stat > r > dae > zipoisson.htm

R Data Analysis Examples: Zero-Inflated Poisson Regression

Zero-inflated poisson regression is used to model count data that has an excess of zero counts. Further, theory suggests that the excess zeros are generated by a separate process from the count values and that the excess zeros can be modeled independently. Thus, the **zip** model has two parts, a poisson count model and the logit model for predicting excess zeros. You may want to review these Data Analysis Example pages, Poisson Regression and Logit Regression.

This page uses the following packages. Make sure that you can load them before trying to run the examples on this page. If you do not have a package installed, run: `install.packages("packagename")` , or if you see the version is out of date, run: `update.packages()` .

```
require(ggplot2)
require(pscl)
require(boot)
```

Version info: Code for this page was tested in R version 3.0.2 (2013-09-25)
On: 2014-02-24
With: boot 1.3-9; ggplot2 0.9.3.1; knitr 1.5; pscl 1.04.4; vcd 1.3-1; gam 1.09; coda 0.16-1; lattice 0.20-24; mvtnorm 0.9-9996; MASS 7.3-29

Please Note: The purpose of this page is to show how to use various data analysis commands. It does not cover all aspects of the research process which researchers are expected to do. In particular, it does not cover data cleaning and verification, verification of assumptions, model diagnostics and potential follow-up analyses.

Examples of Zero-Inflated Poisson regression

Example 1. School administrators study the attendance behavior of high school juniors at two schools. Predictors of the number of days of absence include gender of the student and standardized test scores in math and language arts.

Example 2. The state wildlife biologists want to model how many fish are being caught by fishermen at a state park. Visitors are asked how long they stayed, how many people were in the group, were there children in the group and how many fish were caught. Some visitors do not fish, but there is no data on whether a person fished or not. Some visitors who did fish did not catch any fish so there are excess zeros in the data because of the people that did not fish.

Description of the data

Let's pursue Example 2 from above.

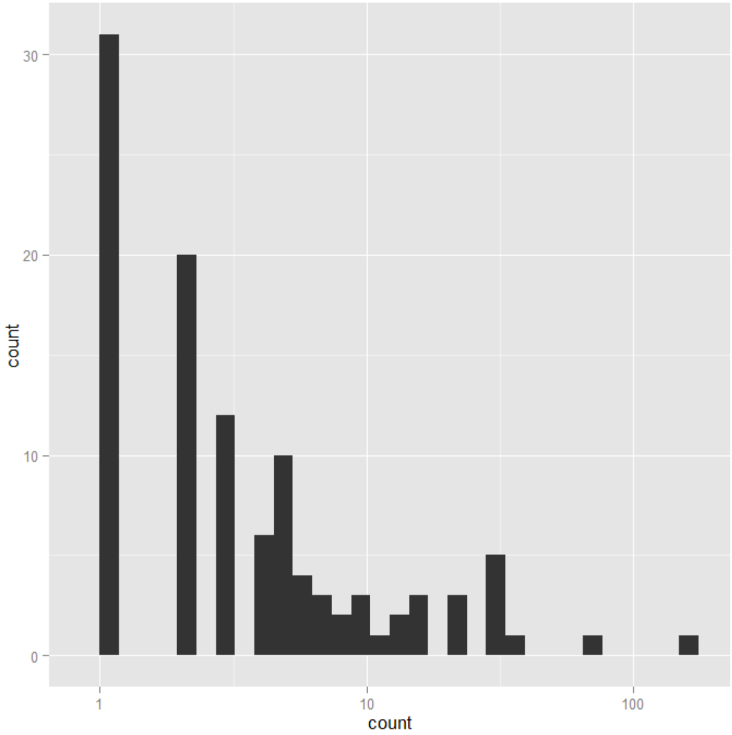
We have data on 250 groups that went to a park. Each group was questioned about how many fish they caught (`count`), how many children were in the group (`child`), how many people were in the group (`persons`), and whether or not they brought a camper to the park (`camper`).

In addition to predicting the number of fish caught, there is interest in predicting the existence of excess zeros, i.e., the probability that a group caught zero fish. We will use the variables `child` , `persons` , and `camper` in our model. Let's look at the data.

```
zinb <- read.csv("http://www.ats.ucla.edu/stat/data/fish.csv")
zinb <- within(zinb, {
    nofish <- factor(nofish)
    livebait <- factor(livebait)
    camper <- factor(camper)
})

summary(zinb)
```

```
##  nofish  livebait camper     persons         child             xb
##  0:176   0: 34    0:103   Min.   :1.00   Min.   :0.000   Min.   :-3.275
##  1: 74   1:216    1:147   1st Qu.:2.00   1st Qu.:0.000   1st Qu.: 0.008
##                           Median :2.00   Median :0.000   Median : 0.955
##                           Mean   :2.53   Mean   :0.684   Mean   : 0.974
##                           3rd Qu.:4.00   3rd Qu.:1.000   3rd Qu.: 1.964
##                           Max.   :4.00   Max.   :3.000   Max.   : 5.353
##        zg             count
##  Min.   :-5.626   Min.   :  0.0
##  1st Qu.:-1.253   1st Qu.:  0.0  
##  Median : 0.605   Median :  0.0  
##  Mean   : 0.252   Mean   :  3.3  
##  3rd Qu.: 1.993   3rd Qu.:  2.0  
##  Max.   : 4.263   Max.   :149.0
```

```
## histogram with x axis in log10 scale
ggplot(zinb, aes(count)) + geom_histogram() + scale_x_log10()
```



Analysis methods you might consider

Below is a list of some analysis methods you may have encountered. Some of the methods listed are quite reasonable while others have either fallen out of favor or have limitations.

- Zero-inflated Poisson Regression - The focus of this web page.
- Zero-inflated Negative Binomial Regression - Negative binomial regression does better with over dispersed data, i.e. variance much larger than the mean.
- Ordinary Count Models - Poisson or negative binomial models might be more appropriate if there are no excess zeros.
- OLS Regression - You could try to analyze these data using OLS regression. However, count data are highly non-normal and are not well estimated by OLS regression.

Zero-inflated Poisson regression

Though we can run a Poisson regression in R using the `glm` function in one of the core packages, we need another package to run the zero-inflated poisson model. We use the `pscl` package.

```
summary(m1 <- zeroinfl(count ~ child + camper | persons, data = zinb))
```

```
## 
## Call:
## zeroinfl(formula = count ~ child + camper | persons, data = zinb)
## 
## Pearson residuals:
##    Min     1Q Median     3Q    Max 
## -1.237 -0.754 -0.608 -0.192 24.085 
##
```

```
## Count model coefficients (poisson with log link):
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)   1.5979     0.0855   18.68   <2e-16 ***
## child        -1.0428     0.1000  -10.43   <2e-16 ***
## camper1       0.8340     0.0936    8.91   <2e-16 ***
##
## Zero-inflation model coefficients (binomial with logit link):
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)    1.297      0.374    3.47  0.00052 ***
## persons       -0.564      0.163   -3.46  0.00053 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Number of iterations in BFGS optimization: 12
## Log-likelihood: -1.03e+03 on 5 Df
```

The output looks very much like the output from two OLS regressions in R.

Below the model call, you will find a block of output containing Poisson regression coefficients for each of the variables along with standard errors, z-scores, and p-values for the coefficients. A second block follows that corresponds to the inflation model. This includes logit coefficients for predicting excess zeros along with their standard errors, z-scores, and p-values.

All of the predictors in both the count and inflation portions of the model are statistically significant. This model fits the data significantly better than the null model, i.e., the intercept-only model. To show that this is the case, we can compare with the current model to a null model without predictors using chi-squared test on the difference of log likelihoods.

```
mnull <- update(m1, . ~ 1)

pchisq(2 * (logLik(m1) - logLik(mnull)), df = 3, lower.tail = FALSE)
```

```
## 'log Lik.' 4.041e-41 (df=5)
```

Since we have three predictor variables in the full model, the degrees of freedom for the chi-squared test is 3. This yields a high significant p-value; thus, our overall model is statistically significant.

Note that the model output above does not indicate in any way if our zero-inflated model is an improvement over a standard Poisson regression. We can determine this by running the corresponding standard Poisson model and then performing a Vuong test of the two models.

```
summary(p1 <- glm(count ~ child + camper, family = poisson, data = zinb))
```

```
##
## Call:
## glm(formula = count ~ child + camper, family = poisson, data = zinb)
##
## Deviance Residuals:
##    Min      1Q  Median      3Q     Max
##  -3.77   -2.23   -1.20   -0.35   24.95
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)   0.9103     0.0812    11.2   <2e-16 ***
## child        -1.2348     0.0803   -15.4   <2e-16 ***
## camper1       1.0527     0.0887    11.9   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 2958.4  on 249  degrees of freedom
## Residual deviance: 2380.1  on 247  degrees of freedom
## AIC: 2723
##
## Number of Fisher Scoring iterations: 6
```

```
vuong(p1, m1)
```

```
## Vuong Non-Nested Hypothesis Test-Statistic: -3.574
## (test-statistic is asymptotically distributed N(0,1) under the
##  null that the models are indistinguishible)
## in this case:
```

```
## model2 > model1, with p-value 0.0001756
```

The Vuong test compares the zero-inflated model with an ordinary Poisson regression model. In this example, we can see that our test statistic is significant, indicating that the zero-inflated model is superior to the standard Poisson model.

We can get confidence intervals for the parameters and the exponentiated parameters using bootstrapping. For the Poisson model, these would be incident risk ratios, for the zero inflation model, odds ratios. We use the `boot` package. First, we get the coefficients from our original model to use as start values for the model to speed up the time it takes to estimate. Then we write a short function that takes data and indices as input and returns the parameters we are interested in. Finally, we pass that to the `boot` function and do 1200 replicates, using snow to distribute across four cores. Note that you should adjust the number of cores to whatever your machine has. Also, for final results, one may wish to increase the number of replications to help ensure stable results.

```
dput(coef(m1, "count"))
```

```
## structure(c(1.59788828690411, -1.04283909332231, 0.834023618148891
## ), .Names = c("(Intercept)", "child", "camper1"))
```

```
dput(coef(m1, "zero"))
```

```
## structure(c(1.29744027908309, -0.564347365357873), .Names = c("(Intercept)",
## "persons"))
```

```
f <- function(data, i) {
  require(pscl)
  m <- zeroinfl(count ~ child + camper | persons, data = data[i, ],
    start = list(count = c(1.598, -1.0428, 0.834), zero = c(1.297, -0.564)))
  as.vector(t(do.call(rbind, coef(summary(m)))[, 1:2]))
}

set.seed(10)
res <- boot(zinb, f, R = 1200, parallel = "snow", ncpus = 4)

## print results
res
```

```
##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = zinb, statistic = f, R = 1200, parallel = "snow",
##     ncpus = 4)
##
##
## Bootstrap Statistics :
##       original    bias    std. error
## t1*   1.59789 -0.056661     0.30307
## t2*   0.08554  0.004257     0.01670
## t3*  -1.04284 -0.002510     0.40557
## t4*   0.09999  0.004395     0.01539
## t5*   0.83402  0.017178     0.40465
## t6*   0.09363  0.004581     0.01536
## t7*   1.29744  0.020810     0.48058
## t8*   0.37385  0.008224     0.03662
## t9*  -0.56435 -0.030103     0.26673
## t10*  0.16296  0.005272     0.02981
```

The results are alternating parameter estimates and standard errors. That is, the first row has the first parameter estimate from our model. The second has the standard error for the first parameter. The third column contains the bootstrapped standard errors, which are considerably larger than those estimated by `zeroinfl`.

Now we can get the confidence intervals for all the parameters. We start on the original scale with percentile and bias adjusted CIs. We also compare these results with the regular confidence intervals based on the standard errors.

```
## basic parameter estimates with percentile and bias adjusted CIs
parms <- t(sapply(c(1, 3, 5, 7, 9), function(i) {
  out <- boot.ci(res, index = c(i, i + 1), type = c("perc", "bca"))
  with(out, c(Est = t0, pLL = percent[4], pUL = percent[5],
     bcaLL = bca[4], bcaLL = bca[5]))
}))
```

```
## add row names
row.names(parms) <- names(coef(m1))
## print results
parms
```

```
##                         Est     pLL      pUL     bcaLL    bcaLL
## count_(Intercept)  1.5979  0.8793  2.07810  1.087354  2.22614
## count_child       -1.0428 -1.7509 -0.17531 -1.618509 -0.02203
## count_camper1      0.8340  0.0596  1.62653  0.001571  1.59995
## zero_(Intercept)   1.2974  0.3503  2.21984  0.293577  2.12070
## zero_persons      -0.5643 -1.1087 -0.07847 -1.008526  0.00633
```

```
## compare with normal based approximation
confint(m1)
```

```
##                      2.5 %  97.5 %
## count_(Intercept)  1.4302  1.7655
## count_child       -1.2388 -0.8469
## count_camper1      0.6505  1.0175
## zero_(Intercept)   0.5647  2.0302
## zero_persons      -0.8838 -0.2449
```

The bootstrapped confidence intervals are considerably wider than the normal based approximation. The bootstrapped CIs are more consistent with the CIs from Stata when using robust standard errors.

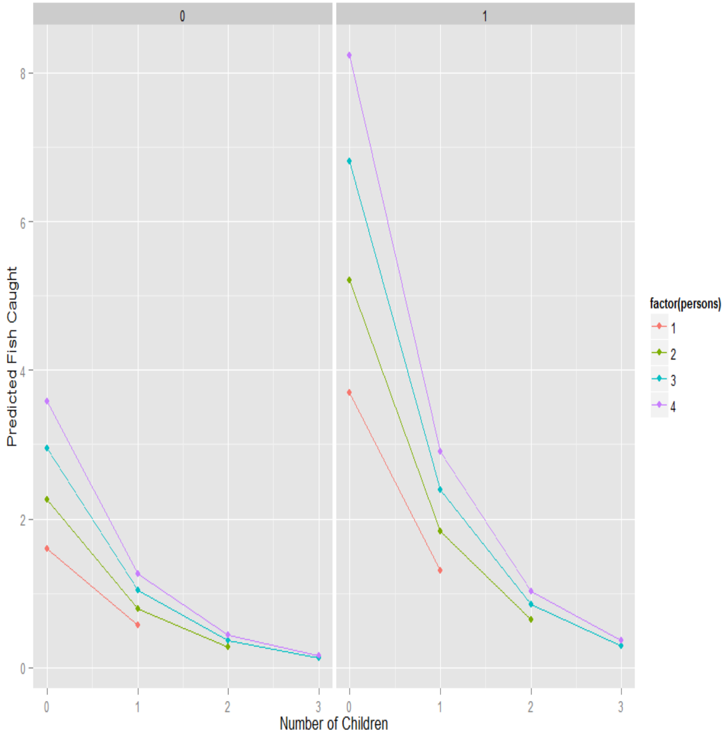
Now we can estimate the incident risk ratio (IRR) for the Poisson model and odds ratio (OR) for the logistic (zero inflation) model. This is done using almost identical code as before, but passing a transformation function to the `h` argument of `boot.ci`, in this case, `exp` to exponentiate.

```
## exponentiated parameter estimates with percentile and bias adjusted CIs
expparms <- t(sapply(c(1, 3, 5, 7, 9), function(i) {
  out <- boot.ci(res, index = c(i, i + 1), type = c("perc", "bca"), h = exp)
  with(out, c(Est = t0, pLL = percent[4], pUL = percent[5],
    bcaLL = bca[4], bcaLL = bca[5]))
}))

## add row names
row.names(expparms) <- names(coef(m1))
## print results
expparms
```

```
##                        Est    pLL    pUL  bcaLL  bcaLL
## count_(Intercept) 4.9426 2.4091 7.9892 2.9664 9.2641
## count_child       0.3525 0.1736 0.8392 0.1982 0.9782
## count_camper1     2.3026 1.0614 5.0862 1.0016 4.9528
## zero_(Intercept)  3.6599 1.4195 9.2058 1.3412 8.3370
## zero_persons      0.5687 0.3300 0.9245 0.3648 1.0063
```

To better understand our model, we can compute the expected number of fish caught for different combinations of our predictors. In fact, since we are working with essentially categorical predictors, we can compute the expected values for all combinations using the `expand.grid` function to create all combinations and then the `predict` function to do it. We also remove any rows where the number of children exceeds the number of persons, which does not make sense logically, using the `subset` function. Finally we create a graph.

```
newdata1 <- expand.grid(0:3, factor(0:1), 1:4)
colnames(newdata1) <- c("child", "camper", "persons")
newdata1 <- subset(newdata1, subset=(child<=persons))
newdata1$phat <- predict(m1, newdata1)

ggplot(newdata1, aes(x = child, y = phat, colour = factor(persons))) +
  geom_point() +
  geom_line() +
  facet_wrap(~camper) +
  labs(x = "Number of Children", y = "Predicted Fish Caught")
```



Things to consider

- Since **zip** has both a count model and a logit model, each of the two models should have good predictors. The two models do not necessarily need to use the same predictors.
- Problems of perfect prediction, separation or partial separation can occur in the logistic part of the zero-inflated model.
- Count data often use exposure variables to indicate the number of times the event could have happened. You can incorporate a logged version of the exposure variable into your model by using the `offset()` option.
- It is not recommended that zero-inflated Poisson models be applied to small samples. What constitutes a small sample does not seem to be clearly defined in the literature.
- Pseudo-R-squared values differ from OLS R-squareds, please see FAQ: What are pseudo R-squareds? for a discussion on this issue.

See Also

R Online Manual

- `zeroinfl`

References

- Long, J. S. 1997. *Regression Models for Categorical and Limited Dependent Variables.* Thousand Oaks, CA: Sage Publications. Everitt, B. S. and Hothorn, T. A Handbook of Statistical Analyses Using R

How to cite this page Report an error on this page or leave a comment

The content of this web site should not be construed as an endorsement of any particular web site, book, or software product by the University of California.

IDRE RESEARCH TECHNOLOGY GROUP

High Performance Computing

Statistical Computing

GIS and Visualization

High Performance Computing
Hoffman2 Cluster
Hoffman2 Account Application
Hoffman2 Usage Statistics
UC Grid Portal
UCLA Grid Portal
Shared Cluster & Storage

GIS
Mapshare
Visualization
3D Modeling
Technology Sandbox
Tech Sandbox Access
Data Centers

Statistical Computing
Classes
Conferences
Reading Materials
IDRE Listserv
IDRE Resources
Social Sciences Data Archive

ABOUT CONTACT NEWS EVENTS OUR EXPERTS