Adv Data Struct & Algorithm: Homework 1 Additional Explanation

Zimian Li

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(a) In class we established that the worst-case complexity of Boston Pool algorithm is $O(n^2)$. For each value of n describe how to construct an instance with at least $2n$ participants (n for each set in the bipartite graph) that elicit this worst case. Make sure to discuss the order in which proposals are made.

Original Answer:
Suppose these two sets are $M = \{m_1, m_2, ..., m_n\}$ and $W = \{w_1, w_2, ..., w_n\}$.
The worst-case scenario could be like the preference list of every m_i is $\{w_1, w_2, ..., w_n\}$, and the preference list of every w_i is $\{m_n, m_{n-1}, ..., m_1\}$. And the proposal order should be from m_1 to m_n.

Explanation:
As I mentioned above, start the Bostion Pool procedure from picking m_1 first, it would be paired with w_1, then m_2, pair it with w_1, m_1 would get single again. Continue it until all done, it's obvious that m_1 attempted n times and at last got paired with w_n, and m_2 attempted $n-1$ times and ended with w_{n-1}, and so forth, until m_n which attempted 1 time and ended with w_1. Finally, the total number of loops is $1 + 2 + 3 + ... + n = (n^2 + n)/2$. That's the worst case.

(g) Consider the following partial preference list of 4 men and 4 women.

Vic:	Amy	Beth	-	-	Amy:	Will	Vic	-	-
Will:	Beth	Amy	-	-	Beth:	Vic	Will	-	-
Xavi:	-	-	Demi	Cora	Cora:	-	-	Xavi	Yan
Yan:	-	-	Cora	Demi	Demi:	-	-	Yan	Xavi

Find a matching that is guaranteed to be stable independent of what the unspecified preferences.

Original Answer:
$\{(Vic, Amy), (Will, Beth), (Xavi, Demi), (Yan, Cora)\}$

Explanation:
Look at the structure of the partial preference list, *Amy* and *Beth* are top 2 in the preference list of Vic and $Will$, on the other hand, Vic and $Will$ are also top 2 in *Amy* and *Beth*'s

preference lists. That means the other 2 men and 2 women can't produce any unstable pairs which can influence the guys in the upper half part of the chart, vice versa. Anyway, the upper half and the lower half are independent which can't influence each other. Then we only need to check if $\{(Vic, Beth), (Will, Amy), (Xavi, Cora), (Yan, Demi)\}$ are unstable pairs. It's obvious that none of them is an unstable pair.