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**Jan 21, 2013**

**HOMEWORK — #1**

**Problem** 1.

(a)

$$f(Y;\mu,\lambda)=\left(\frac{\lambda}{2\pi Y^3}\right)^{1/2}e^{-\lambda[\frac{Y}{2\mu^2}-\frac{1}{\mu}+\frac{1}{2Y}]}$$

$$f(Y;\mu,\lambda)=\left(\frac{\lambda}{2\pi Y^3}\right)^{1/2}e^{-\frac{\lambda}{2Y}}e^{\frac{\frac{-1}{2\mu^2}Y+\frac{1}{\mu}}{1/\lambda}}$$

$$\phi=\lambda$$

$$\theta=\frac{-1}{2\mu^2}$$

$$c(Y,\phi)=\frac{1}{2}\log(\phi)-\frac{1}{2}\log(2\pi Y^3)-\frac{\phi}{2Y}$$

$$a(\phi)=1/\phi$$

$$b(\theta)=-\sqrt{-2\theta}$$

$$T(Y)=Y$$

$$f(Y;\theta,\phi)=e^{\frac{\theta T(Y)-b(\theta)}{a(\phi)}+c(Y,\phi)},\phi>0,\theta<0,Y>0$$

(b)

$$E(Y)=E(T(Y))=b'(\theta)=\frac{1}{\sqrt{-2\theta}}=\mu$$

$$Var(Y)=Var(T(Y))=a(\phi)b''(\theta)=\frac{1}{\phi}(-2\theta)^{-3/2}=\frac{\mu^3}{\lambda}$$

**Problem** 2.

(a)

$$f(Y_i;\lambda,\nu) = \frac{1}{\Gamma(\nu)}\nu^{\nu} Y_i^{\nu-1} e^{-\frac{\nu}{\lambda}Y_i - \nu\log(\lambda)}$$
$$\theta = -\frac{1}{\lambda}$$
$$c(Y_i,\nu) = -\log(\Gamma(\nu)) + \nu\log(\nu) + (\nu-1)\log(Y_i)$$
$$a(\nu) = 1/\nu$$
$$b(\theta) = -\log(-\theta)$$
$$T(Y_i) = Y_i$$
$$f(Y_i;\theta,\nu) = e^{\frac{\theta T(Y_i) - b(\theta)}{a(\nu)} + c(Y_i,\nu)},\ \nu > 0, \theta < 0, Y_i > 0$$
$$b'(\theta) = -1/\theta = \mu$$
$$g(\mu) = -1/\mu$$
$$\mu_i = -\frac{1}{\theta_i}$$
$$\theta_i = x_i^T\beta$$
$$L(\beta) = \prod_{i=1}^{n} e^{\nu[\theta_i Y_i - b(\theta_i)]}$$
$$l(\beta) = \sum_{i=1}^{n} \nu[\theta_i Y_i - b(\theta_i)]$$
$$U(\beta) = \nu\sum_{i=1}^{n}(Y_i - \mu_i)x_i = \nu\sum_{i=1}^{n}(Y_i + \frac{1}{x_i^T\beta})x_i$$
$$J(\beta) = -\nu\sum_{i=1}^{n} -x_i b''(\theta_i)x_i^T = \nu\sum_{i=1}^{n} x_i x_i^T/(x_i^T\beta)^2$$
$$I(\beta) = E(J(\beta)) = \nu\sum_{i=1}^{n} x_i x_i^T/(x_i^T\beta)^2$$

(b) Do the followings until $||\beta^{(t+1)} - \beta^{(t)}|| < 10^{-6}$

$$\beta^{(t+1)} = \beta^{(t)} + I(\beta^{(t)})^{-1}U(\beta^{(t)})$$
$$\beta^{(t+1)} = \beta^{(t)} + \left[\sum_{i=1}^{n} x_i x_i^T/(x_i^T\beta)^2\right]^{-1}\sum_{i=1}^{n}(Y_i + \frac{1}{x_i^T\beta})x_i$$

(c) $\hat\beta = (-1.9041688, -0.4798081, -0.6218624)^T$

(d)

$$(I(\hat\beta))^{-1} = \begin{pmatrix} 0.08724123 & -0.06052363 & -0.03075956 \\ -0.06052363 & 0.07851455 & -0.01274745 \\ -0.03075956 & -0.01274745 & 0.09478218 \end{pmatrix}$$
$$\hat\beta - \beta \sim N(0, (I(\hat\beta))^{-1})$$

95% CI for $\beta_1$ :

$$(\hat\beta_1 - 1.96\sqrt{0.07851455}, \hat\beta_1 + 1.96\sqrt{0.07851455}) = (-1.029009, 0.06939268)$$

95% CI for $\beta_2$ :

$$(\hat\beta_2 - 1.96\sqrt{0.09478218}, \hat\beta_2 + 1.96\sqrt{0.09478218}) = (-1.225282, -0.01844284)$$

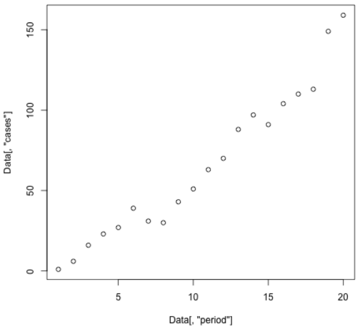
(e) For model $g(\mu_i) = \beta_0'$, $\hat\beta_0' = -2.653$

$2 \times (l(\hat\beta) - l((\beta_0', 0, 0)^T)) = 8.131798 \sim \chi_2^2$

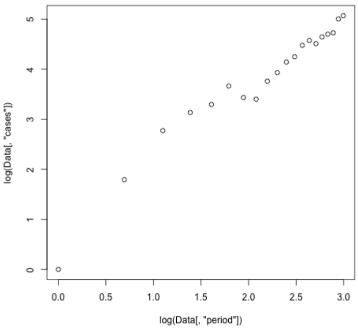
p-value is 0.01714, reject $H_0$ at $\alpha = 0.05$.

**Problem** 3.

(a)




(b)




$\log(y_i)$ and $\log(i)$ seem to have linear relation.

(c)

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Call:
glm(formula = cases ~ log_period_centered, family = poisson(link = "log"),
    data = Data)

Deviance Residuals:
    Min        1Q    Median        3Q       Max
-2.0568   -0.8302   -0.3072    0.9279    1.7310

Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
(Intercept)          4.05063    0.03331  121.61   <2e-16 ***
log_period_centered  1.32661    0.06463   20.52   <2e-16 ***
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(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 677.264  on 19  degrees of freedom
Residual deviance:  21.755  on 18  degrees of freedom
AIC: 138.05

Number of Fisher Scoring iterations: 4
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$\hat{\beta}_0 = 4.05063$ and $\hat{\beta}_1 = 1.32661$.
$\beta_0$: log mean number of cases in period 10.
$\beta_1$: increase in log ratio of mean number of cases when log period increases by 1.

*Submitted by Wen Wang on Jan 21, 2013.*