微积分

- $\lim\limits_{\Delta x \to 0} \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} = f'(x_0) = \text{切线的斜率}$

常见函数的导数

代数，指数和对数函数的导数

$c' = 0$
$(x^n)' = nx^{n-1} \quad (n\text{为自然数})$
$(x^\mu)' = \mu x^{\mu-1} \quad (x>0,\ \mu \neq 0)$
$(e^x)' = e^x$
$(a^x)' = a^x \ln a$
$(\log_a x)' = \frac{1}{x \ln a} \quad (a>0, a\neq 1, x>0)$
$(\ln x)' = \frac{1}{x}$
$(x^x)' = x^x(1+\ln x)$

三角函数的导数

$(\sin x)' = \cos x$
$(\cos x)' = -\sin x$
$(\tan x)' = \sec^2 x$
$(\cot x)' = -\csc^2 x$
$(\sec x)' = \sec x \tan x$
$(\csc x)' = -\csc x \cot x$

反三角函数的导数

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} (|x|<1)$$
$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} (|x|<1)$$
$$(\arctan x)' = \frac{1}{1+x^2}$$
$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$
$$(\operatorname{arcsec} x)' = \frac{1}{|x|\sqrt{x^2-1}} (|x|>1)$$
$$(\operatorname{arccsc} x)' = -\frac{1}{|x|\sqrt{x^2-1}} (|x|>1)$$

双曲函数的导数

$(\sinh x)' = \cosh x = \frac{e^x+e^{-x}}{2}$ $(\operatorname{arsinh} x)' = \frac{1}{\sqrt{x^2+1}}$

$(\cosh x)' = \sinh x = \frac{e^x-e^{-x}}{2}$ $(\operatorname{arcosh} x)' = \frac{1}{\sqrt{x^2-1}}(x > 1)$

$(\tanh x)' = \operatorname{sech}^2 x$ $(\operatorname{artanh} x)' = \frac{1}{1-x^2}(|x| < 1)$

$(\operatorname{sech} x)' = -\tanh x \operatorname{sech} x$ $(\operatorname{arsech} x)' = -\frac{1}{x\sqrt{1-x^2}}(0 < x < 1)$

$(\operatorname{csch} x)' = -\coth x \operatorname{csch} x (x \neq 0)$ $(\operatorname{arcech} x)' = -\frac{1}{x\sqrt{1-x^2}}(0 < x < 1)$

$(\coth x)' = -\operatorname{csch}^2 x (x \neq 0)$ $(\operatorname{arcoth} x)' = \frac{1}{1-x^2}(|x| > 1)$

一般求导法则

线性法则

$[Mf(x)]' = Mf'(x)$
$(u \pm v)' = u' \pm v'$

乘法法则

$(uv)' = u'v + v'u$

除法法则

$(\frac{u}{v})' = \frac{u'v-v'u}{v^2} \quad (v \neq 0)$

倒数法则

$(\frac{1}{v})' = \frac{-v'}{v^2} \quad (v \neq 0)$

复合函数求导法则

$(f \circ g)' = (f' \circ g)\, g'$
$\frac{\mathrm{d}f[g(x)]}{\mathrm{d}x} = \frac{\mathrm{d}f(g)}{\mathrm{d}g}\frac{\mathrm{d}g}{\mathrm{d}x} = f'[g(x)]g'(x)$

反函数的导数

$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{1}{\frac{\mathrm{d}g(f)}{\mathrm{d}f}} = \left[\frac{\mathrm{d}g(f)}{\mathrm{d}f}\right]^{-1} = [g'(f)]^{-1}$

证明：

$\because g(f(x)) = x$，故 $g(f(x))' = 1$，根据复合函数求导法则

$$g(f(x))' = \frac{\mathrm{d}g[f(x)]}{\mathrm{d}x} = \frac{\mathrm{d}g(f)}{\mathrm{d}f}\frac{\mathrm{d}f}{\mathrm{d}x} = 1$$

$$\therefore \frac{\mathrm{d}f}{\mathrm{d}x} = \frac{1}{\frac{\mathrm{d}g(f)}{\mathrm{d}f}} = \left[\frac{\mathrm{d}g(f)}{\mathrm{d}f}\right]^{-1} = [g'(f)]^{-1}$$

同理 $\frac{\mathrm{d}g}{\mathrm{d}x} = \frac{1}{\frac{\mathrm{d}f(g)}{\mathrm{d}g}} = \left[\frac{\mathrm{d}f(g)}{\mathrm{d}g}\right]^{-1} = [f'(g)]^{-1}$