

# X 的奇幻之旅

**O'RLY?**

游戏客人

# 数字

## 罗马数字

```
In[*]:= FromRomanNumeral /@ {"MDCCCVII", "MDCCCLXXIV"} //  
[将罗马数字转换为十进制数字  
( "埃兹拉·康奈尔 (Ezra Cornell, " <> ToString@#[[1]] <> "-" <> ToString@#[[2]] <> ") " &  
[转换为字符串][转换为字符串]  
Out[*]=  
埃兹拉·康奈尔 (Ezra Cornell, 1807-1874)
```

## 牛顿分形

- 代码参考: <https://mathematica.stackexchange.com/questions/100053/my-introduction-to-compile/100055#100055>
- 科普视频: <https://www.youtube.com/watch?v=-RdOwhmqP5s>

(\*牛顿迭代法求 $z^n-1=0$ 的根,  $z$ 为迭代起点,  $maxIterations$ 为最大迭代次数, 返回归一化幅值\*)

newtonFractalCompiled =


Compile[{{n, \_Integer}, {z, \_Complex}, {maxIterations, \_Integer}},

[编译][整数][复数][整数]

Arg[FixedPoint[# - (#^n - 1) / (n #^(n - 1)) &, N[z], maxIterations]] / (2 Pi)]

[辐角][固定点][数值运算][圆周率]

Out[\*]=

CompiledFunction[ Argument count: 3  
Argument types: {\_Integer, \_Complex, \_Integer}]

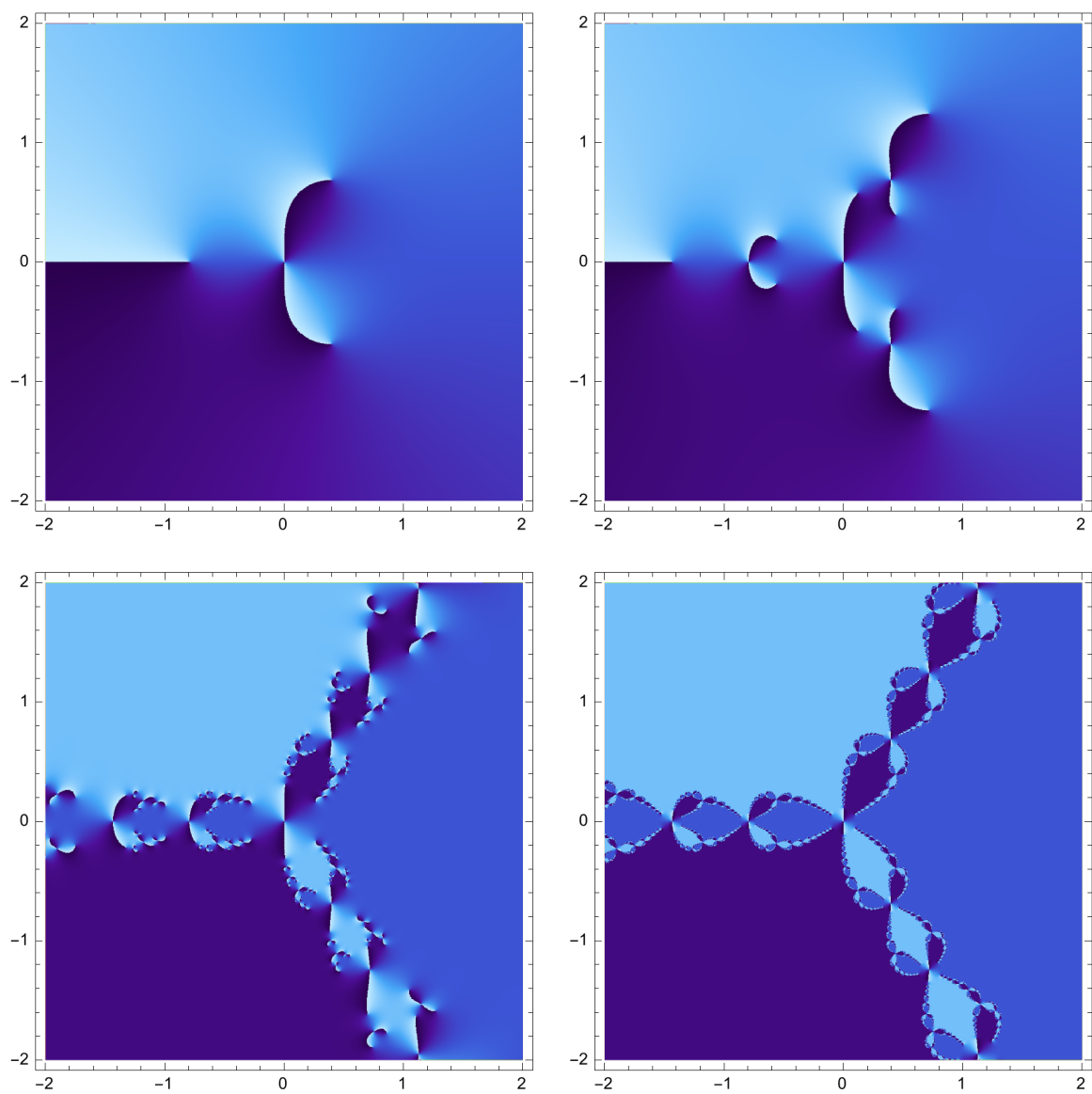
## 改变迭代次数

```

In[ ]:= DensityPlot[newtonFractalCompiled[3, x + I y, #], {x, -2, 2},
  [密度图]                                [虚数单位]
  {y, -2, 2}, PlotPoints -> 300, ColorFunction -> "DeepSeaColors"] & /@
  [绘图点]                                [颜色函数]
  {1, 2, 5, 10} // ArrayReshape[#, {2, 2}] & // GraphicsGrid
  [数组重塑]                                [图形网格]

```

Out[ ]:=



## 改变方程次数

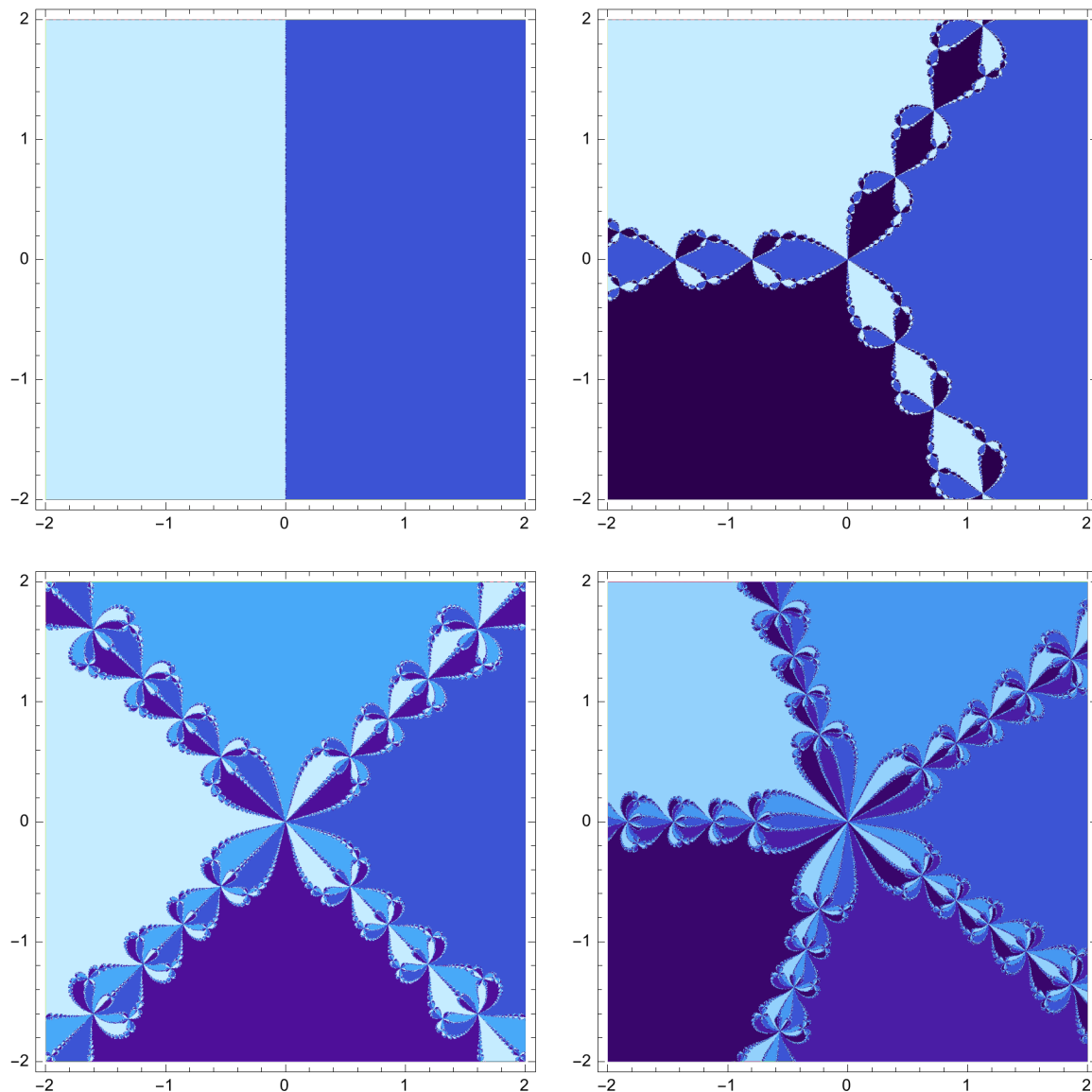
$n = 2$  时不产生分形

```

In[ ]:= DensityPlot[newtonFractalCompiled[#, x + I y, 50], {x, -2, 2},
  [密度图]                                [虚数单位]
  {y, -2, 2}, PlotPoints -> 300, ColorFunction -> "DeepSeaColors"] & /@
  [绘图点]                                [颜色函数]
  {2, 3, 4, 5} // ArrayReshape[#, {2, 2}] & // GraphicsGrid
  [数组重塑]                                [图形网格]

```

Out[ ]:=



上图中的任一点，要么收敛到某个根，要么可以到达所有根  
 设方程有  $n$  个根，在图中画半径任意小的圆，要么只包含一种颜色，要么同时包含  $n$  种颜色，这就是分形的来源

## Britney Gallivan 折纸公式

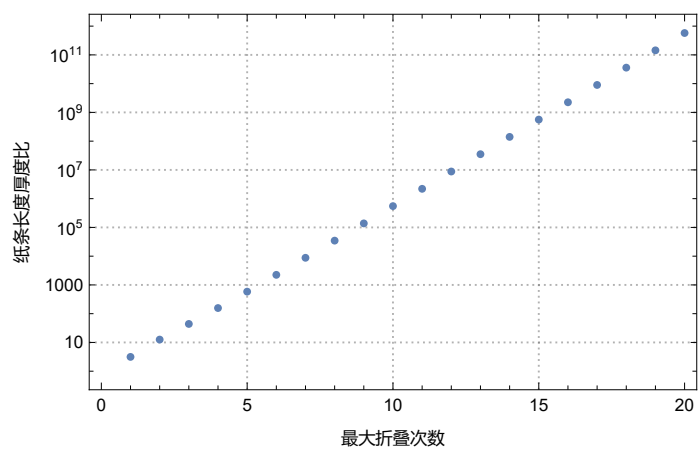
$$L = \frac{\pi T}{6} (2^n + 4) (2^n - 1)$$

```

In[ ]:= ListLogPlot[Table[{n, Pi / 6 (2^n + 4) (2^n - 1)}, {n, 1, 20}],
  点集的对数图 表格 圆周率
  PlotTheme -> "Detailed", FrameLabel -> {"最大折叠次数", "纸条长度厚度比"}]
  绘图主题 边框标签

```

Out[ ]=

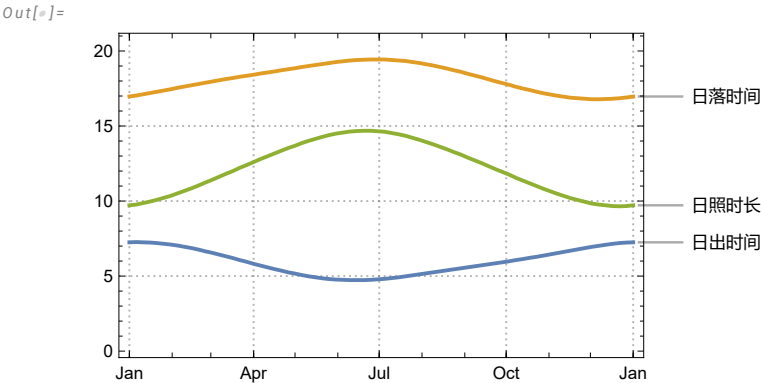


# 形状

## 日照时间

```
In[*]:= {sunrises, sunsets} =  
  Normal@#[DateRange[DateObject[{2022, 1, 1}, "Day", "Gregorian", +8.], DateObject[  
    [转换为… [日期范围 [日期对象 [日期对象  
    {2023, 1, 1}, "Day", "Gregorian", +8.], {5, "Day"}]] & /@ {Sunrise, Sunset};  
                                     [日出 [日落]
```

```
In[*]:= dates = sunrises[[All, 1, 1];  
          [全部  
          {sunriseTimes, sunsetTimes} =  
            DateValue[#, "Hour"] + DateValue[#, "Minute"] / 60. & /@#[All, 2] & /@  
            [日期值 [日期值 [全部  
            {sunrises, sunsets};  
            DateListPlot[{Transpose[{dates, sunriseTimes}], Transpose[{dates, sunsetTimes}],  
              [日期列表图 [转置 [转置  
              Transpose[{dates, sunsetTimes - sunriseTimes}], PlotTheme -> "Detailed",  
                [转置 [绘图主题  
              PlotRange -> Full, PlotLabels -> {"日出时间", "日落时间", "日照时长"}]  
                [绘制范围 [全范围 [数据绘制标签]
```



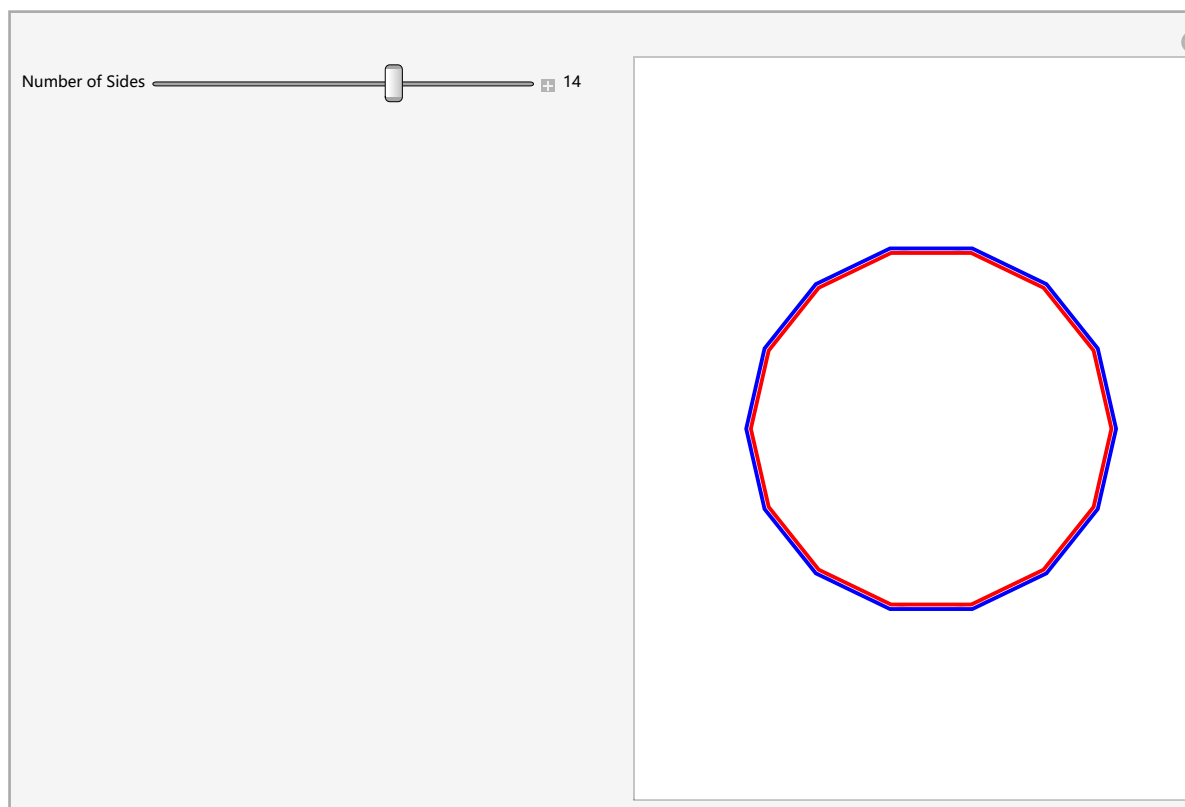
# 割圆术

```

In[ ]:= Manipulate[Graphics[{ {Opacity[0.2], Circle[{0, 0}, 1]}, {EdgeForm[{Thick, Blue}],
  FaceForm[], RegularPolygon[Sec[Pi / n], n] (*外切多边形*)},
  {EdgeForm[{Thick, Red}], FaceForm[], RegularPolygon[n] (*内接多边形*)}},
  PlotRange -> {{-1.5, 1.5}, {-1.5, 1.5}}, AspectRatio -> Automatic],
  {n, 5, "Number of Sides", 3, 20, 1, Appearance -> "Labeled"}]

```

Out[ ]:=

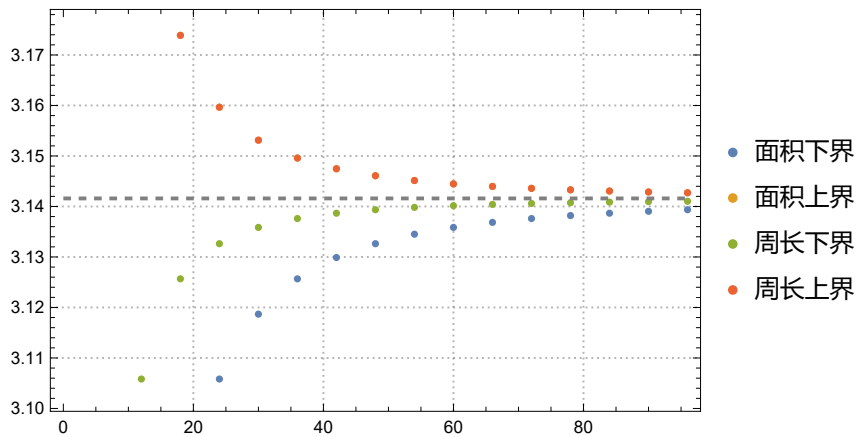


```

In[ ]:= Table[{n, #}, {n, 6, 96, 6}] & /@
  表格
  {Area@RegularPolygon[n], Area@RegularPolygon[Sec[Pi / n], n],
    面积 正多边形 面积 正多边形 正割 圆周率
    ArcLength[#] / 2 &@RegionBoundary@RegularPolygon[n],
    弧长 区域边界 正多边形
    ArcLength[#] / 2 &@RegionBoundary@RegularPolygon[Sec[Pi / n], n]} //
  弧长 区域边界 正多边形 正割 圆周率
  Show[ListPlot[#, PlotTheme -> "Detailed",
    显示 绘制点集 绘图主题
    PlotLegends -> {"面积下界", "面积上界", "周长下界", "周长上界"}],
    绘图的图例
    Plot[Pi, {n, 0, 96}, PlotStyle -> {Dashed, Gray}]] &
  绘图 圆周率 绘图样式 虚线 灰色

```

Out[ ]:=





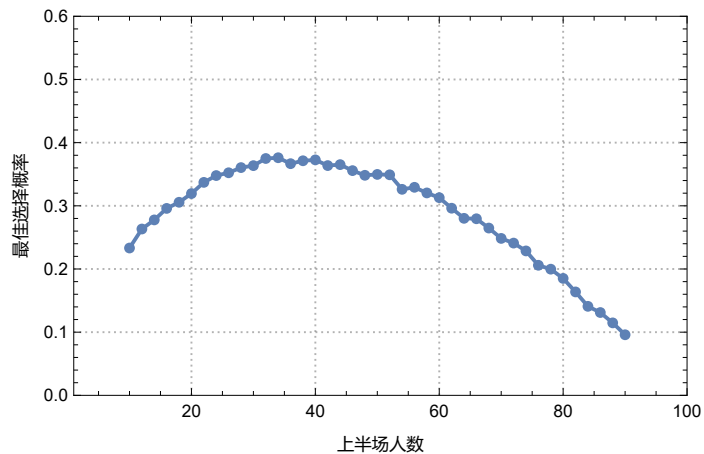
# 变化

## 最优停止问题

### 蒙特卡洛模拟

```
In[*]:= MonteCarloSimulation[k_, n_] :=  
  Module[{candidates, bestInFirstHalf, selected}, candidates = RandomSample[Range[n]];  
    [模块] [伪随机采样] [范围]  
    bestInFirstHalf = Max[Take[candidates, k]];  
    [...] [选取]  
    selected = FirstCase[Drop[candidates, k], x_ /; x > bestInFirstHalf, 0];  
    [第一个匹配] [去掉元素]  
    selected == Max[candidates]]  
    [最大值]  
  
OptimalStoppingProbability[k_, n_, iterations_] :=  
  N[Count[Table[MonteCarloSimulation[k, n], iterations], True] / iterations]  
  [...] [计数] [表格] [真]  
  
In[*]:= ListLinePlot[Table[{i, OptimalStoppingProbability[i, 100, 10000]}, {i, 10, 90, 2}],  
  [绘制点集的线条] [表格]  
  PlotTheme -> "Detailed", PlotLegends -> None, FrameLabel -> {"上半场人数", "最佳选择概率"},  
  [绘图主题] [绘图的图例] [无] [边框标签]  
  PlotMarkers -> {Automatic, 8}, PlotRange -> {{1, 100}, {0, .6}}]  
  [绘制点的标记] [自动] [绘制范围]
```

Out[\*]=



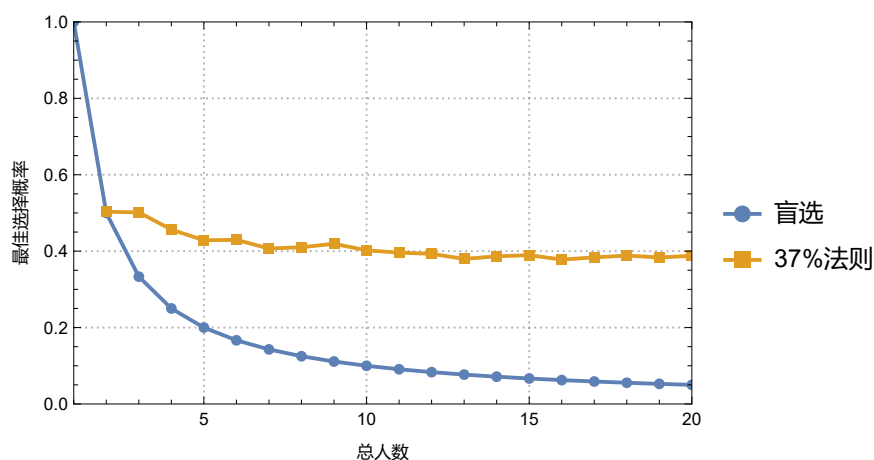
## 小人数的37%法则

```

In[ ]:= ListLinePlot[Table[{i, 1/i}, {i, 1, 20}],
  |绘制点集的线条 |表格
  Table[{i, OptimalStoppingProbability[Round[.37 i], i, 10000]}, {i, 2, 20}],
  |表格 |舍入
  PlotTheme -> "Detailed", PlotLegends -> {"盲选", "37%法则"},
  |绘图主题 |绘图的图例
  FrameLabel -> {"总人数", "最佳选择概率"},
  |边框标签
  PlotMarkers -> {Automatic, 8}, PlotRange -> {{1, 20}, {0, 1}}]
  |绘制点的标记 |自动 |绘制范围

```

Out[ ]=



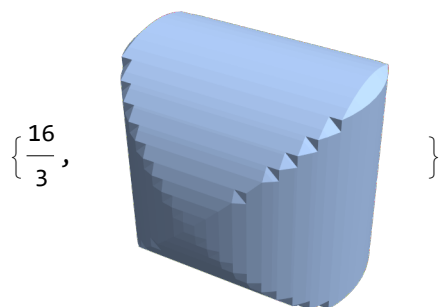
## 相交区域的体积

```

In[ ]:= cy1 = Cylinder[{0, 0, -5}, {0, 0, 5}], 1];
  |圆柱体
cy2 = Cylinder[{-5, 0, 0}, {5, 0, 0}], 1];
  |圆柱体
region = RegionIntersection[cy1, cy2];
  |区域交集
@@region & /@ {Volume, Region}
  |体积 |几何区域

```

Out[ ]=



# 罗密欧与朱丽叶

## 方程的解

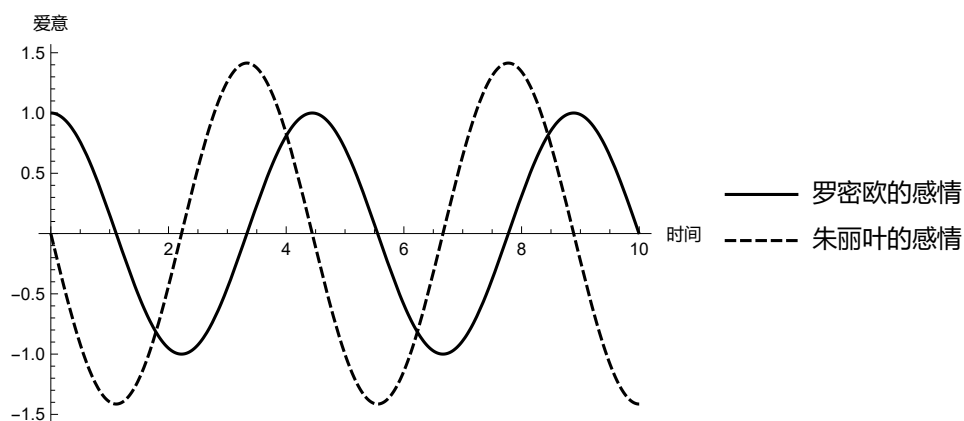
```
In[8]:= sols = DSolveValue[{
  (*微分方程*) R'[t] == a J[t], J'[t] == -b R[t],
  (*初始条件*) R[0] == 1, J[0] == 0
}, {R[t], J[t]}, t]
```

Out[8]=

$$\left\{ \cos[\sqrt{a} \sqrt{b} t], -\frac{\sqrt{b} \sin[\sqrt{a} \sqrt{b} t]}{\sqrt{a}} \right\}$$

```
In[15]:= Plot[Evaluate[sols /. {a -> 1, b -> 2}], {t, 0, 10}, PlotTheme -> "Monochrome",
  (*绘图*) (*计算*) (*绘图主题*)
  AxesLabel -> {"时间", "爱意"}, PlotLegends -> {"罗密欧的感情", "朱丽叶的感情"}]
  (*坐标轴标签*) (*绘图的图例*)
```

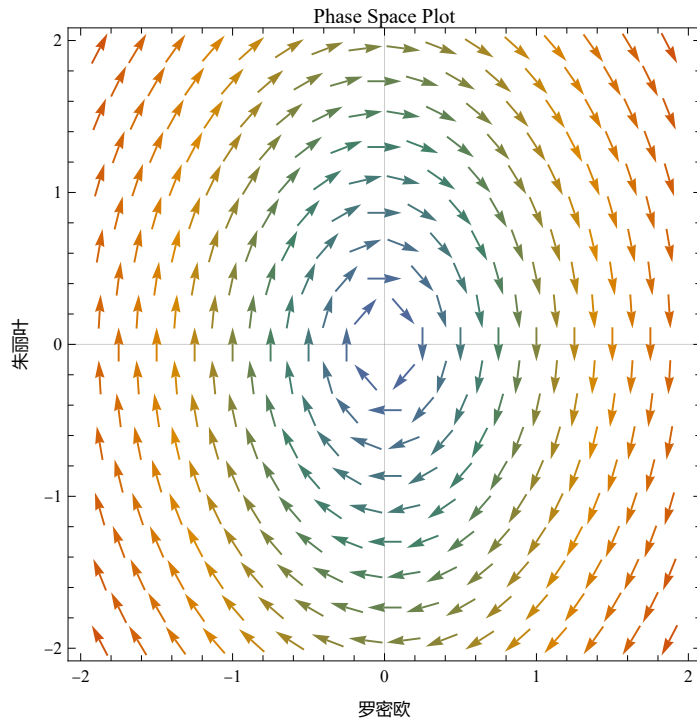
Out[15]=



## 相空间图

```
In[25]:= VectorPlot[{a J[t], -b R[t]} /. {R[t] → x, J[t] → y, a → 1, b → 2},
  | 向量图
  {x, -2, 2}, {y, -2, 2}, FrameLabel → {"罗密欧", "朱丽叶"},
  | 边框标签
  PlotLabel → "Phase Space Plot", PlotTheme → "Scientific"]
  | 绘图标签      | 绘图      | 绘图主题
```

Out[25]=



VectorPlot[

[向量图](#)

{a J[t], -b R[t] - mu J[t] (\*增加阻力项\*)} /. {R[t] → x, J[t] → y, a → 1, b → 2, mu → 2},  
 {x, -2, 2}, {y, -2, 2}, FrameLabel → {"罗密欧", "朱丽叶"},  
[边框标签](#)

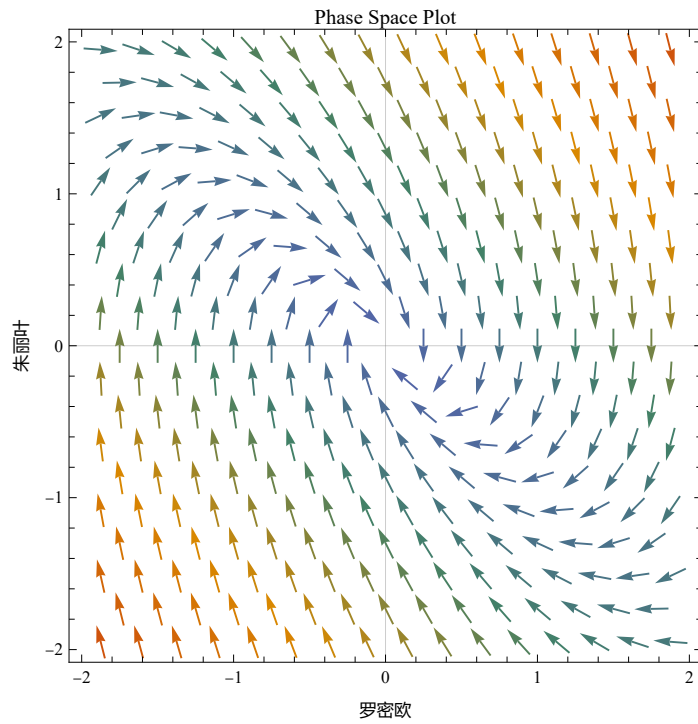
PlotLabel → "Phase Space Plot", PlotTheme → "Scientific"]

[绘图标签](#)

[绘图](#)

[绘图主题](#)

Out[31]=



# 数据

## 谷歌搜索算法

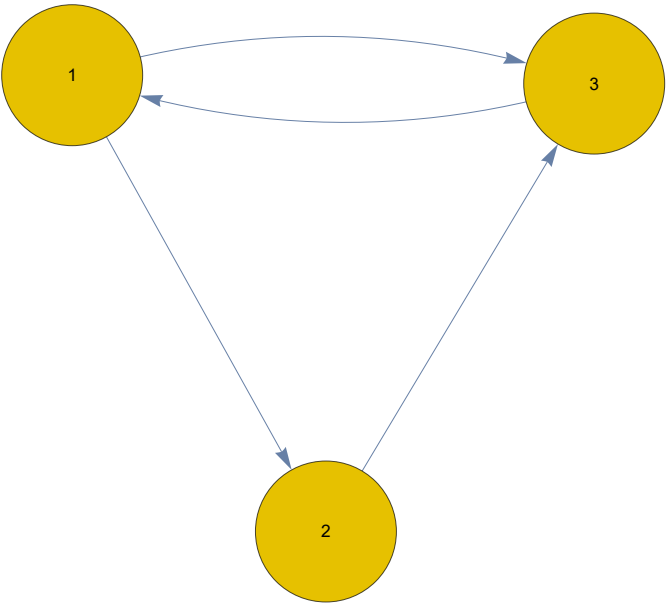
### 离散马尔可夫过程

`In[ ]:= dmp = DiscreteMarkovProcess[{1/3, 1/3, 1/3}, {{0, 1/2, 1/2}, {0, 0, 1}, {1, 0, 0}}];`  
[离散马尔可夫过程]

`Graph@dmp`

[图]

`Out[ ]:=`



`In[ ]:= StationaryDistribution[dmp]`  
[平稳分布]

`Out[ ]:=`

`ProbabilityDistribution[ $\frac{2}{5} \text{Boole}[x == 1] + \frac{1}{5} \text{Boole}[x == 2] + \frac{2}{5} \text{Boole}[x == 3]$ , {x, 1, 3, 1}]`

### 线性代数

`In[ ]:= Limit[MatrixPower[{{0, 1/2, 1/2}, {0, 0, 1}, {1, 0, 0}}, n], n -> Infinity]`  
[极限] [矩阵的幂] [无穷大]

`Out[ ]:=`

`{ $\{\frac{2}{5}, \frac{1}{5}, \frac{2}{5}\}, \{\frac{2}{5}, \frac{1}{5}, \frac{2}{5}\}, \{\frac{2}{5}, \frac{1}{5}, \frac{2}{5}\}}$ }`

## 迭代计算

```

In[*]:= RSolveValue[{x[n + 1] == z[n], y[n + 1] == 1 / 2 x[n],
  |用符号解差分方程
      z[n + 1] == 1 / 2 x[n] + y[n], x[0] == 1 / 3, y[0] == 1 / 3, z[0] == 1 / 3},
      {x[n], y[n], z[n]}, n] // Limit[#, n → Infinity] &
                                     |极限      |无穷大

Out[*]=

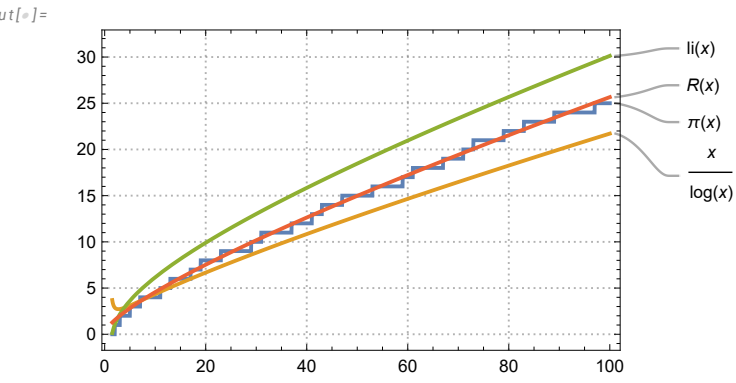
$$\left\{ \frac{2}{5}, \frac{1}{5}, \frac{2}{5} \right\}$$


```

# 前沿

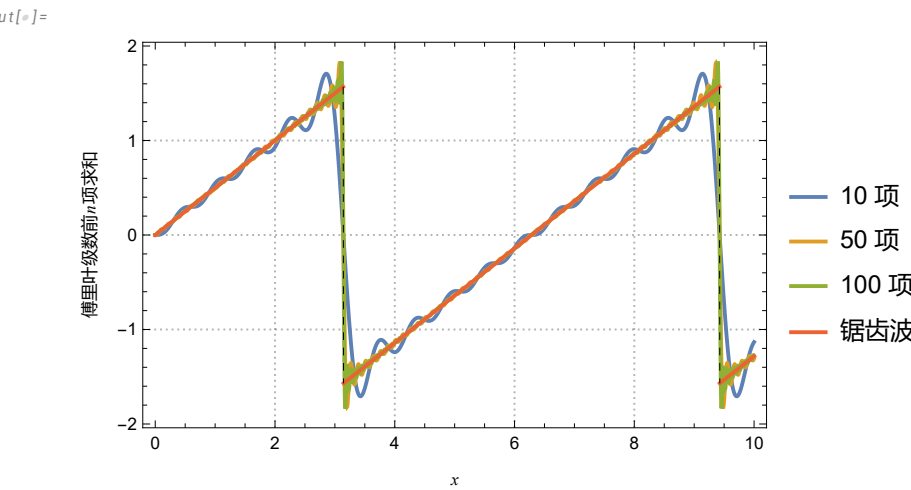
## 质数分布

```
In[ ]:= Plot[{PrimePi[x], x / Log[x], LogIntegral[x], RiemannR[x]}, {x, 1.5, 100},
[绘图] [素数Pi的数目] [对数] [对数积分] [黎曼素数计数函数]
PlotLabels -> "Expressions", PlotTheme -> "Detailed", PlotLegends -> None]
[数据绘制标签] [绘图主题] [绘图的图例] [无]
```



## 吉布斯现象

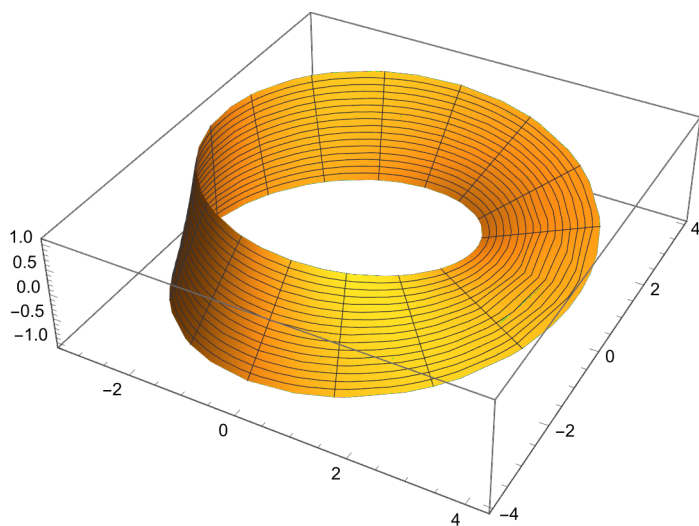
```
In[ ]:= Sum[(-1)^(i-1) Sin[i x] / i, {i, 1, #}] & /@ {10, 50, 100, Infinity} //
[求和] [正弦] [无穷大]
Plot[#, {x, 0, 10}, ExclusionsStyle -> Dashed, PlotTheme -> "Detailed", PlotLegends ->
[绘图] [排除样式] [虚线] [绘图主题] [绘图的图例]
{"10 项", "50 项", "100 项", "锯齿波"}, FrameLabel -> {"x", "傅里叶级数前n项求和"}] &
[边框标签]
```





## 莫比乌斯环

```
In[ ]:= ParametricPlot3D[{Cos[t] (3 + r Cos[t / 2]), Sin[t] (3 + r Cos[t / 2]), r Sin[t / 2]},  
  绘制三维参数图    余弦    余弦    正弦    余弦    正弦  
  {r, -1, 1}, {t, 0, 2 Pi}]  
  圆周率  
Out[ ]=
```



图形上的参考线好像就是“测地线”