

Sec. 9.3 8.

本题 three steps FFT 是指

$$F_8 = \begin{pmatrix} I_4 & D_4 \\ I_4 & -D_4 \end{pmatrix} \begin{pmatrix} F_4 & 0 \\ 0 & F_4 \end{pmatrix} \begin{pmatrix} \text{奇-偶} \\ \text{置换} \end{pmatrix}$$

P

给定

$$C = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow{P} \begin{pmatrix} 1 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow{\begin{pmatrix} F_4 \\ F_4 \end{pmatrix}} \begin{pmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} I_4 & D_4 \\ I_4 & -D_4 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \\ 0 \end{pmatrix} = F_8 C$$

Sec 9.3 14

给定循环矩阵 $C = \begin{pmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{pmatrix}$

令 $J = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$, 则

$$J^2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad J^3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\Rightarrow C = 2I_4 - J - J^3$$

有两种方法求 C 的特征值

① 讲义方法; ② 求 J 的特征值 λ

则 C 的特征值是 $2 - \lambda - \lambda^3$.

一般情形:

$$\text{设 } A = \begin{pmatrix} a_1 & a_2 & a_3 & \cdots & a_n \\ a_n & a_1 & a_2 & \cdots & a_{n-1} \\ a_{n-1} & a_n & a_1 & \cdots & a_{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_2 & a_3 & a_4 & \cdots & a_1 \end{pmatrix}$$

(n 阶循环矩阵)

令 $a_2 = 1, a_1 = a_3 = \cdots = a_n = 0$, 得

$$J = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{pmatrix} = \begin{pmatrix} 0 & I_{n-1} \\ 1 & 0 \end{pmatrix}$$

$$\text{则 } J^k = \begin{pmatrix} 0 & I_{n-k} \\ I_k & 0 \end{pmatrix}$$

$$A = a_1 I_n + a_2 J + \cdots + a_n J^{n-1}$$

$$\triangleq f(x) = a_1 + a_2 x + a_3 x^2 + \cdots + a_n x^{n-1}$$

$$|\lambda I_n - J| = \lambda^n - 1 \Rightarrow \Lambda \text{ 的特征值:}$$

$$1, w_n = e^{i \frac{2\pi}{n}}, w_n^2, \cdots, w_n^{n-1}$$

$\Rightarrow A$ 的特征值是

$$f(1), f(w_n), f(w_n^2), \cdots, f(w_n^{n-1}).$$

$\triangleq F_n$ 是 n 阶 Fourier 阵, $U_n = \frac{1}{\sqrt{n}} F_n$

$$U_n^H J U_n = \begin{pmatrix} 1 & & \\ & w_n & \\ & & \ddots \\ & & & w_n^{n-1} \end{pmatrix}$$

$$= \Lambda$$

$$U_n^H A U_n = f(\Lambda)$$