Lecture 1 Intro sigma algebra

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silent

Foundations

Intro prob & stat 601 /602
Real Analysis (almost all)
Measure Theory (a few)
Complex Analysis

History

How did probability become mathematics?

Gambling
Huygens 1657 "... dice games..."
Fermat, Pascal, Bernoulli,
Laplace, Gauss

$$\text{Prob} = \text{frequency} = \frac{\#\text{ favorable outcomes}}{\text{total }\#\text{ of outcomes}}$$

limits of frequencies, infinite series of trials
von Mises

Von Mises, Borel, Bernstein, Kolmogorov, ...

1933 Kolmogorov "Foundations of Prob. Theory"

Hilbert 1900 problems

6th Hilbert Problem

(DF) Ω a set elementary events (arbitrary)

$\omega \in \Omega$

Events are subsets of Ω

(DF) Algebra (Field, Ring)

$\mathcal{A}$ is a set of subsets of Ω such that

(1) $\Omega \in \mathcal{A}$

(2) Closed over operations of $\cup$, $\cap$, that is:

$A, B \subset \mathcal{A} \Rightarrow A \cap B,\ A \cup B \in \mathcal{A}$

(3) closed over operation of complement

notation

$\bar{A}$ = a Complement of A

Note

Because $\overline{A \cap B} = \bar{A} \cup \bar{B}$

Can read in any direction $\leftarrow$ or $\rightarrow$

and drop either $A \cap B$ or $A \cup B$ from (2)

$\emptyset = \bar{\Omega} \in \mathcal{A}$

(DF) σ-Algebra $\mathcal{F}$ def is similar to Algebra $\mathcal{A}$ except that (2) is replaced by "closed with respect to countable $\cup$s, $\cap$s"

This means

(2) $\rightarrow$ (2') Sequence of events $\{A_n\}$:

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(2) $\rightarrow$ (2') sequence of events $\{A_n\}$:

$$\bigcup_{n=1}^{\infty} A_n \in \mathcal{F}, \quad \bigcap_{n=1}^{\infty} A_n \in \mathcal{F}$$

Note

B/c of the relationship

$$\overline{\bigcap_n A_n} = \bigcup_n \overline{A_n}$$ can only use one of

DF Pair $(\Omega, \mathcal{F})$ is called measurable space

consists of subsets of Ω