$\LaTeX$ Note Template

1 Theorem, Proposition, Proof

Theorem 1.1. *If* $1 < p < \infty$ *and* $m > n/p$, *or* $p = 1$ *and* $m \geq n$, *there exist a constant* $C = C(m, n, \gamma, p)$, *such that*

$$\|R^m u\|_{L^\infty(\Omega)} \leq C d^{m-n/p} |u|_{W_p^m(\Omega)}$$

for all $u \in W_p^m(\Omega)$.

Proof. First, we assume that $u \in C^m(\Omega) \cap W_p^m(\Omega)$. We can use the pointwise representation of $R^m u(x)$.

$$\begin{aligned}
|R^m u(x)| &= m \left| \sum_{|\alpha|=m} \int_{C_x} k_\alpha(x, z) D^\alpha u(z)\, dz \right| \\
&\leq C \sum_{|\alpha|=m} \int_\Omega |x - z|^{-n+m} |D^\alpha u(z)|\, dz \\
&\leq C' d^{m-n/p} |u|_{W_p^m(\Omega)}.
\end{aligned}$$

The proof can be completed via a density argument. □

Theorem 1.2 (xxx)**.** *If* $1 < p < \infty$ *and* $m > n/p$, *or* $p = 1$ *and* $m \geq n$, *there exist a constant* $C = C(m, n, \gamma, p)$, *such that*

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Proposition 1.3.

$$Q^m u(x) = \sum_{|\lambda|<m} \left(\int_B \psi_\lambda(y) u(y)\, dy \right) x^\lambda$$

where $\psi_\lambda \in C_0^\infty(\mathbb{R}^n)$ *and* $\operatorname{supp}(\phi_\lambda) \in \overline{B}$.

Proof. This follows from xxx if we define

$$\psi_\lambda(y) = \sum_{\alpha \geq \lambda, |\alpha|<m} \frac{(-1)^{|\alpha|}}{\alpha!} a_{[\lambda,\alpha-\lambda]} D^\alpha (y^{\alpha-\lambda} \phi(y)).$$

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Proposition 1.4 (xxx)**.**

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2 Corollary, Lemma, Claim

Corollary 2.1. *Under the assumption of xxx, the following inequality holds*

$$\inf_{v\in P^{m-1}} \|u-v\|_{W_p^k(\Omega)} \leq C_{m,n,\gamma} d^{m-k} |u|_{W_p^k(\Omega)}, \;\; k=0,1,\ldots,m,$$

Corollary 2.2 (xxx)**.** *Under the assumption of xxx, the following inequality holds*

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Lemma 2.3. *Let $f \in L^p(\Omega)$ for $p \geq 1$ and $m \geq 1$ and let*

$$g(x) = \int_\Omega |x-z|^{-n+m} |f(z)| \, dz$$

Then

$$\|g\|_{L^p(\Omega)} \leq C_{m,n} d^m \|f\|_{L^p(\Omega)}.$$

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$$\|g\|_{L^p(\Omega)} \leq C_{m,n} d^m \|f\|_{L^p(\Omega)}.$$

Claim 2.5. *$Q^m u$ is a polynomial of degree less than m in x.*

Claim 2.6 (xxx)**.** *$Q^m u$ is a polynomial of degree less than m in x.*

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3 Definition

Definition 3.1. Ω is star-shaped with respect to the ball B if , for all $x \in \Omega$, the closed convex hull of $\{x\} \cup B$ is a subset of Ω.

Definition 3.2 (xxx). Ω is star-shaped with respect to the ball B if , for all $x \in \Omega$, the closed convex hull of $\{x\} \cup B$ is a subset of Ω.

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4 Example

Example 4.1. The integral form of the Taylor remainder for $f \in C^m([0,1])$ is given by

$$f(s) = \sum_{k=0}^{m-1} \frac{1}{k!} f^{(k)}(0) + \int_0^s \frac{1}{(m-1)!} f^{(m)}(t)(s-t)^{m-1}\, dt.$$

Example 4.2 (xxx). The integral form of the Taylor remainder for $f \in C^m([0,1])$ is given by

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5 Problem, Solution

Problem 5.1. Calculate the integral of the function $g(x) = 3x^2$ with respect to x.

Solution 5.1. To calculate the integral of $g(x) = 3x^2$, we use the power rule for integration:

$$\int 3x^2\, dx = x^3 + C$$

where C is the constant of integration. □

Problem 5.2 (xxx). Calculate the integral of the function $g(x) = 3x^2$ with respect to x.

Solution 5.2 (xxx). To calculate the integral of $g(x) = 3x^2$, we use the power rule for integration:

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6 Remark

Remark 6.1. Such a polynomial is not unique, due to the choice od cut-off function ϕ.

Remark 6.2 (xxx). Such a polynomial is not unique, due to the choice od cut-off function ϕ.

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7 Note

Note 7.1. The degree of $Q^m u$ is at most $m - 1$.

Note 7.2 (xxx). The degree of $Q^m u$ is at most $m - 1$.

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8 lstlisting

Listing 1: hello world

```
def hello():
    print("Hello, world!")

hello()
```

Listing 2: hanoi.py

```
step = 1


def hanoi(n, a, b, c, depth=0):
    def move(n, a, c):
        global step
        print("    " * depth, end="")
        print(f"{step=}: move [{n}] from {a} to {c}")
        step += 1

    if n == 1:
        move(n, a, c)
    else:
        hanoi(n - 1, a, c, b, depth=depth + 1)
        move(n, a, c)
        hanoi(n - 1, b, a, c, depth=depth + 1)


if __name__ == "__main__":
    n = int(input("Hanoi Problem, N = "))
    hanoi(n, "A", "B", "C")
```

9 algorithm

Algorithm 1: what

Input: This is some input
Output: This is some output
`/* This is a comment */`

some code here;
$x \leftarrow 0$;
$y \leftarrow 0$;
if $x > 5$ **then**
  x is greater than 5 ; `// This is also a comment`
else
  x is less than or equal to 5;
end
foreach *y in 0..5* **do**
  $y \leftarrow y + 1$;
end
for y *in* 0..5 **do**
  $y \leftarrow y - 1$;
end
while $x > 5$ **do**
  $x \leftarrow x - 1$;
end
return Return something here;