Biostat 830, Homework 1

Due: Thurs, Feb. 9, 2017 (in class)

1. Prove the Chapman-Kolmogorov Equations.

2. Prove Proposition 2, page 6.

3. Prove Proposition 4, page 7.

4. Consider the set function ψ defined in the proof of Proposition 6:

$$\psi(A) = \int_{\mathcal{X}} K_{1/2}(y, A)\varphi(dy).$$

   where φ is a probability measure and the Markov chain $\mathbf{\Phi}$ is $\varphi-$irreducible. Prove that ψ is also a probability measure. That is, show ψ is a measure and that $\psi(\mathcal{X}) = 1$.

5. Suppose ν is a probability measure. Let ϕ be an irreducibility measure with $\phi(\mathcal{X}) < \infty$. Suppose that ν and ϕ are equivalent: $\phi \prec \nu$ and $\nu \prec \phi$. Show that ν is also an irreducibility measure.

6. Complete the proof of Theorem 5. That is, show that the minorization condition holds for the probability measure, ν, and number, δ^*, given in class for both the m-skeleton and the K_ϵ-chain.

7. Finish the proof of Proposition 9(ii). Show that each $\bar{C}(n, m)$ is small and that the $\bar{C}(n, m)$ is a countable covering of $\mathcal{X}$, i.e.

$$\mathcal{X} = \bigcup_{m=1}^{\infty} \bigcup_{n=1}^{\infty} \bar{C}(n, m)$$

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