Mid-term
Due date: 2019.11.04

Consider the electromagnetism (with no sources) Lagrangian density,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \,, \tag{1}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$.

1) From the translation symmetry, compute the energy-momentum tensor $T^{\mu\nu}$ coming from this Lagrangian density. [4 points]

2) The $T^{\mu\nu}$ obtained in 1) is not symmetric in $\mu \leftrightarrow \nu$. However, we can change it to

$$T^{\mu\nu}_{\text{new}} = T^{\mu\nu} + \partial_\lambda Y^{\lambda\mu\nu} \,, \tag{2}$$

requiring that $Y^{\lambda\mu\nu}$ is antisymmetric in the first two indices. Calculate $\partial_\mu \partial_\lambda Y^{\lambda\mu\nu}$. [1 point]

3) Show that $T^{\mu\nu}_{\text{new}}$ is symmetric in $\mu \leftrightarrow \nu$ for the choice $Y^{\lambda\mu\nu} = F^{\mu\lambda} A^\nu$. **Hint:** you might want to consider using the equation of motion. [3 point]

4) Compute the energy density and momentum density coming from $T^{\mu\nu}_{\text{new}}$ for the $Y^{\lambda\mu\nu}$ given in 3), in terms of the electric field $\vec{E}$ and magnetic field $\vec{B}$. (Recall that $F^{i0} \equiv E^i$ and $F^{ij} \equiv -\epsilon^{ijk} B^k$, where ϵ^{ijk} are the Levi-Civita symbol, with the sign convention $\epsilon^{123} = 1$ and $\epsilon_{123} = 1$.) [5 points]