BIOSTAT 602 SUMMARY

1. Data reduction

(1) Sufficient: Within any region of the partition, mass of x changes uniformly (ratio invariant) as θ changes
   (a) Test:
$$f_X(x|\theta) = h(x)g(T(X), \theta)$$
(2) Minimal sufficient: The most coarse partition; also unique
   (a) Test
$$\frac{f_X(x|\theta)}{f_X(y|\theta)} \quad \text{is independent of } \theta \Leftrightarrow T(x) = T(y)$$
(3) Ancillary: For any region of the partition, mass of the whole region is invariant as θ changes.
(4) Complete: No coarser partition is ancillary
   (a) Definition
$$E[g(T)|\theta] = 0 \ \forall\theta \Rightarrow \Pr(g(T) = 0|\theta) = 1 \ \forall\theta$$
   (b) Properties
      (i) A complete sufficient statistic is also minimal sufficient.
      (ii) A complete sufficient statistic is independent of any ancillary statistic.
(5) Exponential family:
   (a) Form:
$$f(x|\theta) = h(x)c(\theta)\exp\left\{\sum_{j=1}^{m} t_j(x)w_j(\theta)\right\}$$
   (b) Statistic:
$$\bigoplus_{j=1}^{m}\sum_{i=1}^{n} t_j(X_i)$$
Sufficient; complete if $\oplus_{j=1}^{m} w_j(\theta)$ contains an open set in $\mathbb{R}^m$.