#### Bayesian Concepts

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#### 1 Introduction

- 1. Prior and posterior distribution
- 2. Predictive probability and application in phase II design
- 3. Credible interval

From the frequentist perspective, we have the data x and the parameter of the distribution  $\theta$  and we make estimation/inference about  $\theta$ . But  $\theta$  is always treated as a fixed parameter. But from bayesian perspective,  $\theta$  is also a random variable. First we introduce some notations:

- The prior distribution  $\pi(\theta|\alpha)$ , where  $\alpha$  is fixed parameters for the distribution of  $\theta$ . This prior distribution of  $\theta$  represents our previous knowledge of  $\theta$  before the data x is collected.
- The data distribution  $f(x|\theta)$ , which is the same as that from frequentist's perspective.
- The posterior distribution  $f_{post}(\theta|x)$ , which is the distribution of  $\theta$  based on (conditional on) the observed data. Note that

$$f_{post}\left(\theta|x\right) = \frac{f\left(\theta,x\right)}{f\left(x\right)} = \frac{f\left(x|\theta\right)\pi\left(\theta|\alpha\right)}{f\left(x\right)} \propto f\left(x|\theta\right)\pi\left(\theta|\alpha\right),$$

where the last  $\propto$  is taken with respect to  $\theta$ . So the kernel of posterior distribution of  $\theta$  given x is determined by  $f(x|\theta)\pi(\theta|\alpha)$ . Sometimes we will write  $f_{post}(\theta|x)$  as  $f_{post}(\theta|x;\alpha)$  to emphasize that this posterior distribution depends on x and parameter  $\alpha$ .

# 2 Credible Interval

equal tail or equal density?

# 3 Predictive distribution

# References