Multivariate Normal Distirbution

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July 22, 2022

For a multivariate normal distribution denoted by

$$\boldsymbol{x} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}),$$

where $\boldsymbol{x} = (x_1, \cdots, x_n)^T \in \mathcal{R}^n$, $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_n)^T$ is the mean vector and

$$\boldsymbol{\Sigma} = (\rho_{ij}\sigma_i\sigma_j)_{n\times n} = \begin{pmatrix} \sigma_1^2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \vdots & \rho_{ij}\sigma_i\sigma_j & \vdots \\ \rho_{n1}\sigma_n\sigma_1 & \cdots & \sigma_n^2 \end{pmatrix}$$

is the $n \times n$ covariance matrix. Here $\sigma_1^2, \cdots, \sigma_n^2$ is the variance of $x_1, \cdots, x_n$ and ρ_{ij} is the correlation between x_i and x_j. Since it's symmetric, $\rho_{ij} = \rho_{ji}$ and $\rho_{ii} = 1$ for $i = 1, \cdots, n$.

The density function is

$$f(\boldsymbol{x}) = (2\pi)^{-n/2} |\boldsymbol{\Sigma}|^{-1/2} \exp\left(-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right).$$

1 Bivariate normal distribution

For a bivarite normal variable (x, y) following

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}\right),$$

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