

# Cox Proportional Hazard Model

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In this note we will talk about the Cox's proportional hazards (Cox's PH) model. Suppose we observe some non-informatively right-censored data  $(U, \delta)$  with covariate vector  $Z$ . That is, for subject  $i$ , the covariate vector is  $Z_i$ , survival time  $T_i$  and censoring time  $C_i$ . The observed data is  $(U_i, \delta_i)$  where  $U_i = \min(T_i, C_i)$  and  $\delta_i = 1(T_i \leq C_i)$ . Also  $T_i \perp C_i | Z_i$ .

And now we want to model the relationship between  $Z$  and  $T$ . One way to do that is to incorporate  $Z$  into the hazard function  $h(\cdot)$ , e.g.,

$$T \sim \text{Exp}(\lambda_Z) \implies h(t) = \lambda_Z \stackrel{\Delta}{=} e^{\alpha + \beta Z} = \lambda_0 e^{\beta Z},$$

where  $\lambda_0 = e^\alpha$  can be viewed as a baseline hazard. If  $\beta = 0$  then  $Z$  is not associated with  $T$ .

We can generalize this idea as

$$h(t|Z) = h_0(t) \times g(Z).$$

So the hazard can be factorized and this model is sometimes called a “multiplicative intensive model” or “multiplicative hazard model” or “proportional hazard model” because this factorization implies that

$$\frac{h(t|Z = z_1)}{h(t|Z = z_2)} = \frac{g(z_1)}{g(z_2)}.$$

## References