Homework 1
Due date: 2019.09.25

Problem 1. In $\hbar = c = 1$ units, the lifetime of a positronium in the ground state is given as

$$\tau = \frac{2}{m_e \alpha^5},$$

where m_e is the mass of the electron and α is the fine structure constant. Take $\alpha = 1/137$ in this problem, and look up the electron mass and other constants from, for example, *http://pdg.lbl.gov/*.

1) In $\hbar = c = 1$ units, what is the value of τ in GeV^{-1}? [1 point]
2) The lifetime expression in SI units is $\tau = \frac{2}{m_e \alpha^5} \hbar^{n_1} c^{n_2}$. Find n_1, n_2. [1 point]
3) What is the value of τ in second? [1 point]

NOTE: Please round your results to two significant figures, and don't worry about uncertainties. Also, please note that although we have derived in class the relations between SI units and $\hbar = c = 1$ units, we only kept one or two significant figures at that time. Therefore, if needed, you might want to re-calculate the relations with more significant figures in order to achieve the correct two significant figure results for this problem.

Problem 2. A pion traveling at speed v decays into a muon and a neutrino, $\pi^+ \to \mu^+ + \nu_\mu$. If the neutrino emerges at 90° to the original pion direction, at what angle, relative to the original pion direction, does the muon come off? (Draw the picture, label the angle you are calculating, and give the answer in terms of v, m_μ and m_π. Assume that the neutrino is massless.) [3 points]

Problem 3. Given two four-vectors, $a^\mu = (2, 0, -1, 9)$ and $b^\mu = (0, -9, 2, 5)$, find: a_μ, b_μ, $\vec{a} \cdot \vec{a}$, $\vec{a} \cdot \vec{b}$, a^2, $a \cdot b$, $(a+b)^2$. [2 points]

NOTE: Please use the convention $g_{00} = -g_{11} = -g_{22} = -g_{33} = 1$ & $g_{\mu\nu} = 0$ (when $\mu \neq \nu$) for the calculations. A three-vector, e.g., $\vec{a}$, is the one appearing in $a^\mu = (a^0, \vec{a})$. A four-vector dot product, e.g., $a \cdot b$, is defined as $a \cdot b \equiv a^\mu b_\mu$. In particular, $a^2 \equiv a \cdot a$.

Problem 4. In a two-body scattering process, A + B → C + D, it is convenient to introduce the *Mandelstam variables* $s \equiv (p_A + p_B)^2$, $t \equiv (p_A - p_C)^2$ and $u \equiv (p_A - p_D)^2$ (note that the square of a four-momentum should be always understood as, for example, $a^2 \equiv a \cdot a \equiv a^\mu a_\mu$).

1) Show that $s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$. [1 point]
2) Find the energy of A in the center-of-momentum (CM) frame, in terms of s, m_A and m_B. [2 points]
3) Find the energy of A in B's rest frame, in terms of s, m_A and m_B. [2 points]

Problem 5. In the perfect vacuum, an electron and a positron can annihilate into two or more photons ($e^+ + e^- \to \mathrm{n}\,\gamma$, with $\mathrm{n} \geq 2$). From the view of conservation of energy and/or momentum, please explain why the single photon creation process cannot happen. [1 point]