Biostat 602 **Assignment 5** **Due: Thursday, February 16**

Turn in the homework problems only.

Homework Problems

1. Let X be a discrete random variable with pmf $f_X(x|\theta)$, where $\theta \in \{1, 2, 3\}$ and $x \in \{1, 2, 3, 4, 5, 6\}$.

$$f_X(x|\theta) = \begin{cases} x/21, & \theta = 1 \\ 1/6, & \theta = 2 \\ I(x = 3), & \theta = 3 \end{cases}$$

   Find a maximum-likelihood estimator of θ. (Note that MLE is a function of x, but may not be represented as an explicit formula.)

2. Let $X_1, X_2, \cdots, X_n$ be i.i.d. random variables from $Uniform(0, \theta)$ with pdf

$$f_X(x|\theta) = \frac{1}{\theta}, \qquad 0 \le x \le \theta$$

   (a) Find a method of moments estimator of θ using the lowest-order moments as possible .

   (b) Calculate the mean and variance of the method of moments estimator.

   (c) Compare the MLE $\hat{\theta}_{MLE} = X_{(n)}$ with the estimator from (a) in terms of bias and variance. Which estimator is better? Justify your answer.

3. Let $X_1, X_2, \cdots, X_n$ be a random sample from a *Double Exponential*(μ, σ) distribution with pdf

$$f_X(x|\mu, \sigma^2) = \frac{1}{2\sigma} \exp\left[-|x - \mu|/\sigma\right], \qquad x \in R, \qquad \mu \in R,\ \sigma > 0$$

   Find MLEs of μ and σ. Show all steps.

   (**Hint:** You may use the fact that for a set of real numbers $x_1, x_2, \ldots, x_n$ the quantity $\frac{1}{n}\sum_{i=1}^{n} |x_i - a|$ is minimized when $a = median\{x_1, x_2, \ldots, x_n\}$.)

4. Let $X_1, \cdots, X_n$ be an *i.i.d.* random sample from the following pdf

$$f_X(x|\theta) = \frac{x}{\theta} \exp\left(-\frac{x^2}{2\theta}\right), \qquad x > 0, \quad \theta > 0$$

   (a) Find a complete sufficient statistic for θ.

   (b) Find the Cramer-Rao lower bound for the variance of any unbiased estimator of θ.

   (c) Can you find a simple function (constant multiple) of the complete sufficient statistic in part (a) which is unbiased?

   (d) Does the estimator in part(c) attain the CRLB obtained in part (b)?

   (**Hint:** You may want to use the fact that the complete sufficient statistic in (a) follows a Gamma distribution.)

5. Let $X_1, \cdots, X_n$ be an *i.i.d.* random sample from pdf

$$f_X(x|\theta) = \theta x^{\theta-1} I(0 < x < 1)$$

   (a) When $\theta \geq 1$, find the maximum likelihood estimator for θ.

   (b) When $\theta > 0$, find the maximum likelihood estimator for $\tau(\theta) = 1/\theta$.

   (c) When $\theta > 0$, find the Cramer-Rao lower bound of the variance of unbiased estimators for $\tau(\theta) = 1/\theta$. Does the MLE in (b) attain the bound?

Practice Problems

(a) C&B Exercise 7.6

(b) C&B Exercise 7.8

(c) C&B Exercise 7.10

(d) C&B Exercise 7.11

(e) C&B Exercise 7.12

(f) C&B Exercise 7.37

(g) C&B Exercise 7.38

(h) C&B Exercise 7.40

(i) C&B Exercise 7.58

(j) C&B Exercise 7.66