Bayesian Concepts

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From the frequentist perspective, we have the data x and the parameter of the distribution θ and we make estimation/inference about θ. But θ is alwyas treated as a fixed parameter. But from bayesian perspective, θ is also a random variable. First we introduce some notations:

- The prior distribution $\pi(\theta|\alpha)$, where α is fixed parameters for the distribution of θ. This prior distribution of θ represents our previous knowledge of θ before the data x is collected.
- The data distribution $f(x|\theta)$, which is the same as that from frequentist's perspective.
- The posterior distribution $f_{post}(\theta|x)$, which is the distribution of θ based on (conditional on) the observed data. Note that

  $$f_{post}(\theta|x) = \frac{f(\theta, x)}{f(x)} = \frac{f(x|\theta)\,\pi(\theta|\alpha)}{f(x)} \propto f(x|\theta)\,\pi(\theta|\alpha),$$

  where the last $\propto$ is taken with respect to θ. So the kernel of posterior distribution of θ given x is determined by $f(x|\theta)\,\pi(\theta|\alpha)$. Sometimes we will write $f_{post}(\theta|x)$ as $f_{post}(\theta|x;\alpha)$ to emphasize that this posterior distribution depends on x and parameter α.

2 Credible Interval

References