HW8. Due 11/29

Friday, November 17, 2017 11:40 AM

1. $\exists\ X_1, \dots, X_n$ are i.i.d., absolutely continuous

$X_1 \sim \underset{\text{CDF}}{F}, \underset{\text{PDF}}{f}$

$\exists\ \hat{F}_n$ is the emirical distribution based on the sample $\{X_i\}_{i=1}^n$

Consider a moving average estimator

$$\hat{f}_n(x) = \frac{\hat{F}_n(x+a_n) - \hat{F}_n(x-a_n)}{2a_n}$$

(a) Show that $\mathbb{E}(\hat{f}_n(x)) \to f(x)$, $a_n \to 0$

(b) Show that $Var(\hat{f}_n(x)) \to 0$, $a_n \to 0$ and $na_n \to \infty$

(c) Argue that $\hat{f}_n$ is a consistent estimator for f under the conditions of (b)

(d) Show that

$$\sqrt{2na_n}\left[\hat{f}_n(x) - \mathbb{E}\hat{f}_n(x)\right] \rightsquigarrow \mathcal{N}(0, f(x))$$

Optional: Show that with some additional assumption on the rate of convergence $a_n \to 0$

$$\sqrt{2na_n} \cdot \left[\hat{f}_n(x) - f(x)\right] \rightsquigarrow \mathcal{N}(0, f(x))$$