Biostat 802 Lab#5

Problem 1: (Example for HW#3 Problem 1) Let P_0, P_1, P_2 be the probability distributions assigning to the integers $1, ..., 4$ the following probabilities:

	1	2	3	4
P_0	0.03	0.02	0.02	0.93
P_1	0.06	0.02	0.04	0.88
P_2	0.12	0.06	0.04	0.78

Determine whether there exists a level α test of $H : P = P_0$ which is UMP against the alternatives P_1 and P_2 when (i) $\alpha = 0.01$; (ii) $\alpha = 0.05$;.

Solution: Consider the table of likelihood ratios:

	1	2	3	4
P_1/P_0	2	1	2	88/93
P_2/P_0	4	3	2	78/93

(i) For $\alpha = 0.01$, first consider a UMP test P_0 against P_1, which has the form according to the **Neyman-Pearson Lemma**:

$$\phi_1(x) = \begin{cases} 1 & P_1/P_0 > k_1 \\ \gamma_1(x) & P_1/P_0 = k_1 \\ 0 & P_1/P_0 < k_1 \end{cases}$$

Construct a similar form for P_0 against P_2

$$\phi_2(x) = \begin{cases} 1 & P_2/P_0 > k_2 \\ \gamma_2(x) & P_2/P_0 = k_2 \\ 0 & P_2/P_0 < k_2 \end{cases}$$

Looking at the order in the likelihood ratio table, in order to have $\phi_1(x) = \phi_2(x)$ we choose $k_1 = 2, \gamma_1(x) = (1/3)1(x = 1)$ and $k_2 = 4, \gamma_2(x) = (1/3)$ So the most powerful tests of P_0 against P_1 and P_2 are of the form

$$\phi_1(x) = \phi_2(x) = (1/3)1(x = 1)$$

(ii) For $\alpha = 0.05$, similar to part (i), the most powerful tests of P_0 against P_1 and P_2 are of the form

$$\phi_1(x) = 1_{(1,3)}(x)$$
$$\phi_2(x) = 1_{(1,2)}(x)$$

You cannot have $\phi_1 = \phi_2$ so there is no UMP test