Dimensional transition from Kitaev spin liquid to decoupled fermion chains

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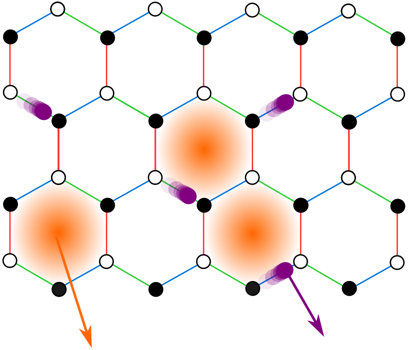
Introduction

• A. Kitaev. Ann. Phys. 321, 2–111 (2006)

$$\mathcal{H} = \sum_{\langle ij \rangle} K_x S_i^x S_j^x + K_y S_i^y S_j^y + K_z S_i^z S_j^z$$



Relevant QSL materials:

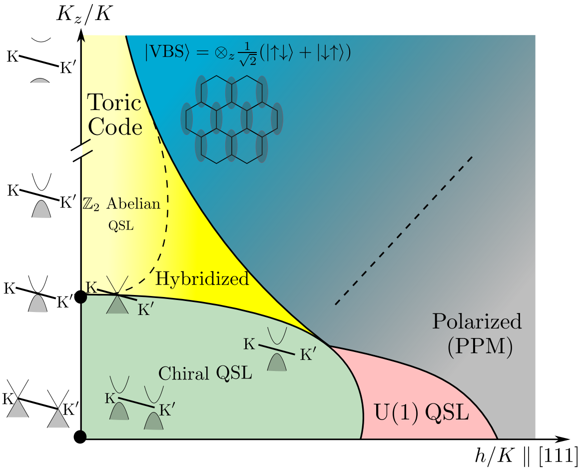
- $\alpha-RuCl_3$
  1. Y. Kasahara, T. Ohnishi, Y. Mizukami, O. Tanaka, Sixiao Ma, K. Sugii, N. Kurita, H. Tanaka, J. Nasu, Y. Motome, T. Shibauchi and Y. Matsuda. Nature 559, 227–231 (2018)

Large in-plane field to destabilize order

- $BaCo_2(AsO_4)_2$
  1. X. Zhang, Y. Xu., T. Halloran, R. Zhong, C. Broholm, R. J. Cava, N. Drichko, N. P. Armitage. A magnetic continuum in the cobalt-based honeycomb magnet BaCo2(AsO4)2. Nat. Mater. 22, 58–63 (2023)

Small [111] field to destabilize order

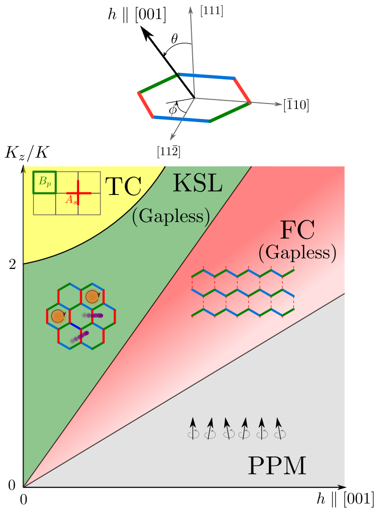
Phase diagram under out-of-plane [111] field



1. N. Patel and N. Trivedi. Magnetic field-induced intermediate quantum spin liquid with a spinon Fermi surface. PNAS 116, 12199 (2019)
2. S. Feng, A. Agarwala, S. Bhattacharjee and N. Trivedi. Anyon dynamics in field-driven phases of the anisotropic Kitaev model, **arXiv:2206.12990**

The simplest [001] field?

Phase diagram under [001] field and exchange anisotropy



$$\mathcal{H} = \sum_i K(\sigma_i^x \sigma_{i+x}^x + \sigma_i^y \sigma_{i+y}^y) + K_z \sum_i \sigma_i^z \sigma_{i+z}^z - h \sum_i \sigma_i^z$$

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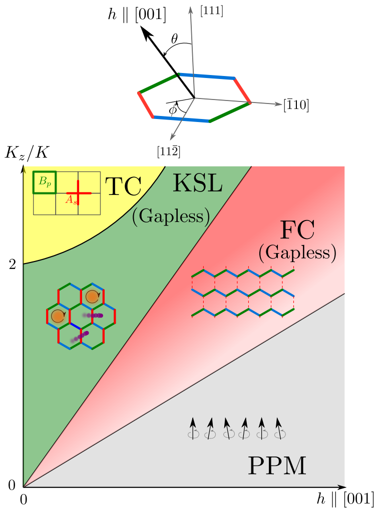

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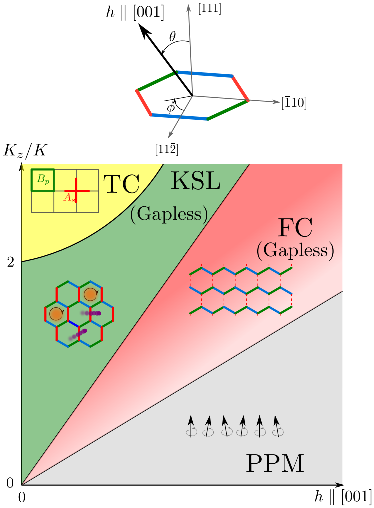
Related works by Majorana MFT:

1. S. Liang, M. H. Jiang, W. Chen, J. X. Li, and Q. H. Wang. Intermediate gapless phase and topological phase transition of the Kitaev model in a uniform magnetic field. Phys. Rev. B 98, 054433 (2018)
2. J. Nasu, Y. Kato, Y. Kamiya, and Y. Motome. Successive Majorana topological transitions driven by a magnetic field in the Kitaev model. Phys. Rev. B 98, 060416(R) (2018)

Phase diagram under [001] field and exchange anisotropy



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Four phases in (K_z, h) plane:

1. Gapless Kitaev spin liquid (KSL) at small h/K_z
2. Decoupled/Weakly coupled fermion chains (FC) at intermediate h/K_z
3. Toric Code (TC) at large K_z
4. Partially polarized magnet (PPM) at large h/K_z as Emergent decoupled boson chains

Discussion by ED, DMRG, MFT, Effective field theory.

Phase diagram (DMRG & ED)

$$\chi = \frac{\partial^2 E_{gs}}{\partial h^2}, \quad \langle W_p \rangle = \langle \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z \rangle . \qquad \langle W_p \rangle = 1 \text{ at } h = 0$$

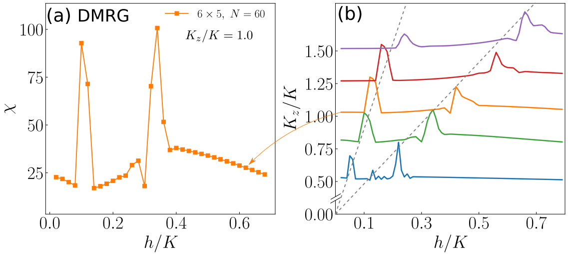

Figure: Magnetic susceptibility χ by DMRG

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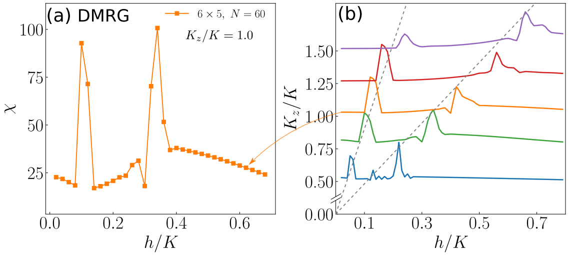


Figure: Magnetic susceptibility χ by DMRG

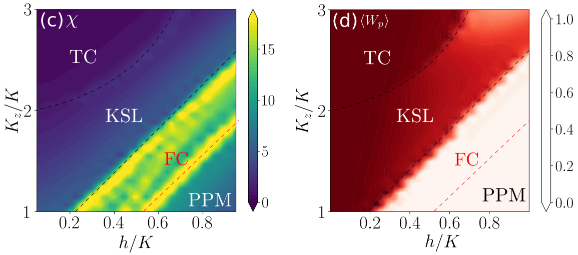
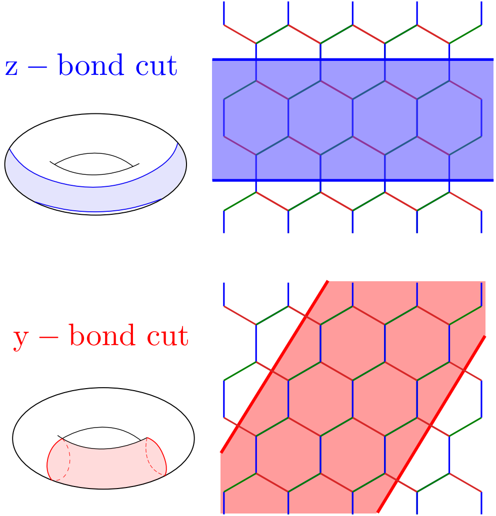


Figure: χ and Z_2 flux expectation by 24-site ED

Entanglement entropy



von-Neumann entropy per α bond S^{α}_{vN}:

$$S^{\alpha}_{vN} = -\frac{1}{|\partial A_\alpha|} \operatorname{Tr} \rho_{A_\alpha} \ln \rho_{A_\alpha}, \ \alpha \in \{z, y\}$$

A_α: subsystem whose edges cut α bonds

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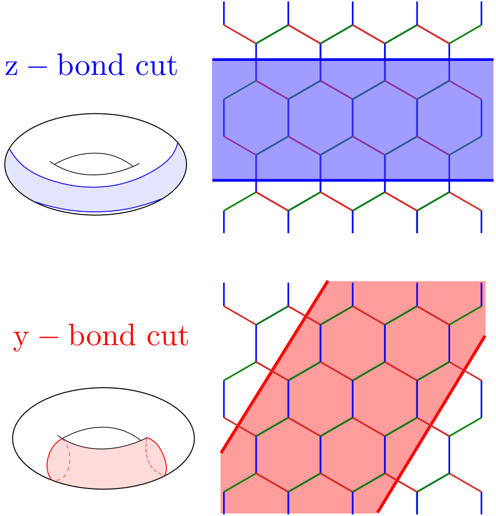

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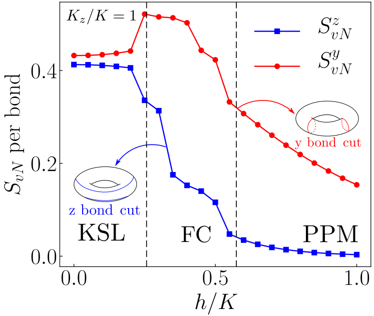

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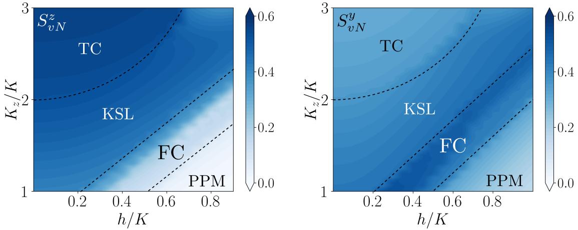

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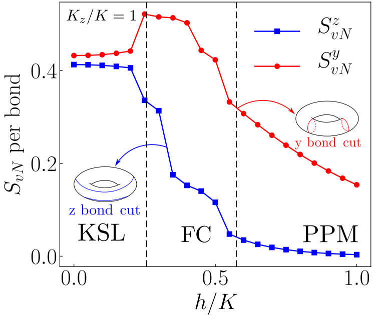


1. KSL: $S^z_{vN} \sim S^y_{vN}(S^x_{vN})$
2. TC: Large S^z_{vN} and small $S^y_{vN}(S^x_{vN})$
3. FC: Small S^z_{vN} and large $S^y_{vN}(S^x_{vN})$
4. PPM: $S^z_{vN} = 0$, $S^y_{vN}(S^x_{vN}) > 0$

Decoupled bosonic chains in PPM

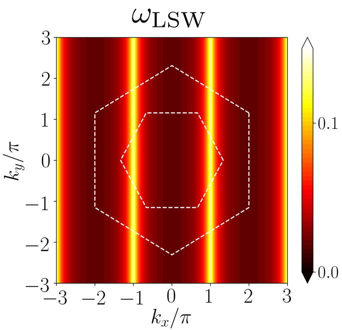


Figure: LSW at $h \sim 0.5$

$$\mathcal{H}_{\mathrm{LSW}} = \frac{1}{2}\sum_{\mathbf{k}} \Psi^{\dagger}_{\mathbf{k}} \mathbf{H}(\mathbf{k}) \Psi_{\mathbf{k}}, \ \mathbf{H}(\mathbf{k}) \equiv \begin{pmatrix} \mathbf{M}(\mathbf{k}) & \mathbf{N}(\mathbf{k}) \\ \mathbf{N}^{\dagger}(\mathbf{k}) & \mathbf{M}(-\mathbf{k}) \end{pmatrix}$$

where we defined $\Psi_{\mathbf{k}} \equiv (a_{\mathbf{k}}, b_{\mathbf{k}}, a^{\dagger}_{-\mathbf{k}}, b^{\dagger}_{-\mathbf{k}})^{\mathrm{T}}$ with a, b boson operators of A and B sublattices, and $\mathbf{N}(\mathbf{k})$ and $\mathbf{M}(\mathbf{k})$ are:

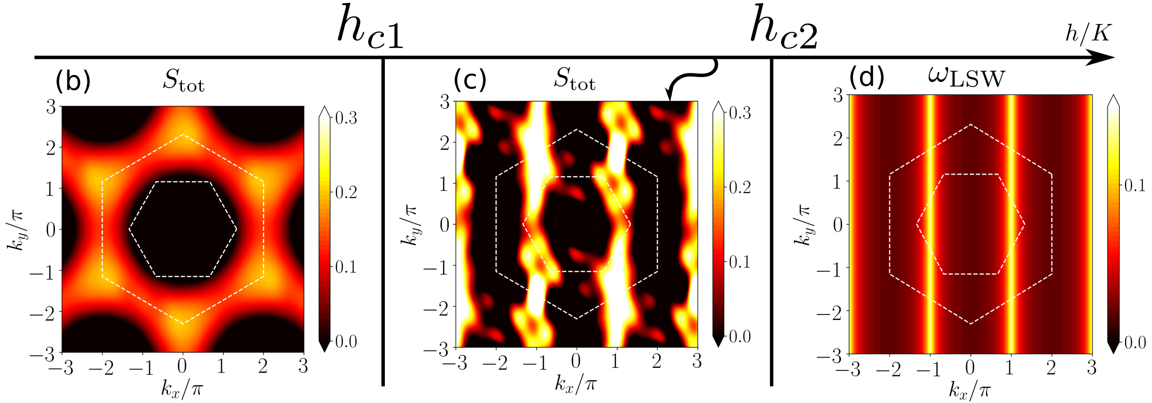
$$\mathbf{N} = \frac{1}{4}\left(K_x e^{i\mathbf{k}\cdot\mathbf{n}_1} - K_y e^{i\mathbf{k}\cdot\mathbf{n}_2}\right)\sigma^x$$

$$\mathbf{M} = (h - \frac{1}{2}K_z)\sigma^z + \frac{1}{4}\left(K_x e^{i\mathbf{k}\cdot\mathbf{n}_1} + K_y e^{i\mathbf{k}\cdot\mathbf{n}_2}\right)\sigma^x$$

Near $h_{c2} \sim 0.5$, K_z exchange on z bonds vanishes in PPM!
$\rightarrow$ Decoupled boson chains.

Structure Factor

$$S_{\text{tot}}(\mathbf{k}) = \left\langle \mathbf{S}_i(\mathbf{k}) \cdot \mathbf{S}_j(-\mathbf{k}) \right\rangle - \left\langle \mathbf{S}_i(\mathbf{k}) \right\rangle \cdot \left\langle \mathbf{S}_j(-\mathbf{k}) \right\rangle , \quad \omega_{\text{LSW}}(\mathbf{k}) : \text{ Linear spin wave of PPM}$$



Weakly Coupled fermionic chains in FC

Kitaev's four majorana decomposition: $\sigma^\alpha = ib^\alpha c$. Convert into canonical fermions and bond fermions:

$$c_{i,A+\hat{z}} = i(f_i - f_i^\dagger),\ c_{i,A} = f_i + f_i^\dagger$$

$$b^\alpha_{i,A} = \chi_{i\alpha} + \chi^\dagger_{i\alpha},\ b^z_{i,A+\hat{\alpha}} = i(\chi_{i\alpha} - \chi^\dagger_{i\alpha})$$

Hence

$$K_z\left(b^z_{i,A} b^z_{i,A+\hat{z}} c_{i,A} c_{i,A+\hat{z}}\right) = K_z(2n_i^f - 1)(1 - 2n_i^z)$$

$$K_x\left(b^x_{i,A} b^x_{i,A+\hat{x}} c_{i,A} c_{i,A+\hat{x}}\right) = K_x(1 - 2n_i^x)$$
$$\times \left(f_i f_{i-\delta_1} - f_i f^\dagger_{i-\delta_1} + f_i^\dagger f_{i-\delta_1} - f_i^\dagger f^\dagger_{i-\delta_1}\right)$$

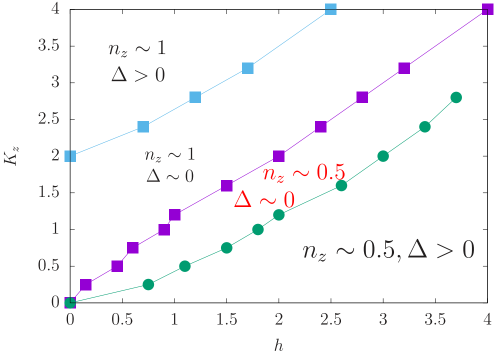


Figure: MFT phase diagram

K_z exchange on z bonds vanishes in the MFT of FC near h^-_{c2}!

Possible effective Theory in FC

Ignoring interchain coupling K_z, $\mathcal{H}$ becomes that of a compass chain, which can be mapped to the critical point of *TFIM*:

$$\mathcal{H} = \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_{i+1}^y \sigma_{i+2}^y \quad \rightarrow \quad \mathcal{H}^c_{\text{TFIM}} = -\sum_i \tau_i^z \tau_{i+1}^z - \sum_i \tau_i^x$$

$\mathcal{H}^c_{\text{TFIM}}$: 1+1D CFT with central charge $c = \frac{1}{2}$, with left and right chiral majoranas $\gamma_{L(R)}$.

$$\mathcal{H}_L = \int dx (i\gamma_L \partial_x \gamma_L), \ \mathcal{H}_R = \int dx(-i\gamma_R \partial_x \gamma_R)$$

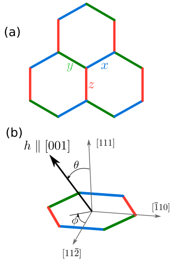
Consider weak coupling between chains via coupling between γ_L and γ_R:

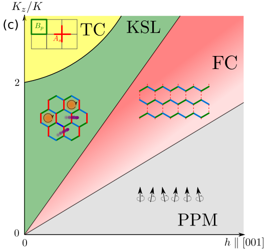
$$\mathcal{H}(\text{FC}) \approx \mathcal{H}_L + \mathcal{H}_R - g \int dx (\gamma_L \partial_x \gamma_L)(\gamma_R \partial_x \gamma_R) \quad \rightarrow \quad \text{stable until finite } g_c$$

A. Rahmani, X. Zhu, M. Franz, and I. Affleck, Phys. Rev. Lett. 115, 166401 (2015)

Summary

1. Weakly coupled fermionic chains from Kitaev model under [001] field $h \sim h_{c2}^-$
2. Decoupled bosonic chains from Kitaev model under large [001] field $\sim h_{c2}^+$
3. Liftshitz transitions by depleting n_i^z bond fermions from 1 to 0.5
4. Proposed effective theory in FC: $\mathcal{H}(\mathrm{FC}) \approx \mathcal{H}_L + \mathcal{H}_R - g \int dx (\gamma_L \partial_x \gamma_L)(\gamma_R \partial_x \gamma_R)$





Outlook:

1. How is (or is not) the FC phase connected to the proposed spinon Fermi surface under [111] field?
2. Other quantum phase transitions as a funciton of angles of field?
3. Dynamics