Multivariate Normal Distirbution

Chao Cheng

July 23, 2022

For a multivariate normal distribution denoted by

$$\boldsymbol{x} \sim N\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right),$$

where $\boldsymbol{x} = \left(x_1, \cdots, x_n\right)^T \in \mathcal{R}^n$, $\boldsymbol{\mu} = \left(\mu_1, \cdots, \mu_n\right)^T$ is the mean vector and

$$\boldsymbol{\Sigma} = \left(\rho_{ij}\sigma_i\sigma_j\right)_{n\times n} = \begin{pmatrix} \sigma_1^2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \vdots & \rho_{ij}\sigma_i\sigma_j & \vdots \\ \rho_{n1}\sigma_n\sigma_1 & \cdots & \sigma_n^2 \end{pmatrix}$$

is the $n \times n$ covariance matrix. Here $\sigma_1^2, \cdots, \sigma_n^2$ is the variance of $x_1, \cdots, x_n$ and ρ_{ij} is the correlation between x_i and x_j. Since it's symmetric, $\rho_{ij} = \rho_{ji}$ and $\rho_{ii} = 1$ for $i = 1, \cdots, n$.

The density function is

$$f\left(\boldsymbol{x}\right) = \left(2\pi\right)^{-n/2} \left|\boldsymbol{\Sigma}\right|^{-1/2} \exp\left(-\frac{1}{2}\left(\boldsymbol{x} - \boldsymbol{\mu}\right)^T \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{x} - \boldsymbol{\mu}\right)\right).$$

1 Conditional distribution

2 Bivariate normal distribution

For a bivarite normal variable (x, y) following

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}\right),$$

A R codes

`R codes`