

# T-test

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## 1 Basic knowledge

**Theorem 1.** *Let  $x_1, \dots, x_n$  be a random sample from a population with mean  $\mu$  and variance  $\sigma^2 < \infty$ . Then*

1.  $E\bar{x} = \mu$ .
2.  $\text{Var}\bar{x} = \sigma^2/n$ .
3.  $ES^2 = \sigma^2$ , where  $S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$ .

**Theorem 2.** *Let  $x_1, \dots, x_n$  be a random sample from  $N(\mu, \sigma^2)$ . Then*

1.  $\bar{X} \sim N(\mu, \sigma^2/n)$ .
2.  $\bar{X}$  is independent of  $S^2$ .
3.  $(n-1)S^2/\sigma^2$  follows a chi-squared distribution with  $n-1$  degree of freedom.

## 2 One-sample test

Consider a random sample  $x_1, \dots, x_n$  from  $N(\mu, \sigma^2)$ .

### 2.1 variance known

### 2.2 variance unknown

## 3 Two sample test

### 3.1 Two-sample, variance known

### 3.2 Two-sample, variance unknown but equal

### 3.3 Two-sample, variance unknown and unequal