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Option A

I. Problem

Better and Tastier Burgers: Given n kinds of burgers, each burger has two attributes: taste value and health value, select the maximum number of burgers that can be arranged in a line, such that each burger in the line is both tastier and healthier than the one before.

II. Analysis

It is similar to the LIS (longest Increasing sequence) problem.

1. In LIS problem, given array A(1…n),we want to find a longest subsequence(not necessarily contiguous)that is strictly increasing. But we only have one attribute. So we define X[i]=longest strictly increasing subsequence that ends at i, X[i]=max{X[j]}+1,j<i and A[j]<A[i].

   To calculate each X[i],we need to traverse from 0 to i-1,the size of array is n, so it takes O(n^2) to calculate all X[i].From all these X[i],we need to find the MAX value, which takes O(n) time, therefore, the overall time complexity is O(n^2).(Could be O(nlogn) if using binary search, but more complicate to implement.)
2. Back to Better and Tastier Burgers problem. Now we want to find a longest subsequence that is strictly increasing in both attributes.

   However, if we sort these burgers by taste value first, we get a new array which is increasing in taste value, the problem becomes---find a longest subsequence(not necessarily continuous) that is strictly increasing in health value, which is exactly the same as LIS problem.

Steps:

1. QUICKSORT the array by taste value in O(nlogn) time.
2. Compute the X[i] of the new array in O(n^2) time.
3. Optional step: use a new array S[i] to save the intermediary result j---max{X[j]}+1,j<i and A[j]<A[i],S[i] is used for printing the result.
4. Find the largest value of all X[i].
5. Return max{X[i]}.

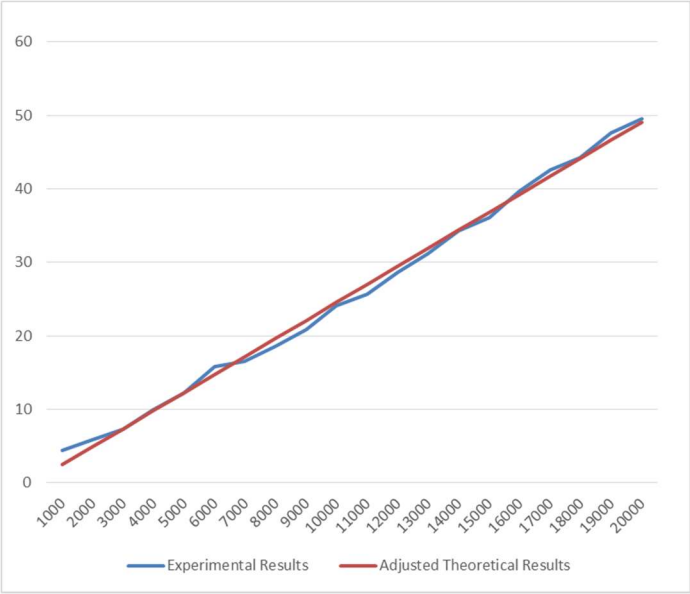
The time complexity is : O(n^2).

III.Coding

Algorithm consists of three funtions: 1. QUICKSORT,
2. Modified version of LIS.
3. function for printing the result.

IV.Result

n	Experimental Result	Theoretical Result	Scaling Constant	Adjusted Theoretical Result
1000	4.47214	1000	0.00245465	2.45465
2000	5.91608	2000	0.00245465	4.9093
3000	7.34847	3000	0.00245465	7.36395
4000	9.89949	4000	0.00245465	9.8186
5000	12.2474	5000	0.00245465	12.27325
6000	15.843	6000	0.00245465	14.7279
7000	16.5529	7000	0.00245465	17.18255
8000	18.5742	8000	0.00245465	19.6372
9000	20.8567	9000	0.00245465	22.09185
10000	24.0624	10000	0.00245465	24.5465
11000	25.7099	11000	0.00245465	27.00115
12000	28.688	12000	0.00245465	29.4558
13000	31.1448	13000	0.00245465	31.91045
14000	34.2783	14000	0.00245465	34.3651
15000	36.0971	15000	0.00245465	36.81975
16000	39.7115	16000	0.00245465	39.2744
17000	42.5911	17000	0.00245465	41.72905
18000	44.238	18000	0.00245465	44.1837
19000	47.676	19000	0.00245465	46.63835
20000	49.5681	20000	0.00245465	49.093
average	25.773779	10500	0.00245465	



The experimental result is the square root of runtime,as we conclude the time complexity of Better and Tastier Burgers algorithm is O(n^2),the sqrt(runtime) should be linear in terms of n,and the graph support this conclusion:the experimental result matches well with adjusted theoretical result,which is a straight line overall.