Are we in a bubble?
A simple time-series-based diagnostic

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Abstract

Time series with bubble-like patterns display an unbalance between growth and acceleration, in the sense that growth in the upswing is "too fast" and then there is a collapse. In fact, such time series show periods where both the first differences (1-L) and the second differences (1-L)^2 of the data are positive-valued, after which period there is a collapse. For a time series without such bubbles, it can be shown that 1-L^2 differenced data should be stable. A simple test based on one-step-ahead forecast errors can now be used to timely monitor whether a series experiences a bubble and also whether a collapse is near. Illustration on simulated data and on two housing prices and the Nikkei index illustrates the practical relevance of the new diagnostic. Monte Carlo simulations indicate that the empirical power of the test is high.

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1. Introduction

There is ample interest in economic bubbles, both in academia and in practice. Usually, economic bubbles are associated with price series (after correction for inflation) showing explosive behavior for a short period, and once the bubble bursts, there is a return to a (much) lower level. Bubbles can occur in almost any price series, and evidence has been documented for stock markets, housing prices, postage stamps, art prices, raw materials and many more. Figure 1 depicts an almost prototypical graph of a time series which displays such a bubble, that is, the Nikkei index, observed annually for 1914-2005, where the bubble burst in 1990.

For analysts it is important to timely diagnose whether a price series is experiencing a bubble-like pattern, and even better, to forecast when the bubble will burst. The latter is of course the Holy Grail, but recent research has shown that time-series-based methods can be informative to diagnose the current state of affairs. Homm and Breitung (2012) evaluate various econometric methods, amongst which the technique developed in Phillips, Wu and Yu (2011), and they rank order their empirical performance using Monte Carlo methods. In this paper I aim to add a new and simple diagnostic for a bubble to the analyst's toolkit.

The new test is based on the notion of the balance between acceleration and growth of a time series, where growth is associated with first differences and acceleration is associated with the first differences of these first differences. Looking again at Figure 1, it is clear that in the years before 1990, the Nikkei data not only showed a positive growth, there also was a long period in which this growth increased. This phenomenon of positive growth and positive acceleration can be coined as positive feedback. Such a feedback drives the data to ever higher levels, mimicking explosive behavior. In contrast, when growth and acceleration are at balance, it can be shown that a specifically transformed time series should be stable and would not show bubble-like patterns. This will be outlined in Section 2. This stability implies that a simple test based on one-step-ahead forecast errors can be used to test and monitor for deviations from stability. The test can even be used to give warning signals for an upcoming collapse. The usefulness of this test will further be illustrated on simulated data and on three actual time series in Section 3. Monte Carlo simulations show that the test has the proper size and also that it can have substantial power. Section 4 concludes with suggestions for further research.

2. The main idea

Consider a price series y_t observed for $t = 1,2\ldots, T$. Explosive behavior, like in Figure 1, entails that for some period the growth of y_t is positive and also that the growth of the growth (to be called: acceleration) is positive. In time series notation, while using the usual lag operator L, this means that $(1\text{-}L)y_t$ and $(1\text{-}L)^2y_t$ both have positive values at the same moment in time. Such positive feedback makes the time series data to explode, until of course the bubble bursts, where after the collapse the data move towards a lower price level. Figure 2 illustrates this positive feedback for an autoregression of order 1 [AR(1)] with parameter ρ equal to 1.10, where a scatter plot of $(1\text{-}L)y_t$ versus $(1\text{-}L)^2y_t$ is presented. Clearly, during the explosive stage, the data quickly move away from the initial cloud of data points. Figure 3 shows that this phenomenon does not occur for stationary data, as the points in the scatter plot are tied towards the cloud of points for an AR(1) with a ρ equal to 0.8.

Hence, for stationary series the two derivative series are at balance, that is, there seems a stable relationship between $(1\text{-}L)y_t$ and $(1\text{-}L)^2y_t$. Such a stable relationship implies that another derivative series does not display bubbles, as will be shown now. Consider an ARMA time series process

$$\varphi(L)y_t = \theta(L)\varepsilon_t$$

where ε_t is a white noise process with mean zero and variance σ_ε^2. The regression line connecting the data of

$$(1-L)y_t$$

with

$$(1-L)(1-L)y_t$$

can be easily seen to reduce to the regression line connecting

$$\varepsilon_t - \varepsilon_{t-1}$$

and

$$\varepsilon_t - 2\varepsilon_{t-1} + \varepsilon_{t-2}$$

as the lag polynomials cancel on both sides. It is easy to derive that the regression coefficient of that line is equal to

$$\frac{\sigma_\varepsilon^2 + 2\sigma_\varepsilon^2}{6\sigma_\varepsilon^2} = 0.5$$

Hence, when growth and acceleration are in balance, one would have that the connected series

$$(1 - L)y_t - 0.5(1 - L)(1 - L)y_t$$
$$= 0.5(1 - L^2)y_t$$

is stable and does not show explosive behavior. Therefore, a simple time-series-based test for the presence of a bubble (and perhaps for a warning of an upcoming collapse) can be based on a regression of the variable $(1 - L^2)y_t$ on an intercept and the associated one-step-ahead forecast errors based on the recursive residuals. This is an option in the Eviews program and is very simple to implement. If lags are needed to take care of residual autocorrelation, these lags can simply be included.

Note that the parameter 0.5 does not have to be estimated as it follows directly from the connection between growth and acceleration. Further, note that a similar connection would exist between y_t and $(1\text{-}L)y_t$, resulting in $0.5(1+L)y_t$. This series however is unlikely to be stable as y_t will most likely have a unit root, and hence a test procedure based on this variable seems less useful. Finally, one could also consider the accumulation of recursive residuals, which in fact would build on the ideas in Homm and Breitung (2012).

3. Illustrations

In this section I illustrate the new diagnostic for two simulated series and for three actual series concerning housing prices and a stock market (Nikkei).

Artificial data

The simulated data were already presented in Figures 2 and 3, and here I consider the Eviews output of the abovementioned regression. Figure 4 displays the recursive forecast errors for the stationary series (ρ = 0.80), and the associated probability, while Figure 5 presents the same tests for the case where ρ = 1.10. From Figure 4 one can see that for 100 artificial observations, only in 6 occasions the p-value is smaller than 0.05. In contrast, Figure 5 shows that from some time onwards the p-values all become exceptionally small.

Three actual time series

The first actual series that I consider is the inflation-corrected price of canal houses in Amsterdam, for the sample 1649 to and including 2004, see Figure 6. Clearly, there have been periods with sharp price increases, most notable around 1975 and again around 2000. Figure 7 shows that for the full sample there is not so much to infer from the scatter plot of growth versus acceleration. However, when only the sample 1950-2004 is considered, as in Figure 8, the collapse of the prices around 1979 is clearly visible. The year 1977 has very large growth and also positive acceleration. Right after 1978 acceleration declines to a negative value. And, already in 1979 both variables take negative values.

Figures 9 and 10 depict the one-step-ahead recursive forecast errors, and clearly towards the end of the sample one can see a range of very small p-values. Figure 10 shows the same graph for the smaller sample, and here it can be seen that in 1977 there is a first worrying signal that there is a bubble, and also 1978 marks a significant deviation from stability. Right after 1978, in 1979, the price series collapsed.

Figure 11 shows the inflation-corrected housing price series for the USA. The bubble around 2005 and the collapse in 2007 are clearly visible. This is amplified in Figure 12 which shows that the price decline in 2007 is preceded by the period 2004-2006 where growth is positive but acceleration changes from positive (2004-2005) to negative in 2006. Figure 13 presents the recursive residuals and it is evident that stability is rejected for 2003, 2004 and 2005, which are clear warning signs of a bubble.

Finally, I return to the Nikkei index, which collapsed in 1990. Figure 14 already marks the years right before the year of the collapse with the same pattern as before with years of positive feedback, followed by a dramatic decline. When analyzing the one-step-ahead forecast errors in Figure 15, it is clear that stability is rejected from 1984 onwards. Hence, signals of a bubble can already be found more than 5 years in advance of the collapse.

4. Monte Carlo simulations

The new and simple test can be run using various statistical packages. Also, no new asymptotic theory is needed, nor new simulated critical values, as the test statistic asymptotically obeys a normal distribution. The further analyze its properties consider the following data generating process

$$y_t = \beta_1 I[t < \tau] + \rho_1 I[t < \tau] y_{t-1} + \beta_2 I[t \geq \tau] + \rho_2 I[t \geq \tau] y_{t-1} + \sigma\varepsilon_t$$

with

$$\varepsilon_t \sim N(0,1)$$

and where $I[.]$ is the usual 1/0 indicator function. Artificial data (10000 runs) are generated for the hypothetical sample 1900-2000. The collapse occurs at time τ. Graphs of the simulated data (unreported) show that when ρ_1 is 1.05, the data already bear similarities with the Nikkei data.

Table 1 presents the empirical size of the test for significance levels 0.1%, 1% and 5%. The first two levels are included as one may want to use a sequence of test values in practice, as is done above. The table presents the empirical rejection frequency under the null hypothesis in case the test date is (i) τ-1, (ii) τ-2 (two years before the collapse), (iii) τ-3, (iv) τ-1 and τ-2, and finally, (v) τ-1, τ-2 and τ-3. Clearly, the numbers in Table 1 show that the test has the proper size.

Table 2 presents the case where there is explosive behavior until 1970 in which year there is a collapse. The degree of explosiveness ranges from 1.01 to 1.10. The empirical power function quickly approaches 1 when nearing 1.10, and of course the power for 1.01 is not large. Table 3 presents similar simulation outcomes, but now for the case where the standard deviation of the error process is much larger (from 500 to 2000). This leads to a loss of power, certainly when the explosiveness is not very far from 1.00. Finally, Table 4 presents the outcomes in case the test sample is reduced from 70 observations to 40. Again, there evidently is loss of power.

Taking Tables 2 to 4 together, it can be learned that the new test can sensibly be used to diagnose the current state of affairs, and also it has predictive content for an upcoming collapse. The empirical examples in Section 3 already indicated that, and the simulation results in these tables seem to support this claim.

5. Conclusion

This paper put forward a simple time-series-based diagnostic that is useful to monitor whether the data show bubble-like behavior and to warn for an upcoming collapse. Three actual cases and extensive Monte Carlo simulations indicated the practical usefulness of the new test. Further applications to a range of time series should provide more experience with this new and useful tool.

Further work can also address a comparison across various methods to test for bubbles that are currently available. And, the new test might also be modified to allow for ARCH like patterns in the data in case one aims to apply it to higher frequency financial data.

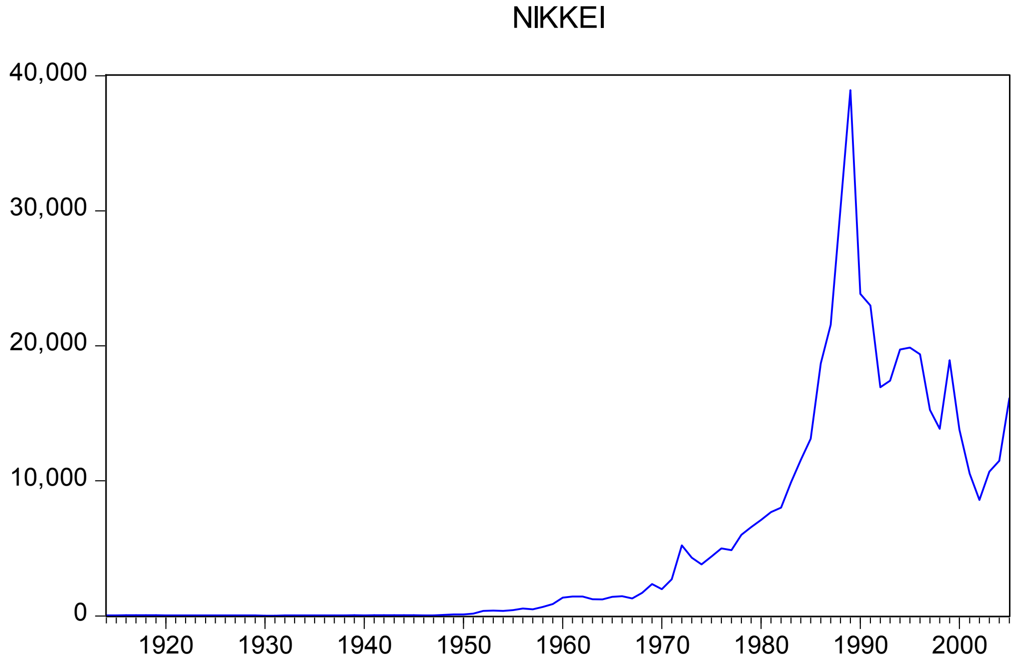

Figure 1: The annual Nikkei index 1914-2005

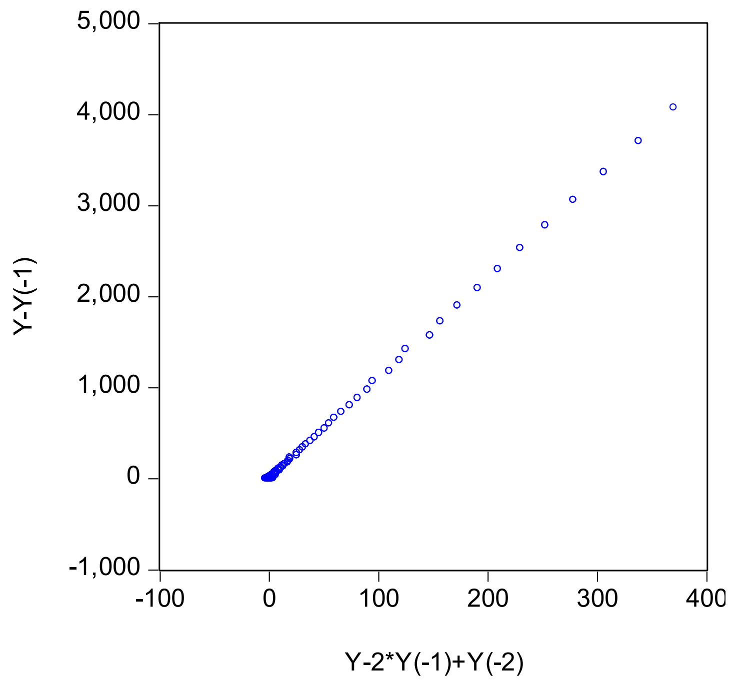

Figure 2: Simulated data for an AR(1) process with $\rho = 1.1$

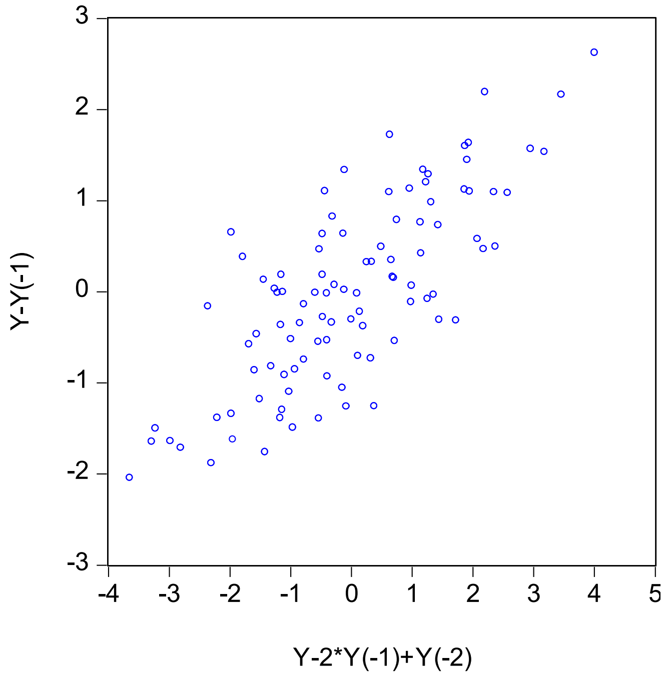

Figure 3: Simulated data for an AR(1) process with $\rho = 0.8$

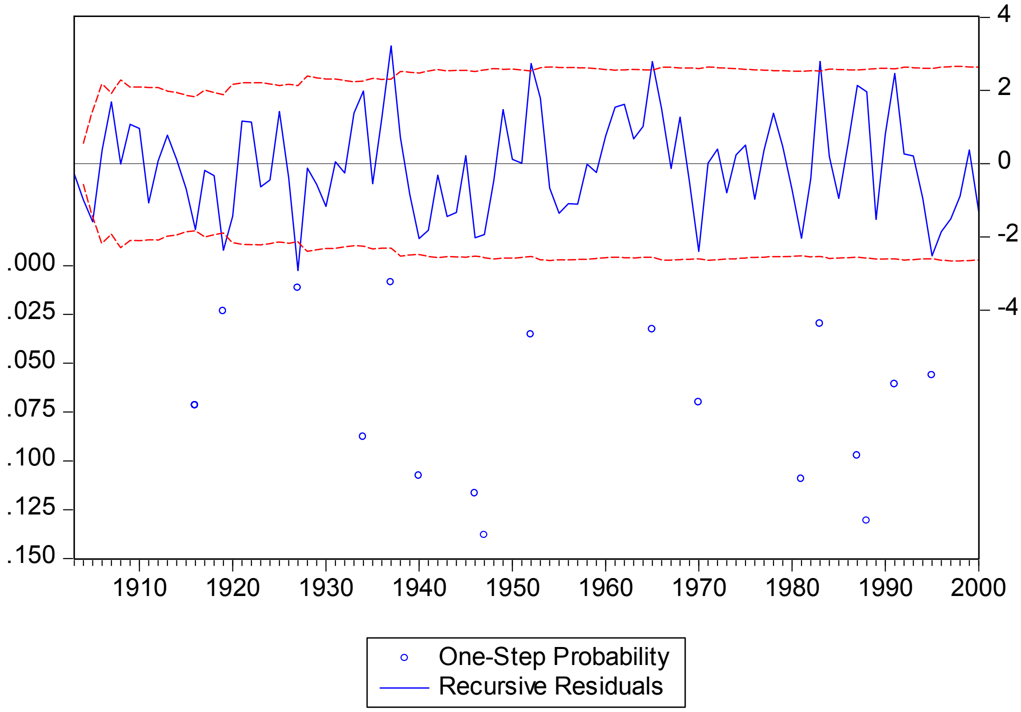

Figure 4: One-step-ahead forecast errors for an AR(1) with ρ = 0.8

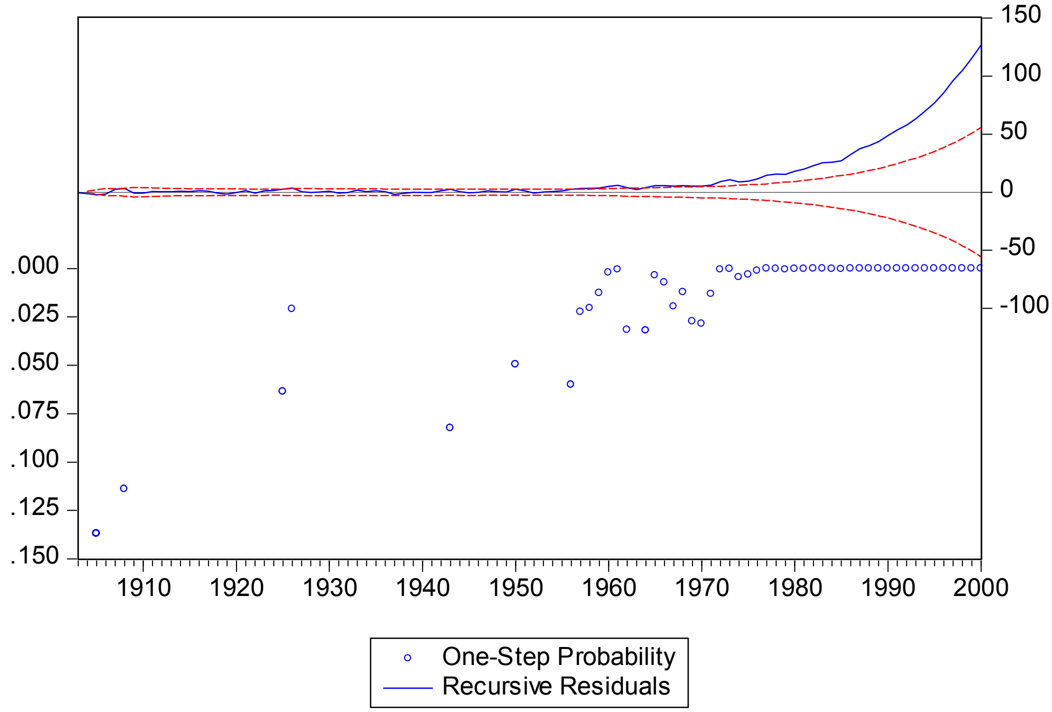

Figure 5: One-step-ahead forecast errors for an AR(1) with ρ = 1.1

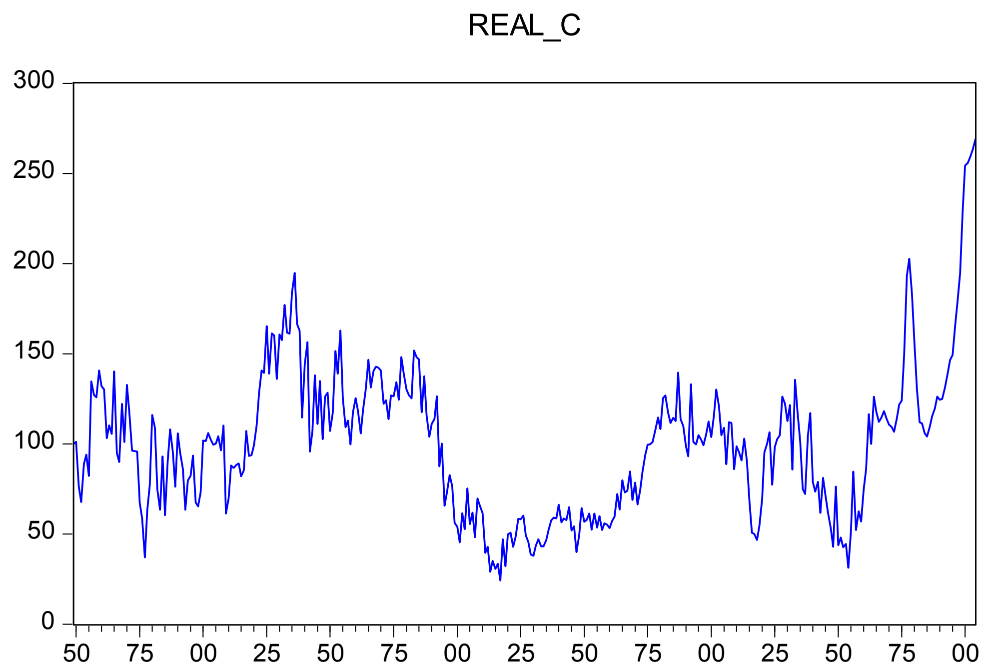

Figure 6: Inflation-corrected prices of canal houses in Amsterdam, after smoothing for two outliers, 1649-2004

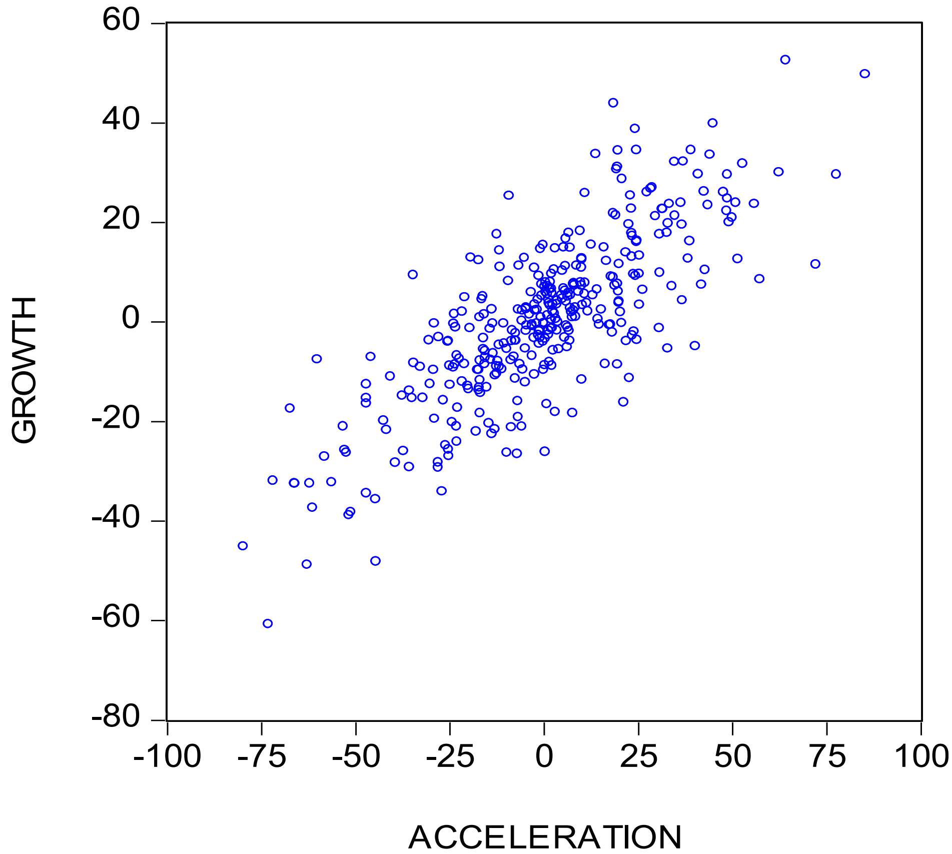

Figure 7: Inflation-corrected prices of canal houses in Amsterdam, growth versus acceleration, all data

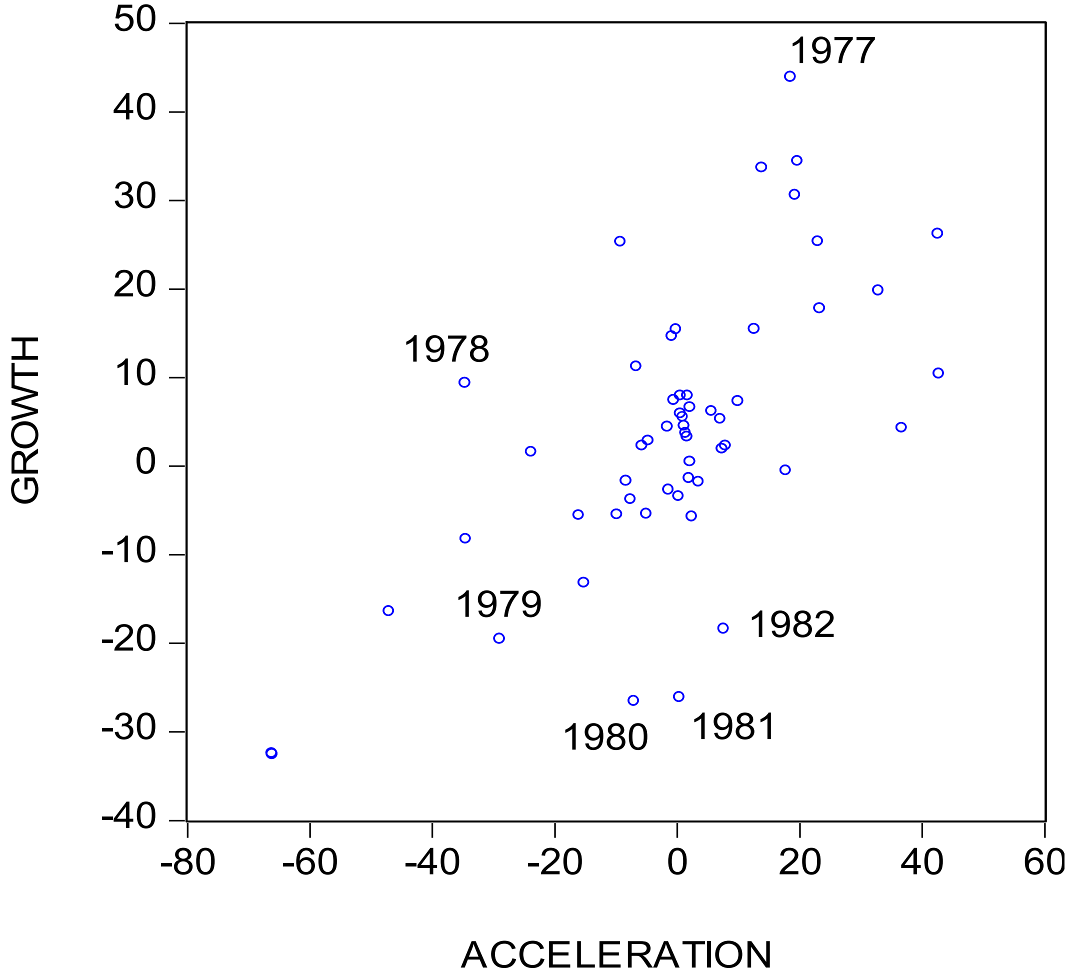

Figure 8: Inflation-corrected prices of canal houses in Amsterdam, growth versus acceleration, 1950-2004

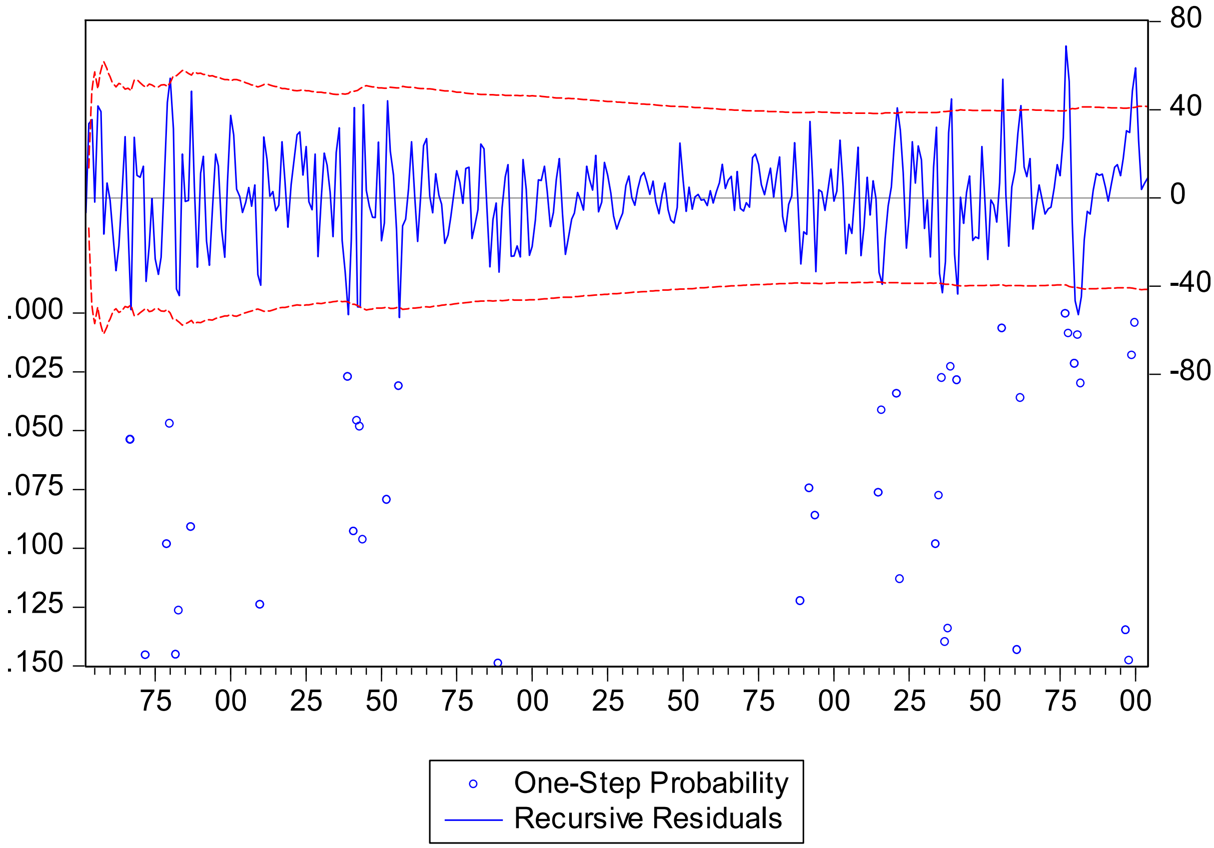

Figure 9: One-step-ahead forecast errors,
recursive estimation of $(1\text{-}L^2)y_t$ on a constant, full sample

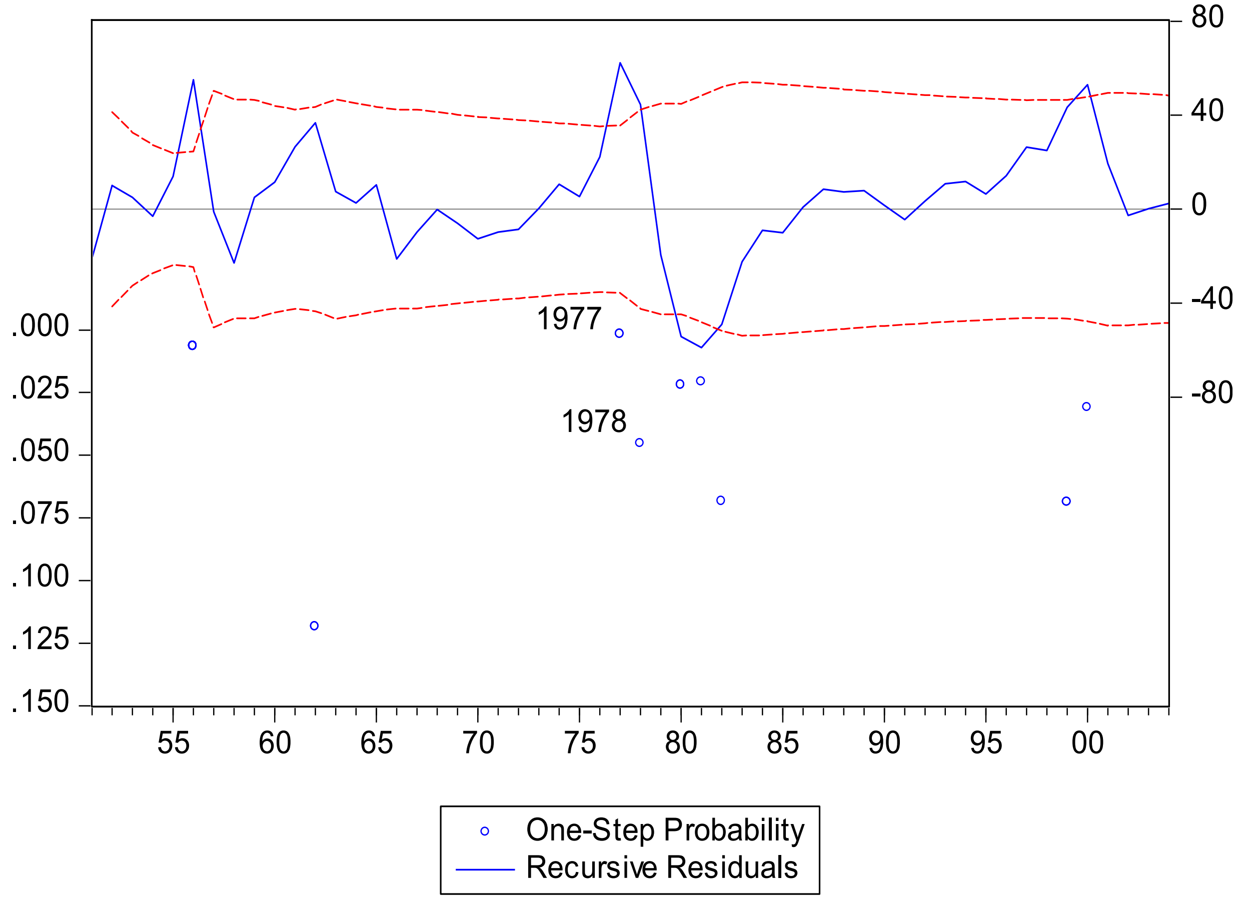

Figure 10: One-step-ahead forecast errors,

recursive estimation of $(1-L^2)y_t$ on a constant, 1950-2004

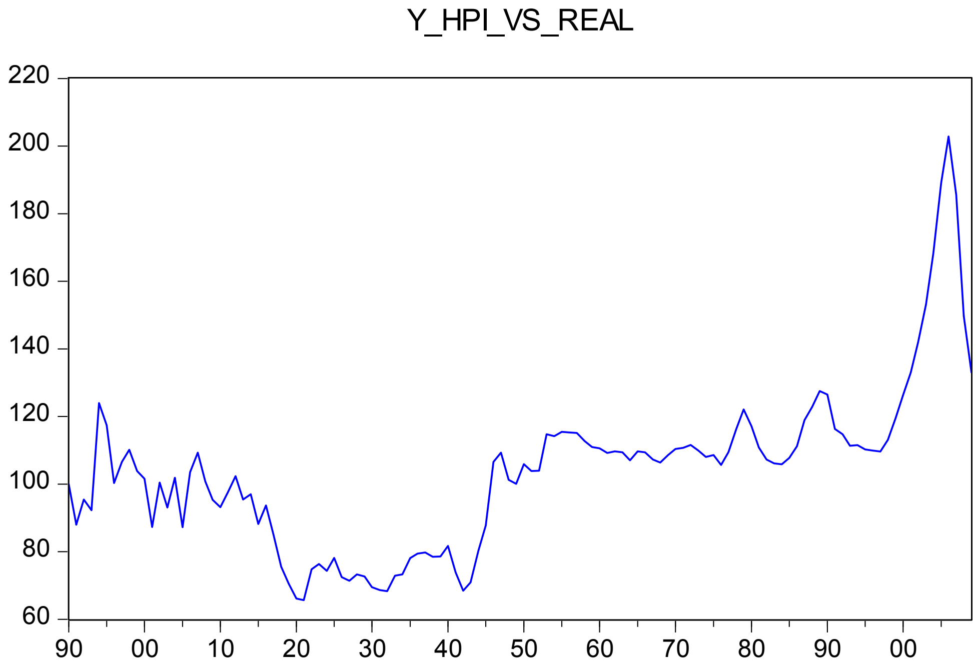

Figure 11: Housing price index USA, 1890-2009, corrected for inflation

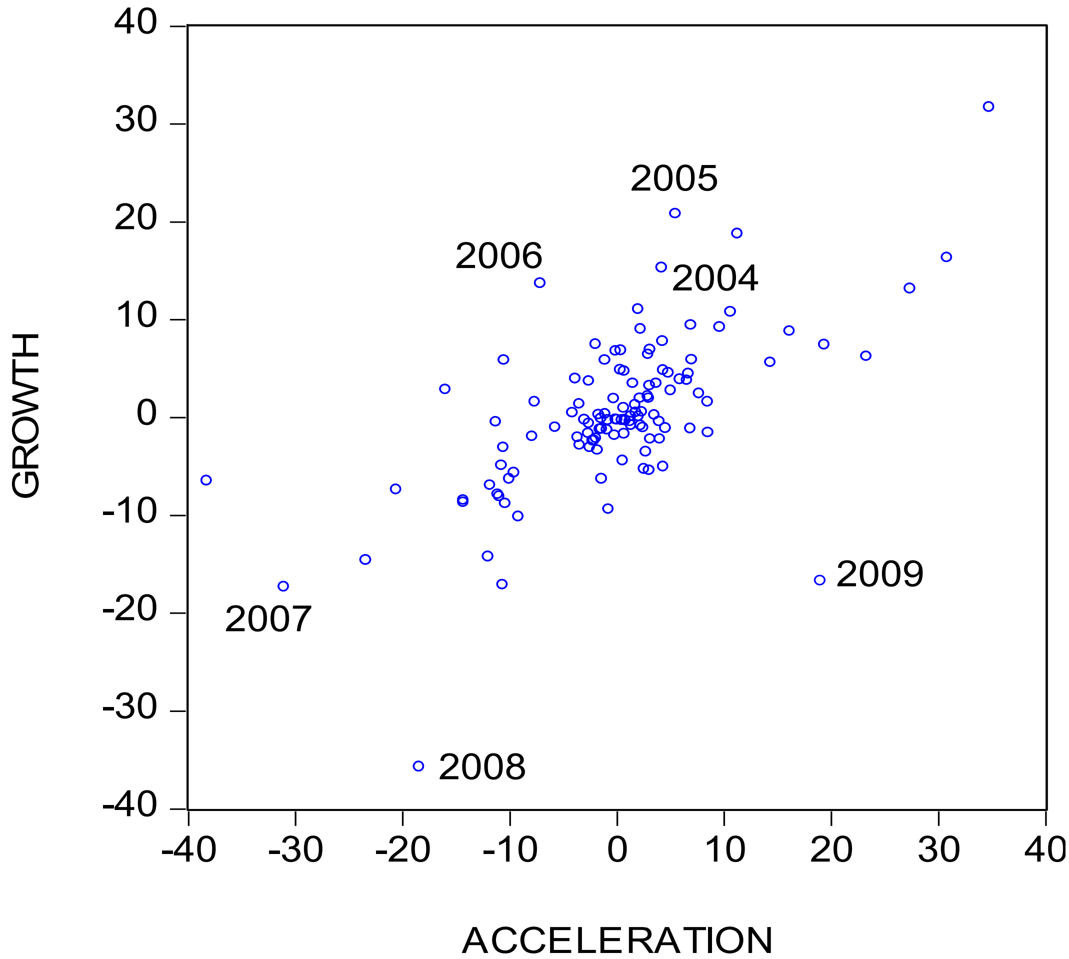

Figure 12: Housing price index USA, 1890-2009, corrected for inflation, growth versus acceleration

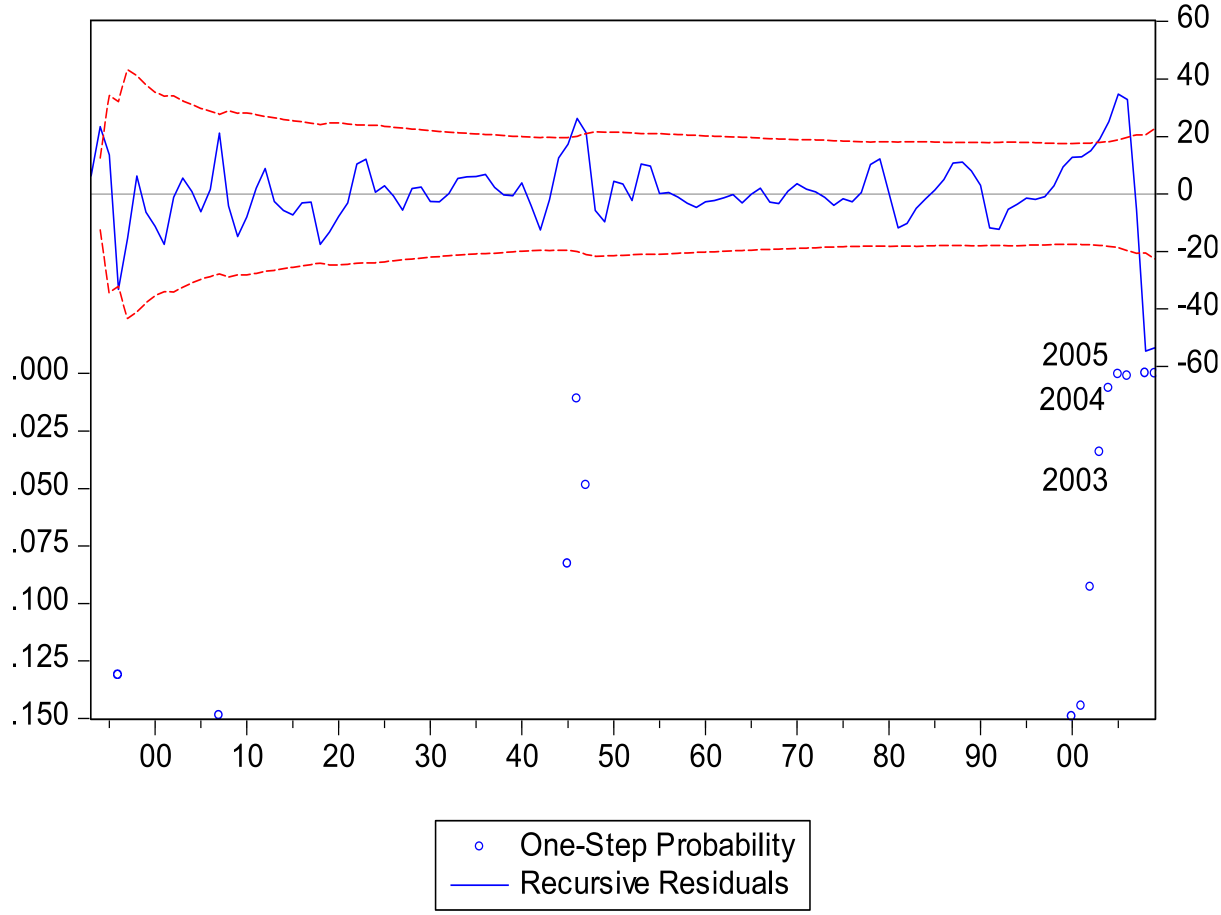

Figure 13: Housing price index USA, 1890-2009, corrected for inflation. One-step-ahead forecast errors, recursive estimation of $(1-L^2)y_t$ on a constant.

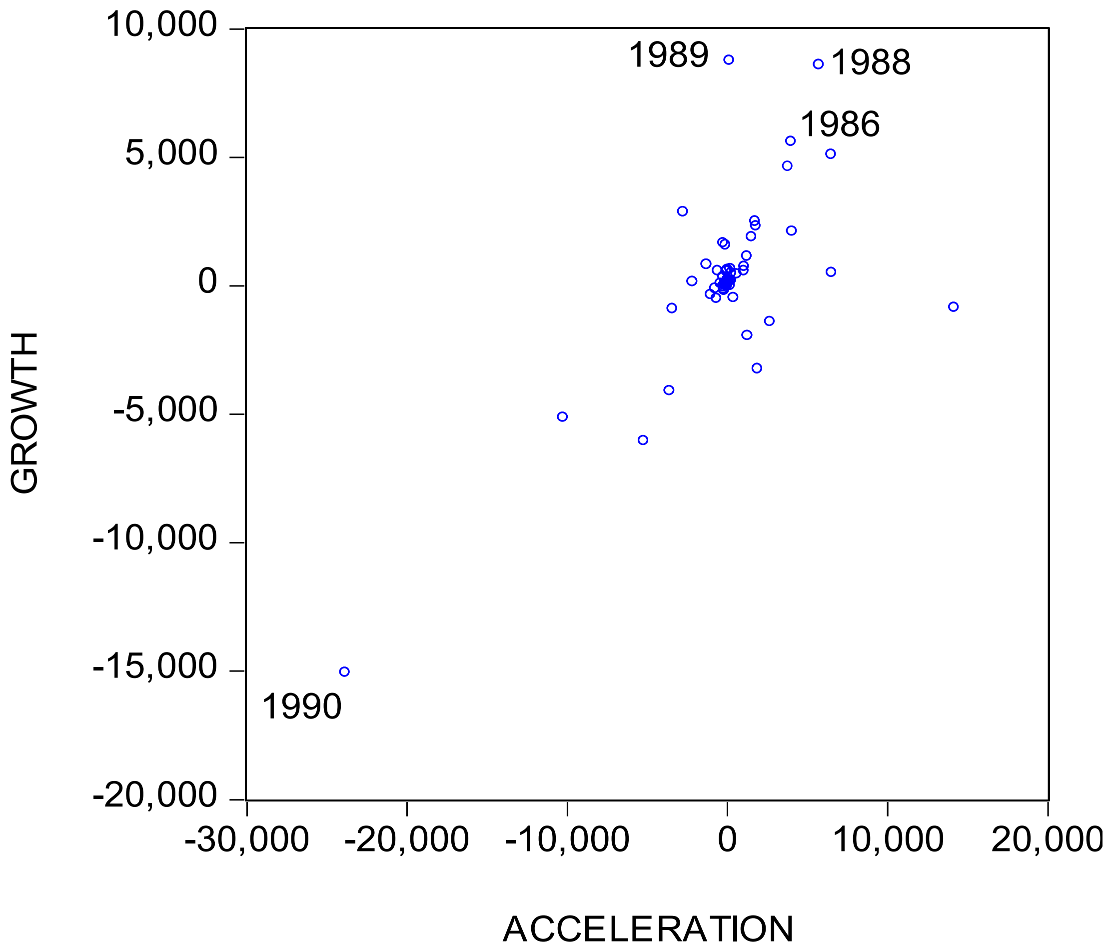

Figure 14: Nikkei Index, 1914-2005, growth versus acceleration

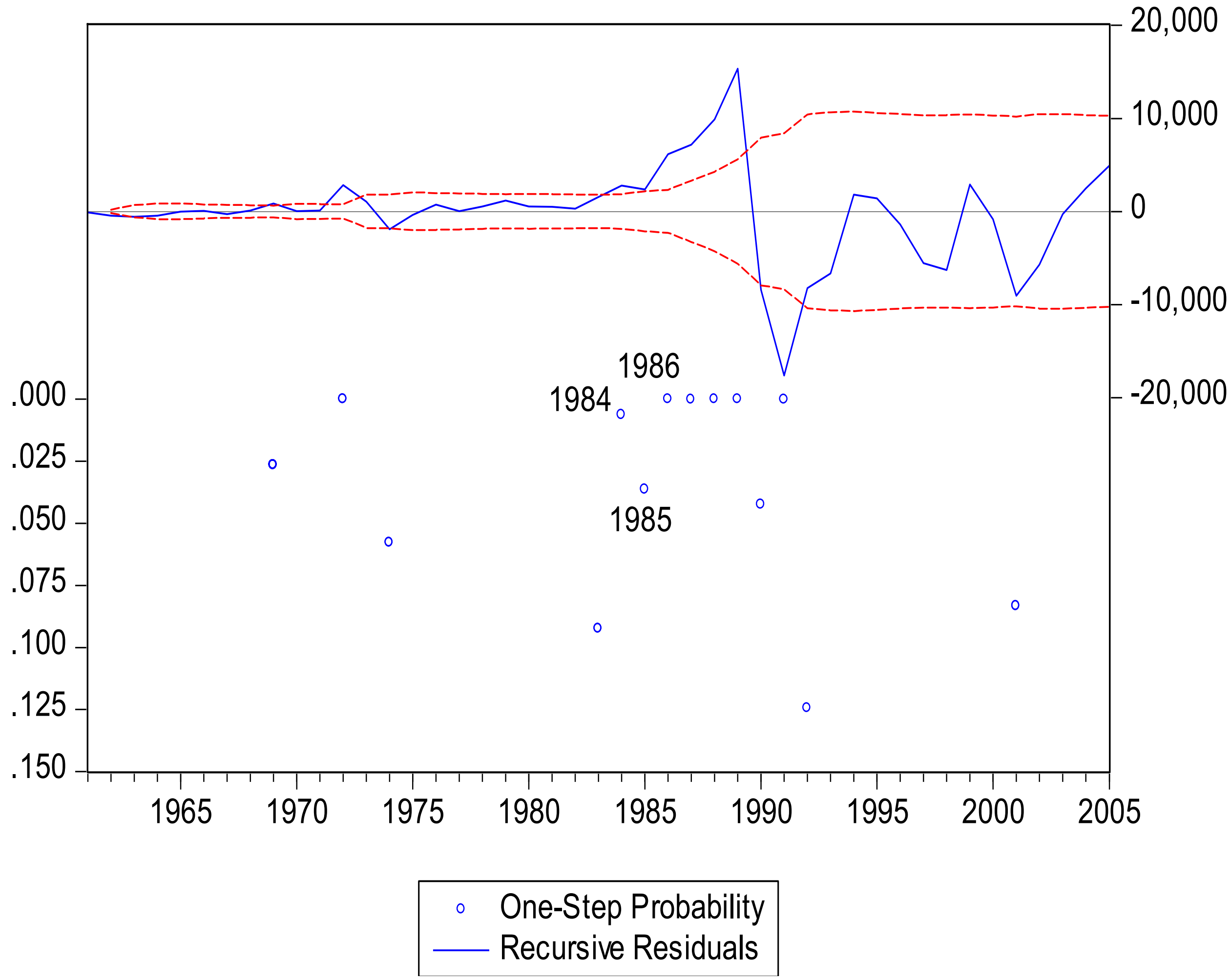

Figure 15: Nikkei Index, 1914-2005,

One-step-ahead forecast errors, recursive estimation of $(1\text{-}L^2)y_t$ on a constant.

Table 1: Empirical size of the test, where τ is set at 1970 (sample size is 70), σ is 500, β_1 is 100 and β_2 is set at 10000, ρ_2 is set at 0.5, 10000 replications

ρ_1	Rejection in years	Significance level		
		0.001	0.01	0.05
0.5	τ-1	0.0003	0.0073	0.0371
	τ-2	0.0006	0.0062	0.0357
	τ-3	0.0007	0.0058	0.0344
	τ-1 and τ-2	0	0.0001	0.0022
	τ-1, τ-2 and τ-3	0	0	0
0.9	τ-1	0.0013	0.0131	0.0525
	τ-2	0.0018	0.0111	0.0539
	τ-3	0.0014	0.0129	0.0522
	τ-1 and τ-2	0	0.0011	0.0002
	τ-1, τ-2 and τ-3	0	0.0012	0
1.0	τ-1	0.0027	0.0124	0.0529
	τ-2	0.0020	0.0124	0.0557
	τ-3	0.0018	0.0138	0.0573
	τ-1 and τ-2	0	0.0007	0.0094
	τ-1, τ-2 and τ-3	0	0.0001	0.0012

Table 2: Empirical power of the test, where τ is set at 1970 (sample size is 70), σ is 500, β_1 is 100 and β_2 is set at 10000, ρ_2 is set at 0.5, 10000 replications

ρ_1	Rejection in years	Significance level		
		0.001	0.01	0.05
1.01	τ-1	0.0018	0.0163	0.0632
	τ-2	0.0020	0.0142	0.0623
	τ-3	0.0017	0.0150	0.0602
	τ-1 and τ-2	0	0.0014	0.0137
	τ-1, τ-2 and τ-3	0	0.0002	0.0010
1.02	τ-1	0.0056	0.0315	0.1103
	τ-2	0.0051	0.0284	0.1058
	τ-3	0.0044	0.0303	0.1009
	τ-1 and τ-2	0.0008	0.0047	0.0328
	τ-1, τ-2 and τ-3	0	0.0051	0.0064
1.03	τ-1	0.0161	0.1076	0.3330
	τ-2	0.0143	0.0982	0.3197
	τ-3	0.0146	0.0917	0.3030
	τ-1 and τ-2	0.0024	0.0313	0.1794
	τ-1, τ-2 and τ-3	0.0001	0.0083	0.0854
1.05	τ-1	0.0100	0.7430	0.9865
	τ-2	0.0092	0.6977	0.9823
	τ-3	0.0107	0.6557	0.9748
	τ-1 and τ-2	0.0018	0.5885	0.9759
	τ-1, τ-2 and τ-3	0.0003	0.4500	0.9603
1.10	τ-1	0.9999	1	1
	τ-2	0.9999	1	1
	τ-3	0.9999	0.9999	1
	τ-1 and τ-2	0.9998	1	1
	τ-1, τ-2 and τ-3	0.9998	0.9999	1

Table 3: Empirical power of the test, where τ is set at 1970 (sample size is 70), σ is 2000, β_1 is 100 and β_2 is set at 10000, ρ_2 is set at 0.5, 10000 replications

ρ_1	Rejection in years	Significance level		
		0.001	0.01	0.05
1.01	τ-1	0.0019	0.0145	0.0559
	τ-2	0.0020	0.0146	0.0561
	τ-3	0.0017	0.0143	0.0602
	τ-1 and τ-2	0	0.0013	0.0102
	τ-1, τ-2 and τ-3	0	0.0001	0.0013
1.02	τ-1	0.0027	0.0163	0.0671
	τ-2	0.0029	0.0150	0.0630
	τ-3	0.0033	0.0154	0.0640
	τ-1 and τ-2	0.0003	0.0023	0.0125
	τ-1, τ-2 and τ-3	0	0.0001	0.0019
1.03	τ-1	0.0062	0.0308	0.1124
	τ-2	0.0064	0.0298	0.1090
	τ-3	0.0061	0.0328	0.1090
	τ-1 and τ-2	0.0005	0.0073	0.0392
	τ-1, τ-2 and τ-3	0.0001	0.0008	0.0107
1.05	τ-1	0.0382	0.3178	0.6101
	τ-2	0.0366	0.2965	0.5921
	τ-3	0.0330	0.2804	0.5685
	τ-1 and τ-2	0.0081	0.1883	0.5052
	τ-1, τ-2 and τ-3	0.0007	0.1052	0.4223
1.10	τ-1	0.9762	0.9868	0.9925
	τ-2	0.9712	0.9853	0.9916
	τ-3	0.9676	0.9840	0.9898
	τ-1 and τ-2	0.9670	0.9823	0.9900
	τ-1, τ-2 and τ-3	0.9582	0.9788	0.9875

Table 4: Empirical power of the test, where τ is set at 1940 (sample size is 40), σ is 500, β_1 is 100 and β_2 is set at 10000, ρ_2 is set at 0.5, 10000 replications

ρ_1	Rejection in years	Significance level		
		0.001	0.01	0.05
1.01	τ-1	0.0027	0.0165	0.0628
	τ-2	0.0027	0.0189	0.0676
	τ-3	0.0030	0.0164	0.0643
	τ-1 and τ-2	0.0002	0.0020	0.0126
	τ-1, τ-2 and τ-3	0	0	0.0011
1.02	τ-1	0.0028	0.0178	0.0663
	τ-2	0.0039	0.0196	0.0668
	τ-3	0.0028	0.0158	0.0671
	τ-1 and τ-2	0.0001	0.0021	0.0139
	τ-1, τ-2 and τ-3	0	0.0002	0.0024
1.03	τ-1	0.0050	0.0269	0.0982
	τ-2	0.0052	0.0279	0.0960
	τ-3	0.0056	0.0264	0.0928
	τ-1 and τ-2	0.0006	0.0050	0.0249
	τ-1, τ-2 and τ-3	0.0001	0.0005	0.0050
1.05	τ-1	0.0169	0.1152	0.3492
	τ-2	0.0162	0.1064	0.3429
	τ-3	0.0193	0.1017	0.2983
	τ-1 and τ-2	0.0026	0.0359	0.1861
	τ-1, τ-2 and τ-3	0.0001	0.0074	0.0822
1.10	τ-1	0.0037	0.9820	0.9991
	τ-2	0.0023	0.9684	0.9993
	τ-3	0.0033	0.9417	0.9989
	τ-1 and τ-2	0.0005	0.9593	0.9988
	τ-1, τ-2 and τ-3	0.0001	0.9154	0.9981

References

Homm, U. and J. Breitung (2012), Testing for speculative bubbles in stock markets: A comparison of alternative methods, *Journal of Financial Econometrics*, 10, 198-231.

Phillips, P.C.B., Y Wu and J Yu (2011), Explosive behavior in the 1990s Nasdaq: When did exuberance escalate asset values? *International Economic Review*, 52, 201-226.