T-test

Chao Cheng

August 2, 2022

1 Basic knowledge

Theorem 1. *Let $x_1, \cdots, x_n$ be an random sample from a population with mean μ and variance $\sigma^2 < \infty$. Then*

1. *$\mathrm{E}\bar{x} = \mu$.*

2. *$\mathrm{Var}\bar{x} = \sigma^2/n$.*

3. *$\mathrm{E}S^2 = \sigma^2$, where $S^2 = \frac{1}{n-1}\sum\limits_{i=1}^{n}\left(x_i - \bar{x}\right)^2$.*

Theorem 2. *Let $x_1, \cdots, x_n$ be an random sample from $N\left(\mu, \sigma^2\right)$. Then*

1. *$\bar{X} \sim N\left(\mu, \sigma^2/n\right)$.*

2. *$\bar{X}$ is independent of S^2.*

3. *$(n-1)S^2/\sigma^2$ follows a chi-squared distribution with $n-1$ degree of freedom.*

2 One-sample test

2.1 variance known

2.2 variance unknown

3 Two sample test

3.1 Two-sample, variance known

3.2 Two-sample, variance unknown but equal

3.3 Two-sample, variance unknown and unequal