Solutions of A Probabilit Path

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1 Solutions to Chapter 1: Sets and Events

1.9.1 $\forall B \in \aleph$, since $\mathcal{C} \subset B$, we have $\{0\} \in B$, therefore $\Omega \setminus \{0\} = \{1\} \in B$. Also $\emptyset \in B$ and $\Omega \in B$. Therefore $\{\emptyset, \{0\}, \{1\}, \Omega\} \subset B$. Note that $\mathcal{P}(\Omega) = \{\emptyset, \{0\}, \{1\}, \Omega\}$. This means

$$\aleph = \{\mathcal{P}(\Omega)\}$$

1.9.2 Like in 1.9.1, we can conclude that

$$\forall B \in \aleph \quad \Rightarrow \{\emptyset, \{0\}, \{1,2\}, \Omega\} \subset B$$

Also note that $\{\emptyset, \{0\}, \{1,2\}, \Omega\}$ is a σ-field itself which means

$$\sigma(\mathcal{C}) = \{\emptyset, \{0\}, \{1,2\}, \Omega\}$$

Those subsets of Ω which are not inclued in $\sigma(\mathcal{C})$ are

$$\{1\}, \quad \{2\}, \quad \{0,1\}, \quad \{0,2\}$$

and it's easy to check that they are all inclued in B if any one of them is inclued. So to sum up, we have

$$\aleph = \{\sigma(\mathcal{C}), \mathcal{P}(\Omega)\}$$

1.9.3