732A62 Time Series Analysis
Division of Statistics and Machine Learning
Department of Computer and Information Science

Computer lab C

Instructions

- The lab is assumed to be done in groups.
- Create a report to the lab solutions in PDF.
- Be concise and do not include unnecessary printouts and figures produced by the software and not required in the assignments.
- **Include all your codes as an appendix into your report.**
- A typical lab report should 2-4 pages of text plus some number of figures plus appendix with codes.
- The group lab report should be submitted via LISAM before the deadline specified in LISAM.
- Use 12345 as a random seed everywhere where the result of the simulation differs with the run unless stated otherwise.

Implementation of Kalman filter

Assignment 1
In table 1 a script for generation of data from simulation o fthe following state space model and implementation of the Kalman filter on the data is given.

$$\begin{aligned}
\mathbf{z}_t &= A_{t-1}\mathbf{z}_{t-1} + e_t, \\
\mathbf{x}_t &= C_t\mathbf{z}_t + \nu_t, \\
\nu_t &\sim N(0, R_t), \\
e_t &\sim N(0, Q_t).
\end{aligned}$$

a. Write down the expression for the state space model that is being simulated.
b. Run this scrip and compare the filtering results with a moving average smoother of order 5.
c. Also, compare the filtering outcome when R in the filter is 10 times smaller than its actual value while Q in the filter is 10 times larger than its actual value. How does the filtering outcome varies?
d. Now compare the filtering outcome when R in the filter is 10 times larger than its actual value while Q in the filter is 10 times smaller than its actual value. How does the filtering outcome varies?
e. Implement your own Kalman filter and replace ksmooth0 function with your script.
f. How do you interpret the Kalman gain?

In Table 2 the Kalman filtering algorithm is given for reference.

Table 1: An R script for the Kalman filtering, smoothing, and prediction

```r
# generate data
set.seed(1); num = 50
w = rnorm(num+1,0,1); v = rnorm(num,0,1)
mu = cumsum(w) # state: mu[0], mu[1],..., mu[50]
y = mu[-1] + v # obs: y[1],..., y[50]
# filter and smooth (Ksmooth0 does both)
ks = Ksmooth0(num, y, A=1, mu0=0, Sigma0=1, Phi=1, cQ=1, cR=1)
# start figure
par(mfrow=c(3,1)); Time = 1:num
plot(Time, mu[-1], main='Predict', ylim=c(-5,10))
lines(Time,y,col="green")
lines(ks$xp)
lines(ks$xp+2*sqrt(ks$Pp), lty=2, col=4)
lines(ks$xp-2*sqrt(ks$Pp), lty=2, col=4)
plot(Time, mu[-1], main='Filter', ylim=c(-5,10))
lines(Time,y,col="green")
lines(ks$xf)
lines(ks$xf+2*sqrt(ks$Pf), lty=2, col=4)
lines(ks$xf-2*sqrt(ks$Pf), lty=2, col=4)
plot(Time, mu[-1], main='Smooth', ylim=c(-5,10))
lines(Time,y,col="green")
lines(ks$xs)
lines(ks$xs+2*sqrt(ks$Ps), lty=2, col=4)
lines(ks$xs-2*sqrt(ks$Ps), lty=2, col=4)
mu[1]; ks$x0n; sqrt(ks$P0n) # initial value info
```

Table 2: Kalman filtering recursion

Inputs: A_t, C_t, Q_t, R_t, m_0, P_0 and $\mathbf{x}_{1:T}$.
initialization
$m_{1|0} \leftarrow m_0$, $P_{1|0} \leftarrow P_0$
for $t = 1$ to T **do**
    observation update step
    $K_t \leftarrow P_{t|t-1} C_t^{\mathrm{T}} (C_t P_{t|t-1} C_t^{\mathrm{T}} + R_t)^{-1}$
    $m_{t|t} \leftarrow m_{t|t-1} + K_t(\mathbf{x}_t - C_t m_{t|t-1})$
    $P_{t|t} \leftarrow (I - K_t C_t) P_{t|t-1}$
    prediction step
    $m_{t+1|t} \leftarrow A_t m_{t|t}$
    $P_{t+1|t} \leftarrow A_t P_{t|t} A_t^{\mathrm{T}} + Q_{t+1}$
end for
Outputs: $m_{t|t}$, $P_{t|t}$ for $t = 1 : T$