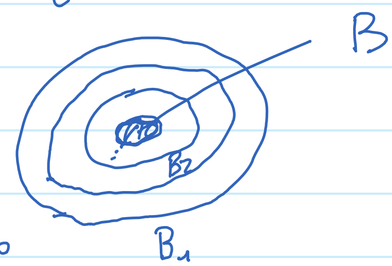
Lecture 3 limits of sets, prob

Tuesday, September 12, 2017 11:15 PM

Let B_n be a decreasing sequence of set

$B_{n+1} \subset B_n$, $n = 1, 2, \dots$



DF B = the limit of B_n $= \bigcap_{n=1}^{\infty} B_n$

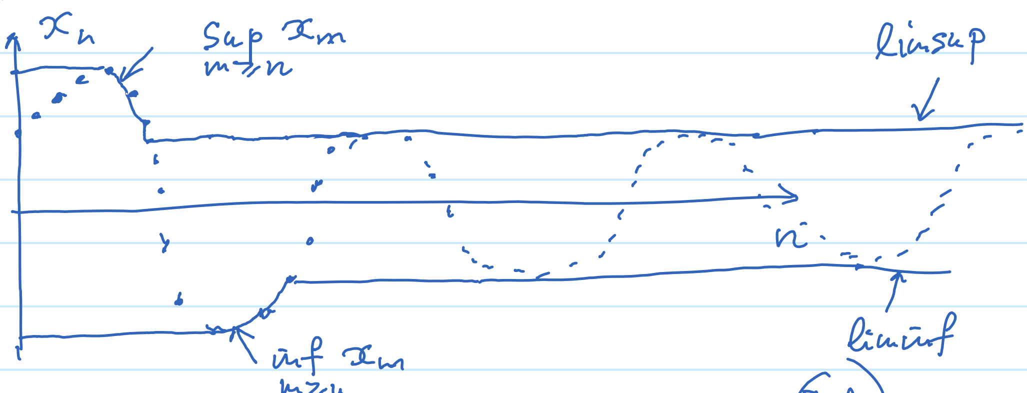
If B_n is an increasing sequence

$B_n \subset B_{n+1}$, $i = 1, 2, \dots$

$B = \lim_{n \to \infty} B_n = \bigcup_{n=1}^{\infty} B_n$

A reminder from calculus: liminf(erior) limsup(erior)

for a sequence $x_n \in \mathbb{R}$



$\sup_{m \geq n} x_m \; (n)$

$\inf_{m \geq n} x_m \; (n)$

Df $\{x_m\}_{m \geq n}$ m-tail of the sequence

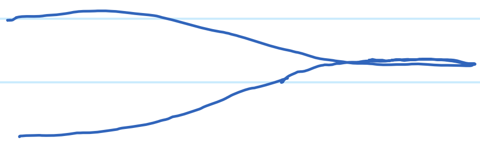
Sequence x_n converges iff all subsequences converge

converge $x_{(n)}$

When $\limsup\limits_{n\to\infty} x_n \neq \liminf\limits_{n\to\infty} x_n \Rightarrow$

I can choose a non-converging subsequence

When $\limsup = \liminf \Rightarrow x_n$ has a $\lim\limits_{n\to\infty}$



$\limsup \geq \liminf$

What is limsup and liminf for sets?

DF Given a sequence of sets A_n

$$\limsup_{n\to\infty} A_n \underbrace{:=}_{\text{equal by definition}} \bigcap_{n=1}^{\infty} \underbrace{\bigcup_{k=n}^{\infty} A_k}_{B_n}$$

What does this mean?

happening $\omega \in A$: A is happening

Experiment is a choice of ω

$$\omega \in \underbrace{\bigcup_{k=n}^{\infty} A_k}_{\text{union over the } n\text{-tail of } \{A_k\}}$$

$\exists$ an A_j for some $j \geq n$: $\omega \in A_j$

"there exists"

B_n = Collection of ω that belong simultaneously to all B_n, $n = 1, 2, \dots$

If $\omega \in \limsup_{n\to\infty} A_n$,

For any n (no matter how large) I can find $\underbrace{A_j : \omega \in A_j}_{A_j \text{ is happening}}$ in the n-th tail

If I had only a finite number of A_ks happening than choosing $n >$ max k for will leave me with the n-tail where not a single A_k is happening

contradicts

$\limsup_{n\to\infty} A_n$ = {infite number of A_ks are happening}

How many A_ks are <u>not</u> happening

Could be finitely or infinitely many.

Now Consider lim inf

DF $\liminf_{n\to\infty} A_n = \bigcup_{n=1}^{\infty} \boxed{\bigcap_{k=n}^{\infty} A_k}$ → B_n

$\omega \in \bigcap_{k=n}^{\infty} A_k$ {<u>all</u> A_ks in the n-tail} are happening

Not a single one is not happening

$\omega \in$ liminf $\Leftrightarrow$ At least one n-tail has all A_ks happening simultaneously

liminf = { all but finitely many A_ks are happening }

Events not happening – can be only finitely many of them in liminf

Story

A company fired Ω employees
ω

No income

nothing to eat

lots of pride

Charity offeres daily meals

on day k $\quad \omega \in A_k$ when ω-subject comes to charity for food

(1) Those who try not to come, but in end always do eventually, and this pattern continues forever

limsup group

(2) The ones in ↑ group who eventually break down and start coming every day and this continues forever

liminf

$$\boxed{\text{liminf} \subset \text{limsup}}$$

(3) After a while find a job

(3) After a while find a job
"finitely many events happening"

Probability

DF

$A \xrightarrow{P} [0,1]$

function of a set $P(A)$
↳ of ωs

defined on
$(\Omega, \mathcal{A})$
where ωs are coming from
↳ collection of subsets of Ω

such that

(1) $P(A) \geq 0$, $\forall A \in \mathcal{A}$
$\forall$ "for any"

(2) $P(\Omega) = 1$

(3) For any countable set $\{A_i\}$, $i=1,2\ldots$
sequence

of disjoint events
$A_i \cap A_j = \emptyset$, $i \neq j$, $i,j = 1,2\ldots$

Such that
$\bigcup_{i=1}^{\infty} A_i \in \mathcal{A}$ (not needed if $\mathcal{A}$ is a σ-algebra)

we have
$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n)$$
Countable additivity property