

### Problem 3

$$\begin{aligned}\pi(\lambda | y, \alpha, \beta) &\propto \pi(\lambda | \alpha, \beta) \pi(y | \lambda, \alpha, \beta) \\ &= \left( \frac{1}{\Gamma(\alpha) \beta^\alpha} \lambda^{\alpha-1} e^{-\frac{\lambda}{\beta}} \right) \left( \frac{e^{-\lambda} \lambda^y}{y!} \right) \propto \lambda^{\alpha+y-1} e^{-(1+\frac{1}{\beta})\lambda}\end{aligned}$$

$$\sim \text{Gamma}(\alpha+y-1, \frac{1}{1+\frac{1}{\beta}})$$

$$\begin{aligned}\pi(\alpha | \lambda, y, \beta) &\propto \pi(\alpha | \beta) \pi(\lambda | \alpha, \beta) \pi(y | \lambda, \alpha, \beta) \\ &= \left( \frac{1}{\Gamma(\alpha) b^a} \alpha^{a-1} e^{-\frac{\alpha}{b}} \right) \left( \frac{1}{\Gamma(\alpha) \beta^\alpha} \lambda^{\alpha-1} e^{-\frac{\lambda}{\beta}} \right) \left( \frac{e^{-\lambda} \lambda^y}{y!} \right) \\ &\propto \alpha^{a-1} e^{-\frac{\alpha}{b}} \frac{1}{\Gamma(\alpha) \beta^\alpha} \lambda^{\alpha-1}\end{aligned}$$

$$\begin{aligned}\pi(\beta | \lambda, y, \alpha) &\propto \pi(\beta | \alpha) \pi(\lambda | \beta, \alpha) \pi(y | \lambda, \beta, \alpha) \\ &= \left( \frac{1}{\Gamma(c) d^c} \beta^{c-1} e^{-\frac{\beta}{d}} \right) \left( \frac{1}{\Gamma(\alpha) \beta^\alpha} \lambda^{\alpha-1} e^{-\frac{\lambda}{\beta}} \right) \left( \frac{e^{-\lambda} \lambda^y}{y!} \right) \\ &\propto \beta^{c-1} e^{-\frac{\beta}{d}} \frac{1}{\beta^\alpha} e^{-\frac{\lambda}{\beta}}\end{aligned}$$