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1 Entanglement Entropy

In this section we describe in detail how Entanglement-related properties are to be calculated. First of all let's see how joint operators act on wavefunctions. Let $|\Psi\rangle$ be an arbitrary pure state from ED, which can be factorized into a bipartite wavefunction:

$$|\psi\rangle = \sum_{l,r} \omega_{lr} |\psi_l\rangle |\psi_r\rangle \equiv \hat{\Omega} \tag{1.1}$$

where we defined $\hat{\Omega}$ to be the matrix written under the bipartite basis. The right reduced density matrix (RDM) is then obtained by tracing out the left dofs:

$$\begin{aligned}
\rho_r = Tr_l[|\psi\rangle\langle\psi|] &= \sum_{l''} \sum_{l,r} \sum_{l',r'} \omega_{lr} \omega^*_{r'l'} \langle\psi_{l''}|\psi_l\rangle |\psi_r\rangle \langle\psi'_r| \langle\psi'_l|\psi_{l''}\rangle \\
&= \sum_{l''} \sum_{l,r} \sum_{r',l'} \omega_{lr} \omega^*_{l'r'} \delta_{l,l''} |\psi_r\rangle \langle\psi'_r| \delta_{l',l''} \\
&= \sum_{lr,r'} \omega_{lr} \omega^*_{r'l} |\psi_r\rangle \langle\psi'_r|
\end{aligned} \tag{1.2}$$

we can rewrite this as:

$$\rho_r = \sum_{lr,r'} \omega^*_{r'l} \omega_{lr} |\psi_r\rangle \langle\psi'_r| = \sum_{r,r'} \left(\sum_l \omega^*_{r'l} \omega_{lr} \right) |\psi_r\rangle \langle\psi'_r| \tag{1.3}$$

that is, in the basis of $|\psi_r\rangle \langle\psi'_r|$ the elements of RDM are

$$\rho_{r',r} = \sum_l \omega^*_{r'l} \omega_{lr} = \hat{\Omega}^\dagger \hat{\Omega} \tag{1.4}$$

In other words, if we can construct the bipartite wavefunction explicitly as a matrix form, the RDM is simply a matrix product of it with its complex conjugation.