Contribution Title*

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Abstract. This paper proved a method to computing the forgetting in CTL which has been submitted to IJCAI, from the resolution proposed by Zhang at all by extending the resolution rules.

Keywords: Forgetting · CTL · Model checking.

1 Introduction

As a logical notion, *forgetting* was first formally defined in propostional and first order logics by Lin and Reiter [13]. Over the last twenty years, researchers have developed forgetting notions and theories not only in classical logic but also in other non-classical logic systems [**?**], such as forgetting in logic programs under answer set/stable model semantics [23,6,20,18,17], forgetting in description logic [19,15,26] and knowledge forgetting in modal logic [25,16,14,8]. In application, forgetting has been used in planning [12], conflict solving [10,24], createing restricted views of ontologies [26], strongest and weakest definitions [9], SNC (WSC) [11] and so on.

Though forgetting has been extensively investigated from various aspects of different logical systems. However, the existing forgetting method in propositional logic, answer set programming, description logic and modal logic are not directly applicable in CTL. Similar with that in [25], we research forgetting in CTL from the semantic forgetting point of view. And it is shown that our definition of forgetting satisfies those four postulates of forgetting.

2 Preliminaries

We start with some technical and notational preliminaries. Throughout this paper, we fix a finite set $\mathcal{A}$ of propositional variables (or atoms), and use V, V' for subsets of $\mathcal{A}$. In the following several parts, we will introduce the structure we use for CTL, syntactic and semantic of CTL and the normal form $\mathrm{SNF}^g_{\mathrm{CTL}}$ (Separated Normal Form with Global Clauses for CTL) of CTL [22].

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2.1 Model structure in CTL

In general, a transition system [4] is described as a *model structure* (or *Kripke structure*)(in this article, we treat transition system and model structure as the same thing), and a model structure is a triple $\mathcal{M} = (S, R, L)$ [7], where

- S is a set of states,
- $R \subseteq S \times S$ is a total binary relation over S, *i.e.* , for each state $s \in S$ there is a state $s' \in S$ such that $(s, s') \in R$, and
- L is an interpretation function $S \to 2^{\mathcal{A}}$ mapping every state to the set of atoms true at that state.

In this article, the same as [4], all of our results apply only to finite Kripke structures. Besides, we restrict ourselves to model structure $\mathcal{M} = (S, R, L, s_0)$ (similar with that in [22]) such that

- there exists a state s_0, called the *initial state*, such that for every state $s \in S$ there is a path π_{s_0} s.t. $s \in \pi_{s_0}$.

We call a model structure $\mathcal{M}$ on a set V of atoms if $L : S \to 2^V$, *i.e.* , the labeling function L map every state to V (not the $\mathcal{A}$). A *path* π_{s_i} start from s_i of $\mathcal{M}$ is a infinite sequence of states $\pi_{s_i} = (s_i, s_{i+1}s_{i+2}, \dots)$, where for each j $(i \le j)$, $(s_j, s_{j+1}) \in R$. By $s' \in \pi_{s_i}$ we mean that s' is a state in the path π_{s_i}.

For a given model structure (S, R, L, s_0) and $s \in S$, the *computation tree* $\mathrm{Tr}_n^{\mathcal{M}}(s)$ of $\mathcal{M}$(or simply $\mathrm{Tr}_n(s)$), that has depth n and is rooted at s, is recursively defined as [4], for $n \ge 0$,

- $\mathrm{Tr}_0(s)$ consists of a single node s with label s.
- $\mathrm{Tr}_{n+1}(s)$ has as its root a node m with label s, and if $(s, s') \in R$ then the node m has a subtree $\mathrm{Tr}_n(s')$[5].

By s_n we mean the node at the nth level in tree $\mathrm{Tr}_m(s)$ $(m \ge n)$.

A K-*structure* (or K-*interpretation*) is a model structure $\mathcal{M} = (S, R, L, s_0)$ associating with a state $s \in S$, which is written as $(\mathcal{M}, s)$ for convenience in the following. In the case s is an initial state of $\mathcal{M}$, the K-structure is *initial*.

2.2 Syntactic and semantic of CTL

In the following we briefly review the basic syntax and semantics of the *Computation Tree Logic* (CTL in short) [5]. The *signature* of $\mathcal{L}$ includes:

- a finite set of Boolean variables, called *atoms* of $\mathcal{L}$: $\mathcal{A}$;
- the classical connectives: $\bot, \vee$ and $\neg$;

[4] According to [1], a *transition system* TS is a tuple $(S, Act, \to, I, AP, L)$ where (1) S is a set of states, (2) Act is a set of actions, (3) $\to \subseteq S \times \mathrm{Act} \times S$ is a transition relation, (4) $I \subseteq S$ is a set of initial states, (5) AP is a set of atomic propositions, and (6) $L : S \to 2^{\mathrm{AP}}$ is a labeling function.

[5] Though some nodes of the tree may have the same label, they are different nodes in the tree.

- the path quantifiers: A and E;
- the temporal operators: X, F, G U and W, that means 'neXt state', 'some Future state', 'all future states (Globally)', 'Until' and 'Unless', respectively;
- parentheses: (and).

The *(existential normal form or ENF in short) formulas* of $\mathcal{L}$ are inductively defined via a Backus Naur form:

$$\phi ::= \bot \mid p \mid \neg\phi \mid \phi \vee \phi \mid \mathrm{EX}\phi \mid \mathrm{EG}\phi \mid \mathrm{E}[\phi\ \mathrm{U}\ \phi] \tag{1}$$

where $p \in \mathcal{A}$. The formulas $\phi \wedge \psi$ and $\phi \rightarrow \psi$ are defined in a standard manner of propositional logic. The other form formulas of $\mathcal{L}$ are abbreviated using the forms of (1). Notice that, according to the above definition for formulas of CTL, each of the CTL *temporal connectives* has the form XY where $X \in \{\mathrm{A}, \mathrm{E}\}$ and $Y \in \{\mathrm{X}, \mathrm{F}, \mathrm{G}, \mathrm{U}, \mathrm{W}\}$. The priorities for the CTL connectives are assumed to be (from the highest to the lowest):

$$\neg, \mathrm{EX}, \mathrm{EF}, \mathrm{EG}, \mathrm{AX}, \mathrm{AF}, \mathrm{AG} \prec \wedge \prec \vee \prec \mathrm{EU}, \mathrm{AU}, \mathrm{EW}, \mathrm{AW}, \rightarrow .$$

We are now in the position to define the semantics of $\mathcal{L}$. Let $\mathcal{M} = (S, R, L, s_0)$ be an model structure, $s \in S$ and ϕ a formula of $\mathcal{L}$. The *satisfiability* relationship between $\mathcal{M}$, s and ϕ, written $(\mathcal{M}, s) \models \phi$, is inductively defined on the structure of ϕ as follows:

- $(\mathcal{M}, s) \not\models \bot$;
- $(\mathcal{M}, s) \models p$ iff $p \in L(s)$;
- $(\mathcal{M}, s) \models \phi_1 \vee \phi_2$ iff $(\mathcal{M}, s) \models \phi_1$ or $(\mathcal{M}, s) \models \phi_2$;
- $(\mathcal{M}, s) \models \neg\phi$ iff $(\mathcal{M}, s) \not\models \phi$;
- $(\mathcal{M}, s) \models \mathrm{EX}\phi$ iff $(\mathcal{M}, s_1) \models \phi$ for some $s_1 \in S$ and $(s, s_1) \in R$;
- $(\mathcal{M}, s) \models \mathrm{EG}\phi$ iff $\mathcal{M}$ has a path $(s_1 = s, s_2, \ldots)$ such that $(\mathcal{M}, s_i) \models \phi$ for each $i \geq 1$;
- $(\mathcal{M}, s) \models \mathrm{E}[\phi_1 \mathrm{U} \phi_2]$ iff $\mathcal{M}$ has a path $(s_1 = s, s_2, \ldots)$ such that, for some $i \geq 1$, $(\mathcal{M}, s_i) \models \phi_2$ and $(\mathcal{M}, s_j) \models \phi_1$ for each $j < i$.

Similar to the work in [4,2], only initial K-structures are considered to be candidate models in the following, unless explicitly stated. Formally, an initial K-structure $\mathcal{K}$ is a *model* of a formula ϕ whenever $\mathcal{K} \models \phi$. Let Π be a set of formulae, $\mathcal{K} \models \Pi$ if for each $\phi \in \Pi$ there is $\mathcal{K} \models \phi$. We denote *Mod*(ϕ) (*Mod*(Π)) the set of models of ϕ (Π). The formula ϕ (set Π of formulae) is *satisfiable* if *Mod*$(\phi) \neq \emptyset$ (*Mod*$(\Pi) \neq \emptyset$). Since both the underlying states in model structure and signatures are finite, *Mod*(ϕ) (*Mod*(Π)) is finite for any formula ϕ (set Π of formulae).

Let ϕ_1 and ϕ_2 be two formulas or set of formulas. By $\phi_1 \models \phi_2$ we denote *Mod*$(\phi_1) \subseteq$ *Mod*(ϕ_2). By $\phi_1 \equiv \phi_2$ we mean $\phi_1 \models \phi_2$ and $\phi_2 \models \phi_1$. In this case ϕ_1 is *equivalent* to ϕ_2.

Let ϕ be a formula or set of formulas. By *Var*(ϕ) we mean the set of atoms occurring in ϕ. Let $V \subseteq \mathcal{A}$. The formula ϕ is V*-irrelevant*, written $\mathrm{IR}(\phi, V)$, if there is a formula ψ with *Var*$(\psi) \cap V = \emptyset$ such that $\phi \equiv \psi$.

2.3 The normal form of CTL

It has proved that any CTL formula φ can be transformed into a set T_φ of $\text{SNF}^g_{\text{CTL}}$ (Separated Normal Form with Global Clauses for CTL) clauses in polynomial time such that φ is satisfiable iff T_φ is satisfiable [21]. An important difference between CTL formulae and $\text{SNF}^g_{\text{CTL}}$ is that $\text{SNF}^g_{\text{CTL}}$ is an extension of the syntax of CTL to use indices. These indices can be used to preserve a particular path context. The language of $\text{SNF}^g_{\text{CTL}}$ clauses is defined over an extension of CTL. That is the language is based on: (1) the language of CTL; (2) a propositional constant **start**; (3) a countably infinite index set Ind; and (4) temporal operators: $\mathrm{E}_{\langle ind\rangle}\mathrm{X}$, $\mathrm{E}_{\langle ind\rangle}\mathrm{F}$, $\mathrm{E}_{\langle ind\rangle}\mathrm{G}$,$\mathrm{E}_{\langle ind\rangle}\mathrm{U}$ and $\mathrm{E}_{\langle ind\rangle}\mathrm{W}$.

The priorities for the $\text{SNF}^g_{\text{CTL}}$ connectives are assumed to be (from the highest to the lowest):

$$\neg, (\mathrm{EX}, \mathrm{E}_{\langle ind\rangle}\mathrm{X}), (\mathrm{EF}, \mathrm{E}_{\langle ind\rangle}\mathrm{F}), (\mathrm{EG}, \mathrm{E}_{\langle ind\rangle}\mathrm{G}), \mathrm{AX}, \mathrm{AF}, \mathrm{AG}$$
$$\prec \wedge \prec \vee \prec (\mathrm{EU}, \mathrm{E}_{\langle ind\rangle}\mathrm{U}), \mathrm{AU}, (\mathrm{EW}, , \mathrm{E}_{\langle ind\rangle}\mathrm{W}), \mathrm{AW}, \rightarrow .$$

Where the operators in the same brackets have the same priority.

Before talked about the sematic of this language, we introduce the $\text{SNF}^g_{\text{CTL}}$ clauses at first. The $\text{SNF}^g_{\text{CTL}}$ clauses consists of formulae of the following forms.

$$
\begin{array}{ll}
\mathrm{AG}(\textbf{start} \supset \bigvee_{j=1}^{k} m_j) & (initial\ clause) \\
\mathrm{AG}(true \supset \bigvee_{j=1}^{k} m_j) & (global\ clause) \\
\mathrm{AG}(\bigwedge_{i=1}^{n} l_i \supset \mathrm{AX} \bigvee_{j=1}^{k} m_j) & (\mathrm{A} - step\ clause) \\
\mathrm{AG}(\bigwedge_{i=1}^{n} l_i \supset \mathrm{E}_{\langle ind\rangle}\mathrm{X} \bigvee_{j=1}^{k} m_j) & (\mathrm{E} - step\ clause) \\
\mathrm{AG}(\bigwedge_{i=1}^{n} l_i \supset \mathrm{AF}l) & (\mathrm{A} - sometime\ clause) \\
\mathrm{AG}(\bigwedge_{i=1}^{n} l_i \supset \mathrm{E}_{\langle ind\rangle}\mathrm{F}l) & (\mathrm{E} - sometime\ clause).
\end{array}
$$

where $k \geq 0$, $n > 0$, **start** is a propositional constant, l_i $(1 \leq i \leq n)$, m_j $(1 \leq j \leq k)$ and l are literals, that is atomic propositions or their negation and ind is an element of Ind (Ind is a countably infinite index set). By clause we mean the classical clause or the $\text{SNF}^g_{\text{CTL}}$ clause unless explicitly stated.

Formulae of $\text{SNF}^g_{\text{CTL}}$ over $\mathcal{A}$ are interpreted in Ind-model structure $\mathcal{M} = (S, R, L, [\_], s_0)$, where S, R, L and s_0 is the same as our model structure talked in 2.1 and $[\_]$: Ind $\rightarrow 2^{(S*S)}$ maps every index $ind \in$ Ind to a successor function $[ind]$ which is a functional relation on S and a subset of the binary accessibility relation R, such that for every

$s \in S$ there exists exactly a state $s' \in S$ such that $(s, s') \in [ind]$ and $(s, s') \in R$. An infinite path $\pi_{s_i}^{\langle ind \rangle}$ is an infinite sequence of states $s_i, s_{i+1}, s_{i+2}, \dots$ such that for every $j \geq i$, $(s_j, s_{j+1}) \in [ind]$.

Similarly, an *Ind-structure* (or *Ind-interpretation*) is a Ind-model structure $\mathcal{M} = (S, R, L, [\_], s_0)$ associating with a state $s \in S$, which is written as $(\mathcal{M}, s)$ for convenience in the following. In the case s is an initial state of $\mathcal{M}$, the Ind-structure is *initial*.

The semantics of $\mathrm{SNF}_{\mathrm{CTL}}^{g}$ is an extension of the semantics of CTL defined in Section 2.2 except using the Ind-model structure $\mathcal{M} = (S, R, L, [\_], s_0)$ replace model structure, $(\mathcal{M}, s_i) \models$ **start** iff $s_i = s_0$ and for all $\mathtt{E}_{\langle ind \rangle} \Gamma$ are explained in the path $\pi_{s_i}^{\langle ind \rangle}$, where $\Gamma \in \{\mathtt{X}, \mathtt{G}, \mathtt{U}, \mathtt{W}\}$. The semantics of $\mathrm{SNF}_{\mathrm{CTL}}^{g}$ is then defined as shown next as an extension of the semantics of CTL defined in Section 2.2. Let φ and ψ be two $\mathrm{SNF}_{\mathrm{CTL}}^{g}$ formulae and $\mathcal{M} = (S, R, L, [\_], s_0)$ be an Ind-model structure, the relation "$\models$" between $\mathrm{SNF}_{\mathrm{CTL}}^{g}$ formulae and $\mathcal{M}$ is defined recursively as follows:

- $(\mathcal{M}, s_i) \models$ **start** iff $s_i = s_0$;
- $(\mathcal{M}, s_i) \models \mathtt{E}_{\langle ind \rangle} \mathtt{X} \psi$ iff for the path $\pi_{s_i}^{\langle ind \rangle}$, $(\mathcal{M}, s_{i+1}) \models \psi$;
- $(\mathcal{M}, s_i) \models \mathtt{E}_{\langle ind \rangle} \mathtt{G} \psi$ iff for every $s_j \in \pi_{s_i}^{\langle ind \rangle}$, $(\mathcal{M}, s_j) \models \psi$;
- $(\mathcal{M}, s_i) \models \mathtt{E}_{\langle ind \rangle} [\varphi \mathtt{U} \psi]$ iff there exists $s_j \in \pi_{s_i}^{\langle ind \rangle}$ such that $(\mathcal{M}, s_j) \models \psi$ and for every $s_k \in \pi_{s_i}^{\langle ind \rangle}$, if $i \leq k < j$, then $(\mathcal{M}, s_k) \models \varphi$;
- $(\mathcal{M}, s_i) \models \mathtt{E}_{\langle ind \rangle} \mathtt{F} \psi$ iff $(\mathcal{M}, s_i) \models \mathtt{E}_{\langle ind \rangle} [\top \mathtt{U} \psi]$;
- $(\mathcal{M}, s_i) \models \mathtt{E}_{\langle ind \rangle} [\varphi \mathtt{W} \psi]$ iff $(\mathcal{M}, s_i) \models \mathtt{E}_{\langle ind \rangle} \mathtt{G} \varphi$ or $(\mathcal{M}, s_i) \models \mathtt{E}_{\langle ind \rangle} [\varphi \mathtt{U} \psi]$.

The semantics of the remaining operators is analogous to that given previously but in the extended Ind-model structure $\mathcal{M} = (S, R, L, [\_], s_0)$. A $\mathrm{SNF}_{\mathrm{CTL}}^{g}$ formula φ is satisfiable, iff for some Ind-model structure $\mathcal{M} = (S, R, L, [\_], s_0)$, $(\mathcal{M}, s_0) \models \varphi$, and unsatisfiable otherwise. And if $(\mathcal{M}, s_0) \models \varphi$ then $(\mathcal{M}, s_0)$ is called a Ind-model of φ, and we say that $(\mathcal{M}, s_0)$ satisfies φ. By $T \wedge \varphi$ we mean $\bigwedge_{\psi \in T} \psi \wedge \varphi$, where T is a set of formulae. Other terminologies are similar with those in section 2.2.

3 Problem Definition

In order to define our problem, *i.e.* forgetting in CTL, we review our definition of V-bisimulation (read ?? for more detials).

Definition 1. *Let $V \subseteq \mathcal{A}$ and $\mathcal{K}_i = (\mathcal{M}_i, s_i)$ $(i = 1, 2)$ be* K*-structures (Ind-structures). Then $(\mathcal{K}_1, \mathcal{K}_2) \in \mathcal{B}$ if and only if*

(i) $L_1(s_1) - V = L_2(s_2) - V$,
(ii) for every $(s_1, s_1') \in R_1$, there is $(s_2, s_2') \in R_2$ such that $(\mathcal{K}_1', \mathcal{K}_2') \in \mathcal{B}$, and
(iii) for every $(s_2, s_2') \in R_2$, there is $(s_1, s_1') \in R_1$

where $\mathcal{K}_i' = (\mathcal{M}_i, s_i')$ with $i \in \{1, 2\}$.

Proposition 1. *Let $i \in \{1, 2\}$, $V_1, V_2 \subseteq \mathcal{A}$, s_i's be two states and π_i's be two pathes, and $\mathcal{K}_i = (\mathcal{M}_i, s_i)$ $(i = 1, 2, 3)$ be* K*-structures (Ind-structures) such that $\mathcal{K}_1 \leftrightarrow_{V_1} \mathcal{K}_2$ and $\mathcal{K}_2 \leftrightarrow_{V_2} \mathcal{K}_3$. Then:*

(i) $s'_1 \leftrightarrow_{V_i} s'_2$ $(i = 1, 2)$ *implies* $s'_1 \leftrightarrow_{V_1 \cup V_2} s'_2$*;*
(ii) $\pi'_1 \leftrightarrow_{V_i} \pi'_2$ $(i = 1, 2)$ *implies* $\pi'_1 \leftrightarrow_{V_1 \cup V_2} \pi'_2$*;*
(iii) for each path π_{s_1} *of* $\mathcal{M}_1$ *there is a path* π_{s_2} *of* $\mathcal{M}_2$ *such that* $\pi_{s_1} \leftrightarrow_{V_1} \pi_{s_2}$*, and vice versa;*
(iv) $\mathcal{K}_1 \leftrightarrow_{V_1 \cup V_2} \mathcal{K}_3$*;*
(v) If $V_1 \subseteq V_2$ *then* $\mathcal{K}_1 \leftrightarrow_{V_2} \mathcal{K}_2$*.*

Now we give the formal definition of forgetting in CTL from the semantic forgetting point view.

Definition 2 (Forgetting). *Let* $V \subseteq \mathcal{A}$ *and* ϕ *a CTL formula. A CTL formula* ψ *with* $Var(\psi) \cap V = \emptyset$ *is a* result of forgetting V from ϕ*, if*

$$Mod(\psi) = \{\mathcal{K} \text{ is initial} \mid \exists \mathcal{K}' \in Mod(\phi) \;\&\; \mathcal{K}' \leftrightarrow_V \mathcal{K}\}. \tag{2}$$

Where $\mathcal{K}$ *and* $\mathcal{K}'$ *are* K*-structures.*

Note that if both ψ and ψ' are results of forgetting V from ϕ then $Mod(\psi) = Mod(\psi')$, *i.e.* , ψ and ψ' have the same models. In the sense of equivalence the forgetting result is unique (up to equivalence).

Similar with the V-bisimulation between K-structures, we define the $\langle V, I\rangle$-bisimulation between Ind-structures as follows:

Definition 3. *(*$\langle V, I\rangle$*-bisimulation) Let* $\mathcal{M}_i = (S_i, R_i, L_i, [\_]_i, s_0^i)$ *with* $i \in \{1, 2\}$ *be two Ind-structures,* V *be a set of atoms and* $I \subseteq Ind$*. The* $\langle V, I\rangle$*-bisimulation* $\beta_{\langle V,I\rangle}$ *between initial Ind-structures is a set that satisfy* $((\mathcal{M}_1, s_0^1), (\mathcal{M}_2, s_0^2)) \in \beta_{\langle V,I\rangle}$ *if and only if* $(\mathcal{M}_1, s_0^1) \leftrightarrow_V (\mathcal{M}_2, s_0^2)$ *and* $\forall j \notin I$ *there is*

(i) $\forall(s, s_1) \in [j]_1$ *there is* $(s', s'_1) \in [j]_2$ *such that* $s \leftrightarrow_V s'$ *and* $s_1 \leftrightarrow_V s'_1$*, and*
(ii) $\forall(s', s'_1) \in [j]_2$ *there is* $(s, s_1) \in [j]_1$ *such that* $s \leftrightarrow_V s'$ *and* $s_1 \leftrightarrow_V s'_1$*.*

Apparently, this definition is similar with our concept V-bisimulation except that this $\langle V, I\rangle$-bisimulation has introduced the index.

Proposition 2. *Let* $i \in \{1, 2\}$*,* $V_1, V_2 \subseteq \mathcal{A}$*,* $I_1, I_2 \subseteq Ind$ *and* $\mathcal{K}_i = (\mathcal{M}_i, s_0^i)$ $(i = 1, 2, 3)$ *be Ind-structures such that* $\mathcal{K}_1 \leftrightarrow_{\langle V_1,I_1\rangle} \mathcal{K}_2$ *and* $\mathcal{K}_2 \leftrightarrow_{\langle V_2,I_2\rangle} \mathcal{K}_3$*. Then:*

(i) $\mathcal{K}_1 \leftrightarrow_{\langle V_1 \cup V_2, I_1 \cup I_2\rangle} \mathcal{K}_3$*;*
(ii) If $V_1 \subseteq V_2$ *and* $I_1 \subseteq I_2$ *then* $\mathcal{K}_1 \leftrightarrow_{\langle V_2,I_2\rangle} \mathcal{K}_2$*.*

Proof. (i) By Proposition 1 we have $\mathcal{K}_1 \leftrightarrow_{V_1 \cup V_2} \mathcal{K}_3$. For (i) of Definition 3 we can prove it as follows: $\forall(s, s_1) \in [j]_1$ there is a $(s', s'_1) \in [j]_2$ such that $s \leftrightarrow_{V_1} s'$ and $s_1 \leftrightarrow_{V_1} s'_1$ and there is a $(s'', s''_1) \in [j]_3$ such that $s' \leftrightarrow_{V_2} s''$ and $s'_1 \leftrightarrow_{V_2} s''_1$, and then we have $\forall(s, s_1) \in [j]_1$ there is a $(s'', s''_1) \in [j]_3$ such that $s \leftrightarrow_{V_1 \cup V_2} s''$ and $s_1 \leftrightarrow_{V_1 \cup V_2} s''_1$. The (ii) of Definition 3 can be proved similarly.

(ii) This can be proved from (i).

4 The Calculus

Resolution in CTL is a method to decide the satisfiability of a CTL formula. In this paper, we will explore a resolution-based method to compute forgetting in CTL. In this part we use the transformation rules Trans(1) to Trans(12) and resolution rules (SRES1), . . . , (SRES8), RW1, RW2, (ERES1), (ERES2) in [22].

The key problems of this method include (1) How to fill the gap between CTL and $\mathrm{SNF}^g_{\mathrm{CTL}}$; and (2) How to eliminate the irrelevant atoms in the formula. We will resolve these two problems by $\langle V, I\rangle$-bisimulation and *substitution* operator. For convenient, we use $V \subseteq \mathcal{A}$ denote the set we want to forget, $V' \subseteq \mathcal{A}$ with $V \cap V' = \emptyset$ the set of atoms (I be the set of index) introduced in the transformation process, φ the CTL formula, T_φ be the set of $\mathrm{SNF}^g_{\mathrm{CTL}}$ clause obtained from φ by using transformation rules and $\mathcal{M} = (S, R, L, [\_], s_0)$ unless explicitly stated. Let T, T' be two set of formulae, I a set of indexes and $V'' \subseteq \mathcal{A}$, by $T \equiv_{\langle V'',I\rangle} T'$ we mean that $\forall(\mathcal{M}, s_0) \in \mathit{Mod}(T)$ there is a $(\mathcal{M}', s'_0)$ such that $(\mathcal{M}, s_0) \leftrightarrow_{\langle V'',I\rangle} (\mathcal{M}', s'_0)$ and $(\mathcal{M}', s'_0) \models T'$ and vice versa.

Let T be a set of $\mathrm{SNF}^g_{\mathrm{CTL}}$ clauses, then NI(T), called *nonInd*-$\mathrm{SNF}^g_{\mathrm{CTL}}$, is a set of formula obtained from T by doing the following steps:

(i) Replacing $P \supset \mathrm{E}_{\langle ind\rangle}\mathrm{X}\varphi_1, \dots, P \supset \mathrm{E}_{\langle ind\rangle}\mathrm{X}\varphi_n \in T$ with $P \supset \mathrm{EX} \bigwedge_{i\in\{0,\dots,n\}} \varphi_i$,

(ii) Replacing $P_1 \supset \mathrm{E}_{\langle ind\rangle}\mathrm{X}\varphi_1, \dots, P_n \supset \mathrm{E}_{\langle ind\rangle}\mathrm{X}\varphi_n \in T$ with $\bigwedge_{e\in 2^{\{0,\dots,n\}}\setminus\{\emptyset\}}(\bigwedge_{i\in e} P_i \supset \mathrm{EX}(\bigwedge_{i\in e} \varphi_i))$,

(iii) Replacing $P \supset \mathrm{E}_{\langle ind\rangle}\mathrm{F}\varphi_1, \dots, P \supset \mathrm{E}_{\langle ind\rangle}\mathrm{F}\varphi_n \in T$ with $P \supset \bigvee \mathrm{EF}(\varphi_{j_1} \wedge \mathrm{EF}(\varphi_{j_2} \wedge \mathrm{EF}(\cdots \wedge \mathrm{EF}\varphi_{j_n})))$, where $(j_1, \dots, j_n)$ are sequences of all elements in $\{0, \dots, n\}$.

Where P, P_i $(1 \le i \le n)$ are conjunction of literals and φ_i $(i \in \{0, \dots, n\})$ are any CTL formulae. We will not consider the case that there is $P_1 \supset \mathrm{E}_{\langle ind\rangle}\mathrm{F}l_1, \dots, P_n \supset \mathrm{E}_{\langle ind\rangle}\mathrm{F}l_n \in T$ due to there is not the case in the process of our computing forgetting.

Lemma 1. ***(NI-BRemain)*** *Let T be a set of $\mathrm{SNF}^g_{\mathrm{CTL}}$ clauses and T' be the nonInd-$\mathrm{SNF}^g_{\mathrm{CTL}}$ of T. If T is satisfiable, then we have $T \equiv_{\langle\emptyset,I\rangle}$ NI(T), where I is the set of indexes in T.*

Proof. It is easy checking that from the definition of NI.

Similarly, let T be a set of $\mathrm{SNF}^g_{\mathrm{CTL}}$ clauses, then we define the following operator:

$$T_{\mathrm{CTL}} = \{C | C' \in T \text{ and } C = D \text{ if } C' \text{is the form } \mathrm{AG}(\mathbf{start} \supset D), \text{else } C = C'\}.$$

Then $T \equiv T_{\mathrm{CTL}}$ by $\varphi \equiv \mathrm{AG}(\mathbf{start} \supset \varphi)$ [3].

The transformation of an arbitrary CTL formula into the set T_φ is a sequence $T_0, T_1, \dots, T_n = T_\varphi$ of formulae with $T_0 = \{\mathrm{AG}(\mathbf{start} \supset p), \mathrm{AG}(p \supset \mathbf{simp}(\mathbf{nnf}(\varphi)))\}$ such that for every i $(0 \le i < n)$, $T_{i+1} = (T_i \setminus \{\psi\}) \cup R_i$ [22], where ψ is a formula in T_i not in $\mathrm{SNF}^g_{\mathrm{CTL}}$ clause and R_i is the result set of applying a matching transformation rule to ψ. Note that throughout the transformation formulae are kept in negation normal form (NNF). Then we have:

Proposition 3. *Let φ be a CTL formula, then $\varphi \equiv_{\langle V',I\rangle} T_\varphi$.*

Proof. (sketch) This can be proved from T_i to T_{i+1} $(0 \leq i < n)$ by using one transformation rule on T_i.

A *derivation* on a set $V \cup V'$ of atoms and T_φ is a sequence $T_0, T_1, T_2, \ldots, T_n = T_\varphi^{r,V\cup V'}$ of sets of $\mathrm{SNF}^g_{\mathrm{CTL}}$ clauses such that $T_0 = T_\varphi$ and $T_{i+1} = T_i \cup R_i$ where R_i is a set of clauses obtained as the conclusion of the application of a resolution rule to premises in T_i. Note that all the T_i $(0 \leq i \leq n)$ are set of $\mathrm{SNF}^g_{\mathrm{CTL}}$ clauses. Besides, if there is a T_i containing **start** $\supset\perp$ or $\top\supset\perp$, then we have $\mathrm{F}_{\mathrm{CTL}}(\varphi, V) =\perp$. Then:

Proposition 4. *Let φ be a CTL formula, then $T_\varphi \equiv_{\langle V\cup V',\emptyset\rangle} T_\varphi^{r,V\cup V'}$.*

Proof. (sketch) This can be proved from T_i to T_{i+1} $(0 \leq i < n)$ by using one resolution rule on T_i.

Proposition 3 and Proposition 4 mean that $\varphi \equiv_{\langle V\cup V',I\rangle} T_\varphi^{r,V\cup V'}$, this resolve the problem (1).

For resolving problem (2), we should pay attention to the fact that by the transformation and resolution rules, we have the following several important properties:

- **(GNA)** for all atom p in *Var*(φ), p do not positively appear in the left hand of the $\mathrm{SNF}^g_{\mathrm{CTL}}$ clause;
- **(PI)** for each atom $p \in V'$, if p appearing in the left hand of a $\mathrm{SNF}^g_{\mathrm{CTL}}$ clause, then p appear positively.

An *instantiate formula* ψ of set V'' of atoms is a formula such that *Var*$(\psi) \cap V'' = \emptyset$. A key point to compute forgetting is eliminate those irrelevant atoms, for this purpose, we define the follow substitution to find out those atoms that do irrelevant.

Definition 4. *[substitution] Let $V'' = V'$ and $\Gamma = T_\varphi^{r,V\cup V'}$, then the process of substitution is as follows:*

(i) for each global clause $C = \top \supset D \vee \neg p \in \Gamma$, if there is one and on one atom $p \in V'' \cap Var(C)$ and $Var(D) \cap V = \emptyset$ then let $C = p \supset D$ and $V'' := V'' \setminus \{p\}$;
(ii) find out all the possible instantiate formulae $\varphi_1, ..., \varphi_m$ of $V \cup V''$ in the $p \supset \varphi_i \in \Gamma$ $(1 \leq i \leq m)$;
(iii) if there is $p \supset \varphi_i$ for some $i \in \{1, \ldots, m\}$, then let $V'' := V'' \setminus \{p\}$, which means p is a instantiate formula;
(iv) for $\bigwedge_{j=1}^m p_j \supset \varphi_i \in \Gamma$ $(i \in \{1, \ldots, m\})$, if there is $\alpha \supset p_1, \ldots, \alpha \supset p_m \in \Gamma$ then let $\Gamma_1 := \Gamma \cup \{\alpha \supset \varphi\}$. if $\Gamma_1 \neq \Gamma$ then let $\Gamma := \Gamma_1$ go to step (i), else return $V \cup V''$.

*Where p, p_i $(1 \leq i \leq m)$ are atoms and α is a conjunction of literals or **start**.*

We denote this process as $\mathrm{Sub}(\Gamma, V')$.

Example 1. Let $\varphi = \mathrm{A}((p \wedge q)\mathrm{U}(f \vee m)) \wedge r$ and $V = \{p\}$. Then we can compute $\mathrm{Sub}(T_\varphi^{r,V\cup V'}, V)$ as follows:

Algorithm 1: Computing $\text{Sub}(\Gamma, V')$

Input: A set Γ of $\text{SNF}^g_{\text{CTL}}$ clauses φ and $V, V' \subseteq \mathcal{A}$
Output: A set of atoms

Let $V'' := V'$;
Let $V_1 = \emptyset$;
Let $\Gamma_1 := \emptyset$;
Let $\Gamma_2 := \Gamma$;
while *($\Gamma_1 \neq \Gamma_2$ or $V_1 \neq V''$)* **do**
  $\Gamma_1 := \Gamma_2$;
  $V_1 := V''$;
  for *($C \in \Gamma_2$)* **do**
    if *(C is a global clause)* **then**
      Let $C := D \vee \neg p$;
      if *($p \in V'' \cap \mathit{Var}(C)$ and $\mathit{Var}(D) \cap V == \emptyset$)* **then**
        $C := p \supset D$;
        $V'' := V'' \setminus \{p\}$;
      end
    end
  end
  for *($C \in \Gamma_2$)* **do**
    if *($C == p \supset \varphi$ and $p \in V''$ and $\mathit{Var}(\varphi) \cap V \cup V'' == \emptyset$)* **then**
      $V'' := V'' \setminus \{p\}$;
    end
  end
  for *($C \in \Gamma_2$)* **do**
    if *($C == \bigwedge_{j=1}^{m} p_j \supset \varphi$ and $\mathit{Var}(\varphi) \cap V \cup V'' == \emptyset$)* **then**
      if *(there is $\alpha \supset p_1, \ldots, \alpha \supset p_m \in \Gamma_2$)* **then**
        $\Gamma_2 := \Gamma_2 \cup \{\alpha \supset \varphi\}$;
      end
    end
  end
end
return $V \cup V''$.

At first, we transform φ into a set of $\text{SNF}^g_{\text{CTL}}$ with $V' = \{x, y, z\}$, which is listed as:

1.**start** $\supset z$	2.$\top \supset \neg z \vee r$	3.$\top \supset \neg x \vee f \vee m$
4.$\top \supset \neg z \vee x \vee y$	5.$\top \supset \neg y \vee p$	6.$\top \supset \neg y \vee q$
7.$z \supset \text{AF}x$	8.$y \supset \text{AX}(x \vee y)$.	

In the second, we compute all the possible resolutions on $V \cup V'$ and is listed as:

(1)**start** $\supset r$	(2)**start** $\supset x \vee y$	$(3)\top \supset \neg z \vee y \vee f \vee m$
$(4) y \supset \mathrm{AX}(f \vee m \vee y)$	$(5)\top \neg z \vee x \vee p$	$(6)\top \neg z \vee x \vee q$
$(7) y \supset \mathrm{AX}(x \vee p)$	$(8) y \supset \mathrm{AX}(x \vee q)$	(9)**start** $\supset f \vee m \vee y$
(10)**start** $\supset x \vee p$	(11)**start** $\supset x \vee q$	$(12)\top \supset p \vee \neg z \vee f \vee m$
$(13)\top \supset q \vee \neg z \vee f \vee m$	$(14) y \supset \mathrm{AX}(p \vee f \vee m)$	$(15) y \supset \mathrm{AX}(q \vee f \vee m)$
(16)**start** $\supset f \vee m \vee p$	(17)**start** $\supset f \vee m \vee q.$	

By the process of substitution we obtain that y is instantiated by $q \wedge \mathrm{AX}(p \vee f \vee m)$, x is instantiated by $f \vee m$ and z is instantiated by r. That is Sub$(T_{\varphi}^{r,V\cup V'}, V') = V$, which means all the introduced atoms are instantiated.

By Sub operator, we guarantee those atoms in $V \cup V''$ are really irrelevant atoms.
Let P be a conjunction of literals, l, l_1 be literals, in which $\mathit{Var}(C_1) \cap V \cup V' = \emptyset$, and C_i ($i \in \{2,3,4\}$) be classical clauses. As we can see that those exist resolution rules cannot deduct all the possible result, so we add following new rules:

(EF1)$\{P \supset \mathrm{AF}l, P \supset \mathrm{E}_{\langle ind\rangle}\mathrm{X}(l_1 \vee C_4), l \supset \neg l_1 \vee C_2, l \supset C_3 \vee C_2\}$
$\rightarrow \{P \supset ((\neg C_3 \wedge \neg C_2) \supset (\mathrm{E}_{\langle ind\rangle}\mathrm{X}(C_3 \wedge \neg(C_2 \vee C_4) \supset \mathrm{AXAF}(C_3 \vee C_2)))))\}$,
(EF2)$\{P \supset \mathrm{AF}l, P \supset \mathrm{AX}(l_1 \vee C_4), l \supset \neg l_1 \vee C_2, l \supset C_3 \vee C_2\}$
$\rightarrow \{P \supset ((\neg C_3 \wedge \neg C_2) \supset (\mathrm{AX}(C_3 \wedge \neg(C_2 \vee C_4) \supset \mathrm{AXAF}(C_3 \vee C_2)))))\}$
(EF3)$\{P \supset \mathrm{E}_{\langle ind\rangle}\mathrm{F}l, P \supset \mathrm{E}_{\langle ind\rangle}\mathrm{X}(l_1 \vee C_4), l \supset \neg l_1 \vee C_2, l \supset C_3 \vee C_2\}$
$\rightarrow \{P \supset ((\neg C_3 \wedge \neg C_2) \supset (\mathrm{E}_{\langle ind\rangle}\mathrm{X}(C_3 \wedge \neg(C_2 \vee C_4) \supset \mathrm{AXAF}(C_3 \vee C_2)))))\}$,
(EF4)$\{P \supset \mathrm{E}_{\langle ind\rangle}\mathrm{F}l, P \supset \mathrm{AX}(l_1 \vee C_4), l \supset \neg l_1 \vee C_2, l \supset C_3 \vee C_2\}$
$\rightarrow \{P \supset ((\neg C_3 \wedge \neg C_2) \supset (\mathrm{E}_{\langle ind\rangle}\mathrm{X}(C_3 \wedge \neg(C_2 \vee C_4) \supset \mathrm{AXAF}(C_3 \vee C_2)))))\}$

Note that the conclusion of EF4 is not stated in normal form. Similarly with that in [22], to present the conclusion of EF4 in normal form, we introduce a set $V_F = \{w_1, w_2, w_3, w_4\}$ of new atomic propositions. Then the conclusion of EF4 can be represented by the following set W of $\mathrm{SNF}^g_{\mathrm{CTL}}$ clauses.

$$\begin{aligned}
&\{\top \supset \neg P \vee C_3 \vee C_2 \vee w_1\} \\
&\cup \{w_1 \supset \mathrm{E}_{\langle ind\rangle}\mathrm{X}(\neg C_{3,i} \vee C_2 \vee C_4 \vee w_2) | C_{3,i} \text{ is a literal in } C_3\} \\
&\cup \{w_2 \supset \mathrm{AX}w_3, w_3 \supset \mathrm{AF}w_4, w_4 \supset C_3 \vee C_2\}
\end{aligned}$$

Similarly, the conclusion of other rules (EF2 to EF4) can be represented by a set of $\mathrm{SNF}^g_{\mathrm{CTL}}$ clauses.
By $\mathrm{EF}(\mathrm{Sub}(T_{\varphi}^{r,V\cup V'}, V'))$ we mean using **(EF1)** to **(EF4)** on $T_{\varphi}^{r,V\cup V'}$ and replace $P \supset \mathrm{E}_{\langle ind\rangle}\mathrm{X}(\neg l \vee C_2 \vee C_4)$ with $P \supset \mathrm{E}_{\langle ind\rangle}\mathrm{X}(\neg l \vee C_2 \vee C_4) \vee \bigwedge W$ when l, C_2, C_3 and C_4 are instantiate formulae of $\mathrm{Sub}(T_{\varphi}^{r,V\cup V'}, V')$ and $\mathit{Var}(l_1) \in V \cup V'$. Let V^F be a set of atom introduced by transforming the result of those rules into $\mathrm{SNF}^g_{\mathrm{CTL}}$. We have:

Proposition 5. *Let $\Gamma = T_{\varphi}^{r,V\cup V'}$, we have $\Gamma \equiv_{\langle V'\cup V^F, \emptyset\rangle} \mathrm{EF}(\mathit{Sub}(\Gamma, V'))$.*

Proof. It is obvious from the **(EF1)** to **(EF4)**.

We prove the (EF1), for other rules can be proved similarly. Let $T_{i+1} = T_i \cup \{\varphi\}$, where $\{\varphi\}$ is obtained from T_i by using rule (EF1) on T_i, *i.e.* $\varphi = P \supset ((\neg C_3 \wedge \neg C_2) \supset (\mathrm{E}_{\langle ind\rangle}\mathrm{X}(C_3 \wedge \neg(C_2 \vee C_4) \supset \mathrm{AXAF}(C_3 \vee C_2))))$. It is apparent that $T_{i+1} \models T_i$ and $T_i \models P \supset \mathrm{E}_{\langle ind\rangle}\mathrm{X}(\neg l \vee C_2 \vee C_4)$. We will show that $\forall(\mathcal{M}, s_0) \in \mathit{Mod}(T_i)$ there is an initial Ind-structure $(\mathcal{M}', s_0')$ such that $(\mathcal{M}', s_0') \models T_{i+1}$ and $(\mathcal{M}', s_0') \leftrightarrow_{\langle V' \cup V^F, \emptyset\rangle} (\mathcal{M}, s_0)$

$\forall(\mathcal{M}, s) \models T_i$ we suppose $(\mathcal{M}, s) \models P \wedge \neg C_3 \wedge \neg C_2$ and $(\mathcal{M}, s_1) \models C_3 \wedge \neg C_2 \wedge \neg C_4$ with $(s, s_1) \in [ind]$ (due to other case can be proved easily). Then we have $(\mathcal{M}, s) \not\models l$ (by $(\mathcal{M}, s) \models l \supset C_3 \vee C_2$) and $(\mathcal{M}, s_1) \models l_1$ (by $(\mathcal{M}, s) \models P \supset \mathrm{E}_{\langle ind\rangle}\mathrm{X}(l_1 \vee C_4)$). If $(\mathcal{M}, s_1) \not\models \mathrm{AXAF}(C_3 \vee C_2)$ then we have $(\mathcal{M}, s_1) \models l$ due to $(\mathcal{M}, s) \models \mathrm{AG}(l \supset C_3 \vee C_2)$ and $(\mathcal{M}, s) \models \mathrm{AF}l$. And then $(\mathcal{M}, s_1) \models \neg l_1$ by $(\mathcal{M}, s) \models \mathrm{AG}(l \supset \neg l_1 \vee C_2)$. It is contract with our supposing. Then $(\mathcal{M}, s_1) \models \mathrm{AXAF}(C_3 \vee C_2)$.

By Proposition 3 we have $\forall(\mathcal{M}, s) \in \mathit{Mod}(\varphi)$ there is an initial Ind-structure $(\mathcal{M}', s')$ such that $(\mathcal{M}', s') \models T_\varphi$ and $(\mathcal{M}, s) \leftrightarrow_{\langle V_F, \emptyset\rangle} (\mathcal{M}, s)$.

Therefore, we can do the following elimination to eliminate them.

Definition 5. ***(Elimination)*** *Let T be a set of formulae, $C \in T$ and V a set of atoms, then the elimination operator, denoted as Elm, is defined as:*

$$\mathit{Elm}(C, V) = \begin{cases} \top, & if\ \mathit{Var}(C) \cap V \neq \emptyset \\ C, & else. \end{cases}$$

For convenience, we let $\mathit{Elm}(T, V) = \{\mathit{Elm}(r, V) | r \in T\}$.

Proposition 6. *Let $V'' = V \cup V'$, $\Gamma = \mathit{Sub}(T_\varphi^{r,V''}, V')$ and $\Gamma_1 = \mathit{Elm}(\mathrm{EF}(\Gamma), \Gamma)$, then $\Gamma_1 \equiv_{\langle V \cup V' \cup V^F, \emptyset\rangle} T_\varphi^{r,V''}$ and $\Gamma_1 \equiv_{\langle V \cup V' \cup V^F, I\rangle} \varphi$.*

Proof. Note the fact that for each clause $C = T \supset H$ in $\mathrm{EF}(\Gamma)$, if $\Gamma \cap \mathit{Var}(C) \neq \emptyset$ then there must be an atom $p \in \Gamma \cap \mathit{Var}(H)$. It is apparent that $\mathrm{EF}(\Gamma) \models \Gamma_1$, we will show $\forall(\mathcal{M}, s_0) \in \mathit{Mod}(\Gamma_1)$ there is a $(\mathcal{M}', s_0)$ such that $(\mathcal{M}', s_0) \models \mathrm{EF}(\Gamma)$ and $(\mathcal{M}, s_0) \leftrightarrow_{\langle \Gamma, \emptyset\rangle} (\mathcal{M}', s_0)$. Let $C = T \supset H$ in $\mathrm{EF}(\Gamma)$ with $\Gamma \cap \mathit{Var}(C) \neq \emptyset$, $\forall(\mathcal{M}, s_0) \in \mathit{Mod}(\Gamma_1)$ we construct $(\mathcal{M}', s_0)$ as $(\mathcal{M}, s_0)$ except for each $s \in S$, if $(\mathcal{M}, s) \not\models T$ then $L'(s) = L(s)$, else:

(i) if $(\mathcal{M}, s) \models H$, then $L'(s) = L(s)$;
(ii) else if $(\mathcal{M}, s) \models T$ with $p \in \mathit{Var}(H) \cap V$, then if p appearing in H negatively, then if C is a global (or an initial) clause then let $L'(s) = L(s) \setminus \{p\}$ else let $L'(s_1) = L(s_1) \setminus \{p\}$ for (each (if C is an A-step or A-sometime clause)) $(s, s_1) \in R$, else if C is a global (or an initial) clause then let $L'(s) = L(s) \cup \{p\}$ else let $L'(s_1) = L(s_1) \cup \{p\}$ for (each (if C is a A-step or A-sometime clause)) $(s, s_1) \in R$.
(iii) for other clause $C = Q \supset H$ with $p \in \mathit{Var}(H) \cap \Gamma$, we can do it as (ii).

It is apparent that $(\mathcal{M}, s_0) \leftrightarrow_{\langle \Gamma, \emptyset\rangle} (\mathcal{M}', s_0)$, we will show that $(\mathcal{M}', s_0) \models \mathrm{EF}(\Gamma)$ from the following two points:

Algorithm 2: Computing $\text{EF}(\Gamma, V)$

Input: A set Γ of $\text{SNF}^g_{\text{CTL}}$ clauses, a set of A-step clauses, a set of E-step clauses and $V \subseteq \mathcal{A}$

Output: A set of formulae

$C_1 := P \supset ((\neg C_3 \wedge \neg C_2) \supset (\text{E}_{\langle ind \rangle}\text{X}(C_3 \wedge \neg(C_2 \vee C_4) \supset \text{AXAF}(C_3 \vee C_2))))$;
$C'_1 := P \supset ((\neg C_3 \wedge \neg C_2) \supset (\text{AX}(C_3 \wedge \neg(C_2 \vee C_4) \supset \text{AXAF}(C_3 \vee C_2))))$;
for *($C \in A$)* **do**
 Let $C == P \supset \text{AF}l$;
 if *($P \supset \text{E}_{\langle ind \rangle}\text{X}(l_1 \vee C_4), l \supset \neg l_1 \vee C_2, l \supset C_3 \vee C_2 \in \Gamma$ and l, C_2, C_3, C_4 are initial formulae)* **then**
  Replacing $P \supset \text{E}_{\langle ind \rangle}\text{X}(\neg l \vee C_2 \vee C_4)$ with $P \supset \text{E}_{\langle ind \rangle}\text{X}(\neg l \vee C_2 \vee C_4) \vee \bigwedge \text{EX}(C_1, L)$;
 end
 if *($P \supset \text{AX}(l_1 \vee C_4), l \supset \neg l_1 \vee C_2, l \supset C_3 \vee C_2 \in \Gamma$ and l, C_2, C_3, C_4 are initial formulae)* **then**
  Replacing $P \supset \text{AX}(\neg l \vee C_2 \vee C_4)$ with $P \supset \text{AX}(\neg l \vee C_2 \vee C_4) \vee \bigwedge \text{AX}(C'_1, L)$;
 end
end
for *($C \in E$)* **do**
 Let $C == P \supset \text{E}_{\langle ind \rangle}\text{F}l$;
 if *($P \supset \text{E}_{\langle ind \rangle}\text{X}(l_1 \vee C_4), l \supset \neg l_1 \vee C_2, l \supset C_3 \vee C_2 \in \Gamma$ or $P \supset \text{AX}(l_1 \vee C_4), l \supset \neg l_1 \vee C_2, l \supset C_3 \vee C_2 \in \Gamma$ and l, C_2, C_3, C_4 are initial formulae)* **then**
  Replacing $P \supset \text{E}_{\langle ind \rangle}\text{X}(\neg l \vee C_2 \vee C_4)$ with $P \supset \text{E}_{\langle ind \rangle}\text{X}(\neg l \vee C_2 \vee C_4) \vee \bigwedge \text{EX}(C_1, L)$;
 end
end
return Γ.

Algorithm 3: $\text{EX}(C, L)$

Input: A formula C

Output: A set $\text{SNF}^g_{\text{CTL}}$ clauses

$C == P \supset ((\neg C_3 \wedge \neg C_2) \supset (\text{E}_{\langle ind \rangle}\text{X}(C_3 \wedge \neg(C_2 \vee C_4) \supset \text{AXAF}(C_3 \vee C_2))))$;
$L_1 := \{\top \supset \neg P \vee C_3 \vee C_2 \vee w_1\}$;
$L_2 := \{w_1 \supset \text{E}_{\langle ind \rangle}\text{X}(\neg l \vee C_2 \vee C_4 \vee w_2) | l \text{ is a literal in } C_3\}$;
$L_3 := \{w_2 \supset \text{AX}w_3, w_3 \supset \text{AF}w_4, w_4 \supset C_3 \vee C_2\}$;
$L := L_1 \cup L_2 \cup L_3$;
return L.

(1) For (ii) talked-above, we show it from the form of $\text{SNF}^g_{\text{CTL}}$ clauses. Supposing C_1 and C_2 are instantiate formula of Γ:

Algorithm 4: $\text{AX}(C, L)$

Input: A formula C
Output: A set $\text{SNF}^g_{\text{CTL}}$ clauses

$C == P \supset ((\neg C_3 \wedge \neg C_2) \supset (\text{AX}(C_3 \wedge \neg(C_2 \vee C_4) \supset \text{AXAF}(C_3 \vee C_2))))$;
$L_1 := \{\top \supset \neg P \vee C_3 \vee C_2 \vee w_1\}$;
$L_2 := \{w_1 \supset \text{AX}(\neg l \vee C_2 \vee C_4 \vee w_2) | l$ is a literal in $C_3\}$;
$L_3 := \{w_2 \supset \text{AX}w_3, w_3 \supset \text{AF}w_4, w_4 \supset C_3 \vee C_2\}$;
$L := L_1 \cup L_2 \cup L_3$;
return L.

(a) If C is a global clause, i.e. $C = \top \supset p \vee C_1$ with C_1 is a disjunction of literals (we suppose p appearing in C positively). If there is a $C' = \top \supset \neg p \vee C_2 \in \text{EF}(\Gamma)$, then there is $\top \supset C_1 \vee C_2 \in \text{EF}(\Gamma)$ by the resolution ($(\mathcal{M}, s) \models C_2$ due to we have suppose $(\mathcal{M}, s) \not\models C$). It is apparent that $(\mathcal{M}', s_0) \models C \wedge C'$.

(b) If $C = T \supset \text{E}_{\langle ind \rangle}\text{X}(p \vee C_1)$. If there is a $C' = T' \supset \text{E}_{\langle ind \rangle}\text{X}(\neg p \vee C_2) \in \text{EF}(\Gamma)$, then there is $T \wedge T' \supset \text{E}_{\langle ind \rangle}\text{X}(C_1 \vee C_2) \in \text{EF}(\Gamma)$ by the resolution ($(\mathcal{M}, s) \models \text{E}_{\langle ind \rangle}\text{X}C_2$ due to we have suppose $(\mathcal{M}, s) \not\models C$). It is apparent that $(\mathcal{M}', s_0) \models C \wedge C'$.

(c) Other cases can be proved similarly.

(2) (iii) can be proved as (ii) due to the fact we point at the beginning.

Therefore, we have $\Gamma_1 \equiv_{\langle V \cup V' \cup V^F, \emptyset \rangle} T^{r, V''}_{\varphi}$ by Proposition 2 and Proposition 5.

And then $\Gamma_1 \equiv_{\langle V \cup V' \cup V^F, I \rangle} \varphi$ follows.

Remember that the auxiliary propositions have been introduced by the process of transforming a formula into a set of $\text{SNF}^g_{\text{CTL}}$ clauses is only to rename those subformulae of φ [3]. For convenience, let Ω be a sequence of sets of $\text{SNF}^g_{\text{CTL}}$ clauses, in which each set can be seen as a formula of conjunction of clauses in it, obtained from *Elm*$(\text{EF}(\Gamma), \Gamma)$ by using distribution law by viewing each $\text{SNF}^g_{\text{CTL}}$ as an atom. Hence we have the following process.

Let $\Gamma = \text{Sub}(T^{r, V \cup V'}_{\varphi}, V')$ and $\Gamma_1 = \text{NI}(\Omega)$, we can easily change Γ_1 into a formula of the form $\Omega_1 = \bigwedge X \wedge \bigwedge_{p \in (V' \setminus \Gamma) \cup V^F} (p \supset \varphi_1 \vee \cdots \vee p \supset \varphi_n)$ with φ is an instantiate formula of Γ and X is a set of formulas. Then $R(\Gamma_1)$ is obtained from Ω_1 by doing the following two steps for each $p \in (V' \setminus \Gamma) \cup V^F$:

- replacing each $p \supset \varphi_1 \vee \cdots \vee p \supset \varphi_n$ with $p \supset \bigvee_{i \in \{1, \ldots, n\}} \varphi_i$;
- replacing $p \supset \varphi_1 \wedge \cdots \wedge p \supset \varphi_m$, φ_j are instantiate formulae of Γ $(j \in \{1, \ldots, m\})$, then let $\psi = \bigwedge_{i=1}^{j_n} \varphi_{j_i}$, where p do not appear in φ_{j_i}, with $p \leftrightarrow \psi$.
- For other formula $C \in \Omega_1$, replacing every p in C with ψ.

Where $\text{NI}(S)$ means do $\text{NI}(e)$ for each $e \in S$ with S is a set of sets of $\text{SNF}^g_{\text{CTL}}$ clause. Apparently, this process is just a process of replacing each atom with an equivalent formula. Then we have:

Proposition 7. *Let $\Gamma = T^{r, V \cup V'}_{\varphi}$, $\Gamma_1 = Sub(\Gamma, V')$ and $\Gamma_2 = Elm(\text{EF}(\Gamma_1), \Gamma_1)$, then $\Gamma_2 \equiv_{\langle (V' \setminus \Gamma_1) \cup V^F, \emptyset \rangle} R(NI(\Gamma_2))$. and $\varphi \equiv_{\langle V \cup V' \cup V^F, I \rangle} R(NI(\Gamma_2))_{CTL}$.*

Proof. For each p talked above is a name of the formula ψ, *i.e.* $p \leftrightarrow \psi$. Then $\Gamma_2 \equiv_{\langle (V' \setminus \Gamma_1) \cup V^F, \emptyset \rangle} R(\Gamma_2)$, and then $\Gamma_2 \equiv_{\langle V \cup V' \cup V^F, \emptyset \rangle} R(\Gamma_2)$ by (V) of Proposition 1.

Therefore, $\varphi \equiv_{\langle V \cup V' \cup V^F, I \rangle} \mathrm{NI}(R(\Gamma_2))_{CTL}$ by Proposition 6 and the definitions of NI and T_{CTL}.

In the case that formula dose not include index, we use model structure $\mathcal{M} = (S, R, L, s_0)$ to interpret formula instead of Ind-model structure. Therefore it is apparent that $\forall(\mathcal{M}, s_0) \in Mod(\varphi)$ there is a $(\mathcal{M}', s_0') \in Mod(\Gamma_1)$ such that $(\mathcal{M}, s_0) \leftrightarrow_{V \cup V'} (\mathcal{M}', s_0')$ and vice versa.

Theorem 1. *Let $V'' = V \cup V'$, $\Gamma = Sub(T_\varphi^{r,V''}, V')$ and $\Gamma_1 = R(NI(Elm(\mathrm{EF}(\Gamma), \Gamma)))_{CTL}$, then*

$$\mathrm{F}_{\mathrm{CTL}}(\varphi, V' \cup V) \equiv \Gamma_1.$$

Proof. $(\Rightarrow)$ $\forall(\mathcal{M}, s_0) \in Mod(\mathrm{F}_{\mathrm{CTL}}(\varphi, V' \cup V))$
$\Rightarrow \exists(\mathcal{M}', s_0') \in Mod(\varphi)$ s.t. $(\mathcal{M}, s_0) \leftrightarrow_{V' \cup V} (\mathcal{M}', s_0')$
$\Rightarrow \exists(\mathcal{M}_1, s_1) \in Mod(Elm(\mathrm{Sub}((T_\varphi^{r,V \cup V'})', V)_{CTL}, V \cup V'))$ s.t. $(\mathcal{M}_1, s_1) \leftrightarrow_{V' \cup V} (\mathcal{M}', s_0')$
$\Rightarrow (\mathcal{M}, s_0) \leftrightarrow_{V' \cup V} (\mathcal{M}_1, s_1)$
$\Rightarrow (\mathcal{M}, s_0) \models Elm(\mathrm{Sub}((T_\varphi^{r,V \cup V'})', V)_{CTL}, V \cup V')$ (IR($Elm(\mathrm{Sub}((T_\varphi^{r,V \cup V'})', V)_{CTL}, V \cup V'), V' \cup V$))

$(\Leftarrow)$ $\forall(\mathcal{M}_1, s_1) \in Mod(Elm(\mathrm{Sub}((T_\varphi^{r,V \cup V'})', V)_{CTL}, V \cup V'))$
$\Rightarrow \exists(\mathcal{M}', s_0') \in Mod(\varphi)$ s.t. $(\mathcal{M}_1, s_1) \leftrightarrow_{V' \cup V} (\mathcal{M}', s_0')$
$\Rightarrow (\mathcal{M}_1, s_1) \models \mathrm{F}_{\mathrm{CTL}}(\varphi, V' \cup V)$ (IR($\mathrm{F}_{\mathrm{CTL}}(\varphi, V' \cup V), V \cup V'$) and $\varphi \models \mathrm{F}_{\mathrm{CTL}}(\varphi, V' \cup V)$)

Then we have the following result:

Theorem 2. ***(Resolution-based CTL-forgetting)*** *Let $V'' = V \cup V'$, $\Gamma = Sub(T_\varphi^{r,V''}, V')$ and $\Gamma_1 = R(NI(Elm(\mathrm{EF}(\Gamma), \Gamma)))_{CTL}$, then*

$$\mathrm{F}_{\mathrm{CTL}}(\varphi, V) \equiv \bigwedge_{\psi \in \Gamma_1} \psi.$$

We can obtain that $\mathrm{F}_{\mathrm{CTL}}(\varphi, V) \equiv \mathrm{F}_{\mathrm{CTL}}(\varphi, V' \cup V)$ by Theorem 1, Proposition **??** and Proposition **??**. Therefore, the Theorem 2 is proved.

Then we can obtain the result of forgetting of Example 1:

$$\begin{aligned}
&\mathrm{F}_{\mathrm{CTL}}(\varphi, \{p\}) \equiv r \wedge (f \vee m \vee q) \wedge \\
&(f \vee m \vee (q \wedge \mathrm{AX}(f \vee m \vee q))) \wedge \mathrm{AG}((q \wedge \mathrm{AX}(f \vee m \vee q) \\
&\supset \mathrm{AX}(f \vee m \vee (q \wedge \mathrm{AX}(f \vee m \vee q))))).
\end{aligned}$$

Given two clauses C and C', we call C and C' are resolvable, the result denote as $res(C, C')$, if there is a resolution rule using C and C' as the premises on some given atom. Then the pseudocode of algorithm resolution-based is as Algorithm 5.

Proposition 8. *Let φ be a CTL formula and $V \subseteq \mathcal{A}$. The time and space complexity of Algorithm 5 are $O((m+1)2^{4(n+n')})$. Where $|Var(\varphi)| = n$, $|V'| = n'$ (V' is set of atoms introduced in transformation) and m is the number of the set Ind of indices introduced during transformation.*

Proof. It follows from that the lines 19-31 of the algorithm, which is to compute all the possible resolution. The possible number of $\mathrm{SNF}^{g}_{\mathrm{CTL}}$ clauses under the give V, V' and Ind is $(m+1)2^{4(n+n')} + (m*(n+n')+n+n'+1)2^{2(n+n')+1})$.

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Algorithm 5: Computing forgetting - A resolution-based method

Input: A CTL formula φ and a set V of atoms
Output: $\mathrm{F}_{\text{CTL}}(\varphi, V)$
$T = \emptyset$ // the initial set of $\mathrm{SNF}^g_{\text{CTL}}$ clauses of φ ;
$T' = \emptyset$ // the set of $\mathrm{SNF}^g_{\text{CTL}}$ clauses without index;
$V' = \emptyset$ // the set of atoms introduced in the process of transforming φ into $\mathrm{SNF}^g_{\text{CTL}}$ clauses;
$OldT = \{$**start** $\supset z, z \supset \varphi\}$;
$V' = \{z\}$;
while $OldT \neq T$ **do**
  $OldT = T$;
  $R = \emptyset$;
  $X = \emptyset$;
  if *Chose a formula* $\psi \in OldT$ *that dose not a* $\mathrm{SNF}^g_{\text{CTL}}$ *clause* **then**
    Using a match rule Rl to transform ψ into a set R of $\mathrm{SNF}^g_{\text{CTL}}$ clauses;
    X is the set of atoms introduced by using Rl;
    $V' = V' \cup X$;
    $T = OldT \setminus \{\psi\} \cup R$;
  end
end
$S = \{C | C \in T$ and $\mathit{Var}(C) \cap V = \emptyset\}$;
$\Pi = T \setminus S$;
for $(p \in V \cup V')$ **do**
  $\Pi' = \{C \in \Pi | p \in \mathit{Var}(C)\}$;
  $\Sigma = \Pi \setminus \Pi'$;
  for *($C \in \Pi'$ s.t. p appearing in C positively)* **do**
    for *($C' \in \Pi'$ s.t. p appearing in C' negatively and C, C' are resolvable)* **do**
      $\Sigma = \Sigma \cup \{res(C, C')\}$;
      $\Pi' = \Pi' \cup \{C'' = res(C, C') | p \in \mathit{Var}(C'')\}$;
    end
  end
  $\Pi = \Sigma$;
end
$Res = \Pi \cup S$;
$\Gamma = \mathrm{Sub}(Res, V')$;
Let Ω, which obtained from $\mathit{Elm}(\mathrm{EF}(\Gamma), \Gamma)$ by using distribution law by viewing each $\mathrm{SNF}^g_{\text{CTL}}$ clause as an atom, be a set of sets of $\mathrm{SNF}^g_{\text{CTL}}$ clauses.;
$\Gamma_1 \leftarrow \mathrm{NI}(\Omega)$;
Transform Γ_1 into a formula [6]:

$$\Gamma_2 = \bigwedge X \wedge \bigwedge_{p \in (V' \setminus \Gamma) \cup V^F} (p \supset \varphi_1 \vee \cdots \vee p \supset \varphi_n)$$

return $\bigwedge_{\psi \in R(\Gamma_2)_{\text{CTL}}} \psi$.