Homework final

Due date: 2019.05.08, 5 pm.
Please bring it to the instructor's office (Red House Building 18, Room 101).

Problem 1. Consider the scattering between a Dirac fermion with mass m and a massless real scalar in the theory described by the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - ig\phi\bar{\psi}\gamma^5\psi \ .$$

1) Derive the tree-level $i\mathcal{M}_{fi}$ for this process. **Please show all your steps, pretending that you don't know the Feynman rules. You can do either fermion + scalar $\rightarrow$ fermion + scalar, or, anti-fermion + scalar $\rightarrow$ anti-fermion + scalar.** [15 points]
2) From the above $i\mathcal{M}_{fi}$, compute the unpolarized cross section for this scattering in the center of mass frame. Please give your results in terms of the the Mandelstam variable s, mass and coupling. [10 points]
3) Give the leading result for the above cross section in the ultra-relativistic limit (i.e., $s \gg m^2$) and non-relativistic limit (i.e., $s \rightarrow m^2$). [5 points]

Problem 2. For the same Lagrangian as in problem 1, but adding a mass term for the real scalar field, $-\frac{1}{2}M^2\phi^2$, where $M > 2m$, calculate the two-body decay rate at tree-level, scalar $\rightarrow$ fermion + anti-fermion, in the rest frame of the decaying particle. **You can start your calculation from $i\mathcal{M}_{fi}$, without re-deriving the Feynman rules.** [10 points]