Cox Proportional Hazard Model

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1 Introduction

In this note we will talk about the Cox's proportional hazards (Cox's PH) model. Suppose we observe some non-informativly right-censored data (U, δ) with covariate vector Z. That is, for subject i, the covariate vector is Z_i, survival time T_i and censoring time C_i. The observed data is (U_i, δ_i) where $U_i = \min(T_i, C_i)$ and $\delta_i = 1(T_i \leq C_i)$. Also $T_i \perp C_i | Z_i$.

And now we want to model the relationship between Z and T. One way to do that is to incorporate Z into the hazard function $h(\cdot)$, e.g.,

$$T \sim Exp(\lambda_Z) \implies h(t) = \lambda_Z \triangleq e^{\alpha+\beta Z} = \lambda_0 e^{\beta Z},$$

where $\lambda_0 = e^{\alpha}$ can be viewed as a baseline hazard. If $\beta = 0$ then Z is not associated with T.

We can generalize this idea as

$$h(t|Z) = h_0(t) \times g(Z).$$

So the hazard can be factorized and this model is sometimes called a "multiplicative intensive model" or "multiplicative hazard model" or "proportional hazard model" because this factorization implies that

$$\frac{h(t|Z = z_1)}{h(t|Z = z_2)} = \frac{g(z_1)}{g(z_2)}.$$

The hazard ratio is constant with respect to t, hence the (constant) proportional hazard. So in our previous model (the exponential survival time), the hazard ratio is

$$\frac{h(t|Z = z_1)}{h(t|Z = z_2)} = e^{\beta(z_1 - z_2)}.$$

2 Estimation

3 Inference

References