

# Minimum Detectable Difference (MDD) in Hypothesis Testing

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February 1, 2023

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## 1 Introduction

In this notes, we talk about minimum detectable difference (MDD) and other related concepts in hypothesis testing. First, we will use a test for normal mean when variance is known as an example. So for an i.i.d. sample  $x_1, \dots, x_n$  with  $x_i \sim N(\mu, \sigma^2)$  with known  $\sigma$ , we want to test

$$H_0 : \mu = \mu_0 \quad \text{v.s.} \quad H_1 : \mu \neq \mu_0.$$

A two-sided test with significant level  $\alpha$  can be constructed as to reject  $H_0$  when

$$\left| \frac{\bar{x} - \mu_0}{\sqrt{\sigma^2/n}} \right| \geq z_{1-\alpha/2},$$

since  $\bar{x} \sim N(\mu_0, \sigma^2/n)$  under  $H_0$ . And for any given underlying  $\mu$  that is not equal to  $\mu_0$ , the probability to correctly reject  $H_0$ , i.e., the power is computed as

$$\begin{aligned} & P \left( \left| \frac{\bar{x} - \mu_0}{\sqrt{\sigma^2/n}} \right| \geq z_{1-\alpha/2} \right) \\ &= P \left( \bar{x} \geq \mu_0 + z_{1-\alpha/2} * \sqrt{\sigma^2/n} \right) + P \left( \bar{x} \leq \mu_0 - z_{1-\alpha/2} * \sqrt{\sigma^2/n} \right) \\ &= P \left( \frac{\bar{x} - \mu}{\sqrt{\sigma^2/n}} \geq \frac{\mu_0 - \mu}{\sqrt{\sigma^2/n}} + z_{1-\alpha/2} \right) + P \left( \frac{\bar{x} - \mu}{\sqrt{\sigma^2/n}} \leq \frac{\mu_0 - \mu}{\sqrt{\sigma^2/n}} - z_{1-\alpha/2} \right) \\ &= \left( 1 - \Phi \left( \frac{\mu_0 - \mu}{\sqrt{\sigma^2/n}} + z_{1-\alpha/2} \right) \right) + \Phi \left( \frac{\mu_0 - \mu}{\sqrt{\sigma^2/n}} - z_{1-\alpha/2} \right). \end{aligned}$$

And a confidence interval for  $\mu$  is constructed as

$$\left[ \bar{x} - z_{1-\alpha/2} * \sqrt{\sigma^2/n}, \quad \bar{x} + z_{1-\alpha/2} * \sqrt{\sigma^2/n} \right].$$

So here we can see that in order to reject  $H_0$ , the critical point is the normalized difference  $\left| \frac{\bar{x} - \mu_0}{\sqrt{\sigma^2/n}} \right|$  to be greater than  $z_{1-\alpha/2}$ . This critical point ( $z_{1-\alpha/2}$ ) is determined by the type of test, which leads to the type of test statistics and the significant level. So in other words, as long as the observed difference  $|\bar{x} - \mu_0|$  is greater than  $z_{1-\alpha/2} * \sqrt{\sigma^2/n}$ ,  $H_0$  would be rejected no matter what the underlying  $\mu$  is. So here  $z_{1-\alpha/2} * \sqrt{\sigma^2/n}$  is the minimum effect size (considering the sample size and variance) that would be just significant, hence the minimum detectable difference (MDD).

## 2 MDD and other concepts

For more details one can see [Mair et al. \[2020\]](#).

## References

Magdalena M. Mair, Mira Kattwinkel, Oliver Jakoby, and Florian Hartig. The minimum detectable difference (MDD) concept for establishing trust in nonsignificant results: A critical review. *Environmental Toxicology and Chemistry*, 39(11):2109–2123, sep 2020. doi: 10.1002/etc.4847.