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A Race Simulation for Strategy Decisions in Circuit Motorsports

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***Abstract*— In motorsports, every racing participant pursues the goal to finish the race in the shortest time possible. There are several ways to influence the race with race strategy decisions, e.g. the timing of pit stops and the choice of tires. In order to be able to evaluate the race strategy before and to quickly adjust it during a race, a tool is required that simulates an entire race in a short time. For this purpose, the paper presents the methodology of a race simulation for circuit motorsports. It simulates races in dependency of various race strategy inputs and is based on a lap-wise discretization. It includes the effects of tire degradation, fuel mass loss, pit stops and overtaking maneuvers. The simulation parameters are chosen in such a way that they can be determined based on publicly accessible lap time data. The Formula 1 2017 Abu Dhabi Grand Prix is analyzed as an exemplary result. An application area is the support of race engineers in present motorsports. Future work aims towards the automation of strategy decisions in motorsports with autonomous racecars.**

I. INTRODUCTION

In circuit motorsports, the race participants drive a certain number of laps against each other on a closed racetrack. The cars are built according to technical regulations that are dependent on the racing series, e.g. Formula 1 or Le Mans Prototype (LMP) in the World Endurance Championship (WEC). The better the rank position at the end of the race, the more points the driver and his team receive. Often two or more cars are operated per team. The points are accumulated over a season to determine the winners of the driver and team championships at the end. Usually a race weekend starts with several practice sessions. Afterwards, in a qualifying session, the starting order of the race is determined based on the best lap time. During a race, drivers can come to their pit to refuel, change tires or repair broken parts of the car. This results in a time loss compared to the other drivers that continue the race.

The timing of a pit stop and the determination of the actions to be carried out in it is part of the race strategy. It is a key factor for a satisfying race outcome. Porsche [1] give an overview of the topics covered by race strategy in the context of endurance races. According to this, it includes the consideration of tire degradation, fuel mass loss, pit stops and the reaction to safety cars and yellow flags. Furthermore, it takes the driver and the pit crew into account. As Fig. 1 visualizes, we divide these aspects into three categories: pit stops, driving strategy and response to race events. Driving strategy includes how aggressively the driver drives the car.

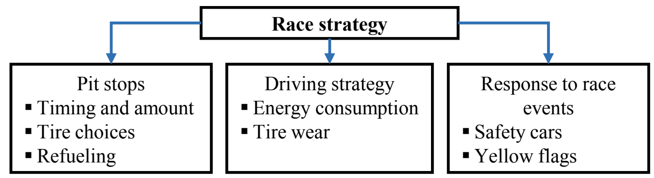

Fig. 1. Aspects of race strategy.

This affects not only lap times but also tire wear and energy consumption.

Race strategies of different racing series differ greatly because they depend heavily on technical and sporting regulations [2]. However, the common characteristic of circuit races is that every race participant tries to finish the race in the shortest time possible. The well-founded definition of a race strategy therefore requires a tool that simulates the race based on various strategy inputs. These tools are known as race simulations.

II. RELATED WORK

At first, we differentiate race simulations from the more popular lap time simulations. The most important aspects are shown in Fig. 2. The goal of a lap time simulation is to calculate the exact lap time with the current car setup. Therefore, it simulates one lap with one car neglecting long-term effects. The underlying models are mostly physically motivated. To summarize, a lap time simulation provides a microscopic view on one race lap. Siegler [3] give an overview of three different types of lap time simulations and compare them. Further research is carried out for a quasi-steady-state approach in Brayshaw [4], for a transient approach in Colunga [5] and for a robust approach based on model predictive control in Timings [6].

A race simulation, in contrast, has a macroscopic view on the race, including race events such as pit stops and long-term effects such as tire degradation. All participating cars and their interactions are simulated together for all the laps. This is done with empirical models to keep the calculation times and amount of required simulation parameters in usable limits. The globally relevant vehicle characteristics are implicitly contained in the simulation parameters, since a fast car is represented by a fast lap time. The goal is to calculate the final race durations of all participants, e.g. for the evaluation of a race strategy. However, both types of simulation work closely together because the lap time simulation can provide many of the required parameters for the race simulation.

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Lap time simulation	Race simulation
▪ Microscopic view on one lap neglecting long-term effects ▪ Physically motivated models (e.g. two-track car model, engine model) ▪ One car simulated for one lap ▪ Goal: Calculation of the exact lap time with the current configuration, e.g. to evaluate the setup	▪ Macroscopic view on the whole race including long-term effects ▪ Empirical models (e.g. tire degradation model, overtaking model) ▪ All participating cars simulated for the whole race with interactions ▪ Goal: Calculation of the final race times, e.g. to evaluate the race strategy

Fig. 2. Comparison between lap time simulation and race simulation.

Little literature can be found when it comes to race strategy. Tulabandhula [7] investigate how to support tire-changing decisions in the NASCAR series based on machine learning. Hirst [8] writes about the analysis and visualization of data for race strategy decisions in Formula 1. McLaren [9] address the fuel mass loss calculation and the pit time loss in a small example. Farroni [10] look at the modelling of tire wear and its effect to the car's performance. Bekker [11] published a holistic race simulation suitable for race strategy evaluation. They use a time-based approach combined with stochastic elements and include passing maneuvers, refueling during pit stops and car failures. In the internet, Porsche [1] give an overview of the facets of race strategy. Bi [12] highlights the significance of big data for race predictions in Formula 1. An important source is Phillips [13], who presents a race simulation for the Formula 1 2014 season on his blog. It uses some of Bekker's ideas, e.g. the time-based approach, and adds missing parts such as a tire degradation model.

III. METHODOLOGY

Our main demands for a race simulation are robust results and a fast calculation time. The former can be achieved by modeling all relevant effects. The latter allows running many simulations for the race preparation on the one hand and the live usage of the simulation during a race on the other hand. However, it does not need to be extremely accurate because the race situation usually changes a few times during a race anyway.

When analyzing the implementation of Bekker [11], there are several reasons why it is unsuitable for our use case. Firstly, the very relevant effect of tire degradation is not included. Secondly, the racetrack is divided into approximately 40 sectors with a length 150 m each. Each of them must be parameterized individually, e.g. with its lap time proportion, fuel consumption proportion and if passing maneuvers are allowed. This is difficult or impossible to do based on publicly accessible data. Thirdly, passing maneuvers must be completed within one sector in order to take place. This means that overtaking on long straights is not simulated realistically. Moreover, it is difficult to set the right minimum time gap required for overtaking for every sector. In addition, only one car can overtake at a time, whereas in reality a slow driver is often overtaken by two cars at once. Fourthly, the Drag Reduction System (DRS) is missing, as the paper was published before its introduction to Formula 1. Since the 2011 season, the driver can activate it under certain conditions to reduce drag on long straights and thus facilitate overtaking.

Due to our requirements and experience, the same applies to the implementation of Phillips [13]. Firstly, he primarily uses average lap times from the second free practice session as basic lap time for the simulation. The disadvantage of this method is the unknown fuel state of the cars. Secondly, he considers only two rubber compounds for his tire degradation model. He assumes that the softer compound is 0.7 s per lap faster for a fresh tire and wears twice as fast as the harder compound. Thirdly, the minimum time gap between two cars is only checked at the end of a lap. However, it is essential to check this again after the completion of the pit stops at the beginning of a lap because drivers may drive back onto the track very close in front or behind another driver. Fourthly, Phillips only considers a time loss for the overtaken driver. Based on our experience, it is important to also include a time loss for the overtaking driver as usually both fight for their positions. Fifthly, he uses a constant time penalty per lap in terms of fuel in his fuel mass model. However, it makes more sense to split the calculation into separate parts. Neither Bekker nor Phillips implement any possibility to consider team orders in the simulation.

Subsequently, we present the principle of our race simulation, which addresses these aspects. After an overview has been given, the submodels are discussed in more detail. The paper concentrates on Formula 1 because it is the most popular racing series. Nevertheless, the race simulation can be adapted to other racing series as well. A basic principle of our design is to keep it as simple as possible and as detailed as necessary. The more detailed the models are, the more input data is required to feed them. This is particularly problematic for us because we are limited to the small amount of data that the FIA [14] (International Automobile Federation) makes publicly accessible.

A. *Overview of the race simulation*

The race simulation is performed lap by lap, as Fig. 3 shows. There are several reasons for discretizing the race in laps instead of time. The most important are that a time discretization would need a lot more input data and the computational effort would be much higher. This is because the various competitors would have to be simulated with their individual racing lines on the track. In contrast, for lap discretization, lap times are sufficient as a basis. The downside of lap discretization is that it prevents the more detailed modeling of some aspects. Overtaking, for example, could be implemented more precisely by simulating the individual racing lines. However, as mentioned previously, this is not necessary.

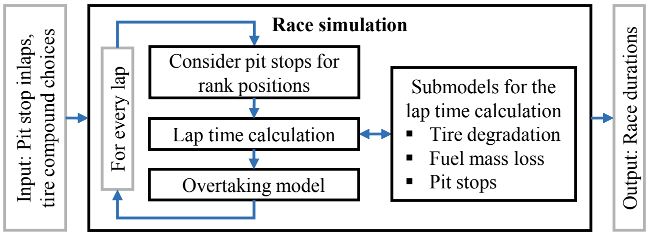

Fig. 3. Race simulation workflow.

As with Bekker [11] and Phillips [13], the user is required to provide the pit stop information for every driver as an input. This includes the expected pit stop inlaps as well as the tire compound choices. It should be emphasized that the race simulation simulates the race depending on the input, but does not optimize it. The output of the simulation are the race durations and thus also the rank positions of all the drivers at the end of the race. At the beginning of each lap, the simulation must check for position changes due to pit stops that began in the previous lap and end in the current lap. Afterward, it calculates the current lap time for every competitor starting from a base lap time. Time losses and gains due to tire wear, fuel mass loss, pit stops and other factors are then added. In the next step, the algorithm checks whether overtaking occurs in the current lap based on the computed lap times and the time gaps between the cars after the previous lap. Overtaking influences driver rank positions as well as their lap times. Afterward, the final lap times and rank positions are known and the next lap can be simulated. This is repeated until the race is completed.

B. Race simulation without competitor interaction

In the first approach, interactions between the competitors are omitted. Therefore, every car can be simulated by itself. The race time after a lap

$$t_{\text{race,currentlap}} = \sum_{lap=1}^{current\,lap} t_{\text{lap}}(lap) \tag{1}$$

follows from the summation of the separate lap times t_{lap} up to this lap. As Eq. 2 shows, the lap times are summed up based on several time elements that are explained in the following paragraphs.

$$\begin{aligned} t_{\text{lap}}(lap) =& t_{\text{base}} + t_{\text{tire}}(a_{\text{tire}}, c_{\text{tire}}) + t_{\text{fuel}}(lap) \\ &+ t_{\text{car}} + t_{\text{driver}} + t_{\text{grid}}(lap) \\ &+ t_{\text{pit,inlap/outlap}}(lap) \end{aligned} \tag{2}$$

1) Base lap time t_{base}: The base lap time is the basis for all lap time calculations. McLaren [9], Bekker [11] and Phillips [13] use a similar approach. However, we define it as the lap time the best racecar of a given racing series needs on a given track for one lap under perfect conditions during the race. To obtain it, the fastest qualifying lap time t_{Q} is utilized. During the qualifying, optimal and comparable conditions can be assumed because the car setups are finalized, they carry a minimum amount of fuel, the tires are in perfect condition and the drivers try to run an optimal lap. Another (but usually less accurate) source is a lap time simulation. As displayed in Eq. 3, the expected delta time between qualifying and race $t_{\text{gap,racepace}}$ of the fastest qualifying driver is then added in order to take into account the fact that the race pace is slower than the qualifying pace. This mainly occurs due to engine durability and fuel consumption limitations. The primary source for the delta time are the free practice sessions. With this approach, we obtain a robust basis for the lap time calculations. In addition, it allows us to quickly modify the lap times for all the drivers at once during a race, e.g. because of bad weather conditions.

$$t_{\text{base}} = t_{\text{Q}} + t_{\text{gap,racepace}} \tag{3}$$

2) Lap time loss due to tire degradation t_{tire}: A racing tire runs through different phases during a race. Usually, the warm-up period is followed by a short peak performance phase. Afterward, one can observe an almost constant performance decrease before the tire completely degrades at the end of its life [10]. This effect is called tire degradation [1], [10]. In Formula 1, Pirelli supplies three out of seven different compounds for dry conditions per race ranging from Hypersoft to Superhard [15]. Typically, the harder the compound, the slower it will be at the beginning [16], but the longer its stable performance phase lasts. Tire degradation can be expressed as lap time loss t_{tire} over tire age a_{tire} in laps. Therefore, the simulation provides empirically based logarithmic and linear formulas for the different rubber compounds c_{tire}:

$$\begin{aligned} t_{\text{tire,log}}(a_{\text{tire}}, c_{\text{tire}}) = & \log\left(a_{\text{tire}} \cdot k_{1,\text{log}}(c_{\text{tire}}) + 1\right) \\ & \cdot k_{2,\text{log}}(c_{\text{tire}}) + k_3(c_{\text{tire}}) \end{aligned} \tag{4}$$

$$t_{\text{tire,lin}}(a_{\text{tire}}, c_{\text{tire}}) = a_{\text{tire}} \cdot k_{2,\text{lin}}(c_{\text{tire}}) + k_3(c_{\text{tire}}) \tag{5}$$

Fig. 4 shows an example for two compounds modelled with Eq. 4. Due to our experience, a logarithmic function represents the tire behavior in the usable range better than the quadratic function that Phillips [13] uses. The simpler linear model is used, if there is only a little data available. The equations show that the degradation factors $k_{1,\text{log}}$ and $k_{2,\text{log}}$ respectively $k_{2,\text{lin}}$ as well as the time offset k_3 depend on the tire compound. k_3 expresses the basic time loss that appears for the medium and harder compounds in comparison to the softer tire [16]. The models are parameterized based on (fuel mass corrected) timing data from the free practices, qualifying and previous races. They are adjusted independently for every driver or at least every team because the degradation is heavily influenced by driving style and vehicle balance.

3) Lap time loss due to fuel mass t_{fuel}: For combustion-powered cars, the additional fuel mass leads to significantly worse lap times at the beginning of a race. As it progresses, time loss decreases due to consumed fuel. In Formula 1, there is a fuel mass loss of about 100 kg during one race. The lap time loss that results from this effect can be calculated by subtracting the consumed fuel mass $m_{\text{fuel,consumed}}$ from the total fuel mass $m_{\text{fuel,tot}}$ and then multiplying it with the mass sensitivity of the lap time $s_{t_{\text{lap}},\text{mass}}$ [9], Eq. 6. $m_{\text{fuel,consumed}}$

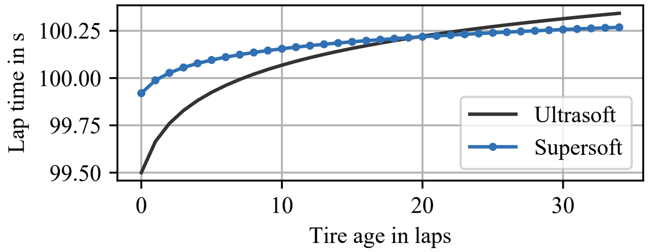

Fig. 4. Lap time effect of the tire degradation modelled with a logarithmic function for the Ultrasoft and Supersoft compounds. As of lap 20, the harder Supersoft leads to a better lap time. This is usually the point where one would pit at the latest to replace the Ultrasoft tires in order to avoid losing time against the Supersoft.

is derived from the fuel consumption per lap $B_{\text{fuel,perLap}}$ as shown in Eq. 7.

$$t_{\text{fuel}}(lap) = \left(m_{\text{fuel,tot}} - m_{\text{fuel,consumed}}(lap)\right) \cdot s_{t_{\text{lap}},\text{mass}} \quad (6)$$

$$m_{\text{fuel,consumed}}(lap) = B_{\text{fuel,perLap}} \cdot lap \quad (7)$$

The mass sensitivity of the lap time and the fuel consumption per lap are usually obtained from a lap time simulation using an engine consumption map. The subdivision of this effect into a mass analysis and the sensitivity allows us to easily include refueling during pit stops in those racing series where it is allowed. Side effects of the fuel mass occur in reality but are currently disregarded in the simulation, e.g. its effect on the tire degradation.

4) Lap time loss due to car abilities t_{car} and driver abilities t_{driver}: In racing series without unified cars, there are obviously differences in the lap times of the different manufacturers, even if all other factors were identical. The same applies to different drivers. Therefore, the simulation considers a constant time offset for both. They can be estimated from the qualifying session or previous races, for example.

5) First lap time loss due to starting grid position t_{grid}: Using the same model as Phillips [13], a handicap is added for every driver based on his grid position p_{grid} at the race start, because cars at the end of the grid are further away from the starting line. t_{firstlap} is added because the first lap takes longer than the following laps due to starting from a standstill:

$$t_{\text{grid}} = p_{\text{grid}} \cdot t_{\text{perGridPos}} + t_{\text{firstlap}} \quad (8)$$

6) Lap time loss due to pit stops $t_{\text{pit,inlap/outlap}}$: Pit stops influence different aspects of the race situation. To begin with, they lengthen the lap times of inlap and outlap by $t_{\text{pit,inlap}}$ respectively $t_{\text{pit,outlap}}$. Both parameters depend on the track layout. The inlap is affected by a few seconds because the lap time is taken at the finish line, which is usually located a slightly behind the pit lane entrance. The pitting cars are already slower at this point due to the pit speed limit of 80 km/h. Similar to Bekker [11] and in contrast to Phillips [13], the outlap lap time increase is split into a pit lane drive-through time under speed limit $t_{\text{pitdrive,outlap}}$ and a standstill time $t_{\text{standstill}}$. $t_{\text{standstill}}$ depends on the actions a team takes

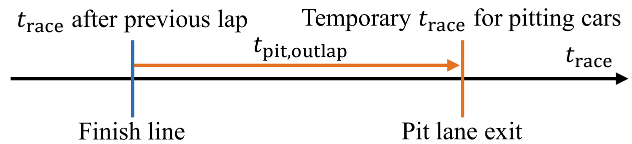

Fig. 5. Pit stop time losses $t_{\text{pit,outlap}}$ are added temporarily to the previous race times t_{race} of pitting cars to obtain comparable race times after pit stops.

during the pit stop. Since refueling was banned in the 2010 season for Formula 1, pit stops mainly involve changing tires or replacing broken spoiler parts. Pit stop penalties t_{penalty} are also important. These usually add five or ten seconds of additional standstill time due to race offenses. As Eq. 9 demonstrates, the simulation provides all the possibilities.

$$t_{\text{pit,outlap}} = t_{\text{pitdrive,outlap}} + t_{\text{standstill}} + t_{\text{penalty}} \quad (9)$$

After having completed the pit stops, the drivers drive back onto the track. The pit lane exit is usually located at the end of the start finish straightaway. This does not match the lap-wise discretization because a normal lap starts and ends at the finish line. Comparable race times are required, however, to be able to consider rank position changes due to the pit stops. As Fig. 5 clarifies, this is solved by adding $t_{\text{pit,outlap}}$ not only to the lap time t_{lap} of the outlap, but temporarily to the race times of the inlap as well. By doing so, the pit cars are virtually released on the finish line. These temporary race times are then used to reorder the rank positions after the pit stops at the beginning of a lap. For the next section, we need to keep in mind that overtaking of drivers in the pit lane occurs without any additional time loss.

C. Race simulation with competitor interaction

After having completed the basic race simulation, overtaking maneuvers are now added:

$$t_{\text{race,currentlap}} = \sum_{lap=1}^{current\,lap} t_{\text{lap}}(lap) + t_{\text{overtaking}}(lap) \quad (10)$$

Modeling overtaking maneuvers and their time loss $t_{\text{overtaking}}$: If the race times of two or more competitors are close together after a lap, it must be checked whether overtaking occurs accordingly Fig. 6. Again, the basic approach is similar to Phillips [13], but has been extended by various aspects. Bekker's implementation [11] differs somewhat more due to their discretization of the racetrack into sectors.

At first, the algorithm adds the calculated lap times for the current lap (without including overtaking) to the latest race times to get the current race times. Afterward, it checks whether the DRS is applicable for any driver. As mentioned, it reduces the drag on some straightaways and therefore the lap time. In Formula 1, drivers may use it when the first two laps are completed and the gap to the driver immediately in front is less than one second, i.e. $t_{\text{DRSwindow}} = 1\,s$. In that case, the lap time and race time of the follower are reduced

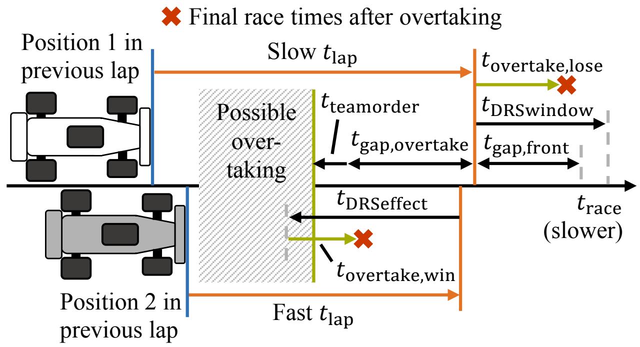

Fig. 6. The overtaking model in detail. The driver in position 2 in the previous lap is fast enough to overtake in the current lap with the help of DRS. As a result, both receive a time malus $t_{overtake,win/lose}$.

TABLE I
EXEMPLARY TEAM ORDER MATRIX.

Follower \ Leader	Driver 1	Driver 2
Driver 1	-	-0.1 s
Driver 2	0.15 s	-

by a track-specific time $t_{DRSeffect}$. The final race time gap between the drivers is then compared to a minimum time gap $t_{gap,overtake}$ that usually allows overtaking on the particular track. This value primarily depends on the track layout. It is a measure of how easy it is to overtake on the track. If, for example, the follower's race time is 1.2 s faster than that of the driver immediately in front of him, then he can easily overtake on a track with a minimum time gap of 0.9 s. As team orders are common in various racing series, the tool also considers a team order matrix as shown in Table I. It contains modifiers $t_{teamorder}$ for the minimum time gap, depending on the driver constellation. If the number one driver of a team drives behind the number two driver of the same team, for example, then he does not need to fulfill the entire minimum time gap to overtake. According to the example, it would be reduced by 0.1 s. Slipstream effects are not considered for overtaking. Depending on the track and the racing series, it can be either an advantage or a disadvantage for the follower because driving in turbulent air is not beneficial to the downforce and can therefore cancel out the effect of reduced drag.

If the calculated race time advantage of the rear driver is large enough, then overtaking occurs. In that case, both competitors receive a time malus $t_{overtake,win}$ respectively $t_{overtake,lose}$ on their lap time as Eq. 11 states. This is done because the racing lines of the involved drivers differ from the time optimal racing line when they are in a position fight. A side effect of this relation is multiple overtaking. If three or more cars drive close by, it can happen, that the second and third car overtake the first car at once due to the overtaking time loss of the first overtaking maneuver.

$$t_{overtaking} = t_{DRSeffect} + t_{overtake,win/lose} \tag{11}$$

On the other hand, if the rear driver is not fast enough to overtake but the calculated race time gap to the car in front is small or negative, then the gap has to be artificially enlarged to a minimum value in the simulation. This is because in reality, the follower cannot drive arbitrarily close to the car in front of him. The minimum time gap $t_{gap,front}$ must be kept between all the cars after overtaking is completed as well as after pit stops are completed.

D. Required parameters

In Table II, an overview of the parameters for the race simulation is provided. Sample values are given for the 2017 Abu Dhabi Grand Prix. They are adjusted based on the lap times published by the FIA [14] and will be used for the simulation in the results section afterward. Table III shows additionally required simulation parameters for the different drivers and manufacturers. For a better overview, we consider only the six drivers of the three top teams, i.e. Mercedes with Bottas and Hamilton, Ferrari with Räikkönen and Vettel and Red Bull with Ricciardo and Verstappen. Because there is only limited data available, the linear tire degradation model is used. The team order matrices are omitted for this example.

IV. RESULTS

The race simulation is implemented in Python. The calculation time for a race is one to two seconds on a common computer (Intel i7 2.7 GHz, 16 GB RAM). In Fig. 7, the simulation result for the 2017 Abu Dhabi Grand Prix with the previously listed parameter settings is shown in comparison to the actual timing data. We prefer to plot the race time gaps by comparing every driver to a virtual driver with a constant lap time. It is chosen so that the race duration of the virtual driver corresponds to the race duration of the leader at the end of the race. This method clearly visualizes where the drivers have gained and lost time against the winners average lap time. Additionally, pit stops appear as a sudden rise, e.g. for Verstappen in lap 14.

The comparison between actual and simulated timing data shows a good match. The curved courses of the gap times indicate the effects of fuel mass loss and tire degradation. Furthermore, one can see that similar lap time increases occur for pit stop inlaps and outlaps in simulation and actuality, e.g. for Bottas in laps 21 and 22. As of lap 25, Hamilton closes in on Bottas after his pit stop, but is obviously not fast enough to overtake. Consequently, a minimum time gap is kept. In the simulation, Ricciardo is able to catch Vettel after his pit stop in lap 22 with the help of DRS. This can be seen from the fact that his time gap decreases a bit instead of increasing due to $t_{overtake,win}$. For him, the effects of fuel mass loss and tire degradation almost cancel each other out after his pit stop, resulting in an almost constant time gap. Therefore, according to our simulation, Vettel would have caught him again later in the race around lap 40. In reality, Ricciardo retired in lap 20 due to a mechanical failure. Looking at the actual data, one can also see that Hamilton stops his hunt for Bottas in lap 52. Of course, this aspect is not included in the simulation data.

TABLE II

Overview of the required simulation parameters. Possible sources: FP – Free Practice, LTS – Lap Time Simulation, PR – Previous Races, Q – Qualifying, R – Rules.

Parameter	Description	Source	Example
Track			
t_{Q}	Fastest qualifying lap time	Q, (LTS)	96.23 s
$t_{\mathrm{gap,racepace}}$	Delta time between qualifying and race pace	FP	3.67 s
$s_{t_{\mathrm{lap}},\mathrm{mass}}$	Mass sensitivity of lap time	LTS	0.033 s/kg
$t_{\mathrm{pit,inlap}}$	Pit stop time loss (inlap)	FP	2.0 s
$t_{\mathrm{pitdrive,outlap}}$	Pit lane drive-through time (outlap)	FP	18.5 s
$t_{\mathrm{perGridPos}}$	First lap time increase per grid position	PR, (LTS)	1.0 s/pos
t_{firstlap}	First lap time increase due to standstill	PR, (LTS)	2.5 s
$t_{\mathrm{gap,overtake}}$	Time gap required for overtaking	PR	1.0 s
$t_{\mathrm{DRSeffect}}$	Time gain due to DRS effect	PR, (LTS)	-0.8 s
Driver			
t_{driver}	Additional time due to driver abilities	Q	0.1 s
p_{grid}	Starting grid position of driver	Q	1
t_{penalty}	Penalty time in pit stop	R	5.0 s
$t_{\mathrm{teamorder}}$	Team order modification of overtaking gap	-	1.0 s
Tire			
$k_{1,\mathrm{log}}$	Parameter of log. tire degradation model	FP, Q, PR	1.0 s/lap
$k_{2,\mathrm{log}}$	Parameter of log. tire degradation model	FP, Q, PR	1.0
$k_{2,\mathrm{lin}}$	Parameter of linear tire degradation model	FP, Q, PR	0.02 s/lap
k_3	Time offset depending on tire compound	FP, Q, PR	0.3 s
Car			
t_{car}	Additional time due to car abilities	Q, (PR)	0.1 s
$m_{\mathrm{fuel,tot}}$	Total fuel mass at race start	R	100.0 kg
$B_{\mathrm{fuel,perLap}}$	Fuel consumption per lap	LTS, FP, R	1.79 kg/lap
$t_{\mathrm{standstill}}$	Average standstill time in pit stop	PR	3.0 s
Race			
$t_{\mathrm{gap,front}}$	Minimum time gap between two cars	PR	0.95 s
$t_{\mathrm{overtake,win}}$	Overtaking time loss (gaining one position)	PR	0.1 s
$t_{\mathrm{overtake,lose}}$	Overtaking time loss (losing one position)	PR	0.6 s
$t_{\mathrm{DRSwindow}}$	DRS window	R	1.0 s

TABLE III

Additional simulation parameters for drivers and cars. Tire compounds: SUS – Supersoft, US – Ultrasoft.

Parameter	Ricciardo	Räikkönen	Hamilton
Driver			
p_{grid}	4	5	2
Tire at start	US	US	US
Pit stops: Inlap / Tire	19 / SUS	15 / SUS	24 / SUS
t_{driver}	0.0 s	0.307 s	0.0 s
Tire			
$k_{2,\mathrm{lin,US}}$	0.050 s/lap	0.029 s/lap	0.020 s/lap
$k_{3,\mathrm{US}}$	0.0 s	0.0 s	0.0 s
$k_{2,\mathrm{lin,SUS}}$	0.050 s/lap	0.039 s/lap	0.030 s/lap
$k_{3,\mathrm{SUS}}$	0.556 s	0.290 s	0.900 s
Car			
t_{car}	0.244 s	0.370 s	0.0 s
	Vettel	Verstappen	Bottas
Driver			
p_{grid}	3	6	1
Tire at start	US	US	US
Pit stops: Inlap / Tire	20 / SUS	14 / SUS	21 / SUS
t_{driver}	0.0 s	0.637 s	0.1 s
Tire			
$k_{2,\mathrm{lin,US}}$	0.020 s/lap	0.018 s/lap	0.020 s/lap
$k_{3,\mathrm{US}}$	0.0 s	0.0 s	0.0 s
$k_{2,\mathrm{lin,SUS}}$	0.013 s/lap	0.036 s/lap	0.020 s/lap
$k_{3,\mathrm{SUS}}$	0.868 s	0.127 s	0.600 s
Car			
t_{car}	0.370 s	0.244 s	0.0 s

TABLE IV

Comparison of the actual and simulated race durations and rank positions at the end of the Abu Dhabi Grand Prix. If Ricciardo is taken out of the score due to his retirement, the rank positions match and are therefore only listed once.

Driver	Actual dur.	Simulated dur.	Δt_{race}	Position
Ricciardo	retired	5687.008 s	retired	retired (4)
Vettel	5674.498 s	5676.086 s	1.588 s	3
Räikkönen	5700.108 s	5698.952 s	-1.156 s	4 (5)
Verstappen	5701.293 s	5701.962 s	0.669 s	5 (6)
Hamilton	5658.582 s	5654.477 s	-4.105 s	2
Bottas	5652.924 s	5653.077 s	0.153 s	1

Table IV allows comparing the actual and simulated race durations and rank positions at the end of the Abu Dhabi Grand Prix. Both aspects show that the simulation results are a good match for the actual race. The slightly higher difference in Hamilton's race time can be explained by his stopped hunt for Bottas. It should be noted, however, that these absolute value results depend primarily on a proper adjustment of the parameters as soon as the most important effects have been modeled. The mean signed deviations of the simulated lap times against the actual lap times range from -1.06 s to 0.73 s with standard deviations ranging between 0.63 s to 1.09 s for the different drivers. The absolute average for all drivers is 0.47 s for the mean deviations and 0.80 s for the standard deviations.

V. Discussion

The example race shows how the different time dependencies fit together and form a complete race simulation. With the developed tool the results, i.e. race durations and rank positions, can be forecasted based on the supplied pit stop information. A race engineer can use it by running several simulations with different race strategies before a race and selecting the one with the best result for his own driver or team. During a race, he can quickly adapt the strategy to unexpected circumstances by defining the current race situation as a starting point and, building on this, testing various action alternatives. Both aspects help to achieve the

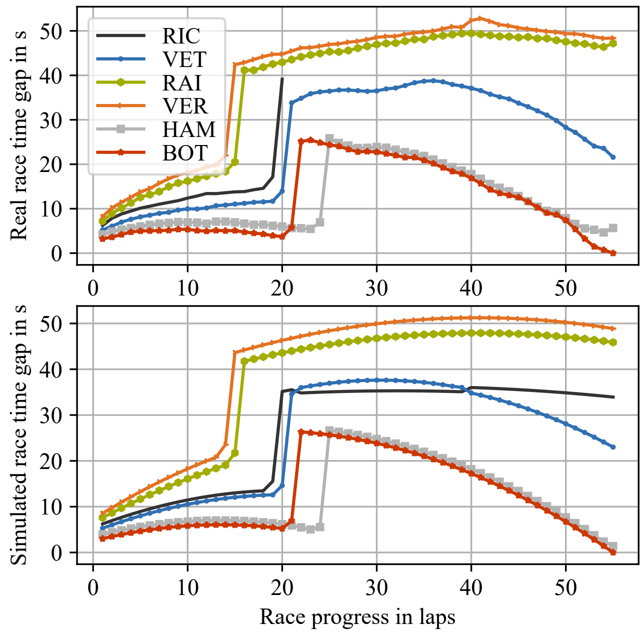

Fig. 7. Actual (upper chart) and simulated (lower chart) race time gaps to a virtual driver with a constant lap time of about 104 s for the 2017 Abu Dhabi Grand Prix. Ricciardo retired in lap 20. Driver abbreviations: RIC – Ricciardo, VET – Vettel, RAI – Räikkönen, VER – Verstappen, HAM – Hamilton, BOT – Bottas.

best possible result. The usage is not limited to racing teams, as the simulation parameters can be determined based on publicly accessible lap time data. The achieved accuracy is sufficient for the application in professional motorsports environments. The points criticized in Bekker's [11] and Phillips' [13] models have been improved. One of the most important enhancements in this context is the tire degradation model, which is individually adjusted to the various compounds, drivers and tracks. Another point to mention is the check of the minimum time gaps between all drivers, which is repeated after pit stops have been completed. Further enhancements are the consideration of the overtaking time loss for both drivers involved in an overtaking maneuver and the implementation of a team order matrix.

There are, however, some limitations in the present state. Firstly, energy consumption (fuel consumption in the context of this paper, which refers to Formula 1) is currently implemented with a constant value per lap. Especially for energy limited racing series such as LMP/WEC and Formula E, it would be beneficial to model this in more detail, e.g. by considering overtaking maneuvers. Secondly, driving strategy is not represented adequately because the driver greatly influences lap time, energy consumption and tire degradation. Thirdly, the simulation does not include a safety car, even though it is often decisive for the race. For this reason, the simulation cannot currently be used for all possible race events. Finally, it should be mentioned that lapping cannot be considered due to the lap-wise discretization. This is because laps have already been calculated in which the car to be overtaken is still on the track when looking at the time axis. However, this does not have much influence because lapping normally takes place without resistance and therefore its influence on the race is negligible.

Future work aims towards the modelling of new aspects, e.g. driving strategy and safety car, as well as detailing existing submodels, e.g. energy consumption and tire degradation. Apart from this, we want to improve the automatic adjustment of parameters based on timing data to relieve the race engineer. We also consider the integration of stochastic elements, as proposed by Bekker [11] and Phillips [13], e.g. for position gains and losses at race start. The final goal is to automatically optimize the race strategy based on the developed race simulation by using the simulation inputs as design variables. The energies available in each lap could also be included as design variables, e.g. to save energy during the race, which can then be used to attack or repel the competitors at the end of the race. The optimization would allow robots to define and adjust their race strategy before and during a race on their own and to react to unforeseen events. Therefore, it could be used for autonomous racing cars in the future, e.g. Roborace [17].

VI. CONCLUSION

In this paper, we presented the methodology of a race simulation that is able to quickly simulate an entire circuit race. It is based on a lap-wise discretization and includes various aspects, such as tire degradation, fuel mass loss, pit stops and overtaking maneuvers. Only publicly accessible lap time data is required as a basis for the simulation parameters. The results shown illustrate the capabilities of the simulation and its suitability to support strategy decisions in motorsports.

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