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离散数学——第八周作业

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5.4 求下列(1)到(5)的前束范式，(6)到(8)的∃前束范式，(9)(10)的Skolem范式（只含∀）。

(4) $(\neg(\exists x)P(x) \vee (\forall y)Q(y)) \rightarrow (\forall z)R(z)$

(8) $(\forall x)(P(x) \rightarrow Q(x)) \rightarrow ((\exists x)P(x) \rightarrow (\exists x)Q(x))$

(9) $(\forall x)(P(x) \rightarrow (\exists y)Q(x, y)) \vee (\forall z)R(z)$

(10) $(\exists y)(\forall y)(\forall z)(\exists u)(\forall v)P(x, y, z, u, v)$

解 (4)

$$
\begin{aligned}
&(\neg(\exists x)P(x) \vee (\forall y)Q(y)) \rightarrow (\forall z)R(z) \\
&= \neg(\neg(\exists x)P(x) \vee (\forall y)Q(y)) \vee (\forall z)R(z) \\
&= ((\exists x)P(x) \wedge (\exists y)\neg Q(y)) \vee (\forall z)R(z) \\
&= (\exists x)(\exists y)(\forall z)(((P(x) \wedge \neg Q(y)) \vee R(z))
\end{aligned}
$$

(8) 在普遍有效的意义下

$$
\begin{aligned}
&(\forall x)(P(x) \rightarrow Q(x)) \rightarrow ((\exists x)P(x) \rightarrow (\exists x)Q(x)) \\
&= (\forall x)(\neg P(x) \vee Q(x)) \rightarrow (\neg(\exists x)P(x) \vee (\exists x)Q(x)) \\
&= (\exists x)(P(x) \wedge \neg Q(x)) \vee (\forall x)\neg P(x) \vee (\exists x)Q(x) \\
&= (\exists x)(\exists z)(\forall y)((P(x) \wedge \neg Q(x)) \vee \neg P(y) \vee Q(z))
\end{aligned}
$$

(9) 在不可满足的意义下

$$
\begin{aligned}
&(\forall x)(P(x) \rightarrow (\exists y)Q(x, y)) \vee (\forall z)R(z) \\
&= (\forall x)(\exists y)(\forall z)(\neg P(x) \vee Q(x, y) \vee R(z)) \\
&= (\forall x)(\forall z)(\neg P(x) \vee Q(x, f(x)) \vee R(z))
\end{aligned}
$$

(10) 在不可满足的意义下

$$
\begin{aligned}
&(\exists y)(\forall y)(\forall z)(\exists u)(\forall v)P(x, y, z, u, v) \\
&= (\forall x)(\forall z)(\exists u)(\forall v)P(x, a, z, u, v) \\
&= (\forall x)(\forall z)(\forall v)P(x, a, z, f(x, z), v)
\end{aligned}
$$

缺2个归结法 -2

5.5 使用推理规则和归结法作推理演算

(1) $(\forall x)(P(x) \vee Q(x)) \wedge (\forall x)(Q(x) \to \neg R(x)) \Rightarrow (\exists x)(R(x) \to P(x))$

(4) 大学里的学生不是本科生就是研究生，有的学生是高材生，John不是研究生但是高材生，从而如果John是学生必是本科生。

解 (1)

$(a)(\forall x)(P(x) \vee Q(x)) \wedge (\forall x)(Q(x) \to \neg R(x))$	Premise
$(b)(\forall x)(P(x) \vee Q(x))$	Simplification of (a)
$(c)(\forall x)(Q(x) \to \neg R(x))$	Simplification of (a)
$(d)(\forall x)(R(x) \to \neg Q(x))$	Transposition of (c)
$(e)(\forall x)(\neg Q(x) \to P(x))$	Material implication of (b)
$(f)(\forall x)(R(x) \to P(x))$	Hypothetical syllogism of $(d),(e)$
$(g)R(c) \to P(c)$	Elimination of $\forall$ from (f)
$(h)(\exists x)(R(x) \to P(x))$	Introduction of $\exists$ to (g)

(4) $P(x): x$ 是学生，$Q(x): x$ 是本科生，$R(x): x$ 是研究生，$S(x): x$ 是高材生，a : John。

前提：

$$(\forall x)(P(x) \to (Q(x) \wedge \neg R(x)) \vee (\neg Q(x) \wedge R(x)))$$

$$(\exists x)S(x)$$

$$\neg R(a)$$

$$S(a)$$

结论：$P(a) \to Q(a)$

证明：

$(a)(\forall x)(P(x) \to (Q(x) \wedge \neg R(x)) \vee (\neg Q(x) \wedge R(x)))$	Premise
$(b)P(a) \to (Q(a) \wedge \neg R(a)) \vee (\neg Q(a) \wedge R(a))$	Elimination of $\forall$ from (a)
$(c)P(a)$	Assumption
$(d)(Q(a) \wedge \neg R(a)) \vee (\neg Q(a) \wedge R(a))$	Modus ponens of $(b)(c)$
$(e)\neg(R(a) \wedge \neg Q(a)) \to (Q(a) \wedge \neg R(a))$	Material implication of (d)
$(f)\neg R(a)$	Premise
$(g)\neg R(a) \vee Q(a)$	Addition of (g)
$(h)\neg(R(a) \wedge \neg Q(a))$	DeMorgan's Law of (g)
$(i)Q(a) \wedge \neg R(a)$	Modus ponens of $(e)(h)$
$(j)Q(a)$	Simplification of (i)
$(k)P(a) \to Q(a)$	Conditional proof