solution

1) Under a translation, $x^\mu \longrightarrow x'^\mu = x^\mu - \delta a^\mu \Rightarrow \delta x^\mu = -\delta a^\mu$

$$A'^\mu(x') = A^\mu(x) \Rightarrow A'^\mu(x) = A^\mu(x+a)$$

$$\Rightarrow \delta A^\mu(x) = A'^\mu(x) - A^\mu(x) = A^\mu(x+a) - A^\mu(x) = \delta a^\nu \partial_\nu A^\mu(x)$$

let $\delta w = \delta a^\nu$

$$\Rightarrow T^\mu{}_\nu \overset{\downarrow}{=} \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\rho)} \frac{\delta A_\rho}{\delta a^\nu} - \delta^\mu{}_\nu \mathcal{L}$$

$$= \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\rho)} \delta^\sigma{}_\nu \partial_\sigma A_\rho - \delta^\mu{}_\nu \mathcal{L}$$

$$= \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\rho)} \partial_\nu A_\rho - \delta^\mu{}_\nu \mathcal{L}$$

using $\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\rho)} = -\frac{1}{4} \times 2 F^{\alpha\beta} \frac{\partial(\partial_\alpha A_\beta - \partial_\beta A_\alpha)}{\partial(\partial_\mu A_\rho)}$

$$= -\frac{1}{2} F^{\alpha\beta} \left(\delta^\mu_\alpha \delta^\rho_\beta - \delta^\mu_\beta \delta^\rho_\alpha \right)$$

$$= -\frac{1}{2} \left(F^{\mu\rho} - F^{\rho\mu} \right)$$

$$= -F^{\mu\rho}$$

$$\Rightarrow T^\mu{}_\nu = -F^{\mu\rho} \partial_\nu A_\rho - \delta^\mu{}_\nu \mathcal{L}$$

$$\Rightarrow \boxed{T^{\mu\nu} = -F^{\mu\rho} \partial^\nu A_\rho - \eta^{\mu\nu} \mathcal{L}}$$

2) $$T^{\mu\nu} = -F^{\mu\rho} \partial^\nu A_\rho - \eta^{\mu\nu} \mathcal{L}$$

$$= -(\partial^\mu A^\rho - \partial^\rho A^\mu) \partial^\nu A_\rho - \eta^{\mu\nu} \mathcal{L}$$

$$= -(\partial^\mu A^\rho)(\partial^\nu A_\rho) + (\partial^\rho A^\mu)(\partial^\nu A_\rho) - \eta^{\mu\nu} \mathcal{L}$$

The first and third terms are symmetric in $\mu \leftrightarrow \nu$, but the second term does not.

$$\partial_\mu \partial_\lambda Y^{\lambda\mu\nu} = -\partial_\mu \partial_\lambda Y^{\mu\lambda\nu} = -\partial_\lambda \partial_\mu Y^{\lambda\mu\nu} = -\partial_\mu \partial_\lambda Y^{\lambda\mu\nu} \boxed{= 0}$$

3) $\partial_\lambda Y^{\lambda\mu\nu} = \partial_\lambda (F^{\mu\lambda} A^\nu) = \partial_\lambda [(\partial^\mu A^\lambda - \partial^\lambda A^\mu) A^\nu]$

$= (\partial_\lambda A^\nu)(\partial^\mu A^\lambda) - (\partial_\lambda A^\nu)(\partial^\lambda A^\mu) + A^\nu \partial_\lambda F^{\mu\lambda}$

From $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$, the equation of motion is

$$\partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} \right] - \frac{\partial \mathcal{L}}{\partial A_\nu} = 0 \Rightarrow \partial_\mu (-F^{\mu\nu}) - 0 = 0 \Rightarrow \partial_\mu F^{\mu\nu} = 0$$

Therefore, the last term of $\partial_\lambda Y^{\lambda\mu\nu}$ vanishes (Since $\partial_\lambda F^{\mu\lambda} = -\partial_\lambda F^{\lambda\mu} = 0$)

$$\Rightarrow T^{\mu\nu}_{new} = -(\partial^\mu A^\rho)(\partial^\nu A_\rho) + (\partial^\rho A^\mu)(\partial^\nu A_\rho) - \eta^{\mu\nu}\mathcal{L}$$
$$+ (\partial_\lambda A^\nu)(\partial^\mu A^\lambda) - (\partial_\lambda A^\nu)(\partial^\lambda A^\mu)$$

The first, third and fifth terms are symmetric in $\mu \leftrightarrow \nu$ individually, while the second term $\longleftrightarrow$ fourth term in $\mu \leftrightarrow \nu$.

Therefore, $T^{\mu\nu}_{new}$ is symmetric in $\mu \leftrightarrow \nu$

Also, notice that $F^{\mu\lambda} F^\nu{}_\lambda = (\partial^\mu A^\lambda - \partial^\lambda A^\mu)(\partial^\nu A_\lambda - \partial_\lambda A^\nu)$

$= (\partial^\mu A^\lambda)(\partial^\nu A_\lambda) - (\partial^\lambda A^\mu)(\partial^\nu A_\lambda) - (\partial^\mu A^\lambda)(\partial_\lambda A^\nu)$
$+ (\partial^\lambda A^\mu)(\partial_\lambda A^\nu)$

$$\Rightarrow T^{\mu\nu}_{new} = -F^{\mu\lambda} F^\nu{}_\lambda - \eta^{\mu\nu}\mathcal{L}$$

and $T^{\nu\mu}_{new} = -F^{\nu\lambda} F^\mu{}_\lambda - \eta^{\nu\mu}\mathcal{L} = -F^\mu{}_\lambda F^{\nu\lambda} - \eta^{\mu\nu}\mathcal{L}$

$= -F^{\mu\lambda} F^\nu{}_\lambda - \eta^{\mu\nu}\mathcal{L}$

$= T^{\mu\nu}_{new}$

4) The energy density is T^{00}_{new},

$T^{00}_{new} = -F^{0\lambda} F^0{}_\lambda - \eta^{00}\mathcal{L} = -F^{0\lambda} F^0{}_\lambda - \mathcal{L}$

$\underset{\uparrow}{=} -F^{0i} F^0{}_i + \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} = +F^{0i} F^{0i} + \frac{1}{4}(F_{0i} F^{0i} + F_{i0} F^{i0} + F_{ij} F^{ij})$

notice that $F^{\alpha\beta}$ is an antisymmetric tensor

$$= (-E^i)(-E^i) + \frac{1}{4}(-2F^{0i}F^{0i} + F^{ij}F^{ij})$$

$$= \frac{1}{2} E^i E^i + \frac{1}{4}(-\varepsilon^{ijk}B^k)(-\varepsilon^{ijl}B^l)$$

$$= \frac{1}{2}|\vec{E}|^2 + \frac{1}{4}\times 2\delta^{kl} B^k B^l = \boxed{\frac{1}{2}(|\vec{E}|^2 + |\vec{B}|^2)}$$

The momentum densities are T^{0i}_{new} $(i = 1, 2, 3)$

$$T^{0i}_{new} = -F^{0\lambda}F^{i}{}_{\lambda} - \eta^{0i}\mathcal{L} = -F^{0\lambda}F^{i}{}_{\lambda} = -F^{0j}F^{i}{}_{j} = F^{0j}F^{ij}$$

$$= (-E^j)(-\varepsilon^{ijk}B^k) = \varepsilon^{ijk}E^jB^k = \boxed{(\vec{E}\times\vec{B})^i}$$