\frac{\partial F}{\partial r}\\
&\quad\left.+\xi_0 H\left(r_1-r\right)\sigma\Delta_b/2\left(1+\exp\left(-i2\theta\right)\right)\partial^2F/\partial r^2\right]\\
&= m_h\int d^3xd^3vE\kappa\cdot\xi_\perp^*\left[\left(-\xi_0\right)H\left(r_1-r\right)\cdot\right.\\
&\quad\left(-\sigma\frac{\partial\Delta_b}{\partial r} + \exp\left(-i2\theta\right)\sigma\left[\frac{\Delta_b}{r} - \frac{\partial\Delta_b}{\partial r}\right]\right)\frac{\partial F}{\partial r}\\
&\quad\left.+\xi_0 H\left(r_1-r\right)\sigma\Delta_b/2\left(1+\exp\left(-i2\theta\right)\right)\partial^2F/\partial r^2\right]\\
&= m_h\int\sqrt{2}\pi d\Lambda dEE^{1/2}\int 2\pi d\theta rdrE\xi_0^2\sigma\left(-\frac{\partial\Delta_b}{\partial r}\frac{\partial F}{\partial r} - \frac{\Delta_b}{2}\frac{\partial^2F}{\partial r^2}\right)
\end{aligned}$$

where $d^3x = R^2/R_0 d\theta drr$, $d^3v = \sqrt{2}\pi\frac{1}{b}d\Lambda dEE^{1/2}$, $R/R_0 = 1+\epsilon\cos\theta$, $1/b = 1-\epsilon\cos\theta$, $\kappa\cdot\hat{\xi}_\perp^* = -\left(\xi_0/R\right)H\left(r_1-r\right)$ and the terms of $\epsilon^2$ order are ignored. And taking the following partly integral

$$\begin{aligned}
&\int drr\frac{\partial\Delta_b}{\partial r}\frac{\partial F}{\partial r}\\
=&\int dr\left(\frac{\partial r\Delta_b}{\partial r}-\Delta_b\right)\frac{\partial F}{\partial r}\\
=&\int dr\frac{\partial r\Delta_b}{\partial r}\frac{\partial F}{\partial r}-\int dr\Delta_b\frac{\partial F}{\partial r}\\
=&\int dr\frac{\partial}{\partial r}\left(r\Delta_b\frac{\partial F}{\partial r}\right)-\int drr\Delta_b\frac{\partial^2 F}{\partial r^2}-\int dr\Delta_b\frac{\partial F}{\partial r}\\
=&\ r_1\Delta_b\frac{\partial F}{\partial r_1}-\int drr\Delta_b\frac{\partial^2 F}{\partial r^2}-\int dr\Delta_b\frac{\partial F}{\partial r},
\end{aligned}$$

one finally obtains

$$\begin{aligned}
&\delta W_{hk}^{(0)}+\delta W_{hf}\left(O\left(\Delta_b/r\right)\right)\\
=&m_h\int\sqrt{2}\pi d\Lambda dEE^{1/2}\int 2\pi d\theta E\xi_0^2\sigma\left[-r_1\Delta_b\frac{\partial F}{\partial r_1}+\frac{1}{2}\int drr\Delta_b\frac{\partial^2 F}{\partial r^2}+\int dr\Delta_b\frac{\partial F}{\partial r}\right]
\end{aligned}\tag{11}$$

which is $m_h\int\sqrt{2}\pi d\Lambda dEE^{1/2}\int 2\pi d\theta E\xi_0^2\sigma\left(-r_1\right)\Delta_b\partial F_h/\partial r_1$ in Graves's. The reason for the differences is that the different form of $\partial F\left(\bar{r}\right)/\partial\bar{r}$, i.e. $\partial F\left(\bar{r}\right)/\partial\bar{r}=\partial F/\partial r-\sigma\Delta_b\cos\theta\partial^2F/\partial r^2$ in Graves's paper.

For $F_h\left(x,E,\Lambda\right)=c_0\left(x\right)\frac{1}{E^{3/2}}\delta\left(\Lambda\right)H\left(E_0-E\right)$, we can get

$$\begin{aligned}
P_h\left(x\right)\ &\equiv\ \frac{1}{2}\int d^3vm_h\left(v_\parallel^2+\frac{1}{2}v_\perp^2\right)F_h\\
&\approx\ \frac{1}{2}\int d^3vm_hv_\parallel^2F_h\\
&=\ \frac{1}{2}m_n\int\sqrt{2}\pi\frac{1}{b\sqrt{1-\frac{\Lambda}{b}}}d\Lambda E^{1/2}dE2EF_h\\
&=\frac{1}{2}\ m_h\int 2^{\frac{3}{2}}\pi E^{\frac{3}{2}}\frac{1}{b\sqrt{1-\frac{\Lambda}{b}}}c_0\left(x\right)\frac{1}{E^{3/2}}\delta\left(\Lambda\right)H\left(E_0-E\right)d\Lambda dE\\
&=\ \pi 2^{\frac{1}{2}}m_hc_0\left(x\right)E_0
\end{aligned}$$

Thus, $F_h=\left[P_h\left(x\right)/\left(\pi\sqrt{2}m_hE_0\right)\right]\frac{1}{E^{3/2}}\delta\left(\Lambda\right)H\left(E_0-E\right)$ with $c_0=P_h\left(x\right)/\left(\pi\sqrt{2}m_hE_0\right)$, which is consistent with Eq.(7) of Graves's paper. The additional $B_0$ of $c_0$ in Graves's paper is due to the use of the pitch angle variable $\lambda=\frac{v_\perp^2/2}{E}$ compared to $\Lambda$ of ours in velocity space integral, i.e. $\sqrt{2}\pi\sum_\sigma dEE^{1/2}Bd\lambda=\sqrt{2}\pi\sum_\sigma\frac{1}{b}d\Lambda dEE^{1/2}$. The first term of Eq. (11) is

$$
\begin{aligned}
\delta W_{hf}' &= m_h \int \sqrt{2}\pi d\Lambda dE E^{1/2} \sum_\sigma \int 2\pi d\theta E \xi_0^2 \sigma \left(-r_1\right) \Delta_b \frac{\partial F}{\partial r_1} \\
&= 2\sqrt{2} m_h \pi^2 \xi_0^2 \left(-r_1\right) \sum_\sigma \sigma \int d\theta d\Lambda dE E^{3/2} \frac{\left|v_\parallel\right| q_1}{\Omega_c} \frac{d}{dr} \frac{\left\langle P_h\left(r\right)\right\rangle_\sigma}{\pi\sqrt{2m_h E_0}} \frac{1}{E^{3/2}} \delta\left(\Lambda\right) H\left(E_0 - E\right) \\
&= 2\sqrt{2} m_h \pi^2 \xi_0^2 \left(-r_1\right) \sum_\sigma \sigma \int d\theta dE \frac{q_1 \left|v_\parallel\right|}{\Omega_c} \frac{1}{\pi\sqrt{2m_h E_0}} \frac{d\left\langle P_h\left(r\right)\right\rangle_\sigma}{dr} \\
&= 4\sqrt{2}\pi^2 \xi_0^2 \left(-r_1\right) \frac{q_1}{\Omega_c} \frac{1}{E_0} \sum_\sigma \sigma \frac{d\left\langle P_h\left(r\right)\right\rangle_\sigma}{dr} \int_0^{E_0} dE \sqrt{E} \\
&= \frac{8}{3}\pi^2 \xi_0^2 \left(-r_1\right) \Delta_{b1}^I \left[\frac{d\left\langle P_h\left(r\right)\right\rangle_{+1}}{dr} - \frac{d\left\langle P_h\left(r\right)\right\rangle_{-1}}{dr}\right] \\
&= \frac{8}{3}\pi^2 \xi_0^2 \left(-r_1\right) \Delta_{b1}^I \frac{d}{dr}\left(A\left\langle P_h\left(r\right)\right\rangle\right) \\
&= \frac{8}{3}\pi^2 \xi_0^2 \left(-r_1\right) \Delta_{b1}^I A \frac{d}{dr}\left\langle P_h\left(r\right)\right\rangle
\end{aligned}
$$

where $A = \frac{\left\langle P_h(r)\right\rangle_{+1} - \left\langle P_h(r)\right\rangle_{-1}}{\left\langle P_h(r)\right\rangle}$ , $\left\langle P_h\left(r\right)\right\rangle = \left\langle P_h\left(r\right)\right\rangle_{+1} + \left\langle P_h\left(r\right)\right\rangle_{-1}$ , $dA/dr = 0$ due to $\left\langle P_h\left(r\right)\right\rangle_{+1} / \left\langle P_h\left(r\right)\right\rangle_{-1} = \mathbf{const}$. Using $\delta\hat{W} = \delta W / \left(2\pi^2 \xi_0^2 \epsilon_1^4 R_0 B_0^2/\mu_0\right)$, we have

$$
\delta\hat{W}_{hf}' = \frac{2}{3}\epsilon_1^{-1}\left(\frac{\Delta_b^I}{r_{ph}}\right)\tilde{\beta}_{ph} A \tag{12}
$$

where $r_{ph} = -\left(d\left\langle P_h\left(r_1\right)\right\rangle/dr\right)^{-1}\left\langle P_h\left(r_1\right)\right\rangle$, $\tilde{\beta}_{ph} = 2\mu_0\left\langle P_h\left(r\right)\right\rangle / \left(\epsilon_1^2 B_0^2\right)$. The above equation is a factor 2 compared to the third term of Eq.(8) of Graves's paper. The second term of Eq. (11) is

$$
\begin{aligned}
\delta W_{hf}'' &= m_h \int \sqrt{2}\pi d\Lambda dE E^{1/2} \sum_\sigma \int 2\pi d\theta E \xi_0^2 \sigma \frac{1}{2} \int dr r \Delta_b \frac{\partial^2 F}{\partial r^2} \\
&= 2\sqrt{2} m_h \pi^3 \xi_0^2 \sum_\sigma \sigma \int d\Lambda dE E^{3/2} \frac{\left|v_\parallel\right| q_1}{\Omega_c} \int dr r \frac{\partial^2}{\partial r^2} \frac{\left\langle P_h\left(r\right)\right\rangle_\sigma}{\pi\sqrt{2m_h E_0}} \frac{1}{E^{3/2}} \delta\left(\Lambda\right) H\left(E_0 - E\right) \\
&= 2\sqrt{2}\pi^2 \xi_0^2 \frac{q_1}{\Omega_c} \frac{1}{E_0} \sum_\sigma \sigma \int dE \sqrt{E} \int dr r \frac{d^2\left\langle P_h\left(r\right)\right\rangle_\sigma}{dr^2} \\
&= \frac{4}{3}\pi^2 \xi_0^2 \Delta_b^I A \int dr r \frac{d^2\left\langle P_h\left(r\right)\right\rangle}{dr^2}
\end{aligned}
$$

The last term of Eq. (11) is

$$\begin{aligned}
\delta W_{hf}''' &= m_h \int \sqrt{2}\pi d\Lambda dE E^{1/2} \sum_\sigma \int 2\pi d\theta E \xi_0^2 \sigma \int dr \Delta_b \frac{\partial F}{\partial r} \\
&= 4\sqrt{2} m_h \pi^3 \xi_0^2 \sum_\sigma \sigma \int d\Lambda dE E^{3/2} \frac{\left|v_\parallel\right| q_1}{\Omega_c} \int dr \frac{d}{dr} \frac{\langle P_h(r)\rangle_\sigma}{\pi\sqrt{2 m_h E_0}} \frac{1}{E^{3/2}} \delta(\Lambda) H(E_0 - E) \\
&= 4\sqrt{2}\pi^2 \xi_0^2 \frac{q_1}{\Omega_c} \frac{1}{E_0} \sum_\sigma \sigma \int dE \sqrt{E} \int dr \frac{d}{dr} \langle P_h(r)\rangle_\sigma \\
&= \frac{8}{3}\pi^2 \xi_0^2 \Delta_b^I A \int dr \frac{d}{dr} \langle P_h(r)\rangle
\end{aligned}$$

For $\delta W_{hf}$ of zero order in $\Delta_b/r$, one has

$$\delta W_{hf}\left(O\left(\Delta_b/r\right)^0\right) = m_h \int d^3x d^3v \vec{\xi}_\perp^* \cdot \vec{\kappa} E(-\xi) \cdot \nabla F_h \tag{13}$$

where $\delta \hat{F}_{hf} = -\xi \cdot \nabla F_h = -\xi_0 H(r_1 - r)\exp(-i\theta)\partial F_h/\partial r$. For $P_\parallel = m_h \int d^3v F_h v_\parallel^2$ and $P_\parallel = \left\langle P_\parallel \right\rangle (1 - \epsilon\cos\theta)$ due to $1/b$. Futhermore,

$$\delta W_{hf}\left(O\left(\Delta_b/r\right)^0\right) = \frac{1}{2}\int d^3x \vec{\xi}_\perp^* \cdot \vec{\kappa}(-\xi) \cdot \nabla P_{h\parallel} = -\frac{1}{2}\int d^3x \vec{\xi}_\perp^* \cdot \vec{\kappa}\xi \cdot \nabla \left\langle P_{h\parallel}\right\rangle (1 - \epsilon\cos\theta) \tag{14}$$

$$\begin{aligned}
&\delta W_{hf}\left(O\left(\Delta_b/r\right)^0\right) \\
&= \frac{1}{2}\int d^3x (-2)\xi_\perp^* \cdot \kappa\xi \cdot \nabla \langle P_h\rangle + \frac{1}{2}\int d^3x 2\vec{\xi}_\perp^* \cdot \vec{\kappa}\xi \cdot \nabla\left(\langle P_h\rangle \epsilon\cos\theta\right)
\end{aligned}$$

where $P = \left(P_\parallel + P_\perp\right)/2 = P_\parallel/2$. The first term of the above equation is the EP-related interchange term of MHD energy principle. For the last term of the above equation, one obtains

$$\begin{aligned}
&\frac{1}{2}\int d^3x 2\vec{\xi}_\perp^* \cdot \vec{\kappa}\xi \cdot \nabla\left(\langle P_h\rangle \epsilon\cos\theta\right) \\
&= \int 2\pi \frac{R^2}{R_0} r dr d\theta \xi_\perp^* \cdot \vec{\kappa}\left(\xi_\theta \nabla\theta + \xi_r \nabla r\right) \cdot \left(\nabla\theta \frac{\partial}{\partial\theta} + \nabla r \frac{\partial}{\partial r}\right)\left(\langle P_h\rangle \epsilon\cos\theta\right) \\
&= -2\pi^2 \xi_0^2 R_0 \int_0^{r_1} dr \epsilon^2 \frac{d}{dr}\langle P_h\rangle - 2\pi^2 \xi_0^2 \int dr \epsilon \langle P_h\rangle
\end{aligned}$$

which is $-2\pi^2\xi_0^2 R_0 \int_0^{r_1} dr\epsilon^2 \frac{d}{dr}\langle P_h\rangle$ in Graves's.

In conclusions,

1. For the form of $\delta W_k$, we agree with Graves.

2. For $\delta W_{hk}^{(0)}+\delta W_{hf}\left(O\left(\Delta_b/r\right)\right)$, we have three terms $\delta W_{hf}'$, $\delta W_{hf}''$, $\delta W_{hf}'''$, while Graves had one term. The possible reason is the different form of $\partial F/\partial r$. Our $\delta W_{hf}'$ is comparable to the Graves's term. However, there is a factor 2 of ours.

3. For $\delta W_{hf}\left(O\left(\Delta_b/r\right)^0\right)$, the EP-related interchange term of MHD energy principle agrees with Graves's. For the anisotropic correction, we have two terms while Graves had one term which is one of our two terms.