Unit 1

Introduction to Logic and Set Theory

Outline of Unit 1

- 1.1 Why Study Logic?
- 1.2 Validity of an Argument
- 1.3 Basics of Set Theory
- 1.4 Venn Diagram Test on Validity

Unit 1.1

Why Study Logic?

What is Logic?

Logic

From Wikipedia, the free encyclopedia

This article is about the systematic study of the form of arguments. For other u

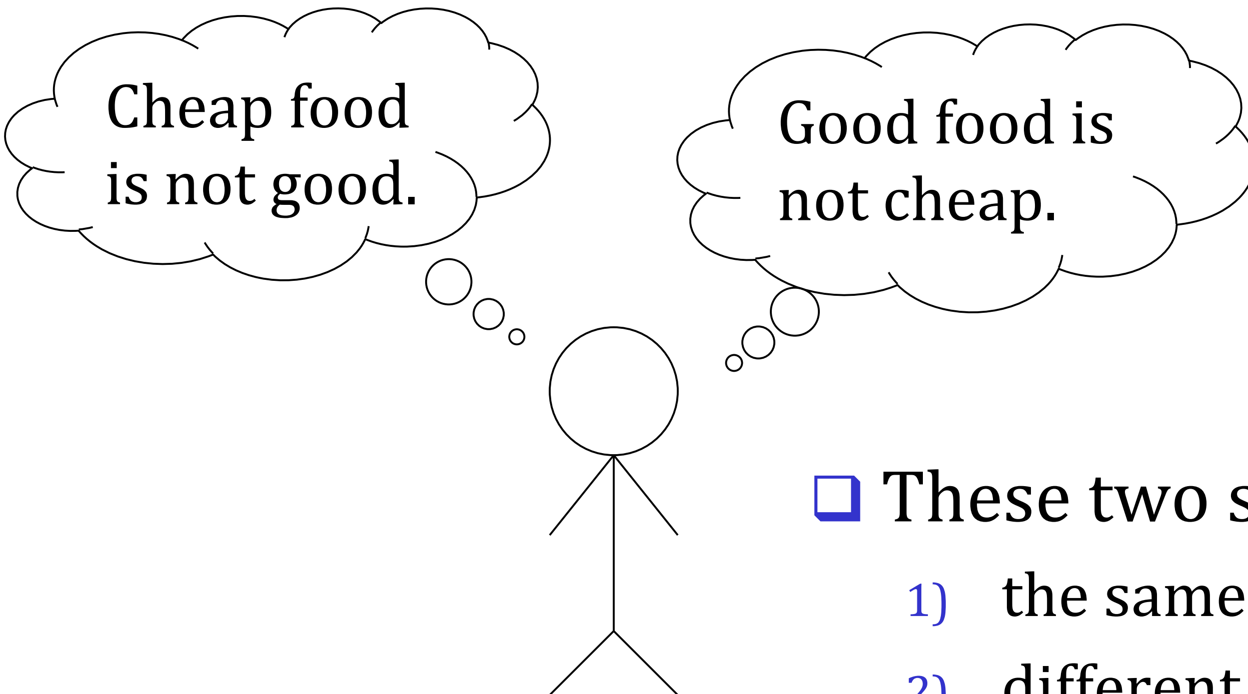
Logic (from the Ancient Greek: λογική, *logikḗ*[1]), originally meaning "the word" or "what is spoken" (but coming to mean "thought" or "reason"), is generally held to consist of the systematic study of the form of arguments. A valid argument is one where there is a specific relation of logical support between the assumptions of the argument and its conclusion. (In ordinary discourse, the conclusion of such an argument may be signified by words like *therefore*, *hence*, *ergo* and so on.)

Q1.1 A Test on Logical Reasoning

- If you overslept, you'll be late.
- You didn't oversleep.
- Therefore,
  1) You're late.
  2) You aren't late.
  3) You did oversleep.
  4) None of these follows.

Which one is correct?

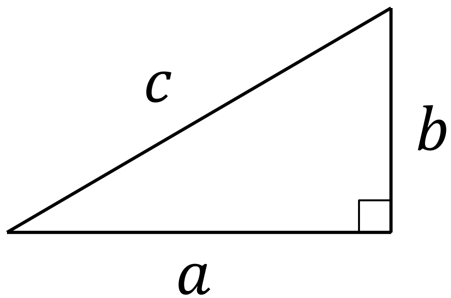
Q1.2 A More Challenging Test



- These two statements say
  1) the same thing.
  2) different things.

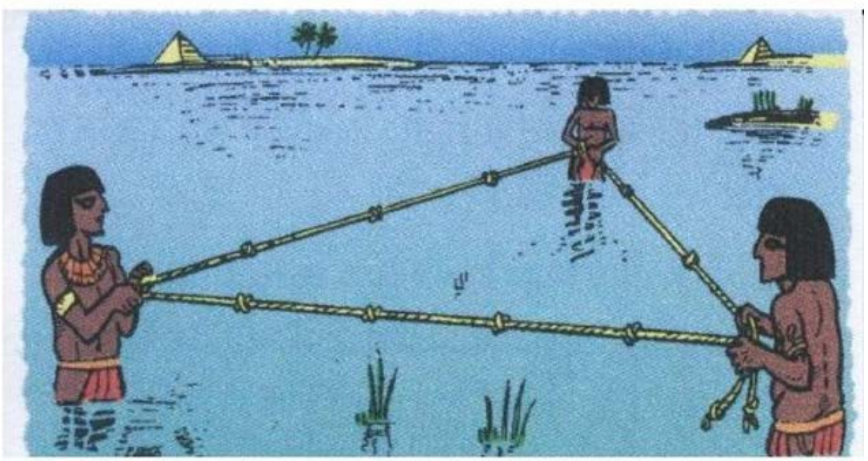
Which one is correct?

Q1.3 What is a Proof?



Claim: $a^2 + b^2 = c^2$

How to prove it?



- The lengths of the sides are 3, 4, 5.
- Since $3^2 + 4^2 = 5^2$, the statement is correct.

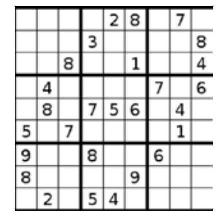
Is this a proof?

1) Yes
2) No

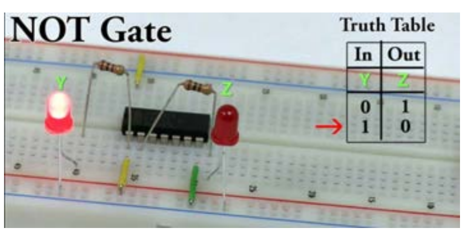
Reasons to Study Logic

1) Logic sharpens reasoning

2) Logic is fun.



3) Logic is useful to
electronic/computer/information engineers



Unit 1.2

Validity of an Argument

Validity: What Follows from What?

Logic

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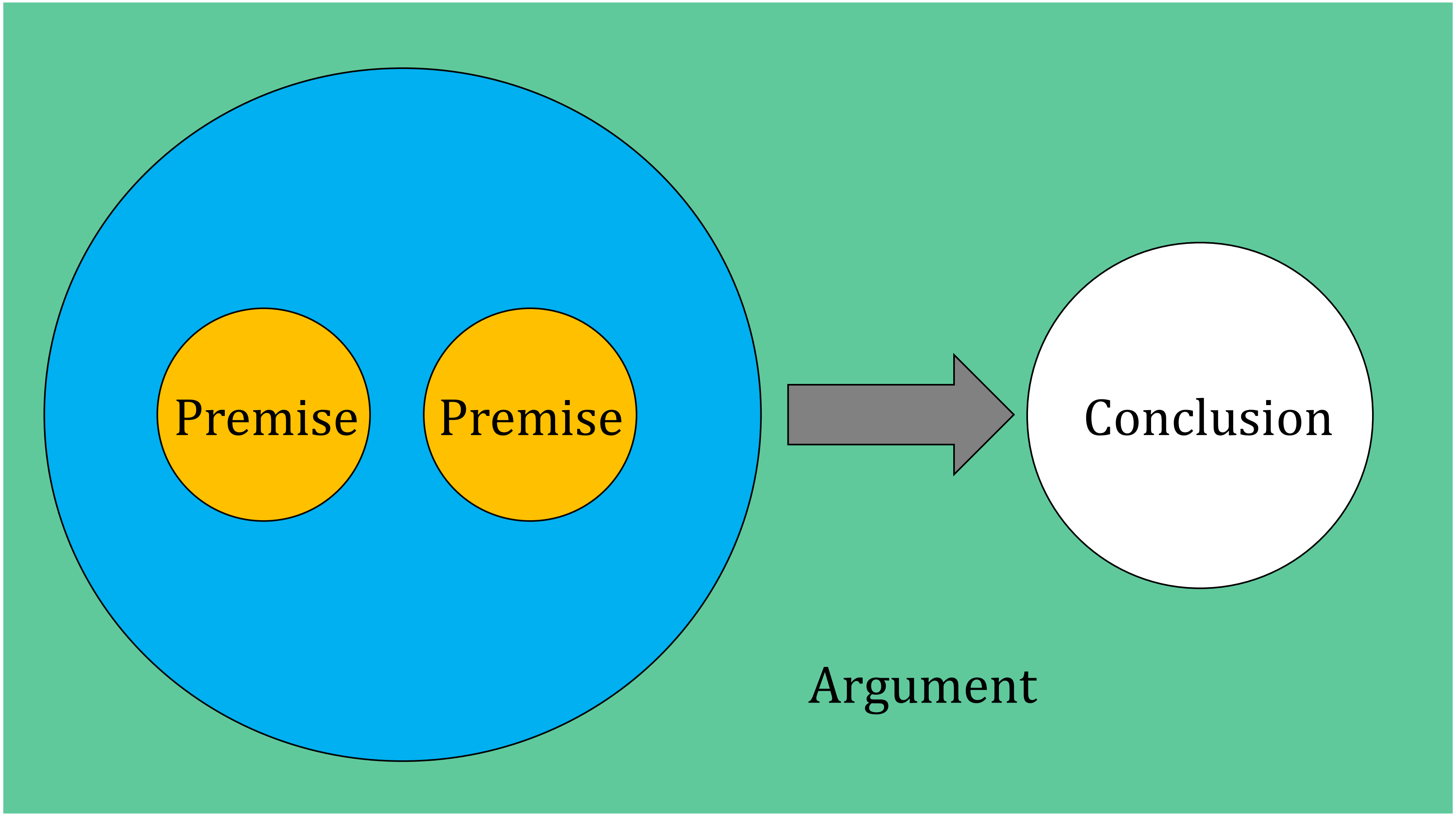
Logical Argument



Logical Argument

- An argument is a list of statements
  - The last statement is called a conclusion.
  - All preceding statements are called premises (or assumptions or hypotheses).

- Example

  If you want to get an A, you should work hard.

  You want to get an A.

  Therefore, you should work hard.

Two Major Types of Arguments

Deductive Argument

- The conclusion follows from the premises necessarily.

All who live in Hong Kong live in Asia.

Vincent lives in Hong Kong.

Therefore, Vincent lives in Asia.

Inductive Argument

- The conclusion follows from the premises probably.

Most who live in Hong Kong speak Cantonese.

Tiffany lives in Hong Kong.

Therefore, Tiffany speaks Cantonese.

The Third Type of Argument

Abductive Argument

- Also called inference to the best explanation.
- The conclusion may or may not be true.

The surprising fact, E, is observed.
H is the best explanation of E.
____
Therefore, H is true.

- Example:

You hear your baby crying and notice a nasty smell.
The best explanation is …

____
Therefore, you should change her diaper .


In this course, we focus on deductive argument.

Valid and Sound Arguments

- An argument is valid if its conclusion is a logical consequence of the premises.
  - i.e. it is impossible for the premises to be true and the conclusion false.
- An argument is sound if it is valid and the premises are true.

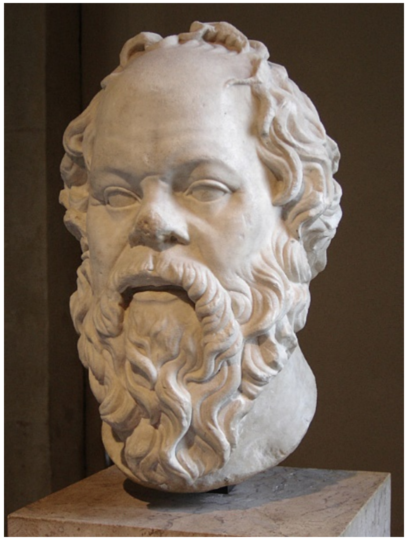
- Caution: An invalid or unsound argument may still have a true conclusion!

The Socrates Argument

All men are mortal.

Socrates was a man.

Therefore, Socrates was mortal.



Socrates (469/470-399BC) was a Greek philosopher and is considered the father of western philosophy.

A deductive argument that has two premises is traditionally called syllogism.

Flying Penguin?

All animals with wings can fly.
Penguins have wings.

Therefore, penguins can fly.

Valid? Sound?

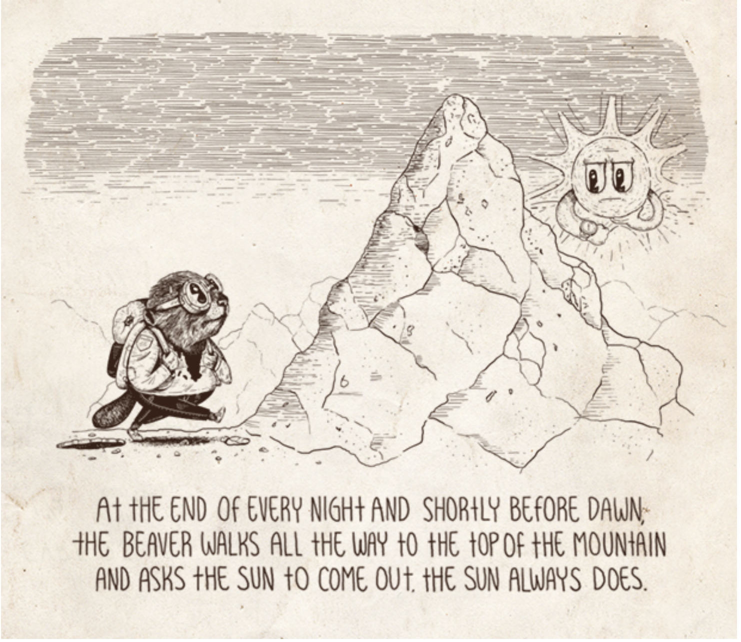
(one-and-a-half-minute video)
https://www.youtube.com/watch?v=9dfWzp7rYR4

Also watch this (3 min):
https://www.youtube.com/watch?v=lzhDsojoqk8&t=98

Fallacies

- A formal fallacy is an error in reasoning that results in an invalid deductive argument.

- An informal fallacy is an error in reasoning that results in an argument whose conclusion lacks adequate support.



Comments on Deductive Arguments

- Our focus is on deductive arguments.
- Usefulness: It reveals consequences of premises.
  - Mathematics is based on deductive arguments.
    - theorems are derived from axioms.
- Limitation: It doesn't tell which premises are actually true.
  - Science is based on inductive arguments.
    - You need to go observe the world and collect empirical evidence.

Unit 1.3

Basics of Set Theory

Set

- A set is a collection of objects.
- Suppose the set A contains an object called x.
  - x is an element (or member) of A.
  - x belongs to A (or x is in A), denoted by $x \in A$.
- The roster notation of a set simply lists all members of the set inside braces { }.
- Example:

$$C = \{10¢, 20¢, 50¢, \$1, \$2, \$5, \$10\}$$

- The order is not important.
- The same element needs not appear more than once. (Duplicate elements are redundant and can be removed.)

Set-Builder Notation

- We can "build" a set by describing what properties its members have.
- Set-builder notation:

$$T = \{\, x \in S \mid P(x) \,\}.$$

such that

- Example:
  - Let $C = \{10¢, 20¢, 50¢, \$1, \$2, \$5, \$10\}$.
  - D is the set whose elements are all elements $x \in C$ such that x is completely bronze in color.
  - $D = \{x \in C \mid x \text{ is completely bronze in color.}\}$
    $= \{10¢, 20¢, 50¢\}$

Cardinality

- The cardinality of a set A is defined as the number of elements in the set.
- It is denoted by $|A|$.

- Example:

  $C = \{10¢, 20¢, 50¢, \$1, \$2, \$5, \$10\}$

  $|C| = 7$.

Subset

- A is a subset of B, written as $A \subseteq B$, if every member of A is also a member of B.
- B is then said to be a superset of A.
- A subset A of B is called a proper subset of B if B contains some elements that are not in A.
  - i.e., A is not the same as B.
- Example:
  - The set of all women is a proper subset of the set of all human beings.

Set Equality

- Two sets are the same (or equal) if and only if
  - they contain the same elements, or equivalently,
  - each is a subset of the other.

$$A = B \quad \Leftrightarrow \quad A \subseteq B \text{ and } B \subseteq A.$$

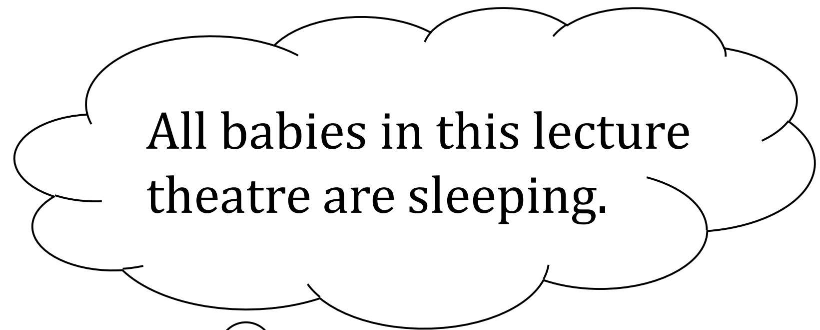
- Example
  - Suppose

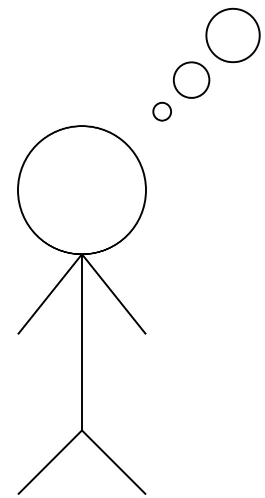
$$A = \{m \in \mathbf{Z} \mid m = 2a \text{ for some integer } a\}$$
$$B = \{n \in \mathbf{Z} \mid n = 2b - 2 \text{ for some integer } b\}$$

  - Are they equal?

The Empty Set

- A set is empty if it contains no elements at all.
- There is only one empty set.
  - If two sets are empty, each set is a subset of the other one, so they are the same set.
- We denote it by $\emptyset$.
- Remark:
  - The empty set $\emptyset$ is different from the set containing $\emptyset$.
    - i.e., $\emptyset \neq \{\emptyset\}$.





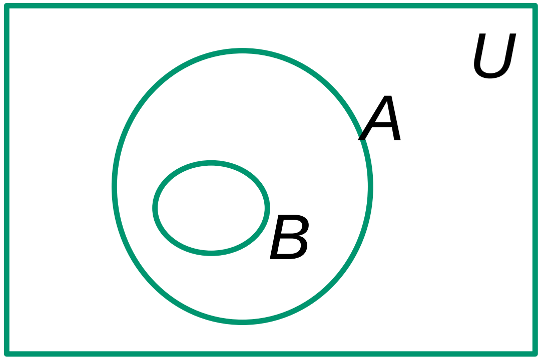
True or False?

Vacuous truth

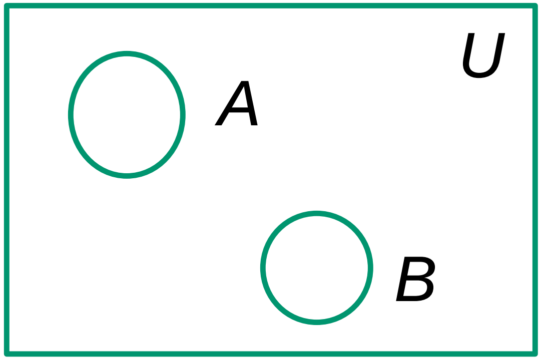
It is universally agreed that given any arbitrary property P, all elements of $\emptyset$ have property P.

Relationship between Sets

- A universal set U is a set containing everything that we are considering.
- Venn diagram
  - U is represented by a rectangular box.
  - Subsets of U (e. g. A and B) are represented by circles (more precisely, regions inside closed curves).
- A and B are disjoint if they have no elements in common.



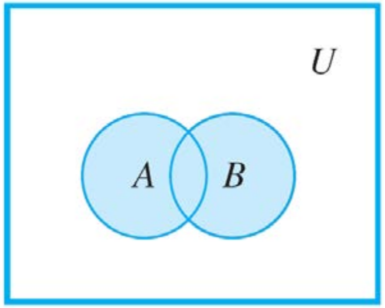
B is a subset of A.



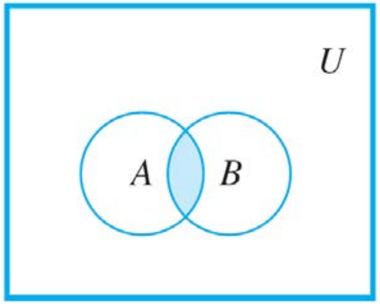
A and B are disjoint.

Three Fundamental Operations

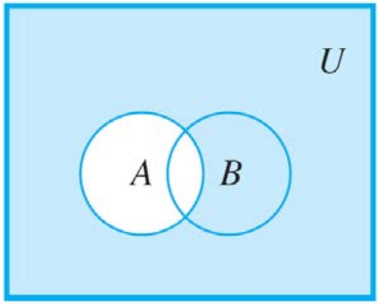
- The union of A and B, denoted by $A \cup B$, is the set of all elements that belong to either A or B, or in both.



- The intersection of A and B, denoted by $A \cap B$, is the set of all elements that are in both A and B.



- The complement of A, denoted by A^c, is the set of all elements in U that do not belong to A.



Power Set

- Given a set A, the set of all its subsets, denoted by $\mathcal{P}(A)$, is called the power set of A.

- Example:
  - Suppose $A = \{1, 2, 3\}$.
  - List all subsets of A:

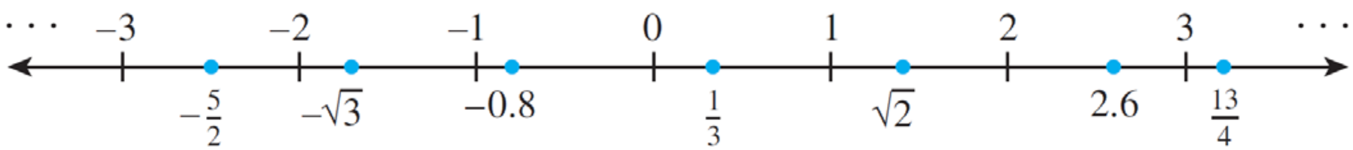
    $\emptyset$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$ and $\{1, 2, 3\}$.
  - Hence,

    $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.

Special Sets

- The following sets are commonly used in mathematics:

Symbol	Set
R	set of all real numbers
Z	set of all integers
Q	set of all rational numbers



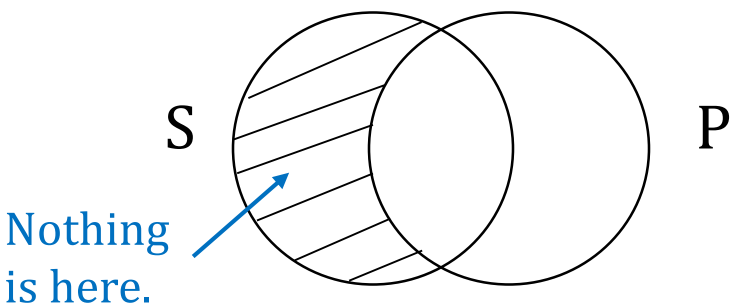
- The numbers represented by blue dots all belong to **R** but not to **Z**.
- Z^+ and Z^- denote the sets of positive and negative integers, respectively.

Unit 1.4

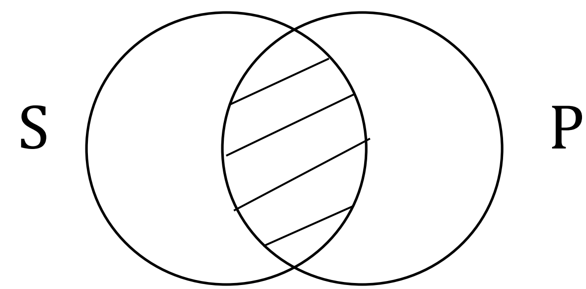
Venn Diagram Test on Validity

Venn Diagram Representations

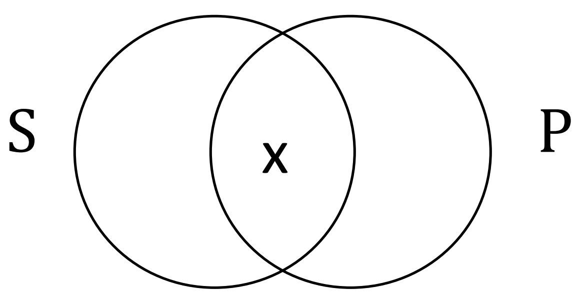
- All S is P.



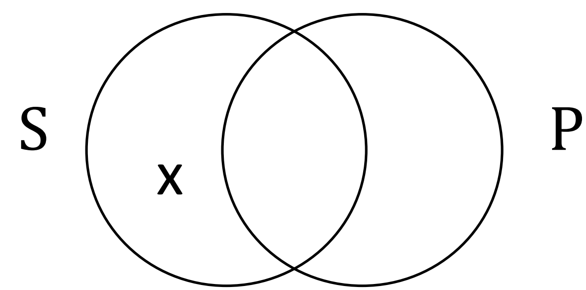
- No S is P.



- Some S is P.



- Some S is not P.



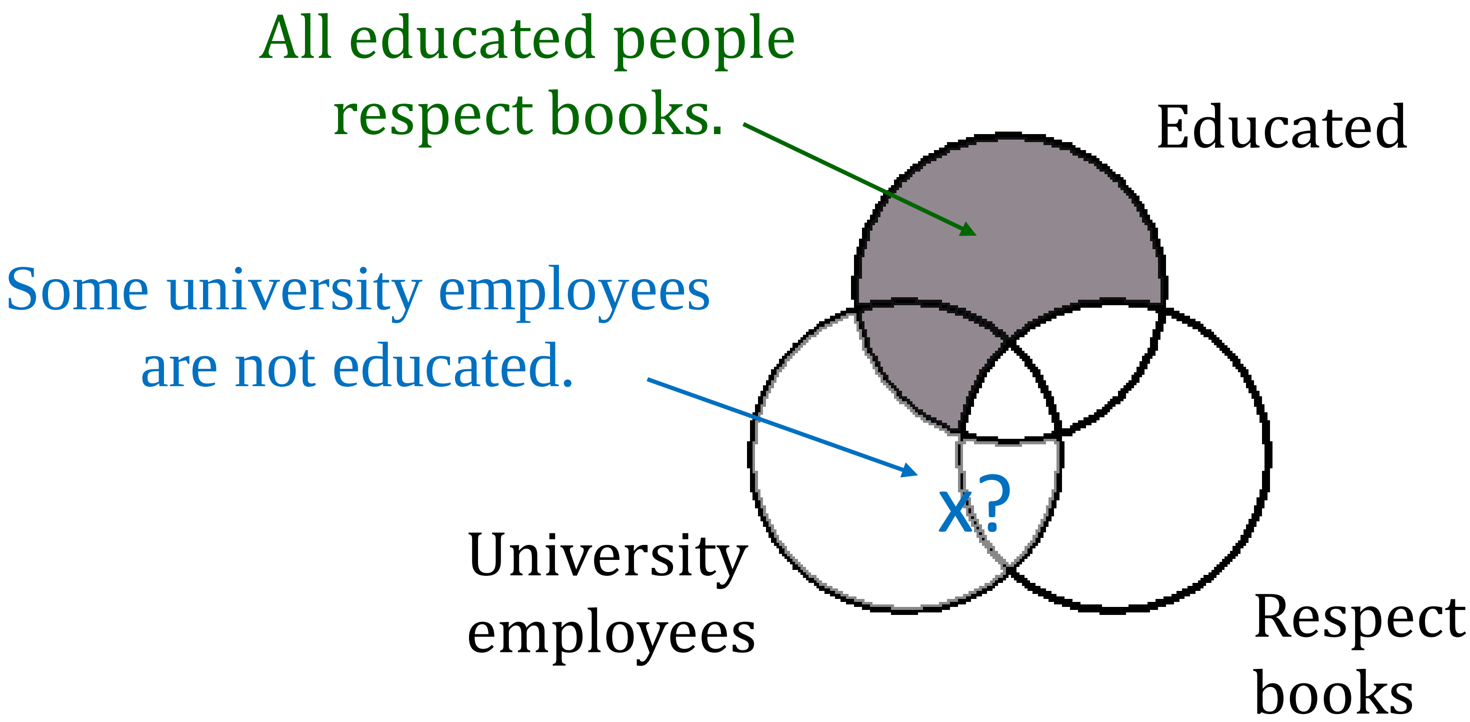
Q1.4

All educated people respect books.

Some university employee are not educated.

Some university employee do not respect books.

Is it valid?



There may be some university employee who do not respect books, but not necessarily. So the argument is invalid.

Q1.5

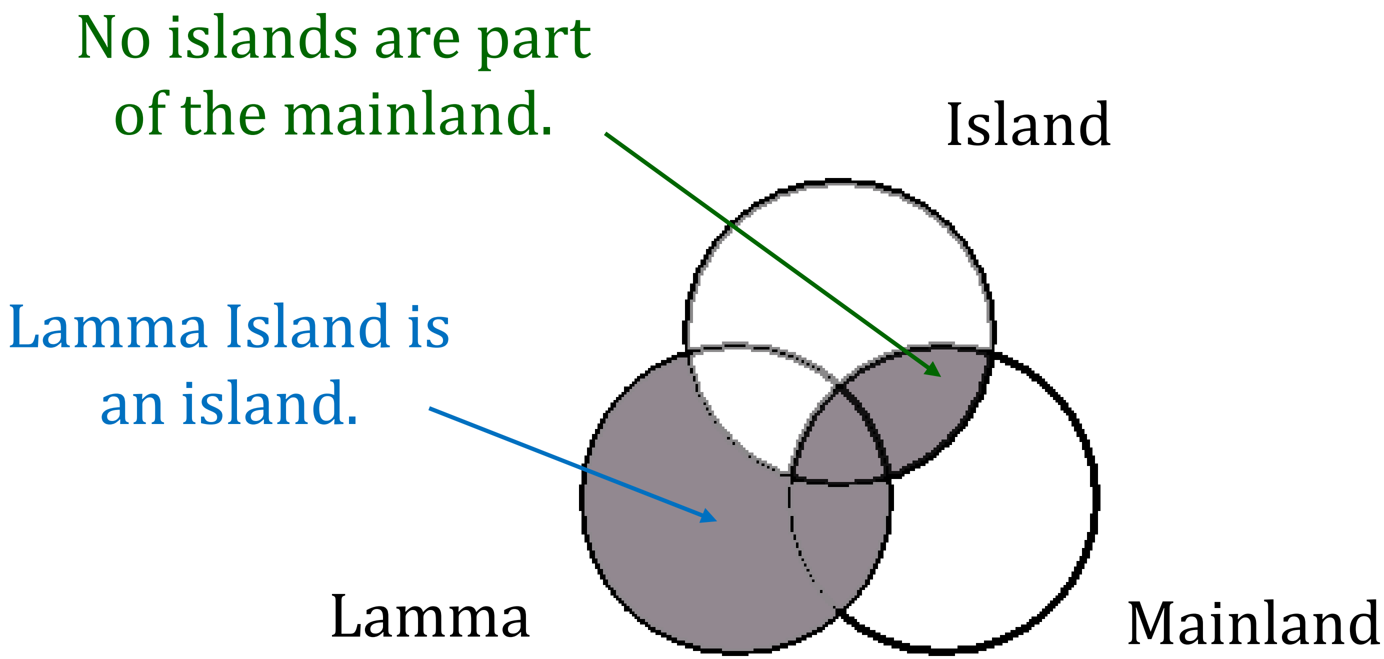
No islands are part of the mainland.

Lamma Island is an island.

Lamma Island is not on the mainland.

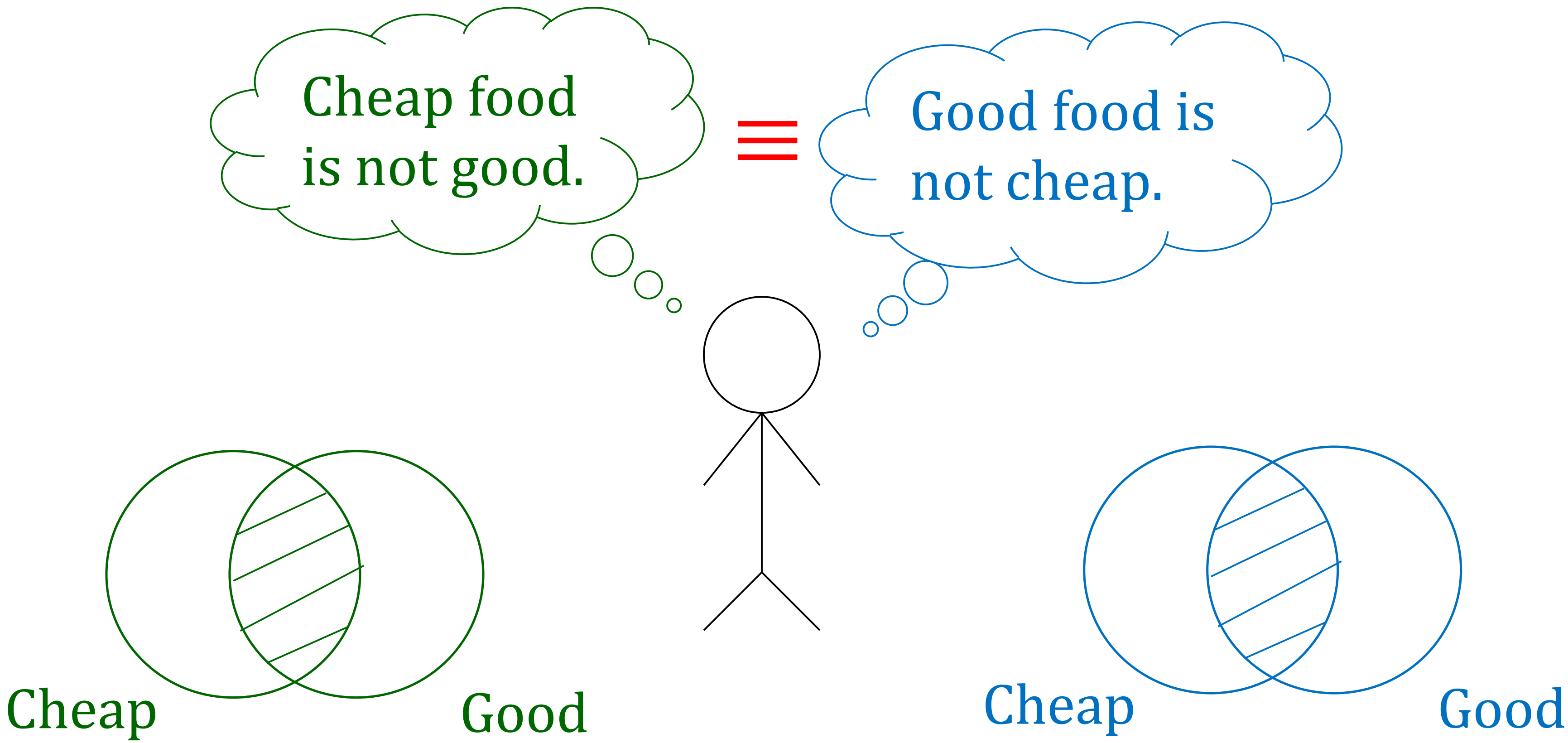
Lamma Winds
南丫風采發電站

Is it valid?



"Lamma Island is not on the mainland" is true.
So the argument is valid.

Example Revisited



The two statements are logically equivalent.