Homework final

Due date: 2019.05.08, 5 pm.
Please bring it to the instructor's office (Red House Building 18, Room 101).

Problem 1. Consider the scattering between a Dirac fermion with mass m and a massless real scalar in the theory described by the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - ig\phi\bar{\psi}\gamma^5\psi \ .$$

1) Derive the tree-level $i\mathcal{M}_{fi}$ for this process. **Please show all your steps, pretending that you don't know the Feynman rules. You can do either fermion + scalar $\rightarrow$ fermion + scalar, or, anti-fermion + scalar $\rightarrow$ anti-fermion + scalar.** [15 points]
2) From the above $i\mathcal{M}_{fi}$, compute the unpolarized cross section for this scattering in the center of mass frame. Please give your results in terms of the the Mandelstam variable s, mass and coupling. [10 points]
3) Give the leading result for the above cross section in the ultra-relativistic limit (i.e., $s \gg m^2$) and non-relativistic limit (i.e., $s \rightarrow m^2$). [5 points]

Problem 2. For the same Lagrangian as in problem 1, but adding a mass term for the real scalar field, $-\frac{1}{2}M^2\phi^2$, where $M > 2m$, calculate the two-body decay rate at tree-level, scalar $\rightarrow$ fermion + anti-fermion, in the rest frame of the decaying particle. **You can start your calculation from $i\mathcal{M}_{fi}$, without re-deriving the Feynman rules, if you are confident of your result. Note that the decay rate is the total one, i.e., sum over the spin polarizations of the final state particles.** [10 points]