BIOSTAT 802 Homework#3

Due on February 27 during the Discussion Session. Hand in to GSI Zac Zhang

Problem 1: Problem 3.6 (Lehmann and Romano) Let P_0, P_1, P_2 be the probability distributions assigning to the integers $1, \dots, 6$ the following probabilities:

	1	2	3	4	5	6
P_0	0.03	0.02	0.02	0.01	0.00	0.92
P_1	0.06	0.05	0.08	0.02	0.01	0.78
P_2	0.09	0.05	0.12	0.00	0.02	0.72

Determine whether there exists a level-α test of $H : P = P_0$ which is UMP against the alternatives P_1 and P_2 when (i) $\alpha = 0.01$; (ii) $\alpha = 0.05$; (iii) $\alpha = 0.07$.

Problem 2: Problem 3.10 (Lehmann and Romano) Consider the problem of testing $H_0 : P = P_0$ against $H_1 : P = P_1$, and suppose that known probabilities $\pi_0 = \pi$ and $\pi_1 = 1 - \pi$ can be assigned to H_0 and H_1 prior to the experiment.

(a) The overall probability of an error resulting form the use of a test ψ is

$$\pi E_0 \psi(X) + (1 - \pi) E_1[1 - \psi(X)].$$

(b) The *Bayes test* is defined as the one that minimizes the above probability, which is given by

$$\psi(x) = \begin{cases} 1, & \text{when } p_1(x) > k p_0(x), \\ 0, & \text{when } p_1(x) < k p_0(x), \end{cases}$$

where $k = \pi_0/\pi_1$.

(c) The conditional probability of H_i given $X = x$, or the posterior probability of H_i, is

$$\frac{\pi_i p_i(x)}{\pi_0 p_0(x) + \pi_1 p_1(x)}, \quad i = 0, 1,$$

and the Bayes test therefore decides in favor of the hypothesis with the larger posterior probability.

Problem 3: Problem 3.9 (Lehmann and Romano) Let X be distributed according to $P_\theta, \theta \in \Omega$, and let T be sufficient for θ. If $\psi(X)$ is any test of a hypothesis concerning θ, then $\phi(T)$ given by $\phi(t) = E[\psi(X)|t]$ is a test depending on T only, and its power function is identical to that of $\psi(X)$.