dwk++ Documentation

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Contents

1 Introduction

dwk++ A small code to get δW_k and solver fishbone dispersion relation. This c++ code is used to calculate delta_W_k in tokamak plasma by 3D integration (r,Lambda,E). A slowing down distribution function and a kink like mode structure is used. The goal of this code is to find the Eigenvalues with a mode structure input.

2 The defintion of δW_k

2.1 The normalized δW_k in the code:

$$\delta W_k = \sum^{p} \int_0^1 dr \int \frac{d\Lambda}{B} \int E^3 dE \tau_b (\omega - \omega_\star) \frac{\partial F}{\partial E} \frac{|Y_p|^2}{n\omega_\phi + p\omega_b - \omega} \tag{1}$$

$\sigma = \pm 1$ is the direction of $v_\parallel$, m and n is poloidal and toroidal mode number, $\Lambda = \frac{\mu}{E}$, E is the fast ion energy, τ_b is the particle bounce time. The slowing down distribution function of fast ions is:

$$F = \frac{1}{C_f} F(r, \epsilon, \Lambda) = \frac{1}{C_f} \frac{1}{E^{3/2} + E_c^{3/2}} erfc\left(\frac{E - E_0}{\Delta E}\right) \exp\left[-(\frac{r - r_0}{\Delta r}^2)\right] \exp\left[-(\frac{\Lambda - \Lambda_0}{\Delta\Lambda})^2\right] \tag{2}$$

$$C_f = \int dE d\Lambda \frac{1}{E^{3/2} + E_c^{3/2}} erfc\left(\frac{E - E_0}{\Delta E}\right) \exp\left[-(\frac{\Lambda - \Lambda_0}{\Delta\Lambda})^2\right] \tag{3}$$

$$\frac{\partial F}{\partial E} = \frac{-1}{C_f}\left[\frac{2\exp(-(\frac{E-E_0}{\Delta E})^2)}{\sqrt{\pi}\Delta E(E^{3/2} + E_c^{3/2})} + \frac{3\sqrt{E} erfc(\frac{E-E_0}{\Delta E})}{2(E^{3/2} + E_c^{3/2})^2} - \frac{2\Lambda(\Lambda - \Lambda_0) erfc\left(\frac{E-E_0}{\Delta E}\right)}{E\Delta\Lambda^2(E^{3/2} + E_c^{3/2})}\right] \tag{4}$$

$$* \quad \exp\left[-(\frac{r - r_0}{\Delta r})^2\right] \exp\left[-(\frac{\Lambda - \Lambda_0}{\Delta\Lambda})^2\right] \tag{5}$$

$$\frac{\partial F}{\partial r} = \frac{1}{C_f} \frac{erfc\left(\frac{E-E_0}{\Delta E}\right)}{E^{3/2} + E_c^{3/2}} \frac{2(r_0 - r)}{\Delta r^2} \exp\left[-(\frac{r - r_0}{\Delta r})^2\right] \exp\left[-(\frac{\Lambda - \Lambda_0}{\Delta\Lambda})^2\right] \tag{6}$$

The diamagnetic frequency:

$$\omega_\star = \frac{m}{2r}\frac{\rho_0}{\varepsilon_0}\frac{\partial F/\partial r}{\partial F/\partial E} \tag{7}$$

where ρ_0 is the gyro radius with injection energy. ε_0 is the inverse aspect-ratio, and $\varepsilon = \frac{r}{R_0}$
The transit frequency for passing particle is given below:

$$\omega_b = \frac{\pi\sqrt{\kappa}}{K(\kappa^{-1})}\frac{\sqrt{\varepsilon\Lambda/2}}{q}\sqrt{E} \tag{8}$$

The transit frequency in toroidal:$\omega_\phi = q\omega_b$. Particle bounce time: $\tau_b = \frac{2\pi}{\omega_b}$. The integral along the particle orbits:

$$Y(r,\Lambda) = \frac{1}{2\pi}\int_0^{2\pi}\chi d\theta B_\Lambda\left(\Lambda_b + 2\left(1-\Lambda_b\right)\right)G(r,\theta)exp\left(-i\chi p\Theta\right) \tag{9}$$

where $\chi(r,\Lambda) = \frac{\sigma\pi\sqrt{\kappa}\sqrt{\varepsilon\Lambda/2}}{K(\kappa^{-1})}$, $\kappa = \frac{1-\Lambda(1-\varepsilon)}{2\varepsilon\Lambda}$. K denotes the complete elliptic integral of the first kind. $\Lambda_b = \frac{\Lambda}{b}$, $B_\Lambda(r,\Lambda,\theta) = \frac{1}{b\sqrt{(1-\Lambda_b)}}$, $b = B/B_0 = 1 + (r/R_0)cos\theta$.

$$G = (g^{\theta\theta}\kappa_\theta + g^{r\theta}\kappa_r)\xi_\theta(\hat{r}(\bar{r},\rho_d,\theta),\theta) + (g^{rr}\kappa_r + g^{r\theta}\kappa_\theta)\xi_r(\hat{r}(\bar{r},\rho_d,\theta),\theta) \tag{10}$$

where $\hat{r} = \bar{r} + \rho_d cos\theta$.

$$\rho_d = \frac{q}{2}\rho_0\sqrt{\frac{E}{1-\Lambda/b}}\left[\frac{\Lambda}{b} + 2(1-\frac{\Lambda}{b})\right] = \frac{q}{2}\rho_0\sqrt{\frac{E}{1-\Lambda_b}}\left[\Lambda_b + 2(1-\Lambda_b)\right] \tag{11}$$

To simplify the code, we use Λ_0 instead of Λin ρ_d. $g^{rr} = 1 + \frac{\varepsilon cos\theta}{2}$, $g^{\theta\theta} = \frac{1}{r^2}\left(1 - \frac{5}{2}\varepsilon cos\theta\right)$, $g^{r\theta} = -\frac{3}{2r}\varepsilon sin\theta$.

$$\Theta(\theta,r,\Lambda) = \int_0^\theta d\theta'\frac{1}{b\sqrt{(1-\Lambda_b)}} = \int_0^\theta B_\Lambda \tag{12}$$

2.2 The mode structure

$\xi_r(r,\theta) = \xi_0(r)exp(-i\theta)$, $\xi_\theta(r,\theta) = -i\xi_0(r)rexp(-i\theta)$

$$\xi_0(r) = \begin{cases} 1 & r \le r_s - \Delta r/2 \\ \frac{\Delta r - 2r + r_s - \Delta r/2}{\Delta r} & r_s - \frac{\Delta r}{2} < r < r_s + \frac{\Delta r}{2} \\ 0 & r \ge r_s + \frac{\Delta r}{2} \end{cases}$$

2.3 The normalized quantities used for δW_k:

$v_0 = \sqrt{2T_h/M}$, T_h is the fast ions injection energy, M is the fast ion's mass. $\omega_0 = \frac{v_0}{R_0}$. $F_0 = \frac{n_0}{v_0^3}$, n_0 is the fast ion density at axis. $r_0 = a$ is the minor radius, $\varepsilon = a/R_0$, $E_0 = T_h/M$, B_0 is the torodial magnetic field at magnetic axis.

3 Fishbone dispersion relation

The dwk++ code calculate the fishbone dispersion relation:

$$\delta W_{mhd} + \delta W_k + \delta I = 0 \tag{13}$$

Assume $\delta W_{mhd} = 0$, the equation reduced to:

$$\frac{4}{\pi}(\frac{r_s}{R_0})^2|\frac{\xi_s}{\xi_0}|^2(-i\frac{\omega}{\omega_A}) + \beta_h\delta\bar{W}_k = 0 \tag{14}$$

$$\omega = -iC\beta_h\delta W_k \tag{15}$$

$$i\omega = C\beta_h\delta W_k \tag{16}$$

$$C = \frac{\omega_A}{\omega_0}\frac{1}{\frac{4}{\pi}(\frac{r_s}{R_0})^2|\frac{\xi_s}{\xi_0}|^2} \tag{17}$$

$$\omega_A = \frac{2}{\tau_{A\theta}s} \tag{18}$$

$$\tau_{A\theta} = \frac{3^{1/2} r_s}{(B_{\theta s}^2/\mu_0 \rho_m)^{1/2}} \tag{19}$$

$$s = r_s \frac{dq}{dr(r = r_s)} \tag{20}$$

$$\beta_h = 2\mu_0 n_0 T_h / B_t^2 \tag{21}$$

4 How to run dwk++

4.1 Compile the code

dwk++ code is using c++ language as the main language, it need a c++ compiler to compile the code, gcc/g++ on max os/linux and intel compiler on linux was tested. To compile the code, libconfig with version >1.5 is needed, and set environment variable LIBCONFIG_DIR to the path where libconfig located.

4.2 Input file

dwk.cfg:

```
/*the input file for dwk++*/
//tokamak parameters
tokamak=
{
        a=0.4;          //minor radius (m).
        R0=1.65;        //major radius (m).
        Bt =1.34;       //Toroidal magnetic field at axis without plasma, (Tesla).
        n0 =1.31e19;    //thermal plasma density, (m^-3).
        mi =2.0;        //ion mass, (protom mass,m_p).
        E_i0 =41.0;     //Fost ion injection energy, (KeV).
        m_ep =2.0;      //fat ion mass
        //q profile, polynomial coefficient, qc[5];
        //q=qc[0] +qc[1]*r +qc[2]*r^2 +qc[3]*r^3 +qc[4]*r^4;
        qc=[0.5, 2.0, 0.0, 0.0, 0.0];
}
//grid parameters
grid=
{
        nx=400; //grid size should be 3n + 1, n is a positive integer.
        nL=100;
        nE=1000;
        ntheta=400;
}
//fast ion distribution
slowing=
{
        r0=0.0;         // (a)
        rd=0.2;         // (a)
        L0=0.28;
        Ld=0.1;
        E0=1.0;         // (E_i0)
        Ed=0.3; // (E_i0)
        Ec=0.25;                // (E_i0)
        rho_d=0.00;             //drift orbit width, (a)
        sigma=1;        //co(1) ? count (-1)
}
//perturbation and omega range for the soultion
mode=
{
        n=1;    //toroidal mode number
        m=1;    //poloidal mode number
        pa=0;   //resounce: sum Yps/(n*omega_phi +p*omega_theta -\omega), sum from pa to pb
        pb=0;
```

```
        r_s=0.5; //q==1 surface.
        q_s=1.0;    //resonance surface
        delta_r=0.001;// step function width for kink, (a).
        omega_0=0.1; //find omega_0(the solution) and scan dwk between omega_0 and omega_1,
                       // (v_0/R_0), v_0 is the injection speed.
        omega_1=1.1;
        omega_i=0.002; //image part of omega
        omega_n=120;    //scan steps
        omega_err=1.0e-5; //residual of omega_0
        max_iter =100;      // maximum iteration number to find the omega_0
        max_iterg =2;
        dw_f=-0.00;   //dw_mhd
}
```

4.3 Output file

4.4 Utilities to plot results

5 Examples

q profile:

$q = 0.5 + 2r^2$
$r_s = 0.5a = 0.2m$
$s = 1$
$R_0 = 1.65m$
$B_{t0} = 4T$
$B_{\theta s} = \frac{r_s B_t}{R q_s} = 0.6T$
$n_0 = 1 \times 10^{20} m^{-3}$
$\rho_m = 3.3452 \times 10^{-7} kgm^{-3}$
$\tau_{A\theta} = 4.6328 \times 10^{-7} s$
$\omega_A = 4.317 \times 10^6$
$T_h = 40KeV$
$v_h = 1.9575 \times 10^6 m/s$
$\omega_0 = 1.1863 \times 10^6$
$C = 28.2486$

$$\frac{\pi B_{\theta s}^2}{\mu_0} R_0 |\xi_s|^2 (\frac{-i\omega}{\omega_A}) + \frac{1}{2}\pi^2 |\xi_0|^2 R_0 n_0 T_h \delta W_k = 0 \tag{22}$$

$$\frac{\pi B_{\theta s}^2}{\mu_0} |\xi_s|^2 (\frac{-i\omega}{\omega_A}) + \frac{1}{2}\pi^2 |\xi_0|^2 n_0 T_h \delta W_k = 0 \tag{23}$$

$$\frac{B_{\theta s}^2}{\mu_0} |\xi_s|^2 (\frac{-i\omega}{\omega_A}) + \frac{1}{2}\pi |\xi_0|^2 n_0 T_h \delta W_k = 0 \tag{24}$$

$$\frac{B_{\theta s}^2}{\mu_0} |\xi_s|^2 (\frac{i\omega}{\omega_A}) = \frac{1}{2}\pi |\xi_0|^2 n_0 T_h \delta W_k \tag{25}$$

$$\frac{B_{\theta s}^2}{\mu_0} |\xi_s|^2 (\frac{\omega}{\omega_A}) = -\frac{1}{2} i\pi |\xi_0|^2 n_0 T_h \delta W_k \tag{26}$$

$$\frac{\omega}{\omega_A} = \frac{-\frac{1}{2} i\pi |\xi_0|^2 n_0 T_h}{\frac{B_{\theta s}^2}{\mu_0} |\xi_s|^2} \delta W_k \tag{27}$$

$$\frac{\omega}{\omega_A} \frac{\omega_0}{\omega_0} = \frac{-\frac{1}{2} i\pi |\xi_0|^2 n_0 T_h}{\frac{B_{\theta s}^2}{\mu_0} |\xi_s|^2} \delta W_k \tag{28}$$

$$\frac{\omega i}{\omega_0} = \frac{\frac{1}{2}\pi |\xi_0|^2 n_0 T_h}{\frac{B_{\theta s}^2}{\mu_0} |\xi_s|^2} \frac{\omega_A}{\omega_0} \delta W_k \tag{29}$$

$$\frac{\omega i}{\omega_0} = \frac{1}{2} \frac{\pi \frac{|\xi_S|^2}{|\xi_0|^2}}{B_{\theta s}^2} \mu_0 n_0 T_h \frac{\omega_A}{\omega_0} \delta W_k \tag{30}$$

$$\frac{\omega i}{\omega_0} = \frac{\pi}{2} \frac{|\xi_S|^2}{|\xi_0|^2} \frac{\mu_0 n_0 T_h}{B_{\theta s}^2} \frac{\omega_A}{\omega_0} \delta W_k \tag{31}$$

$$\frac{\omega i}{\omega_0} = \frac{\pi}{2} \frac{|\xi_S|^2}{|\xi_0|^2} \frac{\mu_0 n_0 T_h R_0 q_s}{B_t^2 r_s^2} \frac{\omega_A}{\omega_0} \delta W_k \tag{32}$$

$$\frac{\omega i}{\omega_0} = \frac{\pi}{4} \frac{|\xi_S|^2}{|\xi_0|^2} \frac{2\mu_0 n_0 T_h R_0^2 q_s^2}{B_t^2 r_s^2} \frac{\omega_A}{\omega_0} \delta W_k \tag{33}$$

$$\frac{\omega i}{\omega_0} = \frac{\pi}{4} \frac{|\xi_S|^2}{|\xi_0|^2} \frac{R_0^2 q_s^2}{r_s^2} \frac{\omega_A}{\omega_0} \beta_h \delta W_k \tag{34}$$

$$\frac{\omega i}{\omega_0} = C \beta_h \delta W_k \tag{35}$$

$$C = \frac{\pi}{4} \frac{|\xi_S|^2}{|\xi_0|^2} \frac{R_0^2 q_s^2}{r_s^2} \frac{\omega_A}{\omega_0}$$