# BIOSTAT 651
# Homework #1
# due: Wednesday, January 25

**- turn in at the start of class**

**- each sub-question=2 points; total 20 points**

1. $Y_i$ has the following density function,

$$f(Y_i; \beta, \alpha) \;=\; \frac{\beta^\alpha}{\Gamma(\alpha)} Y_i^{\alpha-1} e^{-Y_i\beta} \qquad Y_i > 0$$

for $i = 1, \ldots, n$. The parameter $\alpha$ is treated as known.

(a) Derive $L(\beta)$, $\ell(\beta)$ and $U(\beta)$.

(b) Derive the observed information, $J(\beta)$.

2. Consider a simple linear regression model,

$$Y_i \;=\; \beta_1 X_i + \epsilon_i, \quad \epsilon_i \;\sim\; N(0, \sigma^2).$$

All elements in $\boldsymbol{\theta} = (\beta_1, \sigma^2)$ are unknown.

(a) Find the MLE, $\widehat{\boldsymbol{\theta}}$. Are MLE of $(\beta_1, \sigma^2)$ the same as LSE of $(\beta_1, \sigma^2)$ (Yes, NO)?

(b) Determine the asymptotic variance of $\widehat{\boldsymbol{\theta}}$, $Var(\widehat{\boldsymbol{\theta}})$. (Use the fact that $Var(\widehat{\boldsymbol{\theta}}) = I(\boldsymbol{\theta})^{-1}$).

3. The number of male and female births at a rural hospital is recorded over a span of 10 months, with the observed data given in the table below. Of interest is $\theta$, defined as the probability that a given newborn is male.

| month | Male | Female |
|---|---|---|
| 1 | 9 | 11 |
| 2 | 21 | 22 |
| 3 | 34 | 27 |
| 4 | 30 | 35 |
| 5 | 17 | 24 |
| 6 | 34 | 29 |
| 7 | 29 | 26 |
| 8 | 22 | 27 |
| 9 | 30 | 38 |
| 10 | 13 | 14 |

(a) Derive $L(\theta)$, $\ell(\theta)$, $U(\theta)$ and $I(\theta)$.

(b) Compute $\widehat{\theta}$, $\widehat{Var(\widehat{\theta})}$, and 95% confidence interval for $\theta$ (use Normal approximation)

(c) Test $H_0 : \theta = 0.5$ against a two-sided $H_1$ using the score test.

4. Solve 6.3 (a), (b) and (c) on page 119 of Textbook.