Homework 3
Due date: 2019.04.15

Problem 1. Particle A, at rest, decays into particles B and C (A $\to$ B + C). Do the following calculations in particle A's rest frame, and give your results in terms of the masses m_A, m_B and m_C.
1) Find the energy of the outgoing particles E_B and E_C. [2 points]
2) Find the magnitudes of the outgoing momenta $|\vec{p}_B|$ and $|\vec{p}_C|$. [2 points]

Problem 2. Consider the theory described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 \ .$$

Compute the leading order (in λ) amplitude, i.e., $i\mathcal{M}_{fi}$, for the $2 \to 2$ scattering (consider only the nontrivial case in which all the particles participate in the scattering).
Note: Please perform the computation with the same degree of details as those done in class. In particular, don't use Feynman rules directly. [4 points]

Problem 3. Consider the theory described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{m_1^2}{2}\sigma^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m_2^2}{2}\phi^2 - g\phi^4 - g'\sigma^2\phi^2 - g''\sigma^4 - \lambda\sigma\phi^2 - \lambda'\sigma^3 \ .$$

Compute the tree-level amplitude, i.e., $i\mathcal{M}_{fi}$, for the process

$$\phi(q_1) + \phi(q_2) \to \phi(q_3) + \phi(q_4) \ .$$

Please give your results in terms of the couplings and the Mandelstam variables, and consider only the nontrivial case in which all the particles participate in the scattering.
Note: Please perform the computation with the same degree of details as those done in class. In particular, don't use Feynman rules directly. [7 points]