1) $R(A, B, C, D, E, F)$

F	FD 1	$A \rightarrow BC$
	FD 2	$C \rightarrow AD$
	FD 3	$DE \rightarrow F$

a) Derive $C \rightarrow B$:

$C \rightarrow A$	Decomposition	FD 2
$A \rightarrow B$	Decomposition	FD 1
$C \rightarrow B$	Transitivity	

b) Derive $AE \rightarrow F$:

$A \rightarrow C$	Decomposition	FD 1
$C \rightarrow D$	Decomposition	FD 2
$A \rightarrow D$	Transitivity	
$AE \rightarrow F$	Pseudo-transitivity	FD 3

2)

a) $\{A\}^+ = \{A, B, C, D\}$

FD 1   FD 2

b) $\{C, E\}^+ = \{C, E, A, D, B, F\}$

FD 2   FD 1   FD 3

$\{C, E\}$ is a superkey

3) R (A,B,C,D,E,F)

FD 1   AB → CDEF
FD 2   E → F
FD 3   D → B

a) A B E D ~~C~~ ~~F~~

A: only LHS
C, F: only RHS

$\{A\}^+ = \{A\}$

$\{A,B\}^+ = \{A,B,C,D,E,F\}$ ✓ SK

$\{A,E\}^+ = \{A,E,F\}$

$\{A,D\}^+ = \{A,D,B,C,E,F\}$ ✓ SK

{A,B} and {A,D} are CKs

b) Since {A,B} is CK, it is also super key. {E} and {D} are not SKs, FD2 and FD3 violate BCNF condition.

c) FD2:

R1: {E,F}   FD2 E → F   CK: E   in BCNF

R2: {A,B,C,D,E}   decomp. FD1 AB → CDE   CK: AB   not
FD3 D → B   D is not SK

R3: {B,D}   FD3 D → B   CK: D   in

R4: {A,C,D,E}   FD4 AD → CE   CK: AD   in

D → B   FD3
AB → CE   decomp. FD1
AD → CE   Pseudo-trans

or AD is SK
so AD must fd CE

FD3:

R5 {B,D}   FD3 D → B   CK: D   in

R6 {A,C,D,E,F}   FD5 similar with FD4 AD → CFE   CK: AD   not
FD2 E → F   E is not SK

R7 {E,F}   FD2 E → F   CK: E   in

R8 {A,C,D,E}   FD4 AD → CE   CK: AD   in

So R1/R7, R3/R5 and R4/R8 are in BCNF relations.

4)

$R(A, B, C, D, E)$

FD 1 $ABC \rightarrow DE$
FD 2 $BCD \rightarrow AE$
FD 3 $C \rightarrow D$

a) $\{A,B,C\}^+ = \{A, B, C, D, E\}$ ✓ SK
$\{B,C,D\}^+ = \{B, C, D, A, E\}$ ✓ SK
$\{C\}^+ = \{C, D\}$ not SK

R is not in BCNF.

b) FD3:

R1 $\{C,D\}$ FD3 $C \rightarrow D$ CK: C
R2 $\{A,B,C,E\}$ decomp. FD1 $ABC \rightarrow E$ CK: ABC

So R1 and R2 are in the set of BCNF relations.