solution 4-1.

$$\overline{|\mathcal{M}|^2} = \frac{e^4}{4} Tr\left[(\not p_2 + m)\left(\frac{2\gamma^\nu p_1^\mu + \gamma^\nu \not k_1 \gamma^\mu}{2p_1\cdot k_1} - \frac{2\gamma^\mu p_1^\nu - \gamma^\mu \not k_2 \gamma^\nu}{2p_1\cdot k_2} \right)(\not p_1 + m) \right.$$
$$\left. \cdot \left(\frac{2\gamma_\nu p_{1\mu} + \gamma_\mu \not k_1 \gamma_\nu}{2p_1\cdot k_1} - \frac{2\gamma_\mu p_{1\nu} - \gamma_\nu \not k_2 \gamma_\mu}{2p_1\cdot k_2} \right) \right]$$

Since the trace of odd number of γ-matrices is zero, we first pick up only the even terms of γ-matrices in the trace

$$Tr[\quad] = Tr\left[\not p_2 \left(\frac{2\gamma^\nu p_1^\mu + \gamma^\nu \not k_1 \gamma^\mu}{2p_1\cdot k_1} - \frac{2\gamma^\mu p_1^\nu - \gamma^\mu \not k_2 \gamma^\nu}{2p_1\cdot k_2} \right) \not p_1 \right.$$
$$\left. \cdot \left(\frac{2\gamma_\nu p_{1\mu} + \gamma_\mu \not k_1 \gamma_\nu}{2p_1\cdot k_1} - \frac{2\gamma_\mu p_{1\nu} - \gamma_\nu \not k_2 \gamma_\mu}{2p_1\cdot k_2} \right) \right]$$
$$+ m^2 Tr\left[\left(\quad \right)\left(\quad \right) \right]$$

Let's look at the terms in the trace one by one.

$$Tr[\quad] = Tr[\not p_2 (\quad) \not p_1 (\quad)] + m^2 Tr[(\quad)(\quad)]$$
$$= Tr\left[\frac{\not p_2 (2\gamma^\nu p_1^\mu + \gamma^\nu \not k_1 \gamma^\mu) \not p_1 (2\gamma_\nu p_{1\mu} + \gamma_\mu \not k_1 \gamma_\nu)}{(2p_1\cdot k_1)^2} + \frac{\not p_2 (2\gamma^\mu p_1^\nu - \gamma^\mu \not k_2 \gamma^\nu) \not p_1 (2\gamma_\mu p_{1\nu} - \gamma_\nu \not k_2 \gamma_\mu)}{(2p_1\cdot k_2)^2} \right.$$
$$- \frac{1}{2(p_1\cdot k_1)\,2(p_1\cdot k_2)} \Big(\not p_2 (2\gamma^\nu p_1^\mu + \gamma^\nu \not k_1 \gamma^\mu) \not p_1 (2\gamma_\mu p_{1\nu} - \gamma_\nu \not k_2 \gamma_\mu)$$
$$\left. + \not p_2 (2\gamma^\mu p_1^\nu - \gamma^\mu \not k_2 \gamma^\nu) \not p_1 (2\gamma_\nu p_{1\mu} + \gamma_\mu \not k_1 \gamma_\nu) \Big) \right]$$
$$+ m^2 Tr\left[\frac{1}{(2p_1\cdot k_1)^2} (2\gamma^\nu p_1^\mu + \gamma^\nu \not k_1 \gamma^\mu)(2\gamma_\nu p_{1\mu} + \gamma_\mu \not k_1 \gamma_\nu) \right.$$
$$+ \frac{1}{(2p_1\cdot k_2)^2} (2\gamma^\mu p_1^\nu - \gamma^\mu \not k_2 \gamma^\nu)(2\gamma_\mu p_{1\nu} - \gamma_\nu \not k_2 \gamma_\mu)$$
$$- \frac{1}{2(p_1\cdot k_1)\,2(p_1\cdot k_2)} \Big((2\gamma^\nu p_1^\mu + \gamma^\nu \not k_1 \gamma^\mu)(2\gamma_\mu p_{1\nu} - \gamma_\nu \not k_2 \gamma_\mu)$$
$$\left. + (2\gamma^\mu p_1^\nu - \gamma^\mu \not k_2 \gamma^\nu)(2\gamma_\nu p_{1\mu} + \gamma_\mu \not k_1 \gamma_\nu) \Big) \right]$$

use $p_1^{\mu} p_{1\mu} = p_1^2 = m^2$

$$
\begin{aligned}
\Rightarrow Tr[\ \ ] = & \frac{1}{(2p_1\cdot k_1)^2} Tr\Big(4m^2 \not{p}_2 \gamma^\nu \not{p}_1 \gamma_\nu + \not{p}_2 \gamma^\nu \not{k}_1 \gamma^\mu \not{p}_1 \gamma_\mu \not{k}_1 \gamma_\nu \\
& \quad + 2\not{p}_2 \gamma^\nu \not{p}_1 \not{p}_1 \not{k}_1 \gamma_\nu + 2\not{p}_2 \gamma^\nu \not{k}_1 \not{p}_1 \not{p}_1 \gamma_\nu \Big) \\
& + \frac{1}{(2p_1\cdot k_2)^2} Tr\Big(4m^2 \not{p}_2 \gamma^\mu \not{p}_1 \gamma_\mu + \not{p}_2 \gamma^\mu \not{k}_2 \gamma^\nu \not{p}_1 \gamma_\nu \not{k}_2 \gamma_\mu \\
& \quad - 2\not{p}_2 \gamma^\mu \not{p}_1 \not{p}_1 \not{k}_2 \gamma_\mu - 2\not{p}_2 \gamma^\mu \not{k}_2 \not{p}_1 \not{p}_1 \gamma_\mu \Big) \\
& - \frac{1}{2(p_1\cdot k_1)2(p_1\cdot k_2)} Tr\Big(4\not{p}_2 \not{p}_1 \not{p}_1 \not{p}_1 - \not{p}_2 \gamma^\nu \not{k}_1 \gamma^\mu \not{p}_1 \gamma_\nu \not{k}_2 \gamma_\mu \\
& \quad + 2\not{p}_2 \not{p}_1 \not{k}_1 \gamma^\mu \not{p}_1 \gamma_\mu - 2\not{p}_2 \gamma^\nu \not{p}_1 \gamma_\nu \not{k}_2 \not{p}_1 \\
& \quad + 4\not{p}_2 \not{p}_1 \not{p}_1 \not{p}_1 - \not{p}_2 \gamma^\mu \not{k}_2 \gamma^\nu \not{p}_1 \gamma_\mu \not{k}_1 \gamma_\nu \\
& \quad + 2\not{p}_2 \gamma^\mu \not{p}_1 \gamma_\mu \not{k}_1 \not{p}_1 - 2\not{p}_2 \not{p}_1 \not{k}_2 \gamma^\nu \not{p}_1 \gamma_\nu \Big) \\
& + m^2 \frac{1}{(2p_1\cdot k_1)^2} Tr\Big(4m^2 \gamma^\nu \gamma_\nu + \gamma^\nu \not{k}_1 \gamma^\mu \gamma_\mu \not{k}_1 \gamma_\nu \\
& \quad + 2\gamma^\nu \not{p}_1 \not{k}_1 \gamma_\nu + 2\gamma^\nu \not{k}_1 \not{p}_1 \gamma_\nu \Big) \\
& + m^2 \frac{1}{(2p_1\cdot k_2)^2} Tr\Big(4m^2 \gamma^\mu \gamma_\mu + \gamma^\mu \not{k}_2 \gamma^\nu \gamma_\nu \not{k}_2 \gamma_\mu \\
& \quad - 2\gamma^\mu \not{k}_2 \not{p}_1 \gamma_\mu - 2\gamma^\mu \not{p}_1 \not{k}_2 \gamma_\mu \Big) \\
& - m^2 \frac{1}{2(p_1\cdot k_1)2(p_1\cdot k_2)} Tr\Big(4\not{p}_1 \not{p}_1 - \gamma^\nu \not{k}_1 \gamma^\mu \gamma_\nu \not{k}_2 \gamma_\mu \\
& \quad + 2\not{p}_1 \not{k}_1 \gamma^\mu \gamma_\mu - 2\gamma^\nu \gamma_\nu \not{k}_2 \not{p}_1 \\
& \quad + 4\not{p}_1 \not{p}_1 - \gamma^\mu \not{k}_2 \gamma^\nu \gamma_\mu \not{k}_1 \gamma_\nu \\
& \quad + 2\gamma^\mu \gamma_\mu \not{k}_1 \not{p}_1 - 2\not{p}_1 \not{k}_2 \gamma^\nu \gamma_\nu \Big)
\end{aligned}
$$

use $\gamma^\nu \not{p}_1 \gamma_\nu = -2\not{p}_1$, $\quad \gamma^\nu \not{k}_1 \gamma^\mu \not{p}_1 \gamma_\mu \not{k}_1 \gamma_\nu = \gamma^\nu \not{k}_1 (-2\not{p}_1) \not{k}_1 \gamma_\nu$

$\not{p}_1^2 = p_1^2 = m^2$, $\quad \gamma^\nu \not{k}_1 \not{p}_1 \gamma_\nu = 4 k_1 \cdot p_1 \qquad = 4 \not{k}_1 \not{p}_1 \not{k}_1$

$\gamma^\nu \not{k}_1 \gamma^\mu \not{p}_1 \gamma_\nu \not{k}_2 \gamma_\mu = -2 \not{p}_1 \gamma^\mu \not{k}_1 \not{k}_2 \gamma_\mu = -8 k_1 \cdot k_2 \not{p}_1$, and similar identities, $\gamma^\nu \gamma_\nu = 4$, $\not{k}_1^2 = k_1^2 = 0$, $\not{k}_2^2 = k_2^2 = 0$

$$\Rightarrow \mathrm{Tr}[\ ] = \frac{1}{(2p_1\cdot k_1)^2} \mathrm{Tr}\left(-8m^2 \not{p}_2 \not{p}_1 + 4 \not{p}_2 \not{k}_1 \not{p}_1 \not{k}_1 - 4m^2 \not{p}_2 \not{k}_1 - 4m^2 \not{p}_2 \not{k}_1\right)$$

$$+ \frac{1}{(2p_1\cdot k_2)^2} \mathrm{Tr}\left(-8m^2 \not{p}_2 \not{p}_1 + 4 \not{p}_2 \not{k}_2 \not{p}_1 \not{k}_2 + 4m^2 \not{p}_2 \not{k}_2 + 4m^2 \not{p}_2 \not{k}_2\right)$$

$$- \frac{1}{2(p_1\cdot k_1) 2(p_1 \cdot k_2)} \mathrm{Tr}\Big(4m^2 \not{p}_2 \not{p}_1 + 8 k_1\cdot k_2 \not{p}_2 \not{p}_1 - 4 \not{p}_2 \not{p}_1 \not{k}_1 \not{p}_1 + 4 \not{p}_2 \not{p}_1 \not{k}_2 \not{p}_1$$
$$+ 4m^2 \not{p}_2 \not{p}_1 + 8 k_1 \cdot k_2 \not{p}_2 \not{p}_1 - 4 \not{p}_2 \not{p}_1 \not{k}_1 \not{p}_1 + 4 \not{p}_2 \not{p}_1 \not{k}_2 \not{p}_1\Big)$$

$$+ m^2 \frac{1}{(2p_1\cdot k_1)^2} \mathrm{Tr}\left(16m^2 + 0 + 8 p_1 \cdot k_1 + 8 k_1 \cdot p_1\right)$$

$$+ m^2 \frac{1}{(2p_1\cdot k_2)^2} \mathrm{Tr}\left(16m^2 + 0 - 8 k_2 \cdot p_1 - 8 p_1 \cdot k_2\right)$$

$$- m^2 \frac{1}{2(p_1\cdot k_1) 2(p_1\cdot k_2)} \mathrm{Tr}\Big(4m^2 - 4 \not{k}_2 \not{k}_1 + 8 \not{p}_1 \not{k}_1 - 8 \not{k}_2 \not{p}_1$$
$$+ 4m^2 - 4 \not{k}_1 \not{k}_2 + 8 \not{k}_1 \not{p}_1 - 8 \not{p}_1 \not{k}_2\Big)$$

use $Tr[\gamma^\alpha \gamma^\beta] = 4 g^{\alpha\beta}$, $Tr[\gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta] = 4(g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\gamma} g^{\beta\delta} + g^{\alpha\delta} g^{\beta\gamma})$

$Tr[I] = 4$

$$\Rightarrow Tr[\ ] = \frac{1}{(2p_1\cdot k_1)^2}\Big(-32 m^2 p_1\cdot p_2 + 32(p_2\cdot k_1)(p_1\cdot k_1) - 32 m^2 p_2\cdot k_1\Big)$$

$$+ \frac{1}{(2p_1\cdot k_2)^2}\Big(-32 m^2 p_1\cdot p_2 + 32(p_2\cdot k_2)(p_1\cdot k_2) + 32 m^2 p_2\cdot k_2\Big)$$

$$- \frac{1}{2(p_1\cdot k_1)2(p_1\cdot k_2)}\Big[16 m^2 p_1\cdot p_2 + 32(k_1\cdot k_2)(p_1\cdot p_2) - 32(p_2\cdot p_1)(k_1\cdot p_1) + 16 m^2 p_2\cdot k_1 + 32(p_2\cdot p_1)(k_2\cdot p_1) - 16 m^2 p_2\cdot k_2$$
$$+ 16 m^2 p_1\cdot p_2 + 32(k_1\cdot k_2)(p_1\cdot p_2) - 32(p_2\cdot p_1)(k_1\cdot p_1) + 16 m^2 (p_2\cdot k_1) + 32(p_2\cdot p_1)(k_2\cdot p_1) - 16 m^2(p_2\cdot k_2)\Big]$$

$$+ \frac{m^2}{(2p_1\cdot k_1)^2} 4\times(16 m^2 + 16 p_1\cdot k_1)$$

$$+ \frac{m^2}{(2p_1\cdot k_2)^2} 4\times(16 m^2 - 16 p_1\cdot k_2)$$

$$- \frac{m^2}{2(p_1\cdot k_1)2(p_1\cdot k_2)} 4\times\Big(4m^2 - 4k_1\cdot k_2 + 8 p_1\cdot k_1 - 8 k_2\cdot p_1 + 4m^2 - 4k_1\cdot k_2 + 8 k_1\cdot p_1 - 8 p_1\cdot k_2\Big)$$

$$\Rightarrow \mathrm{Tr}[\ \ ] = \frac{1}{(2p_1\cdot k_1)^2}\times 32\left[-m^2 p_1\cdot p_2 + (p_2\cdot k_1)(p_1\cdot k_1) - m^2 p_2\cdot k_1 + 2m^4 + 2m^2 p_1\cdot k_1\right]$$

$$+\frac{1}{(2p_1\cdot k_2)^2}\times 32\left[-m^2 p_1\cdot p_2 + (p_2\cdot k_2)(p_1\cdot k_2) + m^2 p_2\cdot k_2 + 2m^4 - 2m^2 p_1\cdot k_2\right]$$

$$-\frac{1}{2(p_1\cdot k_1)2(p_1\cdot k_2)}\times 32\Big[m^2 p_1\cdot p_2 + 2(k_1\cdot k_2)(p_1\cdot p_2) - 2(p_2\cdot p_1)(k_1\cdot p_1)$$

$$+2(p_2\cdot p_1)(k_2\cdot p_1) + m^2(p_2\cdot k_1) - m^2(p_2\cdot k_2)$$

$$+m^4 - m^2 k_1\cdot k_2 + 2m^2 p_1\cdot k_1 - 2m^2 p_1\cdot k_2\Big]$$

use

$$k_1\cdot k_2 = k_1\cdot(p_1+k_1-p_2) = p_1\cdot k_1 - p_2\cdot k_1 = p_1\cdot k_1 - p_1\cdot k_2$$

$$(p_1+k_1)^2 = (p_2+k_2)^2 \Rightarrow m^2 + 2p_1\cdot k_1 = m^2 + 2p_2\cdot k_2 \Rightarrow p_1\cdot k_1 = p_2\cdot k_2$$

$$p_1\cdot p_2 + p_2\cdot k_1 = p_2\cdot(p_1+k_1) = p_2\cdot(p_2+k_2) = m^2 + p_2\cdot k_2 = m^2 + p_1\cdot k_1$$

$$(p_1-k_2)^2 = (p_2-k_1)^2 \Rightarrow m^2 - 2p_1\cdot k_2 = m^2 - 2p_2\cdot k_1 \Rightarrow p_1\cdot k_2 = p_2\cdot k_1$$

$$p_1\cdot p_2 = p_1\cdot(p_1+k_1-k_2) = m^2 + p_1\cdot k_1 - p_1\cdot k_2$$

$$\Rightarrow \mathrm{Tr}[\ \ ] = \frac{8}{(p_1\cdot k_1)^2}\left[-m^2(m^2+p_1\cdot k_1) + (p_1\cdot k_1)(p_1\cdot k_2) + 2m^4 + 2m^2 p_1\cdot k_1\right]$$

$$+\frac{8}{(p_1\cdot k_2)^2}\Big[-m^2(m^2+p_1\cdot k_1 - p_1\cdot k_2) + (p_1\cdot k_1)(p_1\cdot k_2) + m^2 p_1\cdot k_1$$

$$+2m^4 - 2m^2 p_1\cdot k_2\Big]$$

$$-\frac{8}{(p_1\cdot k_1)(p_1\cdot k_2)}\Big[m^2(m^2+p_1\cdot k_1 - p_1\cdot k_2) + 2(p_1\cdot p_2)\underbrace{\underbrace{(k_1\cdot k_2 - k_1\cdot p_1 + p_1\cdot k_2)}_{k_2\cdot(k_2+p_1)-k_1\cdot p_1}}_{0}$$

$$+m^2 p_1\cdot k_2 - m^2 p_1\cdot k_1 + m^4 - m^2(p_1\cdot k_1 - p_1\cdot k_2)$$

$$+2m^2 p_1\cdot k_1 - 2m^2 p_1\cdot k_2\Big]$$

$$= 8\left\{\frac{m^4}{(p_1\cdot k_1)^2} + \frac{m^2+p_1\cdot k_2}{p_1\cdot k_1} + \frac{m^4}{(p_1\cdot k_2)^2} + \frac{-m^2+p_1\cdot k_1}{p_1\cdot k_2}\right.$$

$$\left. - \frac{2m^4}{(p_1\cdot k_1)(p_1\cdot k_2)} - \frac{-m^2}{(p_1\cdot k_1)} - \frac{m^2}{(p_1\cdot k_2)}\right\}$$

$$\Rightarrow \overline{|\mathcal{M}|^2} = \frac{e^4}{4} \times 8 \left\{ \frac{m^4}{(p_1\cdot k_1)^2} + \frac{m^4}{(p_1\cdot k_2)^2} - \frac{2m^4}{(p_1\cdot k_1)(p_1\cdot k_2)} + \frac{2m^2}{p_1\cdot k_1} - \frac{2m^2}{p_1\cdot k_2} + \frac{p_1\cdot k_2}{p_1\cdot k_1} + \frac{p_1\cdot k_1}{p_1\cdot k_2} \right\}$$

$$= 2e^4 \left[\frac{p_1\cdot k_1}{p_1\cdot k_2} + \frac{p_1\cdot k_2}{p_1\cdot k_1} + 2m^2\left(\frac{1}{p_1\cdot k_1} - \frac{1}{p_1\cdot k_2}\right) + m^4\left(\frac{1}{p_1\cdot k_1} - \frac{1}{p_1\cdot k_2}\right)^2 \right]$$

Done!