

A naïve matrix multiplication

• For a $m \times n$ matrix A and $n \times k$ matrix B, consider calculating a matrix C = AB.

• Then, C_{ii} is

$$C_{ij} = \sum_{s=1}^{n} A_{is} B_{sj}$$

What is the time complexity to calculate C?

An example matrix multiplication

```
 \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ 1 & 2 & 3 & \dots & n \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 2 & 3 & \dots & n \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1/2 & 1/2 & 1/2 & \dots & 1/2 \\ 1/3 & 1/3 & 1/3 & \dots & 1/3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1/n & 1/n & 1/n & \dots & 1/n \end{pmatrix}
```

- What would be the resulting matrix?
- What would be the sum of elements of the resulting matrix?

Implementing the naïve multiplication

```
int main(int argc, char** argv) {
  int32 t n1, n2, n3;
  int32 t i, j, k;
 n1 = atoi(argv[1]);
 n2 = atoi(argv[2]);
 n3 = atoi(argv[3]);
  printTime("Initializing %d by %d matrix", n1, n2);
  vector< vector<double> > A;
  A.resize(n1);
  for(i=0; i < n1; ++i) {</pre>
   A[i].resize(n2);
    for(j=0; j < n2; ++j)
     A[i][j] = j+1.0;
```

```
printTime("Initializing %d by %d matrix", n2, n3);
vector< vector<double> > B;
B.resize(n2);
for(i=0; i < n2; ++i) {
 B[i].resize(n3);
  for(j=0; j < n3; ++j) B[i][j] = 1.0/(i+1.);
printTime("Performing a naive matrix multiplication");
vector< vector<double> > C;
C.resize(n1);
double sum = 0;
for(i=0; i < n1; ++i) {</pre>
  C[i].resize(n3, 0);
  for(j=0; j < n3; ++j) {
    for(k=0; k < n2; ++k) C[i][j] += (A[i][k] * B[k][j]);
    sum += C[i][j];
}
printTime("Finished a naive matrix multiplication. Sum = %lg", sum);
return 0;
```

Modified printTime() function

```
#include <ctime>
#include <svs/time.h>
#include <cstdarg>
void printTime(const char* msg, ...) { // advanced syntax to allow variable
                                   // number of function arguments
 va list ap;
 va start(ap, msq);
 char buff[255];
 struct timeval tv;
 gettimeofday(&tv, NULL);
 time_t current_time = tv.tv_sec;
  strftime(buff, 120, "%Y/%m/%d %H:%M:%S", localtime(&current time));
  fprintf(stderr, "[%s.%06d]\t", buff, tv.tv usec);
 vfprintf(stderr, msg, ap);
 fprintf(stderr, "\n");
 va end(ap);
```

A running example: 131s

```
$ ./mat_mult 1000 10000 1000
[2016/11/09 01:12:32.751429] Initializing 1000 by 10000 matrix
[2016/11/09 01:12:32.819379] Initializing 10000 by 1000 matrix
[2016/11/09 01:12:32.868526] Performing a naive single-threaded (1000 x 10000) by (10000 x 1000) matrix multiplication
[2016/11/09 01:14:43.347102] Finished performing a naive single-threaded matrix multiplication. Sum = 1e+10
```

Parallelizing the computation

• The calculation of C_{ij}

$$C_{ij} = \sum_{s=1}^{\infty} A_{is} B_{sj}$$

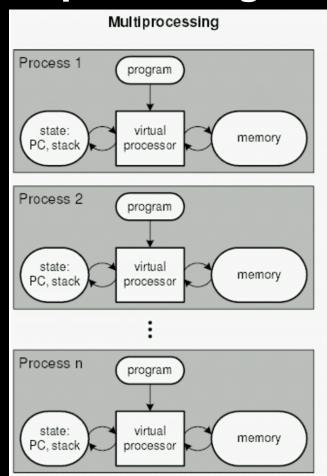
does not depend on other elements in C

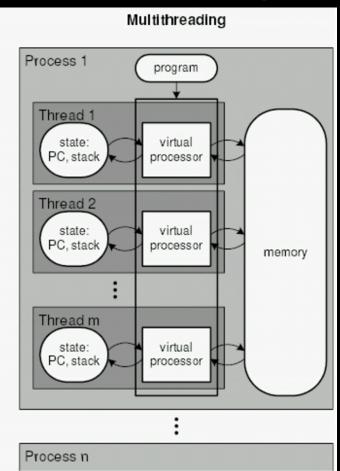
Can we use multiple CPUs to parallelize the process?

Multithreading

- A programming model that allows to use multiple CPUs in a single process.
- Threads run independently, but share the resources of the same process.
- Advantages: Efficiency
- Disadvantages: Programming Difficulty & Synchronization

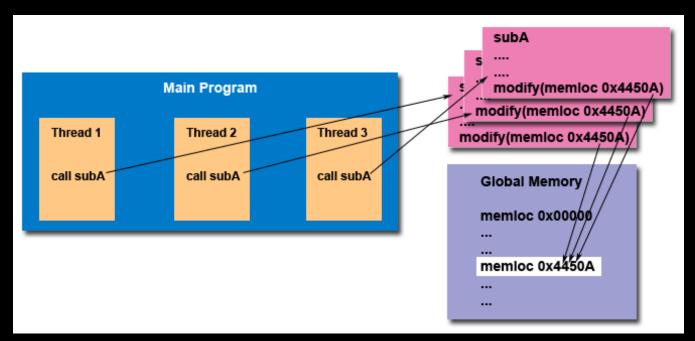
Multiprocessing vs. Multithreading





POSIX Threads (pthreads)

- A language-independent model for multithreading
- A thread is implemented as a subroutine (function)



Steps for using POSIX threads

- 1. Define a number of threads with type <a hread_t
- 2. Create a number of threads using

- thread should be the pointer to each thread to create
- func is the function to execute as the thread
- arg is the pointer to the arguments of the function, but should not have a type
- 3. Wait for the threads to end using pthread join(pthread t thread, void** value ptr)

pthread matrix multiplication : arguments

```
class mult thread args {
public:
  vector< vector<double> >* pA;
  vector< vector<double> >* pB;
  vector< vector<double> >* pC;
  int32 t from n1;
  int32 t to n1;
  int32 t from n3;
  int32 t to n3;
  mult thread args(vector< vector<double> >* pA,
        vector < vector < double > > * pB,
        vector< vector<double> >* _pC,
        int32 t from n1, int32 t Tto n1, int32 t from n3,
       int32 \overline{t} \overline{to} n3\overline{)}: pA(pA), \overline{p}B(\overline{pB}), pC(\overline{pC}),
  from n1( from n1), to n1( to n1), from n3( from n3),
to n3(\overline{to} \overline{n}3) {}
                                                                   13
```

Define the function to invoke

```
void* mult thread(void* args) {
 mult thread args* mt args = (mult thread args*)args;
 const vector< vector<double> >& A = *mt args->pA;
 const vector< vector<double> >& B = *mt args->pB;
 vector< vector<double> >& C = *mt args->pC;
  int32 t i, j, k;
  int32_t n2 = (int32_t)B.size();
  for(i=mt args->from n1; i < mt args->to n1; ++i) {
    for(j=mt args->from n3; j < mt args->to n3; ++j) {
      for(k=0; k < n2; ++k)
     C[i][j] += (A[i][k] * B[k][j]);
  return NULL;
```

Parallelization by pthread

```
pthread_t thread[4];
mult thread args args1(&A, &B, &C, 0, n1/2, 0, n3/2);
if ( pthread_create(&thread[0], NULL, mult_thread, &args1) != 0 ) perror("Can't create");
mult_thread_args args2(&A, &B, &C, 0, n1/2, n3/2, n3);
if ( pthread_create(&thread[1], NULL, mult_thread, &args2) != 0 ) perror("Can't create");
mult_thread_args args3(&A, &B, &C, n1/2, n1, 0, n3/2);
if ( pthread_create(&thread[2], NULL, mult_thread, &args3) != 0 ) perror("Can't create");
mult_thread_args args4(&A, &B, &C, n1/2, n1, n3/2, n3);
if ( pthread_create(&thread[3], NULL, mult_thread, &args4) != 0 ) perror("Can't create");
for(i=0; i < 4; ++i) {
  pthread_join(thread[i], NULL);
                                                                                  15
```

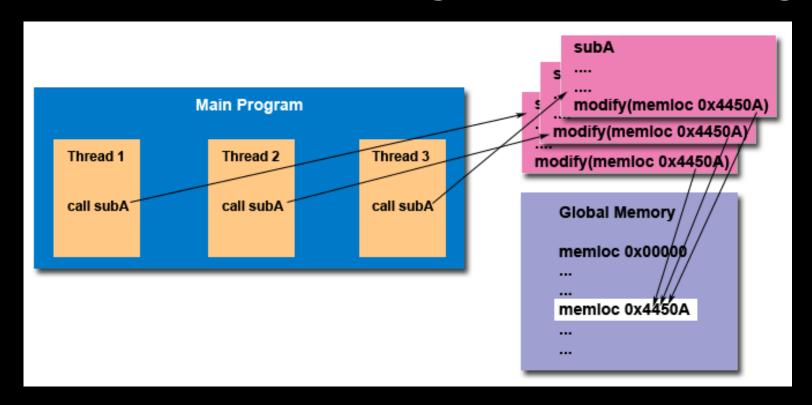
A running example: 30s

- \$./mat_mult 1000 10000 1000
- [2016/11/09 02:49:44.367431] Performing a naive 4-pthreaded (1000 x 10000) by (10000 x 1000) matrix multiplication
- [2016/11/09 02:50:14.271771] Finished performing a naive 4threaded matrix multiplication. Sum = 1e+10

Another possible way to parallelize

```
void* mult thread(void* args) {
 mult thread args* mt args = (mult thread args*)args;
  const vector< vector<double> >& A = *mt args->pA;
  const vector< vector<double> >& B = *mt args->pB;
  vector< vector<double> >& C = *mt args->pC;
  int32 t i, j, k;
  int32 t n1 = (int32 t)A.size(), n3 = (int32 t)B[0].size(),
  for(i=0; i < n1; ++i) {
    for(j=0; j < n3; ++j) {
      for(k=mt args->from n2; k < mt args->to n2; ++k)
      C[i][j] += (A[i][k] * B[k][j]);
  return NULL;
                                    Do we see any problem?
```

Race condition: Challenges in multithreading



To resolve race condition, "mutex" is used to lock variables.

OpenMP: An easier way of multithreading

```
#include <omp.h>
 printTime("Performing a naive 4 omp-threaded
    (%d \times %d) by (%d \times %d) matrix multiplication", n1, n2, n2, n3);
 omp set num threads(4);
#pragma omp parallel for private(i,j,k) schedule(dynamic)
 for(i=0; i < n1; ++i) {</pre>
  for(j=0; j < n3; ++j) {
     for(k=0; k < n2; ++k)
      C[i][j] += (A[i][k] * B[k][j]);
     sum += C[i][j];
```

Compiling with OpenMP & running: 34s

```
$ g++ -fopenmp -02 -o mat_mult mat_mult.cpp -std=c++0x
$ ./mat_mult 1000 10000 1000
[2016/11/09 05:23:19.077628] Performing a naive 4 omp-threaded
(1000 x 10000) by (10000 x 1000) matrix multiplication
[2016/11/09 05:23:53.417607] Finished performing a naive omp-threaded matrix multiplication. Sum = 9.3e+09
```

What was wrong with the code?

Resolving race condition with OpenMP

```
#include <omp.h>
printTime("Performing a naive 4 omp-threaded
    (%d \times %d) by (%d \times %d) matrix multiplication", n1, n2, n2, n3);
 omp set num threads(4);
#pragma omp parallel for private(i,j,k) schedule(dynamic)
 for(i=0; i < n1; ++i) {</pre>
  for(j=0; j < n3; ++j) {
     for(k=0; k < n2; ++k)
      C[i][j] += (A[i][k] * B[k][j]);
#pragma omp critical
     sum += C[i][j];
```

Results with race condition resolution

```
$ ./mat_mult 1000 10000 1000

[2016/11/09 05:36:47.281005] Performing a naive 4 omp-threaded
(1000 x 10000) by (10000 x 1000) matrix multiplication

[2016/11/09 05:37:21.578820] Finished performing a naive omp-
threaded matrix multiplication. Sum = 1e+10
```

Summary: Multithread Programming

- An efficient way to parallelize a program utilizing multiple CPUs.
- POSIX thread provides a standard method for multithread programming across many environments.
- OpenMP provides an easier way to parallelize a loop or a block.
- Multi-thread programming requires a very careful attention of race condition on each variable used within a thread.
 - Simultaneous update may result in unexpected results, and much be locked properly to execute exclusively for each thread