

# Bayesian Concepts

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## Contents

<b>1 Introduction</b>	<b>1</b>
<b>2 Credible Interval</b>	<b>2</b>
<b>3 Predictive distribution</b>	<b>2</b>

## 1 Introduction

1. Prior and posterior distribution
2. Predictive probability and application in phase II design
3. Credible interval

From the frequentist perspective, we have the data  $x$  and the parameter of the distribution  $\theta$  and we make estimation/inference about  $\theta$ . But  $\theta$  is always treated as a fixed parameter. But from bayesian perspective,  $\theta$  is also a random variable. First we introduce some notations:

- The prior distribution  $\pi(\theta|\alpha)$ , where  $\alpha$  is fixed parameters for the distribution of  $\theta$ . This prior distribution of  $\theta$  represents our previous knowledge of  $\theta$  before the data  $x$  is collected.
- The data distribution  $f(x|\theta)$ , which is the same as that from frequentist's perspective.
- The posterior distribution  $f_{post}(\theta|x)$ , which is the distribution of  $\theta$  based on (conditional on) the observed data. Note that

$$f_{post}(\theta|x) = \frac{f(\theta, x)}{f(x)} = \frac{f(x|\theta) \pi(\theta|\alpha)}{f(x)} \propto f(x|\theta) \pi(\theta|\alpha),$$

where the last  $\propto$  is taken with respect to  $\theta$ . So the kernel of posterior distribution of  $\theta$  given  $x$  is determined by  $f(x|\theta) \pi(\theta|\alpha)$ . Sometimes we will write  $f_{post}(\theta|x)$  as  $f_{post}(\theta|x; \alpha)$  to emphasize that this posterior distribution depends on  $x$  and parameter  $\alpha$ .

2 Credible Interval

3 Predictive distribution

References