Biostat 802 Lab 2

Jan 31st 2018

Problem 1: Let $X_1, \ldots, X_n$ be *i.i.d.* with $N(\mu, \sigma^2), \mu \in (-\infty, \infty), \sigma > 0$. The objective is to assess estimation of σ^2 under the squared error loss function $L((\mu, \sigma^2), a) = (a\sigma^{-2} - 1)^2$. Consider three estimators of σ^2:

$$\sigma_1^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2,\ \sigma_2^2 = \frac{1}{n+1}\sum_{i=1}^{n}(X_i - \bar{X})^2,\ \sigma_3^2 = \frac{1}{n+2}\sum_{i=1}^{n}X_i^2,$$

Show that

(a) Calculate the risk function of each of the estimators above.

(b) Which estimators above are admissible? Justify your answer.

(c) Which estimators above are not admissible? Justify your answer.

Problem 2: (From the 2016 Qualifying Exam) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a distribution with the density function,

$$f(x|\theta) = 2x/\theta^2,\ 0 \le x \le \theta,$$

and 0, otherwise, where parameter $\theta > 0$.

(a) First compute $E(X_1|\theta)$ and then, using the sample $X_1, \ldots, X_n$, derive an unbiased estimator of θ and its variance.

(b) Obtain the maximum likelihood estimator of θ. Is it unbiased? What is the mean square error of the maximum likelihood estimator of θ? Justify your answer.

(c) Assuming a prior $\pi(\theta) \propto \theta^{-2}$, find the posterior density of θ and its posterior mean.

(d) As a frequentist, compute the sampling variance of the posterior mean in (c). Compare the properties of the estimators of θ derived in (a), (b) and (c). Which estimator will you choose and why?