HW1-1 Solution

1) using $\alpha = \frac{1}{137}$, $m_e = 0.5109989 \times 10^{-3}$ GeV

$$\Rightarrow \quad \tau = \frac{2}{m_e \alpha^5} \simeq \boxed{1.9 \times 10^{14}\ \mathrm{GeV}^{-1}}$$

2) using $\hbar = 1.0546 \times 10^{-34}\ \mathrm{J \cdot s} \equiv 1$

and $1\,\mathrm{eV} = 1.6022 \times 10^{-19}\ \mathrm{J}$, $1\,\mathrm{GeV} = 10^9\,\mathrm{eV}$

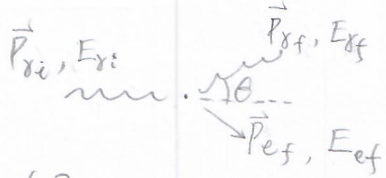
$$\Rightarrow \quad 1\,\mathrm{s} = \frac{1}{1.0546 \times 10^{-34}\ \mathrm{J}} = \frac{1.6022 \times 10^{-19}}{1.0546 \times 10^{-34}\ \mathrm{eV}} = \frac{1.6022 \times 10^{-19} \times 10^9}{1.0546 \times 10^{-34}\ \mathrm{GeV}}$$

$$\Rightarrow \quad \tau = \frac{2}{m_e \alpha^5} = \frac{2}{0.5109989 \times 10^{-3}\,\mathrm{GeV} \left(\frac{1}{137}\right)^5}$$

$$= \frac{2}{0.5109989 \times 10^{-3} \times \left(\frac{1}{137}\right)^5} \times \frac{1.0546 \times 10^{-34}}{1.6022 \times 10^{-19} \times 10^9}\ \mathrm{s}$$

$$\simeq \boxed{1.2 \times 10^{-10}\ \mathrm{s}}$$

HW 1-2 solution




$$(P_{\gamma_f} - P_{\gamma_i})^2 = (P_{ei} - P_{ef})^2$$

$$\Rightarrow \quad 0 + 0 - 2 P_{\gamma_f} \cdot P_{\gamma_i} = m^2 + m^2 - 2 P_{e_i} \cdot P_{ef}$$

in the lab frame

$$\Rightarrow \quad -E_{\gamma_f} E_{\gamma_i} + \vec{P}_{\gamma_f} \cdot \vec{P}_{\gamma_i} = m^2 - m E_{ef}$$

eg; $E_{\gamma_i} = \frac{hc}{\lambda} = \frac{2\pi}{\lambda}$, $\hbar = c = 1$

$$\Rightarrow \quad -\left(\frac{2\pi}{\lambda}\right)\left(\frac{2\pi}{\lambda'}\right)(1 - \cos\theta) = m^2 - m E_{ef}$$

using conservation of energy, $\frac{2\pi}{\lambda} + m = \frac{2\pi}{\lambda'} + E_{ef} \Rightarrow E_{ef} = \frac{2\pi}{\lambda} + m - \frac{2\pi}{\lambda'}$

$$\Rightarrow \quad -\left(\frac{2\pi}{\lambda}\right)\left(\frac{2\pi}{\lambda'}\right)(1 - \cos\theta) = m^2 - m\left(\frac{2\pi}{\lambda} + m - \frac{2\pi}{\lambda'}\right)$$

$$\Rightarrow \quad -\frac{2\pi}{\lambda\lambda'}(1 - \cos\theta) = -m\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$\Rightarrow \quad 2\pi(1 - \cos\theta) = m(\lambda' - \lambda)$$

$$\Rightarrow \quad \boxed{\lambda' = \frac{2\pi(1 - \cos\theta)}{m} + \lambda}$$

HW1-3 solution

$m_e = m_{e^+} = m_{e^-} = 0.51099889 \times 10^{-3}$ GeV,

$m_H = 125.18$ GeV

$m_Z = 91.1876$ GeV.

1) $(P_{e^+} + P_{e^-})^2 = (P_H + P_Z)^2$.

Since both sides are Lorentz invariant quantities, we can evaluate $(P_{e^+} + P_{e^-})^2$ in the lab frame: $(P_{e^+} + P_{e^-})^2 = 2m_e^2 + 2m_e E_{e^+}$

and evaluate $(P_H + P_Z)^2$ in the center-of-mass frame: $(P_H + P_Z)^2 = (E_H + E_Z)^2$

To find the minimum value of $E_{e^+}$, we just need $(E_H + E_Z)^2$ to take its minimum value, which is $(m_H + m_Z)^2$,

$$\Rightarrow \quad 2m_e^2 + 2m_e E_{e^+_{min}} = (m_H + m_Z)^2$$

$$\Rightarrow \quad E_{e^+_{min}} = [(m_H + m_Z)^2 - 2m_e^2]/(2m_e)$$

$$= \frac{(125.18 + 91.1876)^2 - 2(0.51099889 \times 10^{-3})^2}{2 \times 0.51099889 \times 10^{-3}} \text{ GeV}$$

$$\simeq \boxed{4.6 \times 10^7 \text{ GeV.}}$$

2) evaluate $(P_{e^+} + P_{e^-})^2$ in the center-of-mass frame: $(P_{e^+} + P_{e^-})^2 = (E_{e^+} + E_{e^-})^2$

$$\underset{\uparrow}{=} (2E_{e^+})^2$$

Since $m_{e^+} = m_{e^-}$

$|\vec{P}_{e^+}| = |\vec{P}_{e^-}|$

$$\Rightarrow \quad (2E_{e^+_{min}})^2 = (m_H + m_Z)^2$$

$$\Rightarrow \quad E_{e^+_{min}} = \frac{m_H + m_Z}{2} = \frac{125.18 + 91.1876}{2} \text{ GeV} \simeq \boxed{1.1 \times 10^2 \text{ GeV}}$$

3) $P_Z^2 = [(P_{e^+} + P_{e^-}) - P_H]^2$

in the center-of-mass frame,

$$\Rightarrow \quad m_Z^2 = (E_{e^+} + E_{e^-})^2 + m_H^2 - 2E_H(E_{e^+} + E_{e^-})$$

$$\Rightarrow E_H = \frac{(E_{e^+} + E_{e^-})^2 + m_H^2 - m_Z^2}{2(E_{e^+} + E_{e^-})}$$

while $|\vec{P}_H| = (E_H^2 - m_H^2)^{\frac{1}{2}}$

$$|\vec{V}_H| = \frac{|\vec{P}_H|}{E_H}$$

Label the reference frame in which the Higgs is at rest as $S'$, and the center-of-mass frame of the electron and positron as $S$, then

$$\begin{aligned} \Delta x &= \gamma(\Delta x' + |\vec{V}_H| \Delta t') \\ &= \gamma(0 + |\vec{V}_H| \tau) \\ &= \gamma |\vec{V}_H| \tau \end{aligned}$$

using $\tau = \frac{1}{\Gamma}$, $\gamma = (1 - |\vec{V}_H|^2)^{-\frac{1}{2}}$

$$\Rightarrow \Delta x = \frac{|\vec{V}_H|}{(1 - |\vec{V}_H|^2)^{\frac{1}{2}} \Gamma}$$

put in

$$|\vec{V}_H| = \frac{\left\{ \left[ \frac{(250 \times 2)^2 + (125.18)^2 - (91.1876)^2}{2 \times (250 \times 2)} \right]^2 - (125.18)^2 \right\}^{\frac{1}{2}}}{\frac{(250 \times 2)^2 + (125.18)^2 - (91.1876)^2}{2 \times (250 \times 2)}}$$

$$\Gamma = 4.07 \text{ MeV} = 4.07 \times 10^{-3} \text{ GeV}$$

using $C = 2.9979 \times 10^{10} \text{ cm} \cdot s^{-1} \equiv 1$ and $1s = \frac{1.6022 \times 10^{-19} \times 10^9}{1.0546 \times 10^{-34} \text{ GeV}}$

$$\Rightarrow 1 \text{GeV} = \frac{1.6022 \times 10^{-19} \times 10^9}{1.0546 \times 10^{-34} \times 2.9979 \times 10^{10}} \text{ cm}^{-1}$$

$$\Rightarrow \Delta x \simeq \boxed{8.7 \times 10^{-12} \text{ cm}}$$