10.0

离散数学——第七周作业

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2019.10.23

5.1 证明下列等值式和蕴含式

(6) $(\exists x)(P(x) \to Q(x)) = (alx)P(x) \to (\exists x)Q(x)$

(8) $(\exists x)P(x) \wedge (\forall x)Q(x) \Rightarrow (\exists x)(P(x) \vee Q(x))$

(10) $(\exists z)(\exists y)(\exists x)((P(x,z) \to Q(x,z)) \vee ((R(y,z) \to S(y,z))$

$= ((\forall z)(\forall x)P(x,z) \to (\exists z)(\exists x)Q(x,y)) \vee ((\forall z)(\forall y)R(x,y) \to (\exists z)(\exists y)S(y,z))$

证 (6)

$$\begin{aligned}(\exists x)(P(x) \to Q(x)) &= (\exists x)(\neg P(x) \vee Q(x)) \\ &= (\exists x)(\neg P(x)) \vee (\exists x)(Q(x)) \\ &= \neg(\forall x)P(x) \vee (\exists x)Q(x) \\ &= (\forall x)P(x) \to (\exists x)Q(x)\end{aligned}$$

(8)

$$\begin{aligned}(\exists x)P(x) \wedge (\forall x)Q(x) &= (\exists x)P(x) \wedge (\forall y)Q(y) \\ &= (\exists x)(P(x) \wedge (\forall y)Q(y)) \\ &\Rightarrow (\exists x)(P(x) \wedge Q(x))\end{aligned}$$

(10)

$$\begin{aligned}&(\exists z)(\exists y)(\exists x)((P(x,z) \to Q(x,z)) \vee ((R(y,z) \to S(y,z)) \\ &= (\exists z)(\exists y)(\exists x)((P(x,z) \to Q(x,z)) \vee (\exists z)(\exists y)(\exists x)(R(y,z) \to S(y,z)) \\ &= (\exists z)(\exists x)(P(x,z) \to Q(x,z)) \vee (\exists z)(\exists y)(R(y,z) \to S(y,z)) \\ &= (\exists z)(\exists x)(\neg P(x,z) \vee Q(x,z)) \vee (\exists z)(\exists y)(\neg R(y,z) \vee S(y,z)) \\ &= ((\exists z)(\exists x)\neg P(x,z) \vee (\exists z)(\exists x)Q(x,z)) \vee ((\exists z)(\exists y)\neg R(y,z) \vee (\exists z)(\exists y)S(y,z)) \\ &= (\neg(\forall z)(\forall x)P(x,z) \vee (\exists z)(\exists x)Q(x,z)) \vee (\neg(\forall z)(\forall y)R(y,z) \vee (\exists z)(\exists y)S(y,z)) \\ &= ((\forall z)(\forall x)P(x,z) \to (\exists z)(\exists x)Q(x,y)) \vee ((\forall z)(\forall y)R(x,y) \to (\exists z)(\exists y)S(y,z))\end{aligned}$$

5.2 判断下列各公式哪些是普遍有效的并给出证明，不是普遍有效的举出反例。

(3) $((\exists x)P(x) \to (\forall x)Q(x)) \to)(\forall x)P(x) \to (\forall x)Q(x))$

(5) $((\exists x)P(x) \to (\exists x)Q(x)) \to (\exists x)(P(x) \to Q(x))$

(7) $(\exists x)P(x) \land (\exists x)Q(x) \to (\exists x)(P(x) \land Q(x))$

解 (3)

$$\begin{aligned}
&((\exists x)P(x) \to (\forall x)Q(x)) \to (\forall x)(P(x) \to Q(x)) \\
&= (\neg(\exists x)P(x) \lor (\forall x)(Q(x)) \to (\forall x)(P(x) \to Q(x)) \\
&= ((\forall x)\neg P(x) \lor (\forall x)Q(x)) \to (\forall x)(P(x) \to Q(x)) \\
&= ((\forall x)\neg P(x) \lor (\forall x)Q(x)) \to (\forall x)(\neg P(x) \lor Q(x)) \\
&= \mathrm{T}
\end{aligned}$$

则原公式普遍有效。

(5)

$$\begin{aligned}
&((\forall x)P(x) \to (\exists x)Q(x)) \to (\exists x)(P(x) \to Q(x)) \\
&= (\neg(\exists x)P(x) \lor (\exists x)Q(x)) \to (\exists x)(\neg P(x) \lor Q(x)) \\
&= ((\forall x)\neg P(x) \lor (\exists x)Q(x)) \to (\exists x)(\neg P(x) \lor Q(x)) \\
&= ((\exists x)\neg P(x) \to (\exists x)\neg P(x)) \lor (\exists x)Q(x) \\
&= \mathrm{T} \lor (\exists x)Q(x) \\
&= \mathrm{T}
\end{aligned}$$

则原公式普遍有效。

(7) 原公式并非普遍有效。例如在论域$\{1,2\}$上，令$P(1) = Q(2) = \mathrm{T}$, $P(2) = Q(1) = \mathrm{F}$, 则原公式真值为F。