

离散数学——第八周作业

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- **5.4** 求下列(1)到(5)的前束范式,(6)到(8)的∃前束范式,(9)(10)的Skolem范式(只含∀)。
 - $(4) (\neg(\exists x)P(x) \lor (\forall y)Q(y)) \to (\forall z)R(z)$
 - $(8) (\forall x)(P(x) \to Q(x)) \to ((\exists x)P(x) \to (\exists x)Q(x))$
 - $(9) (\forall x)(P(x) \to (\exists y)Q(x,y)) \lor (\forall z)R(z)$
 - $(10) (\exists y)(\forall y)(\forall z)(\exists u)(\forall v)P(x,y,z,u,v)$

解 (4)

$$(\neg(\exists x)P(x) \lor (\forall y)Q(y)) \to (\forall z)R(z)$$

$$= \neg(\neg(\exists x)P(x) \lor (\forall y)Q(y)) \lor (\forall z)R(z)$$

$$= ((\exists x)P(x) \land (\exists y)\neg Q(y)) \lor (\forall z)R(z)$$

$$= (\exists x)(\exists y)(\forall z)(((P(x) \land \neg Q(y)) \lor R(z))$$

(8) 在普遍有效的意义下

$$(\forall x)(P(x) \to Q(x)) \to ((\exists x)P(x) \to (\exists x)Q(x))$$

$$= (\forall x)(\neg P(x) \lor Q(x)) \to (\neg(\exists x)P(x) \lor (\exists x)Q(x))$$

$$= (\exists x)(P(x) \land \neg Q(x)) \lor (\forall x)\neg P(x) \lor (\exists x)Q(x)$$

$$= (\exists x)(\exists z)(\forall y)((P(x) \land \neg Q(x)) \lor \neg P(y) \lor Q(z))$$

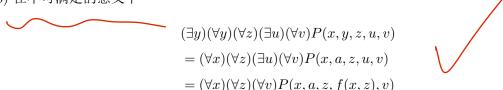
(9) 在不可满足的意义下

$$(\forall x)(P(x) \to (\exists y)Q(x,y)) \lor (\forall z)R(z)$$

$$= (\forall x)(\exists y)(\forall z)(\neg P(x) \lor Q(x,y) \lor R(z))$$

$$= (\forall x)(\forall z)(\neg P(x) \lor Q(x,f(x)) \lor R(z))$$

(10) 在不可满足的意义下



- 5.5 使用推理规则和归结法作推理演算
 - $(1) (\forall x)(P(x) \lor Q(x)) \land (\forall x)(Q(x) \to \neg R(x)) \Rightarrow (\exists x)(R(x) \to P(x))$
- (4) 大学里的学生不是本科生就是研究生,有的学生是高材生,John不是研究生但是高材生,从而如果John是学生必是本科生。

解 (1)

$$(a)(\forall x)(P(x)\vee Q(x))\wedge(\forall x)(Q(x)\to\neg R(x)) \qquad \qquad \text{Premise}$$

$$(b)(\forall x)(P(x)\vee Q(x)) \qquad \qquad \text{Simplification of } (a)$$

$$(c)(\forall x)(Q(x)\to\neg R(x)) \qquad \qquad \text{Simplification of } (a)$$

$$(d)(\forall x)(R(x)\to\neg Q(x)) \qquad \qquad \text{Transposition of } (c)$$

$$(e)(\forall x)(\neg Q(x)\to P(x)) \qquad \qquad \text{Material implication of } (b)$$

$$(f)(\forall x)(R(x)\to P(x)) \qquad \qquad \text{Hypothetical syllogism of } (d), (e)$$

$$(g)R(c)\to P(c) \qquad \qquad \text{Elimination of } \forall \text{ from } (f)$$

$$(h)(\exists x)(R(x)\to P(x)) \qquad \qquad \text{Introduction of } \exists \text{ to } (g)$$

(4) P(x): x 是学生,Q(x): x 是本科生,R(x): x 是研究生,S(x): x 是高材生,a: John。前提:

$$(\forall x)(P(x) \to (Q(x) \land \neg R(x)) \lor (\neg Q(x) \land R(x)))$$

$$(\exists x)S(x)$$

$$\neg R(a)$$

$$S(a)$$

结论: $P(a) \rightarrow Q(a)$ 证明:

 $(k)P(a) \rightarrow Q(a)$

Conditional proof