HW4

1 a)

Y_{1k} : response from subject 1 (treatment)

Y_{2k} : " 2 (placebo)

$\pi_1 = P(Y_{1k}=1)$ $\pi_2 = P(Y_{2k}=1)$.

$H_0 : \pi_1 = \pi_2$ vs $H_1 : \pi_1 \neq \pi_2$

Test statistic

$$X_M^2 = \frac{(18-36)^2}{(18+36} = 6$$

Since $6 > 3.84 = \chi^2_{1,0.95}$, we can reject H_0

Treatment has different effects on the kidny disease.

b) Since $X_{1k} = 1$ and $X_{2k} = 0$, $X_{1k} - X_{2k} = 0$ for all k.

$$L_k(\beta) = \left\{\frac{e^\beta}{1+e^\beta}\right\}^{Y_{1k}(1-Y_{2k})} \cdot \left\{\frac{1}{1+e^\beta}\right\}^{Y_{2k}(1-Y_{1k})}$$

$$l_k(\beta) = Y_{1k}(1-Y_{2k})\beta - \{Y_{1k}(1-Y_{2k}) + Y_{2k}(1-Y_{1k})\}\log(1+e^\beta)$$

$$l(\beta) = \sum_{k=1}^{79} l_k(\beta) = 36\beta - 54\log(1+e^\beta)$$

$$U(\beta) = \frac{\partial l(\beta)}{\partial \beta} = 36 - 54\frac{e^\beta}{1+e^\beta} = 0$$

$$e^{\hat\beta} = \frac{36}{18} = 2$$

$$\hat\beta = \log 2.$$

c) $H_0: \beta = 0$ vs $H_1: \beta \neq 0$.

$$U(\beta)\Big|_{\beta=0} = 36 - \frac{54}{2} = 9.$$

$$J(\beta) = 54 \frac{e^{\beta}}{(1+e^{\beta})^2}$$

$$J(\beta)\Big|_{\beta=0} = \frac{54}{4}$$

$$X_S^2 = U(0)\,J^{-1}(0)\,U(0) = 6 \quad \sim \chi_1^2$$

Since $6 > \chi^2_{1,0.95} = 3.84$, we can reject H_0.

$\Rightarrow$ The test statistic is identical to the test statistic of the McNemar test.

d) $H_0: \beta = 0$ vs $H_1: \beta \neq 0$

$$X_W^2 = \frac{\hat{\beta}^2}{SE(\hat{\beta})^2} = \frac{(\log 2)^2}{J^{-1}(\log 2)} = 5.765 \quad \sim \chi_1^2$$

$5.765 > 3.84 = \chi^2_{1,0.95}$, reject H_0

This test is not identical to the McNemar test.

[redacted]

2.

(a) $\hat{\beta}_{01} = \log\left(\frac{100}{150}\right) = -0.405$

$\hat{\beta}_{11} = \log\left(\frac{70}{80}\right) - \log\left(\frac{100}{150}\right) = 0.272$

$\hat{\beta}_{21} = \log\left(\frac{30}{20}\right) - \log\left(\frac{100}{150}\right) = 0.811$

(b) $\hat{\beta}_{01}$: Estimated log odds of ~~getting~~ moderate obesity for a non-drinker, given that the patient is not severely obese.

$e^{\hat{\beta}_{21}}$: Estimated odds ratio of moderate obesity comparing heavy drinkers to non-drinkers, given that the patient is not severely obese.

(c) $SE(\hat{\beta}_{01}) = \sqrt{\frac{1}{150} + \frac{1}{100}}$

95% CI for β_{01} : $\hat{\beta}_{01} \pm 1.96 \cdot SE(\hat{\beta}_{01}) = (-0.66, -0.15)$

$SE(\hat{\beta}_{21}) = \sqrt{\frac{1}{30} + \frac{1}{20} + \frac{1}{100} + \frac{1}{150}}$

95% CI for β_{21} : $\hat{\beta}_{21} \pm 1.96\, SE(\hat{\beta}_{21}) = (0.19, 1.43)$

95% CI for $e^{\beta_{21}}$: $(e^{0.19}, e^{1.43}) = (1.21, 4.18)$

3.

$Y_i = 0$ progression
$Y_i = 1$ no change
$Y_i = 2$ partial remission
$Y_i = 3$ complete remission
$Trt = 0$ Alt, $Trt = 1$ Seq
$Sex = 0$ Female, $Sex = 1$ male

(a) Model : $\log\left\{\frac{P(Y_i \le j)}{P(Y_i > j)}\right\} = \gamma_{0j} + \gamma_1 Treat_i + \gamma_2 Sex_i$

$\hat{\gamma}_{01} = -0.196 \quad \hat{\gamma}_{02} = 1.371 \quad \hat{\gamma}_{03} = 2.422$

$\hat{\gamma}_1 = -0.581 \quad \hat{\gamma}_2 = -0.541$

Estimated probabilities:

$$P(Y_i = j \mid Trt, Sex) = P(Y_i \le j \mid Trt, Sex) - P(Y_i \le j-1 \mid Trt, Sex),$$

where $P(Y_i \le j \mid Trt, Sex) = \text{expit}(\gamma_{0j} + \gamma_1 Trt_i + \gamma_2 Sex_i)$

Trt	Sex	Progression	No change	Partial remission	Complete remission
Alt	F	0.45	0.35	0.12	0.08
Alt	M	0.32	0.37	0.17	0.13
Seq	F	0.32	0.37	0.18	0.14
Seq	M	0.21	0.35	0.22	0.21

(b) Pearson Chi-square statistic $= 5.35 \sim \chi^2_7$.

$\chi^2_{7,0.95} = 14.07 > 5.35 \Rightarrow$ Fail to reject H_0.

So the model fits well according to Pearson Chi-square statistic.

(c) $H_0: \gamma_{Trt} = 0$ v.s. $H_1: \gamma_{Trt} \neq 0$

$X^2_W = 7.51 \sim \chi^2_1 \quad$ p-value $= 0.0061$

Reject H_0.

So there is significant difference in responses for the 2 treatments.