# Projective Measurememts

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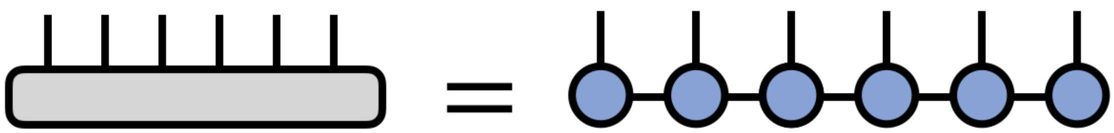
## 1 RDM

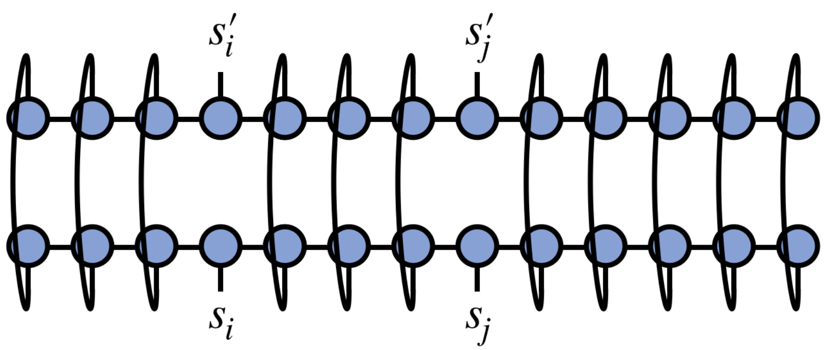
Suppose we'd like a two-site RDM of site $s_i$ and $s_j$:

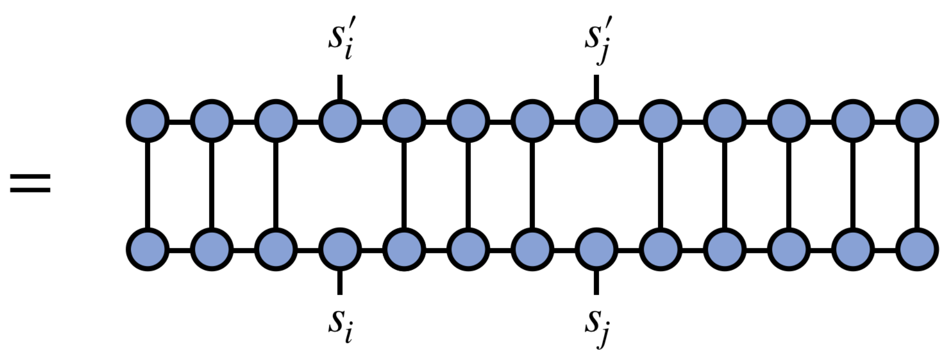
$$\rho = \mathrm{Tr}_{s \neq s_i, s_j} \Big( |\psi\rangle \langle\psi| \Big) \tag{1}$$

the MPS is represented as



RDM is then achieved by contracting physical links:



## 2 Projective Operator

For example, assume a 4-qubit system, and assume a projection into the basis $|1001\rangle$ along z axis. The projector is

$$\mathcal{P} = |1001\rangle \langle 1001| \tag{2}$$

by

$$(A \otimes B) \cdot (C \otimes D) = (A \cdot C) \otimes (B \cdot D) \tag{3}$$

$P$ is equivalent to

$$\mathcal{P} = |10\rangle \langle 10| \otimes |01\rangle \langle 01| = |1\rangle \langle 1| \otimes |0\rangle \langle 0| \otimes |0\rangle \langle 0| \otimes |1\rangle \langle 1| \tag{4}$$

which can be viewed as tensor products of local projectors.

Given a mixed state $\{p_i, |\psi_i\rangle\}$, a projective measurement gives:

1. outcome "i" with probablity
$$p_i = \mathrm{Tr}\Big(\mathcal{P}_i^\dagger \mathcal{P}_i \rho\Big) \tag{5}$$

2. and $\rho$ collapses into:
$$\rho' = \frac{\mathcal{P}_i \rho \mathcal{P}_i^\dagger}{\mathrm{Tr}\Big(\mathcal{P}_i^\dagger \mathcal{P}_i \rho\Big)} \tag{6}$$

In MPS, we can do this in sequence since local projectors $P_j$ commute with each other.

Let $|\psi\rangle$ be a pure quantum state of a lattice Hamiltonian. For an operator $O$ that is supported on one part of the bipartited of lattice the following holds:

$$\langle\psi|O|\psi\rangle = \mathrm{Tr}(\rho_i O) \tag{7}$$

where $\rho_i$ is the (reduced) density matrix of the selected part of lattice, and the trace is taken over either states thereof or of the full system (they give the same result).

That is

$$\mathrm{Tr}(\rho O) = \mathrm{Tr}(\rho_i O) \tag{8}$$