Mid-term
Due date: 2019.11.04

Problem 1. List all states (J^{PC}) with total spin $J = 0, 1, 2$ and P, C parities that cannot be realized as a fermion-antifermion system (i.e., as e^+e^- or quark-antiquark). (Hypothetical particles with such combinations of quantum numbers are called exotic, and are being sought for in experiments, so far unsuccessfully). [3 points]

Problem 2. In class, we have constructed the 2×2 matrices of $\hat{S}_x$, $\hat{S}_y$ and $\hat{S}_z$ for spin $\frac{1}{2}$.
1) Do the same for spin 1 and spin $\frac{3}{2}$, i.e., construct the 3×3 matrices of $\hat{S}_x$, $\hat{S}_y$ and $\hat{S}_z$ for spin 1, and 4×4 matrices for spin $\frac{3}{2}$. [2 points]
2) Compute the $\hat{S}^2$ matrices (i.e., $\hat{S}^2 \equiv \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2$) for spin 1 and spin $\frac{3}{2}$. [1 point]
Hint: similar to spin $\frac{1}{2}$, the eigenstates of $\hat{S}_z$ for spin 1 can be chosen as

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

for the eigenvalues $+\hbar, 0, -\hbar$, respectively.

Problem 3.
1) Find the ratio of the cross sections for the following reactions, assuming the center of momentum energy is such that the isospin $I = \frac{3}{2}$ channel dominates: (a) $\pi^- + p \to K^0 + \Sigma^0$; (b) $\pi^- + p \to K^+ + \Sigma^-$; (c) $\pi^+ + p \to K^+ + \Sigma^+$. [2 points]
2) What if the energy is such that the $I = \frac{1}{2}$ channel dominates? [1 point]