离散数学——第九周作业

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9.1 列出下列集合所有的元素

(4) $A_4 = \{z | z = \{x, y\} \wedge x \in \mathbb{Z} \wedge y \in \mathbb{Z} \wedge 0 \leq x \leq 2 \wedge -2 \leq 2 \leq 1\}$

解 (4) $A_4 = \{\{-2, 0\}, \{-2, 1\}, \{-2, 2\}, \{-1, 0\}, \{-1, 1\}, \{-1, 2\}, \{0\}, \{0, 1\}, \{0, 2\}, \{1\}, \{1, 2\}\}$

9.2 写出下列集合的表达式

(4) $\{3, 5, 7, 11, 13, 17, 19, 23, 29, \cdots\}$

解 (4) $A_4 = \{x | x \in \mathbb{N} \wedge x > 2 \wedge (\forall y)(y \in \mathbb{N} \wedge y > 1 \rightarrow (\forall z)(z \in \mathbb{N} \wedge z > 1 \rightarrow yz \neq x))\}$

9.3 给出集合A, B, C的例子，使$A \in B$, $B \in C$但$A \notin C$。

答 $A = \varnothing, B = \{\varnothing\}, C = \{\{\varnothing\}\}$

9.4 给出集合A, B, C的例子，使$A \in B$, $B \in C$且$A \in C$。

答 $A = \varnothing, B = \{\varnothing\}, C = \{\varnothing, \{\varnothing\}\}$

9.6 对任意的集合A, B和C，下列命题是否为真，若真则证明之。若假则举反例。

(2) 若$A \in B$且$B \subseteq C$，则$A \subseteq C$

(4) 若$A \in B$且$B \nsubseteq C$，则$A \notin C$

解 (2) 假。反例：$A = \{\varnothing\}, B = C = \{\{\varnothing\}\}$

(4) 假。反例：$A = \varnothing, B = \{\varnothing\}, C = \{\varnothing, \{\{\varnothing\}\}\}$

9.7 写出下列集合的幂集和笛卡尔积

(1) $\{a, \{a\}\}$的幂集

(3) $\{\varnothing, a, \{b\}\}$的幂集

(5) $P(P(\varnothing)) \times P(P(\varnothing))$

解 (1) $P(\{a,\{a\}\})=\{\varnothing,\{a\},\{\{a\}\},\{a,\{a\}\}\}$

(3) $P(\{\varnothing,a,\{b\}\})=\{\varnothing,\{\varnothing\},\{a\},\{\{b\}\},\{\varnothing,a\},\{\varnothing,\{b\}\},\{a,\{b\}\},\{\varnothing,a,\{b\}\}\}$

(5)

$$\begin{aligned}&P(P(\varnothing))\times P(P(\varnothing))\\&=P(\{\varnothing\})\times P(\{\varnothing\})\\&=\{\varnothing,\{\varnothing\}\}\times\{\varnothing,\{\varnothing\}\}\\&=\{\langle\varnothing,\varnothing\rangle,\langle\varnothing,\{\varnothing\}\rangle,\langle\{\varnothing\},\varnothing\rangle,\langle\{\varnothing\},\{\varnothing\}\rangle\}\end{aligned}$$

9.8 设$B=P(P(P(\varnothing)))$

(1) 是否$\varnothing\in B$? 是否$\varnothing\subseteq B$?

(3) 是否$\{\{\varnothing\}\}\in B$? 是否$\{\{\varnothing\}\}\subseteq B$?

解

$$B=P(P(P(\varnothing)))=P(P(\{\varnothing\}))=P(\{\varnothing,\{\varnothing\}\})=\{\varnothing,\{\varnothing\},\{\{\varnothing\}\},\{\varnothing,\{\varnothing\}\}\}$$

(1) $\varnothing\in B,\varnothing\subseteq B$

(3) $\{\{\varnothing\}\}\in B,\{\{\varnothing\}\}\subseteq B$

缺(2)小问 -0.5

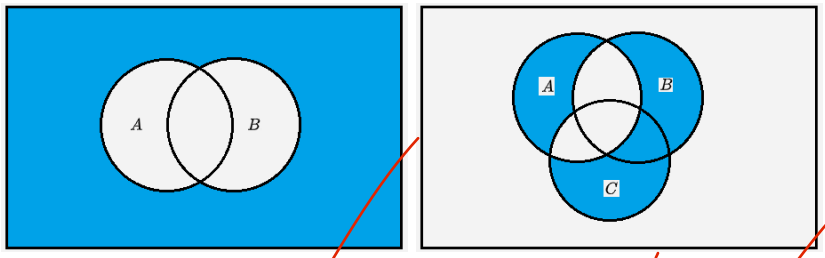
9.9 画出下列集合的文氏图:

(1) $(-A)\cap(-B)$

(3) $A\oplus(B\cup C)$

缺(2)小问 -0.5

解 如图 9.9 所示。



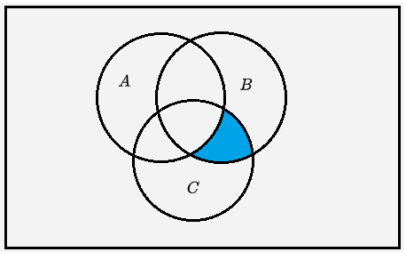
(1) (3)

图 9.9

9.10 用公式表示下列文氏图（题图9.10 (1)）中的集合:

答 (1) $(B\cap C)-A$

缺(2)小问 -0.5



题图 9.10 (1)

9.11 化简下列各式：

(2) $\{\varnothing, \{\varnothing\}\} - \varnothing$

(4) $\{\varnothing, \{\varnothing\}\} - \{\{\varnothing\}\}$

解 (2) $\{\varnothing, \{\varnothing\}\} - \varnothing = \{\varnothing, \{\varnothing\}\}$

(4) $\{\varnothing, \{\varnothing\}\} - \{\{\varnothing\}\} = \{\varnothing\}$

9.12 设全集 $E = \{1, 2, 3, 4, 5\}$，集合$A = \{1, 4\}$, $B = \{1, 2, 5\}$, $C = \{2, 4\}$。求下列集合：

(1) $A \cap -B$

(3) $-(A \cap B)$

(5) $P(A) - P(B)$

解 (1) $A \cap -B = \{1, 4\} \cap \{3, 4\} = \{4\}$

(3) $-(A \wedge B) = -(\{1, 4\} \cap \{1, 2, 5\}) = -\{1\} = \{2, 3, 4, 5\}$

(5)

$$
\begin{aligned}
&P(A) - P(B) \\
&= P(\{1, 4\}) - P(\{1, 2, 5\}) \\
&= \{\varnothing, \{1\}, \{4\}, \{1, 4\}\} - \{\varnothing, \{1\}, \{2\}, \{5\}, \{1, 2\}, \{1, 5\}, \{2, 5\}, \{1, 2, 5\}\} \\
&= \{\{4\}, \{1, 4\}\}
\end{aligned}
$$