

# Bayesian Concepts

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## 1 Introduction

1. Prior and posterior distribution
2. Predictive probability and application in phase II design
3. Credible interval

From the frequentist perspective, we have the data  $x$  and the parameter of the distribution  $\theta$  and we make estimation/inference about  $\theta$ . But  $\theta$  is always treated as a fixed parameter. But from bayesian perspective,  $\theta$  is also a random variable. First we introduce some notations:

- The prior distribution  $\pi(\theta|\alpha)$ , where  $\alpha$  is fixed parameters for the distribution of  $\theta$ . This prior distribution of  $\theta$  represents our previous knowledge of  $\theta$  before the data  $x$  is collected.
- The data distribution  $f(x|\theta)$ , which is the same as that from frequentist's perspective.
- The posterior distribution  $f_{post}(\theta|x)$ , which is the distribution of  $\theta$  based on (conditional on) the observed data. Note that

$$f_{post}(\theta|x) = \frac{f(\theta, x)}{f(x)} = \frac{f(x|\theta) \pi(\theta|\alpha)}{f(x)} \propto f(x|\theta) \pi(\theta|\alpha),$$

where the last  $\propto$  is taken with respect to  $\theta$ . So the kernel of posterior distribution of  $\theta$  given  $x$  is determined by  $f(x|\theta) \pi(\theta|\alpha)$ . Sometimes we will write  $f_{post}(\theta|x)$  as  $f_{post}(\theta|x; \alpha)$  to emphasize that this posterior distribution depends on  $x$  and parameter  $\alpha$ .

## 2 Credible Interval

There are mainly two types of credible interval: equal tail interval (ETI) and highest (posterior) density interval (HDI). Denote the CI as  $[lower, upper]$ , then these two ends satisfies different conditions for ETI and HDI:

- For ETI, the CI has equal probability tails on both sides, that is  $F(lower) = (1 - F(upper)) = \alpha/2$  and the CI is  $[lower, upper]$ .
- For HDI, the CI has equal **density** on both ends, that is  $f(lower) = f(upper)$  and  $F(upper) - F(lower) = 1 - \alpha$ .

**Note:** sometimes we will use SDI (shortest density interval) to mean HDI, since SDI can be seen as a more robust method to compute HDI[Liu et al., 2015].

ETI and HDI can produce same results as long as the distribution is symmetric. But in reality many posterior distribution is skewed, then HDI is a more reasonable CI. A demonstration is shown in Figure 1. Note that HDI can be hard to compute, a starting

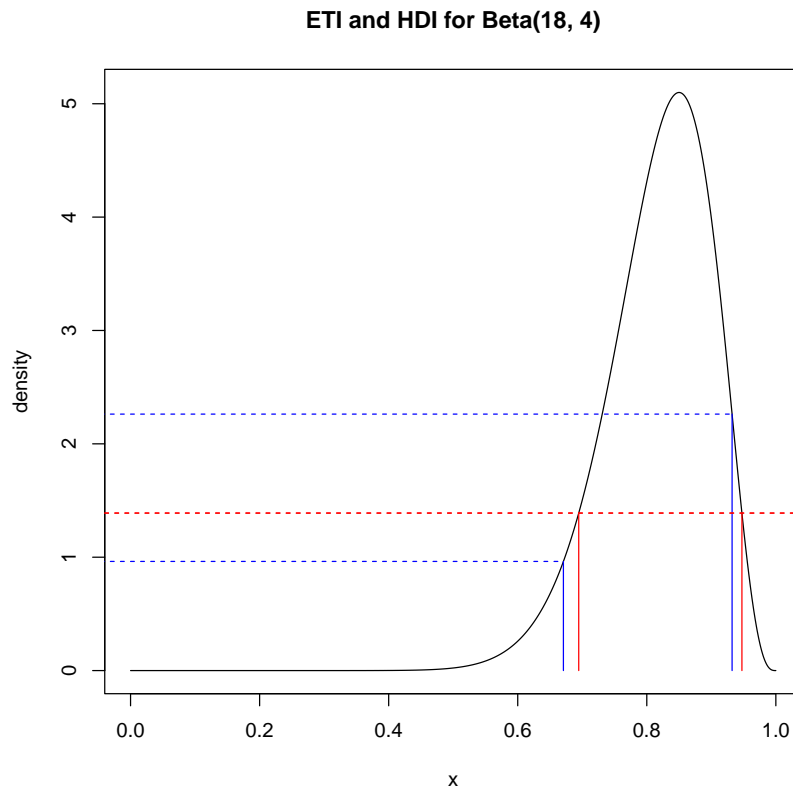


Figure 1: Comparison of 90% ETI and HDI for Beta(18,4)

point of R script is shown here, using package bayestestR:

```
x <- seq(from = 0, to = 1, by = 0.001)
a <- 18
b <- 4
y1 <- dbeta(x, a, b)
```

```

# HDI
tmp <- bayestestR::distribution_beta(n = 100000, shape1 = a, shape2 = b)
tmp2 <- bayestestR::ci(tmp, ci = 0.90, method = "HDI")

# Compare HDI and SDI
bayestestR::ci(tmp, ci = 0.90, method = "HDI")
str(bayestestR::hdi(tmp, ci = 0.9))
str(bayestestR::spi(tmp, ci = 0.9))    # SPI is a more robust HDI

```

### 3 Predictive distribution

### References

Ying Liu, Andrew Gelman, and Tian Zheng. Simulation-efficient shortest probability intervals. *Statistics and Computing*, 25(4):809–819, jun 2015. doi: 10.1007/s11222-015-9563-8.