HW 9 Due Monday Dec 11th

Friday, December 1, 2017 9:36 AM

Continuing the problem of HW 8

Consider

$$X_n = \sqrt{2n a_n} \cdot \underbrace{(\alpha_1, \alpha_2)}_{\alpha^T} \begin{pmatrix} \hat{f}_n(x) - \mathbb{E}\hat{f}_n(x) \\ \hat{f}_n(y) - \mathbb{E}\hat{f}_n(y) \end{pmatrix},$$

$$n \to \infty, \quad a_n \to 0, \quad n a_n \to \infty$$

(1) Find the large-sample limit of $\mathrm{Var}(X_n)$

(2) Prove that $\mathbb{E}|X+Y|^r \le 2^{r-1}\left[\mathbb{E}|X|^r + \mathbb{E}|Y|^r\right]$, $r \ge 1$, $\forall$ r.v. X, Y: the above expectations exist. (the C_r inequality)

(3) Find the limit (in distribution) of

$$Y_n = \sqrt{2n a_n} \cdot \begin{pmatrix} \hat{f}_n(x) - \mathbb{E}\hat{f}_n(x) \\ \hat{f}_n(y) - \mathbb{E}\hat{f}_n(y) \end{pmatrix}$$

Hint: Lyapunov $s=3$ and Cramer-Wold

(4) Find an additional restriction on a_n such that $\mathbb{E}\hat{f}_n$ can be replaced by f in the CLT statement