

12.12 第十次作业答案

(一) 选择题

1. A. $||-y| \leq 1; x-y > 0.$

2. D. $f(x+y, x-y) = (x+y) \cdot (x-y) \Rightarrow f(x, y) = xy$

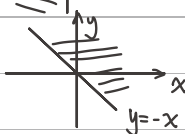
3. C. $z = ye^x \quad \frac{\partial z}{\partial x} = ye^x$

(二) 解答题

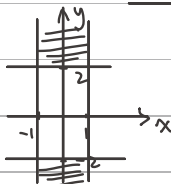
2 (1). $D = \{xy \geq 0\} = \text{第一、三象限含轴}$



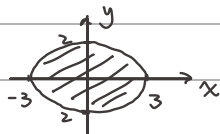
(2) $D = \{x+y > 0\}$



(3). $D = \{|x| \leq 1, |y| \geq 2\}.$



(4) $D = \{\frac{x^2}{9} + \frac{y^2}{4} \leq 1\}.$



3. (1). $\frac{\partial^2 z}{\partial x^2} = 3x^2y^2, \quad \frac{\partial^2 z}{\partial y^2} = 2x^3y$

(2). $\frac{\partial^2 z}{\partial x^2} = 4x^3, \quad \frac{\partial^2 z}{\partial y^2} = 3y^2$

(3). $\frac{\partial^2 z}{\partial x^2} = -\frac{1}{x}, \quad \frac{\partial^2 z}{\partial y^2} = \frac{1}{y}$

(4). $\frac{\partial^2 z}{\partial x^2} = \frac{y^2}{(x+y)^2}, \quad \frac{\partial^2 z}{\partial y^2} = \frac{x^2}{(x+y)^2}$

(5). $\frac{\partial^2 z}{\partial x^2} = ye^{xy} + 2xy, \quad \frac{\partial^2 z}{\partial y^2} = xe^{xy} + x^2$

(6). $\frac{\partial^2 u}{\partial x^2} = x(x^2+y^2+z^2)^{-\frac{1}{2}}, \quad \frac{\partial^2 u}{\partial y^2} = y(x^2+y^2+z^2)^{-\frac{1}{2}}, \quad \frac{\partial^2 u}{\partial z^2} = z(x^2+y^2+z^2)^{-\frac{1}{2}}$

(7). $\frac{\partial^2 u}{\partial x^2} = \frac{z}{y} \cdot x^{\frac{z}{y}-1}, \quad \frac{\partial^2 u}{\partial y^2} = -\frac{z \ln x}{y^2} \cdot x^{\frac{z}{y}}, \quad \frac{\partial^2 u}{\partial z^2} = \frac{\ln x}{y} \cdot x^{\frac{z}{y}}$

(8). $\frac{\partial^2 u}{\partial x^2} = \frac{1}{z} - \frac{y}{x^2}, \quad \frac{\partial^2 u}{\partial y^2} = \frac{1}{x} - \frac{z}{y^2}, \quad \frac{\partial^2 u}{\partial z^2} = \frac{1}{y} - \frac{x}{z^2}$

$$4. (1) \frac{\partial z}{\partial x} = 4x^3 + 6xy.$$

$$\frac{\partial z}{\partial y} = 3x^2 + 3y^2$$

$$(2) \frac{\partial z}{\partial x} = \ln(x+y) + \frac{x}{x+y}$$

$$\frac{\partial z}{\partial y} = \frac{x}{x+y}$$

$$\frac{\partial^2 z}{\partial x^2} = 12x + 6y$$

$$\frac{\partial^2 z}{\partial y^2} = 6y$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{(x+y)} + \frac{y}{(x+y)^2}$$

$$\frac{\partial^2 z}{\partial y^2} = -\frac{x}{(x+y)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = 6x$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{y}{(x+y)^2}$$

$$5. z(x, y) = \ln(x+y) \triangleq \tilde{z}(x, y) \text{ 则 } \frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 \tilde{z}}{\partial x^2} \cdot \left(\frac{dx}{dx}\right) = 2x \frac{\partial^2 \tilde{z}}{\partial x^2}.$$

$$\text{则 } 1 = \frac{x+y}{x+y} = \frac{\partial \tilde{z}}{\partial x} + \frac{\partial \tilde{z}}{\partial y} = 2x \frac{\partial^2 \tilde{z}}{\partial x^2} + 2y \frac{\partial^2 \tilde{z}}{\partial y^2}. \text{ 即证所求.}$$

$$6. \ln z = y \ln x. \triangleq \tilde{z}(x, y). \text{ 则 } \frac{\partial \tilde{z}}{\partial x} = \frac{1}{x} \frac{\partial z}{\partial x} \quad \frac{\partial \tilde{z}}{\partial y} = \frac{1}{x} \frac{\partial z}{\partial y}$$

$$z = \frac{x}{y} \cdot \frac{y}{x} + \frac{1}{\ln x} \ln x = \frac{x}{y} \cdot \frac{\partial \tilde{z}}{\partial x} + \frac{1}{\ln x} \cdot \frac{\partial \tilde{z}}{\partial y} = \frac{1}{x} \left(\frac{x}{y} \frac{\partial z}{\partial x} + \frac{1}{\ln x} \frac{\partial z}{\partial y} \right)$$

即证所求.