Test for the probability of a binomial distribution

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For an i.i.d sample from a bernoulli distribution

$$x_1, \cdots, x_n \overset{\text{i.i.d.}}{\sim} Bernoulli(p),$$

The likelihood of the data is

$$f\left(x_1, \cdots, x_n\right) = \prod_{i=1}^{n} p^{x_i} \left(1-p\right)^{1-x_i} = p^{\sum x_i} \left(1-p\right)^{n-\sum x_i}.$$

MLE for p is $\bar{x} = \frac{1}{n} \sum x_i$ and

$$\sum_{i=1}^{n} x_i \sim Binom(n, p).$$

So here are mainly two situations: One is to test the probability p against some given value p_0. The other is to compare the probability between two independent random samples $x_1, \cdots, x_n$ and $y_1, \cdots, y_m$.

1 Normal approximation

Note that

$$\mathrm{E}X = p, \quad \mathrm{Var}X = p\left(1-p\right).$$

Then by CLT we have

$$\bar{x} \overset{\text{asymp}}{\sim} N\left(p, \ \frac{p\left(1-p\right)}{n}\right).$$