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Ref.:

Ms. No. AMAI-D-21-00168

Knowledge Forgetting in Propositional µ-calculus

Annals of Mathematics and Artificial Intelligence

Dear Dr. Wang,

Reviewers have now commented on your paper. You will see that they are interested in continuing the consideration of your work and are advising that you revise your manuscript. If you are prepared to undertake the work required, the editors would be pleased to reconsider a revised submission.

The reviewers' comments can be found at the end of this email or can be accessed by following the provided link.

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Yours sincerely

Martin Charles Golumbic

Editor-in-Chief

Annals of Mathematics and Artificial Intelligence

Reviewers' comments:

Reviewer #1: Knowledge forgetting has its roots in the works of Boole, but has attracted a considerable amount of interest recently, in particular in the context of logic programming and action logics, but also more generally in knowledge representation and reasoning. So, forgetting has been intensely studied in logical frameworks. This paper elaborates on establishing a corresponding foundation of forgetting in the framework of the \mu-calculus, presenting a representation theorem, and applications of this framework to computing weakest sufficient conditions resp. strongest necessary conditions, and to knowledge update.

The paper is well written, the techniques are developed carefully, the proofs seem to be sound (after a superficial check), and the results are (to the best of my knowledge, though I'm not a specialist in \mu-calculus) novel, interesting, and significant. So, I only have minor comments (see below), except for two more major issues: **First,** a general recommendation: The style of the paper is very mathematical, in particular, in the proofs. This is okay for the proofs in the appendix, but for the proofs within the paper, I would prefer a more readable, verbal style (see, e.g., proof of Ths.3 and 15). Moreover, the authors are quite brief and implicit in some places, e.g., around Th.11: The precise relationships between V, \varphi, and \psi are crucial for this theorem (and also for the (equivalent) problems (i)-(iii) and Cor.12), so they should be part of the theorem resp. problems resp. corollary. **Second,** regarding the WSC resp. SNC, no comparison is made to the related work

[1] (see below), this would be mandatory.

While the English of the paper is generally quite good, there are some awkward phrasings which should be improved (see minor comments below).

Minor comments and linguistic issues:

p.1, l.46: desire -> desirable

p.3, l.59: Please introduce the abbreviation PL properly.

p.3, l.61: various \* logic languages

p.6, l.25: as a subset \*of\* the set of states

p.7, l.37: It has \*been\* shown

p.9, Fig.1: In the RHS, I guess the nodes should be t\_0 and t\_1.

p.9, l.55: It's better to avoid using logical symbols on a meta-level too much in a paper on logics, so I'd prefer to write "such that" instead of "&".

p.10, Th.2 and the following sentence: It's straightforward to generalize Th.2 to sets of atoms, but this should be mentioned.

p.10: **The Forgetting postulates of [10] should not be presented within a sentence (??)**

**p.13, Prop.8: Can you explain on a meta-level what these equivalences mean?**

p.13: Ex.2 is a continuation of Ex.1, right? This should be mentioned, otherwise the reader cannot understand the hint to "ch" in the sentence before Ex.2.

p.14, l.25: Let ... be (not "are", "is").

p.16, l.15: a new proposition - do you mean "a new atom"? The same applies to Prop.16.

p.18f**,** l.14: What do you mean by saying "a minimal subset of atoms that makes \varphi consistent"? Regarding knowledge update, why is the "pam" approach (should it be "pma" = possible models approach?) used for motivation? It does not really fit, why not recalling the original KM approach via general (pre)orders? This would make Th.20 a perfect fit.

——yes, "pma" = possible models approach

p.22, l.25: two Kripke structure\*s\*

p.23: There are some strange phrasings here: l.32, 38: one "iff" is too much; l.56: eliminate "be"; l.61: there is -> we have (also in other places).

p.23, l.58f: In the specification of the s'' - what are s and s' here? The specification of S' seems to be wrong.

p.24, l.24, 39: Pro.1 -> Prop.1.

p.24, **footnote 4: It would be good to have a paper as reference here.**

p.25, l.48: a\*n\* SC

p.26, l.47: two \mu-sentence\*s\*; l.51: wsc should be upper case.

p.27; a\*n\* \mathcal{M} (several times)

p.27, l.39: follows -> implies

Both \phi and \varphi are used. Since both are variants of \phi, I'd suggest to use the same symbol if \phi is meant, and a different symbol if one would like to make a difference (e.g., in Th.3).

[1] Su K, Sattar A, Lv G, Zhang Y (2009) Variable forgetting in reasoning about knowledge. J Artif Intell Res 35:677-716, DOI 10.1613/jair.2750, URL https://doi.org/10.1613/jair.2750

**Reviewer #2:** The article presents a definition and analysis of forgetting in the

mu-calculus. It does not diverge from techniques used in other modal logics: it

extends bisimulation and use it to define forgetting; it then show that

forgetting is characterized by some reasonable properties, and possess other

reasonable properties. It then shows how to apply forgetting to the problems of

weakest necessary condition and knowledge update. A complexity analisys is also

done.

All of this has been done in the past for other logics, and especially other

kinds of modal logics like S5 and temporal logics. To my knowledge, it was not

done so far for the mu-calculus. While not a fundamental study, this one is

adequately done. It advances the state of the art.

The article is mostly comprehensible, but a large number of corrections are

needed. I know about forgetting and complexity but not so much about the

mu-calculus itself; for that part, the authors may want to rely more on what

the other reviewers say.

## Page 1

**L45**: written this way, it seems that it is the whole specification that becomes

irrelevant; regarding this, I also doubt that "becomes" is the correct verb; in

forgetting, what is irrelevant or not depends on the context; if some part of

the specification is irrelevant to a certain task, it may be so from the

beginning

——one may want to discard some atoms (elements) that become irrelevant or unnecessary in a specification .

L46: "may need a (weakest) precondition" is too technical for an abstract; the

concept is introduced only much later in the article; this is also simple to

fix: "one may need to know what makes something true, or the minimal condition

under which something holds"

L48: "in a principled way": buzzword, sound like marketing; would

"non-principled way" make any sense? what would the difference be? (?)

L52: that the mechanism is used for the weakest precondition is already said

before; also the verb "**present**" in "to present knowledge update" is unclear:

maybe the meaning is "to perform knowledge update"?

## Page 2

L16: "essentially" is redundant here; I do not think there is anything else

L17: the comparison with S2S and the small model property seems irrelevant

here; the mu-calculus is well-established now, it does need any justification

**L25: i**t is unclear how part of the specification may become "infeasible due to

practical difficulties"; perhaps a practical example would be help here, a

concrete case where this happens

例如：缺少某种原料。

**L25:** "system update" -> "system updated"

also, why is this? an actual formula and update may help understanding why

**L31:** which property may be "desiderable"? for example?

实例：上述的3个问题用KR中的实例可以吗？

L35: "principled way" again means nothing; if the meaning is that the mechanism

is general or otherwise based on some specific principle, say which

L37: the parenthetic remark "with the well-known notion of uniform

interpolation" does not convey the correct meaning, which should be that

forgetting is related to uniform interpolation in formal logic

L39: what is a signature? saying "a set of variables" is longer but clear to

everybody

L41: forgetting does not change the behavior of the system nor the

specification; the system will still behave as before; it is its formal

description (the formula) that becomes incorrect at describing it

**L44**: the weakest precondition is already mentioned just one paragraph above;

repeating it is unnecessary

这里是为了引入weakest sufficient condition

L46: "Indeed" is out of place here, since the following sentence refers to

something unrelated to what before

L52: the noun "none of the existing forgetting" is incomplete: "none of the

existing forgetting approaches"

the sentence itself is not justified; bisimulation has been used in forgetting;

this is exactly the approach taken here

L59: what this sentence means is **unclear:** "These results are mostly applicable

to uniform interpolation due to its duality with knowledge forgetting"; as it

is written it may imply that these results only applies to uniform

interpolation, or that they also \_extend\_ to uniform interpolation; my guess is

that the second is the correct interpretation

## Page 3

L14: remove either "how" or "that"

The title of Section 2 is misleading: "related work" suggests that a comparison

of the current work with others is done. Since the current approach has not

been described yet, this would be impossible. A title like "previous work" is

more appropriate.

## Page 4

L15: "the results of T on P" are presumably the consequences of T on P

L21: the acronym "PL" seems not to be defined anywhere

**L21:** citation [5] is not the most appropriate to the term **"variable**

**elimination"**

note also that the term "variable elimination" is used only in the specific

context of automated reasoning, as a mechanism for establishing the

satisfiability of a formula all by itself (e.g., like the original Davis-Putnam

procedure does, for example), or as a preprocessing step to simplify the

formula; this is important to say because some people incorrectly believe that

"variable elimination" is standard term and do not understand why using

"knowledge forgetting" instead; forgetting is much more than variable

elimination (e.g., you can forget something different than a variable, such as

a literal or an object), and is now standard when referring to a broader

context (out of propositional logic, and also in propositional logic but out of

automated reasoning)

Section 2.2 contains a lot of facts on forgetting in FOL, which is not much

related to the current context of modal logics (without quantifiers); some of

this stuff can be cut out of the article altogether

instead, forgetting in S5 and CTL should be described in more details, as these

are modal logics like the mu-calculus

## Page 5

L36: one of the bullet points says "the temporal operatorS", but only one

temporal operator is then defined

L56: the condition "free variables are distinct from bound variables" is always

the case according to the definition of free variables above: "free variables

are those not bound"

**L57**: the notation phi(X) has not been defined; nor I think is necessary, since

phi(X) just means a formula phi that contains X; but may also not contain it;

it's just a formula phi

## Page 6

L21: some explanation about the definition is in order here; in particular, the

meaning of the labeling function may not be immediately clear; something like

"its value on a state is the set of atoms that are true in that state"

L26: again, the intuitive meaning of ||phi|| is missing; even a short sentence

like "it is the set of states where phi is true" would suffice

L42: and again, v[X:=S'] is the same valuation of v except X is true exactly in

the states S'

**L52**: add that this replacement is possible because (M,v) |= phi is independent

on v on mu-sentences (not in general)

## Page 7

L24: according to how E and X are defined here, EX is not "an operator" but a

pair of operators

L42: the definition of Fd starts by saying that this is a set of conjunctions,

but the first bullet point include a disjunction in it; I think that the

problem is that the "which" in "... and non-contradictory conjunction of

literals which is closed under" should have been "and"

also "conjunction" should have been "conjunctions"

L57: "variety" -> "a variety"

L59: "the definition of forgetting" is ambitious; in certain logics such as

Answer Set Programming, forgetting has been investigated for years, yet no

consensus has been reached about which form is correct; rather the opposite, it

seems that different forms best suit different applications; I would rather say

that this is a definition of forgetting, not "the"

L60: "the forgetting" -> "forgetting"

## Page 8

L14: the model checking problem is a reasoning problem; as a result, "the

reasoning problem" is unclear here

L19: "the explored logic form systems" are rather the "logics explored in the

past" or "explored so far"

L20: the sentence "some atoms (1) from a given formula (2) should not violate

the existing specification (3) over the remaining signature (4)" is made

unclear by the switch of nouns: atoms->signature formula->specification; stick

to one, like: some atoms from a given formula should maintain the part of the

formula over the remaining atoms"

**L22:** there is a logical jump here, from forgetting to bisumulation; why

bisimulation is used as the base for forgetting is not written; the reason is

of course that "not violating" or "maintaining" something implies a form of

equivalence because the part of the formula/specification that are not

forgotten should semantically remain the same, and bisimulation expresses

exactly that

this also motivates extending the notion to different sets of variables, as

done here without a motivation

L26: say that V are the atoms allowed to take different values, and will

eventually be the atoms to forget; that would clarify the following definition

**L27:** the meaning of the sentence "for convinience..." is unclear; it took me a

bit to get it: Mi stands for the Kripke model (Si, ri, Ri, Li) throughout the

article, where i=1,2,3; there is no need to refer to natural numbers are no M4,

M5 etc, is used in the article, an the only use of Mi with an arbitrary i

starts by re-defining it as (Si, ri, Ri, Li) anyway

L42: the complement of what?

L51: what is an initial k-structure?

## Page 9

**L19: I do not think this is a good example; the only state where forgetting ch**

**makes any difference is s0, and "merging" s1 and s2 is also possible without**

**forgetting**

**L38:** I would rather have (ii) being that V-bisimulation implies V'-bisimulation

whenever V' is a superset of V; this is simpler to express and to understand

(bisimulation is monotonic with respect to V), and the current version of (ii)

is a straightforward consequence of monotony and transitivity

——这个实际上也是成立的，但是这里的(ii)是为了后面的遗忘的证明方便。

增加了这一结论

**L59: many formulae can be the result of forgetting, according to Definition 3;**

**therefore, the function F() cannot be a single formula, as it appears to be**

**used in the following Theorem 2, for example**

**——是的，但是是逻辑等价的**

**## Page 10**

L17: missing letter in "sentenc"

L19: it is important to say that the definition by D'Agostino is equivalent but

not the same as the the definition of forgetting given in this article;

otherwise, the latter would go in the "previous work" section

L25: given the definition of forgetting, I would not call it "closed"; that

would mean that the result is in the mu-calculus, as opposed of not being so;

but the definition mandates the result of forgetting being a mu-calculus

formula; forgetting is closed by definition; rather, what is proved is that

forgetting is defined for all formulae of the mu-calculus; alternatively, the

result of forgetting exists for all formulae of the mu-calculus

L28: this sentence is vague: "the notion of forgetting satisfies the general

postulates and the algebraic properties, including modularity, commutativity,

and homogeneity."; the "general postulates" and the "algebraic properties" are

undefined at this point; what this sentence seems to mean is that forgetting

satisfies certain reasonable postulates and algebraic properties; in other

words, not "the postulates" and "the algebraic properties", but "certain

postulates" and "certain algebraic properties"

L31: "the result of using the O on the elements of L is also in L" is the

correct definition of O being closed; but this is also the case for forgetting

as defined here, since the definition is "the result of forgetting ... is a

mu-formula..."; rather, what is proved is that such a result always exists

L34: Theorem 2 and the definition of forgetting do not match: the former has

the irrelevance of variables (IR), the second has the absence of variables

——在Def 3中，要求\varphi(\psi) \cap V = \emptyset，这是Thm 2中的IR的特殊形式。因为IR(\psi, V) if 存在一个公式\varphi 使得 \Var(\varphi) \cap V =\emptyset 和 \varphi \equiv \psi.

see also the comment about Definition 3; writing formulae==F() is an abuse of

notation; can be done, but should be remarked in some place

L58: "the forgetting results have no effect on formulas that are

independent..." should be "forgetting has no effects on the consequences that

are independent..."

## Page 11

L19: the part "hold if phi, phi'..." is unnecessary and confusing; just "hold

on phi, phi' and V" is enough

L29: rather than "it is evident", I would spell this argument in full, which is

just: F(...) is in the set {phi|...}

L32: "there is" -> "it holds"

L46: the link between IR and the following property is not evident at all; if

IR is necessary to the proof of the lemma, state it here because the proof is

much later in the article

## Page 12

L41: rather than obvious, I would say that the meaning of (iii) is the same (or

similar) as (ii); if the former is obvious, the latter is as well

L60: I would not call this property "commutativity"; the definition of

commutativity is "a binary operation is commutative if changing the order of

the operands does not change the result"; for forgetting, the operands are a

formula and a set of variables; they are not even of the same type

## Page 13

L26: the two cases EXf and AXf are written in full in the previous lines, why

using the more convoluted form QXf with Q in {E,A} here?

L43: **where is this equivalence proved?**

——在Def 3下面给了讨论。

L59: points (1)-(3): how are these forgettings computed?

## Page 14

L15: the following theorem does not state that model checking is intractable;

it only gives an upper bound of complexity (membership, not hardness)

**L56:** **I suspect "equivalent" should be "equivalence"; also "constraints" should**

**be "complexity"**

## Page 15

L47: a point that is easy to miss in this explanation is that a SNC or a WSC

may not exists for a given formula and atom, e.g., {a->q, b->q} for the WSC;

**just saying that the SNC and WSC are unique is misleading: they are unique if**

**they exist**

（定义在给定集合上的SNC和WSC）

## Page 16

**L22:** what happens if either the SNC or the WSC does not exist? maybe the

corresponding result of forgetting does not exist either and vice versa, but

the statement of Theorem 15 does not specify this and its proof does not seem

to cover this case

——在本文中，遗忘的结果总是存在的，最坏的情形是遗忘掉公式中的所有原子，此时，若公式是可满足的，则结果为\top；最好的情形是一个原子都不遗忘，此时，得到的结果为原公式。（如定理 ？）

L40: again, what is an initial K-structure? this concept is defined only in the

following page

## Page 17

L36: is an initial structure the same as an initial K-structure?

L57: this definition is given in reverse: phi is not a given, it is part of the

definition of Vmin; it should not go in "For the ...", which seems to be part

of the given; rather: Vmin is the minimal set of atoms that makes F(...)

consistent

## Page 18

**L54:** isn't this definition the same as always taking V1 equal to V2? if there

is a bisimulation when forgetting less atoms, it should also be when forgetting

more

## Page 19

L52: I do not see how the WSC can possibly be used in knowledge update

——不是knowledge update，而是system update。

## Page 25

L38: this paragraph seems to imply that entailment problem with forgetting

reduces to the emptiness problem on automata; that would prove the problem in

**EXPTIME**, not EXPTIME-complete

## Page 26

**proof of Theorem 17 appears to be wrong:** just because a problem can be reduced

to two problems that are C-complete does not mean the problem itself is

C-complete; example: the problem on QBFs whose answer is always "no" can be

reduced to "is the QBF F satisfiable" and "is the QBF F not satisfiable", which

are both PSPACE-complete; yet, the problem whose answer is always "no" is

trivial, as the answer only take constant time to be produced

## Page 28

citation [5] contains the repetition "In: In"

**## Page 29**

here did article [15] appear? the citation gives three venues

## Page 31

in citation [28], the word "STRIPS" should be all capitalized

write all author names in citation [34]

**Reviewer #3:** The forgetting (or variable elimination) is to obtain a new logical theory from a given logical theory by hiding certain variables. This problem has been studied in various logics and has potential applications in some practical domains. The paper under review investigates the problem of forgetting for propositional mu-calculus. Specifically, the definition of forgetting is provided, some properties of forgetting are presented including a representation theorem, and the relationships of forgetting to weakest sufficient condition, strongest necessary condition and update are also discussed.

The results in the paper looked interesting and I believe they are technically sound. I can follow most part of the paper, while some revisions are required. I'd like to see some more evidences why the proposed results are novel and non-trivial. In particular, as stated in the paper, the notion of interpolation for propositional mu-calculus and the forgetting operators in some other logics have been investigated. For instance, which results in the paper have not be shown for other logics? Which adaption from propositional logic/logic programming to mu-calculus is non-trivial? Some sentences are sloppy and the paper contains many grammatical errors**. A stronger motivation is required for the work. T**o make the paper more readable, more examples could be used to demonstrate some concepts and results. The proofs also need to be double checked and concisely presented.

Detailed comments:

**1.** Some statements are not well supported and many ideas are not explained clearly. E.g., the last two sentences in the 2nd paragraph in Section 1: **"It is also a challenge to find a (weakest) condition for a system ...".** I am not saying they are wrong but they do need to be clearer and more convincing. The statement immediately after "For instance" is not more specific than the general one.

2. Page 2, lines 52-53 "none of existing knowledge forgetting is applicable to μ-calculus": What did you mean by this statement?

3. Page 3, line 59: If I am correct, Lin and Reiter re-named the variable elimination as **"forgetting"** but George Boole had introduced it when he established his Boolean logic.

——谢谢，我查了确实是你说的这样

4. Definition 1: It looked the semantics of disjunctive mu-formulas is defined before the syntax is introduced.

——吸取公式是mu公式的一种特殊形式，这里先介绍cover定义的mu公式的语法和语义，再给出吸取公式的定义。

5. Page 8, lines 41-42: The statement is not quite clear to me.

**6. Page 9, line 45: The "transitivity" is not the standard one. This should be made clear.**

**7. Page 10,** lines 24-28 and many other places: The use of article "the" is incorrect.

8. Page 10, line 38: It may be ok to use "some set" here but I would prefer "a set".

9. Page 12, line 53: It is unclear why this property is called "Modularity", while it has not relevance to **modularity** in the usual sense.

10. Page 15, line 19 "Applications of Forgetting": I did not see why the relationships discussed in this section belong to the category of "**applications**".

Minor issues:

1. Page 16, lines 36-37 (and some other places): "by suppose" --> "by assumption"

2. Page 21, line 51 "This is following the definition ..."

3. Page 23, line 34: "supposing" --> "Suppose (that)"

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