Linear Mixed Effects Models Examples

Biostatistics 653

Applied Statistics III: Longitudinal Data Analysis

Evaluating the Need for Random Effects

- One way to think of the linear mixed effects model is to think of a random coefficient model in which every subject has its own intercept and slope. However, what if the slopes do not vary across units? In this case, we would think of slopes as fixed effects rather than random.

Evaluating the Need for Random Effects

Consider a simple example with only one group. The "full" model with random intercepts and slopes is given by

$$Y_{ij} = \beta_0 + \beta_1 t_{ij} + b_{0i} + b_{1i} t_{ij} + \epsilon_{ij}$$

where

$$V(\boldsymbol{b}_i) = \boldsymbol{D} = \begin{pmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{pmatrix}$$

If the slopes do not vary, then we have the "reduced" model given by

$$Y_{ij} = \beta_0 + \beta_1 t_{ij} + b_{0i} + \epsilon_{ij}$$

where

$$V(b_i) = d_{11}$$

Assume in each model $V(\boldsymbol{\epsilon}_i) = \boldsymbol{R}_i = \sigma^2 \boldsymbol{I}_{n_i}$

Evaluating the Need for Random Effects

- Both models lead to the same specification for the mean $E(\boldsymbol{Y}_i) = \boldsymbol{X}_i\boldsymbol{\beta}$, but they involve different covariance models $\boldsymbol{\Sigma}_i = \boldsymbol{Z}_i\boldsymbol{D}\boldsymbol{Z}_i^T + \sigma^2\boldsymbol{I}_{n_i}$.
- In the full model, $\boldsymbol{\Sigma}_i$ is constructed using

$$\mathbf{Z}_i = \begin{pmatrix} 1 & t_{i1} \\ 1 & t_{i2} \\ \vdots & \vdots \\ 1 & t_{in_i} \end{pmatrix}$$

- In the reduced model, $D = d_{11}, \boldsymbol{Z}_i = (1, \cdots, 1)^T$ and $\boldsymbol{\Sigma}_i$ is compound symmetry

$$\boldsymbol{\Sigma}_i = \begin{pmatrix} d_{11} + \sigma^2 & d_{11} & \cdots & d_{11} \\ d_{11} & d_{11} + \sigma^2 & \cdots & d_{11} \\ \vdots & \vdots & \ddots & \vdots \\ d_{11} & \cdots & d_{11} & d_{11} + \sigma^2 \end{pmatrix}$$

Evaluating the Need for Random Effects

- To determine which model is more appropriate, one might rely on information criteria (AIC, BIC) to select between these models. In addition, noticing that these are nested models, it would be natural to think about doing a likelihood ratio test!

Evaluating the Need for Random Effects

- We are often interested in comparing two nested models, one with q correlated random effects, and one with q+1 correlated random effects. The difference in covariance parameters between these two models is q+1, as we will need one additional variance and q additional covariances in the larger model. One might think about trying to conduct a likelihood ratio test then with q + 1 degrees of freedom, but there is a problem with this approach.

Evaluating the Need for Random Effects

- Testing for significance of random effects is not a standard hypothesis testing problem because the null H_0: $\boldsymbol{b}_i = 0$ is on the boundary (0) of the parameter space because variances cannot be negative. Thus the usual χ^2 asymptotic distribution of the LRT is invalid. However, it has been shown that to test whether a single random effect can be removed (e.g., using q random effects instead of q + 1), a mixture of two 2 distributions can be used to obtain pvalues. In this case, one can calculate p-values based on $0.5\,\chi_q^2 + 0.5\chi_{q+1}^2$. Critical values are available in the Fitzmaurice book, Table C.1.

Evaluating the Need for Random Effects

- Thus if one wished to test whether a random intercepts model was sufficient for the dental data, compared to a model with random intercepts and a random slope for a linear term in age, one would conduct a LRT of the difference in the log-likelihoods and use a 50-50 mixture of a χ_1^2 and a χ_2^2 distribution to obtain the critical value.

Evaluating the Need for Random Effects

- In other situations, including more complex comparisons among nested models for covariance in which the null distribution of the LRT is not well understood (for example, comparing a model with q correlated random effects to one with q + k correlated random effects, where k > 1), Fitzmaurice et al. recommend using $\alpha = 0.10$ with the df equal to the difference in number of covariance parameters (not the difference in number of random effects) instead of $\alpha = 0.05$ to prevent selection of a model that is too parsimonious; however you should be aware that they recommend this as an ad-hoc procedure (and that its theoretical properties have not been extensively studied).

Dental Example

- For the dental data, we can fit the random intercepts model and then use the mixture of chi-square distributions to determine if we need the random slope and random intercept, or if the random intercept only is sufficient.
- For the dental data, consider the reduced model

$$Y_{ij} = \beta_{0G} + \beta_{0B} + \beta_{1G}t_{ij} + \beta_{1B}t_{ij} + b_{0i} + \epsilon_{ij}$$

where

$$V(b_i) = d_{11}$$

and

$$V(\boldsymbol{\epsilon}_i) = \sigma_B^2\boldsymbol{I} \; or \; \sigma_G^2\boldsymbol{I}$$

SAS Code

```
title 'INTERCEPTS RANDOM FOR EACH GENDER';
title2 'R_i is diagonal within-child with gender-specific variance';
title3 'D is unstructured and the same for all children';
proc mixed method=ml data=proyuniv;
class newid gender;
model dist=gender gender*time/noint;
random intercept / type=un subject=newid g gcorr v vcorr;
repeated/group=gender subject=newid rcorr;
run;
```

Model	Cov params	-2 log $\hat{l}_{REML}$	AIC	BIC
Random intercept and slope	5	411.5	421.5	428.0
Random intercept	3	415.2	421.2	425.1

Dental Example

- In order to test whether we need the random slope for time, we compare the difference in -2 times the log-likelihoods to a mixture of a χ_1^2 and χ_2^2 distribution. With $\alpha = 0.05$, the critical value is 5.14. We calculate the value of the test statistic 415.2-411.5 = 3.7, and we conclude that the random slope for time is not needed. The AIC and BIC criteria also prefer the model with only random intercepts. Although we used ML estimation here, the tests could also be carried out using REML.

Varying R and D for Dental Data

- Using the random intercepts and slopes model, we consider various D and Ri structures in the dental data. In particular, we have

$$Y_{ij} = \beta_0 + \beta_1 t_{ij} + b_{0i} + b_{1i} t_{ij} + \epsilon_{ij}$$

where $\boldsymbol{b}_i \sim N_2(0, \boldsymbol{D})$ and $\boldsymbol{\epsilon}_i \sim N_4(0, \boldsymbol{R}_i)$.

- For each model, we will examine the number of parameters estimated, the log-likelihood, AIC, and BIC.

Varying R and D for Dental Data

Model 1 is the model fit previously in the slides, for which

$$V(\boldsymbol{b}_i) = \boldsymbol{D} = \begin{pmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{pmatrix}$$

and

$$V(\boldsymbol{\epsilon}_i) = \boldsymbol{R}_i = \begin{cases} \sigma_B^2 \boldsymbol{I}, & for\ boys \\ \sigma_G^2 \boldsymbol{I}, & for\ girls \end{cases}$$

We thus estimated 5 covariance parameters. The model had

$$-2\hat{l}_{REML} = 411.5$$
$$AIC = 421.5$$
$$BIC = 428.0$$

Varying R and D for Dental Data

Model 2 has

$$V(\boldsymbol{b}_i) = \boldsymbol{D} = \begin{pmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{pmatrix}$$

and

$$V(\boldsymbol{\epsilon}_i) = \boldsymbol{R}_i = \sigma^2 \boldsymbol{I}$$

We thus estimated 4 covariance parameters. The model had

$$-2\hat{l}_{REML} = 432.6$$
$$AIC = 440.6$$
$$BIC = 445.8$$

SAS Code

```
title 'R_i diagonal, constant var sigma^2 for each gender';
title2 ' this is default R_i so no repeated statement needed ';
title3 'D is 2 x2 unstructured, same for both genders';
proc mixed data=proyuniv method=ml;
class newid gender;
model dist=gender gender*time/noint solution;
random intercept time/type=un subject=newid g gcorr v vcorr;
estimate 'diff in mean slope' gender 0 0 gender*time 1 -1;
contrast 'overall gender difference' gender 1 -1, gender*time 1 -1/e chisq;
run;
```

Varying R and D for Dental Data

Model 3 has

$$V(\boldsymbol{b}_i) = \boldsymbol{D} = \begin{pmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{pmatrix}$$

and

$$V(\boldsymbol{\epsilon}_i) = \boldsymbol{R}_i = \sigma^2 \begin{pmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{pmatrix}$$

We thus estimated 5 covariance parameters. The model had

$$-2\hat{l}_{REML} = 428.8$$
$$AIC = 438.8$$
$$BIC = 445.3$$

SAS Code

```
title 'R_i is AR(1), same var and rho for each gender';
title2 ' const var same for each gender ';
title3 'D is 2 x2 unstructured, same for both genders';
proc mixed data=proyuniv method=ml;
class newid gender;
model dist=gender gender*time/noint solution;
repeated / type=ar(1) subject=newid rcorr;
random intercept time/type=un subject=newid g gcorr v vcorr;
estimate 'diff in mean slope' gender 0 0 gender*time 1 -1;
contrast 'overall gender difference' gender 1 -1,
 gender*time 1 -1/   e chisq;
run;
```

Varying R and D for Dental Data

Model 4 has

$$V(\boldsymbol{b}_i) = \boldsymbol{D_h} = \begin{pmatrix} d_{h11} & d_{h12} \\ d_{h12} & d_{h22} \end{pmatrix}$$

where h=1,2 indicates boys and girls, and

$$V(\boldsymbol{\epsilon}_i) = \boldsymbol{R}_i = \begin{cases} \sigma_B^2 \boldsymbol{I}, & for\ boys \\ \sigma_G^2 \boldsymbol{I}, & for\ girls \end{cases}$$

We thus estimated 8 covariance parameters. The model had

$$-2\hat{l}_{REML} = 428.8$$
$$AIC = 438.8$$
$$BIC = 445.3$$

SAS Code

```
title 'R_i is diagonal within child but two vars';
title2 ' \sigma^2_G and \sigma^2_B';
title3 'D is 2 x2 unstructured, different for each gender';
proc mixed data=proyuniv method=ml;
class newid gender;
model dist=gender gender*time/noint solution;
repeated / group=gender subject=newid rcorr;
random intercept time/type=un group=gender
   subject=newid g gcorr v vcorr;
estimate 'diff in mean slope' gender 0 0 gender*time 1 -1;
contrast 'overall gender difference' gender 1 -1,
       gender*time 1 -1/
        e chisq;
run;
```

Varying R and D for Dental Data

Model 5 has

$$V(\boldsymbol{b}_i) = \boldsymbol{D} = \begin{pmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{pmatrix}$$

and

$$V(\boldsymbol{\epsilon}_i) = \boldsymbol{R}_i = \sigma_1^2 \boldsymbol{I} + \sigma^2 \begin{pmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{pmatrix}$$

We thus estimated 6 covariance parameters. The model had

$$-2\hat{l}_{REML} = 428.7$$
$$AIC = 440.7$$
$$BIC = 448.5$$

SAS Code

```
title 'R_i sum of 2 components; an AR(1) component
   for fluctuations ';
title2 'and diagonal component with var sigma^2
   common across gender';
title3 'local option adds diag component to ar(1) structure';
title4 'D is 2 x2 unstructured, samefor each gender';
proc mixed data=proyuniv method=ml;
class newid gender;
model dist=gender gender*time/noint solution;
repeated / type=ar(1) local subject=newid rcorr;
random intercept time/type=un subject=newid g gcorr v vcorr;
estimate 'diff in mean slope' gender 0 0 gender*time 1 -1;
contrast 'overall gender difference' gender 1 -1,
   gender*time 1 -1/ e chisq;
run;
```

Degrees of Freedom for *F* Tests for Mixed Effects Models

- It is not always possible to find an exact F test for mixed and random effects models. When the sample size is reasonably large with respect to the number of parameters being estimated, we can rely on asymptotic results and base tests on the χ^2 distribution (which has an implicit assumption of infinite denominator degrees of freedom). In small samples, one can argue that the use of the χ^2 test is anti-conservative, as the asymptotic approximation can be bad with small N.

Degrees of Freedom for *F* Tests for Mixed Effects Models

- However, using the F can be questionable as well, as in general settings it may also involve approximations. Several approximations have been proposed, with those proposed by Satterthwaite (1946) and Kenward and Roger (1997) preferred. We note that these approximations are not the default in PROC MIXED, though the DDFM option may be used to ask for other approximations (DDFM=SATTERTH or DDFM=KENWARDROGER). With reasonably large N, we do not expect to see big differences in p-values across different approximations.

Degrees of Freedom for *F* Tests for Mixed Effects Models

- The Satterthwaite approximate F test in the mixed-factor ANOVA setting involves developing a linear combination of mean squares that has the same expectation as the mean squares of the factor effect of interest, when H0 is true. This method is computationally intensive and may add substantially to computing time.

Degrees of Freedom for *F* Tests for Mixed Effects Models

- The method developed by Kenward and Roger is based on creating an estimated variance-covariance matrix that better approximates the truth in the small sample setting. In particular, they estimate the amount by which the asymptotic variance-covariance matrix underestimates (in a matrix sense) $V(\widehat{\boldsymbol{b}})$. They then inflate the variance-covariance matrix by this amount and compute the degrees of freedom based on this adjustment, using a Satterthwaitetype approach. This method has been shown to have nice properties in a number of simulation studies.

Degrees of Freedom for *F* Tests for Mixed Effects Models

- The default method for calculating degrees of freedom in PROC MIXED depends on the setting but generally takes less time to compute than the above methods. (The defaults are not problematic in certain simple settings, but for complex covariance structures with relatively small N one may wish to consider using the Kenward and Roger method instead.)

BLUPs for the Dental Data

Using the random intercepts and slopes model with $\boldsymbol{D}$ unstructured and separate $\boldsymbol{R}_i = \sigma_h^2 \boldsymbol{I}_{n_i}$ for each gender, we now obtain the estimated BLUPs.

```
/* outpred gets BLUPs for individual estimated means */
/* ODS statement in second call produces datasets containing fixed */
/* effect estimates and the BLUPs for individual slopes and intercepts */

title 'R_i separate for each gender and diagonal ';
title2 'D unstructured';
proc mixed data=proyuniv method=ml;
class newid gender;

model dist=gender gender*time/noint solution outpred=pdata;
repeated/group=gender subject=newid;
random intercept time/type=un subject=newid solution;
run;
proc print data=pdata; run;
```

BLUPs for the Dental Data

```
title 'R_i separate for each gender and diagonal ';
title2 'D unstructured';
proc mixed data=proyuniv method=ml;
class newid gender;
model dist=gender time*gender/noint solution;
repeated/group=gender subject=newid;
random intercept time/type=un subject=newid solution;
ods listing exclude SolutionF;
ods output SolutionF=fixed1;
ods listing exclude SolutionR;
ods output SolutionR=rand1;
run;

data fixed1; set fixed1;
keep gender effect estimate;
run;

title3 'FIXED EFFECTS data';
proc print data=fixed1; run;

proc sort data=fixed1; by gender; run;
```

BLUPs for the Dental Data

```
data fixed12; set fixed1; by gender;
retain fixint fixslope;
if effect='GENDER' then fixint=estimate;
if effect='time*GENDER' then fixslope=estimate;
if last.GENDER then do;
output; fixint=.; fixslope=.;
end;
drop effect estimate;
run;

title3 'RECONFIGURED FIXED EFFECTS DATA';
proc print data=fixed12; run;

data rand1; set rand1;
gender=1; if newid<12 then gender=0;
keep newid gender effect estimate;
run;

title3 'random effects output data';
proc print data=rand1; run;
```

BLUPs for the Dental Data

```
proc sort data=rand1; by newid; run;

data rand12; set rand1; by newid;
retain ranint ranslope;
if effect='Intercept' then ranint=estimate;
if effect='time' then ranslope=estimate;
if last.newid then do;
output; ranint=.; ranslope=.;
end;
drop effect estimate;
run;

proc sort data=rand12; by gender newid; run;

title3 'reconfigured RE dataset';
proc print data=rand12; run;


data both1; merge fixed12 rand12; by gender;
beta0i=fixint+ranint;
beta1i=fixslope+ranslope;
run;

title3 'random intercepts and slopes';
proc print data=both1; run;
```

BLUPs for the Dental Data

```
                 Model Information

Data Set                      WORK.PROYUNIV
Dependent Variable            dist
Covariance Structures         Unstructured, Variance
                              Components
Subject Effects               newid, newid
Group Effect                  GENDER
Estimation Method             ML
Residual Variance Method      None
Fixed Effects SE Method       Model-Based
Degrees of Freedom Method     Containment


             Class Level Information

Class     Levels    Values

newid         27    1 2 3 4 5 6 7 8 9 10 11 12 13
                    14 15 16 17 18 19 20 21 22 23
                    24 25 26 27
GENDER         2    0 1

            Dimensions
Covariance Parameters             5
Columns in X                      4
Columns in Z Per Subject          2
Subjects                         27
Max Obs Per Subject               4

          Number of Observations

Number of Observations Read              108
Number of Observations Used              108
Number of Observations Not Used            0
```

BLUPs for the Dental Data

Iteration History

Iteration	Evaluations	-2 Log Like	Criterion
0	1	478.24175986	
1	2	418.92503842	1.16632499
2	1	416.18869903	1.23326209
3	1	407.89638533	0.01954268
4	2	406.88264563	0.00645800
5	1	406.10632159	0.00056866
6	1	406.04318997	0.00000764
7	1	406.04238894	0.00000000

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm	Subject	Group	Estimate
UN(1,1)	newid		3.1978
UN(2,1)	newid		-0.1103
UN(2,2)	newid		0.01976
Residual	newid	GENDER 0	0.4449
Residual	newid	GENDER 1	2.6294

Fit Statistics

-2 Log Likelihood	406.0
AIC (smaller is better)	424.0
AICC (smaller is better)	425.9
BIC (smaller is better)	435.7

BLUPs for the Dental Data

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
4	72.20	<.0001

Solution for Fixed Effects

Effect	GENDER	Estimate	Standard Error	DF	t Value	Pr > |t|
GENDER	0	17.3727	0.7386	54	23.52	<.0001
GENDER	1	16.3406	1.1114	54	14.70	<.0001
time*GENDER	0	0.4795	0.06180	54	7.76	<.0001
time*GENDER	1	0.7844	0.09722	54	8.07	<.0001

Solution for Random Effects

Effect	newid	Estimate	Std Err Pred	DF	t Value	Pr > |t|
Intercept	1	-0.4853	1.1744	54	-0.41	0.6811
time	1	-0.06820	0.1017	54	-0.67	0.5052
Intercept	2	-1.1922	1.1744	54	-1.02	0.3146
time	2	0.1420	0.1017	54	1.40	0.1683
Intercept	3	-0.8535	1.1744	54	-0.73	0.4705
time	3	0.1773	0.1017	54	1.74	0.0869
Intercept	4	1.7024	1.1744	54	1.45	0.1530
time	4	0.04017	0.1017	54	0.40	0.6943
Intercept	5	0.9136	1.1744	54	0.78	0.4400
time	5	-0.08680	0.1017	54	-0.85	0.3970
Intercept	6	-0.6740	1.1744	54	-0.57	0.5684
time	6	-0.07292	0.1017	54	-0.72	0.4763
Intercept	7	-0.05461	1.1744	54	-0.05	0.9631
time	7	0.03641	0.1017	54	0.36	0.7217

BLUPs for the Dental Data

Intercept	8	1.9350	1.1744	54	1.65	0.1052
time	8	-0.1149	0.1017	54	-1.13	0.2636
Intercept	9	-0.2190	1.1744	54	-0.19	0.8528
time	9	-0.1151	0.1017	54	-1.13	0.2624
Intercept	10	-2.9974	1.1744	54	-2.55	0.0136
time	10	-0.09085	0.1017	54	-0.89	0.3755
Intercept	11	1.9249	1.1744	54	1.64	0.1070
time	11	0.1530	0.1017	54	1.50	0.1382
Intercept	12	1.3469	1.4342	54	0.94	0.3519
time	12	0.08788	0.1232	54	0.71	0.4786
Intercept	13	-0.8676	1.4342	54	-0.60	0.5478
time	13	-0.04068	0.1232	54	-0.33	0.7424
Intercept	14	-0.3575	1.4342	54	-0.25	0.8041
time	14	-0.02176	0.1232	54	-0.18	0.8605
Intercept	15	1.5946	1.4342	54	1.11	0.2711
time	15	-0.02772	0.1232	54	-0.23	0.8228
Intercept	16	-1.1581	1.4342	54	-0.81	0.4229
time	16	-0.04153	0.1232	54	-0.34	0.7373
Intercept	17	0.8972	1.4342	54	0.63	0.5342
time	17	0.02260	0.1232	54	0.18	0.8551
Intercept	18	-0.6889	1.4342	54	-0.48	0.6329
time	18	-0.02853	0.1232	54	-0.23	0.8177
Intercept	19	-0.1443	1.4342	54	-0.10	0.9202
time	19	-0.07348	0.1232	54	-0.60	0.5533
Intercept	20	-0.1273	1.4342	54	-0.09	0.9296
time	20	0.02544	0.1232	54	0.21	0.8372
Intercept	21	2.5349	1.4342	54	1.77	0.0828
time	21	0.1088	0.1232	54	0.88	0.3811
Intercept	22	-0.2261	1.4342	54	-0.16	0.8753
time	22	-0.08535	0.1232	54	-0.69	0.4913
Intercept	23	-0.6374	1.4342	54	-0.44	0.6585
time	23	0.006510	0.1232	54	0.05	0.9580
Intercept	24	-1.7008	1.4342	54	-1.19	0.2409
time	24	0.1139	0.1232	54	0.92	0.3591
Intercept	25	0.2387	1.4342	54	0.17	0.8684
time	25	-0.03166	0.1232	54	-0.26	0.7981
Intercept	26	0.1180	1.4342	54	0.08	0.9347
time	26	0.06104	0.1232	54	0.50	0.6222
Intercept	27	-0.8223	1.4342	54	-0.57	0.5688
time	27	-0.07545	0.1232	54	-0.61	0.5427

BLUPs for the Dental Data

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
GENDER	2	54	384.72	<.0001
time*GENDER	2	54	62.66	<.0001

Obs	GENDER	ID	newid	time	dist	male	Pred	StdErrPred	DF	Alpha	Lower	Upper	Resid
1	0	1	1	8	21.0	0	20.1783	0.43711	54	0.05	19.3019	21.0546	0.82175
2	0	1	1	10	20.0	0	21.0009	0.33796	54	0.05	20.3234	21.6785	-1.00095
3	0	1	1	12	21.5	0	21.8236	0.34908	54	0.05	21.1238	22.5235	-0.32365
4	0	1	1	14	23.0	0	22.6463	0.46259	54	0.05	21.7189	23.5738	0.35366
5	0	2	2	8	21.0	0	21.1527	0.43711	54	0.05	20.2763	22.0290	-0.15266
6	0	2	2	10	21.5	0	22.3957	0.33796	54	0.05	21.7181	23.0733	-0.89570
7	0	2	2	12	24.0	0	23.6387	0.34908	54	0.05	22.9389	24.3386	0.36126
8	0	2	2	14	25.5	0	24.8818	0.46259	54	0.05	23.9543	25.8092	0.61822
9	0	3	3	8	20.5	0	21.7737	0.43711	54	0.05	20.8974	22.6501	-1.27372
10	0	3	3	10	24.0	0	23.0873	0.33796	54	0.05	22.4098	23.7649	0.91266
11	0	3	3	12	24.5	0	24.4010	0.34908	54	0.05	23.7011	25.1008	0.09905
12	0	3	3	14	26.0	0	25.7146	0.46259	54	0.05	24.7871	26.6420	0.28543

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...
...
...

BLUPs for the Dental Data

Model Information

Data Set	WORK.PROYUNIV
Dependent Variable	dist
Covariance Structures	Unstructured, Variance Components
Subject Effects	newid, newid
Group Effect	GENDER
Estimation Method	ML
Residual Variance Method	None
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Containment

Class Level Information

Class	Levels	Values
newid	27	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27
GENDER	2	0 1

Dimensions

Covariance Parameters	5
Columns in X	4
Columns in Z Per Subject	2
Subjects	27
Max Obs Per Subject	4

BLUPs for the Dental Data

Number of Observations

Number of Observations Read	108
Number of Observations Used	108
Number of Observations Not Used	0

Iteration History

Iteration	Evaluations	-2 Log Like	Criterion
0	1	478.24175986	
1	2	418.92503842	1.16632499
2	1	416.18869903	1.23326209
3	1	407.89638533	0.01954268
4	2	406.88264563	0.00645800
5	1	406.10632159	0.00056866
6	1	406.04318997	0.00000764
7	1	406.04238894	0.00000000

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm	Subject	Group	Estimate
UN(1,1)	newid		3.1978
UN(2,1)	newid		-0.1103
UN(2,2)	newid		0.01976
Residual	newid	GENDER 0	0.4449
Residual	newid	GENDER 1	2.6294

Fit Statistics

-2 Log Likelihood	406.0
AIC (smaller is better)	424.0
AICC (smaller is better)	425.9
BIC (smaller is better)	435.7

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
4	72.20	<.0001

BLUPs for the Dental Data

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
GENDER	2	54	384.72	<.0001
time*GENDER	2	54	62.66	<.0001

FIXED EFFECTS data

Obs	Effect	GENDER	Estimate
1	GENDER	0	17.3727
2	GENDER	1	16.3406
3	time*GENDER	0	0.4795
4	time*GENDER	1	0.7844

RECONFIGURED FIXED EFFECTS DATA

Obs	GENDER	fixint	fixslope
1	0	17.3727	0.47955
2	1	16.3406	0.78438

BLUPs for the Dental Data

Obs	Effect	newid	Estimate	gender
1	Intercept	1	-0.4853	0
2	time	1	-0.06820	0
3	Intercept	2	-1.1922	0
4	time	2	0.1420	0
5	Intercept	3	-0.8535	0
6	time	3	0.1773	0
7	Intercept	4	1.7024	0
8	time	4	0.04017	0
9	Intercept	5	0.9136	0
10	time	5	-0.08680	0
11	Intercept	6	-0.6740	0
12	time	6	-0.07292	0
13	Intercept	7	-0.05461	0
14	time	7	0.03641	0
15	Intercept	8	1.9350	0
16	time	8	-0.1149	0
17	Intercept	9	-0.2190	0
18	time	9	-0.1151	0
19	Intercept	10	-2.9974	0
20	time	10	-0.09085	0
21	Intercept	11	1.9249	0
22	time	11	0.1530	0
23	Intercept	12	1.3469	1
24	time	12	0.08788	1
25	Intercept	13	-0.8676	1
26	time	13	-0.04068	1
27	Intercept	14	-0.3575	1
28	time	14	-0.02176	1
29	Intercept	15	1.5946	1
30	time	15	-0.02772	1
31	Intercept	16	-1.1581	1
32	time	16	-0.04153	1
33	Intercept	17	0.8972	1
34	time	17	0.02260	1
35	Intercept	18	-0.6889	1
36	time	18	-0.02853	1
37	Intercept	19	-0.1443	1
38	time	19	-0.07348	1
39	Intercept	20	-0.1273	1
40	time	20	0.02544	1

BLUPs for the Dental Data

41	Intercept	21	2.5349	1
42	time	21	0.1088	1
43	Intercept	22	-0.2261	1
44	time	22	-0.08535	1
45	Intercept	23	-0.6374	1
46	time	23	0.006510	1
47	Intercept	24	-1.7008	1
48	time	24	0.1139	1
49	Intercept	25	0.2387	1
50	time	25	-0.03166	1
51	Intercept	26	0.1180	1
52	time	26	0.06104	1
53	Intercept	27	-0.8223	1
54	time	27	-0.07545	1

Obs	newid	gender	ranint	ranslope
1	1	0	-0.48526	-0.06820
2	2	0	-1.19224	0.14198
3	3	0	-0.85346	0.17726
4	4	0	1.70243	0.04017
5	5	0	0.91363	-0.08680
6	6	0	-0.67403	-0.07292
7	7	0	-0.05461	0.03641
8	8	0	1.93498	-0.11486
9	9	0	-0.21898	-0.11515
10	10	0	-2.99738	-0.09085
11	11	0	1.92494	0.15297
12	12	1	1.34688	0.08788
13	13	1	-0.86755	-0.04068
14	14	1	-0.35750	-0.02176
15	15	1	1.59462	-0.02772
16	16	1	-1.15811	-0.04153
17	17	1	0.89718	0.02260
18	18	1	-0.68894	-0.02853
19	19	1	-0.14433	-0.07348

BLUPs for the Dental Data

20	20	1	-0.12730	0.02544
21	21	1	2.53489	0.10877
22	22	1	-0.22609	-0.08535
23	23	1	-0.63735	0.00651
24	24	1	-1.70079	0.11392
25	25	1	0.23870	-0.03166
26	26	1	0.11799	0.06104
27	27	1	-0.82229	-0.07545

Obs	GENDER	fixint	fixslope	newid	ranint	ranslope	beta0i	beta1i
1	0	17.3727	0.47955	1	-0.48526	-0.06820	16.8875	0.41135
2	0	17.3727	0.47955	2	-1.19224	0.14198	16.1805	0.62152
3	0	17.3727	0.47955	3	-0.85346	0.17726	16.5193	0.65681
4	0	17.3727	0.47955	4	1.70243	0.04017	19.0752	0.51971
5	0	17.3727	0.47955	5	0.91363	-0.08680	18.2864	0.39274
6	0	17.3727	0.47955	6	-0.67403	-0.07292	16.6987	0.40662
7	0	17.3727	0.47955	7	-0.05461	0.03641	17.3181	0.51595
8	0	17.3727	0.47955	8	1.93498	-0.11486	19.3077	0.36469
9	0	17.3727	0.47955	9	-0.21898	-0.11515	17.1537	0.36440
10	0	17.3727	0.47955	10	-2.99738	-0.09085	14.3753	0.38869
11	0	17.3727	0.47955	11	1.92494	0.15297	19.2977	0.63251
12	1	16.3406	0.78438	12	1.34688	0.08788	17.6875	0.87225
13	1	16.3406	0.78438	13	-0.86755	-0.04068	15.4731	0.74369
14	1	16.3406	0.78438	14	-0.35750	-0.02176	15.9831	0.76262
15	1	16.3406	0.78438	15	1.59462	-0.02772	17.9352	0.75665
16	1	16.3406	0.78438	16	-1.15811	-0.04153	15.1825	0.74285
17	1	16.3406	0.78438	17	0.89718	0.02260	17.2378	0.80697
18	1	16.3406	0.78438	18	-0.68894	-0.02853	15.6517	0.75584
19	1	16.3406	0.78438	19	-0.14433	-0.07348	16.1963	0.71090
20	1	16.3406	0.78438	20	-0.12730	0.02544	16.2133	0.80981
21	1	16.3406	0.78438	21	2.53489	0.10877	18.8755	0.89315
22	1	16.3406	0.78438	22	-0.22609	-0.08535	16.1145	0.69903
23	1	16.3406	0.78438	23	-0.63735	0.00651	15.7033	0.79088
24	1	16.3406	0.78438	24	-1.70079	0.11392	14.6398	0.89830
25	1	16.3406	0.78438	25	0.23870	-0.03166	16.5793	0.75272
26	1	16.3406	0.78438	26	0.11799	0.06104	16.4586	0.84542
27	1	16.3406	0.78438	27	-0.82229	-0.07545	15.5183	0.70893