HW5

Due Friday, Oct. 20th

Thursday, October 12, 2017 1:24 AM

① Let $(\Omega, \mathcal{F}, P)$ be the probability space. Prove that <u>any</u> pair of events in $\mathcal{F}$ are independent <u>iff</u> their probabilities are either 0 or 1.

② $\exists$ X, Y are r.v. : Y^2 and X^2 are independent. Show that X, Y are not necessarily independent

③ Prove that r.v. X is independent of itself <u>iff</u> it is a Const with probability 1

④ $\exists$ $\mathcal{F} \supset \mathcal{B}_1 \supset \mathcal{B}_2 \supset \dots \supset \mathcal{B}_n$ is a decreasing sequence of σ-algebras. For an $\mathcal{F}$-measurable r.v. X, find

$$\mathbb{E}(\dots \mathbb{E}(\mathbb{E}(X \mid \mathcal{B}_1) \mid \mathcal{B}_2) \dots \mid \mathcal{B}_n)$$