Minimum Detectable Difference (MDD) in Hypothesis Testing

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1 Introduction

In this notes, we talk about minimum detectable difference (MDD) and other related concepts in hypothesis testing. First, we will use a test for normal mean when variance is known as an example. So for an i.i.d. sample $x_1, \cdots, x_n$ with $x_i \sim N(\mu, \sigma^2)$ with known σ, we want to test

$$H_0: \ \mu = \mu_0 \quad \text{v.s.} \quad H_1: \ \mu \neq \mu_0.$$

A two-sided test with significant level α can be constructed as to reject H_0 when

$$\left| \frac{\bar{x} - \mu_0}{\sqrt{\sigma^2/n}} \right| \geq z_{1-\alpha/2},$$

since $\bar{x} \sim N(\mu_0, \sigma^2/n)$ under H_0. And for any given underlying μ that is not equal to μ_0, the probability to correctly reject H_0, i.e., the power is computed as

$$\begin{aligned}
&P\left(\left| \frac{\bar{x} - \mu_0}{\sqrt{\sigma^2/n}} \right| \geq z_{1-\alpha/2} \right) \\
=&P\left(\bar{x} \geq \mu_0 + z_{1-\alpha/2} * \sqrt{\sigma^2/n} \right) + P\left(\bar{x} \leq \mu_0 - z_{1-\alpha/2} * \sqrt{\sigma^2/n} \right) \\
=&P\left(\frac{\bar{x} - \mu}{\sqrt{\sigma^2/n}} \geq \frac{\mu_0 - \mu}{\sqrt{\sigma^2/n}} + z_{1-\alpha/2} \right) + P\left(\frac{\bar{x} - \mu}{\sqrt{\sigma^2/n}} \leq \frac{\mu_0 - \mu}{\sqrt{\sigma^2/n}} - z_{1-\alpha/2} \right) \\
=&\left(1 - \Phi\left(\frac{\mu_0 - \mu}{\sqrt{\sigma^2/n}} + z_{1-\alpha/2} \right) \right) + \Phi\left(\frac{\mu_0 - \mu}{\sqrt{\sigma^2/n}} - z_{1-\alpha/2} \right).
\end{aligned}$$

And a confidence interval for μ is constructed as

$$\left[\bar{x} - z_{1-\alpha/2} * \sqrt{\sigma^2/n}, \quad \bar{x} + z_{1-\alpha/2} * \sqrt{\sigma^2/n} \right].$$

References