LLS: Single-variable

$$y = M(x|p) = mx + b$$

$$p = (p_0, p_1) = (m, b)$$

$$L(p) = L(m,b) = \sum_{i=1}^{N} \left(\hat{y}_i - M(\hat{x}_i, m, b)\right)^2$$

$$= \sum_{i=1}^{N} (\hat{y}_i - m x_i - b)^2$$

We need to find $\underset{m,b}{\operatorname{argmin}} \sum_{i=1}^{N} (y_i - m x_i - b)^2$

$$\frac{\partial L(m,b)}{\partial m} = -2 \sum_{i=1}^{N} (y_i - m x_i - b) x_i$$

$$= -2 \sum_{i=1}^{N} x_i y_i + 2m \sum_{i=1}^{N} x_i^2 + 2\beta \sum_{i=1}^{n} x_i$$

$$\frac{\partial L(m,b)}{\partial b} = -2 \sum_{i=1}^{n} (y_i - m x_i - b)$$

$$= -2 \sum_{i=1}^{N} y_i + 2m \sum_{i=1}^{n} x_i - 2nb$$

Let $\frac{\partial L(m,b)}{\partial m} = 0$, $\frac{\partial L(m,b)}{\partial b} = 0$

$$\Rightarrow \bar{y} = m\bar{x} + \beta$$

$$\beta = \bar{y} - m\bar{x}$$

$$\Downarrow$$

$$\sum_{i=1}^{N} x_i y_i = m \sum_{i=1}^{N} x_i^2 + \beta \sum_{i=1}^{N} x_i$$

$$\sum_{i=1}^{N} x_i y_i = m \sum_{i=1}^{N} x_i^2 + (\bar{y} - a\bar{x}) \sum_{i=1}^{N} x_i$$

$$m = \frac{\sum_{i=1}^{N} (x_i y_i - \bar{y} x_i)}{\sum_{i=1}^{N} (x_i^2 - \bar{x} x_i)} = \frac{\sum_{i=1}^{N} x_i y_i - \frac{1}{n}\left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} y_i\right)}{\sum_{i=1}^{N} x^2 - \frac{1}{n}\left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} x_i\right)}$$

$$\beta = \bar{y} - \frac{\mathrm{cov}(x,y)}{\mathrm{Var}(x)} \bar{x}$$

$$= \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N} (x_i - \bar{x})^2} = \frac{\mathrm{cov}(x,y)}{\mathrm{Var}(x)}$$