

Dimensional transition from Kitaev spin liquid to decoupled fermion chains

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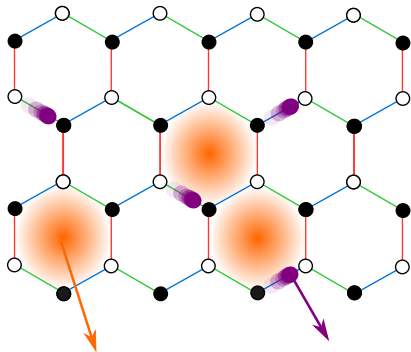
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Introduction

- A. Kitaev. Ann. Phys. 321, 2–111 (2006)

$$\mathcal{H} = \sum_{\alpha} \sum_{\langle ij \rangle} K_x S_i^x S_j^x + K_y S_i^y S_j^y + K_z S_i^z S_j^z$$



Z_2 Flux: W_p

Majorana: c

Relevant QSL materials:

- α -RuCl₃

- 1 Y. Kasahara, T. Ohnishi, Y. Mizukami, O. Tanaka, Sixiao Ma, K. Sugii, N. Kurita, H. Tanaka, J. Nasu, Y. Motome, T. Shibauchi and Y. Matsuda. Nature 559, 227–231 (2018)

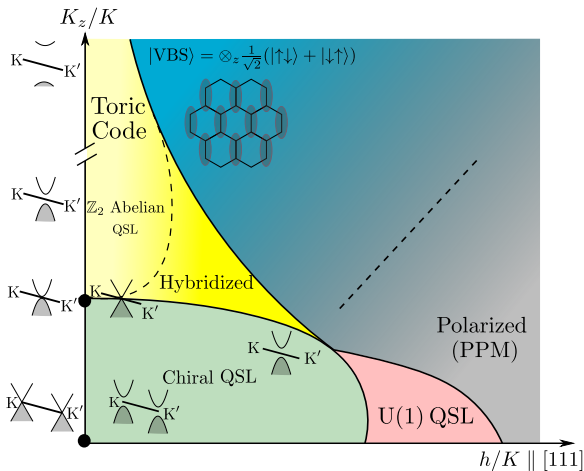
Large in-plane field to destabilize order

- BaCo₂(AsO₄)₂

- 1 X. Zhang, Y. Xu., T. Halloran, R. Zhong, C. Broholm, R. J. Cava, N. Drichko, N. P. Armitage. A magnetic continuum in the cobalt-based honeycomb magnet BaCo₂(AsO₄)₂. Nat. Mater. 22, 58–63 (2023)

Small [111] field to destabilize order

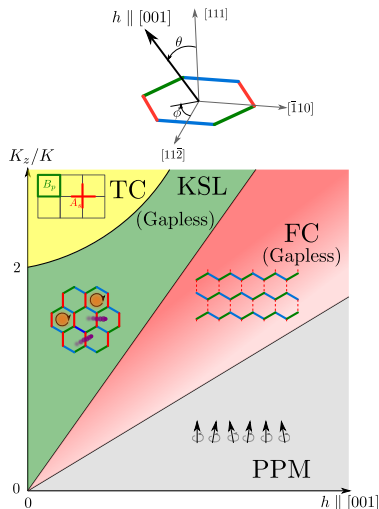
Phase diagram under out-of-plane [111] field



- 1 N. Patel and N. Trivedi. Magnetic field-induced intermediate quantum spin liquid with a spinon Fermi surface. PNAS 116, 12199 (2019)
- 2 S. Feng, A. Agarwala, S. Bhattacharjee and N. Trivedi. Anyon dynamics in field-driven phases of the anisotropic Kitaev model, [arXiv:2206.12990](#)

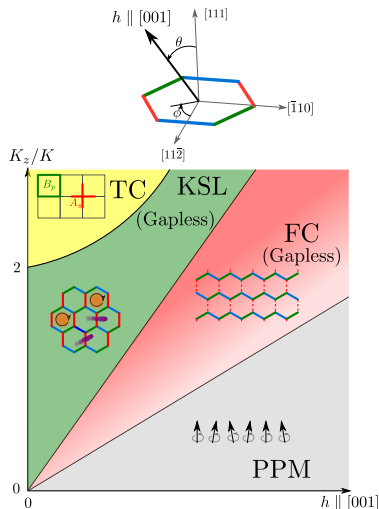
The simplest [001] field?

Phase diagram under [001] field and exchange anisotropy



$$\mathcal{H} = \sum_i K(\sigma_i^x \sigma_{i+x}^x + \sigma_i^y \sigma_{i+y}^y) + K_z \sum_i \sigma_i^z \sigma_{i+z}^z - h \sum_i \sigma_i^z$$

Phase diagram under [001] field and exchange anisotropy

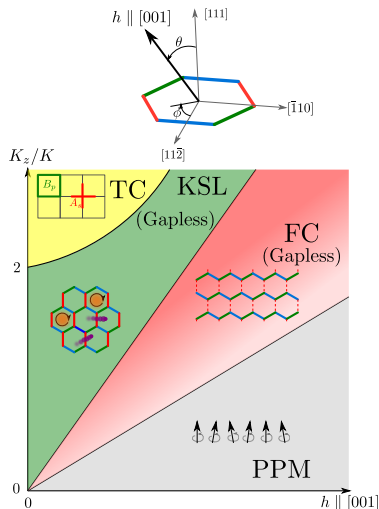


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Related works by Majorana MFT:

- 1 S. Liang, M. H. Jiang, W. Chen, J. X. Li, and Q. H. Wang. Intermediate gapless phase and topological phase transition of the Kitaev model in a uniform magnetic field. Phys. Rev. B 98, 054433 (2018)
- 2 J. Nasu, Y. Kato, Y. Kamiya, and Y. Motome. Successive Majorana topological transitions driven by a magnetic field in the Kitaev model. Phys. Rev. B 98, 060416(R) (2018)

Phase diagram under [001] field and exchange anisotropy



$$\mathcal{H} = \sum_i K(\sigma_i^x \sigma_{i+x}^x + \sigma_i^y \sigma_{i+y}^y) + K_z \sum_i \sigma_i^z \sigma_{i+z}^z - h \sum_i \sigma_i^z$$

Four phases in (K_z, h) plane:

- 1 Gapless Kitaev spin liquid (KSL) at small h/K_z
- 2 **Decoupled/Weakly coupled fermion chains (FC)** at intermediate h/K_z
- 3 Toric Code (TC) at large K_z
- 4 Partially polarized magnet (PPM) at large h/K_z as **Emergent decoupled boson chains**

Discussion by ED, DMRG, MFT, Effective field theory.

Phase diagram (DMRG & ED)

$$\chi = \frac{\partial^2 E_{\text{gs}}}{\partial h^2}, \quad \langle W_p \rangle = \langle \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z \rangle. \quad \langle W_p \rangle = 1 \text{ at } h = 0$$

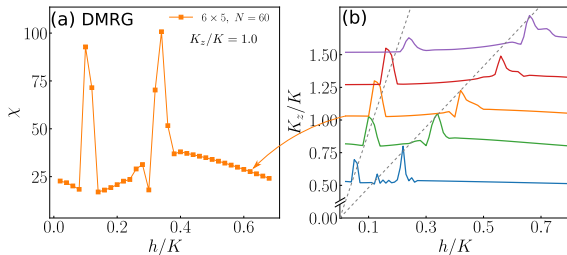


Figure: Magnetic susceptibility χ by DMRG

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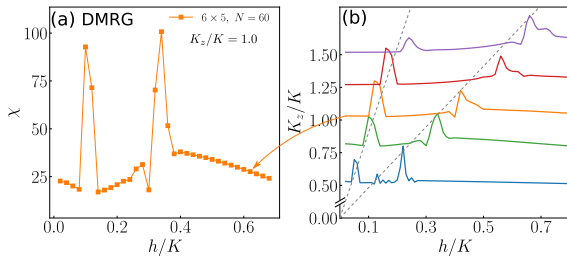


Figure: Magnetic susceptibility χ by DMRG

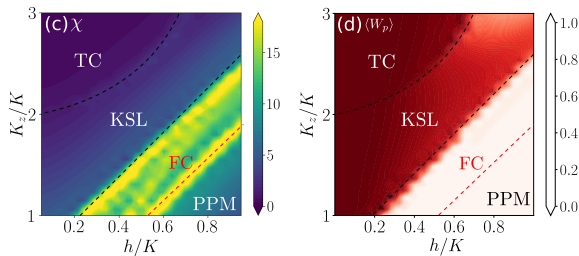
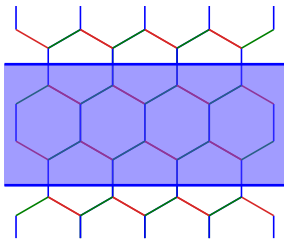
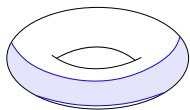


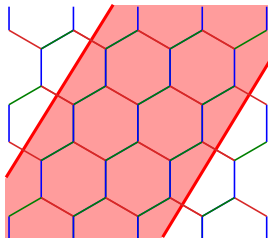
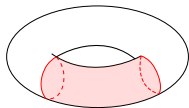
Figure: χ and Z_2 flux expectation by 24-site ED

Entanglement entropy

z - bond cut



y - bond cut



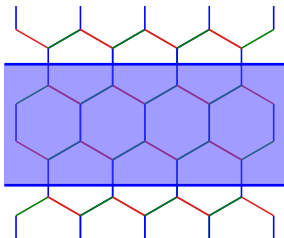
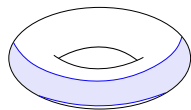
von-Neumann entropy per α bond S_{vN}^α :

$$S_{vN}^\alpha = -\frac{1}{|\partial A_\alpha|} \text{Tr} \rho_{A_\alpha} \ln \rho_{A_\alpha}, \alpha \in \{z, y\}$$

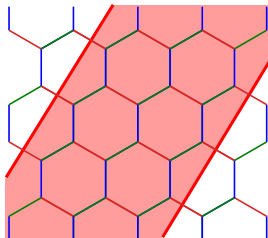
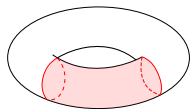
A_α : subsystem whose edges cut α bonds

Entanglement entropy

z - bond cut



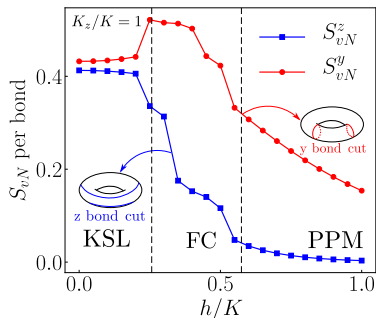
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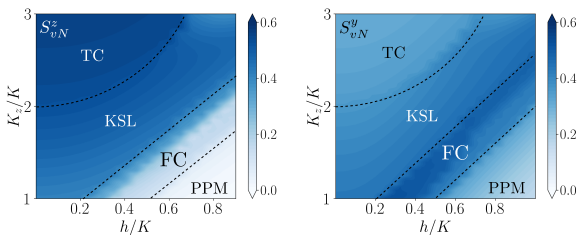
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Entanglement entropy

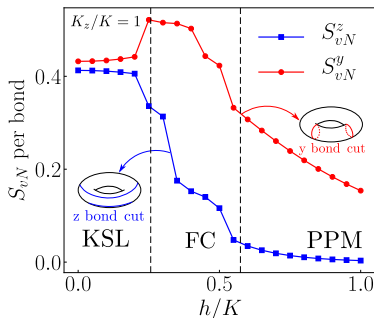


- ① KSL: $S_{vN}^z \sim S_{vN}^y(S_{vN}^x)$
- ② TC: Large S_{vN}^z and small $S_{vN}^y(S_{vN}^x)$
- ③ FC: Small S_{vN}^z and large $S_{vN}^y(S_{vN}^x)$
- ④ PPM: $S_{vN}^z = 0$, $S_{vN}^y(S_{vN}^x) > 0$

von-Neumann entropy per α bond S_{vN}^α :

$$S_{vN}^\alpha = -\frac{1}{|\partial A_\alpha|} \text{Tr} \rho_{A_\alpha} \ln \rho_{A_\alpha}, \quad \alpha \in \{z, y\}$$

A_α : subsystem whose edges cut α bonds



Decoupled bosonic chains in PPM

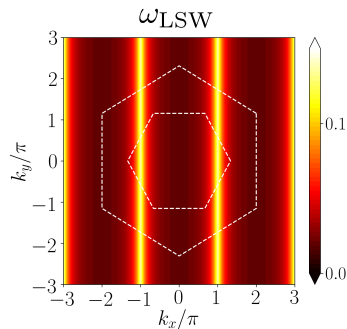


Figure: LSW at $h \sim 0.5$

$$\mathcal{H}_{\text{LSW}} = \frac{1}{2} \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^{\dagger} \mathbf{H}(\mathbf{k}) \Psi_{\mathbf{k}}, \quad \mathbf{H}(\mathbf{k}) \equiv \begin{pmatrix} \mathbf{M}(\mathbf{k}) & \mathbf{N}(\mathbf{k}) \\ \mathbf{N}^{\dagger}(\mathbf{k}) & \mathbf{M}(-\mathbf{k}) \end{pmatrix}$$

where we defined $\Psi_{\mathbf{k}} \equiv (a_{\mathbf{k}}, b_{\mathbf{k}}, a_{-\mathbf{k}}^{\dagger}, b_{-\mathbf{k}}^{\dagger})^T$ with a, b boson operators of A and B sublattices, and $\mathbf{N}(\mathbf{k})$ and $\mathbf{M}(\mathbf{k})$ are:

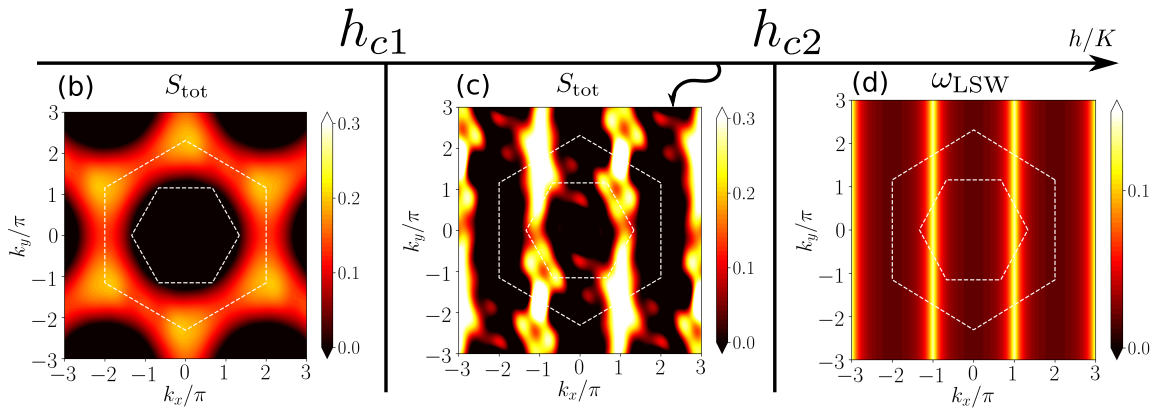
$$\mathbf{N} = \frac{1}{4} \left(K_x e^{i\mathbf{k} \cdot \mathbf{n}_1} - K_y e^{i\mathbf{k} \cdot \mathbf{n}_2} \right) \sigma^x$$

$$\mathbf{M} = \left(\textcolor{red}{h} - \frac{1}{2} K_z \right) \sigma^z + \frac{1}{4} \left(K_x e^{i\mathbf{k} \cdot \mathbf{n}_1} + K_y e^{i\mathbf{k} \cdot \mathbf{n}_2} \right) \sigma^x$$

Near $h_{c2} \sim 0.5$, K_z exchange on z bonds vanishes in PPM!
 → Decoupled boson chains.

Structure Factor

$$S_{\text{tot}}(\mathbf{k}) = \langle \mathbf{S}_i(\mathbf{k}) \cdot \mathbf{S}_j(-\mathbf{k}) \rangle - \langle \mathbf{S}_i(\mathbf{k}) \rangle \cdot \langle \mathbf{S}_j(-\mathbf{k}) \rangle, \quad \omega_{\text{LSW}}(\mathbf{k}) : \text{Linear spin wave of PPM}$$



Weakly Coupled fermionic chains in FC

Kitaev's four majorana decomposition:
 $\sigma^\alpha = ib^\alpha c$. Convert into canonical fermions
 and bond fermions:

$$c_{i,A+\hat{z}} = i(f_i - f_i^\dagger), \quad c_{i,A} = f_i + f_i^\dagger$$

$$b_{i,A}^\alpha = \chi_{i\alpha} + \chi_{i\alpha}^\dagger, \quad b_{i,A+\hat{\alpha}}^z = i(\chi_{i\alpha} - \chi_{i\alpha}^\dagger)$$

Hence

$$K_z \left(b_{i,A}^z b_{i,A+\hat{z}}^z c_{i,A} c_{i,A+\hat{z}} \right) = K_z (2n_i^f - 1)(1 - 2n_i^z)$$

$$K_x \left(b_{i,A}^x b_{i,A+\hat{x}}^x c_{i,A} c_{i,A+\hat{x}} \right) = K_x (1 - 2n_i^x) \\
\times (f_i f_{i-\delta_1} - f_i f_{i-\delta_1}^\dagger + f_i^\dagger f_{i-\delta_1} - f_i^\dagger f_{i-\delta_1}^\dagger)$$

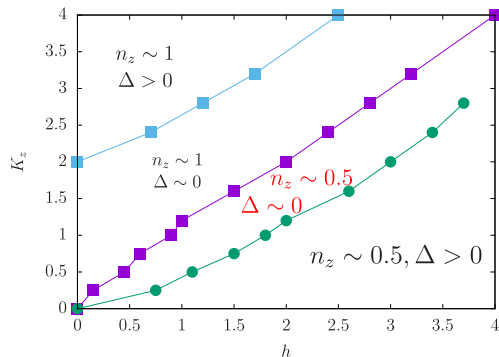


Figure: MFT phase diagram

K_z exchange on z bonds vanishes in the
 MFT of FC near h_{c2}^- !

Possible effective Theory in FC

Ignoring interchain coupling K_z , \mathcal{H} becomes that of a compass chain, which can be mapped to the critical point of *TFIM*:

$$\mathcal{H} = \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_{i+1}^y \sigma_{i+2}^y \rightarrow \mathcal{H}_{\text{TFIM}}^c = - \sum_i \tau_i^z \tau_{i+1}^z - \sum_i \tau_i^x$$

$\mathcal{H}_{\text{TFIM}}^c$: 1+1D CFT with central charge $c = \frac{1}{2}$, with left and right chiral majoranas $\gamma_{L(R)}$.

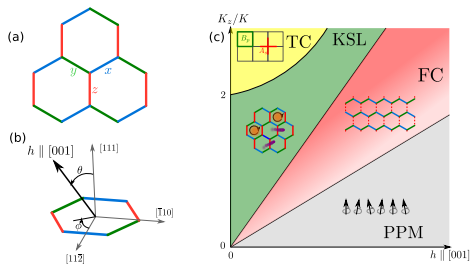
$$\mathcal{H}_L = \int dx (i \gamma_L \partial_x \gamma_L), \quad \mathcal{H}_R = \int dx (-i \gamma_R \partial_x \gamma_R)$$

Consider weak coupling between chains via coupling between γ_L and γ_R :

$$\mathcal{H}(\text{FC}) \approx \mathcal{H}_L + \mathcal{H}_R - g \int dx (\gamma_L \partial_x \gamma_L) (\gamma_R \partial_x \gamma_R) \rightarrow \text{stable until finite } g_c$$

Summary

- 1 Weakly coupled fermionic chains from Kitaev model under [001] field $h \sim h_{c2}^-$
- 2 Decoupled bosonic chains from Kitaev model under large [001] field $\sim h_{c2}^+$
- 3 Lifshitz transitions by depleting n_i^z bond fermions from 1 to 0.5
- 4 Proposed effective theory in FC: $\mathcal{H}(\text{FC}) \approx \mathcal{H}_L + \mathcal{H}_R - g \int dx (\gamma_L \partial_x \gamma_L) (\gamma_R \partial_x \gamma_R)$



Outlook:

- 1 How is (or is not) the FC phase connected to the proposed spinon Fermi surface under [111] field?
- 2 Other quantum phase transitions as a function of angles of field?
- 3 Dynamics