Keys for elementary particle physics

Elementary particle physics addresses two questions: 'What is matter made of at the most fundamental level?' and 'How do they interact with one another?'

Almost all of our experimental information of elementary particles comes from three sources: scatterings, decays, and bound states.

The procedure to build the theories of elementary particles is: guess a form of the theory and compare the resulting theoretical predictions with the experimental data.
Therefore, math, pure field theory, particle phenomenology and experiment are combined to push the progress of particle physics.

In the 1960s and 1970s a theory emerged that described all of the known elementary particle interactions, except gravity. This theory, or, more accurately, this collection of related theories, based on two families of elementary particles (quarks and leptons), and incorporating quantum electrodynamics, the Glashow-Weinberg-Salam theory of electroweak processes, and quantum chromodynamics - has come to be called the Standard Model.

The fundamental interactions in the Standard Model derive from one general principle -- the requirement of local gauge invariance.
Take QED as an example:

So the Lagrangian $\mathcal{L}$ is unchanged under the simultaneous transformation

$$\begin{cases} A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu \Lambda(x) \\ \psi(x) \to \psi'(x) = e^{-iq|e|\Lambda(x)} \psi(x) \end{cases}$$

We can then introduce the covariant derivative

$$D_\mu \equiv \partial_\mu + i q|e| A_\mu$$

$$\Rightarrow \quad \bar\psi i\gamma^\mu D_\mu \psi = \bar\psi i\gamma^\mu(\partial_\mu + iq|e|A_\mu)\psi = \bar\psi i\gamma^\mu \psi - q|e|\bar\psi\gamma^\mu\psi A_\mu$$

$$\Rightarrow \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\psi i\gamma^\mu D_\mu\psi - m\bar\psi\psi.$$

This is the QED Lagrangian, which describes the electromagnetic field and a Dirac field with the Dirac particle (not anti-particle) charge q.

To include, for example, both electron-positron and proton-antiproton, we just write

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\psi_e i\gamma^\mu D_\mu\psi_e - m\bar\psi_e\psi_e + \bar\psi_p i\gamma^\mu D_\mu\psi_p - m\bar\psi_p\psi_p$$

where
$$D_\mu\psi_e = \partial_\mu\psi_e - i|e|A_\mu\psi_e$$
$$D_\mu\psi_p = \partial_\mu\psi_p + i|e|A_\mu\psi_p.$$

Some history about particle physics

1. The question 'What is matter made of?' had a satisfying answer by 1932: it was all just electron, proton, neutron and photon.

electrons (Thomson@1897):
cathode ray deflects in magnetic field, and then apply an electric field to cancel the deflection. The directions of the ray, the magnetic field and electric field perpendicular to each other). The obtained charge-to-mass ratio is much larger than any known ions.

(1) $$E e = e v B \Rightarrow v = \frac{E}{B}$$

(2) $$m\frac{v^2}{R} = e v B \Rightarrow \left(\frac{mv}{R} = eB \Rightarrow R = \frac{mv}{eB} = \frac{mE}{eB^2}\right)$$

$$\Rightarrow \frac{e}{m} = \frac{E}{RB^2}$$

proton (Rutherford@1920):
shoot alpha particle to air (mostly nitrogen), N14 + α → O17 + p.

neutron (Chadwick@1932): solved the puzzle of why alpha particle has two unit of charge but weighs four times, rather than two times, of proton. Neutron is electrically neutral, and it has different penetrating power compared to gamma rays.

photon:
Planck's quanta for black body radiation --> Einstein's explanation of photoelectric effect (but no one like Einstein's idea since people believed that EM radiation should be wave-like)
--> Compton scattering establish Einstein's idea that photon is a particle.

By the way, we now interpret force as 'mediated' by a field. In the case of electrodynamics, the mediator is the photon.
You could think of the mediating particles as 'messengers', and the message can just as well be 'come a little closer' for attraction as 'go away' for repulsion.

2. After these four familiar particles, we come to a next era of particle physics.

Meson:
Why nucleus hold together? Why the protons don't repel each other?
Answer: there must be some other force, more powerful than the force of electrical repulsion, that binds the protons (and neutrons) together --- Yukawa's meson. The short range of the force indicated that the mediator would be rather heavy:
suppose the force is of Yukawa type, i.e., F = coupling/r^2 * exp [- m*r], then from the fact that the range of force is about the size of nucleus, ~ 1 fm, we get the mediator mass m = 1/(1 fm) = 200 MeV.
This particle's mass is smaller than nucleon (i.e., proton and neutron), but larger than electron, so it is called meson (meaning 'middle-weight').

We now know Yukawa's meson is pion.

Yukawa's meson was discovered in cosmic ray observation, where the decay product of charged pion (i.e., muon) was also discovered. Additionally, in the charged pion decay, from conservation of energy, the existence of neutrino can be inferred. Also, from the fact that the energy of the muon is single valued in the decaying pion reference frame, we know that only one neutrino is emitted in the charged pion decay.

By the way, apart from particle accelerators and nuclear reactors, cosmic rays is another source from which we can study particle physics. The advantage is that it is free and the energy can be huge (the highest energy so far recorded is ~ 10^11 GeV), while the disadvantage is that we don't have a controlled experiment since we don't know when and where it will hit the detector.

Positron:
Another achievement of cosmic ray experiment is that Dirac's positron was discovered by Anderson using cloud chamber.

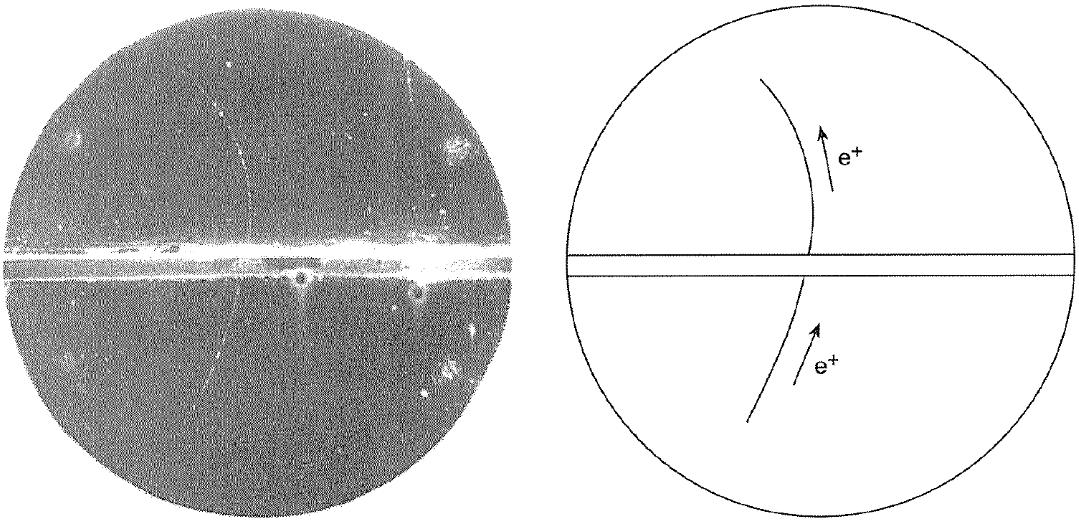


Fig. 1.4 The positron. In 1932, Anderson took this photograph of the track left in a cloud chamber by a cosmic ray particle. The chamber was placed in a magnetic field (pointing into the page), which caused the particle to travel in a curve. But was it a negative charge traveling downward or a positive charge traveling upward? In order to distinguish, Anderson had placed a lead plate across the center of the chamber (the thick horizontal line in the photograph). A particle passing through the plate slows down, and subsequently moves in a tighter circle. By inspection of the curves, it is clear that this particle traveled upward, and hence must have been positively charged. From the curvature of the track and from its texture, Anderson was able to show that the mass of the particle was close to that of the electron. (Photo courtesy California Institute of Technology.)

Neutrino:

(1) Pauli suggested neutrino as a solution to the beta decay problem (i.e., the energy of the electron emitted in beta decay is not single valued).

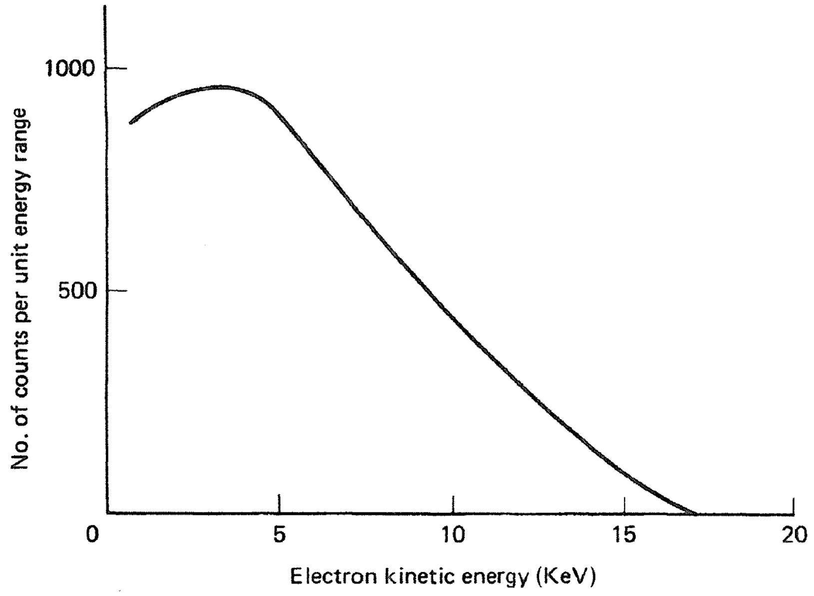


Fig. 1.5 The beta decay spectrum of tritium ($^3_1H \rightarrow ^3_2He$).
(*Source*: Lewis, G. M. (1970) *Neutrinos*, Wykeham, London, p. 30.)

(2) Neutrino was discovered by Cowan and Reines through inverse beta decay experiment: antineutrino + proton --> neutron + positron. The antineutrinos in this experiment came from a nearby nuclear reactor.
(3) Then, from the fact that the reaction 'antineutrino + neutron --> proton + electron' does not happen, while the reaction 'neutrino + neutron --> proton + electron' happens, Davis concluded that neutrino and antineutrino are not the same. This is the evidence of conservation of lepton number.
(4) By conservation of lepton number alone, there is no reason why the reaction 'muon --> electron + photon' cannot happen. This reaction is never observed, and this indicates a separation of electron number and muon number. The separate conservations of electron number and muon number was confirmed by Lederman's experiment, which showed that the reaction 'anti-muon-neutrino + proton --> anti-muon + neutron' happens, while the reaction 'anti-muon-neutrino + proton --> positron+ neutron' does not happen.

By the way, there is also baryon number conservation, otherwise proton decays.

3. Then, a chaos period came --- from cosmic rays and particle accelerators many new mesons and baryons (mesons and baryons together are called hadrons) were discovered.

Many mesons were discovered: kaon, eta, phi, omega, rho, etc..
Many baryons were discovered: Lambda, Delta, Xi, Sigma, etc..

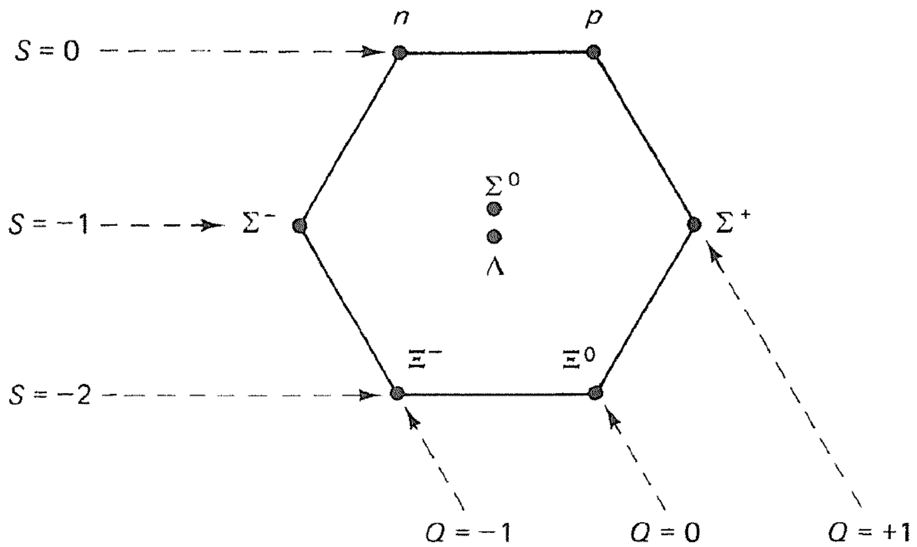
A puzzle: many of the newly discovered hadrons were produced copiously (on a time scale of 10^(-23) seconds), but they decay relatively slowly (on a time scale of 10^(-10) seconds). Why is that so?
Gell-Mann and Nishijima introduced the concept of strangeness to solve the puzzle, and explains which reactions can happen and which cannot.
These strange particles are produced in pairs by strong interaction, while their individual decay is a weak interaction. The former conserve strangeness, while the latter does not.

For example, for the reaction
negative charged pion + proton -> kaon^0 + Lambda,

assign strangeness number +1 for kaon^0 and -1 for Lambda, and strangeness number 0 for pion and proton;
kaon^0 decays into two opposite charged pions,
Lambda decays into proton and negative charged pion.

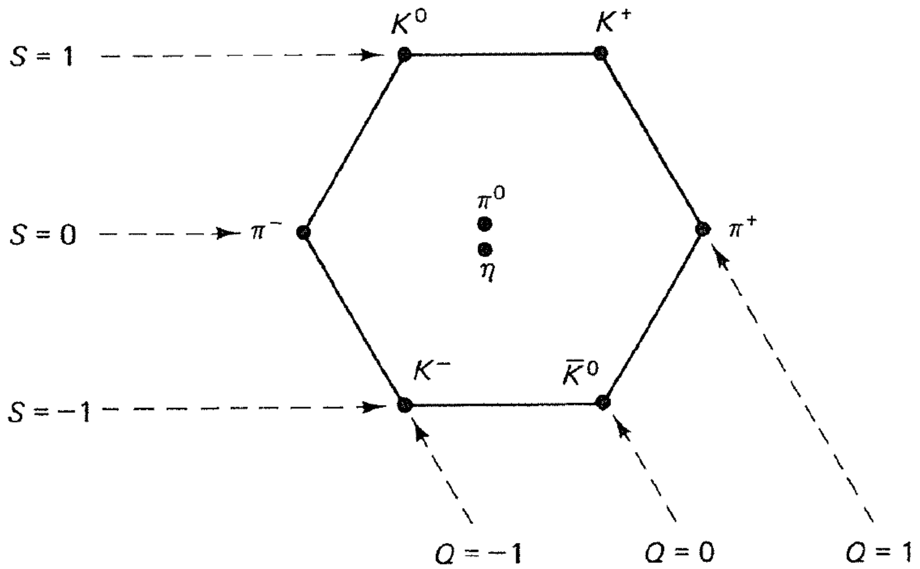
4. The Eightfold-way

It turns out the hadrons form some patterns. Gell-Mann and Ne'eman introduced the Eightfold Way. This is as Mendeleev introduced the periodic table.
The properties of the particles in each of the multiplets have some relations.

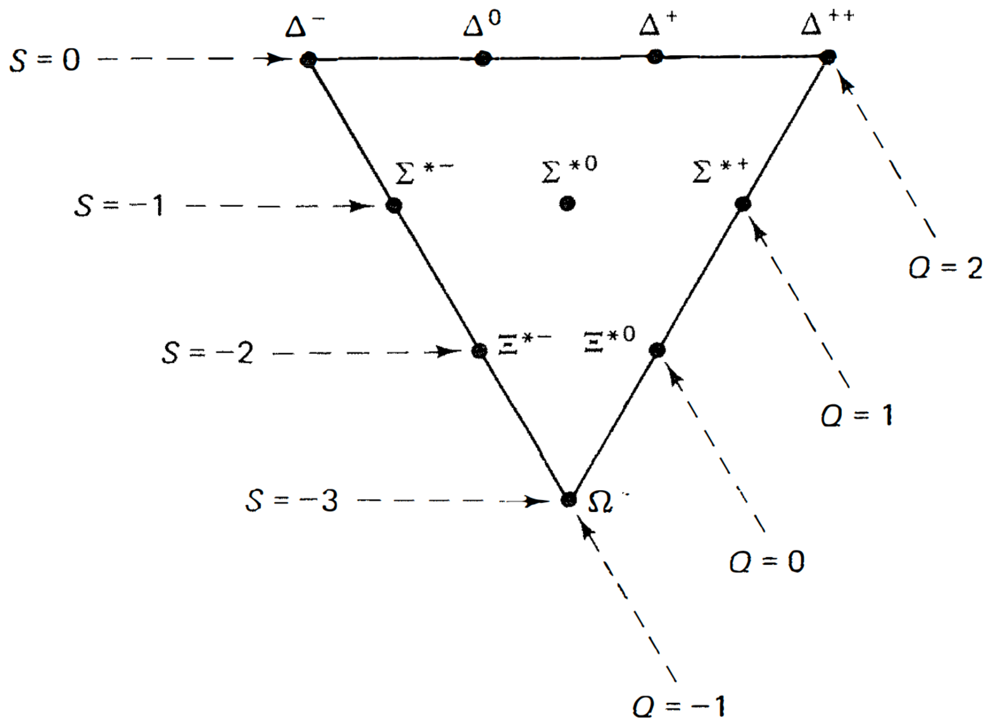


The baryon octet

For the baryon octet, there is the Gell-Mann–Okubo formula: the masses satisfy (approximately)
2 * (nucleon + Xi) = 3 * Lambda + Sigma
(ignoring small differences between p and n; the neutral and charged Sigma; and the neutral and charged Xi).



The meson octet



the baryon decuplet

The average masses of each row have equal spacing (approximately), that is,
Delta – Sigma^star = Sigma^star – Xi^star = Xi^star – Omega

Note that the Omega baryon was first predicted by Gell-Mann (he calculated its mass and lifetime) and then discovered in experiment.

But, where does the Eightfold Way come from?
Gell-Mann and Zweig proposed quark model to explain the Eightfold Way. This is as periodic table is explained by quantum mechanics and the Pauli exclusion principle.

The quark model was confirmed through deep inelastic scattering experiments (similar to Rutherford's experiment to reveal the existence of nucleus in atom, the deep inelastic scattering experiments use, for example, high energy electron to hit proton and revealed that the latter has three lumps inside it).

The baryon decuplet

qqq	Q	S	Baryon
uuu	2	0	Δ^{++}
uud	1	0	Δ^{+}
udd	0	0	Δ^{0}
ddd	-1	0	Δ^{-}
uus	1	-1	Σ^{*+}
uds	0	-1	Σ^{*0}
dds	-1	-1	Σ^{*-}
uss	0	-2	Ξ^{*0}
dss	-1	-2	Ξ^{*-}
sss	-1	-3	Ω^{-}

10 combinations of u, d and s quarks correspond to baryon decuplet. The baryon octet will be explained in later courses.

The meson nonet

$q\bar{q}$	Q	S	Meson
$u\bar{u}$	0	0	π^{0}
$u\bar{d}$	1	0	π^{+}
$d\bar{u}$	-1	0	π^{-}
$d\bar{d}$	0	0	η
$u\bar{s}$	1	1	K^{+}
$d\bar{s}$	0	1	K^{0}
$s\bar{u}$	-1	-1	K^{-}
$s\bar{d}$	0	-1	$\bar{K}^{0}$
$s\bar{s}$	0	0	??

Note that u u^bar, d d^bar and s s^bar all have the same charge and strangeness. Later we will see that the three combinations of them correspond to three mesons.

The quark model suffers one problem: why no one has ever seen an individual quark?
The answer is quark confinement.

The quark model also suffers another problem: three u quarks can form \delta^{++}, isn't it forbidden by Pauli exclusion principle?
The answer is color.

5. All the other fermions in the Standard Model came

The charm quark, bottom quark, tau lepton and tau-neutrino in the 1970s.
The top quark in 1995. Note that unlike other quarks, the top quark is too short lived to form bound state.

6. The W and Z bosons

Originally the weak interaction is described by a contact interaction (the four-fermion interaction). Later, Glashow, Weinberg and Salam set up the electroweak theory, which combine the electromagnetic interaction and weak interaction. The mediators for the weak interaction, i.e., W^+, W^- and Z were discovered in 1983.

7. The Standard Model of particle physics (SM)

The SM has been accepted as the correct theory which describes the elementary particles and their interactions up to the energy scale (~ TeV) so far explored. The last piece of this theory, that is, the long-waited Higgs boson, was discovered in 2012.

Lepton classification

	l	Q	L_e	L_μ	L_r
First generation	e	-1	1	0	0
	ν_e	0	1	0	0
Second generation	μ	-1	0	1	0
	ν_μ	0	0	1	0
Third generation	τ	-1	0	0	1
	ν_τ	0	0	0	1

Quark classification

	q	Q	D	U	S	C	B	T
First generation	d	$-1/3$	-1	0	0	0	0	0
	u	$2/3$	0	1	0	0	0	0
Second generation	s	$-1/3$	0	0	-1	0	0	0
	c	$2/3$	0	0	0	1	0	0
Third generation	b	$-1/3$	0	0	0	0	-1	0
	t	$2/3$	0	0	0	0	0	1

The force mediators are photon, W bosons (80 GeV), Z boson (91 GeV), and gluons.

Quark and lepton masses (in MeV/c^2)

lepton	mass	quark	mass
ν_e	$<2 \times 10^{-6}$	u	2
ν_u	<0.2	d	5
ν_τ	<18	s	100
e	0.511	c	1200
μ	106	b	4200
τ	1777	t	174 000

Possible issues of the SM: why three generations, why the masses of the fermions are these values, etc..

Beyond the SM: Grand Unification Theory (GUT), Supersymmetry (SUSY), String theory, etc..

A sketch of the four interactions

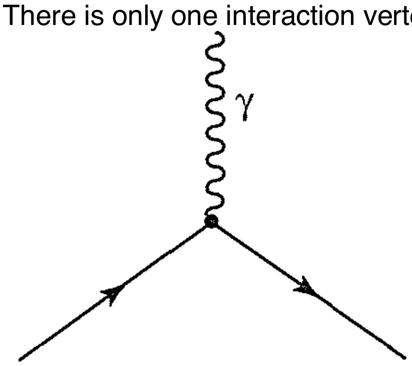
There are both classical and quantum theories for the electromagnetic force.
The classical theory is given by Maxwell's equations, and it is already consistent with special relativity. The quantum theory is quantum electrodynamics (QED), which was perfected by Tomonaga, Feynman and Schwinger.

There is no classical theory for the strong and weak force.
The theories for strong interaction start from Yukawa's theory, and now it is quantum chromodynamics (QCD) that becomes part of the SM.
The theories of the weak interactions start from Fermi's theory, and now it is the GWS (which incorporates QED as well, therefore the GWS theory is also called electroweak theory) that becomes part of the SM.

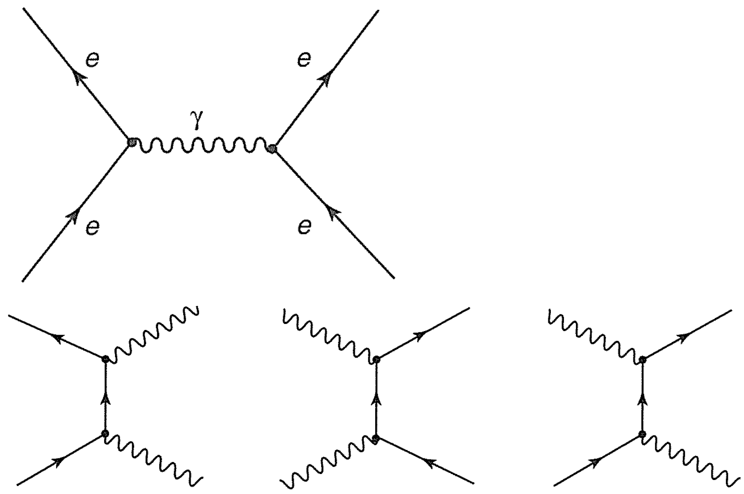
There is no quantum theory for the gravity (at least not confirmed by experiments).

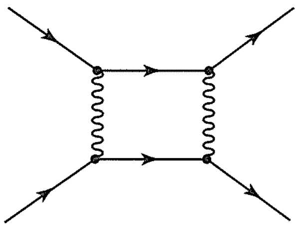
1. QED

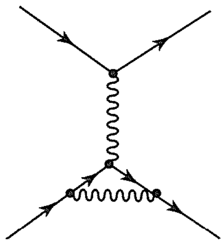
There is only one interaction vertex in this theory:



Some typical Feynman diagrams:



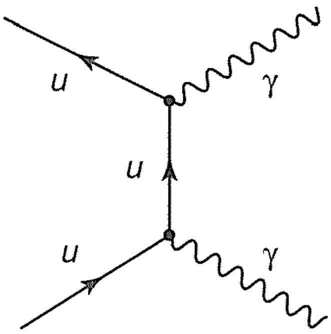




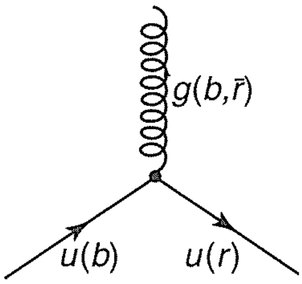
Note that for a given physical process, each Feynman diagram represents a number.

The procedure to calculate a physical process (scattering or decay) is the following: first draw all the needed Feynman diagrams, and then use Feynman rules to do the calculation.
The word “needed” means that evaluate the physical process to the necessary accuracy in order to be able to compare with experimental results.
In particular, each QED vertex contributes a factor of alpha (= 1/137), so usually only the first several orders in perturbation expansions are needed.

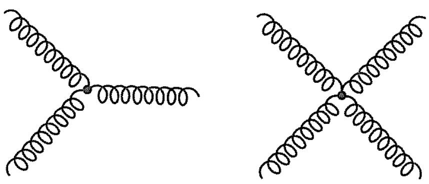
Note that the diagram below could actually represent the decay of pion^0 -> 2 photons, although free quark cannot be seen.



2. QCD
In QCD, color plays the role of charge.
Similar to QED,



Difference compared to QED, gluons have color, so that gluon can interact with gluon.



Another difference comes from the strong interaction coupling, alpha_s (as compared to the QED coupling, i.e., alpha = 1/137).
The alpha_s is large at large distance, for example, at nucleus size. This makes the perturbation expansion fails. On the other hand, at small distance (inside the hadron), alpha_s is small. In

other words, the strong interaction is small at high energy (note that high energy corresponds to small distance). This phenomenon is called asymptotic freedom.

One more difference between QED and QCD is color confinement --- we don't have a free quark or a free gluon, we only have colorless hadrons.

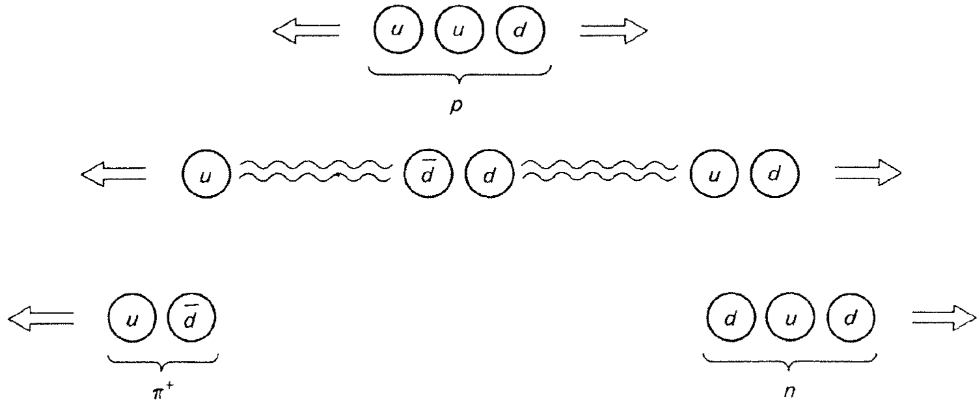
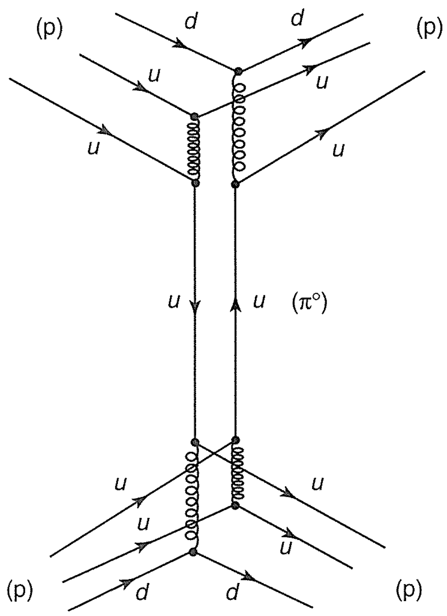


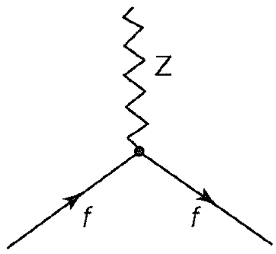
Fig. 2.3 A possible scenario for quark confinement: as we pull a u quark out of the proton, a pair of quarks is created, and instead of a free quark, we are left with a pion and a neutron.

The diagram below can be treated as a realization of Yukawa's theory from QCD.



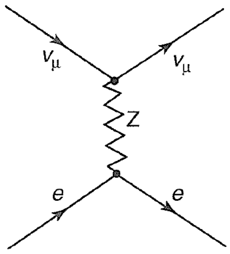
3. Weak Interactions

Here is a fundamental vertex of weak interaction.

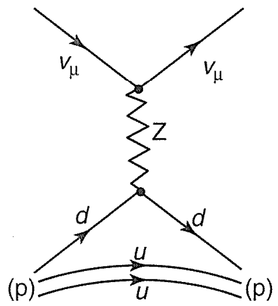


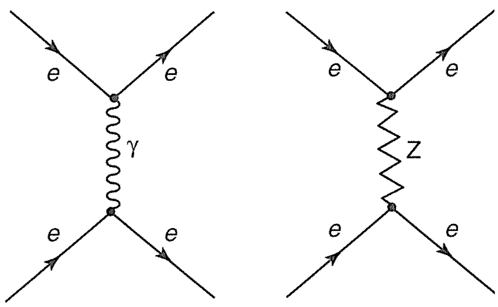
Note that the same type of diagram also appears in QED and QCD.
In fact, the weakness of the weak interactions at relatively low energy is not because of the coupling strength (it is actually larger than the QED fine structure constant), it is due to the large Z and W masses.

A neutrino electron scattering diagram



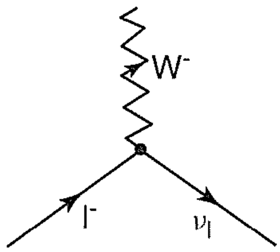
A neutrino proton scattering diagram

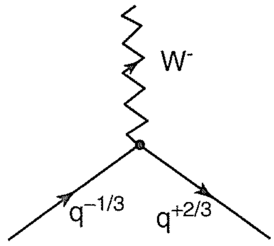




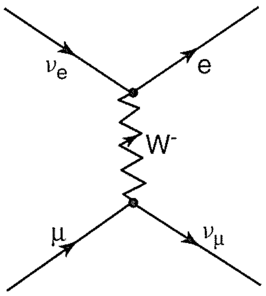
Note that the Z mediated diagram is not just a minor correction to the photon mediated diagram --- the former violates parity conservation.

Here are other fundamental vertices of weak interaction:

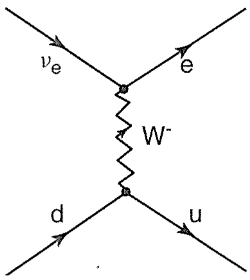


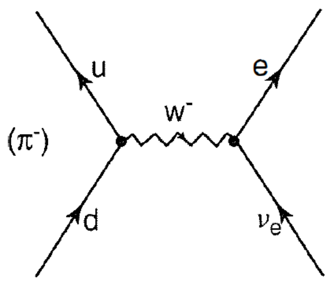


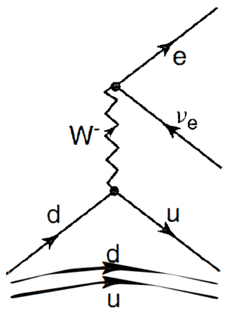
scattering or decay diagrams:



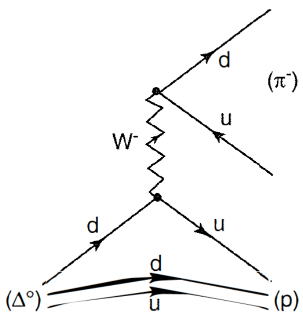
Note that the following can be a charged pion decay diagram, or part of the neutron decay diagram:

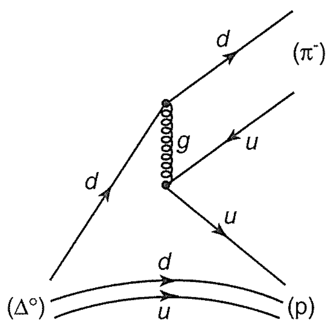




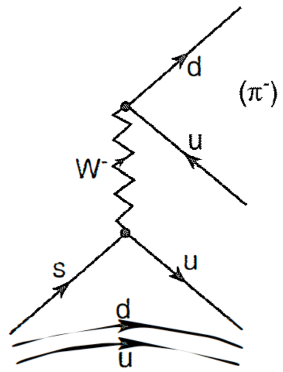


Note that a physical process can have contributions from several interactions.
For example, Delta^0 -> proton + pion^-,



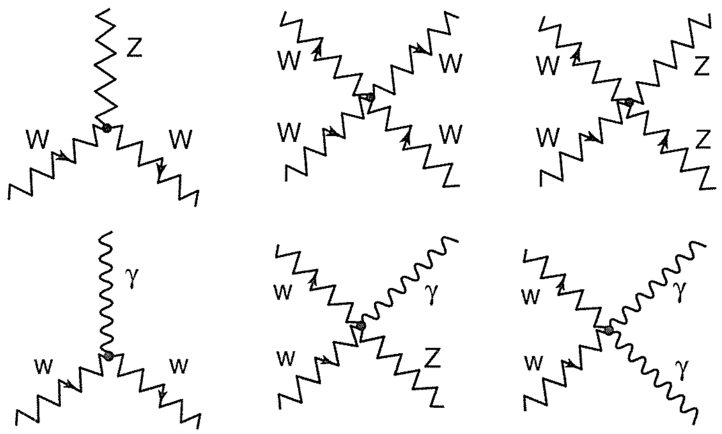


There is one important difference of the W-quark vertex compared to the W-lepton vertex: the former can go across the generations, and this is described by the CKM matrix.
For example, Lambda -> pion + proton



In sum, W boson is the only particle capable of causing a true decay, rather than just a mere repackaging of the quarks or (annihilation) decay of a bound state.
If there is no CKM matrix, the second and third generations of quarks cannot decay into first generation ones.

Finally, the W and Z bosons have pure bosonic vertices:



4. Conservation Law
Energy, momentum and angular momentum conservations are true in all particle physics processes --- a requirement of special relativity.
Electric charge, color, baryon number and lepton number are conserved in SM.
Parity (P) violation, charge conjugation (C) violation and CP violation are built into the GWS theory.

5. Further unification?
The electromagnetic interaction and weak interaction are unified as electroweak theory (i.e., the GWS theory), and the Higgs mechanism is the key in this unification.

Grand Unification Theory (GUT) further unifies the strong and electroweak theory. A hint of this is the possible coupling unification at very high energy scale. This theory needs to be tested by experiments, for example, proton decays.

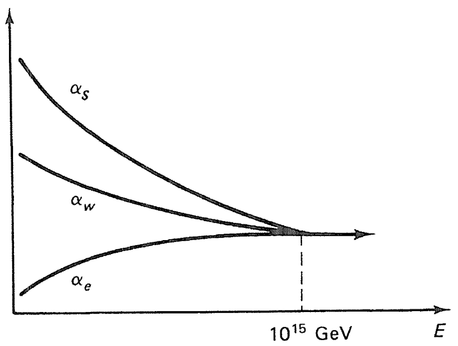


Fig. 2.5 Evolution of the three fundamental coupling constants.

Further unify the gravity with the other three interactions??